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Group Decision-Making Model Using the Exponential Similarity Measure of Confidence Neutrosophic Number Cubic Sets in a Fuzzy Multi-Valued Circumstance

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Abstract: Fuzzy decision-making is a critical research topic in uncertain decision-making issues. Under uncertain scenarios, a group of decision makers/experts presents the fuzzy evaluation data of the criteria to an alternative. In this case, we can use a fuzzy multi-valued set (FMVS) to express them. To solve the operation problem between different fuzzy sequence lengths in FMVSs and ensure some confidence level of fuzzy assessment values from the perspective of probability, this paper first proposes a transformation technique from FMVS to a confidence neutrosophic number cubic set (CNNCS) based on confidence levels and normal distribution of fuzzy values in FMVS. Then, we present an exponential similarity measure between CNNCSs and its group DM model with some confidence levels and normal distribution in a FMVS circumstance. Finally, the developed group DM model is applied to the selection of intelligent manufacturing equipment, and then the decision results corresponding to the 90%, 95%, and 99% confidence levels reveal the decision flexibility and rationality/reliability.

Keywords: fuzzy multi-valued set; confidence neutrosophic number cubic set; exponential similarity measure; group decision-making

1. Introduction

In uncertain decision-making (DM) issues, fuzzy DM is a critical one of DM research topics. Fuzzy sets (FSs) [1] have been applied in various DM areas, such as social science, economics and engineering management [2–6]. As an extension of FS that contains almost one occurrence of each element, Yager [7] presented a fuzzy multi-set (FMS) or bag, where permit multiple occurrences of the elements with identical or different membership degrees. Since then, the fuzzy multisets have been applied to group DM [8, 9] and clustering analysis [10–12] and so on. To avoid aggregation operations between different fuzzy sequence lengths in FMSs, Fu et al. [13] introduced a transformation technique from FMS to an entropy fuzzy set in terms of the mean and Shannon/probability entropy of fuzzy sequences, and then developed a group DM model using the Aczel-Alsina aggregation operators of entropy fuzzy elements and used it for renal cancer surgery options with FMS information.

In view of the hybrid form of interval fuzzy values (uncertain fuzzy values) and fuzzy values (exact fuzzy values), Jun et al. [14, 15] proposed (fuzzy) cubic sets (CSs). Then, CSs have been applied in many DM problems [16–18]. Moreover, there are some extension forms of CSs, such as cubic hesitant fuzzy sets [19–21], fuzzy credibility cubic numbers [22], and cubic fuzzy-consistency sets transformed from cubic fuzzy multi-valued sets [23], and their DM applications in existing literature. Since CS shows its obvious merit in the hybrid information expression of interval fuzzy values and fuzzy values, it is more useful than FS in multi-criteria group DM problems.

In uncertain problems, a neutrosophic number (NN) $N = h + uI = [h + uI^-, h + uI^+]$ for an indeterminacy $I = [I^-, I^+]$ and $h, u \in \Re$ was proposed by Smarandache [24–26]. NN implies its main merit in the indeterminate information representation of changeable interval values or fuzzy values corresponding to different indeterminate ranges of I. Hence, it shows better flexibility and generalization in the representation and processing capability of uncertain information in multi-criteria DM problems [27, 28]. Recently, Lv et al. [29] presented the concepts of NN probability and confidence neutrosophic numbers (CNNs) (confidence intervals) in light of confidence levels and normal and log-normal probability distributions of multi-valued datasets from the perspective of probability, and then developed CNN linear programming methods based on normal and log-normal probability distributions to carry out production planning problems in uncertain scenarios.

In the setting of FMSs, Fu et al. proposed a transformation technique from FMS to entropy fuzzy elements based on the mean and Shannon/probability entropy of fuzzy sequences in FMS. Then, from the perspective of probability estimation, the transformation technique does not consider a confidence level and certain probability distribution of fuzzy sequences/data, which shows its defect. To avoid this defect, this paper proposes a new transformation technique from a fuzzy multi-valued set (FMVS) to a confidence neutrosophic number cubic set (CNNCS) and group DM model using an exponential similarity measure (ESM) of CNNCSs to solve group DM problems in view of the conditions of some confidence levels and normal distribution in a FMVS circumstance.

This paper contains remaining structures. The second section introduces the definitions of FMVS and CNNCS and some basic relationships of CNNCEs. The third section proposes an ESM between CNNCSs and a weighted ESM of CNNCSs. The fourth section develops a group DM model based on the weighted ESM of CNNCSs in a FMVS circumstance. The fifth section utilizes the developed group DM model to perform the selection of intelligent manufacturing equipment. The sixth section provides decision results and discussions corresponding to the 90%, 95%, and 99% confidence levels to reveal the decision flexibility and rationality/reliability. The last section summarizes the conclusions and future research directions.

2. FMVS and CNNCS

This section gives the definitions of FMVS and CNNCS and then defines some basic relationships of confidence neutrosophic number cubic elements (CNNCEs).

Definition 1. A FMVS H on a finite set $Z = \{z_1, z_2, \ldots, z_q\}$ is defined as

$$H = \left\{ \left\langle z_k, M_H(z_k) \right\rangle \mid z_k \in Z \right\}, \tag{1}$$

where $M_H(z_k)$ contains multiple membership degrees of each element z_k to the set H, denoted as a fuzzy sequence $M_H(z_k) = (h_{k1}, h_{k2}, \ldots, h_{kr_k})$ with identical and/or different fuzzy values for $z_k \in Z$ and $h_{ki} \in [0, 1]$ ($k = 1, 2, \ldots, q$; $i = 1, 2, \ldots, r_k$).

