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Decision Making Methods with Linguistic Neutrosophic Information: A Review

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Abstract: Linguistic neutrosophic information and its extension have been long recognized as a useful tool in decision-making problems in many areas. This paper briefly describes the development process of linguistic neutrosophic information expressions, and gives in-depth studies on seven different concepts and tools. At the same time, a brief evaluation and summary of the decision-making methods of its various measures and aggregation operators are also made. A comparative analysis of different linguistic neutrosophic sets is made with examples to illustrate the effectiveness and practicability of decision making methods based on multiple aggregation operators and measures. Finally, according to the analysis of the current situation of linguistic neutrosophic information, the related trends of its future development are discussed.

Keywords: linguistic; neutrosophic; decision making, aggregation operator, measures

1. Introduction

In a complex decision-making problem where humans are accustomed to use language to express their idea, decision makers may use linguistic variables (LVs) to qualitatively evaluate attributes. With this regard, Zadeh [1] first proposed the use of LVs to describe preference information and applied it to fuzzy reasoning, and attracted the attention of scholars at home and abroad. Since then, several studies have been carried out to solve problems in different application area [2-6]. However, previous studies [2-6] have reported that merely incomplete information can effectually expressed, while uncertain and conflicting information, are not. To fill the shortcomings mentioned above, Smarandache proposed the neutrosophic sets [7-8] and neutrosophic numbers (NNs) [7-9]. Since the concept of the neutrosophic set was established, some scholars focused on the combination of neutrosophic set and linguistic set to come up with their new concepts.

Fang and Ye [10] first introduced a linguistic neutrosophic number (LNN) concept. LNN has three-part the truth linguistic probability, indeterminacy linguistic probability, and falsity linguistic probability and can express three kinds of linguistic information in this situation. And they also provided score and accuracy functions and some aggregation operators of LNNs. Fan et al. [11] presented an LNN normalized weighted Bonferroni mean operator and an LNN normalized weighted geometric Bonferroni mean operator and applied them to deal with decision-making(DM) problems in LNN environment. Shi and Ye [12] proposed two cosine measures based on the distance and cosine of the included angle between two vectors of LNNs for describing indeterminate linguistic information. Meanwhile, Shi and Ye [13] presented three correlation coefficients of LNNs and showed how they can apply on multiple attribute group decision-making (MAGDM) problems.

On the basis of combining LNNs and NLNs, Cui et al. [14] defined a linguistic neutrosophic uncertain number (LNUN) and the score and accuracy function of LNUNs and then developed related aggregation operators to tackle MAGDM problems. Cui and Ye [15] further introduced a hesitant linguistic neutrosophic number (HLNN) and put forward a MADM method based on similarity measures for DM problems in HLNN sets. On the other hand, Ye et al. [16] proposed a Q-linguistic neutrosophic variable set (Q-LNVs), which extended linguistic neutrosophic evaluation to two-dimensional universal sets (TDUSs). Then, Fan et al. [17] presented a linguistic neutrosophic multiset (LNM) and two Heronian mean operators to handle the multiplicity information under LNM environment. Besides, Ye [18] originally put forward the concept of a linguistic cubic variable (LCV), which consists both uncertain and certain LV synchronously, then he developed some operators to aggregate linguistic cubic information. Next, Lu and Ye [19] integrated Dombi operators with LCVs to better handle DM problems of linguistic cubic sets. Further, Ye and Cui [20] proposed a linguistic neutrosophic hesitant variable (LCHV), and applied aggregation operators to figure out DM problems with interval and hesitant linguistic information. Then, Lu and Ye [21] presented cosine similarity measures of LCHVs which is characterized by the least common multiple number extension method, and its applications in decision-making with LCHV information. Also, Ye and Cui [22] put forward single-valued linguistic neutrosophic interval linguistic numbers (SVLN-ILN) and correlative aggregation operators together with its decision-making approach. Meanwhile, Ye [23] first proposed a new linguistic neutrosophic notion, named linguistic neutrosophic cubic numbers (LNCNs), which is made up of an inconclusive linguistic neutrosophic number and an LNN. Fan and Ye [24] extended the Heronian mean operator to LNCNs and adopt this idea to solve decision-making problems.

