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Decomposition of Matrix under Neutrosophic Environment

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Abstract: Matrices help for the effective representation of systems of linear equations and analyzing any sort of data. The decomposition of any matrix allows for the efficient implementation of matrix-based algorithms. Spectral decomposition is one of the approaches commonly used for square symmetric matrices in order to spell out variation for each of the involved components. The Neutrosophic environment is based on square symmetric matrices and likely to call Spectral decomposition. Neutrosophic is the branch of philosophy that deals with nature, the scope of neutralities and their associations with changed ideational spectra. It is the generalization of the classical set, classical fuzzy set, and intuitionistic fuzzy set. These set theories often limited to handle the problem of uncertainty. Neutrosophic basically based on three possibilities; like Degree of Truth (T), Degree of Falsehood (F) and Degree of Indeterminacy (I).In real-life uncertainties commonly happened and so neutrosophic plays an important role to measure those uncertainties such as inexplicit statements, specious or inadequate information. In order to measure the indeterminacy, a neutrosophic matrix approach is purposed and matrix named “Square-Symmetric Neutrosophic (SSN) matrix”. The SSN matrix is computed using the spectral decomposition of matrices; which do factorization of a matrix into canonical form. The increasing level of indeterminacy restrains from reaching to exact decision. If indeterminacy in (any two) SSN matrices increases, then this leads to reduce variation in data. The process is checked through the Eigenvectors which suggests that through spectral decomposition the variation of the indeterminacy in SSN matrices can be minimized.

Keywords: Neutrosophic set, Square Neutrosophic matrices, and Spectral decomposition.

1. Introduction

Neutrosophic philosophy was presented by Florentin Smarandache (Smarandache, 1999) which based on three components namely Degree of Truth(T), Degree of Falsehood(F) and Degree of Indeterminacy(I) defined on the sample space X, where these three components are fully independent. This theory has many applications in different fields such as (Ansari, Biswas, & Aggarwal, 2011; Broumi & Smarandache, 2013; Cheng & Guo, 2008; Kharal, 2014) where inconsistent, and indeterminate problems occurred. Two types of measure for bipolar and interval-valued bipolar neutrosophic sets proposed by (Abdel-Basset, Mohamed, Elhoseny, Chiclana, & Zaied, 2019). A robust ranking method with the neutrosophic set theory proposed by (Abdel-Baset, Chang, & Gamal, 2019) study the environmental performance of green supply chain management. The uncertainty mostly handle with the support of set theories but neutrosophic theory generalize these set theories (Azizzadeh, Zadeh, Zahed, & Zadeh, 1965). In decision-making problems the neutrosophic approach is used that deal and overcome the ambiguity (Abdel-Basset, Atef, & Smarandache, 2019). A neutrosophic method for assessment of Hospital medical care systems which based on plithogenic data sets presented by(Abdel-Basset, El-hoseny, Gamal, & Smarandache, 2019). For Supply Chain Sustainability a neutrosophic method is presented by (Abdel-Basset, Mohamed, Zaied, & Smarandache, 2019). Matrices play a big role in science and technology. When uncertainty involved in classical matrix different fuzzy matrices are developed using the fuzzy relation system. For this purpose different square neutrosophic matrices were proposed by (Dhar, Broumi, & Smarandache, 2014). The descriptive neutrosophic statistics using the neutrosophic logic Proposed by (Smarandache, 2014) and Neutrosophic Probability, Set, and Logic also proposed by (Smarandache, 1998). Later on, (Aslam, 2018), (Aslam, Bantan, & Khan) and (Aslam, 2019) introduced the inferential neutrosophic statistics and neutrosophic statistical quality control. (Alhabib, Ranna, Farah, & Salama, 2018) presented Some continuous Neutrosophic Probability models including the Poisson model, Exponential model and Uniform model that are applicable when uncertainty involved in data. The neutrosophic matrix operations first time introduced by (Ye, 2017) and solution methods including addition method, substitution method and inverse method also developed. (Basu & Mondal, 2015) proposed different types of Neutrosophic Soft matrix along with various mathematical operations. In medical science this application is applicable.(Uma, Murugadas, & Sriram) developed the methods of determinant and adjoint of Fuzzy Neutrosophic Matrices. (Varol & Aygün, 2019) proposed a neutrosophic matrix, whose elements are based on single-valued neutrosophic sets. In this paper, they proposed various theorems on neutrosophic matrix with basic operations. (Sumathi & Arockiarani, 2014) discussed some operations on fuzzy neutrosophic matrix and developed a decision method scheme that deal uncertainty. (Kavitha, Murugadas, & Sriram, 2018) studied the powers of a fuzzy neutrosophic soft square matrix under the function of max and min. Our aim in this paper to propose a neutrosophic matrix called "Square-Symmetric Neutrosophic (SSN) matrix, whose entries based on indeterminate part. The SSN matrix is computed using the spectral decomposition of matrices.

