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Single Valued Bipolar Pentapartitioned Neutrosophic Set and Its Application in MADM Strategy

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Abstract

The main objective of this paper is to introduce the notion of single-valued bipolar pentapartitioned neutrosophic set (SVBPNS). We also present some supporting examples and prove some basic properties of SVBPNS. We define score function and accuracy function of SVBPNS, and establish their basic properties. We define the single-valued bipolar pentapartitioned neutrosophic arithmetic mean (SVBPNAM) operator and the single-valued bipolar pentapartitioned neutrosophic geometric mean (SVBPNGM) operator and prove their basic properties. We develop two Multi-Attribute Decision Making (MADM) strategies namely SVBPNS-MADM Strategy based on SVBPNAM operator and SVBPNS-MADM strategy based on SVBPNGM operator under SVBPNS environment. Finally, we present a real world numerical example to illustrate the developed strategies.

Keywords: Single-Valued Pentapartitioned Neutrosophic Set; SVBPNS; MADM-Strategy.

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1. Introduction

Smarandache [1] defined the Neutrosophic Set (NS) to deal with uncertainty, indeterminacy and inconsistency involved in this real world of mathematical objects. NS is the generalization of Fuzzy Set (FS) [2] and intuitionistic fuzzy set (IFS) [3] by incorporating degrees of indeterminacy and rejection (falsity or non-membership) as independent components. In 2010, Wang et al. [4] defined Single Valued NS (SVNS). The SVNSs, its variants and extensions have been utilized in many areas such as air surveillance [5], conflict resolution [6], decision making [7-12] fault diagnosis [13], image segmentation [14], and so on. Details applications and theoretical developments of NSs are depicted in the studies [15-20].

Deli et al. [21] introduced the Single Valued Bipolar NS (SVBNS). Later on, so many researchers applied the notion of SVBNS in the model formation for Multi Attribute Decision making (MADM) [22-26] problems. In 2020, Mallick and Pramanik [27] grounded the notion of Pentapartitioned Neutrosophic Set (PNS) in which five independent components were introduced. In 2021, Das et al. [28] established an MADM strategy using tangent similarity measure under single valued PNS Environment. Recently, Das et al. [29] proposed an MADM strategy based on Grey Relational Analysis (GRA) under the single valued PNS Environment.

Research gap: No report of the investigation dealing with the combination of bipolar neutrosophic set and PNS has been appeared in the literature.

Motivation of the study: The research gap motives us to investigate the possible combination of bipolar neutrosophic set and PNS.

In this study, we introduce the Single-Valued Bipolar Pentapartitioned Neutrosophic Set (SVBPNS) by combing SVBNS and PNS. Then, we establish some basic properties of SVBPNS. Also, few illustrative examples on the SVBPNS are provided. Further, we propose some aggregation operators and prove their basic properties. Also, we develop two new MADM strategies under the SVBPNS environment.

The organization of the remaining part of this article is described as follows:
Section 2 presents some relevant results on PNS. Section 3 devotes to introduce the SVBPNS. In Section 4, we introduce two aggregation operators, namely, single-valued bipolar pentapartitioned neutrosophic arithmetic mean operator and single-valued bipolar pentapartitioned neutrosophic geometric mean operator under the SVBPNS environment. In Section 5, we procure the notion of score function and accuracy function under SVBPNS Environment. In Section 6, we develop an MADM strategy using the single-valued bipolar pentapartitioned neutrosophic arithmetic mean operator under SVBPNS environment. Further, in Section 7, we establish an MADM strategy using the single-valued bipolar pentapartitioned neutrosophic geometric mean operator under SVBPNS environment. In Section 8, we validated the proposed MADM strategies by providing a real world numerical example, and also comparing both the MADM strategies. Finally, in Section 9, we conclude the paper by stating future scope research in newly defined set environment.

2. Some Preliminary Results

We recall some basic definitions on NS, Bipolar NS, and PNS, which are relevant to the main results of this paper.

Definition 2.1.[1]. An NS V over a fixed set ψ is defined as follows:

$$V=\{(\mu,T_V(\mu),I_V(\mu),F_V(\mu)):\mu\in\psi\},$$

where T, I, F : $\psi \rightarrow]^{-}0, 1^{+}[$ are the truth, indeterminacy and falsity membership functions respectively and

.

Example 2.1. Suppose that $\psi = \{x, y\}$ be a fixed set. Then, U={(x,0.2,0.8,0.8), (y,0.3,0.2,0.4)} is an NS over ψ.

Definition 2.2.[21]. A BNS U over a non-empty set ψ is defined as follows:

$$U=\{(\mu,\ T_U^+(\mu),\ I_U^+(\mu),\ F_U^+(\mu),\ T_U^-(\mu),\ I_U^-(\mu),\ F_U^-(\mu)):\mu\in\psi\},$$

where $T_U^+(\mu)$, $I_U^+(\mu)$, $F_U^+(\mu)\in[0, 1]$, and $T_U^-(\mu)$, $I_U^-(\mu)$, $F_U^-(\mu)\in[-1, 0]$.

Here, $T_U^+(\mu)$, $I_U^+(\mu)$, and $F_U^+(\mu)$ denote the positive degree of truth-membership, indeterminacy-membership, falsity-membership respectively for $\mu\in\psi$ corresponding to the BNS U and $T_U^-(\mu)$, $I_U^-(\mu)$,and $F_U^-(\mu)$ denote the negative degree of truth-membership, indeterminacy-membership, falsity-membership respectively of $u\in\psi$ corresponding to the BNS U.

Example 2.2. Suppose that $\psi=\{x, y\}$ be a fixed set. Then, U={(x,0.1,0.6,0.8,-0.3,-0.4,-0.7), (y,0.3,0.4,0.6,-0.5,-0.4,-0.5)} is a bipolar neutrosophic set over ψ.

Definition 2.3.[21]. Assume that $U=\{(\mu,\ T_U^+(\mu),\ I_U^+(\mu),\ F_U^+(\mu),\ T_U^-(\mu),\ I_U^-(\mu),\ F_U^-(\mu)):\mu\in\psi\}$ be a BNS. Then, for each $\mu\in\psi$, $[T_U^+(\mu),\ I_U^+(\mu),\ F_U^+(\mu),\ T_U^-(\mu),\ I_U^-(\mu),\ F_U^-(\mu)]$ is called a Single Valued Bipolar Neutrosophic Number (SVBNN).

Definition 2.4.[27]. Assume that ψ be a fixed set. A PNS Z over ψ is defined by:

$$Z = \{(\mu,\ T_Z(\mu),\ C_Z(\mu),\ G_Z(\mu),\ U_Z(\mu),\ F_Z(\mu)):\mu\in\psi\},$$

where $T_Z(\mu)$, $C_Z(\mu)$, $G_Z(\mu)$, $U_Z(\mu)$, and $F_Z(\mu)\in[0, 1]$ are the truth, contradiction, ignorance, unknown and falsity membership values for each $\mu\in\psi$. So,

$$0 \le T_Z(\mu)+C_Z(\mu)+G_Z(\mu)+U_Z(\mu)+F_Z(\mu) \le 5.$$

Definition 2.5.[27]. Suppose that $M = \{(\mu, T_M(\mu), C_M(\mu), G_M(\mu), U_M(\mu), F_M(\mu)):\mu\in\psi\}$ and $N = \{(\mu, T_N(\mu), C_N(\mu), G_N(\mu), U_N(\mu), F_N(\mu)):\mu\in\psi\}$ be any two PNSs over ψ. Then, $M\subseteq N \Leftrightarrow T_M(\mu)\le T_N(\mu)$, $C_M(\mu)\le C_N(\mu)$, $G_M(\mu)\ge G_N(\mu)$, $U_M(\mu)\ge U_N(\mu)$, $F_M(\mu)\ge F_N(\mu)$, for all $\mu\in\psi$.

Definition 2.6.[27]. The null PNS (0_{PN}) and the absolute PNS (1_{PN}) over ψ are defined as follows:

(i) 0_{PN}={(μ, 0, 0, 1, 1, 1): $\mu\in\psi$};

(ii) 1_{PN}={(μ, 1, 1, 0, 0, 0): $\mu\in\psi$};

It is clearly seen that, $0_{PN}\subseteq X\subseteq 1_{PN}$, where X is a PNS over ψ.

Example 2.3. Consider a PNS X={(n,0.3,0.4,0.5,0.7,0.3), (m,0.3,0.6,0.4,0.8,0.4)} and Y={(n,0.4,0.7,0.1,0.5, 0.2), (m,0.8,0.9,0.2,0.1,0.2)} over $\psi=\{n, m\}$. Then, $X\subseteq Y$.

Definition 2.7.[27]. Suppose that $M = \{(\mu, T_M(\mu), C_M(\mu), G_M(\mu), U_M(\mu), F_M(\mu)): \mu\in\psi\}$ and $N = \{(\mu, T_N(\mu), C_N(\mu), G_N(\mu), U_N(\mu), F_N(\mu)): \mu\in\psi\}$ be any two PNSs over ψ. Then, their intersection $X\cap Y = \{(\mu, \min\{T_M(\mu), T_N(\mu)\}, \min\{C_M(\mu), C_N(\mu)\}, \max\{G_M(\mu), G_N(\mu)\}, \max\{U_M(\mu), U_N(\mu)\}, \max\{F_M(\mu), F_N(\mu)\}): \mu\in\psi\}$.

Example 2.4. Consider two PNSs X= {(n,0.4,0.3,0.7,0.4,0.9), (m,0.5,0.6,0.3,0.8,0.4)} and Y={(n,0.6,0.2,0.8, 0.7,0.8), (m,0.5,0.8,0.7,0.4,0.8)} over $\psi = \{n, m\}$. Then, their intersection is:

$X \cap Y$ = {(n,0.4,0.2,0.8,0.7,0.9), (m,0.5,0.6,0.7,0.8,0.8)}.

