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Fuzzy Hypersoft Expert Set with Application in Decision Making for the Best Selection of Product

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Abstract. Numerous researchers have made a few models dependent on soft set, to tackle issues in decision making and clinical analysis, yet a large portion of these models manage one expert. This causes an issue with the clients, particularly with the individuals who use polls in their work and studies. Accordingly we present another model i.e. fuzzy hypersoft expert set which not just addresses this constraint of fuzzy soft-like models by accentuating the assessment, all things considered, yet additionally settle the deficiency of soft set for disjoint attribute-valued sets comparing to distinct attributes. In this study, the existing concept of fuzzy soft expert set is generalized to fuzzy hypersoft expert set which is more flexible and useful. Some fundamental properties (i.e. subset, not set and equal set), results (i.e. commutative, associative, distributive and D Morgan Laws) and set-theoretic operations (i.e. complement, union, intersection AND, and OR) are discussed. An algorithm is proposed to solve decision-making problems and is applied to select the best product.

Keywords: Soft Set; Fuzzy Soft Set; Fuzzy Soft Expert Set; Hypersoft Set; Fuzzy Hypersoft Expert Set.

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1. **Introduction**

Zadeh [1] initiated fuzzy set theory as a basic model to tackle uncertainties in the data. Molodtsov [2] presented soft set theory that is supposed to be a new parameterized class of subsets of the universe of discourse, which addresses the inadequacy of fuzzy set-like structures for parameterization tools. It has helped the researcher (experts) to resolve efficiently the decision-making problems involving vagueness and uncertainty. The researchers [3–15] studied and broadened the concept of soft set and applied to different fields. The gluing concept of soft set with expert system initiated by Alkhazaleh et al. [16] to emphasize the due status of the opinions of all experts regarding taking any decision in decision-making system. Al-Quran et al. [17] proposed neutrosophic vague soft expert set theory, Alkhazaleh et al. [18] characterized fuzzy soft expert set. and its application. Bashir et al. [19, 20] presented possibility fuzzy soft expert set and fuzzy parameterized soft expert set. Sahin et al. [21] investigated neutrosophic soft expert sets. Alhazaymeh et al. [22,23] studied mapping on generalized vague soft expert set and generalized vague soft expert set. Alhazaymeh et al. [24] explained the application of generalized vague soft expert set in decision making. Hassan et al. [25] reviewed Q-neutrosophic soft expert set and its application in decision making. Ulucay et al. [26] studied generalized neutrosophic soft expert set for multiple-criteria decision-making. Al-Qudah et al. [27] explained bipolar fuzzy soft expert set and its application in decision making. Al-Qudah et al. [28] investigated complex multi-fuzzy soft expert set and its application. Al-Quran et al. [29] presented the com-plex neutrosophic soft expert set and its application in decision making. Pramanik et al. [30] studied the topsis for single valued neutrosophic soft expert set based multi-attribute decision making problems. Abu Qamar et al. [31] investigated the generalized Q-neutrosophic soft ex-pert set for decision under uncertainty. Adam et al. [32] characterized the multi Q-fuzzy soft expert set and its application. Ulucay et al. [33] presented the time-neutrosophic soft expert sets and its decision making problem. Al-Quran et al. [34] studied fuzzy parameterised single valued neutrosophic soft expert set theory and its application in decision making. Hazaymeh et al. [35] researched generalized fuzzy soft expert set.

There are many real life scenarios when we are to deal with disjoint attribute-valued set for distinct attributes. In 2018, Smarandache [36] addressed this inadequacy of soft with the development of new structure hypersoft set by replacing single attribute-valued function to multi-attribute valued function. In 2020, Saeed et al. [37, 38] extended the concept and discussed the fundamentals of hypersoft set such as hypersoft subset, complement, not hypersoft set, aggregation operators along with hypersoft set relation, sub relation,complement relation, function, matrices and operations on hypersoft matrices. In the same year, Mujahid et al. [39] discussed hypersoft points in different fuzzy-like environments. In 2020, Rahman et al. [40] defined complex hypersoft set and developed the hybrids of hypersoft set with complex fuzzy set, complex intuitionistic fuzzy set and complex neutrosophic set respectively. They also discussed their fundamentals i.e. subset, equal sets, null set, absolute set etc. and theoretic operations i.e. complement, union, intersection etc. In 2020, Rahman et al. [41] conceptualized convexity cum concavity on hypersoft set and presented its pictorial versions with illustrative examples. Recently the researchers [42–49] investigated on the theory of hypersoft set and developed certain its hybrids with discussion and applications in decision making.

