ELECTRE Approach for Multi-attribute Decision-making in Refined Neutrosophic Environment

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Abstract: Uncertainty, imprecise, incomplete, and inconsistent information can be found in many real-life systems and may enter some problems in a much more complex way. Neutrosophic set is the effective and useful tool to describe problems with Uncertainty, imprecise, incomplete, and inconsistent information. In this regard, the present study is trying to present a neutrosophic electrode model through an example to demonstrate the efficiency of the proposed model. In this example, 3 alternatives were evaluated on 5 criteria by 4 experts based on the neutrosophic linguising variables. After converting the neutrosophic linguising variables to neutrosophic numbers, it is paid to calculate the integrated matrix and after that, weights of criteria and experts. In the next steps, the concordance and disconcordance matrices are calculated and after that the calculations are done based on the description of section 3. Finally, are ranked the alternatives in this numerical example. The results show that A3, A2 and A1 were ranked first to third respectively.

Keywords: ELECTRE; Multi-attribute Decision Making; Refined Neutrosophic Environment

1. Introduction

In fact, we have partial, approximate or inaccurate information about the phenomena around ourselves. Uncertainty may occur due to addressing to this inaccurate or partial information. Moreover, Xu and Yager (2006) pointed out that lack of awareness about exact result of a particular choice due to lack of time, lack of accessible information, and insufficient attention of decision makers to the information caused uncertainty. It seems a framework is required to overcome this uncertainty [1]. Liu and lin (2006) classified different uncertainty frameworks into following categories: probability, gray system theory, and fuzzy set theory. Fuzzy set theory is one of the widely accepted frameworks for uncertainty [2]. The general form of this theory is considered as the degree of membership for each set of elements from the reference set, so that there is a large distinction between membership and non-membership of the elements. In fact, determining membership degree for elements is difficult and is accompanied with a degree of hesitation. Considering hesitation, Atanassov (1986) introduced the concept of the intuitive fuzzy set as generalization of fuzzy set [3]. The inventive fuzzy set (IFS) will be defined with three continuous members: the degree of membership, the degree of non-membership, and the degree of hesitation [4], which is the most ideal measure of fuzzy set to describe the information of an uncertain and inaccurate decision [3].
Comparing to fuzzy sets, IFS is more efficient in terms of ambiguity and uncertainty. IFS is confusing and unreliable as the intuitive fuzzy set takes into account membership and non-membership degree as well as hesitation degree which seems to be one of the elements of real-world data. On the other hand, it is difficult to identify “exact values” for membership and non-membership degrees of an element due to the complexity and diversity of real-life management conditions. Therefore, presentation of membership and non-membership degrees as distance may provide appropriate measure for uncertainty, inaccuracy or ambiguity. Atanassov and Gargov (1989) introduced the concept of Interval Valued Intuitionistic Fuzzy Sets (IVIFS) with the degree of membership and the degree of non-membership, whose values are relative to real numbers as interval [5]. IVIFS is the development of a normal distance fuzzy set using the concept of the inventive fuzzy set. Intuitional fuzzy set is a new and effective tool for dealing with a variety of obscure and inaccurate variables for solving decision problems that deals with more vague and uncertain data relative to the intuitive fuzzy set [6].

Although fuzzy sets developed and prevailed, in reality, they could not handle problems with a variety of uncertainty conditions; particularly problems with indeterminate and inconsistent information are not solvable by fuzzy sets. In decision-making problems, fuzzy sets could not handle all types of uncertainty, including indeterminate and inconsistent information, in the real world [7]. In many situations, decision makers have incomplete, indeterminate, and inconsistent options relative to criteria. It has been determined that intuitive fuzzy and fuzzy decision-making analyses are inadequate to handle incomplete, indeterminate, and inconsistent information [8]. Recently Smarandache (1999) has proposed the concepts of non-rooted logic and the neutrosophic set to control these conditions [9]. The set is most appropriate tool for dealing with decision-making problems with incomplete, indeterminate, and inconsistent information while the intuitionistic fuzzy set cannot represent and handle indeterminacy and inconsistent information [10]. The neutrosophic set is a powerful framework that incorporates all the concepts of a definitive set, Fuzzy sets and Fuzzy Intuitionistic sets. The neutrosophic set is identified by three independent degrees, called the degree of accuracy, lack of reliability, and the degree of inaccuracy. These three elements are completely independent. One of the important features of this set is that each of the elements of this set not only has a certain degree of membership, but also have a definite degree of inaccuracy and lack of reliability [11]. It is important to note that, unlike IFS and IVIFS, the uncertainty gap in a neutrosophic set is clearly defined. The neutrosophic set has applications in various fields, including image processing ([12-13]), medical artificial ([14-15]), cluster analyses [16] and supplier selection [17]. Other collections have arisen since the neutrosophic collection is not easy to use in the empirical and practical problems. Wang et al. (2010) introduced a single-value neutrosophic set (SVNS) which is a specific example of a non-stereoscopic set used to handle real-life science and engineering problems [7]. The increasing growth of the neutrosophic collection as well as the pervasiveness of decision-making has led neutrosophic set to be used extensively in decision-making problems. Some uses of this collection in the decision-making process are mentioned in the following.