Definition 2.8. [27]. Assume that M = {(μ, T_M(μ), C_M(μ), G_M(μ), U_M(μ), F_M(μ)):μ∈ψ} and N = {(μ, T_N(μ), C_N(μ), G_N(μ), U_N(μ), F_N(μ)):μ∈ψ} be two PNSs over ψ. Then, the union of X and Y is defined by:

$X \cup Y$ = {(μ, max {T_M(μ), T_N(μ)}, max {C_M(μ), C_N(μ)}, min {G_M(μ), G_N(μ)}, min {U_M(μ), U_N(μ)}, min {F_M(μ), F_N(μ)}):μ∈ψ}.

Example 2.5. Consider two PNSs X={(n,0.4,0.5,0.6,0.8,0.9), (m,0.8,0.5,0.9,1.0,0.5)} and Y={(n,0.6,0.7,0.0, 0.5,0.3), (m,1.0,0.9,0.4,0.0,0.1)} over ψ={n, m}. Then, their union is:

$X \cup Y$ = {(n,0.6,0.7,0.0,0.5,0.3), (m,1.0,0.9,0.4,0.0,0.1)}.

Definition 2.9.[27]. Suppose that M = {(μ, T_M(μ), C_M(μ), G_M(μ), U_M(μ), F_M(μ)): μ∈ψ} a PNS over ψ. Then, the complement of M is defined by:

M^c = {(μ, F_M(μ), U_M(μ), 1-G_M(μ), C_M(μ), T_M(μ)): μ∈ψ}.

Example 2.6. Suppose that M={(n,0.5,0.7,0.9,0.7,0.9), (m,0.8,0.1,0.5,0.7,0.0)} be an PNS over a fixed set ψ={n, m}. Then, M^c={(n,0.9,0.7,0.1,0.7,0.5), (m,0.0,0.7,0.5,0.1,0.8)}.

Definition 2.10. Suppose that $u_1, u_2, \ldots, u_n$ be n real numbers. Then, the arithmetic mean (AM) of $u_1, u_2, \ldots, u_n$ is defined by AM ($u_1, u_2, \ldots, u_n$) = $\frac{1}{n}\sum_{i=1}^{n} u_i$.

Definition 2.11. Suppose that $u_1, u_2, \ldots, u_n$ be n real numbers. Then, the geometric mean (GM) of $u_1, u_2, \ldots, u_n$ is defined by GM ($u_1, u_2, \ldots, u_n$) = $(\prod_{i=1}^{n} u_i)^{\frac{1}{n}}$.

3. Single-Valued Bipolar Pentapartitioned Neutrosophic Set

In this section, we procure the notion of SVBPNS. Also, we investigate some different properties of these kind of sets. Also, few illustrative examples are given.

Definition 3.1. A single-valued bipolar pentapartitioned neutrosophic set N over a non-empty set ψ is defined as:

N={(μ, T_N^-(μ), C_N^-(μ), G_N^-(μ), U_N^-(μ), F_N^-(μ), T_N^+(μ), C_N^+(μ), G_N^+(μ), U_N^+(μ), F_N^+(μ)):μ∈ψ}, where T_N^- (μ), C_N^-(μ), G_N^- (μ), U_N^- (μ), F_N^-(μ) ∈ [-1,0] and T_N^+ (μ), C_N^+(μ), G_N^+ (μ), U_N^+ (μ), F_N^+(μ) ∈ [0,1].

The negative membership degrees T_N^-(μ), C_N^-(μ), G_N^-(μ), U_N^-(μ), *and* F_N^-(μ) indicate the degree of truth-membership, contradiction-membership, ignorance-membership, unknown-membership, falsity-membership respectively for μ∈ψ corresponding to an SVBPNS N. Again, the positive membership degrees, T_N^+ (μ), C_N^+ (μ), G_N^+ (μ), U_N^+ (μ), and F_N^+ (μ) indicate the degree of truth-membership, contradiction-membership, ignorance-membership, unknown-membership, falsity-membership respectively for n∈ψ corresponding to an SVBPNS N.

Example 3.1. Let ψ={n, m} be a fixed set. Then, U={(n,-0.2,-0.4,-0.3,-0.4,-0.7,0.1,0.6,0.8,0.4,0.1), (y,-0.5, -0.4,-0.5,-0.3,-0.2,0.5,0.1,0.3,0.4,0.6)} is an SVBPNS over ψ.

Definition 3.2. Let N={(μ, T_N^-(μ),C_N^-(μ),G_N^-(μ),U_N^-(μ),F_N^-(μ),T_N^+(μ),C_N^+(μ),G_N^+(μ),U_N^+(μ),F_N^+(μ)):μ∈ψ} be an SVBPNS. Then, [T_N^-(μ), C_N^-(μ), G_N^-(μ), U_N^-(μ), F_N^-(μ), T_N^+(μ), C_N^+(μ), G_N^+(μ), U_N^+(μ), F_N^+(μ)] is called a single-valued bipolar pentapartitioned neutrosophic number (SVBPNN), for each μ∈ψ.

Definition 3.3. Suppose that A = {(μ, T_A^-(μ), C_A^-(μ), G_A^-(μ), U_A^-(μ), F_A^-(μ), T_A^+(μ), C_A^+(μ), G_A^+(μ), U_A^+(μ), F_A^+(μ)): μ∈ψ} and B = {(μ, T_B^-(μ), C_B^-(μ), G_B^-(μ), U_B^-(μ), F_B^-(μ), T_B^+(μ), C_B^+(μ), G_B^+(μ), U_B^+(μ), F_B^+(μ)): μ∈ψ} be any two SVBPNSs over ψ. Then, A⊆B if and only if T_A^-(μ) ≤ T_B^-(μ), C_A^-(μ) ≥ C_B^-(μ), G_A^-(μ) ≥ G_B^-(μ), U_A^-(μ) ≥ U_B^- (μ), F_A^-(μ) ≥ F_B^-(μ), T_A^+(μ) ≤ T_B^+(μ), C_A^+(μ) ≥ C_B^+(μ), G_A^+(μ) ≥ G_B^+(μ), U_A^+(μ) ≥ U_B^+(μ), F_A^+(μ) ≥ F_B^+(μ), for all μ∈ψ.

Example 3.2. Consider two SVBPNSs X={(x,-0.2,-0.5,-0.3,-0.4,-0.3,0.3,0.4,0.5,0.7,0.3), (y,-0.3,-0.5,-0.4, -0.2,-0.4,0.3,0.6,0.4,0.8,0.4)} and Y={(x,-0.2,-0.6,-0.7,-0.5,-0.5,0.4,0.3,0.1,0.5,0.2), (y,-0.2,-0.6,-0.6,-0.3,-0.5, 0.8,0.5,0.2,0.1,0.2)} over ψ = {x, y}. Then, X⊆Y.

Definition 3.4. Suppose that A = {(μ, T_A^-(μ), C_A^-(μ), G_A^- (μ), U_A^- (μ), F_A^-(μ), T_A^+(μ), C_A^+(μ), G_A^+(μ), U_A^+(μ), F_A^+(μ)): μ∈ψ} and B = {(μ, T_B^- (μ), C_B^-(μ), G_B^- (μ), U_B^- (μ), F_B^-(μ), T_B^+ (μ), C_B^+(μ), G_B^+(μ), U_B^+(μ), F_B^+(μ)):μ∈ψ} are any two SVBPNSs over ψ. Then, the intersection of X and Y is defined by:

X∩Y = {(μ, min {T_A^-(μ), T_B^-(μ)}, max {C_A^-(μ), C_B^-(μ)}, max {G_A^-(μ), G_B^-(μ)}, max {U_A^-(μ), U_B^-(μ)}, max {F_A^-(μ), F_B^-(μ)}, min {T_A^+(μ), T_B^+(μ)}, max {C_A^+(μ), C_B^+(μ)}, max {G_A^+(μ), G_B^+(μ)}, max {U_A^+(μ), U_B^+(μ)}, max {F_A^+(μ), F_B^+(μ)}) : μ∈ψ}.

Example 3.3. Suppose that X and Y are two SVBPNSs over ψ= {x, y} such that X = {(x,-0.3,-0.7,-0.5, -0.1,-0.5,0.5,0.7,0.2,0.4,0.2), (y,-0.5,-0.1,-0.5,-0.3,-0.4,0.4,0.7,0.5,0.7,0.3)} and Y = {(x,-0.1,-0.7,-0.5,-0.4, -0.3,0.2,0.5,0.3,0.5,0.4), (y,-0.4,-0.5,-0.5,-0.2,-0.3,0.4,0.5,0.3,0.4,0.3)}. Then, their intersection is X∩Y = {(x,-0.3,-0.7,-0.5,-0.1,-0.3,0.2,0.7,0.3,0.5,0.4), (y,-0.5,-0.1,-0.5,-0.2,-0.3, 0.4,0.7,0.5,0.7,0.3)}.

Definition 3.5. Suppose that A = {(μ, T_A^-(μ), C_A^-(μ), G_A^- (μ), U_A^- (μ), F_A^-(μ), T_A^+(μ), C_A^+(μ), G_A^+(μ), U_A^+(μ), F_A^+(μ)):μ∈ψ }and B = {(μ, T_B^- (μ), C_B^-(μ), G_B^- (μ), U_B^- (μ), F_B^-(μ), T_B^+ (μ), C_B^+(μ), G_B^+(μ), U_B^+(μ), F_B^+(μ)): μ∈ψ } are any two SVBPNSs over ψ. Then, the union of X and Y is defined by:

X∪Y = {(μ, max {T_A^-(μ), T_B^-(μ)}, min {C_A^-(μ), C_B^-(μ)}, min {G_A^-(μ), G_B^-(μ)}, min {U_A^-(μ), U_B^-(μ)}, min {F_A^-(μ), F_B^-(μ)}, max {T_A^+(μ), T_B^+(μ)}, min {C_A^+(μ), C_B^+(μ)}, min {G_A^+(μ), G_B^+(μ)}, min {U_A^+(μ), U_B^+(μ)}, min {F_A^+(μ), F_B^+(μ)}) : μ∈ψ}.

