



An Intelligent Dual Simplex Method to Solve Triangular Neutrosophic Linear Fractional Programming Problem

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Abstract: This paper develops a general form of neutrosophic linear fractional programming (NLFP) problem and proposed a novel model to solve it. In this method the NLFP problem is decomposed into two neutrosophic linear programming (NLP) problem. Furthermore, the problem has been solved by combination of dual simplex method and a special ranking function. In addition, the model is compared with an existing method. An illustrative example is shown for better understanding of the proposed method. The results show that the method is computationally very simple and comprehensible.

Keywords: Triangular neutrosophic numbers; dual simplex method, ranking function, linear fractional programming, linear programming

1. Introduction

Linear fractional programming (LFP) problem is a special type of linear programming (LP) problem where the constraints are in linear form and the objective functions must be a ratio of two linear functions. Last few years, many researchers have been developed various methods to solve LFP problem in both classical logic and fuzzy logic [1-8]. These methods are interesting, however, in daily life circumstances, due to ambiguous information supplied by decision makers, the parameters are often illusory and it is very hard challenge for decision maker to make a decision. In such a case, it is more appropriate to interpret the ambiguous coefficients and the vague aspirations parameters by means of the fuzzy set theory.

In this manuscript a real life problem was presented, having vague parameters. Perhaps the best task given to mankind is to control the earth inside which they live. Anyway, some guidance's have been given and, limits have been set with the end goal that some law of nature ought not to be abused. During the time spent controlling nature, humanity has developed extremely incredible instruments that permit them to have control of some significant spots like the ocean, air, and ground. For instance, and as a genuine circumstance, an infection called Covid-19 was recognized first in the city of Wuhan China, which is the capital of Hubei territory on December 31, 2019. In the wake of appearing the pneumonia without an unmistakable reason and for which the antibodies or medicines were not found. Further, it is indicated that the transmission of the infection differs from Human to Human. The case spread Wuhan city as well as to different urban areas of China. Moreover, the disease spread to other area of the world, for example, Europe, North America, and Asia. It is obscure to all whether the infection will be spread all world or constrained to some nation. In this point, what is the amount of the affected related to the number of the individuals? It's absolutely blind in regards to everybody and the information's are uncertain. Whether or not it was impacted to each age social occasion or some specific get-together? Everything is questionable and uncertain. Hence, from the above real-life conditions, the values are incomplete and ambiguous. This type of problem can be handled by way of fuzzy sets.

The thought of fuzzy logic was setup by Zadeh [16] and from that point forward it has discovered enormous applications in different fields. When applied the LFP problem with fuzzy numbers, it is termed as fuzzy LFP (FLFP) problem. As yet, exceptional sorts of FLFP problem have already been interpreted within many articles to resolve such kind of problems. Li and Chen [9] developed an approach for solving FLFP problem via triangular fuzzy numbers, inspired by them, a multi-objective LFP problem with the fuzzy strategy is viewed via Luhandjula [10]. Meher et al. [20] proposed an idea to compute an (α, β) optimal solution for finding FLFP problem. Subsequently, Veeramani and Sumathi [15] examined a FLFP problem with triangular fuzzy number by way of multi-objective LFP problem and changed into a single objective linear programming problem. A goal programming approach used to be delivered to solve FLFP problem via Veeramani and

Sumathi [6]. However, Das et al. [11] solved the FLFP problem using the concepts of simple ranking approach between two triangular fuzzy numbers. Das and Mandal [14] introduced a ranking method for solving the FLFP problem. A method was introduced by Pop and Minasian [2] for solving fully FLFP (FFLFP) problem where the cost of objective, constraints and the variables are triangular fuzzy numbers. Later on, some of the mathematicians [3-4,17-19,21-23] proposed a different methods for solving FFLFP problem. A new method of lexicographic optimal solution was proposed by Das et al. [12].

