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Assessment of MCDM problems by TODIM using aggregated weights

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Abstract: Sustainability of sheep and goat production systems is a significant task for any organization that aims for long term goals. Housing and feeding selection for goat farming is the most important factor that should be considered before setting out the goat farm. The decision framework of housing selection should include environmental, social and human impact for the long term, rather than on short-term gains. In the selection process, various parameters are involved such as housing materials, area to prevent water stagnation, ventilation, enough space for the pen and run system, space for feeders and water troughs. Those parameters highlight the quality of housing in relation to aspects of traditional breeding provided by the organizations. However, the process of housing selection is often led by hands on experience which contains vague, ambiguous and uncertain decisions. To overcome this issue it is necessary to frame an efficient algorithm which could remove the entire barrier in the decision making process. In this paper we propose a neutrosophic multi-criteria decision making framework that combines the TODIM method with the SD- HNWA operator. The resulting multi-criteria decision analytical MCDM framework is then applied in selecting the best system in housing and feeding of goats at a mixed farming agrofarm in India. The proposed approach allows us to establish the neutrosophic based value function that measures the degree to which one alternative is superior to others by calculating accurate number of information in pair wise comparison in terms of gain and loss. The outcomes of the proposed method are compared with the use of the TOPSIS method to prove its efficiency and validate the results.

Keywords: MCDM, Hexagonal Neutrosophic numbers, Similarity Degree, Aggregated Weights, TODIM, TOPSIS.

1. Introduction

Live stock management is considered as one of the most important study topic as it plays a vital role in self employment for the younger generation with higher level of educational qualifications in a country like India, with a traditionally high rate of population growth. It is also considered as an employment intervention strategy for the younger generation for the self employment of the youth. Goats are among the main meat producing animal in India where it has huge domestic demand. As a result, goat production system in India is shifting to intensive system of management.The goat rearing using improved management practices concentrates on maximization of the returns from the view of the entrepreneur.

However, without any systematic study it is difficult to assess the economic viability of the goat farming, as the whole system is built upon nature. The good management practice in livestock management is the key for the resilience, social, economical and ecological sustainability and preservation of bio-diversity in pastoral eco-systems, especially in the rural areas where goat production plays a relevant role in the livehood for farmers. For example, Shalander [25] has proposed a multi-disciplinary project on transfer of technology for sustainable goat production in which he indicates that lack of technical knowledge in housing and feeding management system per capita income in goat rearing is not being up to the expected margin of the goat farmers. Biswas et al. [9] shows that the growth rate of goat feeder with supplements by additional concentrate with grazing was more when compared with the normal grazing goats.

In the real world, just like other decision making problem such as supplier selection or candidate selection, the challenge of uncertainty in the process of housing and feeding selection in live-stock management is inevitable owing to the fact that the consequences of events are not precisely known. In addition human judgmental analysis also contributes to its intricacy in the decision making analysis. To overcome this vagueness and intricacy in decision making this study aims to propose an integrated framework under neutrosophic environment to evaluate alternative choices in terms of management system of housing and feeding.

In this research the TODIM and TOPSIS methods will be applied in the processing of selecting such alternatives. The TODIM method (an acronym for Interactive Multi-Criteria Decision Making in Portuguese) is a discrete multi-criteria method founded on prospect theory which underlies a psychological theory in it, while in practice all other discrete multi-criteria methods assume that the decision maker always looks for the solution corresponding to the maximum of some global measure. In this way, the method is based on a descriptive theory, proved by empirical evidence, of how people effectively make decisions when they are under risk. The mathematical structure of TODIM allows measuring the degree to which one alternative is superior to others and then ranking the alternatives by computing the global value of each alternative. That structure is embedded in the paradigm of prospect theory. Gomes and Lima [18] first applied TODIM in its classical formulation as a tool for ranking projects based on the environmental impacts of alternative road standards in Brazil. A number of other applications of TODIM has appeared in the literature since then as it is commented in the section 2.2. Similarly, the TOPSIS method [23] is used to weight and compare alternatives against a set of criteria and then select the best one. The application of both TODIM and TOPSIS are then compared one against the other. The novelty of this framework lies in studying the behavioral risk analysis under neutrosophic environment as pointed out in the above paragraph.

The main contribution of this article is as follows

- A framework is designed that emphasizes the importance of shelter and feeding system for sustainable and productive goat farming.
- Two well established Multi-Criteria Decision Making (MCDM) methods dealing with imprecise information are applied to a quite important problem in India and compared.
- Relevant criteria and sub-criteria are defined for the alternatives to maintain accuracy and consistency in selecting the alternatives.

2. Literature review

2.1. Commercial goat farming

Raising animals lie upon a set of activities that are dependent upon biotic and socio-economic factors. Choudhary et al. [35] highlights that India is the rich in its repository of goat genetic resource with 28 recognized breeds with higher proportion of non-descriptive or mixed breeds. A study was undertaken by Patil et al. [28] to compare the grazing system and stall feeding system in goats in Gulbarga District in Karnataka which highlighted that in stall feeding system of goat rearing, goats are found healthier and weight gain was much faster than grazing system. Kumar [26] investigated on commercial goat farming in India and presented that planned management and technology based system would help in increasing the goat productivity in goat farming and bridge the demand-supply gap. Argüello [8] has presented a review on trends in goat research which talks about the pathology, reproduction, milk and cheese production and quality, production systems, nutrition, hair production, drugs knowledge and meat production.

2.2. Multi Criteria Decision Making

Zadeh [42] put forward the concept of fuzzy sets in 1965. Later the theory of fuzzy sets gradually developed in the further years. The theory of 'intuitionistic fuzzy set' [IFS] was proposed by Atanassov [10] in 1986. Intuitionistic fuzzy set [IFS] was extended to 'Interval intuitionistic fuzzy sets' [IIFS] by Atanassov and Gargov [11]. A number of researchers have contributed their research to the study of MCDM and a commendable accomplishment has been obtained in fuzzy sets. Smarandache [36] proposed neutrosophic set based on Neutrosophy in 1998. The neutrosophic theory takes into account the dynamic features of all limitations to handle uncertain, indeterminate situations. Abdel-Basset et al. [2] proposed uncertainty assessments of linear time-cost tradeoffs using neutrosophic set considering the neutrosophic activity duration of time-cost tradeoffs in project management such as the tradeoffs between the project completion time and the cost and the uncertain conditions of environment of projects. Abdel-Basset [6] developed and applied a novel decision making model for sustainable supply chain under uncertainty environment.

Wang et al. [38] developed 'Single Valued Neutrosophic Set' (SVNS) and proposed various properties of set-theoretic operators to deal with uncertain, indeterminate and inconsistent data. Ye [40] proposed trapezoidal neutrosophic number an extension from SVNS and trapezoidal fuzzy number and defined its score and accuracy function with aggregating operators in [41]. Smarandache [37] introduced the plithogenic set as generalization of crisp, fuzzy, intuitionistic fuzzy and neutrosophic sets whose elements are characterized by many attribute values which have corresponding contradiction degree values between each attribute value and the dominant attribute value. Abdel-Basset [1] developed an evaluation framework based on plithogenic set theory for smart disaster response systems in uncertainty environment that deals more effectively with disaster by the effective communication of the information provided by the sensors with the response teams.

Decision making situations in real life are much complicated when the decision makers (DMs) have to fit in the best alternatives with respect to the given multiple criteria. Biswas et al. [14] established TOPSIS strategy for (MCDM) in trapezoidal neutrosophic environment using the maximum deviation strategy and also developed an optimization model to obtain the weight of the attributes which are incompletely known or completely unknown. Abdel-Basset [5] proposed a decision making problem to solve a supply chain problem of inventory location using the best-worst method based on a novel plithogenic model.