Example 3.4. Suppose that X and Y be two SVBPNSs over ψ = {x, y} such that X = {(x,-0.4,-0.7,-0.5,-0.6,-0.7,0.5,0.7,0.5,0.2,0.3), (y,-0.1,-0.3,-0.7,-0.7,-0.4,0.4,0.7,0.8,0.6,0.4)} and Y = {(x, -0.2, -0.3,-0.4,-0.7,-0.6,0.3,0.8,0.5,0.4,0.7), (y,-0.7,-0.1,-0.4,-0.7,-0.6,0.7,0.8,0.6,0.7,0.9)}. Then, their union is X∪Y = {(x,-0.2,-0.7,-0.5,-0.7,-0.7,0.5,0.7,0.5,0.2,0.3), (y,-0.1,-0.3,-0.7,-0.7,-0.6,0.7,0.7,0.6,0.6,0.4).

Definition 3.6. Let A = {(μ,T_A^-(μ),C_A^-(μ),G_A^-(μ),U_A^-(μ),F_A^-(μ),T_A^+(μ),C_A^+(μ),G_A^+(μ),U_A^+(μ),F_A^+(μ)) : μ∈ψ} be an SVBPNSs over ψ. Then, the complement of A is defined as follows:

A^c= {(μ,-1-T_A^-(μ),-1-C_A^-(μ),-1-G_A^-(μ),-1-U_A^-(μ),-1-F_A^-(μ),1-T_A^+(μ),1-C_A^+(μ),1-G_A^+(μ),1-U_A^+(μ),1-F_A^+(μ)):μ∈ψ}.

Example 3.5. Suppose that A = {(x,-0.4,-0.7,-0.5,-0.6,-0.7,0.5,0.7,0.5,0.2,0.3), (y,-0.1,-0.3,-0.7,-0.7,-0.4,0.4, 0.7,0.8,0.6,0.4)} be an SVBPNS over ψ = {x, y}. Then, the complement of A is A^c ={(x,-0.6,-0.3,-0.5,-0.4, -0.3,0.5,0.3,0.5,0.8,0.7), (y,-0.9,-0.7,-0.3,-0.3,-0.6,0.6,0.3,0.2,0.4,0.6)}.

Definition 3.7. The null SVBPNS (0_{BPN}) and the absolute SVBPNS (1_{BPN}) over ψ are defined as follows:

(i) 0_{BPN} = {(μ, -1, 0, 0, 0, 0, 0, 1, 1, 1, 1) : μ∈ψ};

(ii) 1_{BPN} = {(μ, 0, -1, -1, -1, -1, 1, 0, 0, 0, 0) : μ∈ψ};

It is clearly seen that,

(i) $0_{BPN} \subseteq X \subseteq 1_{BPN}$, where X is an SVBPNS over ψ;

(ii) $0^c_{\mathbf{BPN}} = 1_{BPN}$ & $1^c_{\mathbf{BPN}} = 0_{BPN}$;

(iii) $0_{BPN} \cup 1_{BPN} = 1_{BPN}$;

(iv) $0_{BPN} \cap 1_{BPN} = 0_{BPN}$.

Definition 3.8. Suppose that μ=[T_ψ^-(μ),C_ψ^-(μ),G_ψ^-(μ),U_ψ^-(μ),F_ψ^-(μ),T_ψ^+(μ),C_ψ^+(μ),G_ψ^+(μ),U_ψ^+(μ),F_ψ^+(μ)] and v=[$T_\psi^-(v)$,$C_\psi^-(v)$,$G_\psi^-(v)$,$U_\psi^-(v)$,$F_\psi^-(v)$,$T_\psi^+(v)$,$C_\psi^+(v)$,$G_\psi^+(v)$,$U_\psi^+(v)$,$F_\psi^+(v)$] be two SVBPNNs. Then,

(*i*) k.μ=[-(-T_ψ^-(μ))^k, -(-C_ψ^-(μ))^k, -(-G_ψ^-(μ))^k, -(-U_ψ^-(μ))^k, -(1-(1-(-F_ψ^-(μ)))^k), 1-(1-T_ψ^+(μ))^k, (C_ψ^+(μ))^k, (G_ψ^+(μ))^k, (U_ψ^+(μ))^k, (F_ψ^+(μ))^k], where k >0.

(*ii*) μ^k=[-(1-(1-(-T_ψ^-(μ)))^k), -(-C_ψ^-(μ))^k, -(-G_ψ^-(μ))^k, -(-U_ψ^-(μ))^k, -(-F_ψ^-(μ))^k, (T_ψ^+(μ))^k, 1-(1-C_ψ^+(μ))^k, 1-(1-G_ψ^+(μ))^k, 1-(1-U_ψ^+(μ))^k, 1-(1-F_ψ^+(μ))^k], where k >0.

(*iii*) μ+η=[- T_ψ^- (μ). T_ψ^- (η), -(- C_ψ^- (μ)- C_ψ^- (η)- C_ψ^- (μ) . C_ψ^- (η)), -(- G_ψ^- (μ)- G_ψ^- (η)- G_ψ^- (μ) . G_ψ^- (η)), -(-U_ψ^-(μ)-U_ψ^-(η)- U_ψ^-(μ). U_ψ^-(η)), -(-F_ψ^-(μ)-F_ψ^-(η)- F_ψ^-(μ). F_ψ^-(η)), T_ψ^+(μ)+ T_ψ^+(η)-T_ψ^+(μ).T_ψ^+(η), C_ψ^+(μ). C_ψ^+(η), G_ψ^+(μ). G_ψ^+(η), U_ψ^+(μ). U_ψ^+(η), F_ψ^+(μ).F_ψ^+(η)];

(*iv*) μ.η=[-(-T_ψ^-(μ)-T_ψ^-(η)-T_ψ^-(μ).T_ψ^-(η)), -C_ψ^-(μ). C_ψ^-(η), -G_ψ^-(μ). G_ψ^-(η), -U_ψ^-(μ). U_ψ^-(η), -F_ψ^-(μ). F_ψ^-(η), T_ψ^+ (μ). T_ψ^+ (η), C_ψ^+ (μ)+ C_ψ^+ (η)- C_ψ^+ (μ). C_ψ^+ (η), G_ψ^+ (μ)+ G_ψ^+ (η)- G_ψ^+ (μ). G_ψ^+ (η), U_ψ^+ (μ)+ U_ψ^+ (η)- U_ψ^+ (μ). U_ψ^+ (η), F_ψ^+(μ)+F_ψ^+(η)-F_ψ^+(μ).F_ψ^+(η)].

4. Single-Valued Bipolar Pentapartitioned Neutrosophic Aggregation Operators

Definition 4.1. Assume that u_i=[$T_\psi^-(u_i)$,$C_\psi^-(u_i)$,$G_\psi^-(u_i)$,$U_\psi^-(u_i)$,$F_\psi^-(u_i)$,$T_\psi^+(u_i)$,$C_\psi^+(u_i)$,$G_\psi^+(u_i)$,$U_\psi^+(u_i)$,$F_\psi^+(u_i)$], i=1, 2, 3,…, n, be a collection of SVBPNNs over ψ. Then, the single-valued bipolar pentapartitioned neutrosophic arithmetic mean (SVBPNAM) operator is defined as follows:

$$\text{SVBPNAM}(u_1, u_2,\ldots, u_n) = \frac{1}{n}\sum_{i=1}^{n} u_i \qquad (1)$$

Theorem 4.1. Assume that u_i=[$T_\psi^-(u_i)$,$C_\psi^-(u_i)$,$G_\psi^-(u_i)$,$U_\psi^-(u_i)$,$F_\psi^-(u_i)$,$T_\psi^+(u_i)$,$C_\psi^+(u_i)$,$G_\psi^+(u_i)$,$U_\psi^+(u_i)$,$F_\psi^+(u_i)$], i=1, 2, 3,…, n, be a collection of SVBPNNs over ψ. Then, the aggregated value SVBPNAM (u_1, u_2,…, u_n) is also an SVBPNN.

Proof. Assume that u_i=[$T_\psi^-(u_i)$,$C_\psi^-(u_i)$,$G_\psi^-(u_i)$,$U_\psi^-(u_i)$,$F_\psi^-(u_i)$,$T_\psi^+(u_i)$,$C_\psi^+(u_i)$,$G_\psi^+(u_i)$,$U_\psi^+(u_i)$,$F_\psi^+(u_i)$], i=1, 2, 3,…, n, be a finite collection of SVBPNNs over ψ. Therefore, u_1 is an SVBPNN.

Now,

$\sum_{i=1}^{2} u_i$ = (u_1 + u_2)

=[- T_ψ^- (u_1). T_ψ^- (u_2), -(- C_ψ^- (u_1)- C_ψ^- (u_2)- C_ψ^- (u_1) . C_ψ^- (u_2)), -(- G_ψ^- (u_1)- G_ψ^- (u_2)- G_ψ^- (u_1) . G_ψ^- (u_2)), -(- U_ψ^- (u_1)- U_ψ^- (u_2)- U_ψ^- (u_1) . U_ψ^- (u_2)), -(- F_ψ^- (u_1)- F_ψ^- (u_2)- F_ψ^- (u_1) . F_ψ^- (u_2)), T_ψ^+ (u_1)+ T_ψ^+ (u_2)- T_ψ^+ (u_1). T_ψ^+ (u_2), C_ψ^+(u_1). C_ψ^+(u_2), G_ψ^+(u_1). G_ψ^+(u_2), U_ψ^+(u_1). U_ψ^+(u_2), F_ψ^+(u_1).F_ψ^+(u_2)]

= [T_ψ^-(u_1, u_2), C_ψ^-(u_1, u_2), G_ψ^-(u_1, u_2), U_ψ^-(u_1, u_2), F_ψ^-(u_1, u_2), T_ψ^+(u_1, u_2), C_ψ^+(u_1, u_2), G_ψ^+(u_1, u_2), U_ψ^+(u_1, u_2), F_ψ^+(u_1, u_2)] (say), which is an SVBPNN.