For convenience, each element $\left\langle z_k, M_H(z_k) \right\rangle$ in H is denoted as a fuzzy multi-valued element (FMVE) $h_k = \left\langle z_k, (h_{k1}, h_{k2}, \ldots, h_{kr_k}) \right\rangle$ with increasing fuzzy sequence. Especially when $r_k = 1$, the FMVS H becomes FS.

According to the confidence interval with a $(1-\varphi)100\%$ confidence level [29], we present a transformation technique from FMVS to CNNCS, which is defined below.

Definition 2. Set FMVS as H_1 = {<z_1, (h_{11}, h_{12}, …, h_{1r_1})>, <z_2, (h_{21}, h_{22}, …, h_{2r_2})>,…, <z_q, (h_{q1}, h_{q2}, …, h_{qr_q})>} in a finite set Z = {z_1, z_2, …, z_q}. Thus, CNNCS can be defined as

$$G_{1\varphi}=\left\{\begin{array}{l}\left\langle z_1,\left[h_{11}^-(I_\varphi),h_{11}^+(I_\varphi)\right],h_{m11}\right\rangle,\left\langle z_2,\left[h_{12}^-(I_\varphi),h_{12}^+(I_\varphi)\right],h_{m12}\right\rangle,...,\\ \left\langle z_q,\left[h_{1q}^-(I_\varphi),h_{1q}^+(I_\varphi)\right],h_{m1q}\right\rangle \mid I_\varphi=[-t_{\varphi/2},t_{\varphi/2}]\end{array}\right\}, \quad (2)$$

where $\left[h_{1k}^-(I_\varphi),h_{1k}^+(I_\varphi)\right]$ (k = 1, 2, …, q) is CNN, which is obtained by

$$\left[h_{1k}^-(I_\varphi),h_{1k}^+(I_\varphi)\right]=\left[h_{m1k}+u_{1k}I_\varphi^-,h_{m1k}+u_{1k}I_\varphi^+\right]=\left[h_{m1k}-\frac{\sigma_{1k}}{\sqrt{r_k}}t_{\varphi/2},h_{m1k}+\frac{\sigma_{1k}}{\sqrt{r_k}}t_{\varphi/2}\right]; \quad (3)$$

$I_\varphi=[I_\varphi^-,I_\varphi^+]=[-t_{\varphi/2},t_{\varphi/2}]$ is an indeterminate interval depending on a specified value of $t_{\varphi/2}$; u_{1k} is an indeterminate parameter; then h_{m1k} and σ_{1k} are the average value and standard deviation of a fuzzy sequence in H_1, which are yielded by the formulae:

$$h_{m1k}=\frac{1}{r_k}\sum_{i=1}^{r_k}h_{1i}, \quad (4)$$

$$\sigma_{1k}=\sqrt{\frac{1}{r_k-1}\sum_{i=1}^{r_k}(h_{1i}-h_{m1k})^2}. \quad (5)$$

Remark 1. The specified values of $t_{\varphi/2}$ are related to (1−φ)100% confidence levels [29], which are usually specified as $t_{\varphi/2}$ = 1.645, 1.960, 2.576 for the levels of φ = 0.1, 0.05, 0.01 in actual applications [29].

From a probabilistic viewpoint and the estimation of small example data in some distribution situation, the CNN of Eq. (3) with a (1−φ)100% confidence level reveals the probability of fuzzy values falling within CNN (confidence interval). For example, considering the 90% confidence level, the 90% probability of all fuzzy values will occur within CNN, while the 10% probability of all fuzzy values will occur outside CNN.

Example 1. Assume that there is the FMVS H_1 = {<z_1, (0.5, 0.6, 0.7, 0.9)>, <z_2, (0.6, 0.7, 0.7, 0.8, 0.9)>} in a finite set Z = {z_1, z_2}, where fuzzy data are in the normal distribution situation. Considering the 90% confidence level with the specified value of $t_{\varphi/2}$ = 1.645, the FMVS H_1 can be transformed into the CNNCS $G_{\varphi1}$ by Eqs. (3)–(5), which is described by the calculational process below.

Using Eqs. (4) and (5), the average values and standard deviations of two fuzzy sequences in H_1 are given as follows:

h_{m11} = 0.675, h_{m12} = 0.74, σ_{11} = 0.1708, and σ_{12} = 0.114.

Using Eq. (3), two CNNs are produced as follows:

$$\left[h_{11}^-(I_\varphi),h_{11}^+(I_\varphi)\right]=\left[0.675-\frac{0.1708}{\sqrt{4}}\times1.645,0.675+\frac{0.1708}{\sqrt{4}}\times1.645\right]=[0.5345, 0.8155],$$

$$\left[h_{11}^-(I_\varphi),h_{11}^+(I_\varphi)\right]=\left[0.74-\frac{0.114}{\sqrt{5}}\times1.645,0.74+\frac{0.114}{\sqrt{5}}\times1.645\right]=[0.6561, 0.8239].$$

Thus, the CNNCS $G_{1\varphi}$ for φ = 0.1 is obtained below:

$G_{1\varphi=0.1}$ = {<z_1, [0.5345, 0.8155], 0.675>, <z_2, [0.6561, 0.8239], 0.74>| I_φ = [−1.645, 1.645]}.