Dealing with disjoint attribute-valued sets is of great importance and it is vital for sensible decisions in decision-making techniques. Results will be varied and be considered inclined and odd on ignoring such kind of sets. Therefore, it is the need of the literature to adequate the exiting literature of soft and expert set for multi-attribute function. Having motivation from [10]- [19], new notions of fuzzy hypersoft expert set are developed and an application is discussed in decision making through a proposed method. The pattern of rest of the paper is: section 2 reviews the notions of soft sets, fuzzy soft set, fuzzy soft expert set, hypersoft set and relevant definitions used in the proposed work. Section 3, presents notions of fuzzy hypersoft expert set with properties. Section 4, demonstrates an application of this concept in a decision-making problem. Section 5, concludes the paper.

1.1. *Motivation*

The novelty of fuzzy hypersoft expert set (FHSE-set) is as:

- It is the extension of soft set, fuzzy soft set, soft expert set and fuzzy soft expert set,
- It tackles all the hindrances of soft set, fuzzy soft set, soft expert set and fuzzy soft expert set for dealing with further partitions of attributes into attribute-valued sets,
- It facilitates the decision-makers to have decisions for uncertain scenarios without encountering with any inclined situation.

2. **Preliminaries**

In this section, some basic definitions and terms regarding the main study, are presented from the literature.

Definition 2.1. [1]
Let $P(\Omega)$ denote power set of Ω(universe of discourse) and F be a collection of parameters defining Ω. A *soft set* M is defined by mapping

$$\Psi : F \to P(\Omega)$$

Definition 2.2. [3] Suppose Ω be a set of universe, while F is a set of parameters. Here I^{Ω} represents the power set of all fuzzy subsets of Ω. Let $C \subseteq F$. A pair (R, F) is called a fuzzy soft set with R is a mapping given by

$$R : C \to I^{\Omega}$$

Definition 2.3. [4]
The union of two soft sets (Ψ_1, A_1) and (Ψ_2, A_2) over Ω is the soft set (Ψ_3, A_3) ; $A_3 \doteq A_1 \cup A_2$, and $\forall\ \xi \in A_3$,

$$\Psi_3(\xi) = \begin{cases} \Psi_1(\xi) & ; \xi \in A_1 - A_2 \\ \Psi_2(\xi) & ; \xi \in A_2 - A_1 \\ \Psi_1(\xi) \cup \Psi_2(\xi) & ; \xi \in A_1 \cap A_2 \end{cases}$$

Definition 2.4. [15]

The extended intersection of two soft sets (Ψ_1, A_1) and (Ψ_2, A_2) with Ω is the soft set (Ψ_3, A_3) while $A_3 \doteq A_1 \cup A_2,$; $\xi \in A_3$,

$$\Psi_3(\xi) = \begin{cases} \Psi_1(\xi) & ; \xi \in A_1 - A_2 \\ \Psi_2(\xi) & ; \xi \in A_2 - A_1 \\ \Psi_1(\xi) \cup \Psi_2(\xi) & ; \xi \in A_1 \cap A_2 \end{cases}$$

Definition 2.5. [16]

Assume that Y be a set of specialists (operators) and $\ddot{O}$ be a set of conclusions, $T = F \times Y \times \ddot{O}$ with $S \subseteq T$ where Ω denotes the universe , F a set of parameters. A pair(Φ, S) is known as a *soft expert set* over Ω, where H is a mapping given by

$$\Phi : S \to P(\Omega)$$

Definition 2.6. [16]

A $(\Phi_1, S) \subseteq (\Phi_2, P)$ over Ω, if

(i) $S \subseteq P$,

(ii) $\forall\ \alpha \in S, \Phi_1(\alpha) \subseteq \Phi_2(\alpha)$.

