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Bipolar quadripartitioned single valued neutrosophic rough set

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Abstract. Here bipolar quadripartitioned single valued neutrosophic rough (BQSVNR) set is introduced. Some basic set theoretic terminologies like constant BQSVNR set, subsethood of two BQSVNR sets are shown. Algebraic operations like union, intersection and complement have also been defined. Different types of measure like similarity measure, quasi similarity measure and distance measures between two BQSVNR sets have been discussed with their properties. Again various measures of similarity namely distance based similarity measure, cosine similarity measure, membership function based similarity measure are introduced in this paper. A medical diagonasis problem has been solved using similarity measure at the end.

Keywords: SVN set, SVNR set, QSVNR set, BQSVNR set

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1. Introduction

Neutrosophic set (NS) was introduced by Smarandache in 2005 [7] as a generalization of intuitionstic fuzzy set (IFS) [4]. Here in NS the indeterminacy factor is independent where as indeterminacy in IFS is dependent completely on the truth and falsity values. This is why NS's are more general in nature and can handle various types of data including incomplete, inconsistent and even para consistent data. Wang et. al [10] in 2010 has introduced a new version of NS called single valued neutrosophic set (SVN) which is much easier for the application than NS in solving physical problems. Currently the theory of NS has becomes a very successful and flourishing area of research and many researchers are doing research in different areas of both theory and application [22, 31, 32, 34–38]. In 2016 R. Chatterjee et. al [23] defined another new version of NS set called Quadripartioned SVN (QSVN) set where the indeterminacy factor consists two divisons namely contradiction and ignorance. This QSVN set is expected to give better results and more realistic value as it characterizes the indeterminacy factor into two parts which is based on the notions of four valued logic of Belnap [2]. On the other hand bipolar SVN set [19] is an identification of polarity. Thus bipolar concept which is very useful in many decision making concept as a large number of human decision making is based on double sided or bipolar judgement thinking on a positive side and negative side. Again Rough Set (RS) by Pawlak [3] is a well established technique to express imperfect information by employing vagueness to the boundary region of a set. RS theory has various applications in artificial intelligence and especially in machine learning [5, 6, 8, 9, 14, 22, 24].

Here the idea of BQSVNR set which is an further extension of the articles [17, 23] is introduced. In literature many types of NS exist together with various types of applications [10, 12, 16, 19, 21, 25–30, 33]. However we refer our readers to study NS theory [7], SVN theory [10], QSVN theory [23], RS theory [3], BRS Theory [24] for their convenience. In this manuscript we have defined BQSVNR set with it's various types of operations. Also various similarity measures of BQSVNR set are discussed. Later a uncertainty based real scientific problem has been worked out by using BQSVNR set model. Finally the future work related to our paper is given.

2. BQSVNR set

Throughout this paper we will consider all the definitions over $X \neq \phi$ together with an equivalence relation R and we will denote it by (X, R). For the many other properties i.e. entropy, various types of similarity measures of a NS, SVN sets, BNS etc we refer our readers to follow any of the monograph say [7, 12, 18].

Definition 2.1. Suppose A be a BQSVN set in (X, R) with positive membership degrees $T^+(m)$,$C^+(m)$, $I^+(m)$, $F^+(m)$ respectively and negative membership degrees $T^-(m), C^-(m), I^-(m), F^-(m)$-respectively of an element $m \in X$. The lower and upper approximations of A in (X, R) denoted by $\underline{L}(A)$ and $\overline{L}(A)$ respectively are defined as follows:

$$\underline{L}(A) = \{\langle m, T^+_{\underline{A}}(m), C^+_{\underline{A}}(m), I^+_{\underline{A}}(m), F^+_{\underline{A}}(m), T^-_{\underline{A}}(m), C^-_{\underline{A}}(m), I^-_{\underline{A}}(m), F^-_{\underline{A}}(m)\rangle | m \in [m]_R \subseteq X\}$$

$$\overline{L}(A) = \{\langle m, T^+_{\overline{A}}(m), C^+_{\overline{A}}(m), I^+_{\overline{A}}(m), F^+_{\overline{A}}(m), T^-_{\overline{A}}(m), C^-_{\overline{A}}(m), I^-_{\overline{A}}(m), F^-_{\overline{A}}(m)\rangle | m \in [m]_R \subseteq X\},$$

where,

$$T^+_{\underline{A}}(m) = \wedge_{m\in[m]_R} T^+_A(m), C^+_{\underline{A}}(m) = \wedge_{m\in[m]_R} C^+_A(m), I^+_{\underline{A}}(m) = \vee_{m\in[m]_R} I^+_A(m),$$
$$F^+_{\underline{A}}(m) = \vee_{m\in[m]_R} F^+_A(m), T^-_{\underline{A}}(m) = \vee_{m\in[m]_R} T^-_A(m), C^-_{\underline{A}}(m) = \vee_{m\in[m]_R} C^-_A(m),$$
$$I^-_{\underline{A}}(m) = \wedge_{m\in[m]_R} I^-_A(m), F^-_{\underline{A}}(m) = \wedge_{m\in[m]_R} F^-_A(m), T^+_{\overline{A}}(m) = \vee_{m\in[m]_R} T^+_A(m),$$
$$C^+_{\overline{A}}(m) = \vee_{m\in[m]_R} C^+_A(m), I^+_{\overline{A}}(m) = \wedge_{m\in[m]_R} I^+_A(m), F^+_{\overline{A}}(m) = \wedge_{m\in[m]_R} F^+_A(m),$$
$$T^-_{\overline{A}}(m) = \wedge_{m\in[m]_R} T^-_A(m), C^-_{\overline{A}}(m) = \wedge_{m\in[m]_R} C^-_A(m), I^-_{\overline{A}}(m) = \vee_{m\in[m]_R} I^-_A(m),$$
$$F^-_{\overline{A}}(m) = \vee_{m\in[m]_R} F^-_A(m),$$

