



Fundamental Homomorphism Theorems for Neutrosophic Triplet Module

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Abstract: In this chapter, our aim is to prove neutro-isomorphism theorems. We define the quotient NT quotient Module and prove the fundamental theorem of neutro-homomorphism. Also, we present and prove the first neutro-isomorphism theorem for neutrosopfic triplet Modules, the second neutro-isomorphism theorem for neutrosopfic triplet Modules, the third neutro-Isomorphism theorem for neutrosopfic triplet Modules and a few special cases.

Keywords: NT submodule, NT R – module, NT quotient Module, Neutro- homomorphism, neutro-isomorphism

1. Introduction

In 1980, Smarandache presented neutrosophy, a part of philosophy. Neutrosophy, which is neutrosophic logic, probability depend on the set in [1]. Neutrosophic logic is the logic of some general concepts such as fuzzy logic presented by Zadeh in [2] and Provided by Atanassov intuitive fuzzy logic in [3].Fuzzy sets membership function but has an intuitive fuzzy set membership function and non-function and does not define membership indeterminacy. But; neutrosophic set includes all the functions. Many researchers have studied the concept neutrosophic theory and its application to issue multiple-criteria decision analysis.in [4-11]. Sahin M., and Kargin A., investigated NT metric space and NT normed space in [12]. Lately, Olgun at al. introduced the neutrosophic module in [13]; Şahin at al. presented Neutrosophic soft lattices in [14]; soft normed rings in [15]; centroid single valued neutrosophic triangular number and its applications in [16]; centroid single valued neutrosophic number and its applications in [17]. Ji at al. searched multi – valued neutrosophic environments and its applications in [18]. Also, Smarandache at al. searched NT theory in [19] and NT groups in [20, 21]. A NT has a form $\langle m, \text{neut}(m), \text{anti}(m) \rangle$ where; neut(m) is neutral of “m” and anti(m) is opposite of “m”. Moreover, neut(m) is different from the classical unitary element and NT group is different from the classical group as well. Lately, Smarandache at al. investigated the NT field [22] and the NT ring [23]. Şahin at al. presented NT metric space, NT vector space and NT normed space in [24] and NT inner product in [25]. Smarandache at al. searched NT G- Module in [26]. Bal at al. searched NT cosets and quotient groups in [27]. Şahin at al. presented fixed point theorem for NT partial metric space and Neutrosophic triplet v – generalized

metric space in [28-29]. Çelik at al. searched fundamental homomorphism theorems for NETGs in [30] and Çelik at al. Searched neutrosophic triplet R-module in [31]

The concept of an R – module over a ring is a general term of the notion of vector space. The basic structure of Abelian rings, can be more common. Because modular theory is more complicated than the structure of a vector space. Lately, Ai at al. defined the irreducible modules and fusion rules for parafermion vertex operator algebras in [32] and Creutzig at al. introduced Braided tensor categories of admissible modules for affine lie algebras in [33].

In this study, we examine the concept of NT R-Modules. So we obtain a new algebraic structures on NT groups and NT ring. In section 2, we give basic definitions of NT sets, NT groups, NT ring, NT vector space, Neutro-Monomorphism, Neutro-Epimorphism, and Neutro-Isomorphism . In section 3, we define the quotient NT quotient Module and prove the fundamental theorem of neutro-homomorphism. Also, we present and prove the first neutro-isomorphism theorem for neutrosopfic triplet Modules, the second neutro-isomorphism theorem for neutrosopfic triplet Modules, the third neutro-Isomorphism theorem for neutrosopfic triplet Modules and a few special cases. Also, we explain the NT quotient R-module. Finally, in Chapter 4, we give some results.

2. Preliminaries

In this section, we present the basic definitions that are important for the development of the paper.