While (Φ_2, P) is known as a *soft expert superset* of (Φ_1, S).

Definition 2.7. [18] A pair (H, C) is called a fuzzy soft expert set over Ω where F is a mapping given by

$$H : C \to I^{\Omega}$$

where I^{Ω} the set of all fuzzy subsets of Ω.

Definition 2.8. [36]

Let $h_1, h_2, h_3, ..., h_m$, for $m \geq 1$, be m distinct attributes, whose corresponding attribute values are respectively the sets $H_1, H_2, H_3, ..., H_m$, with $H_i \cap H_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., m\}$. Then the pair (Ψ, G), where $G = H_1 \times H_2 \times H_3 \times ..... \times H_m$ and $\Psi : G \to P(\Omega)$ is called a *hypersoft Set* over Ω.

Definition 2.9. [38]

Let $\Gamma_1, \Gamma_2, \Gamma_3, ...\Gamma_m$ be disjoint attribute-valued sets for m distinct attributes . A pair (Φ, Γ) is called fuzzy hypersoft set over Ω with Γ is the cartesian product of $\Gamma_i, i = 1, 2, ..., m$, and $\Phi : \Gamma \to P(\Omega)$. In general, $\Phi(\alpha) = \{(x, \Phi(\alpha))(x)/x \in \Omega\}; \alpha \in \Gamma$. Here $P(\Omega)$ is the collection of all fuzzy sets.

3. Fuzzy Hypersoft Expert set (FHSE-Set)

Definition 3.1. Fuzzy Hypersoft Expert set (FHSE-Set)
A pair$(\xi, \mathbb{S})$ is known as a *fuzzy hypersoft expert set* over $\coprod$, where

$$\xi : \mathbb{S} \rightarrow I^{\coprod}$$

where

- $I^{\coprod}$ is collection of all fuzzy subsets of $\coprod$
- $\mathbb{S} \subseteq \mathcal{H} = \mathcal{G} \times \mathcal{D} \times \mathbb{C}$
- $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \mathcal{G}_3 \times .... \times \mathcal{G}_p$ where $\mathcal{G}_i$ are disjoint attributive-valued sets corresponding to distinct attributes $g_i, i = 1, 2, 3, ..., p$
- $\mathcal{D}$ be a set of specialists (operators)
- $\mathbb{C}$ be a set of conclusions.

For simplicity, $\mathbb{C} = \{0 = disagree, 1 = agree\}$.

Example 3.2. Suppose that a multi-national company aims to proceed the valuation of certain specialists about its certain products. Let $\coprod = \{m_1, m_2, m_3, m_4\}$ be a set of products and
$\mathcal{G}_1 = \{q_{11}, q_{12}\}$
$\mathcal{G}_2 = \{q_{21}, q_{22}\}$
$\mathcal{G}_3 = \{q_{31}, q_{32}\}$
be disjoint attributive sets for distinct attributes q_1= simple to utilize, q_2= nature, q_3= modest.
Now
$\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \mathcal{G}_3$

$$\mathcal{G} = \left\{ \begin{array}{l} \mu_1 = (q_{11}, q_{21}, q_{31}), \mu_2 = (q_{11}, q_{21}, q_{32}), \mu_3 = (q_{11}, q_{22}, q_{31}), \mu_4 = (q_{11}, q_{22}, q_{32}), \\ \mu_5 = (q_{12}, q_{21}, q_{31}), \mu_6 = (q_{12}, q_{21}, q_{32}), \mu_7 = (q_{12}, q_{22}, q_{31}), \mu_8 = (q_{12}, q_{22}, q_{32}) \end{array} \right\}$$