where $0 \le T^+_{\underline{A}}(m)+C^+_{\underline{A}}(m)+I^+_{\underline{A}}(m)+F^+_{\underline{A}}(m) \le 4$,$-4 \le T^-_{\underline{A}}(m)+C^-_{\underline{A}}(m)+I^-_{\underline{A}}(m)+F^-_{\underline{A}}(m) \le 0$, $0 \le T^+_{\overline{A}}(m) + C^+_{\overline{A}}(m) + I^+_{\overline{A}}(m) + F^+_{\overline{A}}(m) \le 4$, $-4 \le T^-_{\overline{A}}(m) + C^-_{\overline{A}}(m) + I^-_{\overline{A}}(m) + F^-_{\overline{A}}(m) \le 0$ and $\vee, \wedge$ mean "max" and "min" operators respectively, $T_A(m), C_A(m), I_A(m), F_A(m)$ are the respective membership function of m w.r.t A. $\underline{L}(A)$ and $\overline{L}(A)$ are two bipolar QSVN sets in X. The pair $(\underline{L}(A), \overline{L}(A))$ is called BQSVNR set in (X, R).

Example 2.2. Consider the case where four economists m_1, m_2, m_3, m_4 were asked to give their opinion on the statement "Rate of economical growth of India in 2020 will cross the rate of economic growth in the year 2019". Each economists will give concern in terms of degree of agreement, agreement or disagreement both, neither agreement nor disagreement, disagreement together with positive and negative aspects respectively. Let R_1 be a set on U of all economists which may be considered as follows:

$$m, n \in U, mR_1n \text{ iff } m \text{ and } n$$

both belongs to same organization i.e. IMF, London school of economics etc.

The aggregate of their opinion can be very well expressed into the following equivalent class R_1 as following:

$$U/R_1 = \{\{m_1, m_2\}, \{m_3\}, \{m_4\}\}$$

We can develop a BQSVN set A on the basis of the economists opinion as follows:

$$A = \{(m_1, (0.8, 0.6, 0.2, 0.2, -0.4, -0.5, -0.3, -0.7)),$$
$$(m_2, (0.4, 0.6, 0.6, 0.8, -0.6, -0.5, -0.4, -0.8)),$$
$$(m_3, (0.5, 0.5, 0.7, 0.1, -0.8, -0.6, -0.4, -0.6)),$$
$$(m_4, (0.6, 0.7, 0.4, 0.1, -0.5, -0.3, -0.6, -0.4))\}.$$

Now by Definition 2.1, we have,

$$\underline{L}(A) = \{(m_1, (0.4, 0.6, 0.6, 0.8, -0.4, -0.5, -0.4, -0.8)),$$
$$(m_2, (0.4, 0.6, 0.6, 0.8, -0.4, -0.5, -0.4, -0.8)),$$
$$(m_3, (0.5, 0.5, 0.7, 0.1, -0.8, -0.6, -0.4, -0.6)),$$
$$(m_4, (0.6, 0.7, 0.4, 0.1, -0.5, -0.3, -0.6, -0.4))\}$$
$$\overline{L}(A) = \{(m_1, (0.8, 0.6, 0.2, 0.2, -0.6, -0.5, -0.3, -0.7)),$$
$$(m_2, (0.8, 0.6, 0.2, 0.2, -0.6, -0.5, -0.3, -0.7)),$$
$$(m_3, (0.5, 0.5, 0.7, 0.1, -0.8, -0.6, -0.4, -0.6)),$$
$$(m_4, (0.6, 0.7, 0.4, 0.1, -0.5, -0.3, -0.6, -0.4))\}$$

Hence $(\underline{L}(A), \overline{L}(A))$ provides the rate of growth of India in 2020 in comparison with the rate of growth in 2019.

Definition 2.3. Suppose A be a BQSVN set in (X, R). If

(i) $\underline{L}(A) = \overline{L}(A)$, then $(\underline{L}(A), \overline{L}(A))$ is called constant BQSVNR set in (X, R).

(ii) $\forall m \in [m]_R \cap \underline{L}(A)(\text{and } \overline{L}(A))$, $T_A^+(m) = 1 = C_A^+(m)$, $I_A^+(m) = F_A^+(m) = 0$, $T_A^-(m) = 0 = C_A^+(m)$, $I_A^-(m) = F_A^-(m) = 1$, then $(\underline{L}(A), \overline{L}(A))$ is called an unit BQSVNR set in (X, R).

(iii) $\forall m \in [m]_R \cap \underline{L}(A)(\text{and } \overline{L}(A))$, $T_A^+(m) = 0 = C_A^+(m)$, $I_A^+(m) = F_A^+(m) = 1$, $T_A^-(m) = 1 = C_A^+(m)$, $I_A^-(m) = F_A^-(m) = 0$,, then $(\underline{L}(A), \overline{L}(A))$ is called zero BQSVNR set in (X, R) and it is denoted by Φ.

Now some set-theoretic operations on BQSVNR set over (X, R) will be studied.

