NSS

Neutrosophic Sets and Systems, Vol. 47, 2021

University of New Mexico

An Efficient Enumeration Technique for a Transportation Problem in Neutrosophic Environment

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Abstract. In this paper, we have given a technique to solve transportation problems in a neutrosophic environment. The proposed technique gives solutions in the best possible and the worst possible manner. The concept is based on (α, β, γ) cut-sets of neutrosophic sets. It converts the problem into an interval-valued problem, which is further solved to give the best possible and the worst possible solution to the considered problem. Moreover, the technique proposed in this paper produces a direct optimal solution. The gained results ensure that the proposed method is better than the traditional ones as it is computationally much more efficient. The proposed technique has been suitably modified by varying (α, β, γ) according to the decision maker's knowledge of supply and demand requirements. Numerical illustrations have been provided to demonstrate the methodology and to prove our claim on efficiency.

Keywords: Neutrosophic Transportation Problem, Single Valued Trapezoidal Neutrosophic Number, (α, β, γ) Cut-Sets, Decision Making Problem

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1. Introduction

The transportation problem (TP) is a unique magnificence network-based linear programming problem with utmost significance in literature. TP was first introduced by Hitchcock [1]. These days transportation problem is used in many fields like management [2], job scheduling [3], investment [4], inventory [5], production [6], etc. To model these real-life problems we need to know some parameters values such as transportation cost, demand and supply. However, in real-life situations the parameters depend on various factors such as travel time, traffic jam, prices of diesel/petrol, weather condition, and so on. Similarly, the demands of several wearing products depends on the season, discount offers, fashion trends, etc. To deal with these obstacles amicably, the parameters of the problems can be represented as imprecise numbers having some uncertainty and vagueness.

The fuzzy set theory (FS) symbolizes the uncertainty introduced by Zadeh [7] in the given data, which is characterized by the grade of membership. A transportation problem discussed in fuzzy environment is called a fuzzy transportation problem (FTP), which has been solved by many researchers ([8]- [14]). However, sometimes the grade of accuracy or membership is not enough to describe ambiguity of the problem. Thus, Atanassove [15] developed the theory of intuitionistic fuzzy set (IFS). An IFS distinguish between the grade of membership and non-membership of each element in the set. The IFS approach is much applicable in real-life decision-making problems. The solution approach of transportation in IFS has been applied by many researchers ([16]- [20]).

Apart from the uncertainty or vagueness of the parameters of the transportation problem in real life, there is some indeterminacy due to various reasons such as imperfection of the data, ignorance of the problem, poor status forecasting, etc. Inconsistency and indeterminacy in information can not be well handled by an IFS. To overcome such uncertainties, Smarandache [21]developed the concept of neutrosophic set (NS) theory, a generalization of the IFS. In the neutrosophic set, the grade of indeterminacy membership is independent of the grade of truth-membership and the grade of falsity-membership. When the grade of the uncertainty of NS equals the grade of hesitation, NS becomes IFS. The single value neutrosophic set (SVN) was developed by Wang et al. [22] for the use of the neutrosophic set in practical decision-making problems and supply management problems in real life. The transportation problem discussed in the neutrosophic environment is known as the neutrosophic transportation problem. Thamaraiselvi and Santhi [23] presented a technique to solved transportation problems in a neutrosophic environment. Singh et al. [24] developed modified method of [23] by correcting mathematical assumptions and introduced a new method to solve the neutrosophic transportation problems. Later, many researchers have explored neutrosophic set in decision-making problems [25], [26], [27].

In spite of the above-mentioned developments, this article aims at providing a simple yet efficient technique for solving neutrosophic transportation problems with easy application in day-to-day situations. The major advantages of the proposed technique are as follows:

- The proposed technique produces the optimal solution for the considered problem in the best possible and the worst possible solution mode.
- The proposed technique is based on (α, β, γ) cut-set values and decision makes can vary these parameters according to their requirements.