The main purpose of this paper is to carry out research on the decision-making methods under the linguistic neutrosophic environment. Firstly, it will be possible to describe some concepts of NLN, LNS, LNUN, HLNS, Q-LNS, LCS, and LNCS. Secondly, insight will be gained into the decision-making methods of using various measures and aggregation operators. Lastly, it gives conclusions and future study of this paper. These findings have significant implications for solving decision making problems in various field.

2. Linguistic Neutrosophic Information Expressions

2.1. Neutrosophic Linguistic numbers

Smarandache [7-8] originally presented the conception of a neutrosophic number that can express incomplete, indeterminate, inconsonant information, represented by $B=t+vI$, where t stands for the determinate part and vI for the indeterminate part, and $t, v \in R$ (all real numbers), $I \in$ [inf I, sup I] (indeterminacy). To better express uncertainty on linguistic information, Smarandache [25] introduced NNs into the LV and proposed a neutrosophic linguistic number (NLN) concept and described by l_{t+vI} where $t+vI$ is NN.

It can be known that on the above method only a single neutrosophic linguistic number is used to evaluate the linguistic information. However, in a complicated DM environment, decision makers may enforce to give several linguistic term values from a linguistic term set (LTS) due to their hesitancy. It means that a single linguistic term value is not sufficient to express the results of the assessment. Hence, it is clearly that the existing NLN method [26] is not suitable for such case. In order to deal with this situation, Ye began to see hesitant neutrosophic linguistic numbers as key components in linguistic decision-making field. As a result, Ye [27] proposed hesitant neutrosophic linguistic numbers (HNLNs) that consist of a series of NLNs, standing for the decision makers' different proposals respectively. Hence, HNLNs can easily be applied to hesitant decision-making problems involving the NLNs consist of partial determinacy and partial uncertainty.

Definition 2.1.1. [27] Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universe of discourse and $L = \{l_0, l_1, \ldots, l_{2t}\}$ be a finite and fully ordered set of discrete linguistic terms. An HNLN set H_l on X described mathematically as the following form:

$$H_l = \left\{ \left\langle x_j, h_l\left(x_j\right) \right\rangle \middle| x_j \in X \right\}, \tag{1}$$

where $h_l(x_j)$ is the set of x_j NLNs for $x_j \in X$ and L, and x_j is the number of NLNs with $j=1, 2, \ldots, n$. Therefore, $h_l(x_j)$ can be denoted by $h_l\left(x_j\right) = \left\{ l_{a_j^k + b_j^k} \middle| l_{a_j^k + b_j^k} \in L, k = 1, 2, \ldots, s_j \right\}$ for $x_j \in X$ *and* $j=1, 2, \ldots, n$.

2.2. Linguistic neutrosophic sets

The existing NLN can provide useful tools to deal with incomplete, indeterminate, and inconsistent linguistic information. However, it cannot use for DM problems with information expressed with their truth, indeterminacy and false functions. An LNN proposed by Fang and Ye [10] can better address the drawback shown above since it is characterized by the truth, indeterminacy, and falsity LVs respectively rather than exact values. In fact, LNNs can also be considered as a new LV added to LIFN to indicate the degree of indeterminacy and the incomplete and inconsistent linguistic information. LNNs are a useful tool in depicting the indeterminate and inconsistent decision-making information by using three linguistic variables.

Definition 2.2.1. [10] Let $L = \{l_0, l_1, \ldots, l_{2t}\}$ is a finitely linguistic term set. If $g = \langle l_T, l_I, l_F \rangle$ is defined as $l_T, l_I, l_F \in L$ and $T, I, F \in [0, 2t]$, where l_T, l_I and l_F use linguistic terms to show the truth, indeterminacy, and falsity degree, severally, then g is called an LNN.

2.3. Linguistic neutrosophic uncertain numbers/sets

Motivated by NLNs and LNNs, Cui et al. [14] defined a new notion of an LNUN constructed respectively by three uncertain linguistic variables representing linguistic truth, indeterminacy and falsity. In general, the LNUN is the expansion of LNN and NLN with partial linguistic certain and partial linguistic uncertain evaluations. It turns out that LNUNs can describe the different complex linguistic neutrosophic decision-making information under an LNUN environment.

