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A Study on Discrete Mathematics: Sum Distance in Neutrosophic Graphs with Application

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Abstract: Distance is an important parameter in any networks/ graphs. The idea of strong sum distance in the fuzzy graph was introduced by Tom and Sunitha (2015). A generalization of fuzzy graph called neutrosophic graph is more realistic to handle imprecise data. In this study, the weight of an edge of a neutrosophic graph is defined and hence defined sum distance in a neutrosophic graph based on the idea of strong sum distance. This distance is also metric and based on this metric, the concepts of degree, eccentricity, radius, diameter etc. are studied with some properties. At last, a real-life application on the travelling salesman problem is illustrated.

Keywords: Distance, Sum distance, Neutrosophic graph.

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1. Introduction

The motivation of graph theory was started from the problem of seven bridges of Konigsberg. A graph is an essential tool for the presentation of relationships among nodes, and there are many applications of a graph in social networks [1]. So it is the platform for a good representation of data.

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However, crisp graphs do not represent any system because the world is now full of imprecise data. The idea of fuzziness was used first to define the fuzzy graph [2] by Kaufmann (1973).

Fuzzy graph [3] theory was developed by Rosenfeld (1975). In the same time, Yeh and Bang (1975) introduced various connectedness concepts in fuzzy graphs [4]. Also, $\mu - length$ of a path and $\mu - distance$ in a fuzzy graph [3] was introduced by Rosenfeld (1975). Hence Bhattacharya (1987) introduced the idea of eccentricity and centre in the fuzzy graph [5] using $\mu - distance$. Also, the properties of $\mu - distance$ [6] were developed by Sunitha and Vijayakumar (1998). Bhutani and Rosenfeld (2003) introduced the concepts of $g - distance$ in fuzzy graphs [7, 8] and eccentricity, centre etc. [9] were also developed. There were further studies on $g - distance$ [10] by Linda and Sunitha (2012).

Day to day, there were developments on fuzzy graphs. Akram (2011) introduced bipolar fuzzy graphs [11] and the interval-valued fuzzy graph [12] were introduced by Akram and Dudek (2011). Samanta and Pal (2013, 2015) introduced fuzzy k-competition graphs, p-competition graphs [13] and also introduced fuzzy planar graph [14]. Tom and Sunitha (2015) introduced a new definition of the length of a path and strong sum distance in fuzzy graphs [15]. There are many research works on fuzzy graphs. But in all these fuzzy graphs, edge membership value is less than its vertex membership values. To remove this limitation, Samanta and Sarkar (2016) introduced a generalized fuzzy graph [16].

As a generalization of fuzzy set and intuitionistic set theory, Smarandache (1998) introduced the concepts of neutrosophic set [17] that consist of a degree of truth membership, falsity membership and indeterminacy membership. In reality, every uncertainty has some possibility, some risk and some neutral factors. Neutrosophic graphs include all three notions properly. Thus any uncertainty/ ambiguity of networks can be represented by neutrosophic graphs. Broumi et al. (2016) introduced the notion of a single-valued neutrosophic graph [18] as a generalization of fuzzy graphs. After that, there are several research works on neutrosophic graphs [19,20]. Akram and Siddique (2017) introduced the neutrosophic competition graphs [21]. Hence Das et al. (2020) proposed generalized neutrosophic competition graphs [22] with applications to economic competitions among some countries.

Abdel-Basset (2019) utilized the neutrosophic theory to solve the transition difficulties of IoT-based enterprises [23]. Also, there are many real-life applications including evaluation of the green supply chain management practices [24], evaluation Hospital medical care systems based on pathogenic sets [25], decision-making approach with quality function deployment for selecting supply chain sustainability metrics [26], intelligent medical decision support model based on soft computing and IoT [27]. Chakraborty (2020) introduced pentagonal neutrosophic number in shortest path problem [28] and a new score function of the pentagonal neutrosophic number and its application in networking problem [29]. Das and Edalatpanah (2020) proposed a new ranking function of the triangular neutrosophic number and its use in integer programming [30]. The remaining study can be found in [31-40].

