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$\mathbf{N}_{\delta^*\mathbf{g}\alpha}$-Continuous and Irresolute Functions in Neutrosophic Topological Spaces

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Abstract. In this paper, the notions of $N_{\delta^*g\alpha}$-continuous and $N_{\delta^*g\alpha}$-irresolute functions in neutrosophic topological spaces are given. Furthermore, we analyze their characterizations and investigate their properties.

Keywords: $N_{\delta^*g\alpha}$-closed set; $N_{\delta^*g\alpha}$-continuous; $N_{\delta^*g\alpha}$-irresolute; $N_{\delta^*g\alpha}$ – homeomorphism; $N_{\delta^*g\alpha}$c-homeomorphism.

1. Introduction

The notion of fuzzy set (FS) and its logic are investigated and discussed by Zadeh [12]. Next, Chang [3] studied the conception of fuzzy topological space (FTS). After that, Atanassav [8] investigated the intuitionistic fuzzy set (IFS) in 1986. Neutrosophy has extend the grounds for a total family of new mathematical estimations. It is one of the non-classical sets, like fuzzy, nano, soft, permutation sets and so on, see ([17]-[39]). The neutrosophic set (NS) was presented by Smarandache[6]and expounded, (NS) is a popularization of (IFS) in intuitionistic fuzzy topological space $(IFTS)$ by coker [4]. In 2012 [1], the conception of neutrosophic topological space (NTS) is presented. Further the fundamental sets like semi/pre/α-open sets are presented in neutrosophic topological spaces $(NTSs)$, see ([13]-[16]). The neutrosophic closed sets $(NCSs)$ and neutrosophic continuous functions $(NCFs)$ were presented by Salama et al.[2] in 2014. Arokiarani et al.[7] presented the neutrosophic α-closed set (NαCS) in $(NTSs)$. The concepts of δ-closure are auxiliary tools in standard topology in the study of H-closed spaces. Damodharan et al.[9,10] presente the idea of N_δ -closure and N_δ -Interior in $(NTSs)$. Further, N_δ -continuous and Neutrosophic almost continuous in $(NTSs)$ were presented and established some of their related attributes. Recently Damodharan and Vigneshwaran [11] presented the conception of $N_{\delta^*g\alpha}$-closed sets in $(NTSs)$and studied some of its characteristics. In 2020, some applications of (NS) are applied by Abdel-Basset and others, see ([40]) In this work, we presented the $N_{\delta^*g\alpha}$-continuous functions and $N_{\delta^*g\alpha}$-irresolute functions in $(NTSs)$. Furthermore, the conceptions of $N_{\delta^*g\alpha}$ – homeomorphism and $N_{\delta^*g\alpha}$c-homeomorphism are presented and investigate their characteristics.

2. **Preliminaries**

In this section, we mention some pertinent basic preliminaries about neutrosophic sets (NSs)and its operations.

2.1. *Definition [1]*

Assume S is a non-empty fixed set. A neutrosophic set (NS) P is an object having the form:

$P = \{\langle s, \mu_m (P(s)), \sigma_i (P(s)), \nu_{nm} (P(s))\rangle \, \forall s \in S\}$, where $\mu_m (P(s))$ represents the degree of membership, $\sigma_i (P(s))$ represents the degree of indeterminacy and $\nu_{nm} (P(s))$ represents the degree of nonmembership $\forall s \in S$ to P.

2.2. *Remark [1]*

A (NS) $P = \{\langle s, \mu_m (P(s)), \sigma_i (P(s)), \nu_{nm} (P(s))\rangle \, \forall s \in S\}$ can be identified to an ordered triple $\langle \mu_m (P(s)), \sigma_i (P(s)), \nu_{nm} (P(s))\rangle$ in $]^{-}0, 1^{+}[$ on S.

2.3. *Definition [1]*

In (NTS) We have:

$$\begin{array}{ll}
0_N \text{ may is defined as } \forall s \in S & 1_N \text{ may be defined as } \forall s \in S \\
0_N = \langle s, 0, 0, 1\rangle & 1_N = \langle s, 1, 0, 0\rangle \\
0_N = \langle s, 0, 1, 1\rangle & 1_N = \langle s, 1, 0, 1\rangle \\
0_N = \langle s, 0, 1, 0\rangle & 1_N = \langle s, 1, 1, 0\rangle \\
0_N = \langle s, 0, 0, 0\rangle & 1_N = \langle s, 1, 1, 1\rangle
\end{array}$$

2.4. *Definition [1]*

Assume P is (NS) of the form:
$P = \{\langle s, \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s))\rangle \forall s \in S\}$, Then the complement of P $[P^c]$ may be defined as
$P^c = \{\langle s, \nu_{nm}(P(s)), \sigma_i(P(s)), \mu_m(P(s))\rangle \forall s \in S\}$

2.5. *Definition [1]*

Assume P and Q are two (NSs)of the form,
$P = \{\langle s, \mu_m(P(s)), \sigma_i(P(s)), \nu_{nm}(P(s))\rangle \forall s \in S\}$ and
$Q = \{\langle s, \mu_m(Q(s)), \sigma_i(Q(s)), \nu_{nm}(Q(s))\rangle \forall s \in S\}$. Then,

(1) Subsets $P \subseteq Q$ may be defined as follows
$$P \subseteq Q \Leftrightarrow \mu_m(P(s)) \leq \mu_m(Q(s)), \sigma_i(P(s)) \geq \sigma_i(Q(s)), \nu_{nm}(P(s)) \geq \nu_{nm}(Q(s))$$
(2) Subsets $P = Q \Leftrightarrow P \subseteq Q$ and $Q \subseteq P$
(3) Union of subsets $P \cup Q$ may be defined as follows
$$P \cup Q = \{s, max\{\mu_m(P(s), \mu_m(Q(s)))\}, min\{\sigma_i(P(s)), \sigma_i(Q(s))\}, \\ min\{\nu_{nm}(P(s)), \nu_{nm}(Q(s))\} \forall s \in S\},$$
(4) Intersection of subsets $P \cap Q$ may be defined as follows
$$P \cap Q = \{s, min\{\mu_m(P(s), \mu_m(Q(s)))\}, max\{\sigma_i(P(s)), \sigma_i(Q(s))\}, \\ max\{\nu_{nm}(P(s)), \nu_{nm}(Q(s))\} \forall s \in S\},$$