Assume that, $\sum_{i=1}^{n} u_i$ is an SVBPNN over ψ for n = m, i.e. $\sum_{i=1}^{m} u_i$= [T_ψ^-(u_1, u_2,…,u_m), C_ψ^-(u_1, u_2,…,u_m), G_ψ^-(u_1, u_2,…,u_m), U_ψ^-(u_1, u_2,…,u_m), F_ψ^-(u_1, u_2,…,u_m), T_ψ^+(u_1, u_2,…,u_m), C_ψ^+(u_1, u_2,…,u_m), G_ψ^+(u_1, u_2,…,u_m), U_ψ^+(u_1, u_2,…,u_m), F_ψ^+(u_1, u_2,…,u_m)] is an SVBPNN.

Now,

$\sum_{i=1}^{m+1} u_i$

= $\sum_{i=1}^{m} u_i$ + u_{m+1}

= [T_ψ^-(u_1, u_2,…,u_m), C_ψ^-(u_1, u_2,…,u_m), G_ψ^-(u_1, u_2,…,u_m), U_ψ^-(u_1, u_2,…,u_m), F_ψ^-(u_1, u_2,…,u_m), T_ψ^+(u_1, u_2,…,u_m), C_ψ^+(u_1, u_2,…,u_m), G_ψ^+(u_1, u_2,…,u_m), U_ψ^+(u_1, u_2,…,u_m), F_ψ^+(u_1, u_2,…,u_m)]

+ [T_ψ^-(u_{m+1}),C_ψ^-(u_{m+1}),G_ψ^-(u_{m+1}),U_ψ^-(u_{m+1}),F_ψ^-(u_{m+1}),T_ψ^+(u_{m+1}),C_ψ^+(u_{m+1}),G_ψ^+(u_{m+1}),U_ψ^+(u_{m+1}),F_ψ^+(u_{m+1})].

=[- T_ψ^- (u_1, u_2,…,u_m). T_ψ^- (u_{m+1}), -(- C_ψ^- (u_1, u_2,…,u_m)- C_ψ^- (u_{m+1})- C_ψ^- (u_1, u_2,…,u_m) . C_ψ^- (u_{m+1})), -(- G_ψ^- (u_1, u_2,…,u_m)- G_ψ^- (u_{m+1})- G_ψ^- (u_1, u_2,…, u_m). G_ψ^- (u_{m+1})), -(- U_ψ^- (u_1, u_2,…,u_m)- U_ψ^- (u_{m+1})- U_ψ^- (u_1, u_2,…,u_m). U_ψ^- (u_{m+1})), -(- F_ψ^- (u_1, u_2,…,u_m)- F_ψ^- (u_{m+1})- F_ψ^- (u_1, u_2,…,u_m) . F_ψ^- (u_{m+1})), T_ψ^+ (u_1, u_2,…,u_m)+ T_ψ^+ (u_{m+1})- T_ψ^+ (u_1, u_2,…,u_m). T_ψ^+ (u_{m+1}), C_ψ^+ (u_1, u_2,…,u_m). C_ψ^+ (u_{m+1}), G_ψ^+ (u_1, u_2,…,u_m). G_ψ^+ (u_{m+1}), U_ψ^+ (u_1, u_2,…,u_m). U_ψ^+ (u_{m+1}), F_ψ^+(u_1, u_2,…,u_m).F_ψ^+ (u_{m+1})]

=[T_ψ^-(u_1, u_2,…, u_{m+1}), C_ψ^-(u_1, u_2,…, u_{m+1}), G_ψ^-(u_1, u_2,…, u_{m+1}), U_ψ^-(u_1, u_2,…, u_{m+1}), F_ψ^-(u_1, u_2,…, u_{m+1}), T_ψ^+(u_1, u_2,…, u_{m+1}), C_ψ^+(u_1, u_2,…, u_{m+1}), G_ψ^+(u_1, u_2,…, u_{m+1}), U_ψ^+(u_1, u_2,…, u_{m+1}), F_ψ^+(u_1, u_2,…, u_{m+1})] (say), which is an SVBPNN.

Therefore, $\sum_{i=1}^{m+1} u_i$ is an SVBPNN. This implies, $\sum_{i=1}^{n} u_i$ is an SVBPNN for n= m+1.

Hence, $\sum_{i=1}^{n} u_i$ is an SVBPNN for n=1 and 2. Again, $\sum_{i=1}^{n} u_i$ is an SVBPNN for n=m+1, whenever it is an SVBPNN for n=m. Therefore, by the principle of mathematical induction, we can say that $\sum_{i=1}^{n} u_i$ is an SVBPNN for each n. Now, from Definition 3.8. we can say that $\frac{1}{n}\sum_{i=1}^{n} u_i$ is an SVBPNN. Hence,

SVBPNAM (u_1,u_2,…,u_n) = $\frac{1}{n}\sum_{i=1}^{n} u_i$ is an SVBPNN.

Example 4.1. Assume that u=(-0.3,-0.5,-0.3,-0.2,-0.5,0.5,0.3,0.6,0.5,0.2) and v=(-0.8,-0.5,-0.5,-0.3, -0.7,0.3,0.6,0.2,0.5,0.4) be two SVBPNNs. Then, SVBPNAM(u, v) = 0.5 (u+v) = 0.5 (-0.24, -0.75,-0.65, -0.44,-0.85,0.65,0.18,0.12,0.25,0.08) = (-0.49,-0.87,-0.81,-0.66,-0.61,0.41,0.42,0.35,0.5,0.28). It is also an SVBPNN.

Definition 4.2. Assume that u_i=[$T_\psi^-(u_i)$,$C_\psi^-(u_i)$,$G_\psi^-(u_i)$,$U_\psi^-(u_i)$,$F_\psi^-(u_i)$,$T_\psi^+(u_i)$,$C_\psi^+(u_i)$,$G_\psi^+(u_i)$,$U_\psi^+(u_i)$,$F_\psi^+(u_i)$], i=1, 2, 3,…, n, be the family of SVBPNNs over ψ. Then, the Single-Valued Bipolar Pentapartitioned Neutrosophic Geometric Mean (SVBPNGM) operator is defined as follows:

SVBPNGM (u_1, u_2,…, u_n) = $(\prod_{i=1}^{n} u_i)^{\frac{1}{n}}$ (2)

Theorem 4.2. Assume that u_i=[$T_\psi^-(u_i)$,$C_\psi^-(u_i)$,$G_\psi^-(u_i)$,$U_\psi^-(u_i)$,$F_\psi^-(u_i)$,$T_\psi^+(u_i)$,$C_\psi^+(u_i)$,$G_\psi^+(u_i)$,$U_\psi^+(u_i)$,$F_\psi^+(u_i)$], i=1, 2, 3,…, n, be a family of SVBPNNs over ψ. Then the aggregated value SVBPNGM (u_1, u_2,…, u_n) is also an SVBPNN.

Proof. Assume that u_i=[$T_\psi^-(u_i)$,$C_\psi^-(u_i)$,$G_\psi^-(u_i)$,$U_\psi^-(u_i)$,$F_\psi^-(u_i)$,$T_\psi^+(u_i)$,$C_\psi^+(u_i)$,$G_\psi^+(u_i)$,$U_\psi^+(u_i)$,$F_\psi^+(u_i)$], i=1, 2, 3,…, n, be a finite collection SVBPNNs over ψ. Therefore, u_1 is an SVBPNN.

Now, $\prod_{i=1}^{2} u_i = u_1 . u_2$ = [-(- T_ψ^- (u_1)- T_ψ^- (u_2)- T_ψ^- (u_1). T_ψ^- (u_2)), - C_ψ^- (u_1) . C_ψ^- (u_2), - G_ψ^- (u_1) . G_ψ^- (u_2), - U_ψ^- (u_1) . U_ψ^- (u_2), - F_ψ^- (u_1) . F_ψ^- (u_2), T_ψ^+ (u_1). T_ψ^+ (u_2), C_ψ^+ (u_1)+ C_ψ^+ (u_2)- C_ψ^+ (u_1). C_ψ^+ (u_2), G_ψ^+(u_1)+G_ψ^+(u_2)-G_ψ^+(u_1).G_ψ^+(u_2), U_ψ^+(u_1)+U_ψ^+(u_2)-U_ψ^+(u_1).U_ψ^+(u_2), F_ψ^+(u_1)+F_ψ^+(u_2)-F_ψ^+(u_1).F_ψ^+(u_2)]

= [T_ψ^-(u_1, u_2), C_ψ^-(u_1, u_2), G_ψ^-(u_1, u_2), U_ψ^-(u_1, u_2), F_ψ^-(u_1, u_2), T_ψ^+(u_1, u_2), C_ψ^+(u_1, u_2), G_ψ^+(u_1, u_2), U_ψ^+(u_1, u_2), F_ψ^+(u_1, u_2)] (say), which is an SVBPNN.

Suppose that, $\prod_{i=1}^{n} u_i$is an SVBPNN over ψ for n = m, i.e. $\prod_{i=1}^{m} u_i$ = [T_ψ^-(u_1, u_2,…,u_m), C_ψ^-(u_1, u_2,…,u_m), G_ψ^-(u_1, u_2,…,u_m), U_ψ^-(u_1, u_2,…,u_m), F_ψ^-(u_1, u_2,…,u_m), T_ψ^+(u_1, u_2,…,u_m), C_ψ^+(u_1, u_2,…,u_m), G_ψ^+(u_1, u_2,…,u_m), U_ψ^+(u_1, u_2,…,u_m), F_ψ^+(u_1, u_2,…,u_m)] is an SVBPNN.