Also In recent years, several studies have been carried out on multi-criteria decision-making techniques in the neutrosophic environment, including:

Sodenkamp et al., (2018) in a research developed a novel method that uses single-valued neutrosophic sets (NSs) to handle independent multi-source uncertainty measures affecting the reliability of experts’ assessments in group multi-criteria decision-making (GMCDM) problems. In the proposed approach, the neutrosophic indicators are defined to explicitly reflect DMs’ credibility.
(voting power), inconsistencies/errors inherent to the assessing process, and DMs’ confidence in their own evaluation abilities [20]. Liu et al., (2019) in their extended the SS TN and TCN to single-valued numbers (SVNN) and proposed the SS operational laws for SVNNs. Then, they merged the prioritized aggregation (PRA) operator with SS operations, and developed the single valued neutrosophic Schweizer Sklar prioritized weighted averaging (SVNSSPRWA) operator, single valued neutrosophic Schweizer-Sklar prioritized ordered weighted geometric (SVNSSPRWG) operator, and single-valued neutrosophic Schweizer-Sklar prioritized ordered weighted geometric (SVNSSPROWG) operator. Moreover, they study some useful characteristics of these proposed aggregation operators (AOs) and proposed two decision making models to deal with multiple-attribute decision making (MADM) problems under SVN information based on the SVNSSPRWA and SVNSSPRWG operators [21]. Liu & you (2019) in their study defined a new distance measure between two linguistic neutrosophic sets (LNSs), and build a model based on the maximum deviation to obtain fuzzy measure, further, they developed the bidirectional projection-based MCGDM method with LNNs in which a weight model based on fuzzy measure is proposed where the weights of evaluation criteria is partial unknown and the interactions among criteria are considered[22]. Thong et al., (2019) in their study proposed a new concept called the Dynamic Interval-valued Neutrosophic Set (DIVNS) for such the dynamic decision-making applications [23]. In the same vein, Abdul Basset et al., have done many studies in the neutrosophic environment such as: supplier selection with group TOPSIS technique under type-2 neutrosophic number[24], project selection with a hybrid neutrosophic multiple criteria group decision making[25], evaluation Hospital medical care systems based on plithogenic sets[26], selecting supply chain with a hybrid plithogenic decision-making approach[27], solve transition difficulties with Utilizing neutrosophic theory[28], Evaluation of the green supply chain management practices[29].

ELECTRE method was introduced by Benayoun, Roy and Sussmann in 1966[30], and has been successfully and widely used in many decision-making problems including agricultural [31], medical science [32], financial [33], economics [34], project selection [35], communication and transportation ([35-36]). The origin of ELECTRE method dates back to 1965, when an European consulting firm employed a team of researchers to make a decision on real multi-criteria problems on innovation in new activities of institutions [37]. ELECTRE method uses the concept of outranking comparisons. This idea relates to the concepts of coordination, inconsistency, and non-rank, deriving from real world applications [38]. The method uses the consistency and inconsistency indices for analyzing non-ranked comparisons between the options [39]. ELECTRE method was developed and different types of this method which are proposed to overcome in decision making conditions are among these methods ELECTRE I, ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE TRI-C and ELECTRE IS ([37],[39],[40-41]) .

Given the extension of this method, it is worth noting that the ELECTRE method as an efficient and useful method in management research has not yet been developed in the context of the neutrosophic ambiguity. For this purpose, the present paper seeks to develop a neutrosophic ELECTRE method based on intuitive fuzzy ELECTRE method.

2. Refined Neutrosophic Environment

Neutrosophy has been proposed by Smarandache [42-43] as a new branch of philosophy, with ancient roots, dealing with “the origin, nature and scope of neutralities, as well as their interactions
with different ideational spectra”. The fundamental thesis of neutrosophy is that every idea has not only a certain degree of truth, as is generally assumed in many-valued logic contexts, but also a falsity degree and an indeterminacy degree that have to be considered independently from each other. Smarandache seems to understand such “indeterminacy” both in a subjective and an objective sense, i.e. as uncertainty as well as imprecision, vagueness, error, doubtfulness, etc [44]. In this section, some basic concepts and definitions of NSs and SNSs are briefly reviewed.

2.1. NS and SNSs

In this subsection, the definitions and operations of NSs and SNSs are introduced.

Definition 1. Let \( X \) is a space of points (objects), with a generic element in \( X \) denoted by \( x \). A neutrosophic set \( A \) in \( X \) is characterized by a truth-membership function \( T_A(x) \), an indeterminacy-membership function \( I_A(x) \) and a falsity-membership function \( F_A(x) \). The functions \( T_A(x) \), \( I_A(x) \) and \( F_A(x) \) are real standard or nonstandard subsets of \([-1,1]\) [9, 45]. In other words, \( T_A(x): X \rightarrow [-1,1] \), \( I_A(x): X \rightarrow [-1,1] \), and \( F_A(x): X \rightarrow [-1,1] \). We have no restriction on the sum of \( T_A(x) \), \( I_A(x) \) and \( F_A(x) \); thus, \( -1 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3 \) [46].

In other form, the neutrosophic set \( A \) is an object having the following form \( A = \{T_A(X), I_A(X), F_A(X), x \in X\} \).

The set \( I_A(X) \) may represent not only indeterminacy, but also vagueness, uncertainty, imprecision, error, contradiction, undefined, unknown, incompleteness, redundancy, etc.[44],[47]. In order to catch up vague information, an indeterminacy-membership degree can be split into subcomponents, such as “contradiction,” “uncertainty”, and “unknown” [48].

