NSS
Neutrosophic Sets and Systems, Vol. 38, 2020
University of New Mexico

Application of Similarity Measure on m-polar Interval-valued Neutrosophic Set in Decision Making in Sports

Muhammad Saeed[1], Muhammad Saqlain[1*], and Asad Mehmood[2]
[1]Department of Mathematics, University of Management and Technology, Lahore, Pakistan; muhammad.saeed@umt.edu.pk
[1*]Department of Mathematics, Lahore Garrison University, Lahore, Pakistan; msaqlain@lgu.edu.pk
[2]Department of Mathematics, University of Management and Technology, Lahore, Pakistan; a.asadkhan.khi@gmail.com

Abstract. In real life, most of the problems occurred by wrong decision making, while in sports it is mandatory for every player, coach, and technique director to make a good and an ideal decision. In this paper, the concept of similarity measure is used in the neutrosophic environment for decision making in a football game for the selection of players. The data is collected in interval-valued, while the new concept m-polar is illustrated as previous records of m matches played by players. m-polar structures provide multiple data on the concerned problem, so as a result the best solution can be developed for the selection problem. An m-polar Interval-valued Neutrosophic Set (mIVNS) is derived for the targeted task of player selection problem. Then some operations, properties, and distance measures are introduced on m-polar Interval-valued Neutrosophic Set (mIVNS). Distance-base Similarity Measure is illustrated to each player with an ideal set in mIVNS structure. In the end, the Algorithm is given for ideal decision-making in sports for the selection of players.

Keywords: mIVNS Set; Operators on mIVNS; Properties; Distance and Similarity Measure; Decision-Making, Selection of Players

1. Introduction

Zadeh [1] has introduced a fuzzy set which describes membership degrees in [0,1]. They can be modeled to proceed towards including soft set theory [2,3] fuzzy soft set theory [4], probability theory, and also other mathematical tools. This theory has been used in many real-life applications to handle unpredictability. However, this theory doesn't deal with the hesitancy degree. To overcome this situation Atanassov [5] gave the idea of the intuitionistic fuzzy set as a generalized form of fuzzy theory, that handles incomplete data by considering both membership function values and non-membership function values. Intuitionistic Fuzzy theory has been extended by many researchers in dealing with real-life problems. Intuitionistic fuzzy sets fail in grip patchy data because membership, hesitancy, and non-membership functions are dependent in this theory, while neutrosophic sets overcome with the solution where these functions are independent which exists in psychology theory and belief system. For several decades there are more concepts applied in soft set theory, different mathematicians relate their ideas and results on soft set theory in fuzzy set theory and other hybrid structures. To analyze the ability of ideal decision on positive effects and as well as on negative effects Zhang [6] gave bipolar concept on truth or false. It was further extended by Chen [7] to multipolar structure, where multiple data can be studied and evaluated. Akram [8-11] has applied to m-polar structure in decision-making problems and pattern recognition techniques.

Firstly, the concept of Neutrosophic Set (NS) introduced by Smarandache [12] is a tool that handles the problem with inconsistency and imprecise data with indeterminacy. Nowadays the NS has been extended in many hybrid structures. Single-valued NS is proposed in [12] and also Interval-valued Neutrosophic Set (IVNS) truth indeterminacy and falsity are determined by intervals that have huge information in real-life problems. Similarity measure technique is a well-known process to compare two sets. The Similarity measure is also used for the evaluation of pattern recognition of an object, set, and material. It is usually used between two independent objects on the basis of distance measure, while distance measure gives the numeric value of separation between two objects. A Similarity measures on fuzzy sets, soft sets, neutrosophic sets, etc. are done by several authors in their papers [13-26]. Aggregate operators, similarity measure, and a TOPSIS technique and their application in real life are introduced by [27-31] by Saeed et al. Application of fuzzy numbers in mobile selection in metros like Lahore is proposed by Saqlain [32,33]. TOPSIS technique of Multi-Criteria Decision Making (MCDM) can also be used for the prediction of games, and it's applied in FIFA 2018 by [34], prediction of games is a very complex topic and this game is also predicted by [35]. Liu et al. [36] introduced multi-valued neutrosophic numbers and utilized it with Bonferroni operator in multi-valued decision-making problems. Kamal et al. introduced a multi-polar neutrosophic soft structure with some operators and properties in [37]. Abdel-Basset et al. [38-42] proposed the solution to supply change problems, professional selection problem, time-cost tradeoff, and leveling problems in construction using a neutrosophic environment. Several authors [43-47] have done researches in m-polar structure with the fuzzy set, neutrosophic set, soft set topology in the past couple of years. From a scholastic point of view, operators on multi-valued neutrosophic soft sets need to be specified so that concepts can easily be applied in real-life applications. The concept of interval-valued neutrosophic sets was proposed where uncertain, vague, inconsistent, and incomplete data given in interval-valued. In this paper, we introduce m-polar Interval-valued neutrosophic sets that deal with multiple set of data that are used in uncertain, vague, inconsistent, and incomplete data environment. An important issue is how we can represent, m-polar interval-valued neutrosophic set? It's operators and similarity measure? What should be the generalized form of interval-valued neutrosophic sets? What should be the application of this environment? To find the answers to all these questions, this study is done.

In this paper, the concept of Interval-valued neutrosophic set is extended to m-polar Interval-valued Neutrosophic Set (m-polar IVNS);

(1) m-polar IVNS, its definition, and representation.
(2) Aggregate operators of mIVNS and properties on operators of mIVNS.
(3) Distance measure and Similarity measures of mIVNS.
(4) m-polar IVNS Algorithm.
(5) Application of the proposed environment

The paper is organized and structured in the following ways, also shown in Figure 1. In section 1, the introduction and literature review are presented. Section 2, consists of some basic definitions which will help read the rest of the article. In section 3, the definition, representation, and some operations like union, intersection, and complement, etc. of m-polar IVNS have been proposed. In section 4, properties concerned with operators have been studied. In section 5, distances on m-polar IVNS have been introduced and the similarity measure concept is revisited. In section 5, the application of the proposed environment with algorithm is presented. In section 6, the conclusion is presented.

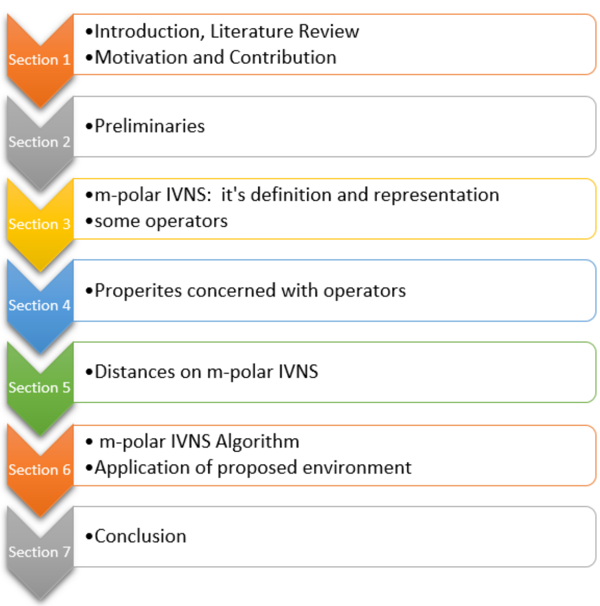

FIGURE 1. Pictorial view for the structure of the article

2. Preliminaries

This section studies some basic definitions related to this article.

2.1. *Neutrosophic Set*

Definition 2.1 [12] Let $\ddot{U}$ be a space of points with a common element denoted by u. A neutrosophic set $\ddot{A}$ over $\ddot{U}$ is characterized by a truth-membership function $\phi_{\ddot{A}}$, an indeterminacy-membership function $\psi_{\ddot{A}}$ and a falsity-membership function $\eta_{\ddot{A}}$. The functions $\phi_{\ddot{A}}$, $\psi_{\ddot{A}}$, and $\eta_{\ddot{A}}$ are real standard belong to interval $[0,1]$. The neutrosophic set can be represented as

$$\ddot{A} = \{u, (\phi_{\ddot{A}}(u), \psi_{\ddot{A}}(u), \eta_{\ddot{A}}(u)) \mid u \in \ddot{U}\}$$
$$\text{where } 0 \leq \phi_{\ddot{A}}(u) + \psi_{\ddot{A}}(u) + \eta_{\ddot{A}}(u) \leq 3.$$