Then, each element $\left\langle z_1,\left[h_{1k}^-(I_\varphi),h_{1k}^+(I_\varphi)\right],h_{m1k}\right\rangle$ in the CNNCS $G_{1\varphi}$ is simply represented as the CNNCE $g_{1k}(I_\varphi)=\left\langle\left[h_{\varphi1k}^-,h_{\varphi1k}^+\right],h_{m1k}\right\rangle$ (k = 1, 2, …, q).

Definition 3. Set two CNNCEs as $g_{1k}(I_\varphi)=\left\langle\left[h_{\varphi1k}^-,h_{\varphi1k}^+\right],h_{m1k}\right\rangle$ and $g_{2k}(I_\varphi)=\left\langle\left[h_{\varphi2k}^-,h_{\varphi2k}^+\right],h_{m2k}\right\rangle$ (k = 1, 2, …, q). Then, their basic relationships are defined below:

(1) $g_{1k}(I_\varphi) \subseteq g_{2k}(I_\varphi) \Leftrightarrow [h^-_{\varphi 1k}, h^+_{\varphi 1k}] \subseteq [h^-_{\varphi 2k}, h^+_{\varphi 2k}]$ and $h_{m1k} \le h_{m2k}$;

(2) $g_{1k}(I_\varphi) = g_{2k}(I_\varphi) \Leftrightarrow g_{1k}(I_\varphi) \subseteq g_{2k}(I_\varphi)$ and $g_{1k}(I_\varphi) \supseteq g_{2k}(I_\varphi)$, i.e., $h^-_{\varphi 1k} = h^-_{\varphi 2k}$, $h^+_{\varphi 1k} = h^+_{\varphi 2k}$, and $h_{m1k} = h_{m2k}$;

(3) $g_{1k}(I_\varphi) \cup g_{2k}(I_\varphi) = \left\langle [h^-_{\varphi 1k} \vee h^-_{\varphi 2k}, h^+_{\varphi 1k} \vee h^+_{\varphi 2k}], h_{m1k} \vee h_{m2k} \right\rangle$;

(4) $g_{1k}(I_\varphi) \cap g_{2k}(I_\varphi) = \left\langle [h^-_{\varphi 1k} \wedge h^-_{\varphi 2k}, h^+_{\varphi 1k} \wedge h^+_{\varphi 2k}], h_{m1k} \wedge h_{m2k} \right\rangle$;

(5) $g^c_{1k}(I_\varphi) = \left\langle [1-h^+_{\varphi 1k}, 1-h^-_{\varphi 1k}], 1-h_{m1k} \right\rangle$ (Complement of $g_{1k}(I_\varphi)$).

3. ESM of CNNCSs

In this section, we present the ESM of CNNCSs, the weighted ESM of CNNCSs, and their characteristics.

Definition 4. Set $G_{1\varphi} = \{g_{11}(I_\varphi), g_{12}(I_\varphi), \ldots, g_{1q}(I_\varphi)\}$ and $G_{2\varphi} = \{g_{21}(I_\varphi), g_{22}(I_\varphi), \ldots, g_{2q}(I_\varphi)\}$ as two CNNCSs, where $g_{1k}(I_\varphi) = \left\langle \left[h^-_{\varphi 1k}, h^+_{\varphi 1k}\right], h_{m1k} \right\rangle$ and $g_{2k}(I_\varphi) = \left\langle \left[h^-_{\varphi 2k}, h^+_{\varphi 2k}\right], h_{m2k} \right\rangle$ (k = 1, 2, …, q) are two collections of CNNCEs. Thus, the ESM of two CNNCSs $G_{\varphi 1}$ and $G_{\varphi 2}$ is defined as

$$E_\varphi(G_{1\varphi}, G_{2\varphi}) = \frac{1}{q}\sum_{k=1}^{q} \frac{\exp\left(-\left((h^-_{\varphi 1k} - h^-_{\varphi 2k})^2 + (h^+_{\varphi 1k} - h^+_{\varphi 2k})^2 + (h_{m1k} - h_{m2k})^2\right)\right) - \exp(-3)}{1-\exp(-3)}. \quad (6)$$

Proposition 1. The ESM $E_\varphi(G_{1\varphi}, G_{2\varphi})$ contains the following characteristics:

(a) $E_\varphi(G_{1\varphi}, G_{2\varphi}) = E_\varphi(G_{2\varphi}, G_{1\varphi})$;

(b) $0 \le E_\varphi(G_{1\varphi}, G_{2\varphi}) \le 1$;

(c) $E_\varphi(G_{1\varphi}, G_{2\varphi}) = 1$ if and only if $G_{1\varphi} = G_{2\varphi}$;

(d) If $G_{1\varphi} \subseteq G_{2\varphi} \subseteq G_{3\varphi}$ for any three CNNCSs $G_{1\varphi}$, $G_{2\varphi}$, and $G_{3\varphi}$, then $E_\varphi(G_{1\varphi}, G_{2\varphi}) \ge E_\varphi(G_{1\varphi}, G_{3\varphi})$ and $E_\varphi(G_{2\varphi}, G_{3\varphi}) \ge E_\varphi(G_{1\varphi}, G_{3\varphi})$ exist.

Proof:

(a) This characteristic is obvious.