Definition 2.3.1. [14] Assume that $L = \{l_0, l_1, \ldots, l_{2t}\}$ is a finite and fully ordered set of linguistic term set. An LNUN in L is constructed as $h = \left\langle l_{T_a + T_b I}, l_{U_a + U_b I}, l_{F_a + F_b I} \right\rangle$ with three uncertain linguistic variables $l_{T_a + T_b I}$, $l_{U_a + U_b I}$, and $l_{F_a + F_b I}$ representing the truth, uncertainty, and falsity NLNs independently, where T_a+T_bI, U_a+U_bI, $F_a+F_bI \in [0, 2t]$ and $I \in [\inf I, \sup I]$.

2.4. Hesitant linguistic neutrosophic sets

It is obvious that much DM information in the real world is fuzzy rather than precise, in which decision-makers may be entangled in a certain decision. However, LNN cannot express the hesitation of decision-makers in the evaluation of linguistic alternatives. A HLNN introduced by Cui and Ye [15] can express much more information given by decision-makers since it is composed of several LNNs related to an objective thing. Essentially, HLNNs are combined form of HFSs and LNNs, which can simultaneously express both the hesitancy information and LNN information of decision-makers.

Definition 2.4.1. [15] Set $X = \{x_1, x_2, \ldots, x_n\}$ as a universe of discourse and a finite linguistic term set $L = \{l_0, l_1, \ldots, l_{2t}\}$, and then an HLNN set N_l on X can be given by

$$N_l = \left\{ \left\langle x_j, E_l\left(x_j\right) \right\rangle \middle| x_j \in X, j = 1, 2, ..., n \right\}, \tag{2}$$

Where $E_l(x_j)$ is a set of x_j LNNs for $x_j \in X$ and L, expressed by an HLNN $E_l\left(x_j\right) = \left\{ \left\langle l_{T_j^k}, l_{U_j^k}, l_{F_j^k} \right\rangle \middle| l_{T_j^k} \in L, l_{U_j^k} \in L, l_{F_j^k} \in L, k = 1, 2, ..., x_j \right\}$ for $\in X$.

2.5. Q-linguistic neutrosophic set

A majority of linguistic concepts only process indeterminate, uncertain and incompatible data of the subject being evaluated in one-dimensional universal sets. This prompted researchers to amplify them to have the ability to depict linguistic arguments in TDUSs. Then, Ye et al. [16] first proposed a Q-LNVS to explain linguistic neutrosophic claims in DM problems of TDUSs. Therefore, Q-LNVS was primarily used to define its linguistic values of truth, indeterminacy and falsity corresponding to TDUSs, respectively.

Definition 2.5.1. [16] Assume that $X = \{x_1, x_2, \ldots, x_n\}$ and $Q = \{q_1, q_2, \ldots, q_m\}$ are two-dimensional universal sets and a finite linguistic term set $L = \{l_0, l_1, \ldots, l_{2t}\}$, and then a Q-LNVS P on X and Q can be denoted by

$$P = \left\{ \begin{array}{c} \left\langle \left(x_i, q_j\right), l_t\left(x_i, q_j\right), l_u\left(x_i, q_j\right), l_v\left(x_i, q_j\right) \middle| x_i \in X, q_j \in Q \right\rangle \\ l_t\left(x_i, q_j\right), l_u\left(x_i, q_j\right), l_v\left(x_i, q_j\right) \in L, i = 1, 2, ..., n; j = 1, 2, ..., m \end{array} \right\}, \tag{3}$$

where $l_t(x_i, q_j)$, $l_u(x_i, q_j)$, $l_v(x_i, q_j)$ denoted the truth, indeterminacy, and falsity LVs, independently, in TDUSs for $t, u, v \in [0, 2t]$.

Then, the basic element $<(x_i, q_j), l_t(x_i, q_j), l_u(x_i, q_j), l_v(x_i, q_j)>$ in L is simply expressed as $l_{ij} = \left\langle \left(x_i, q_j\right), s_{t_{ij}}, s_{u_{ij}}, s_{v_{ij}} \right\rangle$, which is known as a Q-linguistic neutrosophic element (Q-LNE).

Later on, based on the linguistic multiplicity evaluation in some real situations, Fan et al. [17] developed an LNM, which is extended from neutrosophic multiset. An LNM can use pure linguistic value to express and process the multiplicity information and can represent the truth, indeterminacy, and falsity through three values, severally.