Now,

$\prod_{i=1}^{m+1} u_i$

= u_{m+1} . $\prod_{i=1}^{m} u_i$

= [T_ψ^-(u_{m+1}),C_ψ^-(u_{m+1}),G_ψ^-(u_{m+1}),U_ψ^-(u_{m+1}),F_ψ^-(u_{m+1}),T_ψ^+(u_{m+1}),C_ψ^+(u_{m+1}),G_ψ^+(u_{m+1}),U_ψ^+(u_{m+1}),F_ψ^+(u_{m+1})]. [T_ψ^-(u_1, u_2,…,u_m), C_ψ^-(u_1, u_2,…,u_m), G_ψ^-(u_1, u_2,…,u_m), U_ψ^-(u_1, u_2,…,u_m), F_ψ^-(u_1, u_2,…,u_m), T_ψ^+(u_1, u_2,…,u_m), C_ψ^+(u_1, u_2,…,u_m), G_ψ^+(u_1, u_2,…,u_m), U_ψ^+(u_1, u_2,…,u_m), F_ψ^+(u_1, u_2,…,u_m)]

=[-(-T_ψ^-(u_{m+1})-T_ψ^-(u_1, u_2,…,u_m)-T_ψ^-(u_{m+1}).T_ψ^-(u_1, u_2,…,u_m)), -C_ψ^-(u_{m+1}). C_ψ^-(u_1, u_2,…,u_m), -G_ψ^-(u_{m+1}). G_ψ^-(u_1, u_2,…,u_m), -U_ψ^-(u_{m+1}). U_ψ^-(u_1, u_2,…,u_m), -F_ψ^-(u_{m+1}). F_ψ^-(u_1, u_2,…,u_m), T_ψ^+(u_{m+1}).T_ψ^+(u_1, u_2,…,u_m), C_ψ^+(u_{m+1})+C_ψ^+(u_1, u_2,…,u_m)-C_ψ^+(u_{m+1}).C_ψ^+(u_1, u_2,…,u_m), G_ψ^+(u_{m+1})+G_ψ^+(u_1, u_2,…,u_m)-G_ψ^+(u_{m+1}).G_ψ^+(u_1, u_2,…,u_m), U_ψ^+(u_{m+1})+U_ψ^+(u_1, u_2,…,u_m)-U_ψ^+(u_{m+1}).U_ψ^+(u_1, u_2,…,u_m), F_ψ^+(u_{m+1})+F_ψ^+(u_1, u_2,…,u_m)-F_ψ^+(u_{m+1}).F_ψ^+(u_1, u_2,…,u_m)]

=[T_ψ^-(u_1, u_2,…, u_{m+1}), C_ψ^-(u_1, u_2,…, u_{m+1}), G_ψ^-(u_1, u_2,…, u_{m+1}), U_ψ^-(u_1, u_2,…, u_{m+1}), F_ψ^-(u_1, u_2,…, u_{m+1}), T_ψ^+(u_1, u_2,…, u_{m+1}), C_ψ^+(u_1, u_2,…, u_{m+1}), G_ψ^+(u_1, u_2,…, u_{m+1}), U_ψ^+(u_1, u_2,…, u_{m+1}), F_ψ^+(u_1, u_2,…, u_{m+1})] (say), which is an SVBPNN.

Therefore, $\prod_{i=1}^{m+1} u_i$ is an SVBPNN. This implies, $\prod_{i=1}^{n} u_i$ is an SVBPNN for n=m+1.

Hence, $\prod_{i=1}^{n} u_i$ is an SVBPNN for n=1 and 2. Again, $\prod_{i=1}^{n} u_i$ is an SVBPNN for n=m+1, whenever it is an SVBPNN for n=m. Therefore, by the principle of mathematical induction, we can say that $\prod_{i=1}^{n} u_i$

is an SVBPNN for each n. Now, from Definition 3.8. we can say that $(\prod_{i=1}^{n} u_i)^{\frac{1}{n}}$ is an SVBPNN.

Hence, SVBPNGM $(u_1,u_2,\ldots,u_n)$ = $(\prod_{i=1}^{n} u_i)^{\frac{1}{n}}$ is an SVBPNN.

Example 4.2. Let u=(-0.3,-0.5,-0.3,-0.2,-0.5,0.5,0.3,0.6,0.5,0.2), v=(-0.8,-0.5,-0.5,-0.3,-0.7,0.3,0.6,0.2,0.5, 0.4) be two SVBPNNs as shown in Example 4.1. Then, SVBPNGM (u, v) = $(u+v)^{0.5}$ = (-0.86,-0.25,-0.15, -0.06,-0.35,0.15,0.72,0.68,0.75,0.52)$^{0.5}$ = (-0.63,-0.5,-0.39,-0.24,-0.59,0.39,0.47,0.43,0.5,0.31). It is also an SVBPNN.

5. Score & Accuracy Functions under the SVBPNS Environment

Definition 5.1. Suppose that μ = $[T_{\psi}^{-}(\mu), C_{\psi}^{-}(\mu), G_{\psi}^{-}(\mu), U_{\psi}^{-}(\mu), F_{\psi}^{-}(\mu), T_{\psi}^{+}(\mu), C_{\psi}^{+}(\mu), G_{\psi}^{+}(\mu), U_{\psi}^{+}(\mu), F_{\psi}^{+}(\mu)]$ be an SVBPNN over ψ. Then, the score function and accuracy function are defined by:

$$S_f(\mu) = \frac{[\,1+T_{\psi}^{-}(\mu)-C_{\psi}^{-}(\mu)-G_{\psi}^{-}(\mu)-U_{\psi}^{-}(\mu)-F_{\psi}^{-}(\mu)+T_{\psi}^{+}(\mu)+1-C_{\psi}^{+}(\mu)+1-G_{\psi}^{+}(\mu)+1-U_{\psi}^{+}(\mu)+1-F_{\psi}^{+}(\mu)]}{10} \quad (3)$$

$$A_f(\mu) = \frac{[T_{\psi}^{-}(\mu)-C_{\psi}^{-}(\mu)-F_{\psi}^{-}(\mu)+T_{\psi}^{+}(\mu)-C_{\psi}^{+}(\mu)-F_{\psi}^{+}(\mu)]}{3} \quad (4)$$

Example 5.1. Suppose that μ=(-0.3,-0.5,-0.3,-0.2,-0.5,0.5,0.3,0.6,0.5,0.2) be an SVBPNN as defined in Example 4.1. Then, $S_f(\mu)$=0.51 and $A_f(\mu)$=0.233.

Definition 5.2. Suppose that μ=$[T_{\psi}^{-}(\mu), C_{\psi}^{-}(\mu), G_{\psi}^{-}(\mu), U_{\psi}^{-}(\mu), F_{\psi}^{-}(\mu), T_{\psi}^{+}(\mu), C_{\psi}^{+}(\mu), G_{\psi}^{+}(\mu), U_{\psi}^{+}(\mu), F_{\psi}^{+}(\mu)]$ and v=$[T_{\psi}^{-}(v), C_{\psi}^{-}(v), G_{\psi}^{-}(v), U_{\psi}^{-}(v), F_{\psi}^{-}(v), T_{\psi}^{+}(v), C_{\psi}^{+}(v), G_{\psi}^{+}(v), U_{\psi}^{+}(v), F_{\psi}^{+}(v)]$ be any two SVBPNNs over ψ. Then,

(i) $S_f(\mu) > S_f(\eta) \Rightarrow \mu > \eta$;

(ii) $S_f(\mu) = S_f(\eta)$, $A_f(\mu) > A_f(\eta) \Rightarrow \mu > \eta$;

(iii) $S_f(\mu) = S_f(\eta)$, $A_f(\mu) = A_f(\eta)$, $T_{\psi}^{+}(\mu) > T_{\psi}^{+}(\eta)$, $T_{\psi}^{-}(\mu) < T_{\psi}^{-}(\eta) \Rightarrow \mu > \eta$.

Theorem 5.1. The score function and accuracy function of an SVBPNN are bounded.

Proof. Suppose that η=$[T_{\psi}^{-}(\eta), C_{\psi}^{-}(\eta), G_{\psi}^{-}(\eta), U_{\psi}^{-}(\eta), F_{\psi}^{-}(\eta), T_{\psi}^{+}(\eta), C_{\psi}^{+}(\eta), G_{\psi}^{+}(\eta), U_{\psi}^{+}(\eta), F_{\psi}^{+}(\eta)]$ be an SVBPNN.

Therefore, $-1 \le T_{\psi}^{-}(\eta) \le 0$, $-1 \le C_{\psi}^{-}(\eta) \le 0$, $-1 \le G_{\psi}^{-}(\eta) \le 0$, $-1 \le U_{\psi}^{-}(\eta) \le 0$, $-1 \le F_{\psi}^{-}(\eta) \le 0$, $0 \le T_{\psi}^{+}(\eta) \le 1$, $0 \le C_{\psi}^{+}(\eta) \le 1$, $0 \le G_{\psi}^{+}(\eta) \le 1$, $0 \le U_{\psi}^{+}(\eta) \le 1$, $0 \le F_{\psi}^{+}(\eta) \le 1$.

This implies, $0 \le 1+T_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta) \le 2$, $0 \le -C_{\psi}^{-}(\eta)+1-C_{\psi}^{+}(\eta) \le 2$, $0 \le -G_{\psi}^{-}(\eta)+1-G_{\psi}^{+}(\eta) \le 2$, $0 \le -U_{\psi}^{-}(\eta)+1-U_{\psi}^{+}(\eta) \le 2$, $0 \le -F_{\psi}^{-}(\eta)+1-F_{\psi}^{+}(\eta) \le 2$.