The drawback of fuzzy sets, whence its incapacity to successfully symbolize facts as its only take into consideration the truth membership function. To conquer this trouble, Atanassov [24] presented the concept of intuitionistic fuzzy sets (IFS) which is a hybrid of fuzzy sets, he took into consideration both truth and falsity membership functions. However, in real-life situations it's still facing some difficulty in case of decision making. Therefore, new set theory was introduced which dealt with incomplete, inconsistency and indeterminate informations called neutrosophic set (NS).

Neutrosophic logic was introduced by Smarandache [27] as a new generalization of fuzzy logic and IFSs. Neutrosophic set may be characterized by three independent components i.e. (i) truth-membership component (T), (ii) indeterminacy membership component (I), (iii) falsity membership component (F).

The decision makers in neutrosophic set want to increase the degree of truth membership and decrease the degree of both indeterminacy and falsity memberships. The truth membership function is exactly the inverse or in the opposite side of the falsity membership function, while the indeterminate membership function took some of its values from the truth membership function and other values of indeterminacy are took from the falsity membership function, that is mean the indeterminate membership is in the middle position between truth and falsity. For solving practical problems, a single value neutrosophic set (SVNS) was introduced by Wang et al. [45]. Some authors [46-48] considered the problem of SVNS in practical applications like educational sector, social sector. The basic definitions and notions of neutrosophic number (NN) were set up by Samarandache [37]. Recently, Abdel-Basset et al. [38] presented a novel technique for neutrosophic

LP problem by considering trapezoidal neutrosophic numbers. Edalatpanah [34] proposed a direct model for solving LP by considering triangular neutrosophic numbers. A new method to find the optimal solution of LP problem in NNs environment was proposed by Ye et al. [43]. The field of solving LP problem with single objective in NNs environment with the help of goal programming introduced by Banerjee and Pramanik [42]. Again, Pramanik and Dey [41] have solved the problem of linear bi-level-LP problem under NNs. Maiti et al. [39] introduced a strategy for multi-level multi-objective LP problem with NNs. Huda E. Khalid [54] established a new branch in neutrosophic theory named as neutrosophic geometric programming problems with their newly algorithms, and novel definitions and theorems.

Here, we consider NLFP problem in which all the parameters, except crisp decision variables are considered as triangular neutrosophic numbers. We emphasize that there are few manuscripts have used triangular neutrosophic numbers in LFP problems. Recently, an interesting method was proposed by Abdel-Basset et al. [28] for solving neutrosophic LFP (NLFP). The NLFP problem is transformed into an equivalent crisp multi-objective linear fractional programming (MOLFP) problem, where the authors have transformed the crisp MOLFP problem is reduced to a single objective LP problem which can be solved easily by suitable LP technique. However, the above mentioned method has a drawback where the solutions are obtained does not satisfy the constraints, more constraints arise step by step.

In this paper, the NLFP problem is decomposed in two NLP problem. The NLP problem is transformed into crisp LP problem by using ranking function. By using dual simplex method, the crisp LP problem was solved. Consequently, the adequacy of the applied procedure is shown through a numerical example.

The remain parts of this paper were orchestrated as follow: some basic definitions and arithmetical operation with respect to the neutrosophic numbers are introduced in Section 2. The strategy of the proposed technique was contained in Section 3. In section 4, the proposed system applied with representation numerical guide to explain its appropriateness. The article reaches a conclusion containing the finishing up comments introduced in Section 5.

2. Preliminaries

In this section, we present the basic notations and definitions, which are used throughout this paper.

Definition 1. [28]

Assume X is a universal set and $x \in X$. A neutrosophic set N may be defined via three membership

functions for truth, indeterminacy along with falsity and denoted by $T_N(x), I_N(x)$ and $F_N(x)$

.These are real standard or real nonstandard subsets of $]0^-, 1^+[$. That is

$T_N(x): X \rightarrow]^{-}0, 1^{+}[$, $I_N(x): X \rightarrow]^{-}0, 1^{+}[$ and $F_N(x): X \rightarrow]^{-}0, 1^{+}[$. There is no restriction

on the sum of $T_N(x), I_N(x)$ and $F_N(x)$, so $0^- \leq \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \leq 3^+$.

