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The definite neutrosophic integrals and its applications

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Abstract: the purpose of this article is to study the definite neutrosophic integrals, where the neutrosophic integrals are defined, in addition, set of theories and properties related to them were discussed, also, applications of the definite neutrosophic integrals were introduced, such as area of neutrosophic curves, length of neutrosophic curve and volumes of neutrosophic revolution. Where detailed examples were given to clarify each case.

Keywords: definite neutrosophic integrals; area of neutrosophic curves; length of neutrosophic volumes of neutrosophic revolution.

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1. Introduction

As an alternative to the existing logics, Smarandache proposed the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [3-13]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number, and found root index $n \geq 2$ of a neutrosophic real and complex number [2-4], studying the concept of the Neutrosophic probability [3-5], the Neutrosophic statistics [4][6], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1-8]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [9]. Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and right-hand side represented with triangular neutrosophic numbers [10]. Chakraborty used pentagonal neutrosophic number in networking problem, and Shortest Path Problem [11-12]. Y.Alhasan studied the concepts of neutrosophic complex numbers, the general exponential form of a neutrosophic complex, the neutrosophic integrals and integration methods, and the neutrosophic integrals by parts [7-14-18-20]. On the other hand, M.Abdel-Basset presented study in the science of neutrosophic about an approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number [15].

Also, neutrosophic sets played an important role in applied science such as health care, industry, and optimization [16-17].. Smarandache, F, and Khalid, H are studied the neutrosophic precalculus and neutrosophic calculus (second enlarged edition)[19].

Integration is important in human life, and one of its most important applications is the calculation of area, size and arc length. In our reality we find things that cannot be precisely defined, and that contain an indeterminacy part.

Paper consists of 5 sections. In 1th section, provides an introduction, in which neutrosophic science review has given. In 2th section, some definitions and theories of the neutrosophic integrals and are discussed. The 3th section frames the definite neutrosophic integrals, in which set of theories and properties related to them were discussed. In 4th section, applications of the definite neutrosophic integrals were introduced. In 5th section, a conclusion to the paper is given.

2. Preliminaries

2.1. Neutrosophic integration by substitution method [18]

Definition2.1.1

$Let\ \ f: D_f \subseteq R \to R_f \cup \{I\}$, to evaluate $\int f(x)dx$

Put: $x = g(u) \quad \Rightarrow \quad dx = \acute{g}(u)du$

By substitution, we get:

$$\int f(x)dx = \int f\big(g(u)\big)\acute{g}(u)du$$

then we can directly integral it.

Theorme2.1.1:

If $\int f(x, I)dx = \varphi(x, I)\ \ then,$

$$\int f\big((a + bI)x + c + dI)\big)\,dx = \left(\frac{1}{a} - \frac{b}{a(a + b)} I\right) \varphi\big((a + bI)x + c + dI)\big) + C$$

where C is an indeterminate real constant, $a \neq 0\,, a \neq -b\ and\ b, c, d$ are real numbers, while $I\ =$ indeterminacy.

Theorme2.1.2:

Let $\ \ f: D_f \subseteq R \to R_f \cup \{I\}$ then:

$$\int \frac{\acute{f}(x, I)}{f(x, I)} dx = ln|f(x, I)| + C$$

where C is an indeterminate real constant (i.e. constant of the form $a + bI$, where a, b are real numbers, while $I\ =$ indeterminacy).

Theorme2.1.3:

Let $\ \ f: D_f \subseteq R \to R_f \cup \{I\}$, then:

$$\int \frac{\acute{f}(x, I)}{\sqrt{f(x, I)}} dx = 2\sqrt{f(x, I)} + C$$

where C is an indeterminate real constant (i.e. constant of the form $a + bI$, where a, b are real numbers, while $I\ =$ indeterminacy).

Theorme2.1.4:

$f: D_f \subseteq R \to R_f \cup \{I\}$, then:

$$\int [f(x,I)]^n \acute{f}(x)\,dx = \frac{[f(x,I)]^{n+1}}{n+1} + C$$

Where n is any rational number. C is an indeterminate real constant (i.e. constant of the form $a + bI$, where a, b are real numbers, while I = indeterminacy).

