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A Contemporary Approach on Neutrosophic Nano Topological Spaces

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Abstract: In this article, we implement a new notion of sets namely neutrosophic nano j-closed set, neutrosophic nano generalized closed set, neutrosophic nano generalized j-closed set and neutrosophic nano generalized j*-closed set in neutrosophic nano topological spaces. We also provide some appropriate examples to study the properties of these sets. The existing relations between some of these sets in neutrosophic nano topological space have been investigated.

Keywords: Neutrosophic nano j-closed set, neutrosophic nano generalized closed set, neutrosophic nano generalized j-closed set, neutrosophic nano generalized j*-closed set.

I. Introduction

In recent years, Topology plays a vast role in research area. In particular, the concept of neutrosophy is a trending tool in topology. We use fuzzy concept where we consider only the membership value. The intuitionistic fuzzy concept is used where the membership and the non-membership values are considered. But, more real life problems deal with indeterminacy. The suitable concept for the situation where the indeterminacy occurs is neutrosophy which is represented by the degree of membership (truth value), the degree of non-membership (falsity value) and the degree of indeterminacy.

The fuzzy concept was initially proposed by Zadeh [22] in 1965 and Chang [7] introduced Fuzzy topological spaces in 1968. Atanasov [6] defined intuitionistic fuzzy set and Coker [8] developed intuitionistic fuzzy topology. In 2005, Smarandache [17] introduced neutrosophic set and many researchers used this concept in engineering, medicine and many fields where the situation of indeterminacy arises. Abdel-Basset et.al, [1 - 5] working with many practical problems by using neutrosophy concept in the recent days. Salama et.al, [14] introduced the generalization of neutrosophic sets, neutrosophic closed sets and neutrosophic crisp sets in neutrosophic topological spaces.

The nano topology which has the maximum of five elements was introduced by Lellis Thivagar [9]. He applied nano topology for nutrition modelling [11] and medical diagnosis [12]. Zhang et.al [23], worked on neutrosophic rough sets over two universes. Lellis Thivagar initiated [10] neutrosophic nano topology and some closed sets on neutrosophic nano topological spaces were derived by recent researchers.

Sasikala and Arockiarani [15] introduced generalized j-closed set. Sasikala and Radhamani [16] introduced nano j-closed set in nano topological spaces. In this paper, we present a new set called neutrosophic nano j-closed set and work with some interesting examples. Also we investigate some of the properties of the introduced sets.

II. Preliminaries

Definition 2.1[9] Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U , called the indiscernibility relation. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to the relation R is the set of all objects, which can be for certain classified as X and it is denoted by $L_R(X)$. i.e., $L_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \subseteq U\}$, where $R(x)$ denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to the relation R is the set of all objects, which can be possibly classified as X and it is denoted by $U_R(X)$ .i.e., $U_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \varphi\}$

(iii) The boundary region of X with respect to the relation R is the set of all objects, which can be classified neither as X nor as not X and it is denoted by $B_R(X)$ .i.e., $B_R(X) = U_R(X) - L_R(X)$

Remark 2.2[9] If (U,R) is an approximation space and $X,Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\varphi) = U_R(\varphi) = \varphi$ and $L_R(U) = U_R(U) = U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.3[9] Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the properties mentioned in remark 2.2, $\tau_R(X)$ satisfies the following axioms:

(i) U and φ are in $\tau_R(X)$

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$

Then $\tau_R(X)$ forms a topology on U called the nano topology with respect to X . We call $(U,\tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets. The complement of nano open sets are called nano closed sets.

Definition 2.4[9] Let $(U,\tau_R(X))$ be a nano topological space. A subset A is called nano generalized closed (briefly Ng-closed) set if $NCl(A)\subseteq V$ where $A\subseteq V$ and V is nano open in U.

Definition 2.5[16] A subset A of a nano topological space $(U,\tau_R(X))$ is called a nano j-open set if $A\subseteq N\,Int[N\,PCl(A)]$. The complement of nano j-open set is called a nano j-closed (briefly Nj-closed) set.

i.e., if A is Nj-closed, then $NCl[N\,PInt(A)]\subseteq A$.

Definition 2.6[16] A subset A of a nano topological space $(U,\tau_R(X))$ is called a nano generalized j-closed (briefly Ngj-closed) set if $N\,JCl(A)\subseteq V$ where $A\subseteq V$ and V is nano open in U.