Definition 2. [38]

A single-valued neutrosophic set (SVNS) N over X is an object having the form

$N = \{x, T_N(x), I_N(x), F_N(x)\}$, where X be a space of discourse,

$T_N(x): X \rightarrow [0, 1]$, $I_N(x): X \rightarrow [0, 1]$ and $F_N(x): X \rightarrow [0, 1]$ with

$0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3, \forall x \in X$.

Definition 3 [34]. A triangular neutrosophic number (TNNs) is signified via

$N = \langle (a'_1, a'_2, a'_3); (q'_1, q'_2, q'_3) \rangle$ is an extended version of the three membership functions for the

truth, indeterminacy, and falsity of x can be defined as follows:

$$T_N(x) = \begin{cases} \frac{(x - a'_1)}{(a'_2 - a'_1)} & a'_1 \leq x < a'_2, \\ 1 & x = a'_2, \\ \frac{(a'_3 - x)}{(a'_3 - a'_2)} & a'_2 \leq x < a'_3, \\ 0 & \text{something else.} \end{cases}$$

$$I_N(x) = \begin{cases} \frac{(a'_2 - x)}{(a'_2 - a'_1)} & a'_1 \leq x < a'_2, \\ 1 & x = a'_2, \\ \frac{(x - a'_2)}{(a'_3 - a'_2)} & a'_2 \leq x < a'_3, \\ 0 & \text{something else.} \end{cases}$$

$$F_N(x) = \begin{cases} \frac{(a'_2 - x)}{(a'_2 - a'_1)} & a'_1 \leq x < a'_2, \\ 0 & x = a'_2, \\ \frac{(x - a'_2)}{(a'_3 - a'_2)} & a'_2 \leq x < a'_3, \\ 1 & \text{something else.} \end{cases}$$

Where, $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3, x \in N$ Additionally, when $a'_i \geq 0, N$ is called a non negative TNN. Similarly, when $a'_i < 0, N$ becomes a negative TNN.

Definition 4 [28].(Arithmetic Operations)

Suppose $N_1 = \langle (a'_1, a'_2, a'_3); (q'_1, q'_2, q'_3) \rangle$ and $N_2 = \langle (b'_1, b'_2, b'_3); (r'_1, r'_2, r'_3) \rangle$ be two TNNs. Then the

arithmetic relations are defined as:

- (i) $N_1 \oplus N_2 = \langle (a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3); (q'_1 \wedge r'_1, q'_2 \vee r'_2, q'_3 \vee r'_3) \rangle$
- (ii) $N_1 - N_2 = \langle (a'_1 - b'_1, a'_2 - b'_2, a'_3 - b'_3); (q'_1 \wedge r'_1, q'_2 \vee r'_2, q'_3 \vee r'_3) \rangle$
- (iii) $N_1 \otimes N_2 = \langle (a'_1 b'_1, a'_2 b'_2, a'_3 b'_3); (q'_1 \wedge r'_1, q'_2 \vee r'_2, q'_3 \wedge r'_3) \rangle$, if $a'_1 > 0, b'_1 > 0$,
- (iv) $\lambda N_1 = \begin{cases} \langle (\lambda a'_1, \lambda a'_2, \lambda a'_3); (q'_1, q'_2, q'_3) \rangle, & \text{if } \lambda > 0 \\ \langle (\lambda a'_3, \lambda a'_2, \lambda a'_1); (q'_1, q'_2, q'_3) \rangle, & \text{if } \lambda < 0 \end{cases}$