Definition 2.4. Consider $L(A) = (\underline{L}(A), \overline{L}(A))$ is a BQSVNR set in (X, R). We define complement BQSVNR set $L^c(A)$ of $L(A)$ as $L^c(A) = ((\underline{L}(A))^c, (\overline{L}(A))^c)$, where

$$(\underline{L}(A))^c = \{\langle x, F_{\underline{A}}^+(m), 1 - I_{\underline{A}}^+(m), 1 - C_{\underline{A}}^+(m), T_{\underline{A}}^+(m),$$
$$F_{\underline{A}}^-(m), 1 - I_{\underline{A}}^-(m), 1 - C_{\underline{A}}^-(m), T_{\underline{A}}^-(m)\rangle | m \in [m]_R \subseteq X\}$$
$$(\overline{L}(A))^c = \{\langle x, F_{\overline{A}}^+(m), 1 - I_{\overline{A}}^+(m), 1 - C_{\overline{A}}^+(m), T_{\overline{A}}^+(m),$$
$$F_{\overline{A}}^-(m), 1 - I_{\overline{A}}^-(m), 1 - C_{\overline{A}}^-(m), T_{\overline{A}}^-(m)\rangle | m \in [m]_R \subseteq X\}$$

Definition 2.5. Suppose $A = (\underline{L}(A), \overline{L}(A))$ and $B = (\underline{L}(B), \overline{L}(B))$ are two BQSVNR set over X. We say $A \subseteq B$ if $\underline{L}(A) \subseteq \underline{L}(B)$, $\overline{L}(A) \subseteq \overline{L}(B)$ i.e.
$T_{\underline{A}}^+(m) \leq T_{\underline{B}}^+(m), C_{\underline{A}}^+(m) \leq C_{\underline{B}}^+(m), I_{\underline{A}}^+(m) \geq I_{\underline{B}}^+(m), F_{\underline{A}}^+(m) \geq F_{\underline{B}}^+(m), T_{\underline{A}}^-(m) \geq T_{\underline{B}}^-(m), C_{\underline{A}}^-(m) \geq C_{\underline{B}}^-(m), I_{\underline{A}}^-(m) \leq I_{\underline{B}}^-(m), F_{\underline{A}}^-(m) \leq F_{\underline{B}}^-(m),$

$T^+_{\overline{A}}(m) \geq T^+_{\overline{B}}(m), C^+_{\overline{A}}(m) \geq C^+_{\overline{B}}(m), I^+_{\overline{A}}(m) \leq I^+_{\overline{B}}(m), F^+_{\overline{A}}(m) \leq F^+_{\overline{B}}(m), T^-_{\overline{A}}(m) \leq T^-_{\overline{B}}(m), C^-_{\overline{A}}(m) \leq C^-_{\overline{B}}(m), I^-_{\overline{A}}(m) \geq I^-_{\overline{B}}(m), F^-_{\overline{A}}(m) \geq F^-_{\overline{B}}(m)\ \forall m \in [m]_R \subseteq X$.

Definition 2.6. Suppose $A = (\underline{L}(A), \overline{L}(A))$ and $B = (\underline{L}(B), \overline{L}(B))$ are two BQSVNR set over X. Then the union of A, B i.e. $A \cup B = (\underline{L}(A) \cup \underline{L}(B), \overline{L}(A) \cup \overline{L}(B))$ is defined as: $T^+_{\underline{A}}(m) \vee T^+_{\underline{B}}(m), C^+_{\underline{A}}(m) \vee C^+_{\underline{B}}(m), I^+_{\underline{A}}(m) \wedge I^+_{\underline{B}}(m), F^+_{\underline{A}}(m) \wedge F^+_{\underline{B}}(m), T^-_{\underline{A}}(m) \wedge T^-_{\underline{B}}(m), C^-_{\underline{A}}(m) \wedge C^-_{\underline{B}}(m), I^-_{\underline{A}}(m) \vee I^-_{\underline{B}}(m), F^-_{\underline{A}}(m) \vee F^-_{\underline{B}}(m), T^+_{\overline{A}}(m) \wedge T^+_{\overline{B}}(m), C^+_{\overline{A}}(m) \wedge C^+_{\overline{B}}(m), I^+_{\overline{A}}(m) \vee I^+_{\overline{B}}(m), F^+_{\overline{A}}(m) \vee F^+_{\overline{B}}(m), T^-_{\overline{A}}(m) \vee T^-_{\overline{B}}(m), C^-_{\overline{A}}(m) \vee C^-_{\overline{B}}(m), I^-_{\overline{A}}(m) \wedge I^-_{\overline{B}}(m), F^-_{\overline{A}}(m) \wedge F^-_{\overline{B}}(m)\ \forall m \in [m]_R \subseteq X$.

Definition 2.7. Suppose $A = (\underline{L}(A), \overline{L}(A))$ and $B = (\underline{L}(B), \overline{L}(B))$ are two BQSVNR set over X. Then the intersection of A, B i.e. $A \cap B = (\underline{L}(A) \cap \underline{L}(B), \overline{L}(A) \cap \overline{L}(B))$ is defined as: $T^+_{\underline{A}}(m) \wedge T^+_{\underline{B}}(m), C^+_{\underline{A}}(m) \wedge C^+_{\underline{B}}(m), I^+_{\underline{A}}(m) \vee I^+_{\underline{B}}(m), F^+_{\underline{A}}(m) \vee F^+_{\underline{B}}(m), T^-_{\underline{A}}(m) \vee T^-_{\underline{B}}(m), C^-_{\underline{A}}(m) \vee C^-_{\underline{B}}(m), I^-_{\underline{A}}(m) \wedge I^-_{\underline{B}}(m), F^-_{\underline{A}}(m) \wedge F^-_{\underline{B}}(m), T^+_{\overline{A}}(m) \vee T^+_{\overline{B}}(m), C^+_{\overline{A}}(m) \vee C^+_{\overline{B}}(m), I^+_{\overline{A}}(m) \wedge I^+_{\overline{B}}(m), F^+_{\overline{A}}(m) \wedge F^+_{\overline{B}}(m), T^-_{\overline{A}}(m) \wedge T^-_{\overline{B}}(m), C^-_{\overline{A}}(m) \wedge C^-_{\overline{B}}(m), I^-_{\overline{A}}(m) \vee I^-_{\overline{B}}(m), F^-_{\overline{A}}(m) \vee F^-_{\overline{B}}(m)\ \forall m \in [m]_R \subseteq X$.