Pramanik and Mallick [30] proposed a VIKOR method for group Decision Making Problem involving trapezoidal neutrosophic number and they adapted a problem of Investment Company from [16] and provided a comparative analysis. Mondal and Pramanik [29] proposed MCDM approach for teacher recruitment in higher education with unknown weights based on score and accuracy function, hybrid score and accuracy functions under simplified neutrosophic environment. Biswas et al. [12,13] developed a new methodology for neutrosophic MCDM with unknown weight information and a Cosine similarity measure based MCDM with trapezoidal fuzzy neutrosophic numbers. Abdel-Basset [3] designed resource levelling problem to minimize the cost of daily resource fluctuation in construction projects under neutrosophic environment to overcome the ambiguity caused by real world problems.

Based on observations of human behaviour, studies have found that human decision making is not completely rational under practical decision situations. After undertaking a number of surveys and experiments, Kahneman and Tversky [24] proposed Prospect theory partially the subject of the Nobel Prize for Economics awarded in 2002, which belongs to the field of cognitive psychology and describes how people make decision under conditions of risk.

Gomes and Lima [20] used the TODIM method in order to show how human judgements in practical multi-criteria analysis fit in to the framework of Prospect Theory and additive difference model. Gomes et al. [19] used the classical TODIM formulation to recommend alternatives for destination of natural gas reserves recently discovered in Santos Basin in Brazil. Gomes et al. [22] proposed a behavioural multi-criteria decision analysis by using the TODIM method with criteria interactions. Gomes and Rangel [21] developed a novel approach using TODIM method on rental evaluation of residential properties carried out together with real estate agents in the city of Volta Redonda, Brazil which has made many successful applications in selection problems. Zindani et al. [44] proposed a material selection approach using the TODIM method and applied it to find the best suited materials for two products, engine flywheel and metallic gear.Duarte [7] proposed the use of multi criteria decision analysis to valuation of six Brazilian banks by applying the fuzzy TODIM method.

Sang and Liu [34] developed the IT2 FSs-based TODIM method to green supplier selection for automobile manufacturers by introducing a new distance computing method. Wang et al. [39] proposed a likehood-based TODIM approach on multi-hesitant fuzzy linguistic information (MHFLSs) which is an extension of (HFLSs) for selection and evaluation of contractors in logistics outsourcing. Chakraborty and Chakraborty [15] used TODIM in identifying the most attractive and affordable under-construction housing project in the city of Kolkata in India. Rangel et al. [32] used TODIM a multi-criteria decision aiding method in the evaluation of the various types of access to the broadband internet available in Volta Redonda, Brazil. Candidate selection is a significant task for any organization that aims to select the most appropriate candidates who lead the firm forward through his strong organizational skill. To overcome this tough task Abdel-Basset [4] proposed a bipolar neutrosophic multi criteria decision making framework for professional selection that employs a collection of neutrosophic analytical network process and TOPSIS under bipolar neutrosophic numbers.

Lourenzutti and Krohling [27] combined TOPSIS and TODIM methods to propose the Hellinger distance in MCDM which serves as an illustration to both methods. Fan et al. [17] proposed an extension of TODIM (H-TODIM) to solve the hybrid MCDM problem in which attribute values have three forms crisp number, interval number and fuzzy number. Ren et al. [33] proposed a Pythagorean fuzzy TODIM approach to analyse MCDM problem. Qin et al. [31] proposed generalizing of the TODIM method under triangular intuitionistic fuzzy environment. Zhang et al. [43] proposed an extended multiple attribute group decision making based on the TODIM method to solve the MCDM problem in which the attribute values are expressed with neutrosophic number.

3. Preliminaries

3.1. Hexagonal Neutrosophic Weighted Aggregated Operator (HNWA)

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1, e_1, f_1), (l_1, m_1, n_1, p_1, q_1, r_1), (u_1, v_1, w_1, x_1, y_1, z_1)$ be a collection of hexagonal neutrosophic numbers, then the HNWA: $\Omega^n \rightarrow \Omega$ is defined as follows

$$HNWA(\tilde{A}_1, \tilde{A}_2, ...\tilde{A}_n) = \sum_{j=1}^{n} \omega_j \tilde{A}_j$$

$$
= \left\langle \begin{array}{c}
\left(\begin{array}{l} 1-\prod_{j=1}^{n}(1-a_j)^{\omega_j}, 1-\prod_{j=1}^{n}(1-b_j)^{\omega_j}, 1-\prod_{j=1}^{n}(1-c_j)^{\omega_j}, \\ 1-\prod_{j=1}^{n}(1-d_j)^{\omega_j}, 1-\prod_{j=1}^{n}(1-e_j)^{\omega_j}, 1-\prod_{j=1}^{n}(1-f_j)^{\omega_j} \end{array} \right) \\
\left(\prod_{j=1}^{n}(l_j)^{\omega_j}, \prod_{j=1}^{n}(m_j)^{\omega_j}, \prod_{j=1}^{n}(n_j)^{\omega_j}, \prod_{j=1}^{n}(p_j)^{\omega_j}\ \prod_{j=1}^{n}(q_j)^{\omega_j}, j=1\prod_{j=1}^{n}(r_j)^{\omega_j} \right) \\
\left(\prod_{j=1}^{n}(u_j)^{\omega_j}, \prod_{j=1}^{n}(v_j)^{\omega_j}, \prod_{j=1}^{n}(w_j)^{\omega_j}, \prod_{j=1}^{n}(x_j)^{\omega_j}, \prod_{j=1}^{n}(y_j)^{\omega_j}, \prod_{j=1}^{n}(z_j)^{\omega_j} \right) \\
\text{where } w=(w_1,w_2,w_3,w_4......w_n)^T \text{ is a weight vector of } \tilde{A}_j \text{ and } \sum_{j=1}^{n} w_j = 1, w_j > 0
\end{array} \right\rangle --(1)
$$

3.2. Distance between two Hexagonal Neutrosophic numbers

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1, e_1, f_1), (l_1, m_1, n_1, p_1, q_1, r_1), (u_1, v_1, w_1, x_1, y_1, z_1)$

$\tilde{A}_2 = (a_2, b_2, c_2, d_2, e_2, f_2), (l_2, m_2, n_2, p_2, q_2, r_2), (u_2, v_2, w_2, x_2, y_2, z_2)$ be two hexagonal neutrosophic numbers then the weighted distance between $\tilde{A}_1$ *and* $\tilde{A}_2$ is defined as follows.

$$
d(\tilde{A}_1, \tilde{A}_2) = \frac{1}{18} \left(\begin{array}{l} |a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2| + |e_1 - e_2| + |f_1 - f_2| + \\ |l_1 - l_2| + |m_1 - m_2| + |n_1 - n_2| + |p_1 - p_2| + |q_1 - q_2| + |r_1 - r_2| + \\ |u_1 - u_2| + |v_1 - v_2| + |w_1 - w_2| + |x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| \end{array} \right) --(2)
$$

3.3. Similarity Degree between two Hexagonal Neutrosophic numbers

Let $\tilde{A}_1 = [(a_1, b_{1,}, c_{1,}, d_{1,}, e_{1,}, f_{1,}),\ (l_{1,}, m_{1,}, n_{1,}, p_{1,}, q_1, r_{1,}), (u_1, v_1, w_1, x_1, y_1, z_1)]$ *and*