- The method gives direct optimal solution for the considered problem.
- The proposed method is better than the traditional or oriented previous methods as it is computationally much more efficient.

The rest of the paper has been configuration as follows: The basic definitions of SVN-numbers are presented in the next Section. In Sect. 3, the arithmetic operations on Single valued trapezoidal neutrosophic (SVTN) numbers are discussed. The concept of (α, β, γ) cut-sets on SVTN-number is presented in Sect. 4. In Sect. 5, the mathematical formulation of transportation problem in neutrosophic environment is discussed. The developed methodology is presented in Sect. 6. The numerical example is illustrated in Sect. 7. The result and discussion are given in Sect. 8. The article is then concluded in Section 9.

2. **Mathematical preliminaries**

In this section, a brief overview of neutrosophic sets followed by some elementry definitions. Throughout this article, S and R represent the set of all neutrosophic sets and the set of real numbers respectively.

Definition 2.1. [21] The neutrosophic set N is characterized by three membership functions, which are the truth-membership function T_S, indeterminacy-membership function I_S, and falsity-membership function F_S, where P is the universe of discourse and $\forall\ u \in P$, $T_S(u), I_S(u), F_S(u) \subseteq\ ]^{-}0, 1^{+}[$, and $^{-}0 \leq inf\ T_S(u) + inf\ I_S(u) + inf\ F_S(u) \leq sup\ T_S(u) + sup\ I_S(u) + sup\ F_S(u) \leq 3^{+}$.

See that according to Definition 2.1, $T_S(u), I_S(u), F_S(u)$ are real standard or non-standard subsets of $]^{-}0, 1^{+}[$and hence, $T_S(u), I_S(u), F_S(u)$ can be subintervals of $[0, 1]$.

Remark 2.2. [28] If $T_S(u) + I_S(u) + F_S(u) = 1$, where $F_S(u) \leq 1 - T_S(u)$ (i.e. $I_S(u) \geq 0$ may exist) then neutrosophic set is known as IFS.

Remark 2.3. [28] If $T_S(u) + I_S(u) + F_S(u) = 1$, where $F_S(u) = 1 - T_S(u)$ (i.e. $I_S(u) = 0$ does not exist) the neutrosophic set is known as a fuzzy set.

Definition 2.4. [22] The single-valued neutrosophic set N over P is $S = \langle T_S(u), I_S(u), F_S(u); u \in P\rangle$ where $T_S : P \rightarrow [0, 1], I_S : P \rightarrow [0, 1]$, and $F_S : P \rightarrow [0, 1]$, $0 \leq T_S(u) + I_S(u) + F_S(u) \leq 3$.

The single-valued neutrosophic number is symbolized by $N = (t, i, f)$, such that $0 \leq t, i, f \leq$ and $0 \leq t + i + f \leq 3$.

Definition 2.5. [26] A single valued trapezoidal neutrosophic number is defined by $\tilde{p} = \langle(p_1, p_2, p_3, p_4); m_{\tilde{p}}, n_{\tilde{p}}, o_{\tilde{p}}\rangle$, where $m_{\tilde{p}}, n_{\tilde{p}}, o_{\tilde{p}} \in [0, 1]$ and p_1, p_2, p_3, p_4 in $\mathbb{R}$ with condition that $p_1 \leq p_2 \leq p_3 \leq p_4$. The truth-membership, indeterminacy-membership, and falsity-membership functions of $\tilde{p}$ are given as follows:

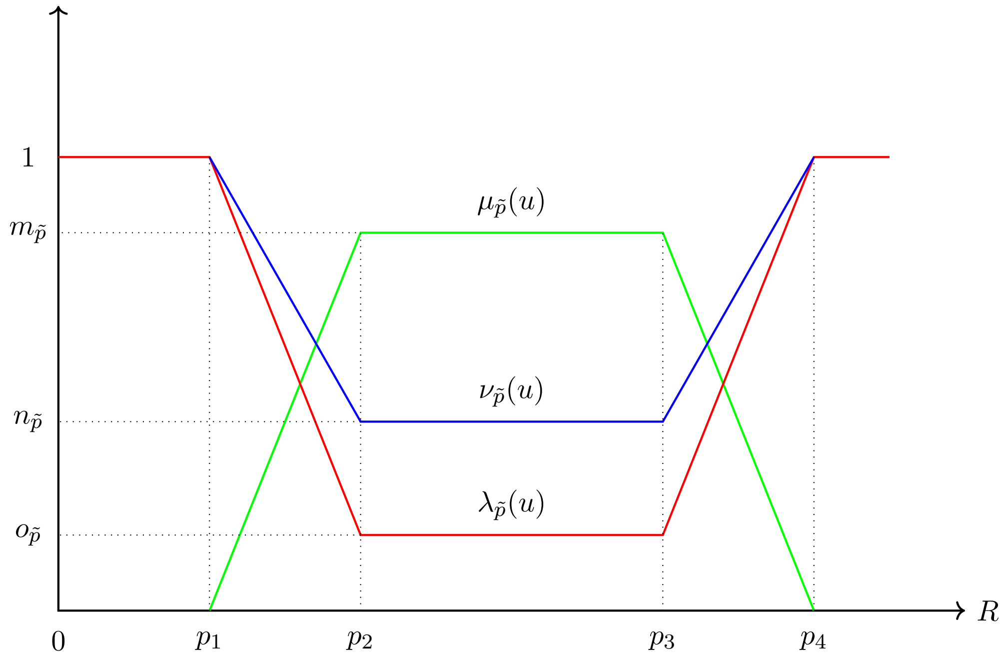

Figure 1. SVTN-number

$$\mu_{\tilde{p}}(u) = \begin{cases} m_{\tilde{p}}(\frac{u-p_1}{p_2-p_1}); & p_1 \leq u \leq p_2 \\ m_{\tilde{p}}; & p_2 \leq u \leq p_3 \\ m_{\tilde{p}}(\frac{p_4-u}{p_4-p_3}); & p_3 \leq u \leq p_4 \\ 0; & \text{otherwise,} \end{cases}$$

$$\nu_{\tilde{p}}(u) = \begin{cases} \frac{p_2-u+n_{\tilde{p}}(u-p_1)}{p_2-p_1}; & p_1 \leq u \leq p_2 \\ n_{\tilde{p}}; & p_2 \leq u \leq p_3 \\ \frac{u-p_3+n_{\tilde{p}}(p_4-u)}{p_4-p_3}; & p_3 \leq u \leq p_4 \\ 1; & \text{otherwise} \end{cases}$$

$$\lambda_{\tilde{p}} = \begin{cases} \frac{p_2-u+o_{\tilde{p}}(u-p_1)}{p_2-p_1}; & p_1 \leq u \leq p_2 \\ o_{\tilde{p}}; & p_2 \leq u \leq p_3 \\ \frac{u-p_3+o_{\tilde{p}}(p_4-u)}{p_4-p_3}; & p_3 \leq u \leq p_4 \\ 1; & \text{otherwise,} \end{cases}$$

where $m_{\tilde{p}}$, $n_{\tilde{p}}$ and $o_{\tilde{p}}$ are represents the maximum truth-membership grade, minimum-indeterminacy membership grade, minimum falsity-membership grade respectively. The geometrical representation of SVTN-number is shown by Fig. 1.