Definition 2.7[17] Let X be an universe of discourse with a general element x, the neutrosophic set is an object having the form $A=\{\prec x,\mu_A(X),\sigma_A(X),\gamma_A(X)\succ, x\in X\}$ where $\mu,\sigma,$ and γ each take the values from 0 to 1 and called as the degree of membership, degree of indeterminacy, and the degree of non-membership of the element $x\in X$ to the set A with the condition $0\le\mu_A(x)+\sigma_A(x)+\sigma_A(x)\le 3$.

Definition 2.8[10] Let U be a nonempty set and R be an equivalence relation on U. Let F be a neutrosophic set in U with the membership function μ_F, the indeterminacy function σ_F, and the non-membership function γ_F. The neutrosophic nano lower, neutrosophic nano upper approximations and neutrosophic nano boundary of F in the approximation (U,R), denoted by $\underline{N},\overline{N}$ and $BN(F)$ are respectively defined as follows:

(i) $\underline{N}(F)=\{\prec x,\mu_{\underline{R}(A)}(x),\sigma_{\underline{R}(A)}(x),\gamma_{\underline{R}(A)}(x)\succ / y\in[x]_R, x\in U\}$

(ii) $\overline{N}(F)=\{\prec x,\mu_{\overline{R}(A)}(x),\sigma_{\overline{R}(A)}(x),\gamma_{\overline{R}(A)}(x)\succ / y\in[x]_R, x\in U\}$

(iii) $BN(F)=\overline{N}-\underline{N}$

Where $\mu_{\underline{R}(A)}(x)=\wedge_{y\in[x]_R}\mu_A(y)$, $\sigma_{\underline{R}(A)}(x)=\wedge_{y\in[x]_R}\sigma_A(y)$, $\gamma_{\underline{R}(A)}(x)=\vee_{y\in[x]_R}\gamma_A(y)$,

$\mu_{\overline{R}(A)}(x)=\vee_{y\in[x]_R}\mu_A(y)$, $\sigma_{\overline{R}(A)}(x)=\vee_{y\in[x]_R}\sigma_A(y)$, $\gamma_{\overline{R}(A)}(x)=\wedge_{y\in[x]_R}\gamma_A(y)$

Definition 2.9[10] Let U be an universe, R be an equivalence relation on U and F be a neutrosophic set in U. If the collection $\tau_N(F)=\{0_N,1_N,\underline{N}(F),\overline{N}(F),BN(F)\}$ forms a topology, then it is said to be a neutrosophic nano topology. We call $(U,\tau_N(F))$ as the neutrosophic nano topological space. The elements of $\tau_N(F)$ are called neutrosophic nano open sets.

Definition 2.10[17] Let U be a nonempty set and the neutrosophic sets A and B are in the form $A=\{\prec x:\mu_A(x),\sigma_A(x),\gamma_A(x)\succ, x\in U\}$, $B=\{\prec x:\mu_B(x),\sigma_B(x),\gamma_B(x)\succ, x\in U\}$. Then the following statements hold:

(i) $0_N=\{\prec x,0,0,1\succ : x\in U\}$ and $1_N=\{\prec x,1,1,0\succ : x\in U\}$

(ii) $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x),\sigma_A(x)\le\sigma_B(x)$ or $\sigma_A(x)\ge\sigma_B(x),\gamma_A(x)\ge\gamma_B(x)$ for all $x\in U$

(iii) $A=B$ iff $A\subseteq B$ and $B\subseteq A$

(iv) $A^C=\{\prec x,\gamma_A(x),1-\sigma_A(x),\mu_A(x)\succ, x\in U\}$

(v) $A \cap B = \{ x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \ for all\ x \in U \}$

(vi) $A \cup B = \{ x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \ for all\ x \in U \}$

(vii) $A - B = \{x, \mu_A(x) \wedge \gamma_B(x), \sigma_A(x) \wedge 1 - \sigma_B(x), \gamma_A(x) \vee \mu_B(x) \ for all\ x \in U \}$

Definition 2.11[10] $[\tau_N(F)]^C$ is called the dual neutrosophic nano topology of $\tau_N(F)$. The elements of $[\tau_N(F)]^C$ are called neutrosophic nano closed (N_N closed) sets. Thus, a neutrosophic set $N(G)$ of U is neutrosophic nano closed iff $U - N(G)$ is neutrosophic nano open in $\tau_N(F)$.