$$(v) \frac{N_1}{N_2} = \begin{cases} \left\langle \left(\frac{a'_1}{b'_3}, \frac{a'_2}{b'_2}, \frac{a'_3}{b'_1} \right); q'_1 \wedge r'_1, q'_2 \vee r'_2, q'_3 \vee r'_3 \right\rangle (a'_3 > 0, b'_3 > 0) \\ \left\langle \left(\frac{a'_3}{b'_3}, \frac{a'_2}{b'_2}, \frac{a'_1}{b'_1} \right); q'_1 \wedge r'_1, q'_2 \vee r'_2, q'_3 \vee r'_3 \right\rangle (a'_3 < 0, b'_3 > 0) \\ \left\langle \left(\frac{a'_3}{b'_1}, \frac{a'_2}{b'_2}, \frac{a'_1}{b'_3} \right); q'_1 \wedge r'_1, q'_2 \vee r'_2, q'_3 \vee r'_3 \right\rangle (a'_3 < 0, b'_3 < 0) \end{cases}$$

Definition 5 [28].Suppose N_1 and N_2 be two TNNs. Then:

- (i) $N_1 \leq N_2$ if and only if $\mathfrak{R}(N_1) \leq \mathfrak{R}(N_2)$.

(ii) $N_1 < N_2$ if and only if $\mathfrak{R}(N_1) < \mathfrak{R}(N_2)$.

Where $\mathfrak{R}(\cdot)$ is a ranking function.

Definition 6 [32].The ranking function for triangular neutrosophic number $N = \langle (a_1^l, a_2^l, a_3^l); (q_1^l, q_2, q_3^l) \rangle$ is defined as:

$$\mathfrak{R}(N) = \frac{a_1^l + a_2^l + a_3^l}{9} (q_1^l + (1 - q_2) + (1 - q_3^l)).$$

3. Proposed Method

One of the main aims of this paper is to extend the linear fractional problem into neutrosophic linear fractional programming problem.

The crisp LFP problem can be presented in the following way:

$$\text{Max (or Min) } z(x) = \frac{N(x)}{D(x)}$$

Subject to (1)

$$x \in S.$$

where $N(x)$ and $D(x)$ are linear functions and the set S is defined as $S = \{x / Ax \leq b, x > 0\}$.

Here A is a fuzzy $m \times n$ matrix.

The problem (1) can be written as:

$$\begin{aligned} & \max (\text{or min}) z(x) = \frac{N(x)}{D(x)} \\ & \text{s.t. } x \in S \\ & \min (\text{or max}) z(x) = \frac{N(x)}{D(x)} \\ & \text{s.t. } x \in S \end{aligned} \tag{2}$$

Now, we consider the neutrosophic linear fractional programming (NLFP) problem with m constraints and n variables:

$$\text{Max (or min) } \tilde{Z} = \frac{\tilde{n}^T \otimes x + \tilde{r}}{\tilde{d}^T \otimes x + \tilde{s}} \tag{3}$$

Subject to

$$\tilde{A} \otimes x \leq \tilde{b}$$

$$x \geq 0, j = 1, 2, \dots, n.$$

where $\tilde{n}^T = [\tilde{n}_{1 \times i}^T]_{1 \times n}$, $\tilde{d}^T = [\tilde{d}_{1 \times i}^T]_{1 \times n}$ and $\text{rank}(\tilde{A}, \tilde{b}) = \text{rank}(\tilde{A}) = m$. \tilde{r} and $\tilde{s} \in \mathbb{R}$ are constants.