(b) Since there is the inequality $0 \le (h^-_{\varphi 1k} - h^-_{\varphi 2k})^2 + (h^+_{\varphi 1k} - h^+_{\varphi 2k})^2 + (h_{m1k} - h_{m2k})^2 \le 3$, the inequality $\exp(0) = 1 \le \exp\left(-\left((h^-_{\varphi 1k} - h^-_{\varphi 2k})^2 + (h^+_{\varphi 1k} - h^+_{\varphi 2k})^2 + (h_{m1k} - h_{m2k})^2\right)\right) \le \exp(-3)$ also exists. Therefore, the value of Eq. (6) belongs to [0, 1], i.e., $0 \le E_\varphi(G_{1\varphi}, G_{2\varphi}) \le 1$.

(c) When $G_{1\varphi}$, = $G_{2\varphi}$, $g_{1k}(I_\varphi) = g_{2k}(I_\varphi)$ (k = 1, 2, …, q) exists. Thus, there are $h^-_{\varphi 1k} = h^-_{\varphi 2k}$, $h^+_{\varphi 1k} = h^+_{\varphi 2k}$, and $h_{m1k} = h_{m2k}$ (k = 1, 2, …, q). In this case, there is exp(0) = 1 in Eq. (6), and then $E_\varphi(G_{1\varphi}, G_{2\varphi}) = 1$ exists.

When $E_\varphi(G_{1\varphi}, G_{2\varphi}) = 1$, there is exp(0) = 1 in Eq. (6). Hence, $h^-_{\varphi 1k} = h^-_{\varphi 2k}$, $h^+_{\varphi 1k} = h^+_{\varphi 2k}$, and $h_{m1k} = h_{m2k}$ exist. In this case, there is $g_{1k}(I_\varphi) = g_{2k}(I_\varphi)$ (k = 1, 2, …, q), and then $G_{1\varphi} = G_{2\varphi}$ can hold.

(d) For $G_{1\varphi} \subseteq G_{2\varphi} \subseteq G_{3\varphi}$, there is $g_{1k}(I_\varphi) \subseteq g_{2k}(I_\varphi) \subseteq g_{3k}(I_\varphi)$, and then $[h^-_{\varphi 1k}, h^+_{\varphi 1k}] \subseteq [h^-_{\varphi 2k}, h^+_{\varphi 2k}] \subseteq [h^-_{\varphi 3k}, h^+_{\varphi 3k}]$ and $h_{m1k} \le h_{m2k} \le h_{m3k}$ (k = 1, 2, …, q) exist. Thus, there are the following inequalities:

$$(h^-_{\varphi 1k} - h^-_{\varphi 2k})^2 \le (h^-_{\varphi 1k} - h^-_{\varphi 3k})^2, (h^+_{\varphi 1k} - h^+_{\varphi 2k})^2 \le (h^+_{\varphi 1k} - h^+_{\varphi 3k})^2,$$

$$(h^-_{\varphi 2k} - h^-_{\varphi 3k})^2 \le (h^-_{\varphi 1k} - h^-_{\varphi 3k})^2, (h^+_{\varphi 2k} - h^+_{\varphi 3k})^2 \le (h^+_{\varphi 1k} - h^+_{\varphi 3k})^2,$$

$$(h_{m1k} - h_{m2k})^2 \le (h_{m1k} - h_{m3k})^2,\ (h_{m2k} - h_{m3k})^2 \le (h_{m1k} - h_{m3k})^2.$$

Since exp(−y) for $y \ge 0$ is a decreasing function, $E_\varphi(G_{1\varphi}, G_{2\varphi}) \ge E_\varphi(G_{1\varphi}, G_{3\varphi})$ and $E_\varphi(G_{2\varphi}, G_{3\varphi}) \ge E_\varphi(G_{1\varphi}, G_{3\varphi})$ can hold.

Considering the weight of $g_{jk}(I_\varphi)$ (k = 1, 2, …, q; j = 1, 2), it is given by $\lambda_k \in$ [0, 1] for $\sum_{k=1}^{q}\lambda_k = 1$. Thus, the weighted ESM of the CNNCSs $G_{1\varphi}$ and $G_{2\varphi}$ is established below:

$$E_{W\varphi}(G_{1\varphi},G_{2\varphi})=\sum_{k=1}^{q}\lambda_k\frac{\exp\left(-\left((h_{\varphi1k}^{-}-h_{\varphi2k}^{-})^2+(h_{\varphi1k}^{+}-h_{\varphi2k}^{+})^2+(h_{m1k}-h_{m2k})^2\right)\right)-\exp(-3)}{1-\exp(-3)}. \quad (7)$$

Proposition 2. The weighted ESM $E_{W\varphi}(G_{1\varphi}, G_{2\varphi})$ also contains these characteristics:

(a) $E_{W\varphi}(G_{1\varphi}, G_{2\varphi}) = E_{W\varphi}(G_{2\varphi}, G_{1\varphi})$;

(b) $0 \le E_{W\varphi}(G_{1\varphi}, G_{2\varphi}) \le 1$;

(c) $E_{W\varphi}(G_{1\varphi}, G_{2\varphi}) = 1$ if and only if $G_{1\varphi} = G_{2\varphi}$;

(d) If $G_{1\varphi} \subseteq G_{2\varphi} \subseteq G_{3\varphi}$ for any three CNNCSs $G_{1\varphi}$, $G_{2\varphi}$, and $G_{3\varphi}$, then there are $E_{W\varphi}(G_{1\varphi}, G_{2\varphi}) \ge E_{W\varphi}(G_{1\varphi}, G_{3\varphi})$ and $E_{W\varphi}(G_{2\varphi}, G_{3\varphi}) \ge E_{W\varphi}(G_{1\varphi}, G_{3\varphi})$.

Based on the similar proof process of Proposition 1, Proposition 2 can be easily verified (omitted).