Definition 2. A neutrosophic set \( A \) is contained in the other neutrosophic set \( B \), denoted by \( A \subseteq B \) if and only if \( \inf T_A(x) \leq \inf T_B(x) \), \( \sup T_A(x) \leq \sup T_B(x) \), \( \inf I_A(x) \geq \inf I_B(x) \), \( \sup I_A(x) \geq \sup I_B(x) \), \( \inf F_A(x) \geq \inf F_B(x) \), and \( \sup F_A(x) \geq \sup F_B(x) \) for every \( x \) in \( X \) [9].

Definition 3. The complement of a neutrosophic set \( A \) is denoted by \( A^c \) and is defined as \( T_A^c(x) = 1 - T_A(x) \), \( I_A^c(x) = 1 - I_A(x) \), and \( F_A^c(x) = 1 - F_A(x) \) for every \( x \) in \( X \) [9].

Since it is hard to use NSs to solve practical problems, so Wang et al introduced Single-valued neutrosophic sets that can be used in real scientific and engineering applications.

2.2. Single-valued neutrosophic sets

Single-valued neutrosophic set is a special case of neutrosophic set. In this section, some basic definitions, operations, and properties regarding single valued neutrosophic sets are introduced.

Definition 4. Let \( X \) be a space of points (objects) with generic elements in \( X \) denoted by \( x \). An \( SVNS \) \( A \) in \( X \) is characterized by the truth-membership function \( T_A(x) \), indeterminacy-membership function \( I_A(x) \), and falsity-membership function \( F_A(x) \). For each point \( x \) in \( X \), \( T_A(x), I_A(x), F_A(x) \in [0,1] \) [7].

Therefore, an \( SVNS \) \( A \) can be written as:

\[
A = \{x, T_A(x), I_A(x), F_A(x)\} \big| x \in X \}
\]

The following expressions are defined in [7] for \( SVNSs \) \( A, B \):

1. \( A \subseteq B \) if and only if \( T_A(x) \leq T_B(x) \), \( I_A(x) \geq I_B(x) \), \( F_A(x) \geq F_B(x) \) for any \( x \) in \( X \),
2. $A = B$ if and only if $A \subseteq B, B \subseteq A$.
3. $A^c = \{ (x, F_A(x), I_A(x), T_A(x)) | x \in X \}$.

For convenience, an SVNS $A$ is denoted by the simplified symbol $A = \{ T_A(x), I_A(x), F_A(x) \}$ for any $x$ in $X$. For two SVNSs $A$ and $B$, the operational relations are defined by [7].

1. $A \cup B = \{ \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \}$ for any $x$ in $X$,
2. $A \cap B = \{ \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \}$ for any $x$ in $X$,
3. $A \oplus B = \{ T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x) \}$ for any $x$ in $X$,
4. $A \otimes B = \{ T_A(x)T_B(x), I_A(x) + I_B(x) - I_A(x)I_B(x), F_A(x) + F_B(x) - F_A(x)F_B(x) \}$ for any $x$ in $X$.

5. $\lambda A = \{ (1 - (1 - T_A(x))^i, (I_A(x))^i, (F_A(x))^i) \}$, $\lambda > 0$ for any $x$ in $X$ [35],
6. $\lambda^2 = \{ (T_A(x))^i, 1 - (1 - I_A(x))^i, 1 - (1 - F_A(x))^i \}, \lambda > 0$ for any $x$ in $X$ [35],
7. $\Delta A = \{ \min(T_A(x) + I_A(x)), 0, F_A(x) \}$ for any $x$ in $X$,
8. $\nabla A = \{ T_A(x), 0, \min(F_A(x) + I_A(x), 1) \}$ for any $x$ in $X$.

2.3. Neutrosophic refined set

Let $A$ be a neutrosophic refined set.

$A = \{ (x, T_A^i(x), I_A^i(x), F_A^i(x)) | x \in X \}$

where $T_A^i(x)$: $X \subseteq [0,1]$, $I_A^i(x)$: $X \subseteq [0,1]$, $F_A^i(x)$: $X \subseteq [0,1]$ such that

$0 \leq \sup T_A^i(x) + \sup I_A^i(x) + \sup F_A^i(x) \leq 3$, $j = 1, 2, \ldots, m$ for any $x \in X$. Now, $(T_A^i(x), I_A^i(x), F_A^i(x))$ are the truth-membership sequence, indeterminacy-membership sequence, and falsity-membership sequence of the element $x$, respectively. Also, $m$ is called the dimension of neutrosophic refined sets $A$ [50].

2.4. Distance between two SVNSs

Majumdar and Samanta [51] studied similarity and entropy measure by incorporating Euclidean distances of neutrosophic sets.

2.4.1. Euclidean distance between two SVNSs

Let $A = \{ (x, T_A^i(x), I_A^i(x), F_A^i(x)) | i = 1, 2, \ldots, n \}$ and

$B = \{ (x, T_B^i(x), I_B^i(x), F_B^i(x)) | i = 1, 2, \ldots, n \}$ be SVNSs. Then the Euclidean distance between two SVNSs $A$ and $B$ can be defined as follows [48]:

$$E(A, B) = \sqrt{\sum_{i=1}^{n} \left( (T_A^i(x) - T_B^i(x))^2 + (I_A^i(x) - I_B^i(x))^2 + (F_A^i(x) - F_B^i(x))^2 \right)}$$

(1)

The normalized Euclidean distance between two SVNSs $A$ and $B$ can be defined as follows:

$$E_N(A, B) = \sqrt{\frac{1}{3n} \sum_{i=1}^{n} \left( (T_A^i(x) - T_B^i(x))^2 + (I_A^i(x) - I_B^i(x))^2 + (F_A^i(x) - F_B^i(x))^2 \right)}$$