Therefore,

$0 \le 1+T_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta)-C_{\psi}^{-}(\eta)+1-C_{\psi}^{+}(\eta)-G_{\psi}^{-}(\eta)+1-G_{\psi}^{+}(\eta)-U_{\psi}^{-}(\eta)+1-U_{\psi}^{+}(\eta)-F_{\psi}^{-}(\eta)+1-F_{\psi}^{+}(\eta) \le 10$

$\Rightarrow 0 \le 1+T_{\psi}^{-}(\eta)-C_{\psi}^{-}(\eta)-G_{\psi}^{-}(\eta)-U_{\psi}^{-}(\eta)-F_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta)+1-C_{\psi}^{+}(\eta)+1-G_{\psi}^{+}(\eta)+1-U_{\psi}^{+}(\eta)+1-F_{\psi}^{+}(\eta) \le 10$

$$\Rightarrow 0 \le \frac{[\,1+T_{\psi}^{-}(\eta)-C_{\psi}^{-}(\eta)-G_{\psi}^{-}(\eta)-U_{\psi}^{-}(\eta)-F_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta)+1-C_{\psi}^{+}(\eta)+1-G_{\psi}^{+}(\eta)+1-U_{\psi}^{+}(\eta)+1-F_{\psi}^{+}(\eta)]}{10} \le 1$$

$\Rightarrow 0 \le S_f(u) \le 1$.

Hence, the score function is bounded.

Again, $-1 \leq T_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta) \leq 1$, $-1 \leq -C_{\psi}^{-}(\eta)-C_{\psi}^{+}(\eta) \leq 1$, $-1 \leq -F_{\psi}^{-}(\eta)-F_{\psi}^{+}(\eta) \leq 1$

This implies,

$-3 \leq T_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta)-C_{\psi}^{-}(\eta)-C_{\psi}^{+}(\eta)-F_{\psi}^{-}(\eta)-F_{\psi}^{+}(\eta) \leq 3$

$\Rightarrow -1 \leq \frac{T_{\psi}^{-}(\eta)-C_{\psi}^{-}(\eta)-F_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta)-C_{\psi}^{+}(\eta)-F_{\psi}^{+}(\eta)}{3} \leq 1$

$\Rightarrow -1 \leq A_f(\eta) \leq 1$.

Hence, the accuracy function is bounded.

Theorem 5.2. The score function and accuracy function of an SVBPNN are monotonic increasing.

Proof. Suppose that $\mu=[T_{\psi}^{-}(\mu), C_{\psi}^{-}(\mu), G_{\psi}^{-}(\mu), U_{\psi}^{-}(\mu), F_{\psi}^{-}(\mu), T_{\psi}^{+}(\mu), C_{\psi}^{+}(\mu), G_{\psi}^{+}(\mu), U_{\psi}^{+}(\mu), F_{\psi}^{+}(\mu)]$ and $\eta=[T_{\psi}^{-}(\eta), C_{\psi}^{-}(\eta), G_{\psi}^{-}(\eta), U_{\psi}^{-}(\eta), F_{\psi}^{-}(\eta), T_{\psi}^{+}(\eta), C_{\psi}^{+}(\eta), G_{\psi}^{+}(\eta), U_{\psi}^{+}(\eta), F_{\psi}^{+}(\eta)]$ be two SVBPNNs over ψ such that $\mu \subseteq \eta$.

Therefore, $T_{\psi}^{-}(\mu) \leq T_{\psi}^{-}(\eta)$, $C_{\psi}^{-}(\mu) \geq C_{\psi}^{-}(\eta)$, $G_{\psi}^{-}(\mu) \geq G_{\psi}^{-}(\eta)$, $U_{\psi}^{-}(\mu) \geq U_{\psi}^{-}(\eta)$, $F_{\psi}^{-}(\mu) \geq F_{\psi}^{-}(\eta)$, $T_{\psi}^{+}(\mu) \leq T_{\psi}^{+}(\eta)$, $C_{\psi}^{+}(\mu) \geq C_{\psi}^{+}(\eta)$, $G_{\psi}^{+}(\mu) \geq G_{\psi}^{+}(\eta)$, $U_{\psi}^{+}(\mu) \geq U_{\psi}^{+}(\eta)$, $F_{\psi}^{+}(\mu) \geq F_{\psi}^{+}(\eta)$.

It is known that,

$$S_f(\mu) = \frac{[1+T_{\psi}^{-}(\mu)-C_{\psi}^{-}(\mu)-G_{\psi}^{-}(\mu)-U_{\psi}^{-}(\mu)-F_{\psi}^{-}(\mu)+T_{\psi}^{+}(\mu)+1-C_{\psi}^{+}(\mu)+1-G_{\psi}^{+}(\mu)+1-U_{\psi}^{+}(\mu)+1-F_{\psi}^{+}(\mu)]}{10};$$

$$S_f(\eta) = \frac{[1+T_{\psi}^{-}(\eta)-C_{\psi}^{-}(\eta)-G_{\psi}^{-}(\eta)-U_{\psi}^{-}(\eta)-F_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta)+1-C_{\psi}^{+}(\eta)+1-G_{\psi}^{+}(\eta)+1-U_{\psi}^{+}(\eta)+1-F_{\psi}^{+}(\eta)]}{10};$$

$$A_f(\mu) = \frac{[T_{\psi}^{-}(\mu)-C_{\psi}^{-}(\mu)-F_{\psi}^{-}(\mu)+T_{\psi}^{+}(\mu)-C_{\psi}^{+}(\mu)-F_{\psi}^{+}(\mu)]}{3};$$

$$A_f(\eta) = \frac{[T_{\psi}^{-}(\eta)-C_{\psi}^{-}(\eta)-F_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta)-C_{\psi}^{+}(\eta)-F_{\psi}^{+}(\eta)]}{3};$$

Now,

$S_f(\eta) - S_f(\mu)$

$$= \frac{[1+T_{\psi}^{-}(\eta)-C_{\psi}^{-}(\eta)-G_{\psi}^{-}(\eta)-U_{\psi}^{-}(\eta)-F_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta)+1-C_{\psi}^{+}(\eta)+1-G_{\psi}^{+}(\eta)+1-U_{\psi}^{+}(\eta)+1-F_{\psi}^{+}(\eta)]}{10} - \frac{[1+T_{\psi}^{-}(\mu)-C_{\psi}^{-}(\mu)-G_{\psi}^{-}(\mu)-U_{\psi}^{-}(\mu)-F_{\psi}^{-}(\mu)+T_{\psi}^{+}(\mu)+1-C_{\psi}^{+}(\mu)+1-G_{\psi}^{+}(\mu)+1-U_{\psi}^{+}(\mu)+1-F_{\psi}^{+}(\mu)]}{10}$$

≥ 0 [since $\mu \subseteq \eta$]

This implies, $S_f(\eta) \geq S_f(\mu)$, i.e. the score function is monotonic increasing.

Now,

$A_f(\eta) - A_f(\mu)$

$$= \frac{[T_{\psi}^{-}(\eta)-C_{\psi}^{-}(\eta)-F_{\psi}^{-}(\eta)+T_{\psi}^{+}(\eta)-C_{\psi}^{+}(\eta)-F_{\psi}^{+}(\eta)]}{3} - \frac{[T_{\psi}^{-}(\mu)-C_{\psi}^{-}(\mu)-F_{\psi}^{-}(\mu)+T_{\psi}^{+}(\mu)-C_{\psi}^{+}(\mu)-F_{\psi}^{+}(\mu)]}{3}$$

≥ 0 [since $\mu \subseteq \eta$]

This implies, $A_f(\eta) \geq A_f(\mu)$, i.e., the accuracy function is monotonic increasing.

Hence, the score and accuracy functions are monotonic increasing functions.

6. SVBPNS-MADM Strategy Based on SVBPNAM Operator

Suppose that A= {A_1, A_2, ..., A_n} be a fixed set of alternatives, and P= {P_1, P_2,..., P_m} be a family of attributes. The decision maker involves in the decision making provides his/her evaluation information of each alternative Q_i (i = 1, 2,…, n) over the attribute P_j (j = 1, 2,..., m) in terms of SVBPNNs. The whole evaluation information of all alternatives can be expressed by a decision matrix.

The proposed SVBPNS-MADM strategy (see Figure 1)is described using the following steps:

Step-1: Construct the decision matrix using SVBPNSs.

The whole evaluation information of each alternative A_i (i = 1, 2,..., n) based on the attributes P_j (j = 1, 2,..., m) is expressed in terms of SVBPNS E_{A_i}= {(P_j,$T_{ij}^-(A_i, P_j)$,$C_{ij}^-(A_i, P_j)$,$G_{ij}^-(A_i, P_j)$,$U_{ij}^-(A_i, P_j)$,$F_{ij}^-(A_i, P_j)$,$T_{ij}^+(A_i, P_j)$,$C_{ij}^+(A_i, P_j)$, $G_{ij}^+(A_i, P_j)$,$U_{ij}^+(A_i, P_j)$, $F_{ij}^+(A_i, P_j)$): $P_j \in P$}, where ($T_{ij}^-(A_i, P_j)$,$C_{ij}^-(A_i, P_j)$,$G_{ij}^-(A_i, P_j)$,$U_{ij}^-(A_i, P_j)$,$F_{ij}^-(A_i, P_j)$,$T_{ij}^+(A_i, P_j)$,$C_{ij}^+(A_i, P_j)$,$G_{ij}^+(A_i, P_j)$,$U_{ij}^+(A_i, P_j)$,$F_{ij}^+(A_i, P_j)$) denote the evaluation information of A_i (i = 1, 2,..., n) based on P_j (j = 1, 2,..., m).