Definition 2.1: [21] Let $\textcolor{blue}{N}$ be a set together with a binary operation $\textcolor{brown}{V}$. Then, $\textcolor{blue}{N}$ is called a NT set if

for any $k \in \textcolor{blue}{N}$ there exists a neutral of “ k ” called $\textcolor{brown}{neut}(k)$ that is different from the classical algebraic unitary element and an opposite of “ k ” called $\textcolor{brown}{anti}(k)$ with $\textcolor{brown}{neut}(k)$ and $\textcolor{brown}{anti}(k)$ belonging to $\textcolor{blue}{N}$, such that

$$k \textcolor{brown}{V} \textcolor{brown}{neut}(k) = \textcolor{brown}{neut}(k) \textcolor{brown}{V} k = k,$$

and

$$k \textcolor{brown}{V} \textcolor{brown}{anti}(k) = \textcolor{brown}{anti}(k) \textcolor{brown}{V} k = \textcolor{brown}{neut}(k).$$

Definition 2.2: [21] Let $(\textcolor{blue}{N}, \textcolor{brown}{V})$ be a NT set. Then, $\textcolor{blue}{N}$ is called a NT group if the following conditions hold.

(1) If $(\textcolor{blue}{N}, \textcolor{brown}{V})$ is well-defined, i.e., for any $k, l \in \textcolor{blue}{N}$, one has $k \textcolor{brown}{V} l \in \textcolor{blue}{N}$.

(2) If $(\textcolor{blue}{N}, \textcolor{brown}{V})$ is associative, i.e., $(k \textcolor{brown}{V} l) \textcolor{brown}{V} m = k \textcolor{brown}{V} (l \textcolor{brown}{V} m)$ for all $k, l, m \in \textcolor{blue}{N}$.

Definition 2.3: [24] Let $(NTF, \nabla_1, \blacksquare_1)$ be a NT field, and let $(NTV, \nabla_2, \blacksquare_2)$ be a NT set together with binary operations “ ∇_2 ” and “ \blacksquare_2 ”. Then $(NTV, \nabla_2, \blacksquare_2)$ is called a NT vector space if the following conditions hold. For all $p, r \in NTV$, and for all $t \in NTF$, such that $p\nabla_2 r \in NTV$ and $p\blacksquare_2 t \in NTV$ [24];

$$(1) (p\nabla_2 r) \nabla_2 s = p\nabla_2 (r\nabla_2 s); p, r, s \in NTV;$$

$$(2) p\nabla_2 r = r\nabla_2 p; p, r \in NTV;$$

$$(3) (r\nabla_2 p) \blacksquare_2 t = (r\blacksquare_2 t) \nabla_2 (p\blacksquare_2 t); t \in NTF and p, r \in NTV;$$

$$(4) (t\nabla_1 c) \blacksquare_2 p = (t\blacksquare_2 p) \nabla_1 (c\blacksquare_2 p); t, c \in NTF and p \in NTV;$$

$$(5) (t\blacksquare_1 c) \blacksquare_2 p = t\blacksquare_1 (c\blacksquare_2 p); t, c \in NTF and p \in NTV;$$

$$(6) \text{ There exists any } t \in NTF \ni p\blacksquare_2 \text{neut}(t) = \text{neut}(t)\blacksquare_2 p = p; p \in NTV.$$

Definition 2.4: [26] Let (G, ∇) be a NT group, $(NTV, \nabla_1, \blacksquare_1)$ be a NT vector space on a NT field $(NTF, \nabla_2, \blacksquare_2)$, and $g \nabla l \in NTV$ for $g \in G$, $l \in NTV$. If the following conditions are satisfied, then $(NTV, \nabla_1, \blacksquare_1)$ is called NT G-module.

a) There exists $g \in G \ni k * \text{neut}(g) = \text{neut}(g) * k = k$, for every $k \in NTV$;

b) $l\nabla_1(g\nabla_1 h) = (l\nabla_1 g)\nabla_1 h, \forall l \in NTV; g, h \in G$;

c) $(r_1\blacksquare_1 s_1 \nabla_1 r_2 \blacksquare_1 s_2)\nabla g = x\blacksquare_1 (h\nabla g)\nabla_1 y\blacksquare_1 (l\nabla g), \forall x, y \in NTF; h, l \in NTV; g \in G$.