The rest of the paper is organized as follows. In Section 2, we discuss the contribution of the study. In section 3, we study some preliminaries related to graph theory. In Section 4, we introduce the sum distance in a neutrosophic graph with some properties. In Section 5, we introduce eccentricity, radius and diameter in a neutrosophic graph with properties. In Section 6, we discuss an application to a travelling salesman problem. In section 7, we conclude the study with future directions.

The gist of contributions of authors (Table 1) are arranged below.

Authors	Year	Contributions
Rosenfeld	1975	Introduce $\mu - length$ of a path and $\mu - distance$in a fuzzy graph.
Bhattacharya	1987	Introduce eccentricity and centre in the fuzzy graph.
Bhutani and Rosenfeld	2003	$g - distance$ in fuzzy graphs and developed eccentricity, centre etc.
Linda and Sunitha	2011	Studied on $g - distance$ in fuzzy graphs.
Tom and Sunitha	2015	Introduce length of a path and strong sum distance in fuzzy graphs.
Das et al.	This paper	Introduce sum distance in neutrosophic graph and eccentricity, radius etc. are studied. An application is illustrated.

Table 1. Contributions of authors

2. Major contributions of the study

The neutrosophic graph is a generalization of the fuzzy graph. The contributions of the study are below.

- This study introduces the concepts of the weight of edges of a neutrosophic graph and weighted sum distance in neutrosophic graph.
- Also the eccentricity, diameter and radius are defined with some properties.
- At last, an application of sum distance in the neutrosophic graph to a travelling salesman problem is illustrated.

3. Preliminaries

Definition 1. A *graph* is an ordered pair (V, E) such that V is the set of vertices and $E \subseteq V \times V$ is the set of edges between vertices. A *path of length* n is a sequence $v_0e_1v_1e_2 \dots \dots . e_nv_n$ where $v_0, v_1, \dots \dots, v_n$ are distinct vertices and $e_1, e_2, \dots \dots, e_n$ are distinct edges. The *distance* between the vertices u and v is the minimum length of the path between u and v. The *eccentricity* of a vertex is the maximum distance to any vertex in the graph. The *radius* of a graph is the minimum eccentricities of all vertices, and the *diameter* of a graph is the maximum eccentricities of vertices.

Definition 2.[3] A *fuzzy graph* G is a triplet (V, σ, μ) in which V is the set of vertices, $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ where $\sigma(x)$ represents the membership value of x and $\mu(x, y)$ represents the membership value of edge (x, y).

Definition 3.[15] *Length* $L(P)$ *of a path* $P: v_0 e_1 v_1 e_2 \ldots \ldots . e_n v_n$ in a connected fuzzy graph $G: (V, \sigma, \mu)$ is given by $L(P) = \sum_{i=1}^{n} \mu(e_i)$ where $\mu(e_i)$ represents membership values of edges e_i.

Definition 4.[15] The strong sum distance between vertices u *and* v is the minimum length of all paths between vertices u *and* v.

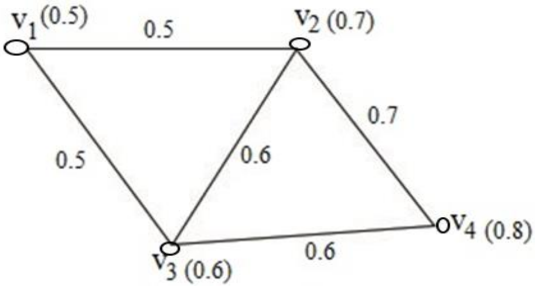

Figure 1. Example of a fuzzy graph

Example 1. The fuzzy graph (Fig.1) has four vertices with five edges. There are three paths from vertex v_1 to vertex v_4. The paths are $P_1: v_1 - v_2 - v_3 - v_4$, $P_2: v_1 - v_2 - v_4$, $P_3: v_1 - v_3 - v_4$. Then $L(P_1) = 1.7, L(P_2) = 1.2, L(P_3) = 1.1$ *and* the strong sum distance between vertices v_1 and v_4 is 1.1.