Definition 2.12[10] Let $(U, \tau_N(A))$ be a neutrosophic nano topological space and $A = \{ \prec x, \mu_A(x), \sigma_A(x), \gamma_A(x) \succ : x \in U \}$ be a neutrosophic set in X. Then the neutrosophic closure and neutrosophic interior of A are defined by $N\,Cl(A) =$ intersection of all closed sets which contains A and $N\,Int(A) =$ union of all open sets which is contained in A.

A is a neutrosophic open set iff $A = N\,Int(A)$ and A is a neutrosophic closed set iff $A = N\,Cl(A)$

III. NEUTROSOPHIC NANO j-CLOSED SETS

Definition 3.1 Let $(U, \tau_N(A))$ be a neutrosophic nano topological space. Then a neutrosophic nano subset A in $(U, \tau_N(A))$ is said to be neutrosophic nano j-closed (briefly N_Nj-closed) set if $N_N Cl(N_N PInt(A)) \subseteq A$.

Theorem 3.2 Every neutrosophic nano closed set is a neutrosophic nano j-closed set.

Proof. Let A be a neutrosophic nano closed set. i.e., $N_N Cl(A) = A$. We know that $N_N Int(A) \subseteq N_N PInt(A) \subseteq A$ which implies $N_N Cl(N_N PInt(A)) \subseteq N_N Cl(A) = A$. Hence every neutrosophic nano closed set is neutrosophic nano j-closed.

Remark 3.3 The converse part of the above theorem need not be true as seen from the following example.

Example 3.4 Let $(U, \tau_N(A))$ be a neutrosophic nano topological space with $U = \{ p1, p2, p3 \}$, the universe of discourse and $R_U = \{\{ p1, p2 \}, \{ p3 \}\}$, the equivalence relation on U.

Let $A = \{ \prec p1, (0.5, 0.4, 0.3) \succ, \prec p2, (0.5, 0.6, 0.4) \succ, \prec p3, (0.2, 0.5, 0.2) \succ \}$ be the neutrosophic nano subset of U.

Now, $N_N L_R(A) = \{ \prec p1, (0.5, 0.4, 0.4) \succ, \prec p2, (0.5, 0.4, 0.4) \succ, \prec p3, (0.2, 0.5, 0.2) \succ \}$,

$N_N U_R(A) = \{ \prec p1, (0.5, 0.6, 0.3) \succ, \prec p2, (0.5, 0.6, 0.3) \succ, \prec p3, (0.2, 0.5, 0.2) \succ \}$,

$N_N B_R(A) = \{ \prec p1, (0.4, 0.6, 0.5) \succ, \prec p2, (0.4, 0.6, 0.5) \succ, \prec p3, (0.2, 0.5, 0.2) \succ \}$ and the neutrosophic nano topology formed by the subset A is $\tau_N(A) = \{ 0_N, 1_N, N_N L_R(A), N_N U_R(A), N_N B_R(A) \}$.

Here the subsets are called neutrosophic nano open sets and the neutrosophic nano closed sets are

$0_N, 1_N, [N_N L_R(A)]^C, [N_N U_R(A)]^C \ and\ [N_N B_R(A)]^C$, where

$[N_N L_R(A)]^C = \{ \prec p1, (0.4, 0.6, 0.5) \succ, \prec p2, (0.4, 0.6, 0.5) \succ, \prec p3, (0.2, 0.5, 0.2) \succ \}$,

$[N_N U_R(A)]^C = \{ \prec p1, (0.3, 0.4, 0.5) \succ, \prec p2, (0.3, 0.4, 0.5) \succ, \prec p3, (0.2, 0.5, 0.2) \succ \}$, and

$[N_N B_R(A)]^C = \{ \prec p1, (0.5, 0.4, 0.4) \succ, \prec p2, (0.5, 0.4, 0.4) \succ, \prec p3, (0.2, 0.5, 0.2) \succ \}$.

Now, $N_N Int(A) = \{ \prec p1, (0.5, 0.6, 0.3) \succ, \prec p2, (0.5, 0.6, 0.3) \succ, \prec p3, (0.2, 0.5, 0.2) \succ \}$ and $N_N PInt(A) = \{ \prec p1, (0.5, 0.6, 0.3) \succ, \prec p2, (0.5, 0.6, 0.3) \succ, \prec p3, (0.2, 0.5, 0.2) \succ \}$.