Definition 2.5: [23] The NT ring is a set endowed with two binary laws $(M, *, \#)$ such that,

a) $(M, *)$ is a abelian NT group; which means that:

- $(M, *)$ is a commutative NT with respect to the law * (i.e. if x belongs to M , then $neut(x)$ and $anti(x)$, defined with respect to the law *, also belong to M)
 - The law * is well – defined, associative, and commutative on M (as in the classical sense);
- b) $(M, *)$ is a set such that the law # on M is well-defined and associative (as in the classical sense);
- c) The law # is distributive with respect to the law * (as in the classical sense)

Definition 2.6: Let $(NTR, \nabla, \blacksquare)$ be a commutative NT ring and let $(NTM, *)$ be a NT abelian group and \circ be a binary operation such that $\circ : NTR \times NTM \rightarrow NTM$. Then $(NTM, *, \circ)$ is called a NT R-Module on $(NTR, \nabla, \blacksquare)$ if the following conditions are satisfied. Where,

$$1) p \circ (r * s) = (p \circ r) * (p \circ s), \forall r, s \in NTM \text{ and } p \in NTR.$$

$$2) (p \nabla k) \circ r = (p \nabla r) \circ (k \nabla r), \forall p, k \in NTR \text{ and } \forall r \in NTM$$

$$3) (p \blacksquare k) \circ r = p \blacksquare (k \circ r), \forall r, s \in NTR \text{ and } \forall m \in NTM$$

$$4) \text{For all } m \in NTM; \text{ there exists at least a } c \in NTR \text{ such that } m \circ neut(c) = neut(c) \circ m = m. \text{ Where,} \\ neut(c) \text{ is neutral element of } c \text{ for } \blacksquare.$$

Definition 2.7: Let $(NTM, *, \circ)$ be a NT R-Module on NT ring $(NTR, \nabla, \blacksquare)$ and $NTSM \subset NTM$. Then $(NTSM, *, \circ)$ is called NT R - submodule of $(NTM, *, \circ)$, if $(NTSM, *, \circ)$ is a NT R – module on NT ring $(NTR, \nabla, \blacksquare)$.

Definition 2.7: $(NTM_1, *, \circ_1)$ be a NT R-module on NT ring $(NTR, \nabla, \blacksquare)$ and $(NTM_2, *, \circ_2)$ be a NT R-module on NT ring $(NTR, \nabla, \blacksquare)$. A mapping $f: NTM_1 \rightarrow NTM_2$ is said to be NT R-module homomorphism when

$$f((r^{\circ}_1 m) *_1 (s^{\circ}_1 n)) = (r^{\circ}_2 f(m)) *_2 (s^{\circ}_2 f(n)), \text{ for all } r, s \in NTR \text{ and } m, n \in NTM_1.$$

Definition 2.8: Assume that $(N_1, *)$ and (N_2, \circ) be two NETG's. If a mapping $f: N_1 \rightarrow N_2$ of NETG is only one to one (injective) f is called neutro-monomorphism.

Definition 2.9: Let $(N_1, *)$ and (N_2, \circ) be two NETG's. If a mapping $f: N_1 \rightarrow N_2$ is only onto (surjective) f is called neutro-epimorphism.

Definition 2.9: Let $(N_1, *)$ and (N_2, \circ) be two NETGs. If a mapping $f: N_1 \rightarrow N_2$ neutro-homomorphism is one to one and onto f is called neutro-isomorphism. Here, N_1 and N_2 are called neutro-isomorphic and denoted as $N_1 \cong N_2$.