2.6. *Proposition [9]*

For any two (NSs) P and Q the following condition holds

i): $(P \cap Q)^c = P^c \cup Q^c$,
ii): $(P \cup Q)^c = P^c \cap Q^c$,

2.7. *Definition [1]*

A neutrosophic topology (NT) on a non-empty set S is a family τ of neutrosophic subsets in S satisfying the following axioms:

i): $0_N, 1_N \in \tau$,
ii): $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
iii): $\cup G_i \in \tau \forall \{G_i : i \in J\} \subseteq \tau$

Then the pair (S, τ) is named a neutroscopic topological space(NTS).

2.8. *Definition [1]*

Assume P is a (NS) in a (NTS) (S, τ). Then

i): $Nint\,(P) = \bigcup\{Q/Q$is a neutrosophic open set (NOS) in (s,τ) and$Q \subseteq P\}$ is named the neutrosophic interior of P;

ii): $Ncl\,(P) = \bigcap\{Q/Q$is a neutrosophic closed set (NCS)in (s,τ) and$Q \supseteq P\}$ is named the neutrosophic closure of P;

2.9. *Definition [7]*

A subset A of (S,τ) is named

i): neutrosophic semi-open set (NSOS)if $P \subseteq Ncl(Nint(P))$.

ii): neutrosophic pre-open set (NPOS)if $P \subseteq Nint(Ncl(P))$.

iii): neutrosophic semi-preopenset (NSPOS)if $P \subseteq Ncl(Nint(Ncl(P)))$.

iv): neutrosophic α-open set (NαOS)if $P \subseteq Nint(Ncl(Nint(P)))$.

v): neutrosophic regular open set (NROS)if $P = Nint(Ncl(P))$.

The complement of a (NSOS) (resp. (NPOS), (NSPOS), (NαOS), (NROS)) set is named (NSCS) (resp. (NPCS), (NSPCS), (NαCS, (NRCS)).

2.10. *Definition [9]*

Assume $\alpha,\beta,\lambda \in [0,1]$ and $\alpha+\beta+\lambda \leq 3$. A neutrosophic point $s_{(\alpha,\beta,\lambda)}$of S is a neutrosophic point (NP) of S which is clarified by

$$s_{(\alpha,\beta,\lambda)}(y) = \begin{cases} (\alpha,\beta,\lambda) & \text{when } y = s, \\ (0,0,1) & \text{when } y \neq s. \end{cases}$$

Here, S is named the support of $s_{(\alpha,\beta,\lambda)}$ and α,β and λ, respectively. A (NP) $s_{(\alpha,\beta,\lambda)}$ is named belong to a (NS)
$P = \langle \mu_m\,(P\,(s))\,, \sigma_i\,(P\,(s))\,, \nu_{nm}\,(P\,(s))\rangle$ in S, denoted by $s_{(\alpha,\beta,\lambda)} \in P$ if $\alpha \leq \mu_m\,(P\,(s))\,, \beta \geq \sigma_i\,(P\,(s))$ and $\lambda \geq \nu_{nm}\,(P\,(s))$ Clearly a (NP) can be represented by an ordered triple of (NP) as follows : $s_{(\alpha,\beta,\lambda)} = (s_\alpha, s_\beta, s_\lambda)$.

2.11. *Definition [9]*

Assume (S,τ) is a (NTS). Assume P is a (NS) and Assume $s_{(\alpha,\beta,\lambda)}$ is a (NP). $s_{(\alpha,\beta,\lambda)}$ is named neutrosophic quasi coincident with P [denoted by $s_{(\alpha,\beta,\lambda)}qP$] if $\alpha + \mu_m(P(s)) > 1; \beta + \sigma_i(P(s)) < 1$ and $\lambda + \nu_{nm}(P(s)) < 1$.

2.12. *Definition [9]*

Assume P and Q are two (NSs). P is named neutrosophic quasi coincident with Q [denoted by PqQ] if $\mu_m(P(s)) + \mu_m(Q(s)) > 1; \sigma_i(P(s)) + \sigma_i(Q(s)) < 1$ and $\nu_{nm}(P(s)) + \nu_{nm}(Q(s)) < 1$.

2.13. *Definition [9]*

Assume (S,τ) is an (NTS). An (NP) $s_{(\alpha,\beta,\lambda)}$ is named an neutroscopic δ-cluster point of an (NS) P if AqP for each neutrosophic regular open q-neighborhood A of $s_{(\alpha,\beta,\lambda)}$. The set of all neutroscopic δ-cluster points of P is named the neutrosophic δ-closure of Pand denoted by $Ncl_\delta\,(P)$. An (NS) P is named an N_δ-closed set (N_δ-CS) if $P = Ncl_\delta\,(P)$. The complement of an (N_δ-CS) is named an N_δ-open set (N_δ-OS).

3. $\mathbf{N_{\delta^* g\alpha}}$-continuous functions

Here, some new conceptions are given by the authors.

3.1. *Definition*

A map $T : (S,\tau) \longrightarrow (Y,\sigma)$ is named a Neutrosophic delta star generalized alpha-continuous map(briefly $N_{\delta^* g\alpha}$-CM) if $T^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS in (S,τ) for any (NCS) in (Y,σ).

3.2. *Theorem*

Any $N_{\delta^* g\alpha}$-CM is N_{gs}-CM.(resp $N_{\alpha g}$-CM, N_{gsp}-CM, N_{gp}-CM). Also converse part is not true as shown through the following examples.