1.1 Fundamental and basic concepts

Definition 1.1.1 (Broumi, Bakali, Talea, Smarandache, & Selvachandran, 2017)(Neutrosophic Set)

Suppose Y be a sample space and let y ε Y. A neutrosophic set $\overline{U}$ in Y based on three components such as truth part $T_{\overline{u}}$, an in determinant part $I_{\overline{u}}$ and falsehood part that is $F_{\overline{u}}$. All these three components are independent to each other and based on standard or on standard subsets such as] 0^- ,1^+[. In real-life applications such as engineering and scientific problems, it is recommended to use the interval [0, 1] instead of]0^- ,1^+[as it reduces the complicity of system. The Neutrosophic set can be defined as

$$\overline{U}=\{((y,T_{\overline{u}}(y),(I_{\overline{u}}(y),(F_{\overline{u}}(y)):y\varepsilon Y\} \quad (1)$$

Where the sum of these three neutrosophic components are
$$0^- \leq T_{\overline{u}}(y) + I_{\overline{u}}(y) + F_{\overline{u}}(y) \leq 3^+ \quad (2)$$

Definition 1.1.2 (Dhar et al., 2014) (Square Neutrosophic Matrix)
Let $A_{m\times m}$ and $B_{n\times n}$ be two square Neutrosophic matrices where indeterminacy involved in the matrices

$$A_{m\times m}=\begin{bmatrix} a_{11}I & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } B_{n\times n}=\begin{bmatrix} b_{11}I & b_{12}I & b_{13} \\ b_{21}I & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

2. Methodology

Spectral Decomposition

The spectral theorem states that any symmetric mx m or nx n matrix which has real entries have exactly m or n real but possibly not different Eigenvalues and analogous to those Eigenvalues there are mutually independent Eigenvectors. Where Eigenvector based on a linear transformation whose direction does not change when a scalar is multiplied and Eigenvalue is a scalar that is used to transform an Eigenvector. Both are used to reduce variation in data. They can also help to improve the model efficiency (LI, 2016).

Consider two square neutrosophic matrices of the same dimension and let λ be an Eigenvalue of these two matrices.

If x any y be two nonzero vectors ($x \neq 0$) and ($y \neq 0$) such that $Ax = \lambda x$ and $By = \lambda y$ (3)

then x is said to be an Eigenvector of the matrix A linked with Eigenvalue λ and y is said to be an Eigenvector of matrix B linked with the Eigen value λ. An equivalent condition for λ to be a solution of the Eigenvalue- Eigenvector equation is $|A - \lambda I| = 0$ and $|B - \lambda I| = 0$.

Let $A_{m\times m}$ and $B_{n\times n}$ be two symmetric matrices. Then these two matrices can be expressed in terms of its m and n Eigen value-Eigen vector pairs (λ_i, e_i) as

$$A_{m\times m}=\sum_{i=1}^{m} \lambda_i e_i e_i{'} \text{ and } B_{n\times n}=\sum_{i=1}^{n} \lambda_i e_i e_i{'} \quad (4)$$

3. Results

The results using the proposed methodology for various values of K and I are given in Table 1.

4 Comparison

In this section, we compare the performance of the proposed method with the method under classical statistics. It is important to note that the proposed methodology of neutrosophic statistics reduces under classical statistics when K=1 and I=0. From Table 1, we note that in matrix A_k where indeterminacy involved in the first variable, so as I is increased, the variation is reduced in the first variable checked through the Eigenvectors. The same two indeterminate variables situation is presented in the matrix B_k where variation in the first two variables also reduces checked through the Eigenvectors as I increase. Therefore, it is concluded that through spectral decomposition the indeterminacy in SSN matrices can be minimized. By this comparison, it is concluded that the proposed methodology under neutrosophic statistics is useful to reduce the variation as compared to classical statistics.

Table 1: Neutrosophic matrices based on different indeterminacy (I) values.