(2)
2.4.2. The Hamming distance between two SVNSs

the Hamming distance between two SVNSs $A$ and $B$ can be defined as follows\[51]:

$$L_{Ham}(A, B) = \sum_{i=1}^{n} |T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|$$

(3)

The normalized Hamming distance between two SVNSs $A$ and $B$ can be defined as follows:

$$L_{Ham(N)}(A, B) = \frac{1}{3n} \sum_{i=1}^{n} |T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|$$

(4)

2.5. Crispification of a neutrosophic set

Let $A = \left\{ x_i : T_A(x_i), I_A(x_i), F_A(x_i) \right\}$, $j = 1, 2, \ldots, n$ be $n$ SVNSs. The equivalent crisp number of each $W_j$ can be defined as [11]:

$$W_j = \frac{1 - \left( (1-T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2 \right)^{\frac{1}{3}}}{\sum_{i=1}^{n} \left( (1-T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2 \right)^{\frac{1}{3}}}$$

(5)

$$W_j \geq 0, \sum_{k=1}^{p} W_j = 1$$

3. ELECTRE approach

The ELECTRE approach is employed to identify the best alternative. The ELECTRE approach can be presented as follows (including 9 steps):

**Step 1.** Determining the decision matrix: Assume that $A = \{A_1, A_2, \ldots, A_m\}$ is the set of alternatives with the set $C$ of $n$ criteria, $C = \{C_1, C_2, \ldots, C_n\}$, $D = (d_{ij})_{m \times n}$ is the decision matrix, and $W = \{W_1, W_2, \ldots, W_n\}$ is the weight vector of criteria that the sum of weight of all criteria is equal to 1.

<table>
<thead>
<tr>
<th>Criteria alternatives</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$\ldots$</th>
<th>$C_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$d_{11}$</td>
<td>$d_{12}$</td>
<td>$\ldots$</td>
<td>$d_{1n}$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$d_{21}$</td>
<td>$d_{22}$</td>
<td>$\ldots$</td>
<td>$d_{2n}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\ddots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$A_m$</td>
<td>$d_{m1}$</td>
<td>$d_{m2}$</td>
<td>$\ldots$</td>
<td>$d_{mn}$</td>
</tr>
<tr>
<td>$W_j$</td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$\ldots$</td>
<td>$w_n$</td>
</tr>
</tbody>
</table>

Table 1. Single-valued neutrosophic set decision matrix

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Here, \( d_{ij} (i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n) \) are all single-valued neutrosophic numbers.

Here, \( \lambda \) is the vector of experts' weight, based on which the opinion of experts is aggregated.

**Step 2.** Aggregate the decision makers (DMs') opinion to construct an neutrosophic decision matrix

Let \( r^k_{ij} = (T^k_{ij}, I^k_{ij}, F^k_{ij}) \) be the neutrosophic number provided by \( DM_k \) on the assessment of \( A_i \) with respect to \( C_j \). The aggregated neutrosophic rating of alternatives with respect to each criterion is calculated based on neutrosophic weighted averaging (NWA) operator as:

\[
r^k_{ij} = \text{NWA}(r^{(1)}_{ij}, r^{(2)}_{ij}, \ldots, r^{(I)}_{ij}) = \left( 1 - \prod_{k=1}^{I} (1 - T^{(k)}_{ij})^{\lambda_k}, \prod_{k=1}^{I} (I^{(k)}_{ij})^{\lambda_k}, \prod_{k=1}^{I} (F^{(k)}_{ij})^{\lambda_k} \right)
\]

**Step 3.** Determining the weights of criteria: There are various ways to determine the weights of the criteria.

Let \( w_j = (T^1_j, I^1_j, F^1_j) \) be the weight of criterion \( C_j \) given by \( K^{th} \) decision-maker \( DM \). The aggregated neutrosophic weights \( w_j \) of criteria are calculated by

\[
w_j = \lambda_1 w_{j(1)}^{(1)} \cup \lambda_2 w_{j(2)}^{(2)} \cup \ldots \cup \lambda_I w_{j(I)}^{(I)} = \left( 1 - \prod_{k=1}^{I} (1 - T^{(k)}_{ij})^{\lambda_k}, \prod_{k=1}^{I} (I^{(k)}_{ij})^{\lambda_k}, \prod_{k=1}^{I} (F^{(k)}_{ij})^{\lambda_k} \right)
\]

where \( w_j = (T^1_j, I^1_j, F^1_j) \), \( j = 1,2,\ldots,n \)

**Step 4.** Determining the concordance and discordance sets: In this step the concordance and discordance sets are determined. The concordance set can be classified in different types of the concordance sets as strong concordance set, moderate concordance set and weak concordance set. It is the same for the discordance sets.