2.2. Integrating products of neutrosophic trigonometric function [18]

I. $\int \sin^m(a+bI)x \cos^n(a+bI)x \ dx$, where m and n are positive integers.
To find this integral, we can distinguish the following two cases:

- Case $n\ is$ odd:
  - Split of $\cos(a+bI)x$
  - Apply $\cos^2(a+bI)x = 1 - \sin^2(a+bI)x$
  - We substitution $u = \sin(a+bI)x$
- Case $m\ is$ odd:
  - Split of $\sin(a+bI)x$
  - Apply $\sin^2(a+bI)x = 1 - \cos^2(a+bI)x$
  - We substitution $u = \cos(a+bI)x$

II. $\int \tan^m(a+bI)x \sec^n(a+bI)x \ dx$, where m and n are positive integers.
To find this integral, we can distinguish the following cases:

- Case $n\ is$ even:
  - Split of $\sec^2(a+bI)x$
  - Apply $\sec^2(a+bI)x = 1 + \tan^2(a+bI)x$
  - We substitution $u = \tan(a+bI)x$
- Case $m\ is$ odd:
  - Split of $\sec(a+bI)x \tan(a+bI)x$
  - Apply $\tan^2(a+bI)x = \sec^2(a+bI)x - 1$
  - We substitution $u = \mathrm{ses}(a+bI)x$
- Case m even and n odd:
  - Apply $\tan^2(a+bI)x = \sec^2(a+bI)x - 1$
  - We substitution $u = \sec(a+bI)x$ or $u = \tan(a+bI)x$, depending on the case.

III. $\int \cot^m(a+bI)x \csc^n(a+bI)x \ dx$, where m and n are positive integers.
To find this integral, we can distinguish the following cases:

- Case $n\ is$ even:
  - Split of $\csc^2(a+bI)x$
  - Apply $\csc^2(a+bI)x = 1 + \cot^2(a+bI)x$
  - We substitution $u = \cot(a+bI)x$
- Case $m\ is$ odd:
  - Split of $\csc(a+bI)x \cot(a+bI)x$
  - Apply $\cot^2(a+bI)x = \csc^2(a+bI)x - 1$
  - We substitution $u = \csc(a+bI)x$

- Case m even and n odd:
  - Apply $\cot^2(a+bI)x = \csc^2(a+bI)x - 1$
  - We substitution $u = \csc(a+bI)x$ or $u = \cot(a+bI)x$, depending on the case.

2.3. Neutrosophic trigonometric identities [18]

1) $sin(a+bI)x\cos(c+dI)\,x = \frac{1}{2}\,[\sin(a+bI+c+dI)x + \sin(a+bI-c-dI)x]$

2) $cos(a+bI)x\sin(c+dI)\,x = \frac{1}{2}\,[\sin(a+bI+c+dI)x - \sin(a+bI-c-dI)\,x]$

3) $cos(a+bI)x\cos(c+dI)\,x = \frac{1}{2}\,[\cos(a+bI+c+dI)x + \cos(a+bI-c-dI)x]$

4) $sin(a+bI)x\sin(c+dI)\,x = \frac{-1}{2}\,[\cos(a+bI+c+dI)x - \cos(a+bI-c-dI)x]$

Where $a \neq c\ (not\ zero)\ and\ b, d$ are real numbers, while I = indeterminacy.

3. The definite neutrosophic integrals

We will choose $I \in]0,1[$, because the undefined(indeterminacy) part in the case of the drawing is usually located in $]0,1[$. Look at pp.20-22 [19]

Theorem 3.1 (Fundamental theorem of neutrosophic integral calculus)
Let be $f(x,I)$ a continuous function defined in the closed interval $[a+a_0I, b+b_0I]$, and let $F(x,I)$ be the anti-derivative of $f(x,I)$, that is $\int f(x,I)dx = F(x,I)$. Then:

$$\int_{a+a_0I}^{b+b_0I} f(x,I)dx = F(\,b+b_0I) - F(a+a_0I)$$

Where a, a_0, b, b_0 are real number, I represent indeterminacy and $I \in]0,1[$.