3. Quotient NTM and Neutro-Isomorphism

In this chapter, We prove neutro-isomorphism theorems. we define the quotient NTM and prove the fundamental theorem of neutro-homomorphism. We also prove the first neutro-isomorphism theorem for neutrosopfic triplet Modules, the second neutro-isomorphism theorem for neutrosopfic triplet Modules, the third neutro-Isomorphism theorem for neutrosopfic triplet Modules and a few special cases.

Definition 3.1: Let NTM, NTM' be neutrosopfic triplet left modules over the neutrosopfic triplet ring R. A map $\delta: NTM \rightarrow NTM'$ is called a neutrosopfic triplet left R-module homomorphism if :

1. δ is a neutrosopfic triplet group neutro-homomorphism, that is if, for every $m, n \in NTM$ we have $\delta(m + n) = \delta(m) + \delta(n)$;
2. For every $r \in R$ and for every $m \in M$ we have $\delta(r \cdot m) = r \cdot \delta(m)$

If $\delta: NTM \rightarrow NTM'$ is a neutrosopfic triplet R-module neutro-homomorphism we say that:

- i) δ is a neutro-monomorphism if the map δ is injective ;
- ii) δ is a neutro-epimorphism if the map δ is surjective ;
- iii) δ is an isomorphism if the map δ is bijective.

We will say that NTM and NTM' are neutro-isomorphic and we will write $NTM \cong NTM'$ if there exists a neutro-isomorphism $\delta: NTM \rightarrow NTM'$. Observe that, in this case, the inverse map of δ , $\delta^{-1}: NTM' \rightarrow NTM$ is also a module isomorphism.

Example 3.2. Let R be a neutrosopfic triplet ring. Given an element $a \in R$ the map

$$\begin{aligned} \delta_a: R &\rightarrow R \\ r &\mapsto r \cdot ra \end{aligned}$$

is a left NTM neutro-homomorphism from rR into rR . Observe that, if

$a \neq \text{neut}(a)$, then δ_a is not a NTR neutro-homomorphism.

Theorem 3.3. Let R be a NTR, let M be a NTM and let H be a neutrosopfic triplet R -Submodule. We define a left NTM structure on the neutrosopfic triplet abelian group M / H by neutrosopfic triplet setting, for every $\dot{r} \in R$ and for every $\dot{m} \in M$, $\dot{r} \cdot (\dot{m} + H) = (\dot{r} \cdot \dot{m}) + H$. Moreover, with respect to this structure, the canonical projection $\delta: H: M \rightarrow M / H$ becomes a surjective neutrosopfic triplet R -module homomorphism.

Proof. We have first to show that (1) is well defined, that is, given any $\dot{r} \in R$, $\dot{m}, \dot{m}' \in M$ such that $\dot{m} + H = \dot{n} + H$ (i.e. $\dot{m} - \dot{n} \in H$), we have that $(\dot{r} \cdot \dot{m}) + H = \dot{r} \cdot \dot{n} + H$ (i.e. $\dot{r} \cdot \dot{m} - \dot{r} \cdot \dot{n} \in H$). But $\dot{m} - \dot{n} \in H$ implies that $\dot{r} \cdot \dot{m} - \dot{r} \cdot \dot{n} = \dot{r} \cdot (\dot{m} - \dot{n}) \in H$ as H is a submodule of M . Let now $k, l \in R$, $\dot{m}, \dot{n} \in R$. We have:

$$\begin{aligned} k \cdot [(\dot{m} + H) + (\dot{n} + H)] &= k \cdot [(\dot{m} + \dot{n}) + H] = (k \cdot (\dot{m} + \dot{n})) + H = (k \cdot \dot{m} + k \cdot \dot{n}) + H = (k \cdot \dot{m} + H) + (k \cdot \dot{n} + H) = \\ k \cdot (\dot{m} + H) + k \cdot (\dot{n} + H); \\ (k + l) \cdot (\dot{m} + H) &= ((k + l) \cdot \dot{m}) + H = (k \cdot \dot{m} + l \cdot \dot{m}) + H = (k \cdot \dot{m} + H) + (l \cdot \dot{m} + H) = k \cdot (\dot{m} + H) + l \cdot (\dot{m} + H); \\ (k \cdot l)(\dot{m} + H) &= ((k \cdot_R l) \cdot \dot{m}) + H = (k \cdot (l \cdot \dot{m})) + H = k \cdot (l \cdot \dot{m} + H) = k \cdot (l \cdot (\dot{m} + H)); \\ neut(k, l)_R \cdot (\dot{m} + H) &= (neut(k, l)_R \cdot \dot{m}) + H = \dot{m} + H. \end{aligned}$$