3. Arithmetic operations on SVTN-number

In this section, the arithmetic operations on single-valued trapezoidal neutrosophic numbers are defined. Let $\tilde{p} = \langle(p_1, p_2, p_3, p_4); m_{\tilde{p}}, n_{\tilde{p}}, o_{\tilde{p}}\rangle$ and $\tilde{q} = \langle(q_1, q_2, q_3, q_4); m_{\tilde{q}}, n_{\tilde{q}}, o_{\tilde{q}}\rangle$ be two single valued trapezoidal neutrosophic numbers and $k \neq 0$ be any number and the operators ($\wedge$,$\vee$) are the max, min respectively then the operations on them are defined as follows [25]:

(1) $\tilde{p} \oplus \tilde{q} = \langle(p_1 + q_1, p_2 + q_2, p_3 + q_3, p_4 + q_4); m_{\tilde{p}} \wedge m_{\tilde{q}}, n_{\tilde{p}} \vee n_{\tilde{q}}, o_{\tilde{p}} \vee o_{\tilde{q}}\rangle$,

(2) $\tilde{p} \ominus \tilde{q} = \langle(p_1 - q_4, p_2 - q_3, p_3 - q_2, p_4 - q_1); m_{\tilde{p}} \wedge m_{\tilde{q}}, n_{\tilde{p}} \vee n_{\tilde{q}}, o_{\tilde{p}} \vee o_{\tilde{q}}\rangle$,

(3)

$$\tilde{p} \otimes \tilde{q} = \begin{cases} \langle(\frac{p_1}{q_4}, \frac{p_2}{q_3}, \frac{p_3}{q_2}, \frac{p_4}{q_1}); m_{\tilde{p}} \wedge m_{\tilde{q}}, n_{\tilde{p}} \vee n_{\tilde{q}}, o_{\tilde{p}} \vee o_{\tilde{q}}\rangle & \text{if } p_4 > 0, q_4 > 0 \\ \langle(\frac{p_4}{q_4}, \frac{p_3}{q_3}, \frac{p_2}{q_2}, \frac{p_1}{q_1}); m_{\tilde{p}} \wedge m_{\tilde{q}}, n_{\tilde{p}} \vee n_{\tilde{q}}, o_{\tilde{p}} \vee o_{\tilde{q}}\rangle & \text{if } p_4 < 0, q_4 > 0 \\ \langle(\frac{p_4}{q_1}, \frac{p_3}{q_2}, \frac{p_2}{q_3}, \frac{p_1}{q_4}); m_{\tilde{p}} \wedge m_{\tilde{q}}, n_{\tilde{p}} \vee n_{\tilde{q}}, o_{\tilde{p}} \vee o_{\tilde{q}}\rangle & \text{if } p_4 < 0, q_4 < 0 \end{cases}$$

(4)

$$c\tilde{p} = \begin{cases} \langle(cp_1, cp_2, cp_3, cp_4); m_{\tilde{p}}, n_{\tilde{p}}, o_{\tilde{p}}\rangle & \text{if } c > 0 \\ \langle(cp_4, cp_3, cp_2, cp_1); m_{\tilde{p}}, n_{\tilde{p}}, o_{\tilde{p}}\rangle & \text{if } c < 0 \end{cases}$$

(5) $\tilde{p}^{-1} = \langle(\frac{1}{p_4}, \frac{1}{p_3}, \frac{1}{p_2}, \frac{1}{p_1}); m_{\tilde{p}}, n_{\tilde{p}}, o_{\tilde{p}}\rangle$, where $\tilde{p} \neq 0$.

4. Concepts of Cut-Sets for SVTN-number

In this section, the (α, β, γ) cut-sets for single valued trapezoidal neutrosophic numbers are discussed [25].

The cut-sets for SVTN-number $\tilde{p} = \langle(p_1, p_2, p_3, p_4); m_{\tilde{p}}, n_{\tilde{p}}, o_{\tilde{p}}\rangle$ are defined as follows:

An (α, β, γ)-cut set of $\tilde{p}$ is a crisp subset of R defined as :

$$\tilde{p}_{\langle\alpha,\beta,\gamma\rangle} = \{u : \mu_{\tilde{p}}(u) \geq \alpha, \nu_{\tilde{p}}(u) \leq \beta, \lambda_{\tilde{p}}(u) \leq \gamma\}$$

which satisfies the conditions as follows:

$0 \leq \alpha \leq m_{\tilde{p}},\ n_{\tilde{p}} \leq \beta \leq 1,\ o_{\tilde{p}} \leq \gamma \leq 1$ and $0 \leq \alpha + \beta + \gamma \leq 3$.