Let us take a closed set in $\tau_N(A)$ and let it be B.
i.e., $B = \{\prec p1,(0.5,0.4,0.4)\succ,\prec p2,(0.5,0.4,0.4)\succ,\prec p3,(0.2,0.5,0.2)\succ\}$.

Clearly $N_N Cl(N_N PInt(B)) = B^C \subseteq B \Rightarrow B$ is NNj-closed.

Let us take an another NNj-closed set $C = \{\prec p1,(0.6,0.4,0.2)\succ,\prec p2,(0.6,0.5,0.3)\succ,\prec p3,(0.3,0.5,0.1)\succ\}$. But C is not a neutrosophic nano closed set. Hence a NNj-closed set need not be a NN closed set.

Theorem 3.5 The union (intersection) of two NNj-closed (open) sets need not be a NNj-closed (open) set as seen in the following example.

Example 3.6 Let $(U,\tau_N(A))$ be a neutrosophic nano topological space with $U = \{p1, p2, p3\}$, the universe of discourse and $R_U = \{\{p1, p2\},\{p3\}\}$, the equivalence relation on U.

Let $A = \{\prec p1,(0.5,0.4,0.3)\succ,\prec p2,(0.5,0.6,0.4)\succ,\prec p3,(0.2,0.5,0.2)\succ\}$ be the neutrosophic nano subset of U. The sets $\{\prec p1,(0.4,0.6,0.5)\succ,\prec p2,(0.4,0.6,0.5)\succ,\prec p3,(0.2,0.5,0.2)\succ\}$ and $\{\prec p1,(0.5,0.4,0.4)\succ,\prec p2,(0.5,0.4,0.4)\succ,\prec p3,(0.2,0.5,0.2)\succ\}$ are NNj-closed sets. But $\{\prec p1,(0.5,0.6,0.4)\succ,\prec p2,(0.5,0.6,0.4)\succ,\prec p3,(0.2,0.5,0.2)\succ\}$ which is the intersection of the above two sets is not a NNj-closed sets.

Theorem 3.7 Every neutrosophic nano j-closed set is a neutrosophic nano pre closed set.

Proof. Let A be a neutrosophic nano j-closed set. i.e., $N_N Cl(N_N PInt(A)) \subseteq A$. We know that $N_N Int(A) \subseteq N_N PInt(A)$ which implies $N_N Cl(N_N Int(A)) \subseteq N_N Cl(N_N PInt(A)) \subseteq A$. Therefore A is a neutrosophic nano pre closed set. Hence every NNj-closed set is NN pre closed.

Remark 3.8 The converse part of the above theorem need not be true as seen from the following example.

Example 3.9 Let $U = \{p1, p2, p3\}$ be the universe with the equivalence relation $R_U = \{\{p1, p3\},\{p2\}\}$ and let the neutrosophic nano subset on U be $A = \{\prec p1,(0.3,0.4,0.2)\succ,\prec p2,(0.4,0.5,0.1)\succ,\prec p3,(0.5,0.2,0.3)\succ\}$. Here $N_N L_R(A) = \{\prec p1,(0.3,0.2,0.3)\succ,\prec p2,(0.4,0.5,0.1)\succ,\prec p3,(0.3,0.2,0.3)\succ\}$, $N_N U_R(A) = \{\prec p1,(0.5,0.4,0.2)\succ,\prec p2,(0.4,0.5,0.1)\succ,\prec p3,(0.5,0.4,0.2)\succ\}$, and $N_N B_R(A) = \{\prec p1,(0.3,0.4,0.3)\succ,\prec p2,(0.1,0.5,0.4)\succ,\prec p3,(0.3,0.4,0.3)\succ\}$. Then the neutrosophic nano topology formed by A is $\tau_N(A) = \{0_N, 1_N, N_N L_R(A), N_N U_R(A), N_N B_R(A)\}$.