Now $\mathcal{H} = \mathcal{G} \times \mathcal{D} \times \mathbb{C}$

$$\mathcal{H} = \left\{ \begin{array}{l} (\mu_1, s, 0), (\mu_1, s, 1), (\mu_1, t, 0), (\mu_1, t, 1), (\mu_1, u, 0), (\mu_1, u, 1), \\ (\mu_2, s, 0), (\mu_2, s, 1), (\mu_2, t, 0), (\mu_2, t, 1), (\mu_2, u, 0), (\mu_2, u, 1), \\ (\mu_3, s, 0), (\mu_3, s, 1), (\mu_3, t, 0), (\mu_3, t, 1), (\mu_3, u, 0), (\mu_3, u, 1), \\ (\mu_4, s, 0), (\mu_4, s, 1), (\mu_4, t, 0), (\mu_4, t, 1), (\mu_4, u, 0), (\mu_4, u, 1), \\ (\mu_5, s, 0), (\mu_5, s, 1), (\mu_5, t, 0), (\mu_5, t, 1), (\mu_5, u, 0), (\mu_5, u, 1), \\ (\mu_6, s, 0), (\mu_6, s, 1), (\mu_6, t, 0), (\mu_6, t, 1), (\mu_6, u, 0), (\mu_6, u, 1), \\ (\mu_7, s, 0), (\mu_7, s, 1), (\mu_7, t, 0), (\mu_7, t, 1), (\mu_7, u, 0), (\mu_7, u, 1), \\ (\mu_8, s, 0), (\mu_8, s, 1), (\mu_8, t, 0), (\mu_8, t, 1), (\mu_8, u, 0), (\mu_8, u, 1) \end{array} \right\}$$

let

$$\mathbb{S}=\left\{\begin{array}{l}(\mu_1,s,0),(\mu_1,s,1),(\mu_1,t,0),(\mu_1,t,1),(\mu_1,u,0),(\mu_1,u,1),\\(\mu_2,s,0),(\mu_2,s,1),(\mu_2,t,0),(\mu_2,t,1),(\mu_2,u,0),(\mu_2,u,1)\\(\mu_3,s,0),(\mu_3,s,1),(\mu_3,t,0),(\mu_3,t,1),(\mu_3,u,0),(\mu_3,u,1),\end{array}\right\}$$

be a subset of $\mathcal{H}$ and $\mathcal{D}=\{s,t,u,\}$ be a set of specialists.

Following survey depicts choices of three specialists:

$\xi_1=\xi(\mu_1,s,1)=\left\{\frac{m_1}{0.2},\frac{m_2}{0.7},\frac{m_3}{0.5},\frac{m_4}{0.1}\right\}$, $\xi_2=\xi(\mu_1,t,1)=\left\{\frac{m_1}{0.4},\frac{m_2}{0.8},\frac{m_3}{0.4},\frac{m_4}{0.2}\right\}$,

$\xi_3=\xi(\mu_1,u,1)=\left\{\frac{m_1}{0.7},\frac{m_2}{0.5},\frac{m_3}{0.6},\frac{m_4}{0.3}\right\}$, $\xi_4=\xi(\mu_2,s,1)=\left\{\frac{m_1}{0.9},\frac{m_2}{0.4},\frac{m_3}{0.7},\frac{m_4}{0.3}\right\}$,

$\xi_5=\xi(\mu_2,t,1)=\left\{\frac{m_1}{0.4},\frac{m_2}{0.8},\frac{m_3}{0.3},\frac{m_4}{0.2}\right\}$, $\xi_6=\xi(\mu_2,u,1)=\left\{\frac{m_1}{0.5},\frac{m_2}{0.3},\frac{m_3}{0.6},\frac{m_4}{0.8}\right\}$,

$\xi_7=\xi(\mu_3,s,1)=\left\{\frac{m_1}{0.2},\frac{m_2}{0.9},\frac{m_3}{0.4},\frac{m_4}{0.5}\right\}$, $\xi_8=\xi(\mu_3,t,1)=\left\{\frac{m_1}{0.4},\frac{m_2}{0.6},\frac{m_3}{0.7},\frac{m_4}{0.9}\right\}$,