An α-cut of $\tilde{p}$ is a crisp subset of R defined as:

$$\tilde{p}_{\alpha} = \{u : \mu_{\tilde{p}}(u) \geq \alpha, u \in R\}$$

where $\alpha \in [0, m_{\tilde{a}}]$.

Clearly, any α-cut set of $\tilde{p}$ for truth-membership function is a closed interval, denoted by

$$\tilde{p}_{\alpha} = [L_{\tilde{p}}(\alpha), R_{\tilde{p}}(\alpha)] = [\tfrac{(m_{\tilde{p}}-\alpha)p_1+\alpha p_2}{m_{\tilde{p}}}, \tfrac{(m_{\tilde{p}}-\alpha)p_4+\alpha p_3}{m_{\tilde{p}}}]$$

An β-cut set of $\tilde{p}$ is a crisp subset of R defined as:

$$\tilde{p}_{\beta} = \{u : \nu_{\tilde{p}}(u) \leq \beta, u \in R\}$$

where $\beta \in [n_{\tilde{p}}, 1]$.

Clearly, β-cut set of $\tilde{p}$ for indeterminacy-membership is a closed interval, denoted by

$$\tilde{p}_\beta = [L^*_{\tilde{p}}(\beta), R^*_{\tilde{p}(\beta)}] = [\tfrac{(1-\beta)p_2+(\beta-n_{\tilde{p}})p_1}{1-n_{\tilde{p}}}, \tfrac{(1-\beta)p_3+(\beta-n_{\tilde{p}})p_4}{1-n_{\tilde{p}}}]$$

An γ-cut set of $\tilde{p}$ is a crisp subset of R defined as:

$$\tilde{p}_\gamma = \{\lambda_{\tilde{p}}(u) \leq \gamma, u \in R\}$$

where $\gamma \in [o_{\tilde{p}}, 1]$.

Clearly, γ-cut set of $\tilde{p}$ for falsity-membership is a closed interval, denoted by

$$\tilde{p}_\gamma = [L^{**}_{\tilde{p}}(\gamma), R^{**}_{\tilde{p}}] = [\tfrac{(1-\gamma)p_2+(\gamma-o_{\tilde{p}})p_1}{1-o_{\tilde{p}}}, \tfrac{(1-\gamma)p_3+(\gamma-o_{\tilde{p}})p_4}{1-o_{\tilde{p}}}]$$

Thus, it can be easily concluded from the definitions of $\tilde{p}_\alpha$, $\tilde{p}_\beta$ and $\tilde{p}_\gamma$ cut sets as follows:

$$\tilde{p}_{\langle\alpha,\beta,\gamma\rangle} = \tilde{p}_\alpha \wedge \tilde{p}_\beta \wedge \tilde{p}_\gamma.$$

5. Mathematical Formulation of Transportation Problem in Neutrosophic Environment

In this section, the transportation problem in a neutrosophic environment is considered. The cost parameter of the problem is taken as SVTN-number. Other parameters supply and demand of problem are assume to be precisely known. Thus, it is assumed that decision maker is indeterminate in considering the value of transportation cost, but there is no hesitation about the demand and supply of the commodity. The mathematical formulation of SVTN- number transportation problem under consideration is as follows:

$$\min \tilde{Z}^N = \sum_{i=1}^{m}\sum_{j=1}^{n} \mathcal{X}_{ij} \otimes \tilde{\mathcal{C}}^{\mathcal{N}}_{ij}$$

Subject to

$$\sum_{j=1}^{n} \mathcal{X}_{ij} = \mathcal{S}_i \;;\; i = 1,2,...,m,$$

$$\sum_{i=1}^{m} \mathcal{X}_{ij} = \mathcal{D}_j \;;\; j = 1,2,...,n,$$

and $\mathcal{X}_{ij} \geq 0$.