The subsets of $\tau_N(A)$ are called neutrosophic nano open sets and the neutrosophic nano closed sets are $0_N, 1_N, [N_N L_R(A)]^C, [N_N U_R(A)]^C$ *and* $[N_N B_R(A)]^C$ where

$[N_N L_R(A)]^C = \{\prec p1,(0.3,0.8,0.3)\succ,\prec p2,(0.1,0.5,0.4)\succ,\prec p3,(0.3,0.8,0.3)\succ\}$,

$[N_N U_R(A)]^C = \{\prec p1,(0.2,0.6,0.5)\succ,\prec p2,(0.1,0.5,0.4)\succ,\prec p3,(0.2,0.6,0.5)\succ\}$, and

$[N_N B_R(A)]^C = \{\prec p1,(0.3,0.6,0.3)\succ,\prec p2,(0.4,0.5,0.1)\succ,\prec p3,(0.3,0.6,0.3)\succ\}$. Then

$N_N Int(A) = \{\prec p1,(0.3,0.4,0.3)\succ,\prec p2,(0.4,0.5,0.1)\succ,\prec p3,(0.3,0.4,0.3)\succ\}$, $N_N PInt(A) = \{\prec p1,(0.3,0.4,0.2)\succ,\prec p2,(0.4,0.5,0.1)\succ,\prec p3,(0.4,0.4,0.3)\succ\}$ and $Cl(A) = 1_N$.

Clearly the set A itself is a neutrosophic nano pre closed set, but not a neutrosophic nano j-closed set, since $N_N Cl(N_N PInt(A)) = 1_N$, which is not contained in A.

Theorem: 3.10 Every neutrosophic nano regular closed set is a neutrosophic nano j-closed set.

Proof. We know that every N_N regular closed set is a N_N closed set and also every N_N closed set is a N_Nj-closed set. Hence every N_N regular closed set is a N_Nj-closed set.

Remark 3.11 The converse part of the above theorem need not be true as seen in the following example.

Example 3.12 Let $U=\{p1,p2,p3\}$ be the universe, $R_U=\{\{p1,p2\},\{p3\}\}$ be the equivalence relation on U, and $A=\{\prec p1,(0.1,0.4,0.2)\succ,\prec p2,(0.4,0.2,0.3)\succ,\prec p3,(0.5,0.3,0.3)\succ\}$ be the neutrosphic nano subset of U. Then $N_N L_R(A)=\{\prec p1,(0.1,0.2,0.3)\succ,\prec p2,(0.1,0.2,0.3)\succ,\prec p3,(0.5,0.3,0.3)\succ\}$, $N_N U_R(A)=\{\prec p1,(0.4,0.4,0.2)\succ,\prec p2,(0.4,0.4,0.2)\succ,\prec p3,(0.5,0.3,0.3)\succ\}$, $N_N B_R(A)=\{\prec p1,(0.3,0.4,0.2)\succ,\prec p2,(0.3,0.4,0.2)\succ,\prec p3,(0.3,0.3,0.5)\succ\}$, and the neutrosophic nano topology formed by A is $\tau_N(A)=\{0_N,1_N,N_N L_R(A),N_N U_R(A),N_N B_R(A)\}$.

Here the subsets are called neutrosophic nano open sets and the neutrosophic nano closed sets are

$0_N,1_N,[N_N L_R(A)]^C,[N_N U_R(A)]^C, and\ [N_N B_R(A)]^C$ where

$[N_N L_R(A)]^C=\{\prec p1,(0.3,0.8,0.1)\succ,\prec p2,(0.3,0.8,0.1)\succ,\prec p3,(0.3,0.7,0.5)\succ\}$,

$[N_N U_R(A)]^C=\{\prec p1,(0.2,0.6,0.4)\succ,\prec p2,(0.2,0.6,0.4)\succ,\prec p3,(0.3,0.7,0.5)\succ\}$, and

$[N_N B_R(A)]^C=\{\prec p1,(0.2,0.6,0.3)\succ,\prec p2,(0.2,0.6,0.3)\succ,\prec p3,(0.5,0.7,0.3)\succ\}$. Then

$N_N Int(A)=\{\prec p1,(0.1,0.2,0.3)\succ,\prec p2,(0.1,0.2,0.3)\succ,\prec p3,(0.5,0.3,0.3)\succ\}$ and $Cl(A)=1_N$.

Let $B=\{\prec p1,(0.2,0.2,0.2)\succ,\prec p2,(0.3,0.4,0.2)\succ,\prec p3,(0.5,0.4,0.2)\succ\}$ be an another neutrosophic nano subset on U. Clearly $N_N Cl(N_N PInt(B))=N_N Cl(N_N L_R(A))=[N_N B_R A)]^C\subseteq B$.

But $N_N Cl(N_N Int(B))\neq B$. Hence a N_Nj-closed set need not be a N_N regular closed.