$\tilde{A}_2 = [(a_2, b_2, c_2, d_2, e_2, f_2),\ (l_2, m_2, n_2, p_2, q_2, r_2), (u_2, v_2, w_2, x_2, y_2, z_2)]$

be two hexagonal neutrosophic numbers and let

$\tilde{A}_2^C = [(u_2, v_2, w_2, x_2, y_2, z_2), (1-l_2, 1-m_2, 1-n_2, 1-p_2, 1-q_2, 1-r_2), (a_2, b_2, c_2, d_2, e_2, f_2)]$ be the complement of $\tilde{A}_2$ then the Degree of Similarity between $\tilde{A}_1$ *and* $\tilde{A}_2$ is defined as follows.

$$
\theta(\tilde{A}_1, \tilde{A}_2) = \left\{ \frac{d(\tilde{A}_1, \tilde{A}_2^C)}{d(\tilde{A}_1, \tilde{A}_2) + d(\tilde{A}_1, \tilde{A}_2^C)} \right\} -------------- (3)
$$

3.4. Hexagonal Neutrosophic Decision Matrix

Let $\tilde{R} = (\tilde{r}_{ij})_{m \times n}$ .If all $\tilde{r}_{ij}$ are hexagonal neutrosophic number then

$$\tilde{R} = \tilde{r}_{ij} = \left[\left(\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij}, \tilde{e}_{ij}, \tilde{f}_{ij}\right), \left(\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{n}_{ij}, \tilde{p}_{ij}, \tilde{q}_{ij}, \tilde{r}_{ij}\right), \left(\tilde{u}_{ij}, \tilde{v}_{ij}, \tilde{w}_{ij}, \tilde{x}_{ij}, \tilde{y}_{ij}, \tilde{z}_{ij}\right)\right]$$

is a hexagonal neutrosophic decision matrix.

3.5. Aggregated Hexagonal Neutrosophic Decision Matrix

Let $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{m\times n}$ $(k = 1,2,3,.....t)$ be a‘t’ neutrosophic decision matrix evaluated by the decision makers $DM_d\,(d = 1,2,3....m)$ respectively, then the aggregated hexagonal neutrosophic decision matrix $\tilde{R} = (\tilde{r}_{ij})_{m\times n}$ is defined as

$$\tilde{r}_{ij} = \left[\left(\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij}, \tilde{e}_{ij}, \tilde{f}_{ij}\right), \left(\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{n}_{ij}, \tilde{p}_{ij}, \tilde{q}_{ij}, \tilde{r}_{ij}\right), \left(\tilde{u}_{ij}, \tilde{v}_{ij}, \tilde{w}_{ij}, \tilde{x}_{ij}, \tilde{y}_{ij}, \tilde{z}_{ij}\right)\right]$$

$$(i = 1,2,3....m), (j = 1,2...n) \quad where \quad \tilde{r} = \frac{1}{t}\sum_{k=1}^{t}\tilde{r}_{ij}^{(k)}$$

3.6. Degree of Similarity

Let $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{m\times n}$ $(k = 1,2,3,......t)$ be a‘t’ neutrosophic decision matrix and $\tilde{R}' = (\tilde{r}'_{ij})_{m\times n}$ be their aggregated hexagonal neutrosophic decision matrix then

$$\theta(\tilde{R}^{(k)}, \tilde{R}') = \frac{1}{m\times n}\sum_{i=1}^{m}\sum_{j=1}^{n}\theta(\tilde{r}_{ij}^{(k)}, \tilde{r}'_{ij}) ---------------- (4)$$

is called the degree of similarity between $\tilde{R}^{(k)}$ and $\tilde{R}'$

3.7. Determine the weight of experts using Degree of Similarity:

If the hexagonal neutrosophic decision matrix $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{m\times n}$ $(k = 1,2,......t)$ are non-identical,then the weight vectors of the experts are expressed as follows.

$$w^{(k)} = \frac{(\theta(\tilde{R}^{(k)}, \tilde{R}))^{\alpha}}{\sum_{k=1}^{t}(\theta(\tilde{R}^{(k)}, \tilde{R}))^{\alpha}} --------------------- (5)$$

4. A Comparative Analysis of TODIM and TOPSIS Methods.

4.1. TODIM

To solve the MCDM problem with hexagonal neutrosophic information’s we propose a hexagonal neutrosophic aggregation TODIM method based on prospect theory under the decision maker’s behavioral risk and arithmetic mean operator.

Let $A_i = (A_1, A_2,....A_m)$ be the alternatives, and $C_j = \{C_1, C_2,.......,C_n\}$ be the criteria.

Let $w = (w_1, w_2, ...w_n)$ be the weights of $C_j, 0 \le w_j \le 1$, and $\sum_{j=1}^{n} w_j = 1$. Let,

$$\tilde{R}_k = \left(\tilde{r}_{ij}^{(k)}\right)_{m \times n} = (T_{ij}, I_{ij}, F_{ij})_{m\times n} = (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij}, \tilde{e}_{ij}, \tilde{f}_{ij}), (\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{n}_{ij}, \tilde{p}_{ij}, \tilde{q}_{ij}, \tilde{r}_{ij}), (\tilde{u}_{ij}, \tilde{v}_{ij}, \tilde{w}_{ij}, \tilde{x}_{ij}, \tilde{y}_{ij}, \tilde{z}_{ij})$$

be a hexagonal neutrosophic decision matrix, where $\tilde{r}_{ij} = (T_{ij}, I_{ij}, F_{ij})$ is an attribute value given by the experts for the alternatives A_i with the criteria C_j, $T_{ij} \in [0,1], I_{ij} \in [0,1], F_{ij} \in [0,1]$,

$0 \le T_{ij} + I_{ij} + F_{ij} \le 3 \quad (i = 1,2,......,m), (j = 1,2,....,n)$

The proposed method is presented as follows.

Stage 1.

Step 1. Construct a decision matrix of dimension $m \times n$ by using the information provided by the decision maker for the alternatives A_i under the criteria C_j. The m^{th} hexagonal neutrosophic decision matrix denoted by the decision maker is defined as follows.

$$\tilde{R}_k = \begin{bmatrix} (a_{11}, b_{11}, c_{11}, d_{11}, e_{11}, f_{11}) & & & & (a_{1n}, b_{1n}, c_{1n}, d_{1n}, e_{1n}, f_{1n}) \\ (l_{11}, m_{11}, n_{11}, p_{11}, q_{11}, r_{11}) & \cdots & \cdots & \cdots & (l_{1n}, m_{1n}, n_{1n}, p_{1n}, q_{1n}, r_{1n}) \\ (u_{11}, v_{11}, w_{11}, x_{11}, y_{11}, z_{11}) & & & & (u_{1n}, v_{1n}, w_{1n}, x_{1n}, y_{1n}, z_{1n}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (a_{m1}, b_{m1}, c_{m1}, d_{m1}, e_{m1}, f_{m1}) & & & & (a_{mn}, b_{mn}, c_{mn}, d_{m}, e_{mn}, f_{mn}) \\ (l_{m1}, m_{m1}, n_{m1}, p_{m1}, q_{m1}, r_{m1}) & \cdots & \cdots & \cdots & (l_{mn}, m_{mn}, n_{mn}, p_{mn}, q_{mn}, r_{mn}) \\ (u_{m1}, v_{m1}, w_{m1}, x_{m1}, y_{m1}, z_{m1}) & & & & (u_{mn}, v_{mn}, w_{mn}, x_{mn}, y_{mn}, z_{mn}) \end{bmatrix}$$