Finally: $\partial H(k \cdot \dot{m}) = k \cdot \dot{m} + H = k \cdot (\dot{m} + H) = k \cdot \partial H(\dot{m})$.

Definition 3.4. Let NTM be a neutrosophic triplet left module over a neutrosophic triplet ring R and let H be a neutrosophic triplet submodule of M . The neutrosophic triplet left R -module having the neutrosophic triplet quotient group M/H for its underlying neutrosophic triplet abelian group is called the neutrosophic triplet quotient module (or a neutrosophic triplet factor module) of NTM modulo $NTSM$ and is denoted by $NTM/NTSM$.

Theorem 3.5. Let R be a neutrosophic triplet ring and let $\delta : NTM \rightarrow NTM'$ be a neutrosophic triplet left R -module neutro-homomorphism. If S is a $NTSM$ of NTM contained in $Ker(\delta)$, then there exists a NTM neutro-homomorphism $\bar{\delta} : NTM/NTSM \rightarrow NTM'$ such that the diagram commutes

$$\text{i.e. } \delta = \bar{\delta} \circ \partial S.$$

Moreover:

1. $\bar{\delta}$ is unique with respect to this property;
2. $Im(\delta) = Im(\bar{\delta})$ and $Ker(\bar{\delta}) = Ker(\delta)/S$;
3. $\bar{\delta}$ is injective $\Leftrightarrow S = Ker(\delta)$.

Proof. In view of the Fundamental Theorem for the a neutrosophic triplet quotient group there exists a a neutrosophic triplet group neutro-homomorphism $\bar{\delta} : NTM/NTSM \rightarrow NTM'$ such that $\delta = \bar{\delta} \circ \partial S$.

Moreover: 1) such a neutrosophic triplet group neutro homomorphism is unique;

$$2) Im(\delta) = Im(\bar{\delta}), Ker(\bar{\delta}) = Ker(\delta)/S;$$

$$3) \bar{\delta} \text{ is injective} \Leftrightarrow S = Ker(\delta).$$

Hence we only have to prove that, for every $m \in NTM$ and $r \in R$:

$$\bar{\delta}(r(m + S)) = r \cdot \bar{\delta}(m + S).$$

It is now an easy calculation to arrive at:

$$\bar{\delta}(r \cdot (m+S)) = \bar{\delta}(r \cdot m + S) = \bar{\delta}(\partial S(r \cdot m)) = \delta(r \cdot m) = r \cdot \delta(m) = r \cdot \bar{\delta}(\partial S(m)) = r \cdot (m+S).$$

Corollary 3.6. (First neutro-Isomorphism Theorem for NTM).

Let R be a NTR and $\delta : NTM \rightarrow NTM'$ be a NTLM neutro-homomorphism. Then the assignment

$$m + Ker(\delta) \rightarrow \delta(m)$$

defines an neutro-isomorphism of neutrosophic triplet left R -modules

$$\tilde{\delta} : NTM/Ker(\delta) \rightarrow Im(\delta)$$

In particular, if δ is surjective, then $\tilde{\delta}$ is an neutro isomorphism and

$$NTM/Ker(\delta) \cong NTM'.$$

Theorem 3.7. (Second neutro-Isomorphism Theorem for NTM)