The strong concordance set is determined as follows:

\[
C_{kl} = \left\{ \left| T_{kj} \geq T_{ij}, F_{kj} < F_{ij}, I_{kj} < I_{ij} \right| \right\}
\]

moderate concordance set is as follows:

\[
C'_{kl} = \left\{ \left| T_{kj} \geq T_{ij}, F_{kj} < F_{ij}, I_{kj} \geq I_{ij} \right| \right\}
\]

weak concordance set is as follows:

\[
C''_{kl} = \left\{ \left| T_{kj} \geq T_{ij}, F_{kj} \geq F_{ij} \right| \right\}
\]

The strong discordance set can be determined in ELECTRE method as follows:

\[
D_{kl} = \left\{ \left| T_{kj} < T_{ij}, F_{kj} \geq F_{ij}, I_{kj} \geq I_{ij} \right| \right\}
\]

moderate discordance set is as follows:

\[
D'_{kl} = \left\{ \left| T_{kj} < T_{ij}, F_{kj} \geq F_{ij}, I_{kj} < I_{ij} \right| \right\}
\]

weak discordance set is as follows:

\[
D''_{kl} = \left\{ \left| T_{kj} < T_{ij}, F_{kj} < F_{ij}, I_{kj} \geq I_{ij} \right| \right\}
\]
$D_{kl}^* = \left\{ j \mid T_{kj} < T_{lj}, F_{kj} < F_{lj} \right\}$

(12)

Decision makers give weights in different sets. $W_C$, $W_{C'}$, $W_{D'}$, $W_{D''}$ and $W_{D'''}$ are the weights of the strong concordance, moderate concordance, weak concordance, strong discordance, moderate discordance and weak discordance sets, respectively.

The concepts of concordance sets and discordance sets are used for calculating concordance sets and discordance matrixes and then determining the aggregate dominance matrix.

**Step 5.** Constructing the concordance and discordance matrixes: The relative value of the concordance set is measured through the concordance index. the concordance index shows that the relative dominance of certain alternative over a competing alternative. The concordance index $g_{kl}$ between $A_k$ and $A_l$ is defined as:

$$C_{kl} = w_C \times \sum_{j \in C_k} w_j + w_{C'} \times \sum_{j \in C'_k} w_j + w_{C''} \times \sum_{j \in C''_k} w_j$$

(13)

The concordance matrix $C$ is defined as follows:

$$C = \begin{bmatrix}
-c_{12} & \cdots & \cdots & -c_{1m} \\
-c_{21} & -c_{23} & \cdots & -c_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
c_{(m-1)1} & \cdots & \cdots & -c_{(m-1)m} \\
c_{m1} & c_{m2} & \cdots & c_{m(m-1)} 
\end{bmatrix}$$

It is obvious that a higher value of $c_{kl}$ indicates that $A_k$ is preferred to $A_l$. The discordance index $d_{kl}$ between $A_k$ and $A_l$ is defined as:

$$d_{kl} = \frac{\max_{j \in D_{kl}} w_{D}^* \times \text{dis}(X_{kj}, X_{lj})}{\max_{j \in l} \text{dis}(X_{kj}, X_{lj})}$$

(14)

$$\text{dis}(X_{kj}, X_{lj}) = \sqrt{\frac{1}{2} \left( (T_{kj} - T_{lj})^2 + (I_{kj} - I_{lj})^2 + (F_{kj} - F_{lj})^2 \right)}$$

$w_{D}^*$ is equal to $W_D$, $W_{D'}$ and $W_{D''}$ depending on the different types of discordance sets. The discordance matrix $D$ is defined as follows:

$$D = \begin{bmatrix}
-c_{12} & \cdots & \cdots & d_{1m} \\
-d_{21} & -d_{23} & \cdots & d_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
d_{(m-1)1} & \cdots & \cdots & -d_{(m-1)m} \\
d_{m1} & d_{m2} & \cdots & d_{m(m-1)} 
\end{bmatrix}$$

**Step 6.** Constructing the concordance and discordance dominance matrixes: The concordance dominance matrix $F$ can be calculated with aid of a threshold value for the concordance index.

When concordance index of $c_{kl}$ does not exceed the minimum specified boundary value, or $c_{kl} \geq \bar{c}$, only $A_k$ has the chance of mastery over $A_l$. 

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Based on the boundary value of Boolean F matrix, each element of this matrix is as follows:

\[
\text{If } c_{kl} \geq \bar{c} \quad f_{kl} = 1
\]

\[
\text{If } c_{kl} < \bar{c} \quad f_{kl} = 0
\]

In this matrix, element 1 indicates mastery of an option with respect to other elements.

The discordance dominance matrix \( G \) can be calculated with aid of a threshold value for the discordance index. This matrix is built for discordance index of \( d_{kl} \) like F matrix with a boundary value of \( d_{kl} \).

Element of discordance dominance matrix \( G \) is measured as follows:

\[
\text{If } d_{kl} \geq \bar{d} \quad g_{kl} = 1
\]

\[
\text{If } d_{kl} < \bar{d} \quad g_{kl} = 0
\]

Each element of matrix \( G \) indicates mastery relations between two options.

**Step 7.** Determining the aggregate dominance matrix: Thus, step is to calculated the intersection of the concordance dominance matrix \( F \) and the discordance dominance matrix \( G \). Each of elements of this matrix \( e_{kl} \) is defined as follows:

\[
e_{kl} = f_{kl} \times g_{kl}
\]

**Step 8.** Eliminate the less favorable alternatives: The aggregate dominance matrix \( E \) provides orders of relative preferences of options. If \( e_{kl} = 1 \), it means that \( A_k \) is preferable to \( A_l \) for both concordance and disharmony criteria, but \( A_k \) still has a chance of mastery over other options.

Conditions where \( A_k \) cannot be mastered in ELECTERE method are as follows:

When at least a 1 is equal to one. \( e_{il} = 1, \ l = 1,2,\ldots, m, \ k \neq l \)

For all of \( i \) \( e_{ii} = 0, \ i = 1,2,\ldots, m, \ i \neq k, i \neq l \)

Application of these conditions seems difficult, but mastery options can be easily identified in \( E \) matrix. If each column of matrix \( E \) has at least an element with value 1, this column is mastered by its other studied rows. Therefore, columns with element 1 will be easily removed.