4. Group DM Model Based on the ESM of CNNCSs

A multi-criteria group DM problem usually contains a group of possible alternatives Me = {Me_1, Me_2, …, Me_p} and a group of main assessment criteria Z = {z_1, z_2, …, z_q}. Taking into account the weights of different criteria, their weight vector is expressed as $\boldsymbol{\lambda}$ = (λ_1, λ_2, …, λ_q). In the group DM problem, the group DM model can be developed and reflected by the decision procedure below.

Step 1: In the suitability assessment of the alternatives, the fuzzy evaluation values of each alternative satisfying the criteria are assigned by a group of experts/decision makers and constructed as the FMVS $H_j = \{h_{jk} \mid k = 1, 2, ..., q\}$ containing the q FMVEs $h_{jk} = \langle z_k, (h_{jk1}, h_{jk2}, ..., h_{jkr_k})\rangle$ (k = 1, 2, …, q; j = 1,2, …, p) for $z_k \in Z$. Then, all FMVSs can be formed as their decision matrix $D_H = (h_{jk})_{p\times q}$.

Step 2: Using Eqs. (3)–(5) for the 90%, 95% and 99% confidence levels with the specified values of $t_{\varphi/2}$ = 1.645, 1.96, 2.576, the FMVSs H_j (j = 1, 2, …, p) can be transformed into the CNNCSs $G_{j\varphi}$ = {$g_{j1}(I_\varphi)$, $g_{j2}(I_\varphi)$, …, $g_{jq}(I_\varphi)$} containing the q CNNCEs $g_{jk}(I_\varphi) = \langle [h_{\varphi jk}^{-}, h_{\varphi jk}^{+}], h_{mjk}\rangle$ (j = 1, 2, …, p; k = 1, 2, …, q) for φ = 0.1, 0.05, 0.01. Thus, their decision matrix is denoted as $D_\varphi = (g_{jk}(I_\varphi))_{p\times q}$.

Step 3: Set the ideal solution/CNNCS as G^* = {<z_1, [1, 1], 1>, <z_2, [1, 1], 1>, …, <z_q, [1, 1], 1>}. Then, the weighted ESM values of $E_{W\varphi}(G_{j\varphi}, G^*)$ (j = 1, 2, …, p) are given by

$$E_{W\varphi}(G_{j\varphi},G^*)=\sum_{k=1}^{q}\lambda_k\frac{\exp\left(-\left((h_{\varphi jk}^{-}-1)^2+(h_{\varphi jk}^{+}-1)^2+(h_{mjk}-1)^2\right)\right)-\exp(-3)}{1-\exp(-3)}. \quad (8)$$

Step 4: The alternatives are sorted, and the optimal choice is determined by the largest weighted ESM value.

Step 5: End.

5. DM Example

5.1 Selection of intelligent manufacturing equipment

This section provides a DM example on the selection of intelligent manufacturing equipment in a manufacturing company to reflect the practicability and efficiency of the developed group DM model in the scenario of FMVSs.

To improve intelligent manufacturing capability in a manufacturing company, the manufacturing company wants to purchase a type of intelligent manufacturing equipment from possible equipment providers. In this case, the technology department preliminarily selects possible six types of intelligent manufacturing equipment (six alternatives) from possible equipment providers, which are denoted as a set of six alternatives Me = {Me_1, Me_2, Me_3, Me_4, Me_5, Me_6}. To assess their suitability, the technology department chooses four assessment criteria: cost (z_1), intelligent degree (z_2), technical advancement level (z_3), and manufacturing performance and capability (z_4). Then, the decision department invites five experts to select the optimal type of intelligent manufacturing equipment (the optimal alternative) by the suitability assessment of each alternative with respect to the four criteria. The weight vector of the four criteria **λ** = (0.2, 0.3, 0.2, 0.3) is presented by experts/decision makers.

For the DM example, the developed group DM model can be applied to the selection problem of intelligent manufacturing equipment and depicted by the decision procedure below.

Step 1: Five experts present their fuzzy evaluation values of each alternative Me_j (j = 1, 2, 3, 4, 5, 6) satisfying the criteria z_k (k = 1, 2, 3, 4). Then, their assessed fuzzy values are constructed as the FMVS decision matrix:

$$D_H=\begin{bmatrix}\langle z_1,(0.7,0.7,0.8,0.8,0.9)\rangle & \langle z_2,(0.6,0.7,0.7,0.7,0.7)\rangle & \langle z_3,(0.7,0.8,0.8,0.9,0.9)\rangle & \langle z_4,(0.7,0.8,0.8,0.8,0.8)\rangle\\ \langle z_1,(0.7,0.7,0.7,0.8,0.8)\rangle & \langle z_2,(0.6,0.6,0.7,0.7,0.8)\rangle & \langle z_3,(0.7,0.7,0.8,0.8,0.8)\rangle & \langle z_4,(0.6,0.7,0.7,0.8,0.8)\rangle\\ \langle z_1,(0.6,0.6,0.6,0.7,0.7)\rangle & \langle z_2,(0.6,0.7,0.8,0.8,0.9)\rangle & \langle z_3,(0.7,0.8,0.8,0.8,0.9)\rangle & \langle z_4,(0.6,0.6,0.6,0.7,0.8)\rangle\\ \langle z_1,(0.6,0.7,0.7,0.7,0.8)\rangle & \langle z_2,(0.6,0.6,0.7,0.8,0.8)\rangle & \langle z_3,(0.6,0.7,0.7,0.7,0.8)\rangle & \langle z_4,(0.6,0.6,0.7,0.7,0.8)\rangle\\ \langle z_1,(0.7,0.7,0.8,0.8,0.8)\rangle & \langle z_2,(0.7,0.7,0.7,0.7,0.7)\rangle & \langle z_3,(0.6,0.7,0.7,0.7,0.7)\rangle & \langle z_4,(0.5,0.6,0.7,0.7,0.7)\rangle\\ \langle z_1,(0.6,0.7,0.7,0.7,0.8)\rangle & \langle z_2,(0.6,0.7,0.7,0.8,0.8)\rangle & \langle z_3,(0.6,0.6,0.6,0.7,0.7)\rangle & \langle z_4,(0.5,0.6,0.8,0.8,0.9)\rangle\end{bmatrix}.$$