Proof. Assume K is a (NCS) in (Y,σ). Since T is $N_{\delta^* g\alpha}$-CM. $T^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS in (S,τ). Since any $N_{\delta^* g\alpha}$CS is N_{gs}-CS (resp $N_{\alpha g}$-CS, N_{gsp}-CS, N_{gp}-CS), therefore $T^{-1}(K)$ is N_{gs}-CS (resp $N_{\alpha g}$-CS, N_{gsp}-CS, N_{gp}-CS) in (S,τ). Hence T is N_{gs}-CM.(resp $N_{\alpha g}$-CM, N_{gsp}-CM, N_{gp}-CM).

3.3. *example*

Assume $S = \{p,q,r\}$. Define the (NSs)D_1, D_2, D_3, D_4 and G_1, G_2, G_3, G_4 as follows:
$D_1 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right)\right\rangle$
$D_2 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right)\right\rangle$
$D_3 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right)\right\rangle$
$D_4 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right)\right\rangle$
and $G_1 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right)\right\rangle$
$G_2 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right)\right\rangle$
$G_3 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right)\right\rangle$
$G_4 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right)\right\rangle$
Then the families $\tau = \{0_N, 1_N, D_1, D_2, D_3, D_4\}$ and $\xi = \{0_N, 1_N, G_1, G_2, G_3, G_4\}$ are neutrosophic topologies (NTs) on S. Thus, (S,τ) and (S,ξ) are (NTSs). Define $T : (S,\tau) \rightarrow (S,\xi)$ as $T\,(p) = p, T\,(q) = q, T\,(r) = r$. Then T is N_{gs}-CM but not $N_{\delta^* g\alpha}$-CM. Hence in (S,τ), $N_{\delta^* g\alpha}$-CS is $\left\langle \left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.5}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}\right)\right\rangle$ and

N_{gs}-CS is $\left\langle\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right)\right\rangle$. Here $T^{-1}\left(G_3^c\right)$ is N_{gs}-CS but not $N_{\delta^* g\alpha}$-CS.

3.4. *example*

Assume $S=\{p,q,r\}$. Define the (NSs)D_1,D_2,D_3,D_4 and H_1,H_2,H_3,H_4 as follows:
$D_1=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$
$D_2=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right)\right\rangle$
$D_3=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right)\right\rangle$
$D_4=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$
and $H_1=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.3},\frac{r}{0.3}\right),\left(\frac{p}{0.3},\frac{q}{0.4},\frac{r}{0.4}\right)\right\rangle$
$H_2=\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.6}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.5},\frac{r}{0.5}\right)\right\rangle$
$H_3=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.3},\frac{r}{0.3}\right),\left(\frac{p}{0.3},\frac{q}{0.5},\frac{r}{0.5}\right)\right\rangle$
$H_4=\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.6}\right),\left(\frac{p}{0.3},\frac{q}{0.2},\frac{r}{0.2}\right),\left(\frac{p}{0.3},\frac{q}{0.4},\frac{r}{0.4}\right)\right\rangle$
Then the families $\tau=\{0_N,1_N,D_1,D_2,D_3,D_4\}$ and $\xi=\{0_N,1_N,H_1,H_2,H_3,H_4\}$ are (NTs) on S. Thus, (S,τ) and (S,ψ) are (NTSs). Define $T:(S,\tau)\to(S,\psi)$ as $T(p)=p, T(q)=q, T(r)=r$. Then T is $N_{\alpha g}$-CM but not $N_{\delta^* g\alpha}$-CM. Hence in (S,τ),
$N_{\delta^* g\alpha}$-CS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.5},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$ and
$N_{\alpha g}$-CS is $\left\langle\left(\frac{p}{0.3},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.4},\frac{q}{0.3},\frac{r}{0.3}\right),\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right)\right\rangle$. Here $T^{-1}\left(H_3^c\right)$ is $N_{\alpha g}$-CS but not $N_{\delta^* g\alpha}$-CS.

3.5. *example*

Assume $Y=\{u,v,w\}$. Define the (NSs)F_1,F_2,F_3,F_4 and I_1,I_2,I_3,I_4 as follows:
$F_1=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$
$F_2=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$
$F_3=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$
$F_4=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$
and $I_1=\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.6}\right),\left(\frac{p}{0.5},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.3}\right)\right\rangle$
$I_2=\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.3}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.6}\right)\right\rangle$
$I_3=\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.6}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.3}\right)\right\rangle$
$I_4=\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.3}\right),\left(\frac{p}{0.5},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.6}\right)\right\rangle$
Then the families $\vartheta=\{0_N,1_N,F_1,F_2,F_3,F_4\}$ and $\zeta=\{0_N,1_N,I_1,I_2,I_3,I_4\}$ are (NTs) on Y. Thus, (Y,ϑ) and (Y,ζ) are (NTSs). Define $g:(Y,\vartheta)\to(Y,\zeta)$ as $g(u)=u, g(v)=v, g(w)=w$. Then g is N_{gp}-CM but not $N_{\delta^* g\alpha}$-CM. Hence in (Y,ϑ),
$N_{\delta^* g\alpha}$-CS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.5},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$ and
N_{gp}-CS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.3}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.6}\right)\right\rangle$. Here $g^{-1}\left(I_3^c\right)$ is N_{gp}-CS but not $N_{\delta^* g\alpha}$-CS.