Then the Decision Matrix (DM[A|P]) can be expressed as:

DM[A|P] =

	P_1	P_2	…	…	P_m
A_1	[$T_{11}^-(A_1, P_1)$, $C_{11}^-(A_1, P_1)$, $G_{11}^-(A_1, P_1)$, $U_{11}^-(A_1, P_1)$, $F_{11}^-(A_1, P_1)$, $T_{11}^+(A_1, P_1)$, $C_{11}^+(A_1, P_1)$, $G_{11}^+(A_1, P_1)$, $U_{11}^+(A_1, P_1)$, $F_{11}^+(A_1, P_1)$]	[$T_{12}^-(A_1, P_2)$, $C_{12}^-(A_1, P_2)$, $G_{12}^-(A_1, P_2)$, $U_{12}^-(A_1, P_2)$, $F_{12}^-(A_1, P_2)$, $T_{12}^+(A_1, P_2)$, $C_{12}^+(A_1, P_2)$, $G_{12}^+(A_1, P_2)$, $U_{12}^+(A_1, P_2)$, $F_{12}^+(A_1, P_2)$]	…	…	[$T_{1m}^-(A_1, P_m)$, $C_{1m}^-(A_1, P_m)$, $G_{1m}^-(A_1, P_m)$, $U_{1m}^-(A_1, P_m)$, $F_{1m}^-(A_1, P_m)$, $T_{1m}^+(A_1, P_m)$, $C_{1m}^+(A_1, P_m)$, $G_{1m}^+(A_1, P_m)$, $U_{1m}^+(A_1, P_m)$, $F_{1m}^+(A_1, P_m)$]
A_2	[$T_{21}^-(A_2, P_1)$, $C_{21}^-(A_2, P_1)$, $G_{21}^-(A_2, P_1)$, $U_{21}^-(A_2, P_1)$, $F_{21}^-(A_2, P_1)$, $T_{21}^+(A_2, P_1)$, $C_{21}^+(A_2, P_1)$, $G_{21}^+(A_2, P_1)$, $U_{21}^+(A_2, P_1)$, $F_{21}^+(A_2, P_1)$]	[$T_{22}^-(A_2, P_2)$, $C_{22}^-(A_2, P_2)$, $G_{22}^-(A_2, P_2)$, $U_{22}^-(A_2, P_2)$, $F_{22}^-(A_2, P_2)$, $T_{22}^+(A_2, P_2)$, $C_{22}^+(A_2, P_2)$, $G_{22}^+(A_2, P_2)$, $U_{22}^+(A_2, P_2)$, $F_{22}^+(A_2, P_2)$]	…	…	[$T_{2m}^-(A_2, P_m)$, $C_{2m}^-(A_2, P_m)$, $G_{2m}^-(A_2, P_m)$, $U_{2m}^-(A_2, P_m)$, $F_{2m}^-(A_2, P_m)$, $T_{2m}^+(A_2, P_m)$, $C_{2m}^+(A_2, P_m)$, $G_{2m}^+(A_2, P_m)$, $U_{2m}^+(A_2, P_m)$, $F_{2m}^+(A_2, P_m)$]
...	…	…	…	…	…
A_n	[$T_{n1}^-(A_n, P_1)$, $C_{n1}^-(A_n, P_1)$, $G_{n1}^-(A_n, P_1)$, $U_{n1}^-(A_n, P_1)$, $F_{n1}^-(A_n, P_1)$, $T_{n1}^+(A_n, P_1)$, $C_{n1}^+(A_n, P_1)$, $G_{n1}^+(A_n, P_1)$, $U_{n1}^+(A_n, P_1)$, $F_{n1}^+(A_n, P_1)$]	[$T_{n2}^-(A_n, P_2)$, $C_{n2}^-(A_n, P_2)$, $G_{n2}^-(A_n, P_2)$, $U_{n2}^-(A_n, P_2)$, $F_{n2}^-(A_n, P_2)$, $T_{n2}^+(A_n, P_2)$, $C_{n2}^+(A_n, P_2)$, $G_{n2}^+(A_n, P_2)$, $U_{n2}^+(A_n, P_2)$, $F_{n2}^+(A_n, P_2)$]	… … … …	… … … …	[$T_{nm}^-(A_n, P_m)$, $C_{nm}^-(A_n, P_m)$, $G_{nm}^-(A_n, P_m)$, $U_{nm}^-(A_n, P_m)$, $F_{nm}^-(A_n, P_m)$, $T_{nm}^+(A_n, P_m)$, $C_{nm}^+(A_n, P_m)$, $G_{nm}^+(A_n, P_m)$, $U_{nm}^+(A_n, P_m)$, $F_{nm}^+(A_n, P_m)$]

Step-2: In this step, the decision maker determines the aggregation values (A_i | P_1, P_2,…, P_m) = SVBPNAM (P_1, P_2,…, P_m) of all the attributes for each alternative by using the eq. (1). After the determination of aggregation values SVBPNAM (P_1, P_2,…, P_m), the decision maker makes an aggregate decision matrix aggregate-D_M.

Step-3: In this step, the decision maker determines the score and accuracy values of each alternative by using the eqs. (3) and (4).

Step-4: In this step, the decision maker ranks the alternatives by using Definition 5.1. and Definition 5.2.

Step-5: End.

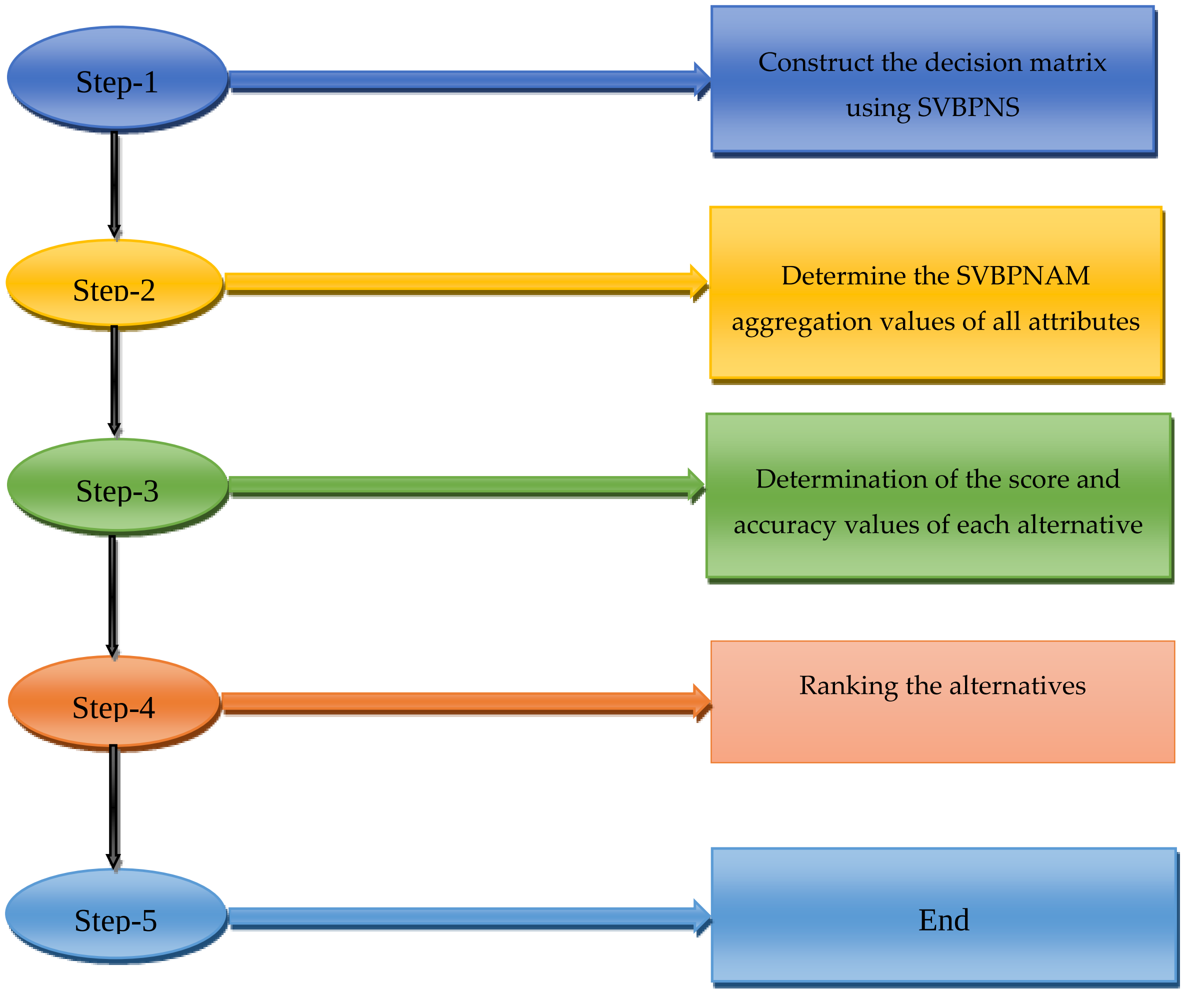

Figure 1: Flow chart of the SVBPNS-MADM Strategy based on SVBPNAM operator

7. SVBPNS-MADM Strategy Based on SVBPNGM Operator

Consider the same MADM problem which is considered in section 6. Then the proposed SVBPNS-MADM strategy (see Figure 2) can be described by the following steps:

Step-1: Construct the decision matrix using SVBPNSs.

It is similar to the step-1 of the section 6.

Step-2: In this step, the decision makers determine the aggregation values (A_i | $P_1, P_2, \ldots, P_m$) = SVBPNGM ($P_1, P_2, \ldots, P_m$) of all the attributes for each alternative by using the eq. (1). After the determination of aggregation values SVBPNGM ($P_1, P_2, \ldots, P_m$), the decision maker makes an aggregate decision matrix aggregate-D_M.

Step-3: In this step, the decision maker determines the score and accuracy values of each alternative by using the eqs. (3) and (4).

Step-4: In this step, the decision maker ranks the alternatives by using Definition 5.1. and Definition 5.2.

Step-5: End.