Let H and B be NTSM of a NTM over a NTR. Then $H \cap B$ and $H + B$ are neutrosophic triplet submodules of NTM and the assignment $m + (H \cap B) \rightarrow m + B$ defines an neutrosophic triplet R -module neutro-isomorphism from $H/(H \cap B)$ into $H + B / B$. Therefore:

$$H/(H \cap B) \cong H + B / B$$

Proof. We know that $H \cap B$ is a NTSM of NTM. Let $r \in R, s \in H \cap B$. Then $rs \in H$ and $rs \in B$, as H and B are neutrosophic triplet submodules of NTM. Therefore $rs \in H \cap B$. We know that $H + B$ is a neutrosophic triplet subgroup of NTM. Let $r \in R, s \in H + B$. Then there exist $m \in H$ and $n \in B$ such that $s = m + n$. Obviously $rm \in H$ and $rn \in B$, and hence $r \cdot s = r \cdot m + r \cdot n \in H + B$. In view of the Second neutro-Isomorphism Theorem for neutrosophic triplet groups, the assignment $m + (H \cap B) \rightarrow m + B$ defines a neutrosophic triplet group neutro-isomorphism $\delta : H/(H \cap B) \rightarrow H + B / B$. Let $r \in R, m \in H$, then we calculate:

$\delta(r(m + (H \cap B))) = \delta(rm + (H \cap B)) = rm + B = r(m + B) = r \delta(m + (H \cap B))$. Therefore δ is a neutrosophic triplet left R -module neutro-isomorphism.

Theorem 3.8. Let R be a NTR, $\delta : NTM \rightarrow NTM'$ be a neutrosophic triplet left R -module neutro-homomorphism. For every neutrosophic triplet submodule S of M containing $Ker(\delta)$ the assignment

$m + S \rightarrow \delta(m) + \delta(S)$ defines a neutro-isomorphism $\hat{\delta}_S : M/S \rightarrow Im(\delta)/\delta(S)$. Therefore

$$M/S \cong Im(\delta)/\delta(S).$$

Proof. We know that the assignment $m + S \rightarrow \delta(m) + \delta(S)$ defines a neutrosophic triplet group neutro-isomorphism $\pi = \hat{\delta}_N : M/S \rightarrow Im(\delta)/\delta(S)$.

Let $r \in R, m \in S$. We have :

$\pi(r(m + S)) = \pi(rm + S) = \delta(rm) + \delta(S) = (r\delta(m)) + \delta(S) = r(\delta(m) + \delta(S)) = r\pi(m + S)$ Therefore π is a neutrosophic triplet left R -module neutro-isomorphism.

Corollary 3.9. (Third neutro-Isomorphism Theorem for NTM)

Let H and B be neutrosophic triplet submodules of a NTM over a NTR and assume that $H \subseteq B$.

Then the assignment $m+ B \rightarrow (m + H)+H/ B$. Defines a neutrosophic triplet left R -module neutro-isomorphism from M/H into $M/H/ B /H$. Therefore

$$M/B \cong M/H/ B /H.$$

Proof. Apply Theorem 3.8 to $\partial_H : M \rightarrow M/H$, recalling that $\partial_H(B) = B / H$.

4. Conclusions

This article mainly focused on fundamental homomorphism theorems for neutrosophic R-modules. We gave and proved the fundamental theorem of neutro-homomorphism, as well as first, second and third neutro-isomorphism theorems explained for NTM. Furthermore, we define neutro-monomorphism, neutro-epimorphism. By applying them to neutrosophic algebraic structures. We looked at it as closely related as different systems. Using the concept of the fundamental theorem of neutro-Homomorphism and neutro-isomorphism theorems, the relationship between neutrosophic algebraic structures was studied.

Abbreviations

NT: Neutrosophic triplet

NTS: Neutrosophic triplet set

NETG: Neutrosophic extended triplet group

NTM: Neutrosophic triplet R-module

NTSM: Neutrosophic triplet R-submodule

NTLM: Neutrosophic triplet left R-module

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