Example3.1:

1) $$\int_{1+2I}^{3-5I} (2x+7I)dx = [x^2+7Ix]_{1+2I}^{3-5I}$$

$$= [(3-5I)^2 + 7I(3-5I)] - [(1+2I)^2 + 7I(1+2I)] = 8 - 38I$$

2) $$\int_{0}^{\pi+3I} cos(x-3I)dx = [sin(x-3I)]_0^{\pi+3I}$$

$$= [sin(\pi+3I-3I)] - [sin(-3I)] = sin(3I)$$

3) $$\int_{2I}^{3+I} 2x(x^2+5I)^2dx = \left[\frac{(x^2+5I)^3}{3}\right]_{2I}^{3+I}$$

$$= \left[\frac{((3+I)^2+5I)^3}{3}\right] - \left[\frac{81I}{3}\right] = \frac{81+279I}{3} = 27+93I$$

4) $\int_{4}^{9+7I} \frac{1}{2\sqrt{x}} dx = [\sqrt{x}]_{4}^{9+7I}$

$$= [\sqrt{9+7I}] - [2] \qquad (*)$$

Let's find $\sqrt{9+7I}$

$$\sqrt{9+7I} = \alpha + \beta I$$
$$9+7I = \alpha^2 + 2\alpha\beta I + \beta^2 I$$

$$9+7I = \alpha^2 + (2\alpha\beta + \beta^2)I$$

then:

$$\begin{cases} \alpha^2 = 9 \\ 2\alpha\beta + \beta^2 = 7 \end{cases}$$

$$\begin{cases} \alpha = \pm 3 \\ \beta^2 + 2\alpha\beta - 7 = 0 \end{cases}$$

Find β:

- When $\alpha = 3 \quad \Rightarrow \beta^2 + 6\beta - 7 = 0$

$$(\beta+7)(\beta-1) = 0 \quad \Rightarrow \beta = -7, \beta = 1$$

$$(3,-7), (3,1)$$

- When $\alpha = -3 \quad \Rightarrow \beta^2 - 6\beta - 7 = 0$

$$(\beta-7)(\beta+1) = 0 \quad \Rightarrow \beta = 7, \beta = -1$$

$$(-3,7), (-3,-1)$$

$$(\alpha, \beta) = (3,-7), (3,1), (-3,7), (-3,-1)$$

$$\sqrt{9+7I} = 3 - 7I \text{ or } 3 + I \text{ or } -3 + 7I \text{ or } -3 - I$$

By substitution in (*), we get the following cases:

$$\int_{4-3I}^{9+7I} \frac{1}{2\sqrt{x}} dx = [\sqrt{x}]_{4-3I}^{9+7I}$$

$$= [\sqrt{9+7I}] - [2] = 3 - 7I - 2 = 1 - 7I$$

$$or \ = [\sqrt{9+7I}] - [\sqrt{4-3I}] = 3 + I - 2 = 1 + I$$

$$or \ = [\sqrt{9+7I}] - [\sqrt{4-3I}] = -3 + 7I - 2 = -5 + 7I$$

$$or \ = [\sqrt{9+7I}] - [\sqrt{4-3I}] = -3 - I - 2 = -5 - I$$

Theorem 3.2 (The mean- value theorem of neutrosophic integral calculus_ part I)

We say that $f(x,I)$ has an anti- derivative on an interval, if $f(x,I)$ is continuous on that interval, then. In specific, if $a+a_0I$ is any point in the interval, then the function $f(x,I)$ defined by:

$$1)\ \frac{d}{dx}\left[\int_{a+a_0I}^{x} f(t,I)dt\right] = f(x,I)$$

$$2)\ \frac{d}{dx}\left[\int_{x}^{a+a_0I} f(t,I)dt\right] = -f(x,I)$$

Example3.2:

$$1)\ \frac{d}{dx}\left[\int_{3I}^{x} (t^2+5I)dt\right] = x^2+5I$$

$$2)\ \frac{d}{dx}\left[\int_{\pi+\frac{\pi}{2}I}^{x} \frac{\sin(t+3I)}{t}dt\right] = \frac{\sin(x+3I)}{x}$$

$$3)\ \frac{d}{dx}\left[\int_{x}^{5-3I} (2It^2+4It)dt\right] = 2Ix^2+4Ix$$

Remarks 3.1:

$$\mathbf{1})\ \frac{d}{dx}\left[\int_{a+a_0I}^{g(x,I)} f(t,I)dt\right] = f\big(g(x,I)\big)\acute{g}(x,I)$$

Proof:

$$\frac{d}{dx}\left[\int_{a+a_0I}^{g(x,I)} f(t,I)dt\right] = \frac{d}{dx}\big[F\big(g(x,I)\big)\big]$$

$$= \acute{F}\big(g(x,I)\big)\acute{g}(x,I)$$

$$= f\big(g(x,I)\big)\acute{g}(x,I)$$

$$\mathbf{2})\ \frac{d}{dx}\left[\int_{g(x,I)}^{a+a_0I} f(t,I)dt\right] = -f\big(g(x,I)\big)\acute{g}(x,I)$$