$\xi_9=\xi(\mu_3,u,1)=\left\{\frac{m_1}{0.7},\frac{m_2}{0.3},\frac{m_3}{0.5},\frac{m_4}{0.2}\right\}$, $\xi_{10}=\xi(\mu_1,s,0)=\left\{\frac{m_1}{0.3},\frac{m_2}{0.2},\frac{m_3}{0.4},\frac{m_4}{0.1}\right\}$,

$\xi_{11}=\xi(\mu_1,t,0)=\left\{\frac{m_1}{0.1},\frac{m_2}{0.9},\frac{m_3}{0.6},\frac{m_4}{0.2}\right\}$, $\xi_{12}=\xi(\mu_1,u,0)=\left\{\frac{m_1}{0.2},\frac{m_2}{0.1},\frac{m_3}{0.3},\frac{m_4}{0.5}\right\}$,

$\xi_{13}=\xi(\mu_2,s,0)=\left\{\frac{m_1}{0.8},\frac{m_2}{0.3},\frac{m_3}{0.5},\frac{m_4}{0.7}\right\}$, $\xi_{14}=\xi(\mu_2,t,0)=\left\{\frac{m_1}{0.7},\frac{m_2}{0.2},\frac{m_3}{0.9},\frac{m_4}{0.4}\right\}$,

$\xi_{15}=\xi(\mu_2,u,0)=\left\{\frac{m_1}{0.6},\frac{m_2}{0.7},\frac{m_3}{0.3},\frac{m_4}{0.2}\right\}$, $\xi_{16}=\xi(\mu_3,s,0)=\left\{\frac{m_1}{0.1},\frac{m_2}{0.4},\frac{m_3}{0.7},\frac{m_4}{0.8}\right\}$,

$\xi_{17}=\xi(\mu_3,t,0)=\left\{\frac{m_1}{0.2},\frac{m_2}{0.9},\frac{m_3}{0.8},\frac{m_4}{0.3}\right\}$, $\xi_{18}=\xi(\mu_3,u,0)=\left\{\frac{m_1}{0.5},\frac{m_2}{0.3},\frac{m_3}{0.6},\frac{m_4}{0.1}\right\}$,

The fuzzy soft expert set can be described as

$$(\xi,\mathbb{S})=\left\{\begin{array}{l}\left((\mu_1,s,1),\left\{\frac{m_1}{0.2},\frac{m_2}{0.7},\frac{m_3}{0.5},\frac{m_4}{0.1}\right\}\right),\left((\mu_1,t,1),\left\{\frac{m_1}{0.4},\frac{m_2}{0.8},\frac{m_3}{0.4},\frac{m_4}{0.2}\right\}\right),\\ \left((\mu_1,u,1),\left\{\frac{m_1}{0.7},\frac{m_2}{0.5},\frac{m_3}{0.6},\frac{m_4}{0.3}\right\}\right),\left((\mu_2,s,1),\left\{\frac{m_1}{0.9},\frac{m_2}{0.4},\frac{m_3}{0.7},\frac{m_4}{0.3}\right\}\right),\\ \left(\mu_2,t,1),\left\{\frac{m_1}{0.4},\frac{m_2}{0.8},\frac{m_3}{0.3},\frac{m_4}{0.2}\right\}\right),\left((\mu_2,u,1),\left\{\frac{m_1}{0.5},\frac{m_2}{0.3},\frac{m_3}{0.6},\frac{m_4}{0.8}\right\}\right),\\ \left((\mu_3,s,1),\left\{\frac{m_1}{0.2},\frac{m_2}{0.9},\frac{m_3}{0.4},\frac{m_4}{0.5}\right\}\right),\left((\mu_3,t,1),\left\{\frac{m_1}{0.4},\frac{m_2}{0.6},\frac{m_3}{0.7},\frac{m_4}{0.9}\right\}\right),\\ \left((\mu_3,u,1),\left\{\frac{m_1}{0.7},\frac{m_2}{0.3},\frac{m_3}{0.5},\frac{m_4}{0.2}\right\}\right),\left((\mu_1,s,0),\left\{\frac{m_1}{0.3},\frac{m_2}{0.2},\frac{m_3}{0.4},\frac{m_4}{0.1}\right\}\right),\\ \left((\mu_1,t,0),\left\{\frac{m_1}{0.1},\frac{m_2}{0.9},\frac{m_3}{0.6},\frac{m_4}{0.2}\right\}\right),\left((\mu_1,u,0),\left\{\frac{m_1}{0.2},\frac{m_2}{0.1},\frac{m_3}{0.3},\frac{m_4}{0.5}\right\}\right),\\ \left((\mu_2,s,0),\left\{\frac{m_1}{0.8},\frac{m_2}{0.3},\frac{m_3}{0.5},\frac{m_4}{0.7}\right\}\right),\left((\mu_2,t,0),\left\{\frac{m_1}{0.7},\frac{m_2}{0.2},\frac{m_3}{0.9},\frac{m_4}{0.4}\right\}\right),\\ \left((\mu_2,u,0),\left\{\frac{m_1}{0.6},\frac{m_2}{0.7},\frac{m_3}{0.3},\frac{m_4}{0.2}\right\}\right),\left((\mu_3,s,0),\left\{\frac{m_1}{0.1},\frac{m_2}{0.4},\frac{m_3}{0.7},\frac{m_4}{0.8}\right\}\right),\\ \left((\mu_3,t,0),\left\{\frac{m_1}{0.2},\frac{m_2}{0.9},\frac{m_3}{0.8},\frac{m_4}{0.3}\right\}\right),\left((\mu_3,u,0),\left\{\frac{m_1}{0.5},\frac{m_2}{0.3},\frac{m_3}{0.6},\frac{m_4}{0.1}\right\}\right),\end{array}\right\}$$