**Step 9.** Using the ranking process proposed by Wu and Chen: Since ELECTERE method cannot rank all options, we use proposed method by Wu and Chen[52] for ranking options. Steps of this method are as follows.

Step 9.1. Determining concordance matrix \( c' \): This step uses ideal TOPSIS solution method. If \( c' \) is the largest value of concordance matrix, matrix \( c' \) will be obtained by calculation of the following equation.

\[
\bar{c} = \frac{\sum_{k=1}^{m} \sum_{l=1}^{m} c_{kl}}{m \times (m - 1)}
\]
\[ c_{ij} = c^* - c_{ij} \]  \hfill (18)

Step 9.2. Determining discordance matrix \( d' \): If \( d^* \) is the largest value of discordance matrix, matrix \( d' \) will be obtained by calculation of the following equation.

\[ d'_{ij} = d^* - d_{ij} \]  \hfill (19)

Step 9.3. Determining the aggregate dominance matrix \( P \):

\[
P = \begin{bmatrix}
- & p_{12} & \cdots & p_{1m} \\
p_{21} & - & p_{23} & p_{2m} \\
\vdots & & - & \\
p_{m1} & p_{m2} & \cdots & p_{m(m-1)} & -
\end{bmatrix}
\]

Each element of matrix \( P \) is defined according to the following equation.

\[
p_{ki} = \frac{d_{ki}}{c_{ki} + d_{ki}}
\]  \hfill (20)

Here, \( c_{ki} \) is the element of concordance dominance matrix, and \( d_{ki} \) is the element of discordance dominance matrix.

Step 9.4. Determining the best alternative: According to results of Step 9-3, we can obtain the combinatorial evaluation of options through Equation 21.

\[
\overline{p}_k = \frac{1}{m-1} \sum_{l=1, l \neq k}^{m} p_{kl}, \quad k = 1, 2, \ldots, m
\]  \hfill (21)

Then, the best option is specified according to Equation 22, and finally options are ranked incrementally.

\[
A^* = \max \{ \overline{p}_k \}
\]  \hfill (22)

\( A^* \) is the best alternative.

The process summary of the proposed method is shown in Figure 1.
4. Numerical example

In this section, we solve a problem to show the effectiveness of the proposed approach. There are three alternatives $A_1, A_2, A_3$ and five criteria $C_1, C_2, C_3, C_4, C_5$. Then, the proposed procedure for solving the problem is provided using the following steps.

Step 1. Constructing the decision matrix: The results of the evaluation of alternatives by four experts, based on the criteria, are shown in the table below:
Table 2. Evaluation of alternatives by neutrosophic numbers

| $D_1$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $D_2$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $D_3$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $D_4$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|       | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
| $A_1$ | (0.7,0.3,0.1) | (0.8,0.4,0.2) | (0.4,0.1,0.1) | (0.5,0.1,0.1) | (0.6,0.4,0.1) | (0.7,0.4,0.2) | (0.3,0.2,0.1) | (0.3,0.1,0.2) | (0.8,0.2,0.2) | (0.7,0.1,0.1) | (0.6,0.2,0.2) | (0.6,0.2,0.2) | (0.8,0.2,0.2) | (0.7,0.1,0.1) | (0.7,0.1,0.1) |
| $A_2$ | (0.8,0.2,0.1) | (0.7,0.4,0.2) | (0.4,0.4,0.4) | (0.6,0.1,0.1) | (0.7,0.1,0.1) | (0.8,0.2,0.1) | (0.6,0.2,0.3) | (0.5,0.1,0.2) | (0.4,0.5,0.2) | (0.7,0.1,0.1) | (0.5,0.5,0.1) | (0.8,0.2,0.1) | (0.6,0.2,0.3) | (0.5,0.1,0.2) | (0.5,0.5,0.1) |
| $A_3$ | (0.6,0.2,0.2) | (0.7,0.2,0.2) | (0.4,0.1,0.1) | (0.7,0.1,0.3) | (0.7,0.3,0.2) | (0.6,0.2,0.2) | (0.7,0.1,0.3) | (0.7,0.1,0.3) | (0.6,0.1,0.2) | (0.7,0.3,0.2) | (0.6,0.2,0.2) | (0.7,0.1,0.3) | (0.7,0.1,0.3) | (0.6,0.1,0.2) | (0.7,0.3,0.2) |
| $A_4$ | (0.9,0.1,0.1) | (0.5,0.3,0.2) | (0.6,0.4,0.1) | (0.2,0.5,0.3) | (0.4,0.4,0.4) | (0.9,0.1,0.1) | (0.5,0.3,0.2) | (0.6,0.4,0.1) | (0.2,0.5,0.3) | (0.4,0.4,0.4) | (0.9,0.1,0.1) | (0.5,0.3,0.2) | (0.6,0.4,0.1) | (0.2,0.5,0.3) | (0.4,0.4,0.4) |
| $D_2$ | (0.8,0.2,0.1) | (0.6,0.3,0.1) | (0.5,0.4,0.1) | (0.4,0.2,0.1) | (0.5,0.3,0.2) | (0.8,0.2,0.1) | (0.6,0.3,0.1) | (0.5,0.4,0.1) | (0.4,0.2,0.1) | (0.5,0.3,0.2) | (0.8,0.2,0.1) | (0.6,0.3,0.1) | (0.5,0.4,0.1) | (0.4,0.2,0.1) | (0.5,0.3,0.2) |
| $D_3$ | (0.8,0.2,0.1) | (0.7,0.1,0.3) | (0.6,0.3,0.3) | (0.8,0.2,0.1) | (0.7,0.2,0.1) | (0.8,0.2,0.1) | (0.7,0.1,0.3) | (0.6,0.3,0.3) | (0.8,0.2,0.1) | (0.7,0.2,0.1) | (0.8,0.2,0.1) | (0.7,0.1,0.3) | (0.6,0.3,0.3) | (0.8,0.2,0.1) | (0.7,0.2,0.1) |
| $D_4$ | (0.6,0.1,0.2) | (0.6,0.1,0.2) | (0.6,0.2,0.1) | (0.7,0.1,0.2) | (0.7,0.3,0.2) | (0.6,0.1,0.2) | (0.6,0.1,0.2) | (0.6,0.2,0.1) | (0.7,0.1,0.2) | (0.7,0.3,0.2) | (0.6,0.1,0.2) | (0.6,0.1,0.2) | (0.6,0.2,0.1) | (0.7,0.1,0.2) | (0.7,0.3,0.2) |