Definition 2.5.2. [17] Set a universe $X = \{x_1, x_2, \ldots, x_n\}$ and $L = \{l_0, l_1, \ldots, l_{2t}\}$ be an LTS, and $Z = \{1, 2, 3, \ldots, \infty\}$, then LNM R represented with the following mathematical expression.

$$R = \left\{ x, \left(\begin{array}{l} \left(f_{R1}, \left\langle l_{\mu_{R1}}(x), l_{\nu_{R1}}(x), l_{\xi_{R1}}(x) \right\rangle\right), \\ \left(f_{R2}, \left\langle l_{\mu_{R2}}(x), l_{\nu_{R2}}(x), l_{\xi R}(x) \right\rangle\right), ..., \\ \left(f_{Ry}, \left\langle l_{\mu_{Ry}}(x), l_{\nu_{Ry}}(x), l_{\xi_{Ry}}(x) \right\rangle\right) \end{array} \right) \middle| x \in X \right\}, \tag{4}$$

where $l_{\mu_{Rt}}(x)$, $l_{\nu_{Rt}}(x)$, $l_{\xi_{Rt}}(x) \in L$, μ_{Rt}, ν_{Rt}, $\xi_{Rt} \in [0, 2t]$. An LNM consists of the truth degree membership function $l_{\mu_{Rt}}(x)$, the indeterminacy degree membership function $l_{\nu_{Rt}}(x)$, and the falsity degree membership function $l_{\xi_{Rt}}(x)$. Among them, $l_{\mu_{R1}}(x)$, $l_{\mu_{R2}}(x)$, …, $l_{\mu_{Ry}}(x) \in [0,1]$, $l_{\nu_{R1}}(x)$, $l_{\nu_{R2}}(x)$, …, $l_{\nu_{Ry}}(x) \in [0,1]$ and $l_{\xi_{R1}}(x)$, $l_{\xi_{R2}}(x)$, …, $l_{\xi_{Ry}}(x) \in [0,1]$, that is,

$0 \le l_{\mu_{Rt}}(x) + l_{\nu_{Rt}}(x) + l_{\xi_{Rt}}(x) \le 3$ (t = 1, 2, …, y), $y \in Z$, f_{R1}, f_{R2}, …, $f_{Ry} \in Z$, and $f_{R1} + f_{R2} + \ldots + f_{Ry} \ge 2$.

The above expression for an LNM R can be simplified to the following form：

$$R = \left\{ x, \left(f_{Rt}, \left\langle l_{\mu_{Rt}}(x), l_{\nu_{Rt}}(x), l_{\xi Rt}(x) \right\rangle \right) \middle| x \in X \right\}, \quad (5)$$

for $t = 1, 2, \ldots, y$.

2.6. Linguistic cubic sets

In reality, some real decision-making problems may contain mixed evaluation information of uncertain and certain linguistic arguments simultaneously. To handle this, Ye [18] proposed an LCV by merging LVs and cubic set together and can be applied apply on can have consists of an uncertain LV and a specific LV.

Definition 2.6.1. [18] Let $L = \{l_0, l_1, \ldots, l_{2t}\}$ is a finite LTS. An LCV V in L is denoted using $V = (\tilde{L}, L_c)$, where $\tilde{L}$ = [L_a, L_b] is a uncertain LV and L_c is an LV for $b \ge a$ and $L_a, L_b, L_c \in L$. If $a \le b \le c$, V = ([L_a, L_b], L_c) is an internal LCV. If $c \notin (a, b)$, V = ([L_a, L_b], L_c) is an external LCV.

However, due to uncertainty and hesitation on the part of decision-makers on the subject of evaluation, in some decision-making problems, information on decision-making is made up of an interval of LV and a hesitant linguistic set. To deal with such a situation, based on the concepts of an LCV and hesitant fuzzy sets, Ye and Cui [20] proposed an LCHV. The proposed LCHV reasonably express the combined information from uncertain and hesitant linguistic arguments and efficiently tackled LCHV problems.