Due to some challenges exists in the crisp methods, these methods cannot be tackling above NLFP problem, and to overcome the challenges, another strategy is proposed. The means of the proposed technique are described in the following algorithm:

Step-1:substituting $\tilde{n}^T = [\tilde{n}_j]_{1 \times n}$, $\tilde{d}^T = [\tilde{d}_j]_{1 \times n}$, $\tilde{A} = [a_{ij}]_{m \times n}$, $\tilde{r} = [\tilde{r}_j]_{1 \times n}$, $\tilde{s} = [\tilde{s}_j]_{1 \times n}$ and $\tilde{b} = [\tilde{b}_i]_{m \times 1}$

the above NLFP problem may be rewritten as :

$$\text{Max (or min)} \quad \tilde{Z}(x) = \frac{\sum_{j=1}^n \tilde{n}_j x_j + \tilde{r}_j}{\sum_{j=1}^n \tilde{d}_j x_j + \tilde{s}_j}$$

Subject to (4)

$$\sum_{i=1}^m \tilde{a}_{ij} x_j \leq \tilde{b}_i$$

$$x_j \geq 0, i = 1, 2, \dots, m$$

Step-2:By considering the following triangular neutrosophic numbers:

$$\tilde{n}_j = (l_j, m_j, n_j); (\mu_j, \nu_j, w_j), \tilde{d}_j = (c_j, d_j, f_j); (\mu_d, \nu_d, w_d), \tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}); (\mu_a, \nu_a, w_a),$$

$$\tilde{r}_j = (r_{j1}, r_{j2}, r_{j3}; \mu_{jr}, \nu_{jr}, w_{jr}), \tilde{s}_j = (s_{j1}, s_{j2}, s_{j3}; \mu_{js}, \nu_{js}, w_{js}) \text{ and } \tilde{b}_i = (p_i, q_i, r_i); (\mu_b, \nu_b, w_b).$$

the NLFP problem may be written as :

$$\text{max (or min)} \quad \tilde{Z}(x) = \frac{\sum_{j=1}^n (l_j, m_j, n_j; \mu_j, \nu_j, w_j) x_j + (r_{j1}, r_{j2}, r_{j3}; \mu_{jr}, \nu_{jr}, w_{jr})}{\sum_{j=1}^n (c_j, d_j, f_j; \mu_d, \nu_d, w_d) x_j + (s_{j1}, s_{j2}, s_{j3}; \mu_{js}, \nu_{js}, w_{js})}$$

subject to

$$\sum_{j=1}^n (a_{ij}, b_{ij}, c_{ij}; \mu_j, \nu_j, w_j) x_j \leq (p_i, q_i, r_i; \mu_j, \nu_j, w_j)$$

$$x_j \geq 0, i = 1, 2, \dots, m.$$

Step-3. To determine the optimal value of the above problem, we take transform the objective function into two neutrosophic linear programming problem and the problem may be written as follows:

$$(E-1) \quad \text{max (or min)} \quad \tilde{Z}(x) = \sum_{j=1}^n (l_j, m_j, n_j; \mu_j, \nu_j, w_j) x_j + (r_{j1}, r_{j2}, r_{j3}; \mu_{jr}, \nu_{jr}, w_{jr})$$

subject to

$$\sum_{j=1}^n (a_{ij}, b_{ij}, c_{ij}; \mu_j, \nu_j, w_j) x_j \leq (p_i, q_i, r_i; \mu_j, \nu_j, w_j)$$

$$x_j \geq 0, i = 1, 2, \dots, m.$$

$$(E-2) \quad \min (\text{or max}) \quad \tilde{Z}(x) = \sum_{j=1}^n (c_j, d_j, f_j; \mu_j, \nu_j, w_j) x_j + (s_{j1}, s_{j2}, s_{j3}; \mu_j, \nu_j, w_j)$$

subject to

$$\sum_{j=1}^n (a_{ij}, b_{ij}, c_{ij}; \mu_j, \nu_j, w_j) x_j \leq (p_i, q_i, r_i; \mu_j, \nu_j, w_j)$$

$$x_j \geq 0, i = 1, 2, \dots, m.$$

Step-4: Using arithmetic operations, defined in definition 5 and 7, the above NLP problems (E-1) and (E-2) are converted into crisp linear programming problems, separately.