Proposition 2.8. *Consider three BQSVNR sets* $\Theta_1, \Theta_2, \Theta_3$ *in* (X, R). *Then for all for BQSVNR sets over* X *we have the following:*

(i) $\Theta_1 \cup \Theta_2 = \Theta_2 \cup \Theta_1;\ \Theta_1 \cap \Theta_2 = \Theta_2 \cap \Theta_1$.
(ii) $\Theta_1 \cup (\Theta_2 \cup \Theta_3) = (\Theta_1 \cup \Theta_2) \cup \Theta_3;\ \ \Theta_1 \cap (\Theta_2 \cap \Theta_3) = (\Theta_1 \cap \Theta_2) \cap \Theta_3$
(iii) $\Theta_3 \cap (\Theta_3 \cup \Theta_2) = \Theta_3;\ \ \Theta_3 \cup (\Theta_3 \cap \Theta_2) = \Theta_3$.
(iv) $(\Theta_1^c)^c = \Theta_1$.
(v) $(\Theta_1 \cup \Theta_2)^c = \Theta_1^c \cap \Theta_2^c; (\Theta_1 \cap \Theta_2)^c = \Theta_1^c \cup \Theta_2^c$
(vi) $\Theta_1 \cup \Theta_1 = \Theta_1;\ \Theta_1 \cap \Theta_1 = \Theta_1$.

We omit the proof of the Proposition 2.8 as it is very straight forward.

3. Different similarity measures of BQSVNR sets

Consider an universal set $X \neq \phi$ and denote the set of BQSVNR set over (X, R) by $\mathcal{B}(X)$.

Definition 3.1. A mapping $s : \mathcal{B}(X) \times \mathcal{B}(X) \to [0, 1]$ is called a similarity measure iff for $W_1, W_2 \in \mathcal{B}(X)$,

(i) $s(W_1, W_2) = s(W_2, W_1)$
(ii) $0 \leq s(W_1, W_2) < 1$ and $s(W_1, W_2) = 1$ iff $W_1 = W_2$.
(iii) for any $W_1, W_2, W_3 \in \mathcal{B}(X)$, $W_1 \subset W_2 \subset W_3$, $s(W_1, W_3) \leq s(W_1, W_2) \wedge s(W_2, W_3)$.

Although in Definition 3.1 the condition (iii) exists but some familiar similarity measure techniques i.e. weighted similarity measure, cosine similarity measures etc fail to satisfy it. On the other hand these similarity techniques has a wide application in real world discission making problems. Thus it is essential to introduce a new definition of similarity measure say "Quasi Similarity Measure" which omits the condition (iii) of Definition 3.1.

Definition 3.2. Consider $\mathcal{B}(X)$, the set of BQSVNR sets over an universe X. Then a function $s' : \mathcal{B}(X) \times \mathcal{B}(X) \to [0, 1]$ is called a quasi similarity measure iff for $W_1, W_2 \in \mathcal{B}(X)$,

(i) $s(W_1, W_2) = s(W_2, W_1)$
(ii) $0 \leq s(W_1, W_2) < 1$ and $s(W_1, W_2) = 1$ iff $W_1 = W_2$.

3.1. *Distance measures between two BQSVNR sets*

Definition 3.3. A function $d_b : \mathcal{B}(X) \times \mathcal{B}(X) \to \mathbb{R}^+$ is called a distance measure for BQSVNR sets iff for $W_1, W_2, W_3 \in \mathcal{B}(X)$,

(i) $d_b(W_1, W_2) = d_b(W_2, W_1)$
(ii) $d_b(W_1, W_2) \geq 0$ and $d_b(W_1, W_2) = 0$ iff $W_1 = W_2$.
(iii) $d_b(W_1, W_2) \leq d_b(W_1, W_3) + d_b(W_3, W_2)$.

Clearly d_b is a metric on $\mathcal{B}(X)$. Suppose $\Theta, \Gamma \in \mathcal{B}(X)$ over an universal set $X = \{x_1, x_2, \ldots, x_n\}$.

Definition 3.4. The Hamming distance $h(\Theta, \Gamma)$ between two BQSVNR sets Θ and Γ is defined as

$$h(\Theta, \Gamma) = \min\left\{\{h(\underline{\Theta}, \underline{\Gamma})\}, \{h(\overline{\Theta}, \overline{\Gamma})\}\right\} \text{ where,}$$

$$\begin{aligned} h(\underline{\Theta}, \underline{\Gamma}) = \{\sum_{j=1}^{n} & |T^+_{\underline{\Theta}}(x_j) - T^+_{\underline{\Gamma}}(x_j)| + |C^+_{\underline{\Theta}}(x_j) - C^+_{\underline{\Gamma}}(x_j)| + |I^+_{\underline{\Theta}}(x_j) - I^+_{\underline{\Gamma}}(x_j)| + \\ & |F^+_{\underline{\Theta}}(x_j) - F^+_{\underline{\Gamma}}(x_j)| + |T^-_{\underline{\Theta}}(x_j) - T^-_{\underline{\Gamma}}(x_j)| + |C^-_{\underline{\Theta}}(x_j) - C^-_{\underline{\Gamma}}(x_j)| + \\ & |I^-_{\underline{\Theta}}(x_j) - I^-_{\underline{\Gamma}}(x_j)| + |F^-_{\underline{\Theta}}(x_j) - F^-_{\underline{\Gamma}}(x_j)|\} \\ h(\overline{\Theta}, \overline{\Gamma}) = \{\sum_{j=1}^{n} & |T^+_{\overline{\Theta}}(x_j) - T^+_{\overline{\Gamma}}(x_j)| + |C^+_{\overline{\Theta}}(x_j) - C^+_{\overline{\Gamma}}(x_j)| + |I^+_{\overline{\Theta}}(x_j) - I^+_{\overline{\Gamma}}(x_j)| + \\ & |F^+_{\overline{\Theta}}(x_j) - F^+_{\overline{\Gamma}}(x_j)| + |T^-_{\overline{\Theta}}(x_j) - T^-_{\overline{\Gamma}}(x_j)| + |C^-_{\overline{\Theta}}(x_j) - C^-_{\overline{\Gamma}}(x_j)| + \\ & |I^-_{\overline{\Theta}}(x_j) - I^-_{\overline{\Gamma}}(x_j)| + |F^-_{\overline{\Theta}}(x_j) - F^-_{\overline{\Gamma}}(x_j)|\} \ \forall x_j \in X. \end{aligned}$$