Step 2: The specified values for φ = 0.1, 0.05, 0.01 are $t_{\varphi/2}$ = 1.645, 1.96, 2.576 [29]. Using Eqs. (3)–(5) with the 90%, 95% and 99% confidence levels, the FMVS decision matrix D_H can be transformed into the following three CNNCS matrices:

$$D_{\varphi=0.1}=\begin{bmatrix}\langle[0.7184, 0.8416], 0.78\rangle & \langle[0.6471, 0.7129], 0.68\rangle & \langle[0.7584, 0.8816], 0.82\rangle & \langle[0.7471, 0.8129], 0.78\rangle\\ \langle[0.6997, 0.7803], 0.74\rangle & \langle[0.6184, 0.7416], 0.68\rangle & \langle[0.7197, 0.8003], 0.76\rangle & \langle[0.6584, 0.7816], 0.72\rangle\\ \langle[0.5997, 0.6803], 0.64\rangle & \langle[0.6761, 0.8439], 0.76\rangle & \langle[0.7480, 0.8520], 0.80\rangle & \langle[0.5942, 0.7258], 0.66\rangle\\ \langle[0.6480, 0.7520], 0.70\rangle & \langle[0.6264, 0.7736], 0.70\rangle & \langle[0.6480, 0.7520], 0.70\rangle & \langle[0.6184, 0.7416], 0.68\rangle\\ \langle[0.7197, 0.8003], 0.76\rangle & \langle[0.7000, 0.7000], 0.70\rangle & \langle[0.6471, 0.7129], 0.68\rangle & \langle[0.5742, 0.7058], 0.64\rangle\\ \langle[0.6480, 0.7520], 0.70\rangle & \langle[0.6584, 0.7816], 0.72\rangle & \langle[0.5997, 0.6803], 0.64\rangle & \langle[0.5991, 0.8409], 0.72\rangle\end{bmatrix}.$$

$$D_{\varphi=0.05}=\begin{bmatrix}\langle[0.7067, 0.8533], 0.78\rangle & \langle[0.6408, 0.7192], 0.68\rangle & \langle[0.7467, 0.8933], 0.82\rangle & \langle[0.7408, 0.8192], 0.78\rangle\\ \langle[0.6920, 0.7880], 0.74\rangle & \langle[0.6067, 0.7533], 0.68\rangle & \langle[0.7120, 0.8080], 0.76\rangle & \langle[0.6467, 0.7933], 0.72\rangle\\ \langle[0.5920, 0.6880], 0.64\rangle & \langle[0.6601, 0.8599], 0.76\rangle & \langle[0.7380, 0.8620], 0.80\rangle & \langle[0.5816, 0.7384], 0.66\rangle\\ \langle[0.6380, 0.7620], 0.70\rangle & \langle[0.6123, 0.7877], 0.70\rangle & \langle[0.6380, 0.7620], 0.70\rangle & \langle[0.6067, 0.7533], 0.68\rangle\\ \langle[0.7120, 0.8080], 0.76\rangle & \langle[0.7000, 0.7000], 0.70\rangle & \langle[0.6408, 0.7192], 0.68\rangle & \langle[0.5616, 0.7184], 0.64\rangle\\ \langle[0.6380, 0.7620], 0.70\rangle & \langle[0.6467, 0.7933], 0.72\rangle & \langle[0.5920, 0.6880], 0.64\rangle & \langle[0.5760, 0.8640], 0.72\rangle\end{bmatrix},$$

$$D_{\varphi=0.01}=\begin{bmatrix}\langle[0.6836, 0.8764], 0.78\rangle & \langle[0.6285, 0.7315], 0.68\rangle & \langle[0.7236, 0.9164], 0.82\rangle & \langle[0.7285, 0.8315], 0.78\rangle\\ \langle[0.6769, 0.8031], 0.74\rangle & \langle[0.5836, 0.7764], 0.68\rangle & \langle[0.6969, 0.8231], 0.76\rangle & \langle[0.6236, 0.8164], 0.72\rangle\\ \langle[0.5769, 0.7031], 0.64\rangle & \langle[0.6286, 0.8914], 0.76\rangle & \langle[0.7185, 0.8815], 0.80\rangle & \langle[0.5570, 0.7630], 0.66\rangle\\ \langle[0.6185, 0.7815], 0.70\rangle & \langle[0.5848, 0.8152], 0.70\rangle & \langle[0.6185, 0.7815], 0.70\rangle & \langle[0.5836, 0.7764], 0.68\rangle\\ \langle[0.6969, 0.8231], 0.76\rangle & \langle[0.7000, 0.7000], 0.70\rangle & \langle[0.6285, 0.7315], 0.68\rangle & \langle[0.5370, 0.7430], 0.64\rangle\\ \langle[0.6185, 0.7815], 0.70\rangle & \langle[0.6236, 0.8164], 0.72\rangle & \langle[0.5769, 0.7031], 0.64\rangle & \langle[0.5307, 0.9093], 0.72\rangle\end{bmatrix}.$$

Step 3: Using Eq. (8), the weighted ESM values of $Ew_{\varphi}(G_{j\varphi}, G^*)$ are shown in Table 1.