$$(E-3) \quad \max (\text{or min}) \quad \tilde{Z}(x) = \mathfrak{R} \left(\sum_{j=1}^n (l_j, m_j, n_j; \mu_j, \nu_j, w_j) x_j + (r_{j1}, r_{j2}, r_{j3}; \mu_j, \nu_j, w_j) \right)$$

subject to

$$\mathfrak{R} \left(\sum_{j=1}^n (a_{ij}, b_{ij}, c_{ij}; \mu_j, \nu_j, w_j) x_j \leq (p_i, q_i, r_i; \mu_j, \nu_j, w_j) \right)$$

$$x_j \geq 0, i = 1, 2, \dots, m.$$

$$(E-4) \quad \min (\text{or max}) \quad \tilde{Z}(x) = \mathfrak{R} \left(\sum_{j=1}^n (c_j, d_j, f_j; \mu_j, \nu_j, w_j) x_j + (s_{j1}, s_{j2}, s_{j3}; \mu_j, \nu_j, w_j) \right)$$

subject to

$$\mathfrak{R} \left(\sum_{j=1}^n (a_{ij}, b_{ij}, c_{ij}; \mu_j, \nu_j, w_j) x_j \leq (p_i, q_i, r_i; \mu_j, \nu_j, w_j) \right)$$

$$x_j \geq 0, i = 1, 2, \dots, m.$$

Step-5: Now solve the above crisp LP problem (E-3) and (E-4) by using the dual simplex method.

Step-6: Find the optimal solution of x_j by solving crisp LP problem obtained in Step-5.

Step-7: Find the fuzzy optimal value by putting x_j in both (E-3) and (E-4) and get crisp linear fractional programming problem.

4. Numerical Example

Here, we select a case of [28] to represent the model alongside correlation of existing technique.

Example-1

In Jamshedpur City, India, A Wooden company is the producer of three kinds of products A, B and C with profit around 8, 7 and 9 dollar per unit, respectively. However the cost for each one unit of the above products is around 8, 9 and 6 dollars respectively. It is assume that a fixed cost of around 1.5 dollar is added to the cost function due to expected duration through the process of production. Suppose the raw material needed for manufacturing product A, B and C is about 4, 3 and 5 units per dollar respectively, the supply for this raw material is restricted to about 28 dollar. Man-hours per unit for the product A is about 5 hour, product B is about 3 hour and C is about 3 hour per unit for manufacturing but total Man-hour available is about 20 hour daily. Determine how many products A, B and C should be manufactured in order to maximize the total profit.

Let x_1, x_2 and x_3 component be the measure of A, B and C, individually to be created.

After prediction of evaluated parameters, the above issue can be defined as the following NLFPP:

$$\text{Max } Z = \frac{8^l x_1 + 7^l x_2 + 9^l x_3}{8^l x_1 + 9^l x_2 + 6^l x_3 + 1.5^l}$$

(5)

Subject to

$$4^l x_1 + 3^l x_2 + 5^l x_3 \leq 28^l$$

$$5^l x_1 + 3^l x_2 + 3^l x_3 \leq 20^l$$

$$x_1, x_2, x_3 \geq 0.$$

Here we consider,

$$8^l = (7, 8, 9; 0.5, 0.8, 0.3), 7^l = (6, 7, 8; 0.2, 0.6, 0.5), 9^l = (8, 9, 10, 8, 0.1, 0.4), 6^l = (4, 6, 8; 0.75, 0.25, 0.1), \\ 1.5^l = (1, 1.5, 2; 0.75, 0.5, 0.25), 4^l = (3, 4, 5; 0.4, 0.6, 0.5), 3^l = (2, 3, 4; 1, 0.25, 0.3), 5^l = (4, 5, 6; 0.3, 0.4, 0.8) \\ 28^l = (25, 28, 30; 0.4, 0.25, 0.6), 20^l = (18, 20, 22; 0.9, 0.2, 0.6)$$