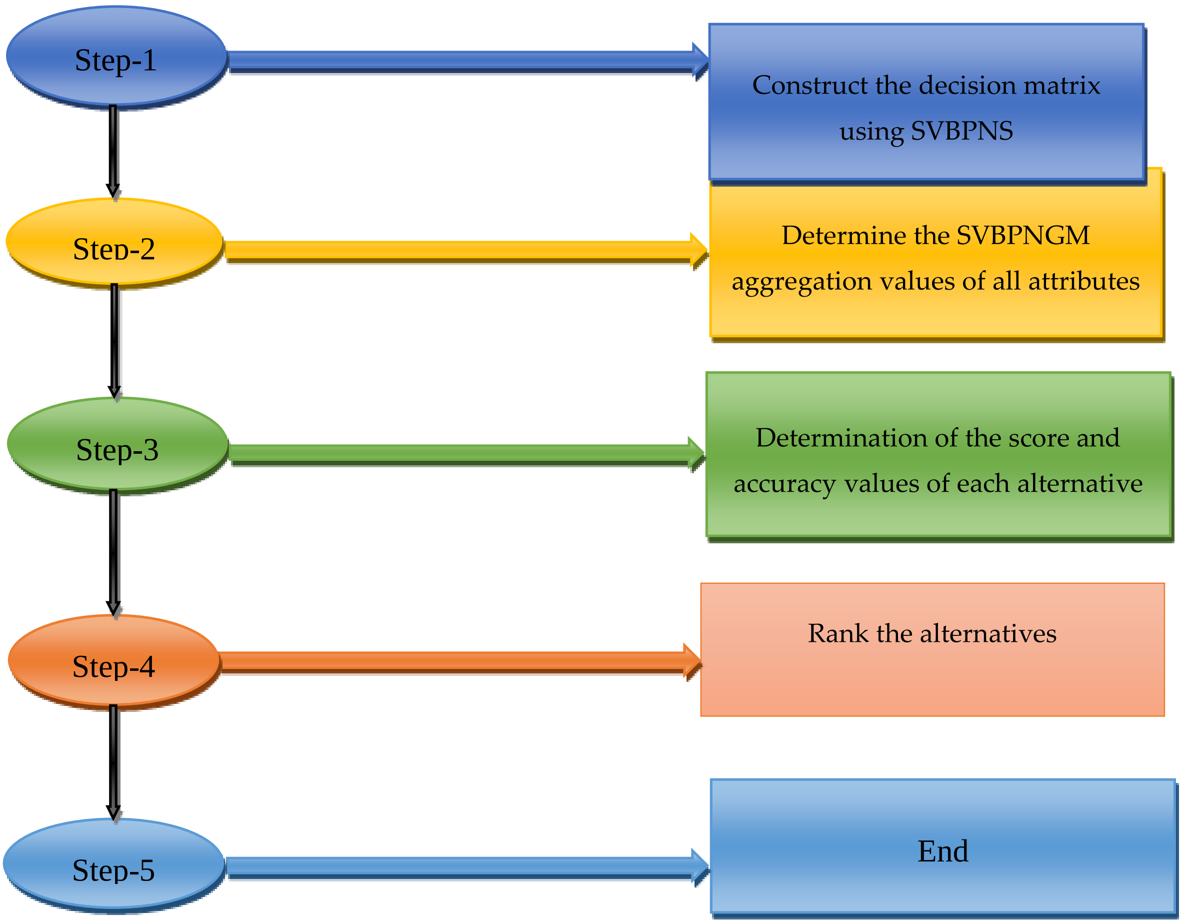

Figure 2: Flow chart of the SVBPNS-MADM Strategy based on SVBPNGM operator

8. Validation of the Proposed SVBPNS-MADM Strategies:

In this section, we present a realistic example of "University Selection for Admission into Various Degree Course" to validate the proposed SVBPNS-MADM strategies based on both SVBPNAM operator and SVBPNGM operator.

8.1. Example: "University Selection for Admission into Various Degree Course".

The selection of university for getting admission for higher education by the students who just have passed the higher secondary or college from any stream can be considered as an MADM problem. To select the best university for higher education, the students must need to select some attributes based on which they select the best university. After the initial screening, the decision maker (student) chooses three alternatives (Universities) for further screening. Suppose the alternatives (Universities) are A_1, A_2, A_3. After the consultation with experts the decision makers (students) can choose three major attributes namely

P_1 **(Faculty):-** In an educational institution, faculty has the most important role for the system as well as students. The number of faculty members and the quality of the faculty members that is the profile of faculty is too important. Only faculty can help and find the creative students for the success of the social. A good quality Teacher encourages students to come to class from time to time with work interest.

P_2 **(NAAC-Grade):-** In India, UGC gives different grades based on their different performance. Higher learning institutes in India are graded for each key aspect/ parameter under different categories such as 'A', 'B', 'C', and 'D'. The NAAC grade indicates the overall performance of an institution such as very good, good, satisfactory, and unsatisfactory.

P_3 **(Government University / Private University):-** Most of the time a central University certificate has more value than a state university. It's generally seen that the government universities charge a lower tuition fee than private universities. There are also more opportunities for a fee reduction in government universities with scholarships and/or quota-based benefits (SC/ST/OBC/EWS, etc.). So there are many issues on this regard that is why we are taking a criterion on this objective.

P_4 **(Infrastructure):** A high-grade university infrastructures [30] must have a dynamic facility. The infrastructure criteria for being a world-class university are:

(1) Physical infrastructure,

(2) Digital infrastructure,

(3) Innovative academic & training Infrastructure for confidence building,

(4) Intellectual property infrastructure,

(5) Emotional infrastructure, and

(6) Network infrastructure,

Based on the rating of the alternatives in terms of SVBPNNs, the decision matrix D_M (see Table-1) is constructed as follows:

Table-1:

D_M	P_1	P_2	P_3	P_4
A_1	(-0.3,-0.5,-0.4,-0.6,-0.3, 0.3,0.6,0.5,0.4,0.2)	(-0.7,-0.2,-0.6,-0.5-0.6, 0.4,0.5,0.5,0.3,0.7)	(-0.4,-0.6,-0.3,-0.5,-0.5, 0.7,0.8,0.5,0.6,0.5)	(-0.1,-0.2,-0.8,-0.1,-0.8, 0.9,0.2,0.8,0.4,0.1)
A_2	(-0.3,-0.7,-0.5,-0.5,-0.3, 0.3,0.5,0.4,0.3,0.2)	(-0.6,-0.6,-0.5,-0.4,-0.5, 0.5,0.4,0.5,0.6,0.5)	(-0.5,-0.5,-0.6,-0.4,-0.2, 0.8,0.6,0.4,0.2,0.6)	(-0.2,-0.2,-0.5,-0.8,-0.9, 1.0,0.7,0.5,0.4,0.4)
A_3	(-0.5,-0.5,-0.7,-0.5,-0.8, 0.6,0.3,0.4,0.6,0.8)	(-0.5,-0.4,-0.7,-0.5,-0.4, 0.8,0.5,0.6,0.5,0.4)	(-0.5,-0.5,-0.2,-0.4,-0.8, 1.0,0.6,0.4,0.2,0.5)	(-0.1,-0.5,-0.4,-0.1,-0.7, 1.0,0.7,0.4,0.3,0.3)

In Table 2, we calculate the aggregation values (A_i | P_1, P_2, P_3) of all attributes for each alternative A_i, by using the SVBPNAM operator.

Table-2: Aggregate-D_M

	(A_i | P_1, P_2, P_3)
A_1	(-0.30274,-0.96634,-0.99149,-0.9767,-0.59094,0.664963,0.468069,0.562341,0.411953,0.289251)
A_2	(-0.36628,-0.98778,-0.98726,-0.99088,-0.59094,1.00000,0.538356,0.447214,0.34641,0.393598)
A_3	(-0.33437,-0.9807,-0.98902,-0.96439,-0.7087,1.00000,0.500997,0.442673,0.366284,0.468069)

By using eq (2), we get $S_f(A_1)$= 0.7156079; $S_f(A_2)$= 0.7465002; $S_f(A_3)$= 0.7530417.

Therefore, $S_f(A_1) < S_f(A_2) < S_f(A_3)$.

The ranking order is obtained as: $A_1 < A_2 < A_3$.

Hence, A_3 is the best university for getting admission among the set of alternatives (universities).

In table 3, we calculate the aggregation values (A_i | P_1, P_2, P_3) of all attributes for each alternative A_i, by using the SVBPNGM operator.

Table-3: Aggregate-D_M

	(A_i | P_1, P_2, P_3)
A_1	(-0.4197,-0.33098,-0.4899,-0.34996,-0.518,0.524361,0.577051,0.602365,0.436537,0.426734)
A_2	(-0.4215,-0.4527,-0.52332,-0.50297,-0.40536,0.588566,0.564412,0.452277,0.39452,0.443368)
A_3	(-0.42085,-0.47287,-0.44496,-0.31623,-0.65063,0.832358,0.547298,0.457839,0.421498,0.547298)

By using eq. (2), we get S_f (A_1)= 0.4750814; S_f (A_2)= 0.5196839; S_f (A_3)= 0.5322265.

Therefore, $S_f(A_1) < S_f(A_2) < S_f(A_3)$.

The ranking order is obtained as: $A_1 < A_2 < A_3$.

Hence, A_3 is the best university for getting admission.

Table 4: Ranking order of alternatives

Strategies	Ranking order	Best alternative
SVBPNS-MADM strategy based on BPNAM operator.	$A_1 < A_2 < A_3$	A_3
SVBPNS-MADM strategy based on BPNAM operator.	$A_1 < A_2 < A_3$	A_3

Both the SVBPNS-MADM strategies offer the same ranking order of the alternatives (See table 4) and A_3 is the best university for getting admission.

9. Conclusions

In this paper, we introduce the notion of SVBPNS, and prove its basic properties and operations. We define the score and accuracy functions of SVBPNNs, and prove their basic properties. Besides, we define two aggregation operators namely, single-valued bipolar pentapartitioned neutrosophic arithmetic mean operator and the single-valued bipolar pentapartitioned neutrosophic geometric mean operator, and prove their basic properties. Based on these two operators, we develop two new MADM strategies and present a numerical example in SVBPNS environment to show the applicability of SVBPNS in MADM. The developed strategies can be further used for the other MADM problems [31-34], medical diagnosis [35-36] , risk analysis [37], and so on.

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