Definition 3.13 Let $(U,\tau_N(A))$ be a neutrosophic nano topological space. Then a neutrosophic nano subset A in $(U,\tau_N(A))$ is said to be neutrosophic nano generalized closed (briefly N_Ng-closed) set if $N_N Cl(A)\subseteq V$ whenever $A\subseteq V$ and V is neutrosophic nano open in U.

Theorem 3.14 Every neutrosophic nano closed set is a neutrosophic nano generalized closed set.

Proof. Let A be the neutrosophic nano closed set. Let $A\subseteq V$ and V is neutrosophic nano open set in U. Since A is N_N closed, $N_N Cl(A)\subseteq A$. i.e., $N_N Cl(A)\subseteq A\subseteq V$. Hence A is N_Ng-closed set. Hence every N_N closed set is N_Ng-closed.

Remark 3.15 The converse of the above theorem need not be true as seen in the following example.

Example 3.16 Let $(U,\tau_N(A))$ be a neutrosophic nano topological space with $U=\{p1,p2,p3\}$, the universe of discourse and $R_U=\{\{p1,p2\},\{p3\}\}$, the equivalence relation on U.

Let $A=\{\prec p1,(0.5,0.4,0.3)\succ,\prec p2,(0.5,0.6,0.4)\succ,\prec p3,(0.2,0.5,0.2)\succ\}$ be the neutrosophic nano subset of U.

Now, $N_N L_R(A)=\{\prec p1,(0.5,0.4,0.4)\succ,\prec p2,(0.5,0.4,0.4)\succ,\prec p3,(0.2,0.5,0.2)\succ\}$, $N_N U_R(A)=\{\prec p1,(0.5,0.6,0.3)\succ,\prec p2,(0.5,0.6,0.3)\succ,\prec p3,(0.2,0.5,0.2)\succ\}$, $N_N B_R(A)=\{\prec p1,(0.4,0.6,0.5)\succ,\prec p2,(0.4,0.6,0.5)\succ,\prec p3,(0.2,0.5,0.2)\succ\}$ and the neutrosophic nano topology formed by the subset A is $\tau_N(A)=\{0_N,1_N,N_N L_R(A),N_N U_R(A),N_N B_R(A)\}$.

Let $V=N_N U_R(A)$ and $B=\{\prec p1,(0.4,0.5,0.6)\succ,\prec p2,(0.3,0.3,0.5)\succ,\prec p3,(0.1,0.4,0.3)\succ\}$.

Clearly B is a NNg-closed set, since $Cl(B)\subseteq V$ whenever $B\subseteq V$. But it is not a NN closed set.

Definition 3.17 Let $(U,\tau_N(F))$ be a neutrosophic nano topological space. Then a neutrosophic nano subset A in $(U,\tau_N(F))$ is said to be neutrosophic nano generalized j-closed (briefly NNgj-closed) set if $N_N JCl\subseteq V$ whenever $A\subseteq V$ and V is neutrosophic nano open in U.

Definition 3.18 Let $(U,\tau_N(F))$ be a neutrosophic nano topological space. Then a neutrosophic nano subset A in $(U,\tau_N(F))$ is said to be neutrosophic nano generalized j*-closed (briefly NNgj*-closed) set if $N_N JCl\subseteq V$ whenever $A\subseteq V$ and V is neutrosophic nano j-open in U.

Theorem 3.19 If A is a neutrosophic nano gj-closed set in $(U,\tau_R(X))$ and $A\subseteq B\subseteq N_N JCl(A)$, then B is neutrosophic nano generalized j-closed set in $(U,\tau_R(X))$.

Proof. Let $B\subseteq V$ where V is neutrosophic nano open in U. Then $A\subseteq B$ implies $A\subseteq V$. Since A is NNgj-closed, $N_N JCl(A)\subseteq V$. Also $A\subseteq N_N JCl(B)$ implies $N_N JCl(B)\subseteq N_N JCl(A)$. Thus $N_N JCl(B)\subseteq V$ and therefore B is NNgj-closed set in U.

Theorem 3.20 Every neutrosophic nano closed set is a neutrosophic nano generalized j-closed.

Proof. Let A be a neutrosophic nano closed set in U. Let $A\subseteq V$ and V is neutrosophic nano open in U. Since A is neutrosophic nano closed, $N_N Cl(A)=A\subseteq V$. Also $N_N JCl(A)\subseteq N_N Cl(A)\subseteq V$, where V is NN open in U. Therefore A is a neutrosophic nano generalized j-closed set. Hence every NN closed set is NNgj-closed.