Definition 5.[18] A graph $G = (V, E)$ where $E \subseteq V \times V$ is said to be neutrosophic graph if

i) there exist functions $\rho_T: V \rightarrow [0,1], \rho_F: V \rightarrow [0,1]$ *and* $\rho_I: V \rightarrow [0,1]$ such that

$$0 \leq \rho_T(v_i) + \rho_F(v_i) + \rho_I(v_i) \leq 3 \text{ for all } v_i \in V \ (i = 1,2,3, \ldots., n)$$

where $\rho_T(v_i), \rho_F(v_i), \ \rho_I(v_i)$ denote the degree of true membership, degree of falsity membership and degree of indeterminacy membership of the vertex $v_i \in V$ respectively.

ii) there exist functions $\mu_T: E \rightarrow [0,1], \mu_F: E \rightarrow [0,1]$ *and* $\mu_I: E \rightarrow [0,1]$ such that

$$\mu_T(v_i, v_j) \leq \min \ [\rho_T(v_i), \rho_T(v_j)]$$
$$\mu_F(v_i, v_j) \geq max[\rho_F(v_i), \rho_F(v_j)]$$
$$\mu_I(v_i, v_j) \geq max[\rho_I(v_i), \rho_I(v_j)]$$
$$\text{and } 0 \leq \mu_T(v_i, v_j) + \mu_F(v_i, v_j) + \ \mu_I(v_i, v_j) \leq 3 \text{ for all } (v_i, v_j) \in E$$

where $\mu_T(v_i, v_j), \ \mu_F(v_i, v_j), \ \mu_I(v_i, v_j)$ denote the degree of true membership, degree of falsity membership and degree of indeterminacy membership of the edge $(v_i, v_j) \in E$ respectively.

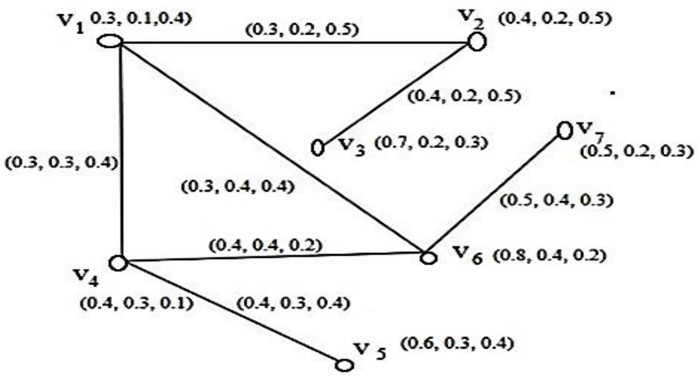

Figure 2. A neutrosophic graph

4. Weighted sum distance in the neutrosophic graph

In the neutrosophic graph, membership values of edges are in neutrosophic nature. So we cannot compare among edges in a neutrosophic graph. To overcome it, we define weight function that maps from the membership value of edges to a crisp value lies between 0 and 1.

Definition 6. Consider a function ω: $[0,1] \times [0,1] \times [0,1] \rightarrow [0,1]$ defined by

$$\omega_{ij}(t, i, f) = w_1 t(1 - f) + w_2 i \text{ where } t, i, f, w_1, w_2 \text{ are the numbers} \in [0,1].$$

The weight of an edge (v_i, v_j) in a neutrosophic graph is a number between 0 and 1 which is obtained from the image of the function ω for corresponding membership value $\left(T_E(v_i, v_j), F_E(v_i, v_j), I_E(v_i, v_j)\right)$ of the edge and it is denoted by ω_{ij}.

Note: This function indicates the overall impression of true , falsity and indeterminacy values. Suppose, in one network, generally predictions are always true of some facts. Then w_1 must be higher value and close to 1. Similarly for the other cases.

Example 2. Weight ω_{23} of edge (v_2, v_3) in the neutrosophic graph (Fig.2) 0.38 where $w_1 = \frac{2}{3}$ and $w_2 = \frac{1}{3}$.

Definition 7. Let P: $u_0 - u_1 - u_2 - \cdots \dots \dots - u_n$ be any path in a neutrosophic graph $G = (V, E)$. Then the length of the path p is the sum of the weights of the edges of the path P in $G = (V, E)$.

$$L_N(p) = \sum_{\substack{0 \le i < n \\ i<j}} \omega_{ij} ,$$

where ω_{ij} is the weight of edge between vertices u_i and u_j.