2.2. *Interval-valued Neutrosophic set*

Definition 2.2 [37] Let $\ddot{U}$ be a space of objects with some element denoted by u. An interval-valued neutrosophic set $\ddot{A}$ over $\ddot{U}$ is characterized by interval-valued truth-membership function ${}^{I}\phi_{\ddot{A}}$, an interval-valued indeterminacy-membership function ${}^{I}\psi_{\ddot{A}}$ and an interval-valued falsity-membership function ${}^{I}\eta_{\ddot{A}}$, such that ${}^{I}\phi_{\ddot{A}}, {}^{I}\psi_{\ddot{A}}$ and ${}^{I}\eta_{\ddot{A}} \subseteq [0,1]$. Thus, an interval-valued neutrosophic sets over $\ddot{U}$ can be represented as

$$\ddot{A} = \{u, ({}^{I}\phi_{\ddot{A}}(u), {}^{I}\psi_{\ddot{A}}(u), {}^{I}\eta_{\ddot{A}}(u)) \mid u \in \ddot{U}\}$$

and

$${}^{I}\phi_{\ddot{A}}(u) = [\phi^{-}_{\ddot{A}}(u), \phi^{+}_{\ddot{A}}(u)]$$
$${}^{I}\psi_{\ddot{A}}(u) = [\psi^{-}_{\ddot{A}}(u), \psi^{+}_{\ddot{A}}(u)]$$
$${}^{I}\eta_{\ddot{A}}(u) = [\eta^{-}_{\ddot{A}}(u), \eta^{+}_{\ddot{A}}(u)]$$
$$\text{where } 0 \leq \sup({}^{I}\phi_{\ddot{A}}(u)) + \sup({}^{I}\psi_{\ddot{A}}(u)) + \sup({}^{I}\eta_{\ddot{A}}(u)) \leq 3$$

2.3. *m-polar Neutrosophic Set*

Definition 2.3 [37] An m-polar neutrosophic set is defined as

$$\ddot{A} = \{u, (\phi^{1}_{\ddot{A}}(u), \phi^{2}_{\ddot{A}}(u), \ldots, \phi^{m}_{\ddot{A}}(u)), (\psi^{1}_{\ddot{A}}(u), \psi^{2}_{\ddot{A}}(u), \ldots, \psi^{m}_{\ddot{A}}(u)), (\eta^{1}_{\ddot{A}}(u), \eta^{2}_{\ddot{A}}(u), \ldots, \eta^{m}_{\ddot{A}}(u)) \mid u \in \ddot{U}\}$$

where $\phi^{i}_{\ddot{A}} : \ddot{U} \to [0,1]$, $\psi^{i}_{\ddot{A}} : \ddot{U} \to [0,1]$, and $\eta^{i}_{\ddot{A}} : \ddot{U} \to [0,1]$; (for all $i = 1, 2, \ldots, m$) denotes the degree of i-th truth-membership, i-th indeterminacy-membership, and i-th falsity-membership respectively for each element $u \in \ddot{U}$ to the set $\ddot{A}$ and,

$$0 \leq \phi^{i}_{\ddot{A}}(u) + \psi^{i}_{\ddot{A}}(u) + \eta^{i}_{\ddot{A}}(u) \leq 3.$$
$$\text{for all } i = 1, 2, \ldots, m$$

2.4. *m-polar Interval-valued Neutrosophic Set*

Definition 2.4 [37] Let $\ddot{U}$ be a space of the objects and an m-polar IVNS $\ddot{A}$ over universe $\ddot{U}$ is defined as

$$\ddot{A} = \{u, ({}^I\phi^1_{\ddot{A}}(u), {}^I\phi^2_{\ddot{A}}(u), \ldots, {}^I\phi^m_{\ddot{A}}(u)), ({}^I\psi^1_{\ddot{A}}(u), {}^I\psi^2_{\ddot{A}}(u), \ldots, {}^I\psi^m_{\ddot{A}}(u)), ({}^I\eta^1_{\ddot{A}}(u), {}^I\eta^2_{\ddot{A}}(u), ldots, {}^I\eta^m_{\ddot{A}}(u)) \mid u \in \ddot{U}\}$$

where,

$$\begin{gathered} {}^I\phi^i_{\ddot{A}}(u) = [\phi^{i-}_{\ddot{A}}(u), \phi^{i+}_{\ddot{A}}(u)], \\ {}^I\psi^i_{\ddot{A}}(u) = [\psi^{i-}_{\ddot{A}}(u), \psi^{i+}_{\ddot{A}}(u)], \\ \text{and } {}^I\eta^i_{\ddot{A}}(u) = [\eta^{i-}_{\ddot{A}}(u), \eta^{i+}_{\ddot{A}}(u)], \\ \text{for all } i = 1, 2, \cdots, m \end{gathered}$$

represents i-th interval-valued truth membership, an i-th interval-valued indeterminacy membership, and i-th interval-valued falsity membership respectively, and

$$\begin{gathered} 0 \leq \sup({}^I\phi^i_{\ddot{A}}(u)) + \sup({}^I\psi^i_{\ddot{A}}(u)) + \sup({}^I\eta^i_{\ddot{A}}(u)) \leq 3. \\ \text{for all } i = 1, 2, \cdots, m \end{gathered}$$

Example 2.1

Let $\ddot{U} = \{u_1, u_2, u_3\}$ be a universal set and we define 3-polar IVNS $\ddot{A}$ over universe $\ddot{U}$ as,

$$\begin{gathered} \ddot{A} = \{u_1, (([0.2, 0.6], [0.3, 0.5], [0.6, 1]), ([0, 0.4], [0.2, 0.6], [0.4, 0.6]), ([0.5, 0.7], [0.8, 1], [0.6, 0.7])), \\ u_2, (([0.3, 0.6], [0.3, 0.7], [0.1, 0.4]), ([0.5, 0.6], [0.8, 1], [0.5, 0.8]), ([0.3, 0.5], [0.6, 0.8], [0.2, 0.5])), \\ u_3, (([0.4, 0.7], [0.5, 0.9], [0.6, 0.8]), ([0.7, 0.9], [0.6, 0.7], [0.5, 0.6]), ([0.2, 0.4], [0.3, 0.5], [0.4, 0.7]))\} \end{gathered}$$

3. **Operations on m-polar interval valued neutrosophic sets**

This section discusses some operators on these sets.

3.1. *m-polar Interval-valued Neutrosophic Subset*

Definition 3.1 Let $\ddot{A}$ and $\ddot{B}$ be two m-polar interval-valued neutrosophic sets over universal set $\ddot{U}$, then $\ddot{A}$ is said to be a subset of $\ddot{B}$ represented as $\ddot{A}\tilde{\subseteq}\ddot{B}$ if

$$\begin{gathered} \phi^{i-}_{\ddot{A}}(u) \geq \phi^{i-}_{\ddot{B}}(u) \text{ and } \phi^{i+}_{\ddot{A}}(u) \leq \phi^{i+}_{\ddot{B}}(u); \\ \psi^{i-}_{\ddot{A}}(u) \geq \psi^{i-}_{\ddot{B}}(u) \text{ and } \psi^{i+}_{\ddot{A}}(u) \leq \psi^{i+}_{\ddot{B}}(u); \\ \eta^{i-}_{\ddot{A}}(u) \leq \eta^{i-}_{\ddot{B}}(u) \text{ and } \eta^{i+}_{\ddot{A}}(u) \geq \eta^{i+}_{\ddot{B}}(u); \end{gathered}$$

Example 3.1

Let $\ddot{U} = \{u_1, u_2, u_3\}$ be a universal set and we define two 3-polar IVNS $\ddot{A}$ and $\ddot{B}$ over universe $\ddot{U}$ as,

$$\begin{gathered} \ddot{A} = \{u_1, (([0.2, 0.6], [0.3, 0.5], [0.6, 1]), ([0, 0.4], [0.2, 0.6], [0.4, 0.6]), ([0.5, 0.7], [0.8, 1], [0.6, 0.7])), \\ u_2, (([0.3, 0.6], [0.3, 0.7], [0.1, 0.4]), ([0.5, 0.6], [0.8, 1], [0.5, 0.8]), ([0.3, 0.5], [0.6, 0.8], [0.2, 0.5])), \\ u_3, (([0.4, 0.7], [0.5, 0.9], [0.6, 0.8]), ([0.7, 0.9], [0.6, 0.7], [0.5, 0.6]), ([0.2, 0.4], [0.3, 0.5], [0.4, 0.7]))\} \end{gathered}$$

and

$\ddot{B} = \{u_1, (([0.1, 0.7], [0.2, 0.7], [0.3, 1]), ([0, 0.6], [0.2, 0.7], [0.2, 0.8]), ([0.6, 0.7], [0.9, 1], [0.6, 0.7])),$
$u_2, (([0.2, 0.7], [0.3, 0.8], [0.1, 0.5]), ([0.2, 0.7], [0.4, 1], [0.3, 0.8]), ([0.3, 0.5], [0.6, 0.7], [0.2, 0.4])),$
$u_3, (([0.2, 0.7], [0.4, 1], [0.3, 0.8]), ([0.6, 0.9], [0.4, 0.7], [0.4, 0.8]), ([0.3, 0.4], [0.5, 0.5], [0.5, 0.6]))\}$

here $\ddot{A}$ is a subset of $\ddot{B}$.