Definition 2.6.2. [20] Set a linguistic variable term set as $L=\{l_j \mid j \in [0,2t]\}$. An LCHV z in L is built by $z = (\tilde{l}_u, \tilde{l}_h)$, where $\tilde{l}_u = [l_a, l_b]$ for $b \ge a$ and $l_a, l_b \in L$ is an interval linguistic variable and $\tilde{l}_h = \{ l_{\lambda_k} \mid l_{\lambda_k} \in L, k = 1,2, \ldots, j\}$ is a set of j possible LVs (i.e., a hesitant LV is listing in an increasing order.)

Furthermore, Ye and Cui [22] presented the idea of an SVLN-ILN, composed entirely of its uncertain / interval linguistic number and its single valued neutrosophic linguistic number. In the case of a DM problem, the SVLN-ILN represents both the linguistic judgment of the decision-maker and the affirmative linguistic judgment of the evaluated object.

Definition 2.6.3 [22] Let a linguistic variable set be $L = \{l_0, l_1, \ldots, l_{2t}\}$. A SVLN-ILN W in L is denoted by $W = \langle [l_a, l_b]; l_T, l_I, l_F \rangle$, where $[l_a, l_b]$ is the interval linguistic number part of W and l_a and l_b are linguistic lower and upper limits of l_j for $l_a \le l_j \le l_b$ and $l_j \in L$, and then $\langle l_T, l_I, l_F \rangle$ is the SVLNN part of W. Here, the truth linguistic function $T_W(l_j)$ of W can be constructed by

$$T_W(l_j) = \begin{cases} l_T, l_a \le l_j \le l_b \\ l_0, \quad otherwise \end{cases}, \quad (6)$$

The indeterminacy linguistic function $I_W(l_j)$ of W can be constructed by

$$I_W(l_j) = \begin{cases} l_I, l_a \le l_j \le l_b \\ l_z, \quad otherwise \end{cases}, \quad (7)$$

The falsity linguistic function $F_W(l_j)$ of W can be constructed by

$$F_W\left(l_j\right)=\begin{cases} l_F, l_a \le l_j \le l_b \\ l_z, \ \ otherwise \end{cases}, \tag{8}$$

where $l_0 \le l_T \le l_z$, $l_0 \le l_I \le l_z$ and $l_0 \le l_F \le l_z$.

2.7. Linguistic neutrosophic cubic sets

A new notion of linguistic neutrosophic cubic set, as presented by Ye, extending the concept of cubic sets to linguistic neutrosophic sets, called linguistic neutrosophic cubic sets. A proposed LNCN contains an uncertain LNN and a single-valued LNN at the same time as the linguistic variables of truth, indeterminacy and falsity [23]. In LNCN, the uncertain LNN expresses the truth, indeterminacy, and falsity values of uncertain LVs, and the single-valued LNN is composed of the truth, indeterminacy, and falsity LVs, which are used to describe their mixed information.

Definition 2.7.1. [23] Let an LTS be $L=\{l_j \mid j \in [0,2t]\}$. An LNCN O in L is defined as $O=(u, c)$, where $u=<[l_{Ta}, l_{Tb}], [l_{Ia}, l_{Ib}], [l_{Fa}, l_{Fb}]>$ is an uncertain LNN with the truth linguistic variables $[l_{Ta}, l_{Tb}]$, indeterminacy linguistic variables $[l_{Ia}, l_{Ib}]$, and falsity uncertain linguistic variables $[l_{Fa}, l_{Fb}]$, for $l_{Ta}, l_{Tb}, l_{Ia}, l_{Ib}, l_{Fa}, l_{Fb} \in L$ and $Ta \le Tb, Ia \le Ib, Fa \le Fb$; $c=<l_T, l_I, l_F>$ is consisted of an LNN with l_T, l_I and l_F each on behalf of the truth, indeterminacy, and falsity LVs, respectively, where $l_T, l_I, l_F \in L$.

3. Decision making methods regarding various measures and aggregation operators

Because of the inherent vagueness of human thinking and the complexity of the objective world, a clear description of decision information is the most crucial part in the real evaluation processes. Hence, to better describe the decision information, the forms of decision information need to be continuously expanded and enriched according to the specific situation. In the process of dealing with information that is incomplete, uncertain and inconsistent, the introduction of linguistic neutrosophic sets play an important role. Smarandache [25] firstly defined NLNs in symbolic neutrosophic theory. Later, to address the problems of neutrosophic linguistic number decision-making, Ye [26] further suggested basic operations and two weighted NLN aggregation operators, namely, the NLN weighted arithmetic average (NLNWAA) operator and the NLN weighted geometric average (NLNWGA) operator. Next, they have been widely used to make alternative manufacturing decisions in flexible manufacturing systems. Then, Ye [27] put forward the concept of HNLNs and the excepted value together with their similarity measure. HNLNs were further developed to use under hesitant and indeterminate linguistic environment. Apart from that, the application is illustrated by taking the problem of manufacturing scheme selection as an example.