Step 2. Aggregate the decision makers' opinion to construct a neutrosophic decision matrix: The aggregated decision matrix can be determined by applying the aggregated operator (6) and is calculated as shown below:

Table 2. The aggregated neutrosophic decision matrix

<table>
<thead>
<tr>
<th></th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$c_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.738,0.144,0.01)</td>
<td>(0.695,0.203,0.187)</td>
<td>(0.570,0.162,0.158)</td>
<td>(0.465,0.244,0.225)</td>
<td>(0.543,0.414,0.193)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.693,0.222,0.067)</td>
<td>(0.650,0.184,0.158)</td>
<td>(0.499,0.259,0.133)</td>
<td>(0.436,0.175,0.144)</td>
<td>(0.559,0.278,0.238)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.693,0.12,0.2)</td>
<td>(0.670,0.144,0.143)</td>
<td>(0.540,0.219,0.201)</td>
<td>(0.593,0.10,0.132)</td>
<td>(0.619,0.201,0.139)</td>
</tr>
</tbody>
</table>

Step 3. Determining the weights of the criteria: The weight matrix (see Table 3) of the criteria described in this problem can be displayed as follows:

Table 3. Weight matrix of criteria

<table>
<thead>
<tr>
<th></th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$c_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>(0.9,0.1,0.2)</td>
<td>(0.8,0.2,0.3)</td>
<td>(0.5,0.4,0.3)</td>
<td>(0.5,0.2,0.15)</td>
<td>(0.5,0.4,0.4)</td>
</tr>
<tr>
<td>$D_2$</td>
<td>(0.8,0.2,0.1)</td>
<td>(0.7,0.1,0.3)</td>
<td>(0.6,0.3,0.3)</td>
<td>(0.8,0.25,0.1)</td>
<td>(0.6,0.3,0.4)</td>
</tr>
<tr>
<td>$D_3$</td>
<td>(0.6,0.3,0.2)</td>
<td>(0.5,0.3,0.2)</td>
<td>(0.8,0.2,0.1)</td>
<td>(0.7,0.2,0.1)</td>
<td>(0.4,0.4,0.4)</td>
</tr>
<tr>
<td>$D_4$</td>
<td>(0.6,0.1,0.2)</td>
<td>(0.6,0.1,0.2)</td>
<td>(0.6,0.2,0.3)</td>
<td>(0.5,0.1,0.2)</td>
<td>(0.3,0.2,0.1)</td>
</tr>
</tbody>
</table>

The aggregated weights for all criteria are presented below:
Table 4. The aggregated weights of criteria

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.725,0.15,0.166)</td>
<td>(0.653,0.15,0.25)</td>
<td>(0.604,0.27,0.241)</td>
<td>(0.608,0.178,0.133)</td>
<td>(0.444,0.31,0.281)</td>
</tr>
</tbody>
</table>

According to Table 4 and equation 5, the crisp of weights of criteria are presented as following:

Table 6. The crisp of weights of criteria

<table>
<thead>
<tr>
<th>CRITERA</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crisp weight</td>
<td>0.204</td>
<td>0.202</td>
<td>0.200</td>
<td>0.202</td>
<td>0.192</td>
</tr>
</tbody>
</table>

Step 4. Determining the concordance and discordance sets: In this step, assume that the subjective importance of attributes, W, is given by the decision maker, the decision maker also gives the relative weight (W’)

\[ W’ = \{w_{c1}, w_{c2}, w_{c3}, w_d, w_{d1}, w_{d2}\} = \left\{\frac{2}{3}, \frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right\} \]

The strong concordance set described in this problem can be displayed as follows:

\[ C = \begin{bmatrix} - & - & C_3 \\ C_4 & - & - \\ C_4, C_5 & C_2, C_4, C_5 & - \end{bmatrix} \]

The moderate concordance set described in this problem can be displayed as follows:

\[ C’ = \begin{bmatrix} - & - & C_1 \\ - & - & - \end{bmatrix} \]

The weak concordance set described in this problem can be displayed as follows:

\[ C'' = \begin{bmatrix} - & C_1, C_2, C_3 & C_2 \\ C_5 & - & - \\ - & C_1, C_3 & - \end{bmatrix} \]

The strong discordance set described in this problem can be displayed as follows:

\[ D = \begin{bmatrix} - & C_4 & C_4, C_5 \\ C_3 & - & - \\ - & C_2, C_4, C_5 \end{bmatrix} \]