Remark 3.21 The converse part of the above theorem need not be true as seen in the following example.

Example 3.22 Let $U=\{p1,p2,p3\}$ be the universe, $R_U=\{\{p1,p2\},\{p3\}\}$ be the equivalence relation on U, and $A=\{\prec p1,(0.1,0.4,0.2)\succ,\prec p2,(0.4,0.2,0.3)\succ,\prec p3,(0.5,0.3,0.3)\succ\}$ be the neutrosphic nano subset of U. Then $N_N L_R(A)=\{\prec p1,(0.1,0.2,0.3)\succ,\prec p2,(0.1,0.2,0.3)\succ,\prec p3,(0.5,0.3,0.3)\succ\}$, $N_N U_R(A)=\{\prec p1,(0.4,0.4,0.2)\succ,\prec p2,(0.4,0.4,0.2)\succ,\prec p3,(0.5,0.3,0.3)\succ\}$, $N_N B_R(A)=\{\prec p1,(0.3,0.4,0.2)\succ,\prec p2,(0.3,0.4,0.2)\succ,\prec p3,(0.3,0.3,0.5)\succ\}$, and the neutrosophic nano topology formed by A is $\tau_N(A)=\{0_N,1_N,N_N L_R(A),N_N U_R(A),N_N B_R(A)\}$. Let the open set $V=\{\prec p1,(0.4,0.4,0.2)\succ,\prec p2,(0.4,0.4,0.2)\succ,\prec p3,(0.5,0.3,0.3)\succ\}$.

Let $B=\{\prec p1,(0.2,0.3,0.3)\succ,\prec p2,(0.2,0.3,0.4)\succ,\prec p3,(0.1,0.2,0.3)\succ\}$. Clearly $B\subseteq V$.

Also $N_N JCl(B)\subseteq V$. Hence B is a NNgj-closed set, but not a NN closed set.

Theorem 3.23 Every neutrosophic nano j-closed set is a neutrosophic nano generalized j-closed set.

Proof. Let A be a NNj-closed set. Let $A\subseteq V$ and V is neutrosophic nano open in U. Since A is NNj-closed, $N_N JCl(A)\subseteq A\subseteq V$. Therefore A is NNgj-closed. Hence every NNj-closed set is NNgj-closed.

Remark 3.24 The converse of the above theorem need not be true as seen in the following example.

Example 3.25 In example 3.22, B is a NNgj-closed set. But $N_N Cl(N_N PInt(B))$ is not contained in V. i.e., B is not a NNj-closed set. Hence every NNgj-closed set need not a NNj-closed set.

Theorem 3.26 Every NNg-closed set is a NNgj-closed set.

Proof. Let A be a NNg-closed set. Then $N_N Cl(A)\subseteq V$ whenever $A\subseteq V$ and V is neutrosophic nano open in U. Since $N_N JCl(A)\subseteq N_N Cl(A)\subseteq V$, we have $N_N JCl(A)\subseteq V$ whenever $A\subseteq V$ and V is NN open in U. Therefore A is NNgj-closed. Hence every NNg-closed set is a NNgj-closed set.

Theorem 3.27 Every NNj-closed set is a NNgj*-closed set.

Proof. Let A be a NNj-closed set. Let $A \subseteq V$ and V is neutrosophic nano j-open in U. Since A is NNj-closed, $N_N JCl(A) = A \subseteq V$, V is NNj- open in U. Therefore A is NNgj*-closed. Hence every NNj-closed set is a NNgj*-closed set.

Theorem 3.28 Every NNgj*-closed set is a NNgj-closed set.

Proof. Let A be a NNgj*-closed set. Let $A \subseteq V$ and V is neutrosophic nano open in U. Since every NN open set is NNj-open, V is NNj-open in U. Since A is NNgj*-closed set, we have $N_N JCl(A) \subseteq V$. Therefore $N_N JCl(A) \subseteq V$ whenever $A \subseteq V$ and V is NNj-open in U. Therefore A is NNgj-closed. Hence every NNgj*-closed set is a NNgj-closed set.

IV. Conclusion

Neutrosophic nano j-closed set, neutrosophic nano generalized closed set, neutrosophic nano generalized j-closed set, neutrosophic nano generalized j*-closed set were introduced and some of their properties were discussed in this paper. The concept can be used for real life decision making problems where the situations of indeterminacy occurs. The practical problems may be solved by finding CORE values through the criterion reduction.

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