To express the truth, falsity, and indeterminacy linguistic information respectively, linguistic neutrosophic numbers containing three independently linguistic variables were presented. After that, some aggregation operators of LNNs, such as the LNN-weighted arithmetic averaging (LNNWAA) and the LNN-weighted geometric averaging (LNNWGA) operators [10], the LNN normalized weighted Bonferroni mean (LNNNWBM) and LNN normalized weighted geometric Bonferroni mean (LNNNWGBM) operators [11], a cosine similarity measure of LNNs [12] and correlation coefficients of LNNs [13] were proposed to tackle decision-making problems in linguistic neutrosophic sets. LNNWAA and LNNWGA operators, as two basic aggregation operators, are often used to select investment alternatives under LNN information. Bonferroni mean (BM), which is an effective aggregation operator that not only considers the importance weights of attributes, but also reflects the interrelationship between attribute values [28] and it is extended to fuzzy sets [29-34] and neutrosophic theory [35-36] to apply Bonferroni mean operators for DM. Motivated by the idea of LNN and Bonferroni mean (BM) operators, Fan proposed the LNNNWBM operator and the LNNNWGBM operator. At the same time, he took different parameter values of p and q to analyze their impact on the decision results. Meanwhile, similarity measures have aroused widespread concerns, which is a vital tool in decision-making process [37-41]. The cosine measures between LNNs were proposed based on distance and the included cosine of the angle between LNNs in vector space that can sort the alternatives and choose the most ideal one(s) [12]. The similarity measure methods have a good application prospect in ideal investment alternatives under linguistic decision-making environments. Further, correlation coefficient is also an available tool for making decisions in complex problems [42-46]. Shi extended correlation coefficients to LNNs and put forward three new correlation coefficients between a substitution and the ideal substitution of LNNs and introduced an example of the investment substitution selection problem.

Also, LNUNs with corresponding weighted aggregation operators were put forward to depict three uncertain linguistic variables for decision-making in the uncertain linguistic environment [14]. Some weighted operators, such as a LNUNWAA operator and a LNUNWGA operator, are raised to aggregate LNUN information and exploited to demonstrate the effectiveness of an investment company decisions.

In fact, the degree of similarity or difference between the research objects plays a decisive role in the DM results. The similarity measures of HLNNs were put forward under the hesitant neutrosophic environment. Cui and Ye [15] presented similarity measures and generalized distance of HLNNs. Then he utilized similarity measures of HLNNs regarding least common multiple cardinality to satisfy the demand of hesitant decision-making and showed their application in investment alternatives.

In vector space, in particular, the Jaccard, Dice, and cosine similarity measures are usually used in diversified fields [41, 47-50]. Applying the Jaccard, Dice, and cosine similarity measures thus improve the decision-making process and produce better results. In this way, a Q-LNVS, which can depict linguistic neutrosophic arguments to two-dimensional universal sets, was presented [16]. And the vector similarity measures that contain Jaccard, Dice, and cosine measures were used for settling linguistic neutrosophic decision-making problems regarding TDUSs. Thereafter, the LNM and its two Heronian mean operators were raised to handle multiplicity information under linguistic neutrosophic multiplicity number environment [17].

On the basis of LCVs, a LCVWAA operator and a LCVWGA operator are presented to aggregate linguistic cubic information [18]. Next, Lu and Ye [19] extended the Dombi operators to LCV, which contain variable operational parameters and more flexible representation of decision information and developed a LCVDWAA operator and a LCVDWGA operator to aggregate linguistic cubic information. These two methods are well applied in the optimal selective problems. Hereafter, a target expansion method of LCHVs using least common multiple/cardinality, and the WAA and WGA operators of LCHVs to reasonably aggregate LCHV information, were proposed [20]. Next, the similarity measures were developed to measure the degree of similarity between LCHVs and an example of engineering selection was used to solve practical problems [21]. Utilizing the mixed information of interval linguistic number and single-valued LNN, SVLN-ILNs and corresponding weighted aggregation operators were given to provide a comprehensively description of interval linguistic parameters and confident linguistic parameters [22].