3.6. *example*

Assume $Y=\{u,v,w\}$. Define the (NSs)F_1,F_2,F_3,F_4 and J_1,J_2,J_3,J_4 as follows:
$F_1=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$
$F_2=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$
$F_3=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$
$F_4=\left\langle\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.5},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.6}\right)\right\rangle$
and $J_1=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right),\left(\frac{p}{0.3},\frac{q}{0.4},\frac{r}{0.3}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.5}\right)\right\rangle$
$J_2=\left\langle\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right)\right\rangle$
$J_3=\left\langle\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.4},\frac{r}{0.3}\right),\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right)\right\rangle$
$J_4=\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right),\left(\frac{p}{0.3},\frac{q}{0.3},\frac{r}{0.2}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.5}\right)\right\rangle$
Then the families $\vartheta=\{0_N,1_N,F_1,F_2,F_3,F_4\}$ and $\zeta=\{0_N,1_N,J_1,J_2,J_3,J_4\}$ are (NTs) on Y. Thus, (Y,ϑ) and (Y,φ) are (NTSs). Define $g:(Y,\vartheta)\to(Y,\varphi)$ as $g(u)=u, g(v)=w, g(w)=v$. Then g is N_{gsp}-C but not $N_{\delta^*g\alpha}$-C. Hence in (Y,ϑ),
$N_{\delta^*g\alpha}$-CS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.5},\frac{q}{0.5},\frac{r}{0.5}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$ and
N_{gsp}-CS is $\left\langle\left(\frac{p}{0.4},\frac{q}{0.6},\frac{r}{0.5}\right),\left(\frac{p}{0.4},\frac{q}{0.4},\frac{r}{0.4}\right),\left(\frac{p}{0.3},\frac{q}{0.4},\frac{r}{0.3}\right)\right\rangle$. Here $g^{-1}\left(J_3^c\right)$ is N_{gsp}-CS but not $N_{\delta^*g\alpha}$-CS.

3.7. *Theorem*

The composition of two $N_{\delta^*g\alpha}$-CMs is also a $N_{\delta^*g\alpha}$-CM. Proof. Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are two $N_{\delta^*g\alpha}$-CMs. Assume l is a NCS in (Z,η). Since g is a $N_{\delta^*g\alpha}$-CM, $g^{-1}(l)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since any $N_{\delta^*g\alpha}$-CS is NCS, $g^{-1}(l)$ is $NCSSin(Y,\sigma)$. Since T is a $N_{\delta^*g\alpha}$-CM, $T^{-1}\left(g^{-1}(l)\right)=goT(l)$ is $N_{\delta^*g\alpha}$-CS in (S,τ), therefore goT is also $N_{\delta^*g\alpha}$-CM.

4. $\mathbf{N_{\delta^*g\alpha}}$-Irresolute functions

Here, some new conceptions are given by the authors.

4.1. *Definition*

A map $T:(S,\tau)\longrightarrow(Y,\sigma)$ is named a Neutrosophic delta star generalized alpha-Irresolute map (briefly $N_{\delta^*g\alpha}$-IMM) if $T^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS in (S,τ) for any $N_{\delta^*g\alpha}$-CS in (Y,σ).

4.2. *Theorem*

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are any two functions, then

(i) $goT:(S,\tau)\longrightarrow(Z,\eta)$ is $N_{\delta^*g\alpha}$-CM if g is N-CM and T is $N_{\delta^*g\alpha}$-CM.
(ii) $goT:(S,\tau)\longrightarrow(Z,\eta)$ is $N_{\delta^*g\alpha}$-IM if both g and T $N_{\delta^{**}g\alpha}$-IM.
(iii) $goT:(S,\tau)\longrightarrow(Z,\eta)$ is $N_{\delta^*g\alpha}$-CM if g is $N_{\delta^*g\alpha}$-CM and T is $N_{\delta^{**}g\alpha}$-IM.

Proof.

(i) Assume K is a(NCS) in (Z,η). Since g is N-CM, $g^{-1}(K)$ is NCS in (Y,σ). Since T is $N_{\delta^* g\alpha}$-CM, $T^{-1}\left(g^{-1}(K)\right)=(goT)^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS in (S,τ), Therefore goT is $N_{\delta^* g\alpha}$-CM.

(ii) Assume K is a $N_{\delta^* g\alpha}$-CS in (Z,η). Since g is $N_{\delta^{**} g\alpha}$-IM, $g^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS in (Y,σ). Since T is $N_{\delta^{**} g\alpha}$-IM, $T^{-1}\left(g^{-1}(K)\right)=(goT)^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS in (S,τ), Therefore goT is $N_{\delta^{**} g\alpha}$-IM.

(iii) Assume K is a(NCS) in (Z,η). Since g is $N_{\delta^* g\alpha}$-CM, $g^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS in (Y,σ). Since T is $N_{\delta^{**} g\alpha}$-IM, $T^{-1}\left(g^{-1}(K)\right)=(goT)^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS in (S,τ), Therefore goT is $N_{\delta^* g\alpha}$-CM.

4.3. *Theorem*

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is $N_{\delta^* g\alpha}$-CM (N_{gs}-CM, $N_{\alpha g}$-CM, N_g-CM). If (S,τ) is an $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space ($N_{{}^{**}_{\delta\alpha} T_{\frac{1}{2}}}$-space, $N_{\delta\alpha T^{**}_c}$-space, $N_{\delta T^{**}_c}$-space) then T is contiuous.

Proof. Assume K is a(NCS) of (Y,σ). Since T is $N_{\delta^* g\alpha}$-CM (N_{gs}-CM, $N_{\alpha g}$-CM, N_g-CM), then $T^{-1}(K)$ is $N_{\delta^* g\alpha}$-CS (N_{gs}-CS, $N_{\alpha g}$-CS, N_g-CS) in (S,τ). Since (S,τ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space ($N_{{}^{**}_{\delta\alpha} T_{\frac{1}{2}}}$-space, $N_{\delta\alpha T^{**}_c}$-space, $N_{\delta T^{**}_c}$-space), then $T^{-1}(K)$ is N_δ-CS in (S,τ). Any N_δ-CS is (NCS) in (S,τ). Therefore T is continuous.

4.4. *Theorem*

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is a surjective, $N_{* g\alpha}$-IM and N_δ-CM. Then $T(A)$ is $N_{\delta^* g\alpha}$-CS of (Y,σ) for any $N_{\delta^* g\alpha}$-CS A of (S,τ).