Step 2: Find the aggregated hexagonal neutrosophic decision matrix of all the three decision makers..The aggregated hexagonal neutrosophic decision matrix $\tilde{R}' = (\tilde{r}_{ij}')_{m\times n}$ is defined as given below.

$$\tilde{r}_{ij}' = \left\lfloor \left(\bar{a}_{ij}', \bar{b}_{ij}', \bar{c}_{ij}', \bar{d}_{ij}', \bar{e}_{ij}', \bar{f}_{ij}'\right), \left(\bar{l}_{ij}', \bar{m}_{ij}', \bar{n}_{ij}', \bar{p}_{ij}', \bar{q}_{ij}', \bar{r}_{ij}'\right), \left(\bar{u}_{ij}', \bar{v}_{ij}', \bar{w}_{ij}', \bar{x}_{ij}', \bar{y}_{ij}', \bar{z}_{ij}'\right) \right\rfloor$$

$$(i = 1,2,3.....,m), (j = 1,2....,n)$$

$$where\ \bar{r}' = \frac{1}{t} \sum_{k=1}^{t} \tilde{r}_{ij}^{(k)}$$

Step 3. Calculate the normalized hamming distance for each $(\tilde{R}, \tilde{R}')$ using the equation (2)

Step 4. Calculate the Degree of Similarity between $\bar{A}_1\ and\ \bar{A}_2'$ using equation (3) and (4)

Step 5. Calculate the weight vector $w^{(k)}$ using equation (5)

Step 6. Using equation (1) calculate HNWA operator

Step 7. Calculate the score value using the equation

$$S(A) = \frac{1}{3}\left[2 + \frac{a+b+c+d+e+f}{6} - \frac{l+m+n+p+q+r}{6} - \frac{u+v+w+x+y+z}{6}\right] ------(6)$$

Step 8. Calculate the normalized hamming distance for the aggregated decision matrix using (2)

Step 9. When the aggregated matrix is brought into expression (7), matrix $\rho(A_i, A_p)$ will be derived .The function $\rho(A_i, A_p)$ is used to represent the degree to which alternative i is better than j. $\varepsilon_j(A_i, A_p)$ is the sum of the sub-function where $j = 1....,n$. Sub-function $\varepsilon_j(A_i, A_p)$ indicates the degree to which i is better than j when a particular criteria c is given

$$\varepsilon_j(A_i, A_p) = \begin{cases} \sqrt{\dfrac{w_{jr} d(\tilde{r}_{ij} - \tilde{r}_{pj})}{\sum\limits_{j=1}^{n} w_{jr}}} & if \quad \tilde{r}_{ij} - \tilde{r}_{pj} \succ 0 \\ 0 & if \quad \tilde{r}_{ij} - \tilde{r}_{pj} = 0 \\ \dfrac{-1}{\upsilon}\sqrt{\dfrac{\sum\limits_{j=1}^{n} w_{jr} d(\tilde{r}_{ij} - \tilde{r}_{pj})}{w_{jr}}} & if \quad \tilde{r}_{ij} - \tilde{r}_{pj} \prec 0 \end{cases} ---------------(7)$$

The parameter υ shows the dilution factor of the loss. If $\tilde{r}_{ij} - \tilde{r}_{pj} \succ 0$ then $\varepsilon_j(A_i, A_p)$ represents the gain and if $\tilde{r}_{ij} - \tilde{r}_{pj} \prec 0$ then $\varepsilon_j(A_i, A_p)$ represents the loss.

Step 10. On the basics of the above equation the overall dominance degree is obtained as

$$\rho_x = \sum_{j=1}^{n} \varepsilon_j (A_i, A_p), (i, p = 1,2,....m)$$

Step 11. Calculate the aggregated dominance matrix

$$\rho(A_i, A_p) = \sum_{x=1}^{n} \lambda_x \rho_x (A_i, A_p), (i, p = 1,2, \ldots m) ---------(8)$$

Step 12. Calculate the overall dominance degree matrix $\rho = [\rho(A_i, A_p)]_{m \times n}$

Step 13. Then the overall value of each A_i can be calculated using the equation

$$\rho(A_i)=\frac{\sum_{p=1}^{m}\rho(A_i,A_p)-\min_i\left\{\sum_{p=1}^{m}\rho(A_i,A_p)\right\}}{\max_i\left\{\sum_{p=1}^{m}\rho(A_i,A_p)\right\}-\min_i\left\{\sum_{p=1}^{m}\rho(A_i,A_p)\right\}}---------(9)$$

Step 14. Rank all alternatives and select the most desirable one in accordance with $\rho(A_i)$. The alternative with maximum value is the best one.

4.2. TOPSIS

Stage 2: Applying the information's derived from step 1 to 6 in stage 1, move on to step 7 of stage 2

Step 7: Let B_1 be the set of benefit attributes and B_2 be the set of cost attributes, of the alternatives respectively. Let B^+ be the hexagonal neutrosophic positive ideal solution and B^- be the hexagonal neutrosophic negative ideal solution. Then B^+ and B^- are defined as follows.

$$B^+=\left[\left\langle r_j^+=(1,1,1,1,1,1),(0,0,0,0,0,0),(0,0,0,0,0,0)\right\rangle j\in B_1\right],\left[\left\langle r_j^+=(0,0,0,0,0,0),(1,1,1,1,1,1),(1,1,1,1,1,1)\right\rangle j\in B_2\right]$$

$$B^-=\left[\left\langle r_j^-=(0,0,0,0,0,0),(1,1,1,1,1,1),(1,1,1,1,1,1)\right\rangle j\in B_1\right],\left[\left\langle r_j^-=,(1,1,1,1,1,1),(0,0,0,0,0,0),(0,0,0,0,0,0)\right\rangle j\in B_2\right]$$

Step 8: Calculate the separation measures, S_i^+ *and* S_i^- of each alternative from the hexagonal neutrosophic positive ideal solution and the hexagonal neutrosophic negative ideal solution as follows.

$$S_i^+=\frac{1}{n}\sum_{j=1}^{n}w_j d(r_{ij},r_j^+)----------------------(10)$$

$$S_i^-=\frac{1}{n}\sum_{j=1}^{n}w_j d(r_{ij},r_j^-)----------------------(11)$$

Step 9: Calculate the relative closeness coefficient of the hexagonal neutrosophic ideal solution. The relative closeness coefficient of the alternative A_i is given as follows.

$$C_i=\frac{S_i^-}{S_i^++S_i^-},0\le C_i\le 1---------------(12)$$

Step 10: Make a decision for selecting the preference alternative by ranking the closeness coefficient in the descending order of C_i to select the best choice.

5. Case Analysis:

In this section, a case study is represented for the proposed multi-criteria group decision-making method. This is related to assessing the best system of housing and feeding of goats in the existing

Goat farm rearing in which goats grow healthier, gain better body weight, and are safer on health grounds. A group of three decision-makers (D_1, D_2 and D_3) are requested to assess the four alternatives ($A_1$to A_4) with respect to the four criteria's, (C_1 to C_4) defined by this group of decision-makers to appraise the alternatives. These criteria and their definitions are represented as follows:

Alternatives:

A_1 - Stall feeding system with normal flooring (intensive system)

A_2 - Grazing system (extensive system)

A_3 - Elevated floor shed with rotational grazing system

A_4 - A part of both extensive and intensive grazing system

The consideration of the criteria and sub criteria's after a brief study on the previous literature review and discussion with the experts are stated below.