Proof:

$$\frac{d}{dx}\left[\int_{g(x,I)}^{a+a_0I} f(t,I)dt\right] = \frac{d}{dx}\left[-F\big(g(x,I)\big)\right]$$

$$= -\acute{F}\big(g(x,I)\big)\acute{g}(x,I)$$

$$= -f\big(g(x,I)\big)\acute{g}(x,I)$$

3) $\frac{d}{dx}\left[\int_{g_1(x,I)}^{g_2(x,I)} f(t,I)dt\right] = f(g_2(x,I))\,\acute{g}_2(x,I) - f(g_1(x,I))\,\acute{g}_1(x,I)$

Proof:

$$\frac{d}{dx}\left[\int_{g_1(x,I)}^{g_2(x,I)} f(t,I)dt\right] = \frac{d}{dx}\left[\int_{g_1(x,I)}^{0+0I} f(t,I)dt + \int_{0+0I}^{g_2(x,I)} f(t,I)dt\right]$$

$$= f(g_2(x,I))\,\acute{g}_2(x,I) - f(g_1(x,I))\,\acute{g}_1(x,I)$$

Example3.3:

1) $\frac{d}{dx}\left[\int_{1+I}^{sin(x+2I)} (3I + t^2)dt\right] = (3I + sin^2(x + 2I))cos(x + 2I)$

2) $\frac{d}{dx}\left[\int_{4+2I}^{\sqrt{3x+7I}} (t - 2I)dt\right] = (\sqrt{3x + 7I} - 2I)\frac{3}{2\sqrt{3x + 7I}}$

3) $\frac{d}{dx}\left[\int_{tan(2x+4I)}^{7-6I} \frac{t^2}{1 + t^2}dt\right] = -\frac{tan^2(2x + 4I)}{1 + tan^2(2x + 4I)}tan^2(2x + 4I) = -tan^2(2x + 4I)$

4) $\frac{d}{dx}\left[\int_{3x+I}^{x^2+2I} \frac{4 - 5I}{t + 2I}dt\right] = \frac{4 - 5I}{x^2 + 2I + 2I}(2x) - \frac{4 - 5I}{3x + I + 2I}(3)$

$$= \frac{(8 - 10I)x}{x^2 + 4I} - \frac{12 - 15I}{3x + 3I}$$

Theorem 3.3 (The mean- value theorem of neutrosophic integral calculus_ part II)

If $f(x,I)$ is continuous on a closed interval $[a + a_0I, b + b_0I]$, then there is at least one point $x^* = x_0 + x_1I$ in $[a + a_0I, b + b_0I]$ such that:

$$\int_{a+a_0I}^{b+b_0I} f(x,I)dx = f(x^*,I)\big(\, b + b_0I - (a + a_0I)\big)$$

Where x_0, x_1 are real numbers, I represent indeterminacy and $I \in]0,1[$

Example3.4:

Findx^* that satisfy The Mean-Value Theorem of Integral Calculus for $f(x,I) = 2x + 3I$ on the interval $[1 + 2I, 3 + 4I]$.

Solution:

$$\int_{a+a_0I}^{b+b_0I} f(x,I)dx = f(x^*,I)\big(\, b + b_0I - (a + a_0I)\big)$$

$$\int_{1+2I}^{3+4I} (2x + 3I)dx = f(x^*,I)(\,2 + 2I)$$

$$[x^2 + 3Ix]_{1+2I}^{3+4I} = (2x^* + 3I)(\,2 + 2I)$$

$$8 + 44I = (2x^* + 3I)(\,2 + 2I)$$

$$2x^* + 3I = \frac{8 + 44I}{2 + 2I}$$

$$2x^* + 3I = \frac{4 + 22I}{1 + I}$$

$$2x^* + 3I = 4 + 9I$$

$$2x^* = 4 + 6I$$

$$x^* = 2 + 3I \quad \in [1 + 2I, 3 + 4I]$$

Example3.4:

Find x^* that satisfy The Mean-Value Theorem of Integral Calculus for $f(x,I) = \sqrt{x}$ on the interval $[0 + 0I, 3 + 2I]$.