Proof. Assume A is a $N_{\delta^* g\alpha}$-CS of (S,τ). Assume U is a $N_{* g\alpha}$-OS of (Y,σ). such that $T(A)\subseteq U$. Since T is surjective and $N_{** g\alpha}$-IM, $T^{-1}(U)$ is $N_{* g\alpha}$-OS in (S,τ). Since $A\subseteq T^{-1}(U)$ and A is $N_{\delta^* g\alpha}$-CS of (S,τ), $Ncl_\delta(A)\subseteq T^{-1}(U)$. Then $T[Ncl_\delta(A)]\subseteq T\left[T^{-1}(U)\right]=U$, since T is N_δ-CS, $T[Ncl_\delta(A)]=Ncl_\delta[T[Ncl_\delta(A)]]$. This implies $Ncl_\delta[T(A)]\subseteq Ncl_\delta[T[Ncl_\delta(A)]]=T[Ncl_\delta(A)]\subseteq U$, Therefore $T(A)$ is a $N_{\delta^* g\alpha}$-CS of (Y,σ)

4.5. *Theorem*

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is a surjective, $N_{\delta^* g\alpha}$-IM and N_δ-CM. If (S,τ) is an $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space, then (Y,σ) is also an $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space.

Proof. Assume A is a $N_{\delta^* g\alpha}$-CS of (Y,σ). Since T is $N_{\delta^{**} g\alpha}$-IM, $T^{-1}(A)$ is $N_{\delta^* g\alpha}$-CS in (S,τ). Since (S,τ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space, $T^{-1}(A)$ is N_δ-CS of (S,τ). Since T is N_δ-CM and surjective, $T\left[T^{-1}(A)\right]=A$ is N_δ-CS in (Y,σ). Thus A is N_δ-CS in (Y,σ), Therefore (Y,σ) is an $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space.

5. $N_{\delta^* g\alpha}$-Homeomorphism

Here, some new conceptions are given by the authors.

5.1. *Definition*

A map $T : (S,\tau) \longrightarrow (Y,\sigma)$ is named a neutrosophic delta star generalized alpha-homeomorphism (briefly $N_{\delta^* g\alpha}$-H) if T is bijective, $N_{\delta^* g\alpha}$-CM and $N_{\delta^* g\alpha}$-OM.

5.2. *Theorem*

Any $N_{\delta^* g\alpha}$-H is N_{gs}-H.

Proof. Let f : (X;) ! (Y;) be N g -H then f is bijective, N g -continuous and N g -OM. Let V be N-CS in (Y;), then f 1 (V) is N g -CS in (X;). Since every N g -CS is N_{gs}-CS, then f 1 (V) is N_{gs}-CS in (X;), Therefore f is N_{gs}-continuous. Let U be N-OS in (X;), then f (U) is N g -OS in (Y;). Since every N g -OS is N_{gs}-OS, then f (U) is N_{gs}-OS in (Y;), Therefore f is N_{gs}-OM. Hence f is N_{gs}-H.

5.3. *example*

Assume $S = \{p,q,r\}$. Define the (NSs)D_1, D_2, D_3, D_4 and G_1, G_2, G_3, G_4 as follows:

$D_1 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$

$D_2 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right) \right\rangle$

$D_3 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right) \right\rangle$

$D_4 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$

and $G_1 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right) \right\rangle$

$G_2 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$

$G_3 = \left\langle \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right) \right\rangle$

$G_4 = \left\langle \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}, \frac{r}{0.6}\right) \right\rangle$

Then the families $\tau = \{0_N, 1_N, D_1, D_2, D_3, D_4\}$ and $\xi = \{0_N, 1_N, G_1, G_2, G_3, G_4\}$ are (NTs) on S. Thus, (S,τ) and (S,ξ) are (NTSs). Define $T : (S,\tau) \to (S,\xi)$ as $T(p) = p, T(q) = q, T(r) = r$. Then T is N_{gs}-H but not $N_{\delta^* g\alpha}$-H. Hence in (S,τ),

$N_{\delta^* g\alpha}$-CS is $\left\langle \left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.5}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}\right) \right\rangle$ and

N_{gs}-CS is $\left\langle \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right) \right\rangle$. Here $T^{-1}(G_3^c)$ is N_{gs}-CS but not $N_{\delta^* g\alpha}$-CS.

$N_{\delta^* g\alpha}$-OS is $\left\langle \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.5}, \frac{r}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.5}\right) \right\rangle$ and

N_{gs}-OS is $\left\langle \left(\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.4}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right), \left(\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.2}\right) \right\rangle$ is N_{gs}-OS but not $N_{\delta^* g\alpha}$-OS.

5.4. *Theorem*

For any bijective map $T : (S,\tau) \longrightarrow (Y,\sigma)$ the following statement are equivalent.

(i) $T^{-1} : (Y,\tau) \longrightarrow (S,\sigma)$ is $N_{\delta^* g\alpha}$-CM.

(ii) T is an $N_{\delta^* g\alpha}$-OM.

(iii) T is an $N_{\delta^* g\alpha}$-CM.

Proof.

$(i) \Rightarrow (ii)$ Assume U is an (NOS)in (S,τ), then S-U is (NCS) in (S,τ) Since T^{-1} is $N_{\delta^*g\alpha}$-CM, then $\left(T^{-1}\right)^{-1}(U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). that is $T(S-U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ), that is Y-$T(U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). This implies that $T(U)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). Thus T is $N_{\delta^*g\alpha}$-OM.

$(ii) \Rightarrow (iii)$ Assume F is an (NCS) in (S,τ), then S-F is N-OS in (S,τ). Since T is $N_{\delta^*g\alpha}$-OM, then $T(S-F)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). That is Y-$T(F)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). This implies that $T(F)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). hence T is $N_{\delta^*g\alpha}$-CM.

$(iii) \Rightarrow (i)$ Assume K is an (NCS) in (S,τ), Since T is $N_{\delta^*g\alpha}$-CM, then $T(K)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). That is $[T^{-1}]^{-1}(K)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). hence T^{-1} is $N_{\delta^*g\alpha}$-CM.

5.5. *Theorem*

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is bijective and $N_{\delta^*g\alpha}$-CM. then the following statement are equivalent.

(i) T is an $N_{\delta^*g\alpha}$-OM.
(ii) T is an $N_{\delta^*g\alpha}$-H.
(iii) T is an $N_{\delta^*g\alpha}$-CM.

