



Fundamental Homomorphism Theorems for Neutrosophic Triplet Module

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Abstract: In this chapter, our aim is to prove neutro-isomorphism theorems. We define the quotient NT quotient Module and prove the fundamental theorem of neutro-homomorphism. Also, we present and prove the first neutro-isomorphism theorem for neutrosophic triplet Modules, the second neutro-isomorphism theorem for neutrosophic triplet Modules, the third neutro-Isomorphism theorem for neutrosophic triplet Modules and a few special cases.

Keywords: NT submodule, NT R – module, NT quotient Module, Neutro- homomorphism, neutro-isomorphism

1. Introduction

In 1980, Smarandache presented neutrosophy, a part of philosophy. Neutrosophy, which is neutrosophic logic, probability depend on the set in [1]. Neutrosophic logic is the logic of some general concepts such as fuzzy logic presented by Zadeh in [2] and Provided by Atanassov intuitive fuzzy logic in [3]. Fuzzy sets membership function but has an intuitive fuzzy set membership function and non-function and does not define membership indeterminacy. But; neutrosophic set includes all the functions. Many researchers have studied the concept neutrosophic theory and its application to issue multiple-criteria decision analysis in [4-11]. Sahin M., and Kargin A., investigated NT metric space and NT normed space in [12]. Lately, Olgun at al. introduced the neutrosophic module in [13]; Şahin at al. presented Neutrosophic soft lattices in [14]; soft normed rings in [15]; centroid single valued neutrosophic triangular number and its applications in [16]; centroid single valued neutrosophic number and its applications in [17]. Ji at al. searched multi – valued neutrosophic environments and its applications in [18]. Also, Smarandache at al. searched NT theory in [19] and NT groups in [20, 21]. A NT has a form $\langle m, \text{neut}(m), \text{anti}(m) \rangle$ where; $\text{neut}(m)$ is neutral of “m” and $\text{anti}(m)$ is opposite of “m”. Moreover, $\text{neut}(m)$ is different from the classical unitary element and NT group is different from the classical group as well. Lately, Smarandache at al. investigated the NT field [22] and the NT ring [23]. Şahin at al. presented NT metric space, NT vector space and NT normed space in [24] and NT inner product in [25]. Smarandache at al. searched NT G- Module in [26]. Bal at al. searched NT cosets and quotient groups in [27]. Şahin at al. presented fixed point theorem for NT partial metric space and Neutrosophic triplet v – generalized

metric space in [28-29]. Çelik at al. searched fundamental homomorphism theorems for NETGs in [30] and Çelik at al. Searched neutrosophic triplet R-module in [31]

The concept of an R – module over a ring is a general term of the notion of vector space. The basic structure of Abelian rings, can be more common. Because modular theory is more complicated than the structure of a vector space. Lately, Ai at al. defined the irreducible modules and fusion rules for parafermion vertex operator algebras in [32] and Creutzig at al. introduced Braided tensor categories of admissible modules for affine lie algebras in [33].

In this study, we examine the concept of NT R-Modules. So we obtain a new algebraic structures on NT groups and NT ring. In section 2, we give basic definitions of NT sets, NT groups, NT ring, NT vector space, Neutro-Monomorphism, Neutro-Epimorphism, and Neutro-Isomorphism . In section 3, we define the quotient NT quotient Module and prove the fundamental theorem of neutro-homomorphism. Also, we present and prove the first neutro-isomorphism theorem for neutrosopfic triplet Modules, the second neutro-isomorphism theorem for neutrosopfic triplet Modules, the third neutro-Isomorphism theorem for neutrosopfic triplet Modules and a few special cases. Also, we explain the NT quotient R-module. Finally, in Chapter 4, we give some results.

2. Preliminaries

In this section, we present the basic definitions that are important for the development of the paper.

Definition 2.1: [21] Let N be a set together with a binary operation ∇ . Then, N is called a NT set if for any $k \in N$ there exists a neutral of “ k ” called $neut(k)$ that is different from the classical algebraic unitary element and an opposite of “ k ” called $anti(k)$ with $neut(k)$ and $anti(k)$ belonging to N , such that

$$k \nabla neut(k) = neut(k) \nabla k = k,$$

and

$$k \nabla anti(k) = anti(k) \nabla k = neut(k).$$

Definition 2.2: [21] Let (N, ∇) be a NT set. Then, N is called a NT group if the following conditions hold.