Example 3. Length of the path $v_1 - v_4 - v_6 - v_7$ in the neutrosophic graph (Fig.2) is 0.8 where $w_1 = \frac{2}{3}$ and $w_2 = \frac{1}{3}$.

Definition 8. Let $G = (V, E)$ be a neutrosophic graph and P be the collection of all paths between two nodes u *and* v *i.e.* P = $\{p_i , i = 1, 2, 3, \dots \dots, n\}$. Then the weighted distance between the nodes u *and* v is denoted by $d_N(u, v)$ and is defined by

$$d_N(u,v) = min\{L_N(p_i): p_i \in P,\ i = 1,2,3,\dots\dots,n\},$$

where $L_N(p_i)$ is the length of the path p_i.

Example 4. Sum distance between the nodes v_1 and v_7 in the neutrosophic graph (Fig.2) is 0.55 where $w_1 = \frac{2}{3}$ and $w_2 = \frac{1}{3}$. .

Theorem 1. Let $G = (V,E)$ be a neutrosophic graph and $d_N(u,v)$ be weighted sum distance between any two nodes u and v. Then $\forall u,v,w \in V$

i) $d_N(u,v) \geq 0$

ii) $d_N(u,v) = 0\ if\ and\ only\ if\ u = v$

iii) $d_N(u,v) = d_N(v,u)$

iv) $d_N(u,v) \leq d_N(u,w) + d_N(w,v)$.

Proof. (i) It clears from the definition that $d_N(u,v) \geq 0$.

(ii) It clears from the definition that $d_N(u,v) = 0\ if\ and\ only\ if\ u = v$.

(iii) $d_N(u,v)$denotes the strong sum distance from u to v. Then there exists a path whose length is minimum among all the path between u to v. Hence the length should be the same from v to u. So $d_N(u,v) = d_N(v,u)$.

(iv) Let p be a path $u - w$ such that $L_N(p) = d_N(u,w)$ and q be a path $w - v$ such that $L_N(q) = d_N(w,v)$. Then $u - v$ is a walk and it is a strong path whose length is at most $d_N(u,w) + d_N(w,v)$. Thus $d_N(u,v) \leq d_N(u,w) + d_N(w,v)$.

5. Eccentricity, Radius and Diameter

The parameters eccentricity, radius and diameter are crucial in graph theory. We studied these important parameters in neutrosophic graph considering the concepts of sum distance. The relations among radius, diameters, eccentricity and distance are studied as follows.

Definition 9. The eccentricity $e_N(u)$ of a node, u is the distance from u to the furthest node in the neutrosophic graph G. Thus

$$e_N(u) = max\{d_N(u,v): \forall v \in V\}.$$

Example 5. Consider a neutrosophic graph (Fig.3). The eccentricity $e_N(v_1)$ of the vertex v_1 is calculated by the following:

$$e_N(v_1) = \max\{d_N(v_1,v_2), d_N(v_1,v_3), d_N(v_1,v_4)\}$$
$$= \max\{0.33, 0.41, 0.87\} = 0.87$$

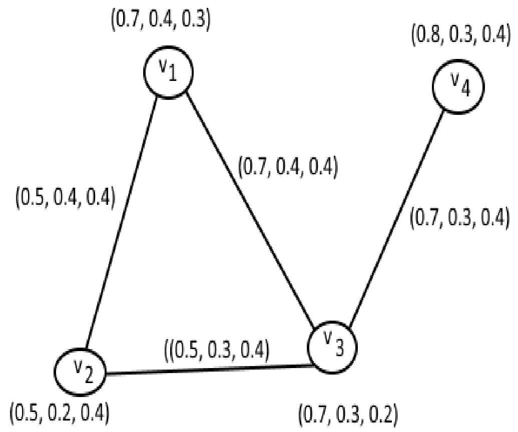

Fig. 3. An example of a neutrosophic graph

Theorem 2. Let $G = (V, E)$ be a connected neutrosophic graph and u, v be any two nodes of G. Then $|e_N(u) - e_N(v)| \leq d_N(u, v)$.