Proof.

$(i) \Rightarrow (ii)$ Assume T is an $N_{\delta^*g\alpha}$-OM. Since T is bijective and $N_{\delta^*g\alpha}$-CM, T is $N_{\delta^*g\alpha}$-H.

$(ii) \Rightarrow (iii)$ Assume T is an $N_{\delta^*g\alpha}$-H. Then T is $N_{\delta^*g\alpha}$-OM. If F is (NCS) in S, then $T(S-F)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). That is Y-$T(F)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ). This implies that $T(F)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). hence T is $N_{\delta^*g\alpha}$-CM.

$(iii) \Rightarrow (i)$ Assume U is an (NOS) in (S,τ), Then S-U is (NCS) in (S,τ). Since T is $N_{\delta^*g\alpha}$-CS, then $T(S-U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). That is $Y-T(U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Hence $T(U)$ is $N_{\delta^*g\alpha}$-OS in (Y,σ).

5.6. *Theorem*

The composition of two $N_{\delta^*g\alpha}$-Hs is also a $N_{\delta^*g\alpha}$-H.

Proof. Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are two $N_{\delta^*g\alpha}$-CM. Assume U is a (NCS) in (Z,η). Since g is a $N_{\delta^*g\alpha}$-CM, $g^{-1}(U)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since any $N_{\delta^*g\alpha}$-CS is (NCS), $g^{-1}(U)$ is (NCS) in (Y,σ). Since T is a $N_{\delta^*g\alpha}$-CM, $T^{-1}\left(g^{-1}(U)\right) = goT(U)$ is $N_{\delta^*g\alpha}$-CS in (S,τ), therefore goT is also $N_{\delta^*g\alpha}$-CM.

Assume A is a (NCS) in (S,τ) then S-A is a (NOS)in (S,τ). Since T is $N_{\delta^*g\alpha}$-H, then $T(S-A)$ is a $N_{\delta^*g\alpha}$-OS in (Y,σ), implies $T(A)$ is $N_{\delta^*g\alpha}$-CS in (Y,σ). Since any $N_{\delta^*g\alpha}$-CS is (NCS), then $T(A)$ is (NCS) in (Y,σ), then $Y-T(A)$ is N-OS in (Y,σ). Since g is $N_{\delta^*g\alpha}$-H.

$g\left(Y-T\left(A\right)\right)$ is $N_{\delta^{*}g\alpha}$-OS in (Z,η), implies $g\left(T\left(A\right)\right)=goT\left(A\right)$ is $N_{\delta^{*}g\alpha}$-CS in (Z,η) therefore goT is $N_{\delta^{*}g\alpha}$-CM and $N_{\delta^{*}g\alpha}$-OM, implies goT is $N_{\delta^{*}g\alpha}$-H.

5.7. *Definition*

A map $T:(S,\tau)\longrightarrow(Y,\sigma)$ is named $N_{\delta^{*}g\alpha}$c-H if T is bijective, T and T^{-1} are $N_{\delta^{*}*g\alpha}$-IM.

5.8. *Theorem*

The composition of two $N_{\delta^{*}g\alpha}$c-Hs is also a $N_{\delta^{*}g\alpha}$c-H.

Proof. Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are two $N_{\delta^{*}g\alpha}$c-Hs. Assume U is a $N_{\delta^{*}g\alpha}$-CS in (Z,η). Since g is a $N_{\delta^{*}g\alpha}$-IM, $g^{-1}\left(U\right)$ is $N_{\delta^{*}g\alpha}$-CS in (Y,σ). Since U is $N_{\delta^{*}*g\alpha}$-IM, $T^{-1}\left(g^{-1}\left(U\right)\right)$ is $N_{\delta^{*}g\alpha}$-CS in (S,τ). that is $\left(goT\right)^{-1}\left(T\right)$ is $N_{\delta^{*}g\alpha}$-CS in (S,τ), therefore $goT:(Y,\sigma)\longrightarrow(Z,\eta)$ is $N_{\delta^{*}g\alpha}$-IM.
Assume G is a $N_{\delta^{*}g\alpha}$-CS in (S,τ), since T^{-1} is a $N_{\delta^{*}g\alpha}$-IM, $\left(T^{-1}\right)^{-1}\left(G\right)$ is $N_{\delta^{*}g\alpha}$-CS in (Y,σ), that is $T\left(G\right)$ is $N_{\delta^{*}g\alpha}$-CS in (Y,σ). Since g^{-1} is $N_{\delta^{*}*g\alpha}$-IM, $\left(g^{-1}\right)^{-1}\left(T\left(G\right)\right)$ is $N_{\delta^{*}g\alpha}$-CS in (Z,η), that is $g\left(T\left(G\right)\right)$ is $N_{\delta^{*}g\alpha}$-CS in (Z,η), therefore $\left(goT\right)\left(G\right)$ is $N_{\delta^{*}g\alpha}$-CS in (Z,η). This implies that $\left(\left(goT\right)^{-1}\right)^{-1}\left(G\right)$ is a $N_{\delta^{*}g\alpha}$-CS in (Z,η). This shows that $\left(goT\right)^{-1}:(Y,\sigma)\longrightarrow(Z,\eta)$ is $N_{\delta^{*}*g\alpha}$-IM. Hence $\left(goT\right)$ is a $N_{\delta^{*}g\alpha}$c-H.

5.9. *Theorem*

Any $N_{\delta^{*}g\alpha}$-H from a $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space into another $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space is a homeomorphism

Proof.

Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is a $N_{\delta^{*}g\alpha}$-H. Then T is bijective, $N_{\delta^{*}g\alpha}$-OM and $N_{\delta^{*}g\alpha}$-CM. Assume U is an (NOS)in (S,τ). Since T is $N_{\delta^{*}g\alpha}$-OM and since (Y,σ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, $T\left(U\right)$ is (NOS)in (Y,σ). This implies that T is N-open map. Assume K is a (NCS) in (Y,σ), since T is $N_{\delta^{*}g\alpha}$-CM and since (S,τ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, $T^{-1}\left(K\right)$ is (NCS) in (S,τ). Therefore T is continuous. Hence T is a homeomorphism.