Definition 3.5. The Normalized Hamming distance between Θ and Γ is defined as $h_N(\Theta, \Gamma) = \frac{1}{8n}(h(\Theta, \Gamma))$.

Definition 3.6. The Euclidean distance $E(\Theta,\Gamma)$ is defined as follows:

$$E(\Theta,\Gamma)=\min\{\{E(\underline{\Theta},\underline{\Gamma})\},\{E(\overline{\Theta},\overline{\Gamma})\}\} \text{ where,}$$

$$E(\underline{\Theta},\underline{\Gamma})=\{\sum_{j=1}^{n}|T^{+}_{\underline{\Theta}}(x_j)-T^{+}_{\underline{\Gamma}}(x_j)|^2+|C^{+}_{\underline{\Theta}}(x_j)-C^{+}_{\underline{\Gamma}}(x_j)|^2+|I^{+}_{\underline{\Theta}}(x_j)-I^{+}_{\underline{\Gamma}}(x_j)|^2+ |F^{+}_{\underline{\Theta}}(x_j)-F^{+}_{\underline{\Gamma}}(x_j)|^2+|T^{-}_{\underline{\Theta}}(x_j)-T^{-}_{\underline{\Gamma}}(x_j)|^2+|C^{-}_{\underline{\Theta}}(x_j)-C^{-}_{\underline{\Gamma}}(x_j)|^2+ |I^{-}_{\underline{\Theta}}(x_j)-I^{-}_{\underline{\Gamma}}(x_j)|^2+|F^{-}_{\underline{\Theta}}(x_j)-F^{-}_{\underline{\Gamma}}(x_j)|^2\}^{\frac{1}{2}}$$

$$E(\overline{\Theta},\overline{\Gamma})=\{\sum_{j=1}^{n}|T^{+}_{\overline{\Theta}}(x_j)-T^{+}_{\overline{\Gamma}}(x_j)|^2+|C^{+}_{\overline{\Theta}}(x_j)-C^{+}_{\overline{\Gamma}}(x_j)|^2+|I^{+}_{\overline{\Theta}}(x_j)-I^{+}_{\overline{\Gamma}}(x_j)|^2+ |F^{+}_{\overline{\Theta}}(x_j)-F^{+}_{\overline{\Gamma}}(x_j)|^2+|T^{-}_{\overline{\Theta}}(x_j)-T^{-}_{\overline{\Gamma}}(x_j)|^2+|C^{-}_{\overline{\Theta}}(x_j)-C^{-}_{\overline{\Gamma}}(x_j)|^2+ |I^{-}_{\overline{\Theta}}(x_j)-I^{-}_{\overline{\Gamma}}(x_j)|^2+|F^{-}_{\overline{\Theta}}(x_j)-F^{-}_{\overline{\Gamma}}(x_j)|^2\}^{\frac{1}{2}}\ \forall x_j\in X.$$

Definition 3.7. The normalized Euclidean distance $Q(\Theta,\Gamma)$ is defined as follows:

$$Q(\Theta,\Gamma)=\frac{1}{2\sqrt{2n}}E(\Theta,\Gamma).$$

Gradually distance measurement process which gives an idea about the similarities between two BQSVNR sets becomes the main attraction among the researchers. Also different MCDM problems can be solved using similarity measures technique [20, 21]. On the other hand many mathematicians have used a variety of distance-based operators say induced weighted aggregation distance (IOWAD) operators, an extended version of common OWA operators to solve various problems [11, 13, 15]. However we will only concentrate only on the following distance oriented similarity measures.

3.2. *Distance oriented similarity measure between two BQSVNR sets*

Consider two BQSVNR set Θ_1,Θ_2 over $\mathcal{B}(X)$. Based on all previously defined distances two new similarity measures $S_1(\Theta_1,\Theta_2),S_2(\Theta_1,\Theta_2)$ for a pair of BQSVNR set Θ_1,Θ_2 can be defined:

$$S_1(\Theta_1,\Theta_2)=\frac{1}{1+h(\Theta_1,\Theta_2)},\ S_2(\Theta_1,\Theta_2)=e^{-\alpha.h(\Theta_1,\Theta_2)},$$

where $\alpha\in\mathbb{R}^+$ is the *steepness measure* of $S_2(\Theta_1,\Theta_2)$. In a similar way using Euclidian distance, we define another pair of similarity measure $S'_1(\Theta_1,\Theta_2),S'_2(\Theta_1,\Theta_2)$ as follows:

$$S'_1(\Theta_1,\Theta_2)=\frac{1}{1+E(\Theta_1,\Theta_2)},\ S'_2(\Theta_1,\Theta_2)=e^{-\beta.E(\Theta_1,\Theta_2)}.$$

where $\beta\in\mathbb{R}^+$ is the *steepness measure* of $S'_2(\Theta_1,\Theta_2)$. one can easily seen that $S_1(\Theta_1,\Theta_2),S_2(\Theta_1,\Theta_2),S'_1(\Theta_1,\Theta_2),S'_2(\Theta_1,\Theta_2)$ satisfies the axioms of Definitions 3.1 for two BQSVNR sets Θ_1,Θ_2.