3.2. *m-polar Interval-valued Neutrosophic Equal Set*

Definition 3.2 Let $\ddot{A}$ and $\ddot{B}$ be two m-polar Interval-valued Neutrosophic Set over universal set $\ddot{U}$. Then two set $\ddot{A}$ and $\ddot{B}$ is said to be equal, represented as $\ddot{A}\tilde{=}\ddot{B}$ if and only if

$$\ddot{A}\tilde{\subseteq}\ddot{B} \text{ and } \ddot{B}\tilde{\subseteq}\ddot{A}$$

3.3. *m-polar Interval-valued Neutrosophic Null Set*

Definition 3.3 An m-polar Interval-valued Neutrosophic Null Set $\ddot{\Phi}$ on universal set $\ddot{U}$ is defined as

$\ddot{\Phi} = \{u, (([1, 0], [1, 0], \ldots, [1, 0]), ([1, 0], [1, 0], \ldots, [1, 0]), ([0, 1], [0, 1], \ldots, [0, 1]))$

3.4. *m-polar Interval-valued Neutrosophic Absolute Set*

Definition 3.4 An m-polar Interval-valued Neutrosophic Absolute Set $\tilde{U}$ on universal set $\ddot{U}$ is defined as

$\tilde{U} = \{u, (([0, 1], [0, 1], \ldots, [0, 1]), ([0, 1], [0, 1], \ldots, [0, 1]), ([1, 0], [1, 0], \ldots, [1, 0]))$

3.5. *Union of m-polar Interval-valued Neutrosophic Set*

Definition 3.5 Let $\ddot{A}$ and $\ddot{B}$ be two m-polar IVNS over a same universe $\ddot{U}$ then the union of $\ddot{A}$ and $\ddot{B}$ defined as $\ddot{A}\tilde{\cup}\ddot{B} = \ddot{C}$ where $\ddot{C} = \{u, ({}^I\phi^i_{\ddot{C}}, {}^I\psi^i_{\ddot{C}}, {}^I\eta^i_{\ddot{C}}) \mid u \in \ddot{U}\}$ such that

$$\begin{gathered}
{}^I\phi^i_{\ddot{C}} = [\inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}), \sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}})], \\
{}^I\psi^i_{\ddot{C}} = [\inf(\psi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}}), \sup(\psi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}})], \\
{}^I\eta^i_{\ddot{C}} = [\sup(\eta^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}), \inf(\eta^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}})], \\
\text{for all } i = 1, 2, \ldots, m
\end{gathered}$$

3.6. *Intersection of m-polar Interval-valued Neutrosophic Set*

Definition 3.6 Let $\ddot{A}$ and $\ddot{B}$ be two m-polar IVNS over a same universe $\ddot{U}$ then the union of $\ddot{A}$ and $\ddot{B}$ defined as $\ddot{A}\tilde{\cap}\ddot{B} = \ddot{C}$ where $\ddot{C} = \{u, ({}^I\phi^i_{\ddot{C}}, {}^I\psi^i_{\ddot{C}}, {}^I\eta^i_{\ddot{C}}) \mid u \in \ddot{U}\}$ such that

$$\begin{gathered}
{}^I\phi^i_{\ddot{C}} = [\sup(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}), \inf(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}})], \\
{}^I\psi^i_{\ddot{C}} = [\sup(\psi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}}), \inf(\psi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}})], \\
{}^I\eta^i_{\ddot{C}} = [\inf(\eta^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}), \sup(\eta^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}})], \\
\text{for all } i = 1, 2, \ldots, m
\end{gathered}$$

3.7. *Complement of m-polar Interval-valued Neutrosophic Set*

Definition 3.7 Let $\ddot{U}$ be a universal set, and $\ddot{A}$ be m-polar IVNS over universe $\ddot{U}$ then the complement of $\ddot{A}$ denoted by $\ddot{A}^c$ and defined as

$$\ddot{A}^c = \{u, ({}^I\eta^i_{\ddot{A}}, [0-1] - {}^I\psi^i_{\ddot{A}}, {}^I\phi^i_{\ddot{A}} \mid u \in \ddot{U}\}$$
$$\text{for all } i = 1, 2, \ldots, m$$

Example 3.2

Let $\ddot{U} = \{u_1, u_2, u_3\}$ be a universal set and we define two 3-polar IVNS $\ddot{A}$ and $\ddot{B}$ over universe $\ddot{U}$ as,

$$\ddot{A} = \{u_1, (([0.2, 0.6], [0.3, 0.5], [0.6, 1]), ([0.2, 0.4], [0, 0.3], [0.3, 0.6]), ([0.4, 0.7], [0.3, 1], [0.2, 0.4])),$$
$$u_2, (([0.3, 0.6], [0.3, 0.7], [0.2, 0.5]), ([0.5, 0.6], [0.6, 1], [0.6, 0.9]), ([0.5, 0.7], [0.6, 0.9], [0.4, 0.9])),$$
$$u_3, (([0.4, 0.7], [0.5, 0.9], [0.6, 0.8]), ([0.7, 0.9], [0.7, 0.9], [0.1, 0.4]), ([0.3, 0.6], [0.4, 0.5], [0.2, 0.5]))\}$$

and

$$\ddot{B} = \{u_1, (([0.3, 0.5], [0.4, 0.7], [0, 0.6]), ([0.2, 0.4], [0, 0.1], [0.3, 0.6]), ([0, 0.4], [0.1, 0.4], [0.2, 0.3])),$$
$$u_2, (([0.3, 0.4], [0.2, 0.5], [0.5, 1]), ([0.6, 0.7], [0.7, 0.9], [0.4, 0.6]), ([0.5, 0.5], [0.7, 0.9], [0.8, 1])),$$
$$u_3, (([0.1, 0.4], [0.2, 0.7], [0.2, 0.6]), ([0.5, 0.8], [0.4, 0.7], [0.4, 0.6]), ([0.2, 0.4], [0.3, 0.6], [0.3, 0.5]))\}$$

then

$$\ddot{A}\tilde{\cup}\ddot{B} = \{u_1, (([0.2, 0.6], [0.3, 0.7], [0, 1]), ([0.2, 0.4], [0, 0.3], [0.3, 0.6]), ([0.4, 0.4], [0.3, 0.4], [0.2, 0.3])),$$
$$u_2, (([0.3, 0.6], [0.2, 0.7], [0.2, 0.1]), ([0.5, 0.7], [0.6, 1], [0.4, 0.9]), ([0.5, 0.5], [0.7, 0.9], [0.8, 0.9])),$$
$$u_3, (([0.1, 0.7], [0.2, 0.9], [0.2, 0.8]), ([0.5, 0.9], [0.4, 0.9], [0.1, 0.6]), ([0.3, 0.4], [0.4, 0.5], [0.3, 0.5]))\}$$

$$\ddot{A}\tilde{\cap}\ddot{B} = \{u_1, (([0.3, 0.5], [0.4, 0.5], [0.6, 0.6]), ([0.2, 0.4], [0, 0.1], [0.3, 0.6]), ([0, 0.7], [0.1, 1], [0.2, 0.4])),$$
$$u_2, (([0.3, 0.4], [0.3, 0.5], [0.5, 0.5]), ([0.6, 0.6], [0.7, 0.9], [0.6, 0.6]), ([0.5, 0.7], [0.6, 0.9], [0.4, 1])),$$
$$u_3, (([0.4, 0.4], [0.5, 0.7], [0.6, 0.6]), ([0.7, 0.8], [0.7, 0.7], [0.4, 0.4]), ([0.2, 0.6], [0.3, 0.6], [0.2, 0.5]))\}$$

$$\ddot{A}^c = \{u_1, (([0.4, 0.7], [0.3, 1], [0.2, 0.4]), ([0, 0.2) \cup (0.4, 1], (0.3, 1], [0, 0.3) \cup (0.6, 1]), ([0.2, 0.6], [0.3, 0.5], [0.6, 1])),$$
$$u_2, (([0.5, 0.7], [0.6, 0.9], [0.4, 0.9]), ([0, 0.5) \cup (0.6, 1], [0, 0.6), [0, 0.6) \cup (0.9, 1]), ([0.3, 0.6], [0.3, 0.7], [0.2, 0.5])),$$
$$u_3, (([0.3, 0.6], [0.4, 0.5], [0.2, 0.5]), ([0, 0.7) \cup (0.9, 1], [0, 0.7) \cup (0.9, 1], [0, 0.1) \cup (0.4, 1]), ([0.4, 0.7], [0.5, 0.9], [0.6, 0.8]))$$

4. Properties on m-polar IVNS set Operators

4.1. *Idempotent Laws*

(i) $\ddot{A}\tilde{\cap}\ddot{A} = \ddot{A}$

(ii) $\ddot{A}\tilde{\cup}\ddot{A} = \ddot{A}$

4.2. *Identity Laws*

(iii) $\ddot{A}\tilde{\cup}\ddot{\Phi} = \ddot{A} = \ddot{\Phi}\tilde{\cup}\ddot{A}$