5.10. *Theorem*

Assume (Y,σ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space. If $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are $N_{\delta^{*}g\alpha}$-H then (goT) is $N_{\delta^{*}g\alpha}$-H.

Proof. Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ and $g:(Y,\sigma)\longrightarrow(Z,\eta)$ are two $N_{\delta^{*}g\alpha}$-H. Assume U is an (NOS)in (S,τ). Since T is $N_{\delta^{*}g\alpha}$-OM, $T\left(U\right)$ is $N_{\delta^{*}g\alpha}$-OS in (Y,σ). Since (Y,σ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, $T\left(U\right)$ is N-OS in (Y,σ). Also since g is $N_{\delta^{*}g\alpha}$-OM, $g\left(T\left(U\right)\right)$ is $N_{\delta^{*}g\alpha}$-OS in (Z,η). Hence goT is $N_{\delta^{*}g\alpha}$-OM. Assume v is a (NCS) in (Z,η). Since g is $N_{\delta^{*}g\alpha}$-CM and since (Y,σ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-space, $g^{-1}\left(V\right)$ is (NCS) in (Y,σ). Since T is $N_{\delta^{*}g\alpha}$-CM,

$T^{-1}\left(g^{-1}\left(V\right)\right)=\left(goT\right)^{-1}\left(V\right)$ is $N_{\delta^{*}g\alpha}$-CS in (S,τ). That is (goT) is $N_{\delta^{*}g\alpha}$-contnuous. Hence (goT) is $N_{\delta^{*}g\alpha}$-H.

5.11. *Theorem*

Any $N_{\delta^{*}g\alpha}$-H from $(N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-S)into another $(N_{\alpha\delta T^{**}_{\frac{3}{4}}g\alpha}$-S)is a $N_{\delta^{*}g\alpha}$c-H.

Proof. Assume $T:(S,\tau)\longrightarrow(Y,\sigma)$ is $N_{\delta^{*}g\alpha}$-H. Assume U be $N_{\delta^{*}g\alpha}$-CS in (Y,σ). Then U is (NCS) in (Y,σ). Since T is $N_{\delta^{*}g\alpha}$-CM, $T^{-1}\left(U\right)$ is $N_{\delta^{*}g\alpha}$-CS in (S,τ). Then T is a $N_{\delta^{*}g\alpha}$-IM. Let K be $N_{\delta^{*}g\alpha}$-OS in (S,τ). Then K is (NOS) in (S,τ). Since T is $N_{\delta^{*}g\alpha}$-OM, $T\left(K\right)$ is $N_{\delta^{*}g\alpha}$-OS in (Y,σ). That is $\left(T^{-1}\right)^{-1}\left(K\right)$ is $N_{\delta^{*}g\alpha}$-OS in (Y,σ) and hence T^{-1} is $N_{\delta^{*}*g\alpha}$-IM. Thus T is $N_{\delta^{*}g\alpha}$c-H.

6. Conclusion

The notions of $N_{\delta^{*}g\alpha}$-continuous and $N_{\delta^{*}g\alpha}$-irresolute functions in (NTS) are given in this work. Next, their characterizations and investigate their properties are analyzed. In future work, we will use the soft sets theory to investigate new classes of neutrosophic soft maps and then we can study these new classes of (NTS)in soft setting.

References

[1] A. A. Salama, and S. A. Alblowi. Neutrosophic Set and Neutrosophic Topological Spaces, IOSR-Journal of Mathematics. (2012), Vol. 3, No. 4, 31–35.

[2] A. A. Salama, F. Smarandache, and K.Valeri. Neutrosophic Closed Set and Neutrosophic Continuous Functions, Neutrosophic Sets and Systems. (2014), Vol. 4, 4–8.

[3] C. L. Chang. Fuzzy Topological Spaces, Journal of Mathematical Analysis and Applications. (1968), Vol. 24, pp. 182–190.

[4] D. Coker. An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets and Systems. (1997), Vol. 88, No. 1, pp. 81–89.

[5] F. Smarandache. A Unifying Field of Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, American Research Press, Rehoboth, 1998.

[6] F. Smarandache. Neutrosophic Set - A Generalization of the Intuitionistic Fuzzy Set, International Journal of Pure and Applied Mathematics, (2005), Vol. 24, No. 3, 287–297.

[7] I. Arokiarani, R. Dhavaseelan, S. Jafari, and M. Parimala. On Some New Notions and Functions in Neutrosophic Topological Spaces, Neutrosophic Sets and Systems. (2017), Vol. 16, 16–19.

[8] K. Atanassov. Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems. (1986), Vol. 20, No. 1, 87–96.

[9] K.Damodharan, M.Vigneshwaran and R. Dhavaseelan. N_{δ}-Closure and N_{δ}-Interior in Neutrosophic Topological Spaces, Asian European Journal of Mathematics, communicated.

[10] K.Damodharan and M.Vigneshwaran. N_{δ}-continuous and neutrosophic almost continuous in Neutrosophic Topological Spaces, Applications and applied Mathematics, communicated.

[11] K.Damodharan and M.Vigneshwaran. $N_{\delta^{*}g\alpha}$-closed sets in neutrosophic topological spaces, Advances in Mathematics: A Scientific Journal. (2020), Vol. 9, No. 3, 819–829

[12] L. A. Zadeh. Fuzzy Sets, Information and Control, (1965), Vol. 8, No. 3, 338–353.

[13] P. Ishwarya and K. Bageerathi , "On Neutrosophic semi-open sets in Neutrosophic topological spaces", International Jour. of Math. Trends and Tech, 2016, 214–223.

[14] Qays Hatem Imran, Smarandache et. al, "On Neutrosophic semi alpha open sets", Neutrosophic sets and systems, 2017, 37–42.