The moderate discordance set described in this problem can be displayed as follows:

\[ D’ = \begin{bmatrix} - & - & - \\ C_1 & - & - \end{bmatrix} \]

The weak discordance set described in this problem can be displayed as follows:
Step 5. Calculating the concordance and discordance matrices: The concordance matrix described in this problem can be calculated as follows:

\[
C = \begin{bmatrix}
- & 0.202 & 0.403 \\
0.266 & - & 0.136 \\
0.394 & 0.733 & -
\end{bmatrix}
\]

The discordance matrix described in this problem can be calculated as follows:

\[
D = \begin{bmatrix}
- & 0.578 & 0.999 \\
0.289 & - & 0.650 \\
0.111 & 0 & -
\end{bmatrix}
\]

Step 6. Determining the concordance and discordance dominance matrixes: The concordance dominance matrix can be determined. The average concordance index is:

\[
\overline{c} = \frac{\sum_{k=1}^{3} \sum_{i=1}^{3} C_{ik}}{3 \times 2} = 0.356
\]

\[
F = \begin{bmatrix}
- & 0 & 1 \\
0 & - & 0 \\
1 & 1 & -
\end{bmatrix}
\]

The discordance dominance matrix can be determined. The average discordance index is:

\[
\overline{d} = \frac{\sum_{k=1}^{3} \sum_{i=1}^{3} D_{ik}}{3 \times 2} = 0.438
\]

\[
G = \begin{bmatrix}
- & 0 & 0 \\
1 & - & 0 \\
1 & 1 & -
\end{bmatrix}
\]

Step 7. Determining the aggregate dominance matrix: The aggregate dominance matrix can be determined.

\[
E = \begin{bmatrix}
- & 0 & 0 \\
0 & - & 0 \\
1 & 1 & -
\end{bmatrix}
\]

Step 8. Eliminating the less favourable alternatives: Using the seventh step, we remove the undesirable alternative. Matrix E provides the following ranking Figure 2.
It is obvious that $A_3$ is preferred to $A_1$ and $A_2$. But two alternatives of $A_1$ and $A_2$ cannot be ranked. This condition appears difficult to apply, but the dominated alternatives can be easily identified in the $E$ matrix. In this section it used ranking process proposed by Wu and Chen. This process is as following:

Step 9. Using the ranking process:

9.1. Determining concordance matrix $c^-$: The concordance dominance matrix can be calculated as follows: ($c^- = 0.733$)

$$C^- = \begin{bmatrix} - & 0.531 & 0.330 \\ 0.467 & - & 0.597 \\ 0.339 & 0 & - \end{bmatrix}$$

9.2. Determining discordance matrix $d^-$: The discordance dominance matrix can be calculated as follows: ($d^- = 0.999$)

$$D^- = \begin{bmatrix} - & 0.421 & 0 \\ 0.710 & - & 0.349 \\ 0.888 & 0.999 & - \end{bmatrix}$$

9.3. Determining the aggregate dominance matrix $P$: The aggregate dominance matrix can be calculated as follows:

$$P = \begin{bmatrix} - & 0.442 & 0 \\ 0.603 & - & 0.369 \\ 0.724 & 1 & - \end{bmatrix}$$

9.4. Determining the best alternative: According to the values of $P$ the best alternative is determined.

$P_1 = 0.221, P_2 = 0.486, P_3 = 0.862$

The optimal ranking order of the alternatives is given by $A_3 > A_2 > A_1$. The best alternative is $A_3$.

5. Conclusion

This paper has proposed an approach for solving MCDM problems using neutrosophic and ELECTRE method. In many cases, it is difficult for decision-makers to precisely express a preference when solving Multi-attribute decision making (MADM) problems with uncertain information. SVNSES is an effective and useful decision-making tool to describe indeterminate and inconsistent
information and it is also possible for a user to view the opinions of all experts in a single model. Since SVNNs reflect not only the degrees of truth (membership) and falsity (non-membership), but also indeterminacy, the evaluation information was described more comprehensively in the proposed approach. This paper is devoted to present a new ELECTERE-based approach for MADM under neutrosophic environment. In the evaluation process, the ratings of each alternative with respect to each attribute are given as linguistic variables characterized by single-valued neutrosophic numbers. After the formation and integration of the decision matrix, the weights of the criteria were calculated. After that, were determined concordance and discordance sets and matrixes, respectively. Then were formed the concordance and discordance dominance matrixes. In the next step, was created the aggregate dominance matrix and then was paid to eliminating the less favourable alternatives. Finally, by using concordance and discordance matrixes and the aggregate dominance matrix, was donned the ranking of alternatives and it was found the best alternative. The results showed that the A3 was the best. The advantage of the proposed method is more suitable for solving multiple attribute decision-making problems with neutrosophic information because neutrosophic sets can handle indeterminate and inconsistent information and are the extension of intuitionistic fuzzy sets. The future work is to develop other aggregated algorithms for some other practical decision-making problems, such as supply chain management, personal selection in academia, project evaluation, manufacturing systems, and many other areas of management systems. Also, in the future, the proposed method can be used for dealing with interval-valued neutrosophic soft expert based MCDM problems. Also, this approach can be applied to other multi-criteria decision-making methods, including VIKOR, DEMTEL, PROMOTHEE and etc, also weight determination techniques; It can also be comparing the results of solving these methods with the results of these techniques in fuzzy and intuitionistic fuzzy environments.

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**References**


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