Solution:

$$\int_{a+a_0I}^{b+b_0I} f(x,I)dx = f(x^*,I)\big(b + b_0I - (a + a_0I)\big)$$

$$\int_{0}^{3+2I} \sqrt{x}\, dx = (3 + 2I)\sqrt{x^*}$$

$$\left[\frac{2}{3}x\sqrt{x}\right]_0^{3+2I} = (3+2I)\sqrt{x^*}$$

$$\frac{2}{3}(3+2I)\sqrt{3+2I} = (3+2I)\sqrt{x^*}$$

$$\sqrt{x^*} = \frac{2\sqrt{3+2I}}{3}$$

By squared, we get:

$$x^* = \frac{4(3+2I)}{9} = \frac{12+8I}{9}$$

$$x^* = \frac{4}{3} + \frac{8}{9}I \quad \in [0+0I, 3+2I]$$

If we take several values of I in the $]0,1[$, we find:

I	$[0+0I, 3+2I]$	x^*	$x^* \in [0+0I, 3+2I]$
0.1	$[0, 3.2]$	1.43	Satisfied
0.3	$[0, 3.6]$	1.61	Satisfied
0.5	$[0, 4]$	1.78	Satisfied

3.1 Properties of definite neutrosophic integrals.

Let $f: D_f \subseteq R \to R_f \cup \{I\}$, and $g: D_g \subseteq R \to R_f \cup \{I\}$ then:

1) $$\int_{a+a_0I}^{b+b_0I} f(x,I)dx = \int_{a+a_0I}^{b+b_0I} f(t,I)dt$$

2) $$\int_{a+a_0I}^{b+b_0I} f(x,I)dx = \int_{a+a_0I}^{c+c_0I} f(x,I)dx + \int_{c+c_0I}^{b+b_0I} f(x,I)dx \quad ; \quad a+a_0I \le c+c_0I \le b+b_0I$$

3) $$\int_{a+a_0I}^{a+a_0I} f(x,I)dx = 0$$

4) $$\int_{a+a_0I}^{b+b_0I} f(x,I)dx = -\int_{b+b_0I}^{a+a_0I} f(x,I)dx$$

5) $$\int_{a+a_0I}^{b+b_0I} (c+c_0I)f(x,I)dx = (c+c_0I)\int_{a+a_0I}^{b+b_0I} f(x,I)dx$$

6) $$\int_{a+a_0I}^{b+b_0I} [f(x,I) \pm g(x.I)]dx = \int_{a+a_0I}^{b+b_0I} f(x,I)dx \pm \int_{a+a_0I}^{b+b_0I} g(x,I)dx$$

7) $$\int_{-(a+a_0I)}^{a+a_0I} f(x,I)dx = \begin{cases} 2\int_{0}^{a+a_0I} f(x,I)dx & ; if\ f(x,I)\ is\ even\ function \\ 0 & ; if\ f(x,I)\ is\ odd\ function \end{cases}$$

Example3.1.1:

1) $$\int_{5+6I}^{5+6I} (x^4 + 2Ix - 4I)dx = 0$$

2) $$\int_{5+6I}^{9+2I} 5I \sin^5 x\, cos^4 x\, dx$$

Let $f(x,I) = 5I \sin^5 x\, cos^4 x$, then:

$f(-x,I) = 5I \sin^5(-x) cos^4(-x)$

$$= 5I(\sin(-\text{x}))^5(\cos(-x))^4 = -5I \sin^5 x\, cos^4 x$$

$$= -f(x,I)$$

Thus $f(x,I)$ is an odd function and so by property 7, we get:

$$\int_{5+6I}^{9+2I} 5I \sin^5 x\, cos^4 x\, dx = 0$$

4. Applications of the definite neutrosophic integrals

4.1 The area under neutrosophic curves

Theorem 4.1.1
Let $f(x,I)$ be a continuous function defined in the interval $[a + a_0I, b + b_0I]$. Then the area of the region below the neutrosophic curve of $f(x,I)$, above the $x-\ axis$, between $x = a + a_0I\ and$ $x = b + b_0I$ $(b > a)$, is given by formula:

$$A = \int_{a+a_0I}^{b+b_0I} |f(x,I)|\, dx$$

Where a, a_0, b, b_0 are real numbers, I represent indeterminacy and $I \in]0,1[$

Theorem 4.1.2

Let $f(y,I)$ be a continuous function defined in the interval $[c+c_0I, d+d_0I]$. Then the area of the region below the neutrosophic curve of $f(y,I)$, above the $y-$ $axis$, between $y=c+c_0I$ and $y=\ d+d_0I$ $(d>c)$, is given by formula:

$$A=\int_{c+c_0I}^{d+d_0I}|f(y,I)|\,dy$$

Where c, c_0, d, d_0 are real numbers, I represent indeterminacy and $I\in]0,1[$

Example4.1.1:

Find the area of the region bounded by the line $f(x,I)=x+4-3I$, the $x-axis$ and the lines $x=2+3I$ and $x=4+I$.