Proof. Let $u, v \in G$ be two nodes such that $e_N(u) \geq e_N(v)$ and $x \in G$ be anode such that $e_N(u) = d_N(u, x)$. Then $d_N(u, x) \leq d_N(u, v) + d_N(v, x)$, by theorem 3.7 (iv). Also $d_N(v, x) \leq e_N(v)$. Thus $e_N(u) = d_N(u, x) \leq d_N(u, v) + e_N(v)$, this implies $0 \leq e_N(u) - e_N(v) \leq d_N(u, v)$. Similarly, if we take $e_N(u) \leq e_N(v)$, we will get $-d_N(u, v) \leq e_N(u) - e_N(v)$. Thus $|e_N(u) - e_N(v)| \leq d_N(u, v)$.

Definition 10. The radius $r_N(G)$ of a neutrosophic graph, G is the minimum among all eccentricity of nodes. Thus

$$r_N(G) = min\{e_N(u): \forall u \in G\}.$$

Example 6. Consider the neutrosophic graph (Fig. 3). The radius $r_N(G)$ of the graph G is calculated by the following:

$$r_N(G) = \min\{e_N(v_1), e_N(v_2), e_N(v_3), e_N(v_4)\}$$
$$= \min\{0.87, 0.83, 0.46, 0.87\} = 0.46$$

Definition 11. The diameter $d_N(G)$ of a neutrosophic graph, G is the maximum among all eccentricity of nodes. Thus

$$d_N(G) = max\{e_N(u): \forall u \in G\}.$$

Example 7. Consider the neutrosophic graph (Fig.3). The diameter $d_N(G)$ of graph G is calculated by the following:

$$d_N(G) = \max\{e_N(v_1), e_N(v_2), e_N(v_3), e_N(v_4)\}$$
$$= \max\{0.87, 0.83, 0.46, 0.87\} = 0.87$$

Definition 12. A node in a neutrosophic graph is called a central node if its eccentricity is equal to the radius of the graph. Thus for a central node u,

$$e_N(u) = r_N(G).$$

Example 8. Consider the neutrosophic graph (Fig. 3). The node $v_3$is a central node, since

the eccentricity $e_N(v_3) =$ the radius $r_N(G)$

Definition 13. A node in a neutrosophic graph is called a peripheral node if its eccentricity is equal to the diameter of the graph. Thus for a peripheral node u,

$$e_N(u) = d_N(G).$$

Example 9. Consider the neutrosophic graph (Fig. 3). The nodes v_1 and v_4 are peripheral nodes, since the eccentricity $e_N(v_1) =$ the diameter $d_N(G) =$ the eccentricity $e_N(v_4)$.

Theorem 3. Let $G = (V, E)$ be a connected neutrosophic graph with $r_N(G)$ and $d_N(G)$ be the radius and diameter respectively, then $r_N(G) \leq d_N(G) \leq 2r_N(G)$.

Proof. From the definition, it follows that $r_N(G) \leq d_N(G)$. Let $u, v, w \in V$ such that u be central node i.e. $e_N(u) = r_N(G)$ and v, w be peripheral node i.e. $e_N(v) = e_N(w) = d_N(G)$. Now $d_N(v, w) \leq d_N(v, u) + d_N(u, w)$, by theorem (iv). This implies $d_N(G) \leq r_N(G) + r_N(G) = 2r_N(G)$. Thus $d_N(G) \leq 2r_N(G)$. Therefore, $r_N(G) \leq d_N(G) \leq 2r_N(G)$.

6. Application to travelling salesman problem

Suppose there are few places in a city and roads connect the places. Hence the places and roads together form a network. But the problem is to find a way that a salesman can visit all the planes once with the lowest travelling cost. Now the travelling cost is directly proportional to the road distance travel by salesman. But all the roads are not in the same smooth conditions to measure road distance in practical. So the real travelling distance with cost may be effected the bad road, non-pucca roads, water path etc. Thus to calculate the path distance, it is generally ignored the current condition of the paths. The true value indicates the expected distance on good road. The falcity indicates the current false parameter like general traffic on the routes, muddied on road. Indeterminacy includes delay due to road construction, political movement and any other factors like water path. Therefore Travelling salesman problem should be presented by neutrosophic environment. Hence the travelling distance between the places should be taken as neutrosophic value.