(iv) $\ddot{A}\tilde{\cap}\tilde{U} = \ddot{A} = \tilde{U}\tilde{\cap}\ddot{A}$

4.3. *Domination Laws*

(v) $\ddot{A}\tilde{\cap}\ddot{\Phi} = \ddot{\Phi} = \ddot{\Phi}\tilde{\cap}\ddot{A}$

(vi) $\ddot{A}\tilde{\cup}\tilde{U} = \tilde{U} = \tilde{U}\tilde{\cup}\ddot{A}$

4.4. *Complement Laws*

(vii) $\ddot{\Phi}^c = \tilde{U}$

(viii) $\tilde{U}^c = \ddot{\Phi}$

4.5. *Double Complementation Law*

(ix) $(\ddot{A}^c)^c = \ddot{A}$

4.6. *Commutative Laws*

(x) $\ddot{A}\tilde{\cup}\ddot{B} = \ddot{B}\tilde{\cup}\ddot{A}$

(xi) $\ddot{A}\tilde{\cap}\ddot{B} = \ddot{B}\tilde{\cap}\ddot{A}$

4.7. *Associative Laws*

(xii) $\ddot{A}\tilde{\cup}(\ddot{B}\tilde{\cup}\ddot{C}) = (\ddot{A}\tilde{\cup}\ddot{B})\tilde{\cup}\ddot{C}$

(xiii) $\ddot{A}\tilde{\cap}(\ddot{B}\tilde{\cap}\ddot{C}) = (\ddot{A}\tilde{\cap}\ddot{B})\tilde{\cap}\ddot{C}$

4.8. *Distributive Laws*

(xiv) $\ddot{A}\tilde{\cup}(\ddot{B}\tilde{\cap}\ddot{C}) = (\ddot{A}\tilde{\cup}\ddot{B})\tilde{\cap}(\ddot{A}\tilde{\cup}\ddot{C})$

(xv) $\ddot{A}\tilde{\cap}(\ddot{B}\tilde{\cup}\ddot{C}) = (\ddot{A}\tilde{\cap}\ddot{B})\tilde{\cup}(\ddot{A}\tilde{\cap}\ddot{C})$

4.9. *De morgan's Laws*

(xvi) $(\ddot{A}\tilde{\cup}\ddot{B})^c = \ddot{A}^c\tilde{\cap}\ddot{B}^c$

(xvii) $(\ddot{A}\tilde{\cap}\ddot{B})^c = \ddot{A}^c\tilde{\cup}\ddot{B}^c$

The Proof of Commutative Laws, Associative Laws, Distributive Laws, and De Morgan's Laws are presented in this paper. They are the following:

Proof(x)

$$\ddot{A}\tilde{\cup}\ddot{B} = \{u, ([\inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}), \sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}})], [\inf(\psi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}}), \sup(\psi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}})], [\sup(\eta^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}), \inf(\eta^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}})])$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}\tilde{\cup}\ddot{B} = \{u, ([\inf(\phi^{i-}_{\ddot{B}}, \phi^{i-}_{\ddot{A}}), \sup(\phi^{i+}_{\ddot{B}}, \phi^{i+}_{\ddot{A}})], [\inf(\psi^{i-}_{\ddot{B}}, \psi^{i-}_{\ddot{A}}), \sup(\psi^{i+}_{\ddot{B}}, \psi^{i+}_{\ddot{A}})], [\sup(\eta^{i-}_{\ddot{B}}, \eta^{i-}_{\ddot{A}}), \inf(\eta^{i+}_{\ddot{B}}, \eta^{i+}_{\ddot{A}})])$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}\tilde{\cup}\ddot{B} = \ddot{B}\tilde{\cup}\ddot{A}$$

Proof(xii)

$$\ddot{A}\tilde{\cup}(\ddot{B}\tilde{\cup}\ddot{C}) = \{u, ([\inf(\phi^{i-}_{\ddot{A}}, \inf(\phi^{i-}_{\ddot{B}}, \phi^{i-}_{\ddot{C}})), \sup(\phi^{i+}_{\ddot{A}}, \sup(\phi^{i+}_{\ddot{B}}, \phi^{i+}_{\ddot{C}}))],$$
$$[\inf(\phi^{i-}_{\ddot{A}}, \inf(\psi^{i-}_{\ddot{B}}, \psi^{i-}_{\ddot{C}})), \sup(\phi^{i+}_{\ddot{A}}, \sup(\psi^{i+}_{\ddot{B}}, \psi^{i+}_{\ddot{C}}))], [\sup(\phi^{i-}_{\ddot{A}}, \sup(\eta^{i-}_{\ddot{B}}, \eta^{i-}_{\ddot{C}})), \inf(\phi^{i+}_{\ddot{A}}, \inf(\eta^{i+}_{\ddot{B}}, \eta^{i+}_{\ddot{C}}))])$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}\tilde{\cup}(\ddot{B}\tilde{\cup}\ddot{C}) = \{u, ([\inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}, \phi^{i-}_{\ddot{C}}), \sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}}, \phi^{i+}_{\ddot{C}})],$$
$$[\inf(\phi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}}, \psi^{i-}_{\ddot{C}}), \sup(\phi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}}, \psi^{i+}_{\ddot{C}})], [\sup(\phi^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}, \eta^{i-}_{\ddot{C}}), \inf(\phi^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}}, \eta^{i+}_{\ddot{C}})])$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}\tilde{\cup}(\ddot{B}\tilde{\cup}\ddot{C}) = \{u, ([\inf(\inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}), \phi^{i-}_{\ddot{C}}), \sup(\sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}}), \phi^{i+}_{\ddot{C}})],$$
$$[\inf(\inf(\phi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}}), \psi^{i-}_{\ddot{C}})), \sup(\sup(\phi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}}), \psi^{i+}_{\ddot{C}}))], [\sup(\sup(\phi^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}), \eta^{i-}_{\ddot{C}})), \inf(\inf(\phi^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}}), \eta^{i+}_{\ddot{C}}))])$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}\tilde{\cup}(\ddot{B}\tilde{\cup}\ddot{C}) = (\ddot{A}\tilde{\cup}\ddot{B})\tilde{\cup}\ddot{C}$$

Proof(xiv)

$$\ddot{A}\tilde{\cup}(\ddot{B}\tilde{\cap}\ddot{C}) = \{u, ([\inf(\phi^{i-}_{\ddot{A}}, \sup(\phi^{i-}_{\ddot{B}}, \phi^{i-}_{\ddot{C}})), \sup(\phi^{i+}_{\ddot{A}}, \inf(\phi^{i+}_{\ddot{B}}, \phi^{i+}_{\ddot{C}}))],$$
$$[\inf(\phi^{i-}_{\ddot{A}}, \sup(\psi^{i-}_{\ddot{B}}, \psi^{i-}_{\ddot{C}})), \sup(\phi^{i+}_{\ddot{A}}, \inf(\psi^{i+}_{\ddot{B}}, \psi^{i+}_{\ddot{C}}))], [\sup(\phi^{i-}_{\ddot{A}}, \inf(\eta^{i-}_{\ddot{B}}, \eta^{i-}_{\ddot{C}})), \inf(\phi^{i+}_{\ddot{A}}, \sup(\eta^{i+}_{\ddot{B}}, \eta^{i+}_{\ddot{C}}))])$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}\tilde{\cup}(\ddot{B}\tilde{\cap}\ddot{C}) = \{u, ([\sup(\inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}), \inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{C}})), \inf(\sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}}, \sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{C}}))],$$
$$[\sup(\inf(\psi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}}), \inf(\psi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{C}})), \inf(\sup(\psi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}}), \sup(\psi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{C}}))],$$
$$[\inf(\sup(\eta^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}), \sup(\eta^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{C}})), \sup(\inf(\eta^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}}), \inf(\eta^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{C}}))])$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}\tilde{\cup}(\ddot{B}\tilde{\cap}\ddot{C}) = (\ddot{A}\tilde{\cup}\ddot{B})\tilde{\cap}(\ddot{A}\tilde{\cup}\ddot{C})$$