Solution:

$$A=\int_{a+a_0I}^{b+b_0I}|f(x,I)|\,dx=\int_{2+3I}^{4+I}|x+4-3I|\,dx$$

$$x+4-3I>0\ \ on\ [2+3I, 4+I]\ \text{ for } I\in]0,1[$$

$$\Longrightarrow\qquad A\ =\int_{2+3I}^{4+I}(x+4-3I)dx=\left[\frac{x^2}{2}+(4-3I)x\right]_{2+3I}^{4+I}=8-4I$$

Clearly that: $8-4I>0$ for $I\in]0,1[$

4.2 Area between two neutrosophic curves

Theorem 4.2.1 (Area between two neutrosophic curves (attributed to $x-axis$))

The area A of the region bounded by the curves $f(x,I), g(x,I)$, and the lines $x=a+a_0I$ and $x=b+b_0I$ $(b>a)$, where f and g are continuous and $f(x,I)\geq\ g(x,I)$ for all x in $[a+a_0I, b+b_0I]$, is given by formula:

$$A=\int_{a+a_0I}^{b+b_0I}[f(x,I)-g(x.I)]dx$$

Where a, a_0, b, b_0 are real numbers, I represent indeterminacy and $I\in]0,1[$

Theorem 4.2.2 (Area between two neutrosophic curves (attributed to $y-axis$))

The area A of the region bounded by the curves $f(y,I), g(y,I)$, and the lines $y=c+c_0I$ and $y=\ d+d_0I$ $(d>c)$, where f and g are continuous and $f(y,I)\geq\ g(y,I)$ for all x in $[c+c_0I,\ d+d_0I]$, is given by formula:

$$A=\int_{c+c_0I}^{d+d_0I}[f(y,I)-g(y.I)]dy$$

Where c, c_0, d, d_0 are real numbers, I represent indeterminacy and $I \in]0,1[$

Example4.2.1:

Evaluate the area of the region bounded by $y = e^{x+7I}$, $y = x - 3I$, and the lines $x = 0$, $x = 1 + I$

Solution:

$y = e^{x+7I} > y = x - 3I$ on $[0, 1 + I]$ for $I \in]0,1[$, then:

$$A = \int_{0}^{1+I} [e^{x+7I} - x + 3I]dx = \left[e^{x+7I} - \frac{x^2}{2} + 3Ix\right]_0^{1+I}$$

$$= e^{1+8I} - \frac{1}{2} + \frac{13I}{2} - e^{7I}$$

Clearly that $e^{1+8I} - \frac{1}{2} + \frac{13I}{2} - e^{7I} > 0$ for $I \in]0,1[$

Example4.2.1:

Evaluate the area of the region bounded by $x = y^2$, $x = (-2 - I)x + 2I$, and the lines
N $x = -2 + I$, $x = -2I$

Solution:

$x = y^2 \geq x = (-2 - I)x + 2I$ on $[-2 + I, -2I]$ for $I \in]0,1[$, then:

$$A = \int_{-2+\mathrm{I}}^{-2I} [y^2 + (2 + I)x - 2I]dy = \left[\frac{y^3}{3} + \frac{(2 + I)}{2} y^2 - 2Iy\right]_{-2+I}^{-2I}$$

$$= \left[\frac{(-2I)^3}{3} + \frac{(2 + I)}{2}(-2I)^2 - 2I(-2I)\right] - \left[\frac{(-2 + I)^3}{3} + \frac{(2 + I)}{2}(-2 + I)^2 - 2I(-2 + I)\right]$$

$$= \frac{-4}{3} + \frac{68}{15} I$$

Clearly that: $\frac{-4}{3} + \frac{68}{15} I > 0$ for $I \in]0,1[$

4.3 Length of neutrosophic curve

Definition 4.3.1

If $y = f(x, I)$ is a smooth curve on the interval $[a + a_0 I, b + b_0 I]$, then the arc length L of this curve over $[a + a_0 I, b + b_0 I]$ is defined as:

$$L = \int_{a+a_0 I}^{\mathrm{b}+b_0 I} \sqrt{1 + \left[\acute{f}(x, I)\right]^2}\, dx$$