- (1) If (N, ∇) is well-defined, i.e., for any $k, l \in N$, one has $k \nabla l \in N$.
- (2) If (N, ∇) is associative, i.e., $(k \nabla l) \nabla m = k \nabla (l \nabla m)$ for all $k, l, m \in N$.

Definition 2.3: [24] Let $(NTF, \nabla_1, \blacksquare_1)$ be a NT field, and let $(NTV, \nabla_2, \blacksquare_2)$ be a NT set together with binary operations " ∇_2 " and " \blacksquare_2 ". Then $(NTV, \nabla_2, \blacksquare_2)$ is called a NT vector space if the following conditions hold. For all $p, r \in NTV$, and for all $t \in NTF$, such that $p \nabla_2 r \in NTV$ and $p \blacksquare_2 t \in NTV$ [24];

$$(1) (p \nabla_2 r) \nabla_2 s = p \nabla_2 (r \nabla_2 s); p, r, s \in NTV;$$

$$(2) p \nabla_2 r = r \nabla_2 p; p, r \in NTV;$$

$$(3) (r \nabla_2 p) \blacksquare_2 t = (r \blacksquare_2 t) \nabla_2 (p \blacksquare_2 t); t \in NTF \text{ and } p, r \in NTV;$$

$$(4) (t \nabla_1 c) \blacksquare_2 p = (t \blacksquare_2 p) \nabla_1 (c \blacksquare_2 p); t, c \in NTF \text{ and } p \in NTV;$$

$$(5) (t \blacksquare_1 c) \blacksquare_2 p = t \blacksquare_1 (c \blacksquare_2 p); t, c \in NTF \text{ and } p \in NTV;$$

$$(6) \text{ There exists any } t \in NTF \ni p \blacksquare_2 \text{neut}(t) = \text{neut}(t) \blacksquare_2 p = p; p \in NTV.$$

Definition 2.4: [26] Let (G, ∇) be a NT group, $(NTV, \nabla_1, \blacksquare_1)$ be a NT vector space on a NT field $(NTF, \nabla_2, \blacksquare_2)$, and $g \nabla l \in NTV$ for $g \in G, l \in NTV$. If the following conditions are satisfied, then $(NTV, \nabla_1, \blacksquare_1)$ is called NT G-module.

$$a) \text{ There exists } g \in G \ni k * \text{neut}(g) = \text{neut}(g) * k = k, \text{ for every } k \in NTV;$$

$$b) l \nabla_1 (g \nabla_1 h) = (l \nabla_1 g) \nabla_1 h, \forall l \in NTV; g, h \in G;$$

$$c) (r_1 \blacksquare_1 s_1 \nabla_1 r_2 \blacksquare_1 s_2) \nabla g = x \blacksquare_1 (h \nabla g) \nabla_1 y \blacksquare_1 (l \nabla g), \forall x, y \in NTF; h, l \in NTV; g \in G.$$

Definition 2.5: [23] The NT ring is a set endowed with two binary laws $(M, *, \#)$ such that,

$$a) (M, *) \text{ is a abelian NT group; which means that:}$$

- $(M, *)$ is a commutative NT with respect to the law $*$ (i.e. if x belongs to M , then $neut(x)$ and $anti(x)$, defined with respect to the law $*$, also belong to M)
 - The law $*$ is well – defined, associative, and commutative on M (as in the classical sense);
- b) $(M, *)$ is a set such that the law $\#$ on M is well-defined and associative (as in the classical sense);
- c) The law $\#$ is distributive with respect to the law $*$ (as in the classical sense)

Definition 2.6: Let $(NTR, \nabla, \blacksquare)$ be a commutative NT ring and let $(NTM, *, \circ)$ be a NT abelian group and \circ be a binary operation such that $\circ: NTR \times NTM \rightarrow NTM$. Then $(NTM, *, \circ)$ is called a NT R-Module on $(NTR, \nabla, \blacksquare)$ if the following conditions are satisfied. Where,

- 1) $p \circ (r*s) = (p \circ r)* (p \circ s), \forall r, s \in NTM$ and $p \in NTR$.
- 2) $(p \nabla k) \circ r = (p \nabla r) \circ (k \nabla r), \forall p, k \in NTR$ and $\forall r \in NTM$
- 3) $(p \blacksquare k) \circ r = p \blacksquare (k \circ r), \forall r, s \in NTR$ and $\forall m \in NTM$
- 4) For all $m \in NTM$; there exists at least a $c \in NTR$ such that $m \circ neut(c) = neut(c) \circ m = m$. Where, $neut(c)$ is neutral element of c for \blacksquare .