Table 1. Decision results corresponding to the 90%, 95% and 99% confidence levels

φ	$t_{\varphi/2}$	$Ew_{\varphi}(G_{j\varphi}, G^*)$	Sorting order	Optimal choice
0.1	1.645	0.8220, 0.7735, 0.7587, 0.7361, 0.7318, 0.7397	$Me_1 > Me_2 > Me_3 > Me_6 > Me_4 > Me_5$	Me_1
0.05	1.96	0.8203, 0.7715, 0.7557, 0.7336, 0.7306, 0.7354	$Me_1 > Me_2 > Me_3 > Me_6 > Me_4 > Me_5$	Me_1
0.01	2.576	0.8163, 0.7666, 0.7485, 0.7274, 0.7278, 0.7250	$Me_1 > Me_2 > Me_3 > Me_5 > Me_4 > Me_6$	Me_1

Step 4: The six alternatives are sorted and the optimal choice is determined by the largest weighted ESM value, then all decision results corresponding to the 90%, 95%, and 99% confidence levels are shown in Table 1.

5.2 Results and discussions

In view of the decision results in Table 1, different confidence levels can impact on the sorting orders of the six alternatives, then the optimal alternative always is Me_1. By comparing existing DM models in the scenarios of FMSs and CSs [13, 16, 17, 18], our new DM model reveals the following main merits:

(i) The proposed information transformation technique from FMVSs to CNNCSs can make the information expression more reasonable and confident and avoid operation problems between different fuzzy sequence lengths in FMVSs since CNNCS contains CNNs (confidence intervals) and average values. Then, CNN can reflect the probabilistic estimation of fuzzy values related to some confidence level to ensure the probabilistic reliability of fuzzy values falling within CNN.

(ii) Our new group DM model based on the weighted ESM of CNNCSs can reflect its decision flexibility depending on specified confidence levels. Then, decision makers can choose their optimal alternative according to their preference for confidence levels so as to satisfy some actual applications or requirements.

(iii) To some extent, existing CS is only a special case of CNNCS. In terms of a probabilistic viewpoint, existing CSs lack a confidence level in group DM problems, which shows its defect in the probabilistic estimation of the group evaluation values; while CNNCS contains both CNNs and average values, which can reflect the confidence level and magnitude of the group evaluation values. Therefore, our new group DM model indicates its obvious superiority over the existing DM models in the scenarios of FMSs and CSs.

6. Conclusions

Based on a confidence level of small sample data (the collection of several fuzzy values), this paper proposed a transformation technique from FMVSs to CNNCSs to reasonably express the mixed information of CNN and mean of fuzzy sequences. In the group DM process, the advantage of CNNCSs is that CNNCSs can effectively ensure the group evaluation data and mean falling within CNNs (confidence intervals) in light of a confidence level and a distribution status of the group evaluation data and solve the operational issue between different fuzzy sequence lengths in the scenario of FMVSs. Then, the proposed ESM of CNNCSs can make the similarity measure more reasonable and confident since it is closely related to confidence levels and normal distribution. Moreover, it also implies the measure flexibility corresponding to different confidence levels. The developed group DM model based on the proposed ESM of CNNCSs can not only make decision results more flexible and confident depending on certain confidence level, but also ensure the credibility and effectiveness of the DM results from the perspective of probability estimation in the scenario of FMVSs. It is obvious that the developed group DM model of CNNCSs reveals its obvious superiority over the existing DM models of FMSs/CSs in the information conversion/expression and DM methods.

Since this original study proposed the transformation technique from FMVSs to CNNCSs and the group DM model of CNNCSs for the first time, they are only suitable for group DM problems under the normal distribution condition of the group evaluation data (FMVSs), which shows their limitation in group DM applications. Therefore, we shall further develop other transformation techniques and group DM models and their applications, such as medical diagnosis, image processing, and production programming problems, as future research directions.