Where a, a_0, b, b_0 are real numbers, I represent indeterminacy and $I \in]0,1[$

Definition 4.3.2

If $x = g(y, I)$ is a smooth curve on the interval $[c + c_0 I,\ d + d_0 I]$, then the arc length L of this curve over $[c + c_0 I,\ d + d_0 I]$is defined as:

$$L = \int_{c+c_0 I}^{d+d_0 I} \sqrt{1 + [\acute{g}(y, I)]^2}\, dy$$

Example4.3.1:

Find the arc length of the curve of $y = f(x, I) = ln(secx)$ on the interval $\left[0,\ \frac{\pi}{4} + 3I\right]$.

Solution:

$f(x, I) = ln(sec(x - 3I)) \ \Rightarrow\ \acute{f}(x, I) = tan(x - 3I)$

$$L = \int_{a+a_0 I}^{b+b_0 I} \sqrt{1 + \left[\acute{f}(x, I)\right]^2}\ dx$$

$$L = \int_{0}^{\frac{\pi}{4}+3I} \sqrt{1 + tan^2(x - 3I)}\ dx$$

$$= \int_{0}^{\frac{\pi}{4}+3I} \sqrt{sec^2(x - 3I)}\ dx$$

$$= \int_{0}^{\frac{\pi}{4}+3I} \sec(x - 3I)\ dx = [ln|sec(x - 3I) + \tan(x - 3I)|]_0^{\frac{\pi}{4}+3I}$$

$$= \left[ln \left|sec\left(\frac{\pi}{4}\right) + \tan(\frac{\pi}{4})\right|\right] - [ln|sec(-3I) + \tan(-3I)|]$$

$$= ln\left|\sqrt{2} + 1\right| - [ln|sec(-3I) + \tan(-3I)|]$$

$$= ln\left|\sqrt{2} + 1\right| - ln|sec(3I) + \tan(3I)|$$

Clearly that: $ln\left|\sqrt{2} + 1\right| - ln|sec(3I) + \tan(3I)| > 0$ for $I \in\,]0,1[$

4.4 Volumes of neutrosophic revolution

Definition 4.4.1

Suppose that $f(x, I) \geq 0$ and it is continuous on the interval $[a + a_0 I, b + b_0 I]$, the volume of the resulting solid of revolution the region under the curve $y = f(x, I)$ for the interval $[a + a_0 I, b + b_0 I]$ about the $x - axis$ is given by:

$$V = \int_{a+a_0 I}^{b+b_0 I} \pi[f(x, I)]^2\, dx$$

Where a, a_0, b, b_0 are real numbers, I represent indeterminacy and $I \in]0,1[$

Definition 4.4.2

Suppose that $g(y, I) \geq 0$ and it is continuous on the interval $[c + c_0 I,\ d + d_0 I]$, the volume of the resulting solid of revolution the region under the curve $x = g(y, I)$ for the interval $[c + c_0 I,\ d + d_0 I]$ about the $y - axis$ is given by:

$$V = \int_{c+c_0 I}^{d+d_0 I} \pi[g(y, I)]^2 \, dy$$

Example4.4.1:

Find the volume of the solid resulting from rotating the region bounded by the curves $y = f(x, I) = \sqrt{x + 2 + 3I}$ from $x = 0\ to\ x = 4 + 5I$ about the $x - axis$.

Solution:

$$V = \int_{a+a_0 I}^{\mathrm{b}+b_0 I} \pi[f(x, I)]^2 \, dx = \int_{0}^{4+5I} \pi\left[\sqrt{x + 2 + 3I}\,\right]^2 dx$$

$$= \int_{0}^{4+5I} \pi[x + 2 + 3I\,] \, dx = \pi\left[\frac{x^2}{2} + (2 + 3I)x\right]_0^{4+5I}$$

$$= \pi\left[\frac{(4 + 5I)^2}{2} + (2 + 3I)(4 + 5I)\ \right] - [0\,]$$

$$= \left(16 + \frac{139}{2} I\right)\pi$$

Example4.4.2:

Find the volume of the solid resulting from rotating the region bounded by the curves $x = g(y, I) = \sqrt{4 + 6I - y}$ from $y = 1 + \mathrm{I}\ to\ y = 4 + 4I$ about the $y - axis$.