Meanwhile, by the combined form of uncertain linguistic and certain linguistic neutrosophic numbers, LNCNs and related aggregation operators, like two weighted aggregation and Heronian mean operators were introduced to work out linguistic decision-making problems [23-24]. The DM method based on a LNCNWAA operator and a LNCNWGA operator was constructed in machinal design schemes problems. All of the above methods assume that the set variables are independent of each other. However, because of the complexity of the real world, most of the information variables are related to each other. This correlation will directly affect the decision results. To overcome the shortcomings, Fan combined the Heronian mean operator with the LNCN to develop a MADM method of mechanical design schemes using the LNCNGWHM operator or LNCNTPWHM operator under LNCN setting.

These various measures and aggregation operators are further shown in Table 1.

Table 1. Regarding Various Measures and Aggregation Operators.

Authors	Sets	Tools and approaches
Jun Ye	NLNs	NLNWAA and NLNWGA operators to aggregate NLN information
Jun Ye	NLNs	the expected value and the similarity measure of HNLNs
Zebo Fang; Jun Ye	LNS	LNNWAA and LNNWGA operators to aggregate LNN information
Changxing Fan et al.	LNS	extend Bonferroni mean to LNN, and propose LNNNWBM and LNNNWGBM operators
Lilian Shi; Jun Ye	LNS	extend cosine similarity measures to LNNs
Lilian Shi; Jun Ye	LNS	put forward three new correlation coefficients of LNNs
Wenhua Cui et al.	LNUNs	LNUNWAA and LNUNWGA operators to aggregate LNUN information
Wenhua Cui, Jun Ye	HLNS	present similarity measures of HLNNs based on LCMC
Jun Ye et al.	Q-LNS	put forward vector similarity measures of Q-LNVSs
Changxing Fan et al.	Q-LNS	extend Heronian mean to LNM, and propose LNMNGWHM and LNMNIGWHM operators
Jun Ye	LCS	LCVWAA and LCVWGA operators to aggregate linguistic cubic information
Jun Ye; Wenhua Cui	LCS	WAA and WGA operators of LCHVs
Jun Ye; Wenhua Cui	LCS	SVLN-ILNWAA and SVLN-ILNWGA operators to aggregate SVLN-ILN information
Jun Ye	LNCS	expand neutrosophic cubic sets to linguistic neutrosophic arguments, and propose LNCNWAA and LNCNWGA operators
Changxing Fan; Jun Ye	LNCS	extend Heronian mean to LNCN, and propose LNCNGWHM and LNCNTPWHM operators
Lilian Shi; Jun Ye	LNS	extend cosine similarity measures to LNNs
Lilian Shi; Jun Ye	LNS	put forward three new correlation coefficients of LNNs

Obviously, the main advantage of NLN is that it can express and process ubiquitous imprecise, incomplete, and indeterminate linguistic information under a linguistic DM environment, which is more suitable for practical scientific and engineering applications. However, in the event of complex DM problems due to the hesitation and uncertainty of the cognition of decision-makers, this method cannot accurately reflect the actual meaning of the decision makers. They may not put their evaluation of a certain attribute with a single NLN. In such a case, the hesitation and uncertain evaluation are expressed by a series of NLNs known as HNLN which is an effective method in a hesitant linguistic environment. Through a comparative analysis of the two MADM methods proposed under the HNLN setting and the present MADM methods proposed in the NLN environment, it is found that the best choice is the same. But it can also be known that their ranking order is slightly different. This is because the MADM method in the HNLN and NLN environments differs in the information expression and algorithm, which explains that there may be differences in the sort order under HNLN and NLN environments.