Definition 2.7: Let $(NTM, *, \circ)$ be a NT R-Module on NT ring $(NTR, \nabla, \blacksquare)$ and $NTSM \subset NTM$. Then $(NTSM, *, \circ)$ is called NT R - submodule of $(NTM, *, \circ)$, if $(NTSM, *, \circ)$ is a NT R – module on NT ring $(NTR, \nabla, \blacksquare)$.

Definition 2.7: (NTM_1, \circ_1) be a NT R-module on NT ring $(NTR, \nabla, \blacksquare)$ and $(NTM_2, *_2, \circ_2)$ be a NT R-module on NT ring $(NTR, \nabla, \blacksquare)$. A mapping $f: NTM_1 \rightarrow NTM_2$ is said to be NT R-module homomorphism when

$$f((r \circ_1 m) *_1 (s \circ_1 n)) = (r \circ_2 f(m)) *_2 (s \circ_2 f(n)), \text{ for all } r, s \in \text{NTR and } m, n \in \text{NTM}_1.$$

Definition 2.8: Assume that $(N_1, *)$ and (N_2, \circ) be two NETG's. If a mapping $f: N_1 \rightarrow N_2$ of NETG is only one to one (injective) f is called neutro-monomorphism.

Definition 2.9: Let $(N_1, *)$ and (N_2, \circ) be two NETG's. If a mapping $f: N_1 \rightarrow N_2$ is only onto (surjective) f is called neutro-epimorphism.

Definition 2.9: Let $(N_1, *)$ and (N_2, \circ) be two NETGs. If a mapping $f: N_1 \rightarrow N_2$ neutro-homomorphism is one to one and onto f is called neutro-isomorphism. Here, N_1 and N_2 are called neutro-isomorphic and denoted as $N_1 \cong N_2$.

3. Quotient NTM and Neutro-Isomorphism

In this chapter, We prove neutro-isomorphism theorems. we define the quotient NTM and prove the fundamental theorem of neutro-homomorphism. We also prove the first neutro-isomorphism theorem for neutrosophic triplet Modules, the second neutro-isomorphism theorem for neutrosophic triplet Modules, the third neutro-Isomorphism theorem for neutrosophic triplet Modules and a few special cases.

Definition 3.1: Let NTM, NTM' be neutrosophic triplet left modules over the neutrosophic triplet ring R. A map $\delta: \text{NTM} \rightarrow \text{NTM}'$ is called a neutrosophic triplet left R-module homomorphism if :

1. δ is a neutrosophic triplet group neutro-homomorphism, that is if, for every $m, n \in \text{NTM}$ we have $\delta(m + n) = \delta(m) + \delta(n)$;
2. For every $r \in R$ and for every $m \in M$ we have $\delta(r \cdot m) = r \cdot \delta(m)$

If $\delta: \text{NTM} \rightarrow \text{NTM}'$ is a neutrosophic triplet R-module neutro-homomorphism we say that:

- i) δ is a neutro-monomorphism if the map δ is injective ;
- ii) δ is a neutro-epimorphism if the map δ is surjective ;
- iii) δ is an isomorphism if the map δ is bijective.

We will say that NTM and NTM' are neutro-isomorphic and we will write $\text{NTM} \cong \text{NTM}'$ if there exists a neutro-isomorphism $\delta: \text{NTM} \rightarrow \text{NTM}'$. Observe that, in this case, the inverse map of δ , $\delta^{-1}: \text{NTM}' \rightarrow \text{NTM}$ is also a module isomorphism.

Example 3.2. Let R be a neutrosophic triplet ring. Given an element $a \in R$ the map

$$\begin{aligned} \delta_a: R &\rightarrow R \\ r &\rightarrow r \cdot a \end{aligned}$$

is a left NTM neutro-homomorphism from ${}_R R$ into ${}_R R$. Observe that, if $a \neq \text{neut}(a)$, then δ_a is not a NTR neutro-homomorphism.