3.3. *Cosine similarity measure of BQSVNR sets*

Here we will discuss the cosine similarity measure, a basic similarity measure technique between two BQSVNR sets. To obtain this similarity measure we represent two BQSVNR sets as vectors. We illustrate our proposed new cosine similarity measure C_{BQSVNR} of BQSVNR sets as following:

Definition 3.8. Consider $A, B \in \mathcal{B}(X)$. For each $x_j \in X,\ j = 1, 2, \ldots, n,$ we define

$$C_{BQSVNR}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\frac{S_1}{S_2.S_3}, where,$$

$$S_1 = \partial T_A(x_j)\partial T_B(x_j) + \partial C_A(x_j)\partial C_B(x_j) + \partial I_A(x_j)\partial I_B(x_j) +$$
$$\partial F_A(x_j)\partial F_B(x_j),$$

$$S_2 = \sqrt{\partial T_A(x_j)^2 + \partial C_A(x_j)^2 + \partial I_A(x_j)^2 + \partial F_A(x_j)^2},$$

$$S_3 = \sqrt{\partial T_B(x_j)^2 + \partial C_B(x_j)^2 + \partial I_B(x_j)^2 + \partial F_B(x_j)^2},$$

where $\partial T^+_X(x_j) = \dfrac{T^+_{\underline{X}}(x_j) + T^+_{\overline{X}}(x_j)}{2}, \partial T^-_X(x_j) = \dfrac{T^-_{\underline{X}}(x_j) + T^-_{\overline{X}}(x_j)}{2}$

$$\partial C^+_X(x_j) = \frac{C^+_{\underline{X}}(x_j) + C^+_{\overline{X}}(x_j)}{2}, \partial C^-_X(x_j) = \frac{C^-_{\underline{X}}(x_j) + C^-_{\overline{X}}(x_j)}{2}$$

$$\partial I^+_X(x_j) = \frac{I^+_{\underline{X}}(x_j) + I^+_{\overline{X}}(x_j)}{2}, \partial I^-_X(x_j) = \frac{I^-_{\underline{X}}(x_j) + I^-_{\overline{X}}(x_j)}{2}$$

$$\partial F^+_X(x_j) = \frac{F^+_{\underline{X}}(x_j) + F^+_{\overline{X}}(x_j)}{2}, \partial F^-_X(x_j) = \frac{F^-_{\underline{X}}(x_j) + F^-_{\overline{X}}(x_j)}{2}$$

$$\partial T_X(x_j) = \frac{\partial T^+_X(x_j) + \partial T^-_X(x_j)}{2}, \partial C_X(x_j) = \frac{\partial C^+_X(x_j) + \partial C^-_X(x_j)}{2}$$

$$\partial I_X(x_j) = \frac{\partial I^+_X(x_j) + \partial I^-_X(x_j)}{2}, \partial F_X(x_j) = \frac{\partial F^+_X(x_j) + \partial F^-_X(x_j)}{2},$$

where $X \in \{A, B\}$.

Theorem 3.9. *$C_{BQSVNR}(A,B)$ is a similarity measure between two BQSVNR sets $A, B \in \mathcal{B}(X)$.*

We omit the proof as it is very simple.

3.4. *Similarity measures of BQSVNR sets based on membership values*

Consider $A, B \in \mathcal{B}(X)$. For each $x_j \in X, j = 1, 2, \ldots, n$ and for $k = 1, 2, \ldots, 4$ define the functions $\underline{h_k}^+, \underline{h_k}^-, \overline{h_k}^+, \overline{h_k}^- : X \to [0, 1]$ respectively as

$$\underline{h_1}^{\pm}(x_j) = |T^{\pm}_{\underline{A}}(x_j) - T^{\pm}_{\underline{B}}(x_j)|,$$
$$\underline{h_2}^{\pm}(x_j) = |F^{\pm}_{\underline{A}}(x_j) - F^{\pm}_{\underline{B}}(x_j)|,$$
$$\underline{h_3}^{\pm}(x_j) = \frac{1}{3}\big(h^{\pm}_1(x_j) + h^{\pm}_2(x_j) + |C^{\pm}_{\underline{A}}(x_j) - C^{\pm}_{\underline{B}}(x_j)|\big),$$
$$\underline{h_4}^{\pm}(x_j) = |I^{\pm}_{\underline{A}}(x_j) - I^{\pm}_{\underline{B}}(x_j)|,$$
$$\overline{h_1}^{\pm}(x_j) = |T^{\pm}_{\overline{A}}(x_j) - T^{\pm}_{\overline{B}}(x_j)|,$$
$$\overline{h_2}^{\pm}(x_j) = |F^{\pm}_{\overline{A}}(x_j) - F^{\pm}_{\overline{B}}(x_j)|,$$
$$\overline{h_3}^{\pm}(x_j) = \frac{1}{3}\big(h^{\pm}_1(x_j) + h^{\pm}_2(x_j) + |C^{\pm}_{\overline{A}}(x_j) - C^{\pm}_{\overline{B}}(x_j)|\big),$$
$$\overline{h_4}^{\pm}(x_j) = |I^{\pm}_{\overline{A}}(x_j) - I^{\pm}_{\overline{B}}(x_j)|$$