Where,

- m and n denote total number of supply sources and total number of demand points, respectively.
- $\mathcal{S}_i$ denotes available commodity at ith source.
- $\mathcal{D}_j$ denotes demand of the commodity at jth destination.
- $\tilde{\mathcal{C}}^{\mathcal{N}}_{ij} = (c_{ij,1}, c_{ij,2}, c_{ij,3}, c_{ij,4} \;; w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}})$ denotes the neutrosophic transportation cost of a unit commodity from ith source to jth destination.
- $\mathcal{X}_{ij}$ denotes number of units of the commodity to be transported from ith source to jth destination.

6. Proposed technique

In this section, the developed methodology to solve the transportation problem is described in which cost parameter is SVTN-number, supply and demand are precisely known. The step-by-step procedures of proposed algorithm is as follows:

Step 1 Consider the transportation problem in a neutrosophic environment where the cost parameter is taken as SVTN-number.

Step 2 Apply the cut set ranking function as defined in section 4 to convert the transportation cost into interval.

Step 3 Take the most optimistic (least value) members of the interval from each cell of the table to convert the interval-valued transportation problem to crisp transportation problem.

Step 4 Select the minimum element in each row and subtract it from each cell of the corresponding row.

Step 5 Select the minimum element from each column and subtract it from each cell of the corresponding column.

Step 6 In this process, at least one zero value cell in each row and each column is obtained. Calculate S_{ij} by using the following formula corresponding to each zero cell:

$$S_{ij} = \frac{\text{Sum of cost of cell adjacent to the (i,j)-cell}}{\text{Number of non-zero value ranked cells adjacent to (i,j)-cell}}$$

Step 7 Choose the cell which has a maximum value of S_{ij} and assign the maximum possible demand to that particular cell. Delete either row/column for which the demand is exhausted.

Step 8 If there occurs a situation in which two or more cell have the same rank then choose a cell which assigns the maximum possible demand.

Step 9 Repeat the process by applying Step 6 to Step 8 until the total demand fulfilled.

Step 10 The required optimal solution is denoted by $\mathcal{X}_{ij}$ and corresponding optimal value can be obtained by $\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{\mathcal{C}}_{ij}^{\mathcal{N}} \otimes \mathcal{X}_{ij}$. This is the best possible solution of the given transportation problem in the neutrosophic environment for given α, β, γ.

The worst possible solution of the given transportation problem in the neutrosophic environment for the values of the given α, β, γ can be obtained by considering the most pessimistic (greatest value) member of the interval from each cell in step 3 and hence following steps 4 to 10.

7. Numerical Example

In this section, a neutrosophic transportation problem with three sources A, B, C and four destination W, X, Y, Z is considered. The parameters of the problem are taken as single valued trapezoidal neutrosophic numbers. The input data of the problem SVTN-TP is denoted in Table 1 as follows [24]:

Table 1. SVTN transportation problem

Destination → Sources ↓	W	X	Y	Z	Supply
A	$(3,5,6,8\,;\,0.6,0.5,0.6)$	$(5,8,10,14\,;\,0.3,0.6,0.6)$	$(12,15,19,22\,;\,0.6,0.4,0.5)$	$(14,17,21,28;0.8,0.2,0.6)$	26
B	$(0,1,3,6\,;\,0.7,0.5,0.3)$	$(5,7,9,11\,;\,0.9,0.7,0.5)$	$(15,17,19,22\,;\,0.4,0.8,0.4)$	$(9,11,14,16\,;\,0.5,0.4,0.7)$	24
C	$(4,8,11,15\,;\,0.6,0.3,0.2)$	$(1,3,4,6\,;\,0.6,0.3,0.5)$	$(5,7,8,10\,;\,0.5,0.4,0.7)$	$(5,9,14,19\,;\,0.3,0.7,0.6)$	30
Demand	17	23	28	12	

For $(\alpha, \beta, \gamma) = (0.3, 0.8, 0.7)$, applying the cut set ranking function as discussed in section 4. The problem is converted to an inter-valued problm as shown in table 2.