6.1. Steps to find the sum distance of the travelling salesman problem.

To find the minimum travelling cost in travelling salesman problem in the neutrosophic environment, all the necessary steps are given below as an algorithm.

Step-1: Input all edge membership values between the places.

Step-2: Evaluate the weight of edges.

Step-3: Find all the Hamiltonian cycles between the requird places.

Step-4: Evaluate length of the said cycles.

Step-5: Find the minimum length among the cycles.

6.2. Numerical example

Suppose there are four places and six roads are connecting the places in a city. A salesman wants to visit all the places once and returning back to the starting place. The problem is to find a cycle with minimum cost of travelling.

The edge membership values (Table 2) between the places are given in the figure 4 where the membership value $(0.5, 0.3, 0.2)$ between two places P_1 and P_2 represent that distance of good road between P_1 and P_2 is 0.5, distance of bad road between P_1 and P_2 is 0.3 and distance of non-constructed road between P_1 and P_2 is 0.2 and similar for the other values.

Places	Distance between places
$P_1 - P_2$	$(0.5, 0.3, 0.2)$
$P_1 - P_3$	$(0.4, 0.6, 0.4)$
$P_1 - P_4$	$(0.8, 0.3, 0.2)$
$P_4 - P_2$	$(0.5, 0.3, 0.6)$
$P_3 - P_2$	$(0.6, 0.2, 0.3)$
$P_4 - P_3$	$(0.7, 0.2, 0.4)$

Table 2. Distace between two places

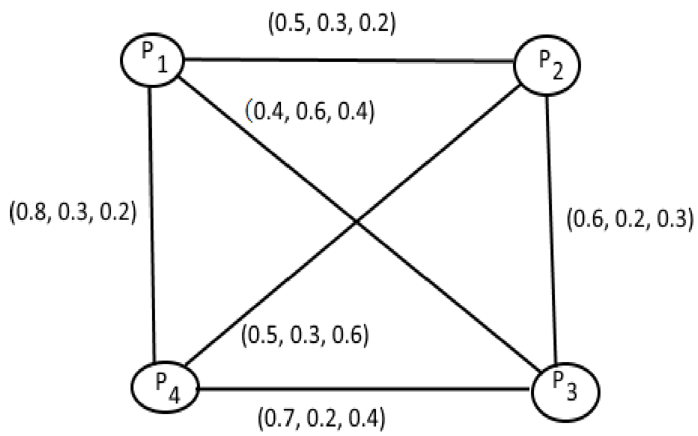

Figure 4: A graph among four cities.

The weight ω_{ij} between the places P_i and P_j are given below:

$$\omega_{12}(P_1, P_2) = 0.3, \quad \omega_{13}(P_1, P_3) = 0.24,$$
$$\omega_{14}(P_1, P_4) = 0.44, \quad \omega_{23}(P_2, P_3) = 0.42,$$
$$\omega_{24}(P_2, P_4) = 0.43, \quad \omega_{34}(P_3, P_4) = 0.51.$$

There are four possible cycles to visit all the places once from starting point to that point. These cycles are:

$$C_1: v_1 - v_2 - v_3 - v_4 - v_1$$

$$C_2: v_1 - v_2 - v_4 - v_3 - v_1$$

$$C_3: v_1 - v_3 - v_2 - v_4 - v_1$$

$$C_4: v_1 - v_3 - v_4 - v_2 - v_1$$

The length $L_N(C_i)$ travelled by the salesman for the above cycles C_i are:

$$L_N(C_1) = 1.67,\ L_N(C_2) = 1.48\ ,\ L_N(C_3) = 1.53,\ L_N(C_4) = 1.48$$

Since the value 1.48 is minimum length for the cycles C_2 and C_4, hence these cycles give the minimum travelling cost to the salesman.

7. Conclusions

In this article, sum distance, eccentricity, radius etc. in a neutrosophic graph has been developed. Some definitions, examples and theorems give a clear idea about the proposed study. A neutrosophic graph is recently a very important topic. There are many scopes to research on that topic. One can develop this study to the generalized neutrosophic graph. The real application inthe travelling salesman problem has been illustrated with a numerical example. This idea also gives us to develop future research in neutrosophic graphs.

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