Now based on the above functions a new similarity measure $\widetilde{S}(A, B)$ can be defined as follows:

$$\widetilde{S}(A, B) = 1 - \frac{1}{4n}[\sum_{j=1}^{n}\sum_{k=1}^{4}\overline{h_k}^+(x_j) + \sum_{j=1}^{n}\sum_{k=1}^{4}\overline{h_k}^-(x_j) + \sum_{j=1}^{n}\sum_{k=1}^{4}\underline{h_k}^+(x_j) +$$
$$\sum_{j=1}^{n}\sum_{k=1}^{4}\underline{h_k}^-(x_j)].$$

The following theorem is obvious:

Theorem 3.10. *$\widetilde{S}(A, B)$ is a similarity measure between $A, B \in \mathcal{B}(X)$.*

Proof. For a BQSVNR set all the positive membership values of $\underline{T}, \underline{C}, \underline{I}, \underline{F}$, $\overline{T}, \overline{C}, \overline{I}, \overline{F}$ lie between $[0, 1]$ and the negative membership values of $\underline{T}, \underline{C}, \underline{I}, \underline{F}$, $\overline{T}, \overline{C}, \overline{I}, \overline{F}$ lie between $[-1, 0]$. Among all these quantities, all has maximum value 1 and the minimum value -1. As a result $0 \leq \widetilde{S}(A, B) \leq 1$. Again $\widetilde{S}(A, B) = 1$ implies that

$$T^+_{\underline{A}}(x_j) = T^+_{\underline{B}}(x_j), C^+_{\underline{A}}(x_j) = C^+_{\underline{B}}(x_j), I^+_{\underline{A}}(x_j) = I^+_{\underline{B}}(x_j), F^+_{\underline{A}}(x_j) = F^+_{\underline{B}}(x_j),$$
$$T^-_{\underline{A}}(x_j) = T^-_{\underline{B}}(x_j), C^-_{\underline{A}}(x_j) = C^-_{\underline{B}}(x_j), I^-_{\underline{A}}(x_j) = I^-_{\underline{B}}(x_j), F^-_{\underline{A}}(x_j) = F^-_{\underline{B}}(x_j),$$
$$T^+_{\overline{A}}(x_j) = T^+_{\overline{B}}(x_j), C^+_{\overline{A}}(x_j) = C^+_{\overline{B}}(x_j), I^+_{\overline{A}}(x_j) = I^+_{\overline{B}}(x_j), F^+_{\overline{A}}(x_j) = F^+_{\overline{B}}(x_j),$$
$$T^-_{\overline{A}}(x_j) = T^-_{\overline{B}}(x_j), C^-_{\overline{A}}(x_j) = C^-_{\overline{B}}(x_j), I^-_{\overline{A}}(x_j) = I^-_{\overline{B}}(x_j), F^-_{\overline{A}}(x_j) = F^-_{\overline{B}}(x_j)$$
$$\forall x_j \in X.$$

Lastly for $A, B, C \in \mathcal{B}(X)$ we suppose that $A \subseteq B \subseteq C$. Now by the Definition 2.5 we have $\forall\ x_j \in X,\ \forall\ \ j = 1, 2, \ldots, 4$

$$\widetilde{S}(A, C) < \widetilde{S}(A, B) \wedge \widetilde{S}(B, C).$$

Hence the result follows. □

3.5. *Weighted similarity measure*

The weighted similarity measure between $A, B \in \mathcal{B}(X)$ is defined as follows:

$$S^w(A,B) = 1 - \frac{1}{4n}[\sum_{j=1}^{n}\sum_{k=1}^{4} w_j \overline{h_k}^+(x_j) + \sum_{j=1}^{n}\sum_{k=1}^{4} w_j \overline{h_k}^-(x_j) + \sum_{j=1}^{n}\sum_{k=1}^{4} w_j \underline{h_k}^+(x_j) + \sum_{j=1}^{n}\sum_{k=1}^{4} w_j \underline{h_k}^-(x_j)]^{\frac{1}{l}}.$$

where l is any integer defined to be the order of similarity, w_i are the weights corresponding with x_j, $j = 1, 2, \ldots, n$ s.t. $\sum_{j=1}^{n} w_j = 1$. Using the same proof procedure of the Theorem 3.10, $S^w(A,B)$ is also a measure of similarity between the two BQSVNR sets $A, B \in \mathcal{B}(X)$.

4. **An application of BQSVNR sets**

By using a BQSVNR set model a real world medical diagnosis problem can be represented very well. To solve these type of medical problem similarity measure technique between two BQSVNR set is quite powerful procedure. Using these similarity measure technique anyone can detect whether a patient is being suffered with a disease or not. In between June to September it is seen that H1N1 virus spreads out rapidly in Kolkata and its sub-urban area of West Bengal India. The patient of these particular virus effected decease has primarily 4 symptoms, namely headache, high fever, cough, red rashes in the body. But in every patient the primary symptoms are not clearly visible. Also in many different viral infections these symptoms are common. The process of classification of patients by considering a variety of symptoms is a difficult task. Our similarity measurement technique which considers patients versus symptoms record provides an approximate way to treat a patient. The basic feature of our study considers only the positive as well as negative value of truth, ignorance, contradiction and false value respectively of each element of the BQSVNR sets.