Proof(xvi)

$$\ddot{A}^c\tilde{\cap}\ddot{B}^c = \{u, [\eta^{i-}_{\ddot{A}}, \eta^{i+}_{\ddot{A}}], [0,1] - [\psi^{i-}_{\ddot{A}}, \psi^{i+}_{\ddot{A}}], [\phi^{i-}_{\ddot{A}}, \phi^{i+}_{\ddot{A}}]\}\tilde{\cap}\{u, [\eta^{i-}_{\ddot{B}}, \eta^{i+}_{\ddot{B}}], [0,1] - [\psi^{i-}_{\ddot{B}}, \psi^{i+}_{\ddot{B}}], [\phi^{i-}_{\ddot{B}}, \phi^{i+}_{\ddot{B}}]\}$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}^c\tilde{\cap}\ddot{B}^c = \{u, [\eta^{i-}_{\ddot{A}}, \eta^{i+}_{\ddot{A}}], [0, \psi^{i-}_{\ddot{A}}) \cup (\psi^{i+}_{\ddot{A}}, 1], [\phi^{i-}_{\ddot{A}}, \phi^{i+}_{\ddot{A}}]\}\tilde{\cap}\{u, [\eta^{i-}_{\ddot{B}}, \eta^{i+}_{\ddot{B}}], [0, \psi^{i-}_{\ddot{B}}) \cup (\psi^{i+}_{\ddot{B}}, 1], [\phi^{i-}_{\ddot{B}}, \phi^{i+}_{\ddot{B}}]\}$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}^c\tilde{\cap}\ddot{B}^c = \{u, [\sup(\eta^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}), \inf(\eta^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}})], [\sup(0,0), \inf(\psi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}})) \cup$$
$$(\sup(\psi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}}), \inf(1,1)], [\inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}), \sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}})]\}$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}^c\tilde{\cap}\ddot{B}^c = \{u, ([\sup(\eta^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}), \inf(\eta^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}})], [0, \inf(\psi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}})) \cup (\sup(\psi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}}), 1], [\inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}), \sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}})])$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}^c\tilde{\cap}\ddot{B}^c = \{u, ([\sup(\eta^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}), \inf(\eta^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}})], [0,1] - [\inf(\psi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}}), \sup(\psi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}})], [\inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}), \sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}})])$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}^c\tilde{\cap}\ddot{B}^c = (\{u, ([\inf(\phi^{i-}_{\ddot{A}}, \phi^{i-}_{\ddot{B}}), \sup(\phi^{i+}_{\ddot{A}}, \phi^{i+}_{\ddot{B}})], [\inf(\psi^{i-}_{\ddot{A}}, \psi^{i-}_{\ddot{B}}), \sup(\psi^{i+}_{\ddot{A}}, \psi^{i+}_{\ddot{B}})], [\sup(\eta^{i-}_{\ddot{A}}, \eta^{i-}_{\ddot{B}}), \inf(\eta^{i+}_{\ddot{A}}, \eta^{i+}_{\ddot{B}})]\}))^c$$
$$\text{for all } i = 1, 2, \ldots, m$$
$$\ddot{A}^c\tilde{\cap}\ddot{B}^c = (\ddot{A}\tilde{\cup}\ddot{B})^c$$

Similarly, Other Laws can be proved.

5. Distance measure for m-polar Interval-valued Neutrosophic Sets

5.1. *Distances*

Let $\ddot{A}$ and $\ddot{B}$ be two m-polar Interval-valued Neutrosophic Sets corresponds to a universal set $\ddot{U} = \{u_1, u_2, \ldots, u_n\}$ such that

$$\ddot{A} = \{u_j, ([\phi^{i-}_{\ddot{A}}(u_j), \phi^{i+}_{\ddot{A}}(u_j)], [\psi^{i-}_{\ddot{A}}(u_j), \psi^{i+}_{\ddot{A}}(u_j), [\eta^{i-}_{\ddot{A}}(u_j), \eta^{i+}_{\ddot{A}}(u_j)]\} \text{ and}$$
$$\ddot{B} = \{u_j, ([\phi^{i-}_{\ddot{B}}(u_j), \phi^{i+}_{\ddot{B}}(u_j)], [\psi^{i-}_{\ddot{B}}(u_j), \psi^{i+}_{\ddot{B}}(u_j), [\eta^{i-}_{\ddot{B}}(u_j), \eta^{i+}_{\ddot{B}}(u_j)]\}$$
$$\text{for all } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n$$

then distances between $\ddot{A}$ and $\ddot{B}$ is defined as;

(1) Hamming distance:

$$H|\ddot{A}, \ddot{B}| = \frac{1}{3m} \sum_{i=1}^{m} \sum_{j=1}^{n} \{|\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j)| + |\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j)| + |\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j)|\} \quad (1)$$

(2) Normalized Hamming distance

$$NH|\ddot{A}, \ddot{B}| = \frac{1}{3mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \{|\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j)| + |\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j)| + |\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j)|\} \quad (2)$$

(3) Euclidean distance

$$E|\ddot{A}, \ddot{B}| = (\frac{1}{3m} \sum_{i=1}^{m} \sum_{j=1}^{n} \{(\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j))^2 + (\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j))^2 + (\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j))^2\})^{\frac{1}{2}} \quad (3)$$

(4) Normalized Euclidean distance

$$NE|\ddot{A}, \ddot{B}| = \frac{1}{3mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \{(\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j))^2 + (\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j))^2 + (\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j))^2\})^{\frac{1}{2}} \quad (4)$$

where

$$\begin{aligned} \bar{\phi}^i_{\ddot{A}}(u_j) &= \tfrac{\phi^{i-}_{\ddot{A}}(u_j) + \phi^{i+}_{\ddot{A}}(u_j)}{2}, & \quad & & \bar{\phi}^i_{\ddot{B}}(u_j) &= \tfrac{\phi^{i-}_{\ddot{B}}(u_j) + \phi^{i+}_{\ddot{B}}(u_j)}{2}, \\ \bar{\psi}^i_{\ddot{A}}(u_j) &= \tfrac{\psi^{i-}_{\ddot{A}}(u_j) + \psi^{i+}_{\ddot{A}}(u_j)}{2}, & & \text{and} & \bar{\psi}^i_{\ddot{B}}(u_j) &= \tfrac{\psi^{i-}_{\ddot{B}}(u_j) + \psi^{i+}_{\ddot{B}}(u_j)}{2}, \\ \bar{\eta}^i_{\ddot{A}}(u_j) &= \tfrac{\eta^{i-}_{\ddot{A}}(u_j) + \eta^{i+}_{\ddot{A}}(u_j)}{2} & & & \bar{\eta}^i_{\ddot{B}}(u_j) &= \tfrac{\eta^{i-}_{\ddot{B}}(u_j) + \eta^{i+}_{\ddot{B}}(u_j)}{2} \end{aligned}$$
$$\text{for all } i = 1, 2, \ldots, m, \text{ and } j = 1, 2, \ldots, n.$$

Theorem 5.1 The distance between two mIVNS $\ddot{A}$ and $\ddot{B}$ satisfy the following inequalities;

(1) $H|\ddot{A}, \ddot{B}| \leq n$,
(2) $NH|\ddot{A}, \ddot{B} \leq 1$,
(3) $E|\ddot{A}, \ddot{B}| \leq \sqrt{n}$,
(4) $NE|\ddot{A}, \ddot{B}| \leq 1$

Theorem 5.2 Distance between any two mIVNS $\ddot{A}$ and $\ddot{B}$ is a metric distance

Proof

(i)$H|\ddot{A}, \ddot{B}| \geq 0$

(ii)$H|\ddot{A}, \ddot{B}| = 0$

$$\Leftrightarrow \frac{1}{3m} \sum_{i=1}^{m} \sum_{j=1}^{n} \{|\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j)| + |\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j)| + |\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j)|\} = 0$$

$$\Leftrightarrow |\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j)| + |\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j)| + |\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j)| = 0$$

for all $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

$$\Leftrightarrow |\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j)| = 0,$$
$$|\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j)| = 0,$$
$$|\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j)| = 0$$

for all $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

$$\Leftrightarrow \bar{\phi}^i_{\ddot{A}}(u_j) = \bar{\phi}^i_{\ddot{B}}(u_j),$$
$$\bar{\psi}^i_{\ddot{A}}(u_j) = \bar{\psi}^i_{\ddot{B}}(u_j),$$
$$\bar{\eta}^i_{\ddot{A}}(u_j) = \bar{\eta}^i_{\ddot{B}}(u_j)$$

for all $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

$$\Leftrightarrow \ddot{A} = \ddot{B}$$

(iii)$H|\ddot{A}, \ddot{B}| = H|\ddot{B}, \ddot{A}|$

(iv) For any three sets $\ddot{A}$, $\ddot{B}$, and $\ddot{C}$

$$|\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j)| + |\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j)| + |\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j)|$$

for all $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

$$= |\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{C}}(u_j) + \bar{\phi}^i_{\ddot{C}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j)|$$
$$+|\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{C}}(u_j) + \bar{\psi}^i_{\ddot{C}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j)|$$
$$+|\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{C}}(u_j) + \bar{\eta}^i_{\ddot{C}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j)|$$

for all $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

$$\leq |\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{C}}(u_j)| + |\bar{\phi}^i_{\ddot{C}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j)|$$
$$+|\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{C}}(u_j)| + |\bar{\psi}^i_{\ddot{C}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j)|$$
$$+|\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{C}}(u_j)| + |\bar{\eta}^i_{\ddot{C}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j)|$$

for all $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

$$= |\bar{\phi}^i_{\ddot{A}}(u_j) - \bar{\phi}^i_{\ddot{C}}(u_j)| + |\bar{\psi}^i_{\ddot{A}}(u_j) - \bar{\psi}^i_{\ddot{C}}(u_j)| + |\bar{\eta}^i_{\ddot{A}}(u_j) - \bar{\eta}^i_{\ddot{C}}(u_j)|$$
$$+|\bar{\phi}^i_{\ddot{C}}(u_j) - \bar{\phi}^i_{\ddot{B}}(u_j)| + |\bar{\psi}^i_{\ddot{C}}(u_j) - \bar{\psi}^i_{\ddot{B}}(u_j)| + |\bar{\eta}^i_{\ddot{C}}(u_j) - \bar{\eta}^i_{\ddot{B}}(u_j)|$$

for all $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

$$H|\ddot{A}, \ddot{B}| \leq H|\ddot{A}, \ddot{C}| + H|\ddot{C}, \ddot{B}|$$

5.2. *Similarity Measure*

Definition 5.2 [31] Similarity measures between two mIVNS is defined as

$$SM(\ddot{A}, \ddot{B}) = \frac{1}{1 + |\ddot{A}, \ddot{B}|} \tag{5}$$

where $|\ddot{A}, \ddot{B}|$ is any distance that are discussed above.