Presently the problem is modified as follows:

$$\text{Max } Z = \frac{(7, 8, 9; 0.5, 0.8, 0.3)x_1 + (6, 7, 8; 0.2, 0.6, 0.5)x_2 + (8, 9, 10, 8, 0.1, 0.4)x_3}{(7, 8, 9; 0.5, 0.8, 0.3)x_1 + (8, 9, 10, 8, 0.1, 0.4)x_2 + (4, 6, 8; 0.75, 0.25, 0.1)x_3 + (1, 1.5, 2; 0.75, 0.5, 0.25)}$$

Subject to

$$(3, 4, 5; 0.4, 0.6, 0.5)x_1 + (2, 3, 4; 1, 0.25, 0.3)x_2 + (4, 5, 6; 0.3, 0.4, 0.8)x_3 \leq (25, 28, 30; 0.4, 0.25, 0.6) \\ (4, 5, 6; 0.3, 0.4, 0.8)x_1 + (2, 3, 4; 1, 0.25, 0.3)x_2 + (2, 3, 4; 1, 0.25, 0.3)x_3 \leq (18, 20, 22; 0.9, 0.2, 0.6)$$

$$x_1, x_2, x_3 \geq 0.$$

Utilizing Step 2 the NFP problem can be transformed into two NLP problem as:

$$\text{Max } Z = (7, 8, 9; 0.5, 0.8, 0.3)x_1 + (6, 7, 8; 0.2, 0.6, 0.5)x_2 + (8, 9, 10, 8, 0.1, 0.4)x_3$$

(E-1) Subject to

$$(3, 4, 5; 0.4, 0.6, 0.5)x_1 + (2, 3, 4; 1, 0.25, 0.3)x_2 + (4, 5, 6; 0.3, 0.4, 0.8)x_3 \leq (25, 28, 30; 0.4, 0.25, 0.6) \\ (4, 5, 6; 0.3, 0.4, 0.8)x_1 + (2, 3, 4; 1, 0.25, 0.3)x_2 + (2, 3, 4; 1, 0.25, 0.3)x_3 \leq (18, 20, 22; 0.9, 0.2, 0.6)$$

$$x_1, x_2, x_3 \geq 0.$$

$$\text{Max } Z = (7, 8, 9; 0.5, 0.8, 0.3)x_1 + (8, 9, 10, 8, 0.1, 0.4)x_2 + (4, 6, 8; 0.75, 0.25, 0.1)x_3 + (1, 1.5, 2; 0.75, 0.5, 0.25)$$

(E-2) Subject to

$$(3, 4, 5; 0.4, 0.6, 0.5)x_1 + (2, 3, 4; 1, 0.25, 0.3)x_2 + (4, 5, 6; 0.3, 0.4, 0.8)x_3 \leq (25, 28, 30; 0.4, 0.25, 0.6) \\ (4, 5, 6; 0.3, 0.4, 0.8)x_1 + (2, 3, 4; 1, 0.25, 0.3)x_2 + (2, 3, 4; 1, 0.25, 0.3)x_3 \leq (18, 20, 22; 0.9, 0.2, 0.6)$$

$$x_1, x_2, x_3 \geq 0.$$

Using Step-3, the ranking function the problem (E-1) and (E-2) can be written as follows:

$$\text{Max } Z = 3.73x_1 + 2.56x_2 + 6.9x_3$$

(E-1) Subject to

$$1.73x_1 + 2.45x_2 + 1.83x_3 \leq 14.29$$

$$1.83x_1 + 2.45x_2 + 2.45x_3 \leq 14$$

$$x_1, x_2, x_3 \geq 0.$$

$$\text{Max } Z = 3.73x_1 + 6.9x_2 + 4.8x_3 + 1$$

(E-2) Subject to

$$1.73x_1 + 2.45x_2 + 1.83x_3 \leq 14.29$$

$$1.83x_1 + 2.45x_2 + 2.45x_3 \leq 14$$

$$x_1, x_2, x_3 \geq 0.$$

Now solve the problem (E-1) by dual simplex method

$$\text{Max } Z = 3.73x_1 + 2.56x_2 + 6.9x_3$$

(E-3) Subject to

$$1.73x_1 + 2.45x_2 + 1.83x_3 \leq 14.29$$

$$1.83x_1 + 2.45x_2 + 2.45x_3 \leq 14$$

$$x_1, x_2, x_3 \geq 0.$$

Now the problem (E-3) is solved by dual simplex method and get the optimal solution is as:

$$x_1 = 0, x_2 = 0, x_3 = 5.71 \text{ and the objective solution is } z = 39.42.$$

$$\text{Max } Z = 3.73x_1 + 6.9x_2 + 4.8x_3 + 1$$

(E-4) Subject to

$$1.73x_1 + 2.45x_2 + 1.83x_3 \leq 14.29$$

$$1.83x_1 + 2.45x_2 + 2.45x_3 \leq 14$$

$$x_1, x_2, x_3 \geq 0.$$

Now the problem (E-4) is solved by dual simplex method and get the optimal solution is as:

$$x_1 = 0, x_2 = 2.65, x_3 = 3.05 \text{ and the objective solution is } z = 33.92.$$

Finally, the optimum solution of crisp linear fractional programming problem is obtained.

Thus,

$$\max z = \frac{39.42}{33.92} = 1.16$$

By comparing the results of objective solutions, we can conclude that our solution is more maximize the cost.

$$1.16 = z_{(\text{Proposed Method})} > z_{(\text{Existing Method}[28])} = 1$$

By comparing the results of the proposed method with existing method [28] based on ranking function and ordering by using Definition 6, we can conclude that our result is more effective, because:

$$0.208 = z_{(\text{Proposed Method})} > z_{(\text{Existing Method}[28])} = 0.069 .$$

This example has been solved by the proposed method to show that one can overcome the limitations of the existing method [28] by using the proposed method. Earlier this problem was also solved by Abel-Basset [28]. Obtained result of the present method has been compared with the results of existing method [28]. It is worth mentioning that one may check that the results obtained by the existing method may not satisfy the constraints properly where the results obtained by the present method satisfied those constraints exactly. Based on the ranking function the proposed method is higher optimized the value as compare to the existing method. In the proposed methodology the FFLP problem turns into a crisp linear programming problem and that problem is solved by using LINGO Version 13.0.

Result Analysis:

In this segment, we give an outcome examination of the proposed strategy with existing technique. In the above writing perusing, we infer that there is exceptionally less exploration paper for taking care of neutrosophic LFP issue. In this manner, we consider the traditional LFP issue and fuzzy LFP issue for correlation with our proposed strategy.

- Our proposed outcomes are better than traditional LFP [22] and fluffy LFP [56] model. The objective solution of our proposed technique is 1.16, anyway in the current strategy [22,56] the objective solution is 1.09. Obviously our target arrangement is maximized.
- In real-life problem, the leaders faces numerous issue to take choice as truth, not truth and bogus. In any case, in Das et al. [56] the fluffy model the leaders consider just truth work. This is the

fundamental downside of Das et al. fluffy model. Taking these points of interest, we proposed new technique.

- Our model is applied in any genuine issue.
- In the above conversation, we reason that our model is another approach to deal with the vulnerability and indeterminacy in genuine issue.

5. Conclusions

This paper introduced a novel method for solving NLFP problem where all the parameters are triangular neutrosophic numbers except decision variables. In our proposed method, NLFP problem is transformed into two equivalent NLP problems and the resultant problem is converted into crisp LP problem by using ranking function. Dual simplex method is used for solving the crisp LP problem. From the computational discussion, we conclude that with respect to the existing method [28], proposed method has less computational steps and the optimum solution is maximize the values. The proposed method NLFP problem has successfully overcome the drawbacks of the existing work [28]. Finally, from the procured results, it might be derived that the model is capable and supportive.

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