Theorem 3.3. Let R be a NTR, let M be a NTM and let H be a neutrosophic triplet R -Submodule. We define a left NTM structure on the neutrosophic triplet abelian group M/H by neutrosophic triplet setting, for every $\dot{r} \in R$ and for every $\dot{m} \in M$, $\dot{r} \cdot (\dot{m} + H) = (\dot{r} \cdot \dot{m}) + H$. Moreover, with respect to this structure, the canonical projection $\delta: M \rightarrow M/H$ becomes a surjective neutrosophic triplet R -module homomorphism.

Proof. We have first to show that (1) is well defined, that is, given any $r \in R, m, m' \in M$ such that $m+H = m'+H$ (i.e. $m-m' \in H$), we have that $(r \cdot m)+H = r \cdot m'+H$ (i.e. $r \cdot m - r \cdot m' \in H$). But $m - m' \in H$ implies that $r \cdot m - r \cdot m' = r \cdot (m - m') \in H$ as H is a submodule of M . Let now $k, l \in R, m, n \in M$. We have:

$$k \cdot [(m + H) + (n + H)] = k \cdot [(m + n) + H] = (k \cdot (m + n)) + H = (k \cdot m + k \cdot n) + H = (k \cdot m + H) + (k \cdot n + H) = k \cdot (m + H) + k \cdot (n + H);$$

$$(k + l) \cdot (m + H) = ((k + l) \cdot m) + H = (k \cdot m + l \cdot m) + H = (k \cdot m + H) + (l \cdot m + H) = k \cdot (m + H) + l \cdot (m + H); (k \cdot l) \cdot (m + H) = ((k \cdot l) \cdot m) + H = (k \cdot (l \cdot m)) + H = k \cdot (l \cdot m + H) = k \cdot (l \cdot (m + H)); neut(k, l)_R \cdot (m + H) = (neut(k, l)_R \cdot m) + H = m + H.$$

Finally: $\partial H (k \cdot m) = k \cdot m + H = k \cdot (m + H) = k \cdot \partial H (m)$.

Definition 3.4. Let NTM be a neutrosophic triplet left module over a neutrosophic triplet ring R and let H be a neutrosophic triplet submodule of M . The neutrosophic triplet left R -module having the neutrosophic triplet quotient group M/H for its underlying neutrosophic triplet abelian group is called the neutrosophic triplet quotient module (or a neutrosophic triplet factor module) of NTM modulo $NTSM$ and is denoted by $NTM/NTSM$.

Theorem 3.5. Let R be a neutrosophic triplet ring and let $\delta : NTM \rightarrow NTM'$ be a neutrosophic triplet left R -module neutro-homomorphism. If S is a $NTSM$ of NTM contained in $Ker(\delta)$, then there exists a NTM neutro-homomorphism $\bar{\delta} : NTM/NTSM \rightarrow NTM'$ such that the diagram commutes

i.e. $\delta = \bar{\delta} \circ \partial S$.

Moreover:

1. $\bar{\delta}$ is unique with respect to this property;

2. $Im(\delta) = Im(\bar{\delta})$ and $Ker(\bar{\delta}) = Ker(\delta)/S$;

3. $\bar{\delta}$ is injective $\Leftrightarrow S = Ker(\delta)$.

Proof. In view of the Fundamental Theorem for the a neutrosophic triplet quotient group there exists a a neutrosophic triplet group neutro-homomorphism $\bar{\delta} : NTM/NTSM \rightarrow NTM'$ such that $\delta = \bar{\delta} \circ \partial S$.

Moreover: 1) such a neutrosophic triplet group neutro homomorphism is unique;

2) $Im(\delta) = Im(\bar{\delta}), Ker(\bar{\delta}) = Ker(\delta)/S$;

3) $\bar{\delta}$ is injective $\Leftrightarrow S = Ker(\delta)$.

Hence we only have to prove that, for every $m \in NTM$ and $r \in R$:

$$\bar{\delta} (r(m + S)) = r \cdot \bar{\delta} (m + S).$$

It is now an easy calculation to arrive at:

$$\bar{\delta} (r \cdot (m+S)) = \bar{\delta} (r \cdot m+S) = \bar{\delta} (\partial S (r \cdot m)) = \delta (r \cdot m) = r \cdot \delta (x) = r \cdot \bar{\delta} (\partial S (m)) = r \cdot (m+S).$$

Corollary 3.6. (First neutro-Isomorphism Theorem for NTM).