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References

1. Zadeh., L.A. Fuzzy sets. *Information and Control* **1965**, 8, 338–353.
2. Capuano, N.; Chiclana, F.; Fujita, H.; Herrera-Viedma, E.; Loia, V. Fuzzy group decision making with incomplete information guided by social influence. *IEEE Transactions on Fuzzy Systems* **2017**, 26(3), 1704–1718.
3. Wang, Y. M.; Elhag, T. M. A fuzzy group decision making approach for bridge risk assessment. *Computers & Industrial Engineering* **2007**, 53(1), 137–148.
4. Cabrerizo, F. J.; Moreno, J. M.; Pérez, I. J.; Herrera-Viedma, E. Analyzing consensus approaches in fuzzy group decision making: advantages and drawbacks. *Soft Computing* **2010**, 14(5), 451–463.
5. Banaeian, N.; Mobli, H.; Fahimnia, B.; Nielsen, I. E.; Omid, M. Green supplier selection using fuzzy group decision making methods: A case study from the agri-food industry. *Computers & Operations Research* **2018**, 89, 337–347.
6. Capuano, N.; Chiclana, F.; Herrera-Viedma, E.; Fujita, H.; Loia, V. Fuzzy group decision making for influence-aware recommendations. *Computers in Human Behavior* **2019**, 101, 371–379.
7. Yager, R.R. On the theory of bags. *International Journal of General Systems* **1986**, 13(1), 23–37.
8. Miyamoto, S. Generalized bags, bag relations, and applications to data analysis and decision making. *Modeling Decisions for Artificial Intelligence* **2009**, 5861, 37–54.
9. Pękala, B.; Bentkowska, U.; Szkoła, J.; Rząsa, W.; Kosior, D.; Fernandez, J.; Miguel, L.D.; Bustince, H. *General local properties of fuzzy relations and fuzzy multisets used to an algorithm for group decision making*. In: 2020 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Glasgow, UK, **2020**, pp. 1–8.
10. Miyamoto, S. *Fuzzy multisets and fuzzy clustering of documents*. In: The 10th IEEE International Conference on Fuzzy Systems. Melbourne, Australia, **2001**, pp. 1539–1542.
11. Fatma Taher , Ahmed Abdelaziz, Neutrosophic C-Means Clustering with Optimal Machine Learning Enabled Skin Lesion Segmentation and Classification, International Journal of Neutrosophic Science, Vol. 19 , No. 1 , (2022) : 177-187
12. Mizutani, K.; Inokuchi, R.; Miyamoto, S. Algorithms of nonlinear document clustering based on fuzzy multiset model. *International Journal of Intelligent Systems* **2008**, 23(2), 176–198.
13. Fu, J.; Ye, J.; Xie, L. Group decision-making model of renal cancer surgery options using entropy fuzzy element Aczel-Alsina weighted aggregation operators under the environment of fuzzy multi-sets. *CMES-Computer Modeling in Engineering and Sciences* **2021**, 130(3), 1751–1769.
14. Jun, Y.B.; Jung, S.T.; Kim, M.S. Cubic subgroups. *Annals of Fuzzy Mathematics and Informatics* **2011**, 2(1), 9–15.
15. Jun, Y.B.; Kim, C.S.; Yang, K.O. Cubic sets. *Annals of Fuzzy Mathematics and Informatics* **2012**, 4(1), 83–98.
16. Fahmi, A.; Amin, F.; Abdullah, S.; Ali, A. Cubic fuzzy Einstein aggregation operators and its application to decision-making. *International Journal of Systems Science* **2018**, 49(11), 2385–2397.
17. Ayub, S.; Abdullah, S.; Ghani, F.; Qiyas, M.; Yaqub Khan, M. Cubic fuzzy Heronian mean Dombi aggregation operators and their application on multi-attribute decision-making problem. *Soft Computing* **2021**, 25(6), 4175–4189.
18. Fahmi, A.; Abdullah, S.; Amin, F.; Ali, A. Precursor selection for Sol–Gel synthesis of titanium carbide nanopowders by a new cubic fuzzy multi-attribute group decision-making model. *Journal of Intelligent Systems* **2019**, 28(5), 699–720.
19. Fu, J.; Ye, J.; Cui, W. An evaluation method of risk grades for prostate cancer using similarity measure of cubic hesitant fuzzy sets. *Journal of biomedical informatics* **2018**, 87, 131–137.
20. Dragisa Stanujkic , Darjan Karabasevic , Florentin Smarandache , Gabrijela Popovic, A Novel Approach for Assessing the Reliability of Data Contained in a Single Valued Neutrosophic Number and its Application in Multiple Criteria Decision Making, International Journal of Neutrosophic Science, vol. 11 , No.1 , (2020) : 22-29

21. Mahmood, T.; Mehmood, F.; Khan, Q. Some generalized aggregation operators for cubic hesitant fuzzy sets and their applications to multi criteria decision making. *Punjab University Journal of Mathematics* **2017**, 49(1), 31–49.
22. Ye, J.; Du, S.G.; Yong, R.; Zhang, F. W. Weighted aggregation operators of fuzzy credibility cubic numbers and their decision making strategy for slope design schemes. *Current Chinese Computer Science* **2020**, 1(1), 28–34.
23. Du, C.; Ye, J. Hybrid weighted aggregation operator of cubic fuzzy-consistency elements and their group decision-making model in cubic fuzzy multi-valued setting. *Journal of Intelligent & Fuzzy Systems* **2021**, 41(6), 7373–7386.
24. Smarandache, F. *Neutrosophy: Neutrosophic probability, set, and logic*. American Research Press, Rehoboth, **1998**.
25. Smarandache, F. *Introduction to neutrosophic measure, neutrosophic integral, and neutrosophic probability*. Sitech & Education Publishing, Columbus, **2013**.
26. Smarandache, F. *Introduction to neutrosophic statistics.* Sitech & Education Publishing, Columbus, **2014**.
27. Ye, J. Bidirectional projection method for multiple attribute group decision making with neutrosophic numbers. *Neural Computing and Applications* **2017**, 28(5), 1021–1029.
28. Du, C.; Ye, J. Weighted parameterized correlation coefficients of indeterminacy fuzzy multisets and their multicriteria group decision making method with different decision risks. *CMES-Computer Modeling in Engineering & Sciences* **2021**, 129(1), 341–354.
29. Lv, G.D.; Du, S.G.; Ye, J. Confidence neutrosophic number linear programming methods based on probability distributions and their applications in production planning problems. *Mathematical Problems in Engineering*, **2022**, Article ID 5243797, https://doi.org/10.1155/2022/5243797

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