Criteria:

C_1 - Floor space requirements

(Covered area, Open area, Ventilation, Bedding, Confinement, Site location)

C_2 - Feeding (Feeder) and watering space requirement

(Feeder size, Fodder type, Quantity, Food Schedule, immunization feeder, feed storage room)

C_3 - Maintenance of health and sanitization

(Nutritional ratio, Vaccination, Climate pattern, Temperature, Supplementary feeding, Cleanliness)

C_4 - Productivity

(Capital, Typologies of farms, Technology integration, Agro climatic characteristics, Market value, Place of selling)

A questionnaire is prepared and handed over to the domain experts. These experts further graded the degree of the statement as given below.

Statement	Very high	High	Fair	Average	Medium	Satisfactory	Low	Very low	Not sure
Score	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1

Table 5.1**.** Rating scale used by experts

Solution.

Step 1. The judgment of the three decision makers for the alternatives A_i under the four criteria were presented using hexagonal neutrosophic number as shown in Table 5.2 .

	Criteria				
DMs	Alternatives	C_1	C_2	C_3	C_4
	A_1	[(.1,.2,.3,.4,.5,.6), (.1,.1,.1,.1,.1,.1), (.5,.6,.7,.8,.9,.9)]	[(.1,.2,.3,.4,.5,.6), (.1,.1,.1,.1,.1,.1), (.5,.6,.7,.8,.9,.9)]	[(.8,.8,.8,.8,.9,.9), (.3,.4,.5,.6,.7,.8), (.1,.1,.1,.1,.1,.1)]	[(.2,.3,.4,.5,.6,.7), (.1,.2,.2,.3,.3,.3), (.5,.5,.6,.6,.7,.7)]

D1	A2	[(.3,.4,.5,.6,.7,.8), (.2,.3,.4,.5,.5,.6), (.6,.6,.7,.8,.8,.8)]	[(.2,.2,.2,.2,.2,.2), (.5,.6,.7,.8,.8,.8), (.6,.6,.7,.7,.8,.8)]	[(.8,.8,.8,.9,.9,.9), (.2,.3,.3,.3,.4,.5), (.1,.1,.2,.2,.2,.2)]	[(.2,.3,.4,.5,.6,.7), (.1,.2,.2,.3,.3,.3), (.5,.5,.6,.6,.7,.7)]
	A3	[(.4,.5,.6,.7,.8,.9), (.2,.3,.4,.6,.7,.8), (.2,.2,.2,.2,.2,.2)]	[(.1,.1.1,.1,.1,.1) ,(.3,.4,.5,.6,.7,.8) (.4,.4,.5,.5,.6,.6)]	[(.1..2,.3,.5,.6,.7), (.3,.3,.4,.5,.8,.9.) (.5,.5,.5,.6,.6,.6)]	[(.9,.9,.9,.9,.9,.9) (.1,.2,.3,.4,.5,.6) (.3,.4,.4,.5,.6,.7)]
	A4	[(.1,.2,.2,.3,.3,.3) ,(.2,.2,.3,.3,.5,.5) (.4,.5,.6,.7,.8,.9,)]	[(.3,.3,.4,.4,.4,.5), (.1,.2,.2,.3,.4,.4) (.5,.5,.5,.6,.6,.6)]	[(.1,.2,.3,.4,.5,.6) ,(.3,.3,.4,.6,.6,.6) (.3,.4,.5,.5,.6,.6)]	[(.6,.6,.6,.7,.7,.8), (.5,.5,.5,.6,.6,.6) (.2,.2,.3,.3,.3,.3)]
D2	A1	[(.6,.7,.8,.9,.9,.9), (.1,.2,.3,.3,.4,.5) (.3,.4,.4,.4,.5,.5)]	[(.3,.4,.5,.6,.7,.8) ,(.2,.3,.4,.5,.6,.7) (.4,.5,.6,.7,.8,.9)]	[(.3,.4,.4,.4,.4,.4) ,(.2,.2,.2,.2,.2,.2) (.4,.4,.5,.5,.5,.5)]	[(.8,.8,.8,.8,.8,.8) ,(.2,.2,.3,.3,.4,.4) (.2,.3,.3,.4,.4,.5)]
	A2	[(.5,.5,.5,.5,.5,.5), (.2,.3,.4,.5,.6,.7), (.5,.5,.5,.5,.5,.5)]	[(.4,.5,.6,.7,.8,.8), (.3,.3,.3,.4,.4,.4), (.2,.3,.4,.5,.6,.7)]	[(.5,.6,.6,.7,.7,.8), (.3,.4,.5,.6,.6,.7), (.5,.5,.6,.7,.8,.8)]	[(.8,.8,.9,.9,.9,.9), (.2,.2,.3,.3,.4,.4), (.2,.3,.3,.4,.4,.5)]
	A3	[(.2,.3,.4,.5,.6,.7), (.6,.6,.6,.6,.6,.6), (.2,.2,.2,.3,.4,.5)]	[(.5,.6,.7,.8,.9,.9), (.3,.4,.4,.4,.4,.4), (.4,.4,.4,.5,.5,.5)]	[(.1,.3,.3,.3,.4,.4), (.3,.3,.3,.3,.3,.3), (.4,.5,.5,.6,.6,.6)]	[(.6,.7,.7,.8,.8,.8), (.1,.2,.3,.4,.5,.6), (.4,.4,.5,.5,.6,.6)]
	A4	[(.3,.4,.4,.5,.5,.6), (.1,.1,.1,.2,.2,.2), (.1,.2,.3,.4,.5,.6)]	[(.4,.4,.5,.6,.7,.7), (.2,.2,.3,.3,.4,.4), (.1,.2,.3,.4,.4,.5)]	[(.8,.9,.9,.9,.9,.9), (.1,.1,.2,.2,.3,.3), (.1,.1,.1,.1,.1,.1)]	[(.5,.5,.5,.6,.6,.7), (.2,.2,.2,.3,.3,.3), (.3,.3,.4,.5,.6,.7)]
D3	A1	[(.1,.2,.2,.2,.3,.3), (.1,.1,.1,.2,.3,.4), (.5,.6,.7,.7,.7,.7)]	[(.4,.4,.5,.6,.7,.8), (.4,.5,.6,.6,.7,.7), (.3,.4,.4,.5,.5,.5)]	[(.6,.7,.8,.8,.9,.9), (.3,.3,.3,.4,.4,.5), (.4,.5,.6,.6,.6,.6)]	[(.3,.3,.4,.4,.4,.5), (.4,.5,.6,.7,.8,.9), (.3,.4,.4,.5,.5,.6)]
	A2	[(.4,.5,.5,.6,.6,.7) ,(.4,.4,.5,.6,.6,.7) (.4,.5,.6,.7,.8,.9)]	[(.7,.7,.7,.8,.8,.8), (.5,.6,.6,.6,.6,.6), (.4,.4,.5,.5,.6,.6)]	[(.6,.7,.7,.8,.8,.8), (.4,.5,.5,.6,.6,.7), (.2,.3,.4,.5,.6,.7)]	[(.2,.2,.2,.3,.3,.3), (.2,.2,.2,.3,.4,.5), (.4,.5,.5,.6,.7,.8)]
	A3	[(.6,.7,.7,.8,.9,.9), (.5,.6,.7,.7,.8,.8), (.3,.4,.5,.5,.6,.6)]	[(.4,.5,.6,.7,.8,.9), (.7,.7,.7,.8,.8,.8), (.1,.2,.3,.4,.5,.6)]	[(.8,.8,.8,.9,.9,.9), (.5,.5,.6,.6,.7,.7), (.2,.3,.3,.4,.4,.5)]	[(.6,.7,.7,.8,.8,.9), (.1,.1,.1,.1,.1,.1), (.2,.2,.2,.3,.3,.3)]
	A4	[(.6,.6,.6,.7,.7,.7), (.4,.5,.6,.6,.7,.7), (.4,.5,.5,.5,.5,.6)]	[(.8,.8,.8,.9,.9,.9), (.4,.4,.4,.4,.4,.4), (.3,.4,.4,.5,.5,.5)]	[(.3,.4,.5,.6,.7,.8), (.5,.5,.5,.5,.5,.5), (.4,.5,.5,.6,.7,.8)]	[(.2,.3,.3,.3,.4,.4), (.6,.6,.6,.7,.7,.7), (.8,.8,.8,.9,.9,.9)]