Table 2. Obtained interval valued transportation problem

Destination → Sources ↓	W	X	Y	Z	Supply
A	$[4, 7]$	$[8, 10]$	$[13.8, 20.2]$	$[16.2, 22.7]$	26
B	$[.4, 4.7]$	$[6.3, 9.6]$	$[17, 19]$	$[11, 14]$	24
C	$[6, 13]$	$[2.7, 4.8]$	$[7, 8]$	$[9, 14]$	30
Demand	17	23	28	12	

Now, taking the most optimistic and most pessimistic values of intervals to get two separate transportation problems as shown in Table 3 and Table 4 respectively.

By solving problem in Table 3 by our proposed technique we will get best possible solution for the problem and by solving problem in Table 4 by our methodology we will get the worst possible solution for the given problem.

After solving the problem in Table 3, we have obtained optimal solution $\{(370, 543, 694, 938); 0.3, 0.7, 0.7\}$, which represent the best possible solution of the problem. Similarly,

Table 3. Optimistic values problem

Destination → Sources ↓	W	X	Y	Z	Supply
A	4	8	13.8	16.2	26
B	.4	6.3	17	11	24
C	6	2.7	7	9	30
Demand	17	23	28	12	

Table 4. Pessimistic values problem

Destination → Sources ↓	W	X	Y	Z	Supply
A	7	10	20.2	22.7	26
B	4.7	9.6	19	14	24
C	13	4.8	8	14	30
Demand	17	23	28	12	

solving problem in Table 4 by our technique, we have obtained optimal solution $\{(380, 549, 696, 940); 0.7, 0.6, 0.7\}$.

8. **Result and discussion**

The result of the above numerical example gives two solutions the best and the worst possible solutions. For $(\alpha, \beta, \gamma) = (0.3, 0.8, 0.7)$, the best possible solution for the problem is $\{(370, 543, 694, 938); 0.3, 0.7, 0.7\}$. The obtained solution represents 30 percent level of truthfulness, 70 level of percent indeterminacy and 70 percent level of falsity. The other values for level (grade) of truthfulness or acceptance $\mu(x)$, indeterminacy $\nu(x)$ and falsity $\lambda(x)$ are

$$\mu(x) = \begin{cases} 0.3(\frac{x-370}{543-370}), & 370 \leq x \leq 543 \\ 0.3, & 543 \leq x \leq 694 \\ 0.3(\frac{938-x}{938-694}), & 694 \leq x \leq 938 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \nu(x) = \lambda(x) = \begin{cases} \frac{543-x+0.7(x-370)}{543-370}, & 370 \leq x \leq 694 \\ 0.7, & 543 \leq x \leq 694 \\ \frac{x-694+0.7(938-x)}{938-694}, & 694 \leq x \leq 938 \\ 1, & \text{otherwise.} \end{cases}$$

respectively.

For $(\alpha, \beta, \gamma) = (0.3, 0.8, 0.7)$, the worst possible solution of the problem is $\{(380, 549, 696, 940); 0.7, 0.6, 0.7\}$. The solution represent 70 percent level of truthfulness, 60 percent level of indeterminacy and 70 percent level of falsity. The other values for $\mu(x)$, $\nu(x)$ and $\lambda(x)$ are

$$\mu(x) = \begin{cases} 0.7(\frac{x-380}{549-380}), & 380 \leq x \leq 549 \\ 0.7, & 549 \leq x \leq 696 \\ 0.7(\frac{940-x}{940-696}), & 696 \leq x \leq 940 \\ 0, & \text{otherwise} \end{cases}$$