Lemma 5.3 Consider two mIVNS $\ddot{A}$ and $\ddot{B}$ corresponds to universal set $\ddot{U}$ then we have the following properties

- $SM(\ddot{A}, \ddot{B}) = SM(\ddot{B}, \ddot{A})$
- $0 \leq SM(\ddot{A}, \ddot{B}) \leq 1$
- $SM(\ddot{A}, \ddot{B}) = 1$ iff $\ddot{A} = \ddot{B}$

5.3. *Similarity*

Definition 5.3 [31] Consider $N(U)$ be the set of all mIVNS corresponds to $\ddot{U}$. Suppose $\ddot{A}$ and $\ddot{B} \in N(U)$. If $SM(\ddot{A}, \ddot{B} \geq \breve{\alpha}, \breve{\alpha} \in [0, 1]$ then two $\ddot{A}$ and $\ddot{B}$ are said to be $\breve{\alpha}$ similar and we denote the relation $\ddot{A} \cong_{\breve{\alpha}} \ddot{B}$.

Lemma 5.4 Two mIVNS $\ddot{A}$ and $\ddot{B}$ are said to be significantly similar if

$$SM(\ddot{A}, \ddot{B}) > \frac{1}{2}$$

6. **Case Study**

Similarity Measure is well-known criteria for the solution of Decision Making Problems. The use of similarity measure can be very helpful in the selection of the best alternative. This criteria is considered the best tool for the comparison of two objects, set, and pattern. While it will better to compare some set with an ideal set to find the similarity with the idealness. This method will conclude that at which stage (from 0 to 1) an object can be ranked, which helps in ranking analysis and selection problem. From the ranking an object, a candidate can be selected more accurately.

In Sports, every game is important for a team, the selection of players for a team can affect the result of a game, so it is mandatory for the team, coach, and technique director to select good and excellent players for a game. Following is the algorithm of our proposed method, also shown in Figure 2.

6.1. *Algorithm of Decision Making on mIVNS structure*

- Construct set of attribute $\ddot{U} = \{u_1, u_2, \ldots, u_n\}$ as n number of attribute of players.
- Construct t m-polar IVNS $\ddot{A}_k$, $(k = 1, 2, \ldots, t)$ corresponds to $\ddot{U}$ based on previous m matches records of t Players

- Construct best ideal performance m-polar IVNS $\ddot{B}$ corresponds to $\ddot{U}$, that is $\ddot{B} = \{u_j, ([1,1],[1,1],\ldots,[1,1]),([0,0],[0,0],\ldots,[0,0]),([0,0],[0,0],\ldots,[0,0])\}$ where $j = 1,2,\ldots,n$
- Calculate distance between each $\ddot{A}_k$ and $\ddot{B}$ using Euclidean distance formula $E|\ddot{A},\ddot{B}| = (\frac{1}{3m}\sum_{i=1}^{m}\sum_{j=1}^{n}\{(\bar{\phi}^i_{\ddot{A}}(u_j)-\bar{\phi}^i_{\ddot{B}}(u_j))^2+(\bar{\psi}^i_{\ddot{A}}(u_j)-\bar{\psi}^i_{\ddot{B}}(u_j))^2+(\bar{\eta}^i_{\ddot{A}}(u_j)-\bar{\eta}^i_{\ddot{B}}(u_j))^2\})^{\frac{1}{2}}$
- Calculating the similarity measures between two m-polar IVNS by using the formula $SM(\ddot{A},\ddot{B}) = \frac{1}{1+E|\ddot{A},\ddot{B}|}$
- Arrange the similarity measure results in descending order as to rank players for next match.

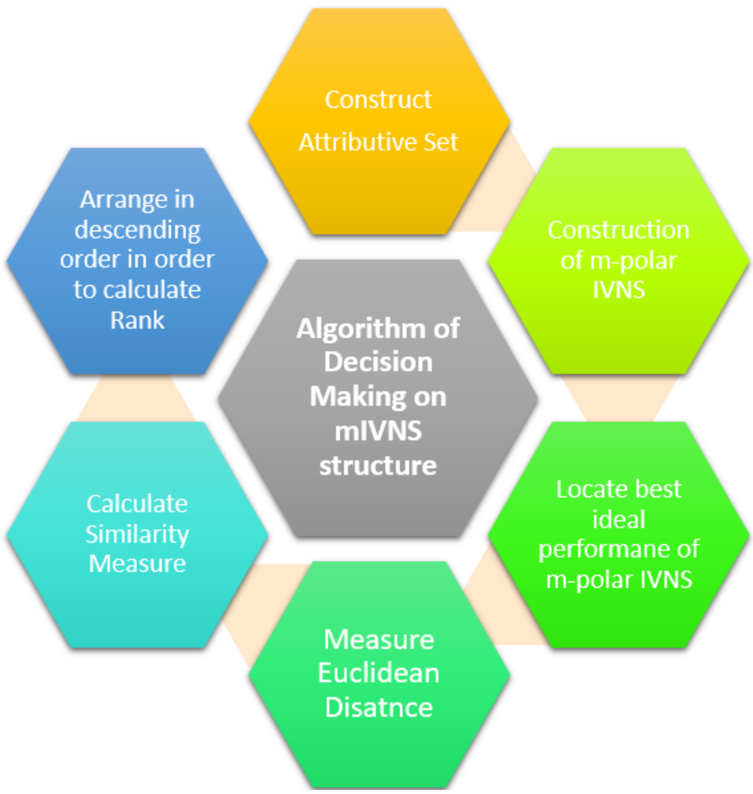

FIGURE 2. Pictorial view of Algorithm

6.2. *Limitation of the Method*

There are several limitations of the method that must be assured before implementing the similarity measure criteria.

(1) Similarity measure can be found between two sets at a time to find comparison among themselves.
(2) The two sets must be independent of each other and must be from the same structure.

6.3. *Problem Formulation and Assumption*

Here we illustrate a similarity measure for the selection of the best three forward players combination of Paris Saint-Germain Club (Football) as looking at previous matches records of forward players who played 90 minutes game. Since PSG is very strong in attacking and they have very good players in forward, mid and defending positions. But the PSG club is weak in scoring the good goals and misses good opportunities of scoring goals. In this paper, we took previous records of some matches of all forward players of PSG who played 90 minutes game for our decision making problem, we illustrate similarity measure based on Euclidean distance for the best 3 forward player combination.

6.4. *Calculation*

The data of previous matches are collected in distinct attributes (quality of and best ideal forward players), attribute set of players is defined as $\ddot{U}$ such that,

$$\ddot{U} = \{u_1, u_2, u_3, u_4\}$$

where u_1 represents "shooting ability", u_2 represents "first touch ability", u_3 represents "speed", and u_4 represents "switch with other forward players",.
Although PSG has six forward players that are Edinson Cavani, Kylian Mbappe, Neymar Jr., Mauro Icardi, Pablo Sarabia, and Eric Maxim Chupo-Moting, but mostly three of them plays a match on the field.
Now we took data from the site of the last 3 matches played by six players mentioned above corresponds to attribute set $\ddot{U}$, then we construct six different 3-polar IVNS $\ddot{A}_1$, $\ddot{A}_2$, $\ddot{A}_3$, $\ddot{A}_4$, $\ddot{A}_5$, and $\ddot{A}_6$, shown in Table 1 where these sets represents the last 3 matches performances of Edinson Cavani, Kylian Mbappe, Neymar Jr., Mauro Icardi, Pablo Sarabia, and Eric Maxim Chupo-Moting,respectively,

We take the best ideal performance of any forward player as mIVNS $\ddot{B}$, that is

$$\ddot{B} = \{u_1, (([1,1][1,1][1,1]), ([0,0][0,0][0,0]), ([0,0][0,0][0,0])), u_2, (([1,1][1,1][1,1]), ([0,0][0,0][0,0]), ([0,0][0,0][0,0])), u_3, (([1,1][1,1][1,1]), ([0,0][0,0][0,0]), ([0,0][0,0][0,0])), u_4, (([1,1][1,1][1,1]), ([0,0][0,0][0,0]), ([0,0][0,0][0,0]))\}$$

Now We calculate the Euclidean distance between $\ddot{A}_k$, $(k = 1,2,3,4,5,6)$ and $\ddot{B}$, and then the Similarity measure is calculated as shown in Table 2.