Table 5.2 Opinion of decision makers on performance values

Step 2. Normalize the hexagonal neutrosophic decision matrix $\tilde{R}^k = (\tilde{r}_{ij}^k)_{m\times n}$ given by the experts $D_k\ (k=1,2,3)$ to get the matrix $\tilde{R}' = (\tilde{r}'_{ij})_{m\times n}$

Criteria				
Alternative	C_1	C_2	C_3	C_4

$\bar{A}_1'$	[(.3,.4,.5,.6,.7,.7), (.1,.1,.2,.2,.3,.3), (.4,.5,.6,.6,.7,.7)]	[(.3,.3,.4,.5,.6,.7), (.3,.4,.4,.5,.6,.6), (.4,.5,.6,.7,.7,.7)]	[(.6,.6,.7,.7,.7,.7), (.3,.3,.3,.4,.4,.5), (.3,.3,.4,.4,.4,.4)]	[(.4,.5,.5,.6,.6,.7), (.2,.3,.3,.4,.4,.4), (.4,.4,.4,.5,.5,.6)]
$\bar{A}_2'$	(.4,.5,.5,.6,.6,.7), (.3,.3,.4,.5,.6,.7), (.5,.5,.6,.7,.7,.7)]	[(.4,.5,.5,.6,.6,.6), (.4,.5,.5,.6,.6,.6), (.4,.4,.5,.6,.6,.7)]	[(.6,.7,.7,.8,.8,.8), (.3,.4,.4,.5,.6,.7), (.3,.3,.4,.5,.5,.6)]	[(.5,.5,.6,.6,.6,.7). (.2,.2,.3,.3,.4,.4), (.2,.3,.3,.4,.5,.6)]
$\bar{A}_3'$	[(.4,.5,.6,.7,.8,.8), (.4,.5,.5,.6,.6,.7), (.2,.3,.3,.3,.4,.4)]	[(.3,.4,.5,.5,.6,.6), (.4,.5,.5,.6,.6,.7), (.3,.3,.4,.5,.5,.6)]	[(.3,.4,.5,.6,.6,.7), (.4,.4,.4,.5,.6,.6), (.3,.4,.4,.5,.5,.6)]	[(.7,.8,.8,.8,.8,.9), (.1,.2,.2,.3,.4,.4), (.3,.3,.4,.4,.5,.5)]
$\bar{A}_4'$	[(.3,.4,.4,.5,.5,.5), (.2,.3,.3,.4,.5,.5), (.3,.4,.5,.6,.6,.7)]	[(.5,.5,.6,.6,.7,.7), (.2,.3,.3,.3,.4,.4), (.3,.4,.4,.5,.5,.5)]	[(.4,.5,.6,.6,.7,.8), (.3,.3,.4,.4,.5,.5), (.3,.3,.3,.4,.4,.5)]	[(.4,.5,.5,.5,.6,.6), (.4,.4,.4,.5,.5,.5), (.4,.4,.5,.6,.6,.6)]

Table 5.3 Normalized hexagonal neutrosophic decision matrix

Step 3.

Once the decision makers provide the decision matrix we calculate the relative weight of each criterion C_j Consider the weight of each criterion as $w=(0.15,0.15,0.20,0.50)$

$$w_r = \max\{w_j / j=1,2,.....n\}$$
$$w_r = \max\{0.15,0.15,0.20,0.50\}$$
$$w_r = 0.50$$

Since $w_r = 0.50$ then C_4 is the reference criterion and the reference criterion weight is 0.50. Then calculate the relative weights of the criterion $C_j (j=1,2,3,4)$ as

$$w_{1r} = \frac{w_1}{w_r} = \frac{0.15}{0.50} = 0.3, w_{2r} = 0.3, w_{3r} = 0.4, w_{4r} = 1$$

The parameter υ the dilution factor of the loss is

$$\upsilon = \sum_{j=1}^{4} w_{jr} = 0.3+0.3+0.4+1 = 2$$

Step 4. Consider the alternative A_1 of DM_1 and the criteria C_1

Calculate the distance between $\bar{A}_1$ *and* $\bar{A}_1'$, $\bar{A}_1$ *and* $\bar{A}_1'^C$ *of* DM_1

$\bar{C}_1 = [(.1,.2,.3,.4,.5,.6),(.1,.1,.1,.1,.1,.1)\ (.5,.6,.7,.8,.9,.9)]$, $\bar{C}_1' = [(.3,.4,.4,.5,.6,.6),(.1,.1,.2,.2,.3,.3)\ (.4,.5,.6,.6,.7,.7)]$

$\bar{C}_1'^C = [(.4,.5,.6,.6,.7,.7)\ (.9,.9,.8,.8,.7,.7)\ (.3,.4,.4,.5,.6,.6)]$ $d(\bar{C}_1,\bar{C}_1') = \frac{1}{18}(2.2)$, $d(\bar{C}_1,\bar{C}_1'^C) = \frac{1}{18}(6.9)$

Step 5. The Degree of Similarity between $\bar{A}_1$ *and* $\bar{A}'$ is defined as follows.

$$\theta(\overline{C}_1, \overline{C}_1') = \left\{ \frac{d(\overline{C}_1, \overline{C}_1'^C)}{d(\overline{C}_1, \overline{C}_1') + d\left(\overline{C}_1, \overline{C}_1'^C\right)} \right\} = \frac{\frac{1}{18}(6.9)}{\frac{1}{18}(2.2+6.9)} = 0.76$$

Continuing the above process for all decision makers the consolidated Degree of Similarity is tabulated below.

Degree of Similarity	A_1 of DM_1	A_2 of DM_1	A_3 of DM_1	A_4 of DM_1	A_1 of DM_2	A_2 of DM_2	A_3 of DM_2	A_4 of DM_2	A_1 of DM_3	A_2 of DM_3	A_3 of DM_3	A_4 of DM_3
$\theta(\overline{C}_1, \overline{C}_1')$	**0.76**	**0.70**	**0.76**	**0.68**	**0.56**	**0.69**	**0.66**	**0.51**	**0.77**	**0.77**	**0.66**	**0.42**
$\theta(\overline{C}_1, \overline{C}_2')$	**0.68**	**0.53**	**0.53**	**0.57**	**0.74**	**0.56**	**0.50**	**0.81**	**0.45**	**0.57**	**0.59**	**0.72**
$\theta(\overline{C}_3, \overline{C}_3')$	**0.66**	**0.61**	**0.60**	**0.48**	**0.54**	**0.61**	**0.38**	**0.65**	**0.72**	**0.83**	**0.56**	**0.45**
$\theta(\overline{C}_4, \overline{C}_4')$	**0.62**	**0.57**	**0.81**	**0.50**	**0.68**	**0.78**	**0.80**	**0.51**	**0.44**	**0.54**	**0.80**	**0.45**

Table 5.4 Degree of Similarity between the alternatives compared with the criteria