	Consider a Neutrosophic square and symmertic matrix			
	$A_k = \begin{bmatrix} 2.2I & 0.4 \\ 0.4 & 2.8 \end{bmatrix}$		$B_k = \begin{bmatrix} 2.2I & 0.4I & 0.2 \\ 0.4I & 2.8 & 1.5 \\ 0.2 & 1.5 & 1.5 \end{bmatrix}$	
	Eigen values	**Eigen vectors**	**Eigen values**	**Eigen vectors**
K=1 and I=0	λ_1=2.856 λ_2=-0.056	e_1'=[0.139,0.99] e_2'=[-0.99,0.139]	λ_1=3.79 λ_2=0.564 λ_3=-0.052	e_1'=[-0.03,-0.83,-0.55] e_2'=[-0.28,0.53,-0.79] e_3'=[-0.96,0.132,-0.25
K=2 and I=1	λ_1=3 λ_2=2	e_1'=[0.45,0.89] e_2'=[-0.8,0.44]	λ_1=3.9 $\lambda_2 = 2.1$ λ_3=0.5	e_1'=[-0.25,-0.81,-0.53] e_2'=[0.97,-0.19,-0.17] e_3'=[0.03,-0.55,0.83]
K=3 and I=2	λ_1=4.5 λ_2=2.7	e_1'=[-.097,-0.23] e_2'=[0.23,-0.97]	λ_1=4.9 $\lambda_2 = 3.3$ λ_3=0.5	e_1'=[0.83,0.49,0.26] e_2'=[0.55,-0.66,-0.50] e_3'=[0.07,-0.56,0.82
K=4 and I=3	λ_1=6.6 λ_2=2.8	e_1'=[-099,-0.10] e_2'=[0.10,-0.99]	λ_1=7.02 $\lambda_2 = 3.41$ λ_3=0.47	e_1'=[0.94,0.31,0.12] e_2'=[0.32,-0.76,-0.56] e_3'=[0.09,-0.57,0.82]
K=5 and I=5	λ_1=11.02 λ_2=2.78	e_1'=[-0.99,-0.05] e_2'=[0.05,-0.99]	λ_1=11.49 λ_2 =3.38 λ_3=0.43	e_1'=[0.971,0.232,0.054] e_2'=[0.219,-0.775,-0.593] e_3'=[0.096,-0.588,0.830]
K=6 and I=10	λ_1=22.01 λ_2=2.79	e_1'=[-0.99,-0.021] e_2'=[0.021,-0.99]	λ_1=22.8 λ_2 =3.19 λ_3=0.29	e_1'=[0.980,0.198,0.023] e_2'=[0.166,-0.748,-0.642] e_3'=[0.109,-0.633,0.767]
K=7 and I=20	λ_1=44 λ_2=2.79	e_1'=[-0.999,-0.009] e_2'=[0.009,-0.999)]	λ_1=45.5 λ_2 =2.83 λ_3=-0.04	e_1'=[0.98,0.18,0.01] e_2'=[0.134,-0.669,-0.730] e_3'=[0.128,-0.719,0.683]
K=8 and I=50	λ_1=110 λ_2=2.79	e_1'=[-0.999,-0.004] e_2'=[0.004,-0.99]	λ_1=113.6 λ_2 =2.2 λ_3=-1.5	e_1'=[0.984,0.178,0.004] e_2'=[-0.081,0.425,0.901] e_3'=[0.158,-0.887,0.433]
K=9 and I=100	λ_1=220 λ_2=2.79	e_1'=[-0.999,-0.002] e_2'=[0.002,-0.999]	λ_1=227.13 λ_2 =1.84 λ_3=-4.67	e_1'=[0.984,0.176,0.002] e_2'=[-0.042,0.224,0.974] e_3'=[0.17,-0.96,0.23]
K=10 and I=200	λ_1=440 λ_2=2.79	e_1'=[-0.99,-0.0009] e_2'=[0.0009,-0.99]	λ_1=454.18 λ_2 =1.66 λ_3=-11.54	e_1'=[0.984,0.175,0.001] e_2'=[-0.020,0.108,0.994] e_3'=[0.173,-0.979,0.109]

5 Conclusions

Sometime the simple matrix theory often limited to handle the problem of uncertainty. The neutrosophic matrix deals the uncertainty, which based on three components including truth component, an indeterminate component and falsehood component. This paper focused on SSN matrices where indeterminacy involved in its variables. So the spectral decomposition analysis is performed that requires a square and symmetric matrix. The proposed method is quite effective to be applied in indeterminacy. The increasing level of indeterminacy restrains from reaching to exact decision. If indeterminacy in two SSN matrices increases, then this leads to reduce variation in data. The process is checked through the Eigenvectors, which suggests that through spectral decomposition the variation of the indeterminacy in SSN matrices can be minimized.

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Conflicts of Interest

The authors declare no conflict of interest.

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