Now we can easily rank the players as $\ddot{A}_3 \succ \ddot{A}_2 \succ \ddot{A}_4 \succ \ddot{A}_1 \succ \ddot{A}_5 \succ \ddot{A}_6$ as highest to lowest value from Table 2, so it shows that Neymar Jr., Kylian Mbappe, and Mauro Icardi are three key players for the very next match as they had a good performance in last 3 matches.

7. **Discussion**

Similarity Measure is a well-known tool for the evaluation of comparisons and finding the similarity between two objects, patterns, and sets. This tool is applicable in every structure

	u_1	u_2	u_3	u_4
$\ddot{A}_1$	([0.71,0.76][0.83,0.90][0.80,0.85]), ([0.23,0.30][0.14,0.22][0.19,0.23]), ([0.30,0.35][0.17,0.20][0.30,0.32])	([0.57,0.63][0.65,0.72][0.62,0.67]), ([0.32,0.38][0.40,0.43][0.36,0.40]), ([0.32,0.39][0.42,0.52][0.41,0.49])	([0.74,0.80][0.75,0.80][0.70,0.76]), ([0.23,0.30][0.17,0.22][0.25,0.29]), ([0.27,0.34][0.19,0.28][0.22,0.30])	([0.50,0.57][0.52,0.60][0.50,0.55]), ([0.42,0.48][0.43,0.50][0.48,0.55]), ([0.50,0.53][0.49,0.55][0.55,0.60])
$\ddot{A}_2$	([0.90,0.97][0.87,0.92][0.86,0.91]), ([0.10,0.18][0.16,0.22][0.20,0.24]), ([0.20,0.24][0.23,0.30][0.18,0.25])	([0.82,0.87][0.84,0.90][0.87,0.93]), ([0.12,0.20][0.20,0.25][0.13,0.20]), ([0.25,0.32][0.18,0.24][0.30,0.33])	([0.60,0.66][0.58,0.64][0.53,0.60]), ([0.33,0.40][0.38,0.45][0.34,0.40]), ([0.38,0.40][0.40,0.50][0.43,0.52])	([0.75,0.80][0.70,0.76][0.68,0.75]), ([0.23,0.30][0.30,0.35][0.33,0.39]), ([0.28,0.34][0.30,0.36][0.39,0.46])
$\ddot{A}_3$	([0.94,0.99][0.96,1][0.95,1]), ([0.13,0.20][0.14,0.22][0.18,0.24]), ([0.20,0.26][0.23,0.27][0.19,0.26])	([0.95,1][0.96,1][0.95,0.98]), ([0.20,0.22][0.15,0.21][0.17,0.23]), ([0.20,0.26][0.21,0.25][0.23,0.29])	([0.65,0.70][0.68,0.76][0.74,0.80]), ([0.39,0.47][0.41,0.46][0.38,0.42]), ([0.40,0.47][0.42,0.50][0.46,0.54])	([0.90,0.95][0.96,1][0.98,1]), ([0.11,0.15][0.14,0.19][0.16,0.24]), ([0.30,0.33][0.25,0.30][0.20,0.30])
$\ddot{A}_4$	([0.78,0.84][0.78,0.84][0.83,0.89]), ([0.20,0.25][0.22,0.29][0.14,0.20]), ([0.34,0.40][0.33,0.35][0.23,0.27])	([0.58,0.64][0.60,0.65][0.61,0.67]), ([0.30,0.36][0.28,0.30][0.35,0.40]), ([0.40,0.45][0.39,0.43][0.33,0.37])	([0.65,0.71][0.63,0.68][0.60,0.67]), ([0.50,0.54][0.34,0.40][0.40,0.47]), ([0.29,0.35][0.31,0.39][0.35,0.40])	([0.54,0.60][0.54,0.59][0.57,0.60]), ([0.37,0.40][0.39,0.44][0.42,0.46]), ([0.43,0.45][0.40,0.44][0.30,0.33])
$\ddot{A}_5$	([0.74,0.80][0.70,0.76][0.70,0.76]), ([0.20,0.24][0.23,0.28][0.30,0.34]), ([0.23,0.31][0.25,0.31][0.30,0.34])	([0.60,0.68][0.58,0.65][0.56,0.62]), ([0.34,0.39][0.38,0.45][0.40,0.48]), ([0.32,0.35][0.45,0.49][0.37,0.42])	([0.50,0.57][0.38,0.45][0.37,0.42]), ([0.37,0.40][0.44,0.51][0.38,0.48]), ([0.41,0.44][0.50,0.54][0.59,0.63])	([0.73,0.82][0.75,0.80][0.80,0.85]), ([0.30,0.32][0.17,0.25][0.22,0.26]), ([0.20,0.26][0.20,0.26][0.19,0.22])
$\ddot{A}_6$	([0.64,0.70][0.62,0.70][0.69,0.74]), ([0.31,0.37][0.28,0.33][0.22,0.29]), ([0.21,0.25][0.23,0.25][0.24,0.30])	([0.60,0.69][0.59,0.65][0.61,0.69]), ([0.20,0.30][0.34,0.40][0.31,0.40]), ([0.32,0.38][0.30,0.34][0.34,0.37])	([0.48,0.54][0.40,0.48][0.40,0.46]), ([0.51,0.55][0.52,0.58][0.48,0.50]), ([0.42,0.45][0.48,0.53][0.50,0.52])	([0.42,0.50][0.55,0.62][0.65,0.70]), ([0.52,0.57][0.43,0.47][0.47,0.53]), ([0.39,0.44][0.38,0.42][0.45,0.50])

TABLE 1. Represents six 3−IVNS $\ddot{A}_1$, $\ddot{A}_2$, $\ddot{A}_3$, $\ddot{A}_4$, $\ddot{A}_5$, and $\ddot{A}_6$

	Distance Measure	Similarity Measure (SM)	Rank
$(\ddot{A}_1, \ddot{B})$	0.2393	0.806	4
$(\ddot{A}_2, \ddot{B})$	0.1943	0.837	2
$(\ddot{A}_3, \ddot{B})$	0.1690	0.855	1
$(\ddot{A}_4, \ddot{B})$	0.2383	0.807	3
$(\ddot{A}_5, \ddot{B})$	0.2459	0.802	5
$(\ddot{A}_6, \ddot{B})$	0.2747	0.784	6

TABLE 2. Represents Distance Measures and Similarity Measures

of fuzzy, intuitionistic fuzzy, neutrosophic, and their hybrid structures while the first distance measure has to be derived for certain structures. It is mandatory that the two sets must be independent of each other and should be from the same structure family. In this paper, a new structure m-polar interval-valued neutrosophic set is derived and some operators are proposed on the derived structure. Furthermore, properties based on operators are discussed, and using distance-based similarity measure an application of player selection problems is dealt with the algorithm.

8. **Conclusion**

m-polar Interval-valued Neutrosophic set has wide application in decision making real-life problems. In this article, m-polar IVNS has been revisited, also some operators and properties on m-polar IVNS have been introduced. Furthermore, distance measure and similarity measure is introduced on m-polar IVNS, and evaluation of selecting of best three forward player combination of PSG club for the very next match as watching their previous match records has been discussed using a similarity measure tool. The concept of m-polar is used as several number of previous m matches played by a player, so the m-polar concept can be used in many different fields of decision-making problems. In future, the this study can be extended and TOPSIS, VIKOR, etc can be applied to this new structure for decision making.