Step 6. Calculate the weight vectors of the decision makers using degree of similarity

$$\theta(\tilde{R}^{(k)}, \tilde{R}') = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} \theta(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}')$$

$$\theta(\tilde{R}^{(1)}, \tilde{R}') = \frac{10.524}{12} = .877, \theta(\tilde{R}^{(2)}, \tilde{R}') = \frac{10..097}{12} = 0.8814, \theta(\tilde{R}^{(3)}, \tilde{R}') = \frac{9.9000}{12} = 0.825$$

$$w^{(1)} = \frac{(\theta(\tilde{R}^{(1)}, \tilde{R}')}{\sum_{k=1}^{t} (\theta(\tilde{R}^{(k)}, \tilde{R}')} = \frac{.877}{2.59} = .34 \quad w^{(2)} = \frac{(\theta(\tilde{R}^{(2)}, \tilde{R}')}{\sum_{k=1}^{t} (\theta(\tilde{R}^{(k)}, \tilde{R}'))} = \frac{.8814}{2.59} = .33$$

$$w^{(3)} = \frac{(\theta(\tilde{R}^{(2)}, \tilde{R}')}{\sum_{k=1}^{t} (\theta(\tilde{R}^{(k)}, \tilde{R}'))} = \frac{.825}{2.59} = .33$$

Step 7. Using equation (1) HNWA the aggregated decision matrix is as follows.

	Criteria			
Alternative	C_1	C_2	C_3	C_4
$\overline{A}_1$	[(.4,.5,.6,.7,.8,.9), (0,.1,.2,.3,.4,.5), (.1,.1,.1,.1,.1,.1)]	[(0,.1,.2,.3,.4,.5) (0,.1,.2,.3,.4,.5) (.2,.3,.4,.5,.6,.7)]	[(.3,.4,.5,.6,.7,.8), (0,.1,.2,.3,.4,.5), (.1,.1,.1,.1,.1,.1)]	[(.3,.4,.5,.6,.7,.8), (.1,.1,.1,.1,.1,.1), (.1,.2,.3,.4,.5,.6)]
$\overline{A}_2$	[(.3,.4,.5,.6,.7,.8), (.1,.1,.2,.2,.3,.3), (0,.1,.1,.1,.2,.2)]	[(.1,.2,.3,.4,.5,.6), (0,.1,.1,.2,.3,.3), (0,.1,.1,.1,.2,.2)]	[(0,.1,.1,.2,.2,.2), (.1,.1,.1,.1,.1,.1), (.5,.6,.6,.7,.7,.8)]	[(.3,.4,.5,.5,.6,.6), (0,0,.1,.1,.1,.1), (.1,.1,.1,.1,.1,.1)]

$\overline{A}_3$	[(.1,.1,.1,.1,.2,.2), (.2,.2,.2,.2,.3,.3), (.4,.4,.5,.5,.6,.6)]	[(.2,.2,.2,.3,.3,.3), (0,.1,.2,.2,.3,.3), (.4,.5,.6,.7,.8,.9)]	[(.3,.4,.5,.6,.7,.8), (0,.1,.2,.3,.4,.5), (.1,.2,.2,.2,.3,.3)]	[(.1,.2,.3,.4,.5,.6), (.2,.2,.2,.2,.2,.2), (.4,.5,.6,.7,.8,.9)]
$\overline{A}_4$	[(.1,.2,.3,.4,.5,.6), (0,.1,.1,.1,.2,.4), (.1,.1,.2,.2,.4,.4)]	[(.5,.5,.5,.5,.5,.5), (.2,.3,.3,.4,.4,.5), (.1,.1,.1,.1,.2,.2)]	[(.4,.5,.5,.6,.7,.8), (.1,.1,.1,.1,.1,.1), (.1,.2,.3,.4,.4,.4)]	[(.1,.1,.1,.1,.1,.1), (0,.1,.1,.2,.3,.3), (.5,.5,.6,.6,.7,.8)]

Table. 5.5 Aggregated decision matrix

Step 8 . Calculate the score value using the equation (6)

$$S(A)=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\left(\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ 0.59 & 0.48 & 0.48 & 0.62 \\ 0.49 & 0.47 & 0.65 & 0.66 \\ 0.58 & 0.54 & 0.56 & 0.73 \\ 0.55 & 0.64 & 0.67 & 0.54 \end{array}\right)$$

Step 9. Using the score function we check for the conditions and find $\left[d\left(r_{ij},r_{pj}\right)\right]$

Here we consider $j=1, i=1,2,3,4$ *and* $p=1,2,3,4$ and check for the conditions in (7)

1) $\tilde{r}_{ij}>\tilde{r}_{pj}$ *or* 2) $\tilde{r}_{ij}=\tilde{r}_{pj}$ *or* 3) $\tilde{r}_{ij}<\tilde{r}_{pj}$ *for* $i=1,2,3,4$ $j=1$ *and* $p=1,2,3,4$

$$\left[d\left(r_{ij},r_{pj}\right)\right]_{m\times n}=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\left[\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ 0 & 0.1222 & 0.2266 & 0.1055 \\ 0.1222 & 0 & 0.1666 & 0.111 \\ 0.2266 & 0.1666 & 0 & 0.2111 \\ 0.1055 & 0.111 & 0.2111 & 0 \end{array}\right]$$

To construct the dominance matrix we check for $(\tilde{r}_{ij}\ is >,<or=to\,\tilde{r}_{pj})$

Since we have

$(r_{11}=r_{11})$, $\varepsilon_1(A_1,A_1)=0$ and as $(r_{11}>r_{21})$ $\varepsilon_1(A_1,A_2)=\sqrt{\dfrac{w_{1r}\,d(r_{11},r_{21})}{\sum_{j=1}^{4} w_{jr}}}=0.1105$

and $(r_{21}<r_{11})$, $\varepsilon_1(A_2,A_1)=\sqrt{\dfrac{\sum_{j=1}^{4} w_{jr}d(r_{21},r_{11})}{w_{1r}}}=-0.4401$

Using equation (7) calculate the dominance matrix $\varepsilon_1\left(A_i,A_p\right)$ as follows.

$$\left[\varepsilon_1\left(A_i,A_p\right)\right]_{m\times n}=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\left[\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ 0 & 0.1105 & 0.1632 & 0.1027 \\ -0.4401 & 0 & -0.5155 & -0.4214 \\ -0.6523 & 0.1290 & 0 & 0.1452 \\ -0.4108 & 0.1053 & -0.5810 & 0 \end{array}\right]$$

Similarly for the values $j=2, i=1,2,3,4$ *and* $p=1,2,3,4$,

$j=3, i=1,2,3,4$ *and* $p=1,2,3,4$ and $j=4, i=1,2,3,4$ *and* $p=1,2,3,4$ the dominance matrix are calculated.

$$\left[\varepsilon_2\left(A_i, A_p\right)\right]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{bmatrix} C_1 & C_2 & C_3 & C_4 \\ 0 & 0.1393 & -0.2508 & -0.3154 \\ -0.2229 & 0 & -0.1977 & -0.3040 \\ 0.1624 & 0.1235 & 0 & -0.2598 \\ 0.1971 & 0.1900 & 0.1624 & 0 \end{bmatrix}$$

$$\left[\varepsilon_3\left(A_i, A_p\right)\right]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{bmatrix} C_1 & C_2 & C_3 & C_4 \\ 0 & 0.1581 & 0.1786 & 0.1624 \\ -0.2529 & 0 & 0.1624 & -0.2304 \\ -0.2858 & -0.2598 & 0 & -0.2665 \\ -0.2598 & 0.1440 & 0.1440 & 0 \end{bmatrix}$$