$$\nu(x) = \begin{cases} \frac{549-x+0.6(x-380)}{549-380}, & 380 \leq x \leq 696 \\ 0.6, & 549 \leq x \leq 696 \\ \frac{x-696+0.6(940-x)}{940-696}, & 696 \leq x \leq 940 \\ 1, & \text{otherwise.} \end{cases}$$

and

$$\lambda(x) = \begin{cases} \frac{549-x+0.7(x-380)}{549-380}, & 380 \leq x \leq 696 \\ 0.7, & 549 \leq x \leq 696 \\ \frac{x-696+0.7(940-x)}{940-696}, & 696 \leq x \leq 940 \\ 1, & \text{otherwise.} \end{cases}$$

respectively. Therefore, with the help of $\mu(x)$, $\nu(x)$ and $\lambda(x)$, the decision maker can decide the total neutrosophic transportation cost to schedule the transportation and budget allocation.

The proposed approach in comparison to the methods of Thamaraiselvi and Sathi [23] and Singh et.al [24] is computationally much more efficient as it is producing direct optimal solution without finding an initial basic feasible solution. Also, our method gives the best and the worst possible solutions under neutrosophic of transportation problem, which enables the decision maker to choose the compromise solution. The proposed model gives direct optimal solution, which makes it computationally less time consuming than other existing methods. Moreover, the proposed technique can be modified by the decision maker by choosing the different values of (α, β, γ) to get satisfactory result. With different values of (α, β, γ), the neutrosophic optimal solutions of the considered problem are shown in Table 5.

9. **Conclusion**

In this article, we have discussed a transportation problem under a neutrosophic environment and proposed a technique to obtaining an optimal solution of the considered problem. The proposed technique has been useful to solve transportation problems in which the cost parameters are taken as single-valued trapezoidal neutrosophic numbers. In this article, we have used cut sets to convert the given problem to an interval-valued problem and is then solved by the proposed technique. The proposed technique is easy to apply in real-life transportation problems. The proposed approach in comparison to the methods of Thamaraiselvi

Table 5. Solutions for different values of (α, β, γ)

α	β	γ	**Best possible solution**	**Worst possible solution**
0.3	0.8	0.7	$(370, 543, 694, 938); 0.3, 0.7, 0.7$	$(380, 549, 696, 940); 0.7, 0.6, 0.7$
0	0.8	0.7	$(370, 543, 694, 938); 0.3, 0.7, 0.7$	$(370, 543, 694, 938); 0.3, 0.7, 0.7$
0	1	1	$(370, 543, 694, 938); 0.3, 0.7, 0.7$	$(370, 539, 676, 890); 0.3, 0.6, 0.7$
0.1	0.9	1	$(370, 543, 694, 938); 0.3, 0.7, 0.7$	$(438, 620, 770, 1044); 0.3, 0.7, 0.7$
0.3	0.8	0.9	$(370, 539, 676, 890); 0.3, 0.6, 0.7$	$(380, 549, 696, 940); 0.3, 0.6, 0.7$
0.1	0.9	1	$(370, 543, 694, 938); 0.3, 0.7, 0.7$	$(370, 543, 694, 930); 0.3, 0.7, 0.7$
0.2	0.9	0.7	$(370, 543, 694, 938); 0.3, 0.7, 0.7$	$(380, 549, 696, 940); 0.7, 0.6, 0.7$

and Sathi [23] and Singh et.al [24] is computationally much more efficient as it is producing direct optimal solution for the problem. Also, our method gives the best and the worst possible solutions under neutrosophic of transportation problem, which enables the decision maker to choose the compromise solution. Moreover, the proposed technique can be modified by the decision maker by choosing the different values of (α, β, γ) to get satisfactory result.

Funding: "This research received no external funding".

Acknowledgments: "Research of first author is supported by University Grants Commission (UGC), India".

Conflict of interest: "Authors Ashok Kumar, Ritika Chopra, Ratnesh Rajan Saxena declare that they have no conflict of interest".

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Received: Aug 30, 2021. Accepted: Dec 1, 2021