Solution:

$$V = \int_{c+c_0 I}^{d+d_0 I} \pi[g(y, I)]^2 \, dy = \int_{1+I}^{4+4I} \pi\left[\sqrt{4 + 6I - y}\right]^2 dy$$

$$= \int_{1+I}^{4+4I} \pi[4 + 6I - y] dy = \pi\left[(4 + 6I)y - \frac{y^2}{2}\right]_{1+I}^{4+4I}$$

$$= \pi\left[(4 + 6I)(4 + 4I) - \frac{(4 + 4I)^2}{2}\right] - [0\,]$$

$$= \left(\frac{9}{2} + \frac{51}{2} I\right)\pi$$

Definition 4.4.3

Suppose that $f(x,I), g(x,I)$ are continuous and non-negative on the interval $[a + a_0 I, b + b_0 I]$, and $f(x,I) \geq g(x,I)$ for all x in the interval $[a + a_0 I, b + b_0 I]$, the volume of the resulting solid of revolution the region bounded between tow the curves $f(x,I), g(x,I)$ for the interval $[a + a_0 I, b + b_0 I]$ about the $x - axis$ is given by:

$$V = \int_{a+a_0 I}^{\mathrm{b}+b_0 I} \pi([f(x,I)]^2 - [g(x,I)]^2)\, dx$$

Definition 4.4.4

Suppose that $w(y,I), v(y,I)$ are continuous and non-negative on the interval $[c + c_0 I,\ d + d_0 I]$, and $w(y,I) \geq v(y,I)$ for all x in the interval $[c + c_0 I,\ d + d_0 I]$, the volume of the resulting solid of revolution the region bounded between tow the curves $w(y,I), v(y,I)$ for the interval $[c + c_0 I,\ d + d_0 I]$ about the $y - axis$ is given by:

$$V = \int_{c+c_0 I}^{d+d_0 I} \pi([w(y,I)]^2 - [v(y,I)]^2)\, dy$$

Example4.4.3:

Find the volume of the solid resulting from rotating the region bounded between tow the curves $f(x,I) = x^2 + 3I$ *and* $g(x,I) = 3I + x$ from $x = 1 + I$ *to* $x = 4 + 2I$ about the $x - axis$.

Solution:

$f(x,I) = x^2 + 3I > g(x,I) = 3I + x$ *on* $[1 + I, 4 + 2I]$ for $I \in]0,1[$, then:

$$V = \int_{a+a_0 I}^{\mathrm{b}+b_0 I} \pi([f(x,I)]^2 - [g(x,I)]^2)\, dx$$

$$= \int_{1+I}^{4+2I} \pi([x^2 + 3I]^2 - [3I + x]^2)\, dx$$

$$= \int_{1+I}^{4+2I} \pi([x^4 + 6Ix^2 + 9I] - [9I + 6Ix + x^2])\, dx$$

$$= \int_{1+I}^{4+2I} \pi(x^4 + (6I - 1)x^2 - 6Ix)\, dx$$

$$=\left[\pi\left(\frac{x^5}{5}+(6I-1)\frac{x^3}{3}-3Ix^2\right)\right]_{1+I}^{4+2I}$$

$$=\pi\left[\left(\frac{(4+2I)^5}{5}+(6I-1)\frac{(4+2I)^3}{3}-3I(4+2I)^2\right)-\left(\frac{(1+I)^5}{5}+(6I-1)\frac{(1+I)^3}{3}-3I(1+I)^2\right)\right]$$

$$=\pi\left[\left(\frac{2752}{15}+\frac{3424}{3}I\right)-\left(\frac{-2}{15}-\frac{32}{15}I\right)\right]$$

$$=\pi\left(\frac{2754}{15}+\frac{3456}{3}I\right)$$

5. Conclusions

This paper is an extension of the papers I presented in the field of neutrosophic integrals. Integrals are important in our life, as they facilitate many mathematical operations in our reality, and this is what led us to study the definite neutrosophic integrals, and its applications, the most important of which are area of neutrosophic curves, length of neutrosophic curve and volumes of neutrosophic revolution. In addition, this paper is considered important in continuing the study of neutrosophic integrals.

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