Step 10. On the basics of the above equation the overall dominance degree is obtained as

$$\rho(A_i, A_p) = \sum_{j=1}^{n} \varepsilon_j (A_i, A_p), (i, p = 1,2,....m)$$

$$\delta = \begin{bmatrix} 0 & 0.238 & -0.1331 & -0.1656 \\ -0.7789 & 0 & -0.7508 & -0.7298 \\ -0.5598 & 0.1927 & 0 & -0.0963 \\ -0.656 & 0.2133 & -0.8454 & 0 \end{bmatrix}$$

Now $\sum_{j=1}^{4} \varepsilon_j (A_i, A_p), (i, p = 1,2,....m$ are (0.2363,-2.2266,-0.4654,-1.000)

Step 12. Then the overall value of each A_i can be calculated using the equation (9)

$\rho(A_1) = 1.000, \quad \rho(A_2) = 0, \quad \rho(A_3) = 0.7150, \quad \rho(A_4) = 0.4980$

Step 13. Ranking the values of all alternatives $\rho(A_i)$ and selecting the most desirable alternatives in accordance with $\rho\left(A_i\right)$, among the four alternatives A_1 is the best choice and the ranking order is

$A_1 > A_3 > A_4 > A_2$

Stage 2.

Step 7. Floor space requirement C_1, Feeding (Feeder) and watering space requirement C_2 are benefiting type criteria $B_1 = \langle C_1, C_2 \rangle$. Maintenance of health and sanitization C_3 and Productivity C_4 are cost type $B_2 = \langle C_3, C_4 \rangle$ .The hexagonal neutrosophic positive-ideal solution B^+ and hexagonal neutrosophic negative-ideal solution B^- are obtained as follows

$$B^+ = \left[\left\langle \begin{matrix} (1,1,1,1,1,1)(0,0,0,0,0,0)(0,0,0,0,0,0), (1,1,1,1,1,1)(0,0,0,0,0,0)(0,0,0,0,0,0), \\ ((0,0,0,0,0,0)(1,1,1,1,1,1)(1,1,1,1,1,1)), ((0,0,0,0,0,0)(1,1,1,1,1,1)(1,1,1,1,1,1)) \end{matrix} \right\rangle \right]$$

$$B^- = \left[\left\langle \begin{matrix} (0,0,0,0,0,0)(1,1,1,1,1,1)(1,1,1,1,1,1), (0,0,0,0,0,0)(1,1,1,1,1,1)(1,1,1,1,1,1) \\ (, (1,1,1,1,1,1)(0,0,0,0,0,0)(0,0,0,0,0,0)), ((1,1,1,1,1,1)(0,0,0,0,0,0)(0,0,0,0,0,0)) \end{matrix} \right\rangle \right]$$

Step 8. The vector of the attribute weight is $w = (0.15, 0.15, 0.20, 0.50)$. By using equation (10) calculate the separation measure S_i^+ of the each alternative from the hexagonal neutrosophic positive ideal solution where $d(r_{ij}, r_j^+)$ is calculated using equation (2).

The calculated values are as follows

$S_1^+ = 0.1482$ $S_2^+ = 0.1408$ $S_3^+ = 0.1370$ $S_4^+ = 0.1164$

By using equation (11) calculate the separation measure S_i^- of the each alternative from the hexagonal neutrosophic negative ideal solution. The calculated values are as follows

$S_1^- = 0.0989$ $S_2^- = 0.1094$ $S_3^- = 0.1129$ $S_4^- = 0.1335$

Step 9. Using equation (12) calculate the relative closeness coefficient of the hexagonal neutrosophic ideal solution. The relative closeness coefficient values are as follows
$C_1 = 0.4002$ $C_2 = 0.4372$ $C_3 = 0.4517$ $C_4 = 0.5342$

Step 10. Rank the alternatives in the decreasing order of closeness coefficient values.

$$A_4 > A_3 > A_2 > A_1$$

6. Graphical Representation of the Comparative study

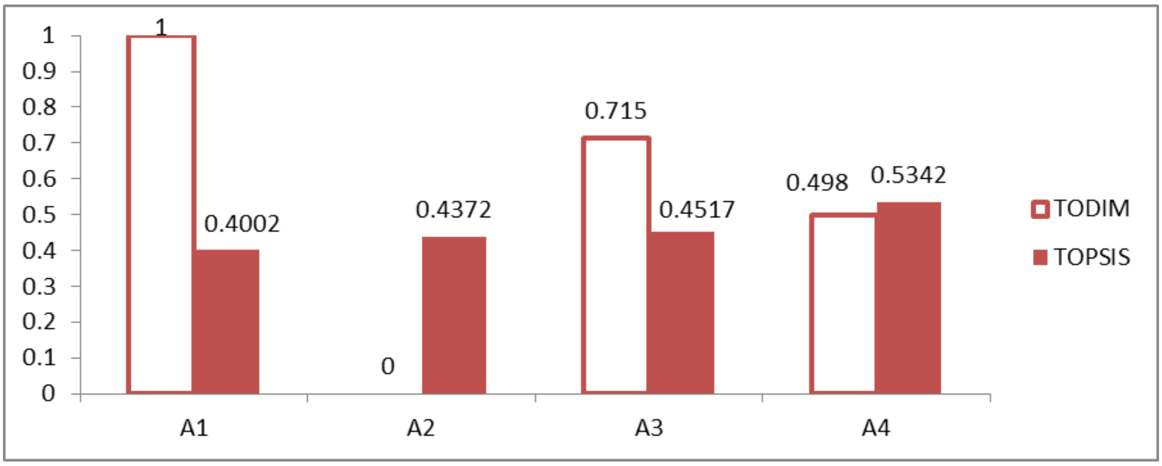

Figure 6.1 Ranking of the four alternatives using TODIM and TOPSIS

- The ranking results of TODIM show that A_1 is the best alternative with maximum global value $\rho(A_1)=1$ and the least global value is $\rho(A_2)=0$ The ranking of the four alternatives using TODIM is $A_1 > A_3 > A_4 > A_2$
- The ranking result using TOPSIS shows that A_4 is the best suited alternative as it ranking is in first position and A_1 is considered to be last as it takes fourth position in ranking .
- The ranking of the four alternatives using TOPSIS is $A_4 > A_3 > A_2 > A_1$.
- In both the methods A_3 take the same position and A_4 is in the third level in TODIM which is nearest to the ranking of TOPSIS. Similarly, A_2 is in the fourth level in TODIM which is very close to the ranking of TOPSIS.
- Both the MCDM ranking results shows that they are similar by large percentage which provides decision maker to increase the flexibility in choosing the optimal alternative.

Conclusion

The research presented in this article is an assessment study of the sustainability of commercial goat farming and its recent impact on self-employment for youth has been carried out in a context characterized by two MCDM methods, TODIM and TOPSIS. Using those methods the social, economic and ecological sustainability in housing and feeding systems of goat farming are evaluated by three experts and the evaluation was considered as hexagonal neutrosophic numbers in order to remove the ambiguity and increase the accuracy in the decision making process. Using the TODIM approach which is able to distinguish between risks based alternative and definite alternative in uncertain circumstances is analyzed .At the same time, by using the TOPSIS method the ranking is performed based on distance of each alternatives to its positive and negative ideal solutions. The ranking results of TODIM show a large percentage of similarity with ranking resulting from TOPSIS.The result shows that stall feeding system with normal flooring and a part with both intensive and extensive grazing system are best suited for sustainable commercial goat farming. This study may be applied in several other fields like livestock management systems with technology adaptation as well as in the economics of goat farming and other livestock sectors..

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