[15] R. Dhavaseelan, and S. Jafari. Generalized Neutrosophic Closed sets, New Trends in Neutrosophic Theory and Applications, (2017), Vol. II, 261–273.

[16] Yeon Seok Eom and Seok Jong Lee, Delta Closure and Delta Interior in Intuitionistic Fuzzy Topological Spaces, International Journal of Fuzzy Logic and Intelligent Systems, (2012), Vol. 12, No. 4, pp. 290–295.

[17] S. M. Khalil, & F. H. Khadhaer, An algorithm for the generating permutation algebras using soft spaces. Journal of Taibah University for Science, 12(3),(2018), 299-308.

[18] S. M. Khalil, M. H. Hasab, Decision Making Using New Distances of Intuitionistic Fuzzy Sets and Study Their Application in The Universities, INFUS, Advances in Intelligent Systems and Computing, (2020), $doi.org/10.1007/978-3-030-51156-2_46$.

[19] S. M. Khalil and A. Rajah, Solving Class Equation $x^d = \beta$ in an Alternating Group for each $\beta \in H \cap C^\alpha$ and $n \notin \theta$, Journal of the Association of Arab Universities for Basic and Applied Sciences, 10,(2011), 42-50.

[20] S. M. Khalil and A. Rajah, Solving Class Equation $x^d = \beta$ in an Alternating Group for all $n \in \theta$ & $\beta \in H_n \cap C^\alpha$, journal of the Association of Arab Universities for Basic and Applied Sciences, 16 (2014), 38–45.

[21] N. M. A. Abbas and S. M. Khalil, On $\alpha^\star-$Open Sets in Topological Spaces, IOP Conference Series: Materials Science and Engineering, 571 (2019) 012021, doi:10.1088/1757-899X/571/1/012021.

[22] S. M. Khalil, The Permutation Topological Spaces and their Bases, Basrah Journal of Science (A), 32(1), (2014), 28-42.

[23] S. M. Khalil and N. M. A. Abbas, On Nano with Their Applications in Medical Field, AIP, (2020) to appear.

[24] S. M. Khalil, New category of the fuzzy d-algebras, Journal of Taibah University for Science, 12(2), (2018), 143-149. doi.org/10.1080/16583655.2018.1451059

[25] S. M. Khalil and A. N. Hassan, New Class of Algebraic Fuzzy Systems Using Cubic Soft Sets with their Applications, IOP Conf. Series: Journal of Physics, (2020), to appear.

[26] S. M. Khalil and N. M. A. Abbas, Characteristics of the Number of Conjugacy Classes and P-Regular Classes in Finite Symmetric Groups, IOP Conference Series: Materials Science and Engineering, 571 (2019) 012007, doi:10.1088/1757-899X/571/1/012007.

[27] M. M. Torki and S. M. Khalil, New Types of Finite Groups and Generated Algorithm to Determine the Integer Factorization by Excel, AIP, (2020) to appear.

[28] S. M. Khalil and N. M. Abbas, Applications on New Category of the Symmetric Groups, AIP, (2020) to appear.

[29] S. M. Khalil and F. Hameed, Applications on Cyclic Soft Symmetric Groups, IOP Conf. Series: Journal of Physics, 1530 (2020) 012046,doi:10.1088/1742-6596/1530/1/012046.

[30] S. M. Khalil, F. Hameed, Applications of Fuzzy $\rho-$Ideals in $\rho-$Algebras, Soft Computing, 24(18), (2020), 13997-14004. doi.org/10.1007/s00500-020-04773-3.

[31] S. A. Abdul-Ghani, S. M. Khalil, M. Abd Ulrazaq, and A. F. Al-Musawi, New Branch of Intuitionistic Fuzzification in Algebras with Their Applications, International Journal of Mathematics and Mathematical Sciences, Volume 2018, Article ID 5712676, 6 pages.

[32] S. M. Khalil, Decision making using algebraic operations on soft effect matrix as new category of similarity measures and study their application in medical diagnosis problems, Journal of Intelligent & Fuzzy Systems, 37, (2019), 1865-1877. doi: 10.3233/JIFS-179249.

[33] A. Firas Muhamad Al –Musawi, S. M. Khalil, M. Abd Ulrazaq, Soft (1,2)-Strongly Open Maps in Bi-Topological Spaces, IOP Conference Series: Materials Science and Engineering, 571 (2019) 012002, doi:10.1088/1757-899X/571/1/012002.

[34] S. M. Khalil, S. A. Abdul-Ghani, Soft M-Ideals and Soft S-Ideals in Soft S-Algebras, J. Phys.: Conf. Ser., 1234 (2019) 012100, doi:10.1088/1742-6596/1234/1/012100.

[35] S. M. Khaliland F. Hameed, An algorithm for generating permutations in symmetric groups using soft spaces with general study and basic properties of permutations spaces. J Theor Appl Inform Technol, 96(9), (2018), 2445-2457.

[36] S. M. Khalil, M. Ulrazaq, S. Abdul-Ghani, Abu Firas Al-Musawi, $\sigma-$Algebra and $\sigma-$Baire in Fuzzy Soft Setting, Advances in Fuzzy Systems, Volume 2018, Article ID 5731682, 10 pages.

[37] M. Abdel-Basset, A. Gamal, L. H. Son and F. Smarandache, (2020). A Bipolar Neutrosophic Multi Criteria Decision Making Framework for Professional Selection. Applied Sciences, 10(4), 1202.

[38] M. Abdel-Basset, Mohamed, R., Zaied, A. E. N. H., Gamal, A., and Smarandache, F. (2020). Solving the supply chain problem using the best-worst method based on a novel Plithogenic model. In Optimization Theory Based on Neutrosophic and Plithogenic Sets (pp. 1-19). Academic Press.

[39] M. Abdel-Basset, et al. "An integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries." Risk Management (2020): 1-27.

[40] M. Abdel-Basset, R. Mohamed and M. Elhoseny, (2020). A novel framework to evaluate innovation value proposition for smart product-service systems. Environmental Technology and Innovation, 101036.

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