In fact, LNNs can express uncertain and inconsistent linguistic information corresponding to human's vague thinking on intricate problems, particularly the qualitative evaluation of some attributes, which solve the problem of uncertain and inconsistent linguistic information. After comparison, it is found that the two sorting orders and the ideal choice based on the LNNWAA and LNNWGA operators are the same, which is consistent with the result in the literature [51]. The LNNNWBM and LNNNWGBM operators take into account the influence of the parameters p and q on the decision results. By diverse values of the parameters p and q, we can know that the arrangement order of the study is the same. Therefore, these two parameters have little effect on this decision problem [11]. The ranking results of this example are consistent, but in contrast, the LNNNWBM operator and LNNNWGBM operator consider the correlation between attributes for MAGDM, making the information aggregation more objective and reliable. The cosine similarity measures of LNNs are simpler than the LNNWGA operator and LNNWAA operator. In addition, the correlation coefficients of LNN are compared with LNNWGA and LNNWAA operators, and it can be seen from the literature [13] that the sort order based on these three new correlation coefficients is consistent with the results proposed in the literature [10]. What counts is that the correlation coefficients of LNNs are relatively simple and can even further avoid some unreasonable phenomena existing in LNNWGA and LNNWAA operators.

Similarity measures of HLNNs based on the LCMC extension method can reflect the indecisiveness of decision-makers under a HLNN environment. The similarity measures not just process the HLNN, but also the LNN as LNN is just a special case of the HLNN without decision-makers hesitation. LNUNWAA and LNUNWGA operators are two types of LNUN information aggregation operators, in which the indeterminacy range of I will lead to different order of the schemes. Therefore, with the MAGDM method based on LNUN information, decision makers can pick disparate indeterminacy ranges according to their own preferences or actual needs, making the actual decision-making problem more flexible. It is worth noting that if the indeterminacy I is not considered (i.e., $I = 0$), LNN is just a special case of LNUN.

Compared with the LNNs decision-making method [10], LNCNs contain more information which can simultaneously express uncertain LNNs and certain LNNs under linguistic DM environment. The aggregation of linguistic neutrosophic cubic information can performed by the LNCNWAA operator and LNCNWGA operator. Therefore, decision-makers have two choices of LNCNs weighted set operators to settle the linguistic neutrosophic cubic decision problem depend on their own preferences and actual needs. The MADM method based on a LNCNGWHM operator and a LNCNTPWHM operator combine the LNCN with Heronian mean operator which can reflect the interaction between attributes. The literature [24] analyzed the possibility that the various parameters p, q, r may could affect decision results differently. Therefore, sort the operation results by adjusting the values of the three parameters. The results show that the parameters in the LNCNGWHM or LNCNTPWHM operator have little effect on the decision of this example. Compared with the results of LNCNWAA and LNCNWGA [23], their sort order is the same. However, LNCNGWHM and LNCNTPWHM operators reflect the interactions between attributes, and take into account different p, q, and r values, making the outcome more convincing and comprehensive than those of LNCNWAA and LNCNWGA.

A linguistic neutrosophic MADM method based on Q-LNVs includes the Jaccard, Dice, and cosine similarity measures. Then LNV is a particular case of Q-LNVS for a general set.

The LNMNGWHM and LNMNIGWHM operators represent and deal with the problem of multiplicity, and can obtain more complicated results by considering the interrelationship between attributes, which make the results more realistic. The ranking results are analyzed by different values of d and f that show no matter how these two values are taken, the sort orders are consistent, so d and f have tiny effect on the ranking results of the study. Compared with the proposed operators in the literature [10], it is found that their results are coincident, but the operators of LNM have the advantage of expressing and handling the multiplicity problems. Therefore, this method can make the decision result more reliable and has certain practicability in practical application.

4. Conclusions

Linguistic neutrosophic information has been extended to various types and these extensions have been used in many areas of decision making. This review paper mainly focused on the overview of the development process of linguistic neutrosophic information expressions from seven aspects (NLNs, LNS, LNUNs, HLNS, Q-LNS, LCS, LNCS), and makes in-depth research on its application in decision-making. Analysis shows that they can be combined with commonly used mathematical tools, such as aggregation operators, measures, etc. These methods are being employed increasingly for the evaluation of alternatives and comparative analysis in different decision problems. Despite their advantage of getting a better result, the currently proposed linguistic neutrosophic information hasn't widely used outside MADM problems.

As a result, in the future study, we will further combine with other fuzzy theories (such as rough sets, etc.) to develop new linguistic sets and expand its application to other domains, such as fault diagnosis, medical diagnosis, picture analysis, and pattern recognition.

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