Let R be a NTR and $\delta : NTM \rightarrow NTM'$ be a NTLM neutro-homomorphism. Then the assignment

$$m + Ker(\delta) \rightarrow \delta (m)$$

defines an neutro-isomorphism of neutrosophic triplet left R -modules

$$\tilde{\delta} : NTM/Ker(\delta) \rightarrow Im(\delta)$$

In particular, if δ is surjective, then $\tilde{\delta}$ is an neutro isomorphism and

$$NTM/Ker(\delta) \cong NTM'.$$

Theorem 3.7. (Second neutro-Isomorphism Theorem for NTM)

Let H and B be NTSM of a NTM over a NTR. Then $H \cap B$ and $H + B$ are neutrosophic triplet submodules of NTM and the assignment $m + (H \cap B) \rightarrow m + B$ defines an neutrosophic triplet R -module neutro-isomorphism from $H / (H \cap B)$ into $H + B / B$. Therefore:

$$H / (H \cap B) \cong H + B / B$$

Proof. We know that $H \cap B$ is a NTSM of NTM. Let $r \in R, s \in H \cap B$. Then $rs \in H$ and $rs \in B$, as H and B are neutrosophic triplet submodules of NTM. Therefore $r \cdot s \in H \cap B$. We know that $H + B$ is a neutrosophic triplet subgroup of NTM. Let $r \in R, s \in H + B$. Then there exist $m \in H$ and $n \in B$ such that $s = m + n$. Obviously $rm \in H$ and $rn \in B$, and hence $r \cdot s = r \cdot m + r \cdot n \in H + B$. In view of the Second neutro-Isomorphism Theorem for neutrosophic triplet groups, the assignment $m + (H \cap B) \rightarrow m + B$ defines a neutrosophic triplet group neutro-isomorphism $\delta : H / (H \cap B) \rightarrow H + B / B$. Let $r \in R, m \in H$, then we calculate:

$\delta (r(m + (H \cap B))) = \delta (rm + (H \cap B)) = rm + B = r(m + B) = r \delta (m + (H \cap B))$. Therefore δ is a neutrosophic triplet left R -module neutro-isomorphism.

Theorem 3.8. Let R be a NTR, $\delta : NTM \rightarrow NTM'$ be a neutrosophic triplet left R -module neutro-homomorphism. For every neutrosophic triplet submodule S of M containing $Ker(\delta)$ the assignment

$m + S \rightarrow \delta (m) + \delta (S)$ defines a neutro-isomorphism $\hat{\delta} : M/S \rightarrow Im(\delta)/\delta(S)$. Therefore

$$M/S \cong Im(\delta)/\delta(S).$$

Proof. We know that the assignment $m + S \rightarrow \delta (m) + \delta(S)$ defines a neutrosophic triplet group neutro-isomorphism $\pi = \hat{\delta}_N : M/S \rightarrow Im(\delta)/S$.

Let $r \in R, m \in S$. We have :

$\pi (r(m + S)) = \pi (rm + S) = \delta (rm) + \delta (S) = (r \delta (m)) + \delta (S) = r(\delta (m) + \delta(S)) = r \pi (m + S)$ Therefore π is a neutrosophic triplet left R -module neutro-isomorphism.

Corollary 3.9. (Third neutro-Isomorphism Theorem for NTM)

Let H and B be neutrosophic triplet submodules of a NTM over a NTR and assume that $H \subseteq B$.

Then the assignment $m + B \rightarrow (m + H) + H / B$. Defines a neutrosophic triplet left R -module neutro-isomorphism from M/H into $M/H / B / H$. Therefore

$$M/B \cong M/H / B / H.$$

Proof. Apply Theorem 3.8 to $\partial_H : M \rightarrow M/H$, recalling that $\partial_H (B) = B / H$.

4. Conclusions

This article mainly focused on fundamental homomorphism theorems for neutrosophic R -modules. We gave and proved the fundamental theorem of neutro-homomorphism, as well as first, second and third neutro-isomorphism theorems explained for NTM. Furthermore, we define neutro-monomorphism, neutro-epimorphism. By applying them to neutrosophic algebraic structures. We looked at it as closely related as different systems. Using the concept of the fundamental theorem of neutro-Homomorphism and neutro-isomorphism theorems, the relationship between neutrosophic algebraic structures was studied.

Abbreviations

NT: Neutrosophic triplet

NTS: Neutrosophic triplet set

NETG: Neutrosophic extended triplet group

NTM: Neutrosophic triplet R -module

NTSM: Neutrosophic triplet R -submodule

NTLM: Neutrosophic triplet left R -module

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