Suppose P_2 and P_3 be two persons who are suspected to be infected by H1N1 virus. Let $D = \{$headache, high fever, cough, red rashes in the body$\}$ be a set of symptoms. Consider P_1 is a model patient who are infected by H1N1 virus. Our solution is to examine the condition of P_2, P_3 w.r.t. the symptoms of P_1 in BQSVNR environment. We have represented our problem as a BQSVNR set model as follows:

$$P_1 = (\overline{N(P_1)}, \underline{N(P_1)})$$
$$= \langle(0.6, 0.4, 0.2, 0.4, -0.4, -0.5, -0.1, -0.8), (0.8, 0.6, 0.7, 0.1, -0.4, -0.3, -0.4, -0.5)\rangle/x_1$$
$$+ \langle(0.5, 0.5, 0.6, 0.4, -.4, -0.3, -0.2, -0.9), (0.7, 0.4, 0.4, 0.1, -0.3, -0.2, -0.7, -0.6)\rangle/x_2$$
$$+ \langle(0.7, 0.5, 0.6, 0.8, -0.5, -0.4, -0.8, -0.7), (0.6, 0.5, 0.5, 0.4, -0.2, -0.4, -0.6, -0.8)\rangle/x_3$$
$$+ \langle(0.8, 0.6, 0.7, 0.1, -0.4, -0.2, -0.6, -0.3), (0.5, 0.6, 0.2, 0.3, -0.1, -0.4, -0.6, -0.9)\rangle/x_4.$$
$$P_2 = (\overline{N(P_2)}, \underline{N(P_2)})$$
$$= \langle(0.4, 0.3, 0.3, 0.4, -0.5, -0.6, -0.1, -0.6), (0.4, 0.6, 0.6, 0.2, -0.4, -0.4, -0.4, -0.5)\rangle/x_1$$
$$+ \langle(0.6, 0.5, 0.4, 0.3, -.4, -0.3, -0.1, -0.8), (0.6, 0.6, 0.3, 0.2, -0.4, -0.2, -0.5, -0.4)\rangle/x_2$$
$$+ \langle(0.8, 0.5, 0.5, 0.6, -0.6, -0.6, -0.6, -0.5), (0.6, 0.4, 0.5, 0.5, -0.3, -0.5, -0.4, -0.6)\rangle/x_3$$
$$+ \langle(0.7, 0.7, 0.6, 0.2, -0.4, -0.3, -0.8, -0.3), (0.5, 0.6, 0.4, 0.5, -0.5, -0.7, -0.4, -0.2)\rangle/x_4.$$
$$P_3 = (\overline{N(P_3)}, \underline{N(P_3)})$$
$$= \langle(0.7, 0.8, 0.9, 0.1, -0.4, -0.4, -0.5, -0.2), (0.6, 0.4, 0.7, 0.6, -0.5, -0.1, -0.9, -0.7)\rangle/x_1$$
$$+ \langle(0.9, 0.2, 0.3, 0.7, -.1, -0.4, -0.7, -0.8), (0.5, 0.6, 0.7, 0.2, -0.4, -0.4, -0.7, -0.6)\rangle/x_2$$
$$+ \langle(0.1, 0.4, 0.8, 0.7, -0.4, -0.5, -0.5, -0.6), (0.4, 0.7, 0.4, 0.2, -0.5, -0.6, -0.4, -0.1)\rangle/x_3$$
$$+ \langle(0.5, 0.6, 0.4, 0.8, -0.1, -0.4, -0.6, -0.9), (0.6, 0.4, 0.7, 0.4, -0.2, -0.6, -0.5, -0.2)\rangle/x_4.$$

Now from Definition 3.4 to Definition 3.7 respectively, we have

$$h(P_1, P_2) = 3.2, \quad h(P_1, P_3) = 7.1$$
$$h_N(P_1, P_2) = 0.1, \quad h_N(P_1, P_3) = 0.845$$
$$E(P_1, P_2) = 0.693, \quad E(P_1, P_3) = 1.609$$

Now we calculate the following measures (as given by Section 3.2) between the pair of persons P_1, P_2 and P_1, P_3 as follows:

$$S_1'(P_1, P_2) = 0.591, \; S_1'(P_1, P_3) = 0.621$$

Since any effected area the probability of infecting a healthy people by H1N1 virus is 80% [**?**] hence we have taken the steepness measure i.e. α, β as 0.8. From this we have,

$$S_2'(P_1, P_2) = 0.574, \; S_2'(P_1, P_3) = 0.76$$

Since in between any two BQSVNR sets there must be similarity thus we restrict ourselves if the similarity measure is > 0.6. Thus from the similarity measures S_1', S_2' we can conclude that the patient P_3 has a higher chance to be infected by H1N1 virus than the patient P_2.

5. Conclusion

The fuzzy set theory (FST) [1] was introduced almost 55 years ago. After its invention, in next half a decade time, many generalizations of FST has been proposed such as intuitionistic fuzzy sets, interval valued fuzzy sets, hesitant fuzzy sets, bipolar fuzzy sets etc and also many other new theories like rough sets, soft sets, neutrosophic sets etc. has came into existence. The chief purpose of all these theories is to model real life situations under different uncertainties using available tools. But it is now a well established fact that no single theory is capable of modeling all different types of uncertainty. For example, fuzzy set can't model uncertainty due to incompleteness; intuitionistic fuzzy sets can't handle para consistent information, rough set is not suitable for handling situations with graded belongingness, soft set is not useful in modeling situations with vague boundaries. Therefore it is a common practice to combine two or more such sets to form a hybrid set. Hybrid set possesses the characteristics of more than one set and therefore has greater capabilities in handling uncertain situations. On the other hand four valued logic has multiple uses in many areas such as digital circuits and data transmission. The QSVN sets utilize the power of four valued logic in modeling uncertainty. We here introduced and investigated a new type of hybrid set called BQSVNR. The BQSVNR set is an extended version of QSVN set, bipolar set as well as rough set. It can handle uncertain situation arisen due to factors like fuzziness, incompleteness, vagueness, haziness, para compactness and bipolarity. Therefore our set is more capable of modeling uncertainty in a better way than any other existing set. In future, one can apply our newly developed hybrid set to model different real life problems. Also one may try to extend our set to bipolar multi-partitioned single valued neutrosophic rough sets and study its properties.

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