References

1. L. A. Zadeh, *Fuzzy sets*, Information and Control, **8**, No. 3 (1965) 338-353.
2. D. A. Molodtsov, *Soft set theory-first results*, Computers & Mathematics with Applications, **37**, No. 4-5 (1999) 19-31.
3. P. K. Maji, R. Biswas, and A. R. Roy *Soft set theory*, Computers & Mathematics with Applications, **45**, No. 37 (2003) 555-562.
4. P. Majumdar, and S. K. Samanta, *Generalised fuzzy soft sets*, Computers & Mathematics with Applications, **59**, No. 4 (2010) 1425-1432.
5. K. T. Atanassov *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20**, No. 1 (1986) 87-96.
6. W. R. Zhang, *Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multiagent decision analysis*, Proceeding of IEEE Conference, (1994) 305-309.
7. J. Chen, S. Li, S. Ma, and X. Wang, *mpolar fuzzy sets: an extension of bipolar fuzzy sets*, The Scientific World Journal 2014, Article Id 416530 (2014) 8.
8. M. Akram A. Adeel, *Novel hybrid decision making based on mF rough information*, Granular Computing, **5**, No. 2 (2020) 185-201.
9. M. Akram, A. Adeel, and J. C. R. Alcantud, *Fuzzy N-soft sets:a novel model with applications*, Journal of Intelligent & Fuzzy Systems, **35**, No. 4 (2018) 4757-4771.
10. M. Akram, M. G. Ali, and N. O. Ashehri, *A new multi-attribute dicision-making methods based on m-polar fuzzy soft rough sets*, Symmetry, **9**, No. 11 (2017) 271.
11. M. Akram, M. G. Ali, N. Waseem, and B. Davvaz, *Dicision-making methods based on hybrid mF models*, Journal of Intelligent & Fuzzy Systems, **35**, No. 3 (2018) 3387-3403.
12. F. Smarandache, *Neutrosophic Set – A Generalization of the Intuitionistic Fuzzy Set*, International journal of pure and applied mathematics, 24, No. 3 (2004) 287.
13. I. Beg, and S. Ashraf, *Similarity measures for fuzzy sets*, Appl. and Comput. Math, **8**, No. 2 (2009) 192-202.
14. S. M. Chen, *Measure of similarity between vague sets*, Fuzzy Sets and Systems, **74**, No. 2 (1995) 217-223.
15. P. Majumdar, and S. K. Samanta, *Similarity measures of Soft sets*, New mathematics and natural computation, **4**, No. 1 (2008) 1-12.
16. P. Majumdar, and S. K. Samanta, *On distance based Similarity measures between intuitionistic fuzzy soft sets*, International Journal of Advance Soft Computing and Applications, **3**, No. 2 (2011) 1-8.
17. S. Broumi, I. Deli, and F. Smarandache, *N-valued Interval Neutrosophic Sets and Their Application in Medical Diagnosis*, Critical Review **10**, (2015) 46–69.
18. I. Deli, S. Broumi, and S.F. Smarandache, *On neutrosophic refined sets and their applications in medical diagnosis*, Journal of New Theory **6**, (2015) 88–98.
19. F. Smarandache, *Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off-Logic, Probability, and Statistics*, Infinite Study, (2016).
20. M. Akram, and N. Waseem, *Similarity Measures for New Hybrid Models: mF Sets and mF Soft Sets*, Punjab University Journal of Mathematics, **51** No. 6 (2019) 115-130

21. A. Kharal, *Distance and similarity measures for soft set*, New mathematics and natural computation, **6** No. 3 (2010) 321-334.
22. J. C. Bezdek, R. Ehrlich, and W. Full, *FCM: The fuzzy c-means clustering algorithm*, Computers & Geosciences, **10**, No. 2-3 (1984) 191-203.
23. D. H. Hongand, and C. Kim, *A note on similarity measures between vague sets and between elements*, Information Sciences, **115**, No. 1-4 (1999) 83-96.
24. F. Feng, Y. B. Jun, X. Liu, and L. Li, *An adjustable approach to fuzzy soft set based decision making*, Journal of Computational and Applied Mathematics, 234, No. 1 (2010) 10-20.
25. I. Deli, *Interval-valued neutrosophic soft sets and its decision making*, International Journal of Machine Learning and Cybernetics, **8**, No. 2 (2017) 665-676.
26. S. M. Chen, M. S. Yeh, and P. Y. Hsiao, *A comparison of similarity measures of fuzzy values*, Fuzzy Sets and Systems, **72**, No. 1 (1995) 79-89.
27. M. Saqlain, M. Saeed, M. R. Ahmad, and F. Smarandache, *Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application*, Neutrosophic Sets and Systems, **27**, (2019) 131-137.
28. M. Saqlain, M. Sana, M. Saeed and F. Smarandache, *Aggregate Operators of Neutrosophic Hypersoft Set*, Neutrosophic Sets and Systems, **32**, (2020) 294-306.
29. M. Saqlain, M. Sana, M. Saeed, and B. Said, *Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets*, Neutrosophic Sets and Systems, **32**, (2020) 317-329.
30. M. Zulqarnain, F. Dayan, and M. Saeed, *Topsis analysis for the prediction of diabetes based on general characteristics of humans*, Int. J. Pharm. Sci. Res., **9**, No. 7 (2018) 2932-2939.
31. M. Saeed, M. Saqlain, A. Mehmood, K. Naseer, S. Yaqoob, *MultiPolar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making*, Neutrosophic Sets and Systems, **33**, (2020) 183-207.
32. M. Saqlain, N. Jafar, and A. Riffat, *Smart phone selection by consumers' in Pakistan: FMCGDM fuzzy multiple criteria group decision making approach*, Gomal University Journal of Research, **34**, No. 1 (2019) 27-31.
33. M. Saqlain N. Jafar, and M. Riaz, *A New Approach of Neutrosophic Soft Set with Generalized Fuzzy TOPSIS in Application of Smart Phone Selection*, Nuetrosophic Sets and Systems, **32**, (2020) 307-316.
34. M. Riaz, M. Saqlain, and M. Saeed, *Application of Generalized Fuzzy TOPSIS in Decision Making for Neutrosophic Soft set to Predict the Champion of FIFA 2018: A Mathematical Analysis*, Punjab University Journal of Mathematics, **51**, No. 8 (2019) 111-126.
35. M. Saqlain, N. Jafar, R. Hamid, and A. Shahzad, *Prediction of Cricket World Cup 2019 by TOPSIS Technique of MCDM-A Mathematical Analysis*, International Journal of Scientific & Engineering Research, **10**, No. 2 (2019) 789-792.
36. P. Liu, L. Zhang, X. Liu, and P. Wang, *Multi-valued neutrosophic number Bonferroni mean operators with their applications in multiple attribute group decision making* International Journal of Information Technology & Decision Making, **15**, No. 5 (2016) 1181-1210.
37. M. Kamal, L. Abdullah, I. Abdullah, S. Alkhazaleh, and F. Karaaslan, *Multi-Valued Interval Neutrosophic Soft Set: Formulation and Theory*, Neutrosophic Sets and Systems, **30**, (2019) 149-170.
38. M. Abdel-Basset, R. Mohamed, M. Elhoseny, and V. Chang, *Evaluation framework for smart disaster response systems in uncertainty environment*, Mechanical Systems and Signal Processing, **145**, (2020) 106941.
39. M. Abdel-Basset, M. Ali, and A. Atef, *Uncertainty assessments of linear time-cost tradeoffs using neutrosophic set*, Computers & Industrial Engineering, **141**, (2020) 106286.
40. M. Abdel-Basset, M. Ali, and A. Atef, *Resource levelling problem in construction projects under neutrosophic environment*, The Journal of Supercomputing, **76**, No. 2 (2020) 964-988.

41. M. Abdel-Basset, A. Gamal, L. H. Son, and F. Smarandache, *A Bipolar Neutrosophic Multi Criteria Decision Making Framework for Professional Selection*, Applied Sciences, **10**, No. 4 (2020) 1202.
42. M. Abdel-Basset, R. Mohamed, A. E. H. Zaied, A. Gamal, and F. Smarandache, *Solving the supply chain problem using the best-worst method based on a novel Plithogenic model*, Optimization Theory Based on Neutrosophic and Plithogenic Sets, Academic Press, (2020) 1-19.
43. M. Kamal, L. Abdullah, M. Saqlain, *Multi-Valued Interval Neutrosophic Linguistic Soft Set Theory and Its Application in Knowledge Management*, CAAI Transactions on Intelligence Technology, (2020) 1-13.
44. M. R. Hashmi, M. Riaz, and F. Smarandache, *m-polar Neutrosophic Generalized Weighted and m-polar Neutrosophic Generalized Einstein Weighted Aggregation Operators to Diagnose Coronavirus (COVID-19)*, Journal of Intelligent and Fuzzy Systems, (2020) 1-21.
45. M. Riaz, and M. R. Hashmi, *MAGDM for agribusiness in the environment of various cubic m-polar fuzzy averaging aggregation operators*, Journal of Intelligent and Fuzzy Systems, **37**, No. 3 (2019) 3671-3691.
46. M. R. Hashmi, M. Riaz, and F. Smarandache, *m-Polar Neutrosophic Topology with Applications to Multi-criteria Decision-Making in Medical Diagnosis and Clustering Analysis*, International Journal of Fuzzy Systems, **22**, No. 1 (2020) 273-292.
47. M. Kamal, and L. Abdullah, *Multi-valued neutrosophic soft set*, Malaysian Journal of Mathematical Sciences, **13** (2019) 153-168.

Received: June 5, 2020. Accepted: Nov 25, 2020