Definition 3.3. Fuzzy Hypersoft Expert subset

A hypersoft expert set $(\xi_1,\mathbb{S})$ is said to be fuzzy hypersoft expert subset of (ξ_2,R) over $\coprod$, if

(i) $\mathbb{S}\subseteq R$,

(ii) $\forall\ \alpha\in\mathbb{S},\xi_1(\alpha)\subseteq\xi_2(\alpha)$.

and denoted by $(\xi_1,\mathbb{S})\subseteq(\xi_2,R)$. Similarly (ξ_2,R) is said to be *fuzzy hypersoft expert superset* of $(\xi_1,\mathbb{S})$.

Example 3.4. Considering Example 3.2, Suppose

$$A_1=\left\{(\mu_1,s,1),(\mu_3,s,0),(\mu_1,t,1),(\mu_3,t,1),(\mu_3,t,0),(\mu_1,u,0),(\mu_3,u,1)\right\}$$

$$A_2=\left\{(\mu_1,s,1),(\mu_3,s,0),(\mu_3,s,1),(\mu_1,t,1),(\mu_3,t,1),(\mu_1,t,0),(\mu_3,t,0),(\mu_1,u,0),(\mu_3,u,1),(\mu_1,u,1)\right\}$$

It is clear that$A_1 \subset A_2$.Suppose (ξ_1, A_1) and (ξ_2, A_2) be defined as following

$$(\xi_1, \mathrm{A}_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.6}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \left((\mu_1, t, 1), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.6}, \frac{m_3}{0.2}, \frac{m_4}{0.1}\right\}\right), \\ \left((\mu_3, t, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.5}, \frac{m_3}{0.6}, \frac{m_4}{0.8}\right\}\right), \left((\mu_3, u, 1), \left\{\frac{m_1}{0.6}, \frac{m_2}{0.2}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.1}, \frac{m_3}{0.2}, \frac{m_4}{0.4}\right\}\right), \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.3}, \frac{m_3}{0.6}, \frac{m_4}{0.7}\right\}\right), \\ \left((\mu_3, t, 0) \left\{\frac{m_1}{0.1}, \frac{m_2}{0.8}, \frac{m_3}{0.7}, \frac{m_4}{0.2}\right\}\right) \end{array} \right\}$$

$$(\xi_2, \mathrm{A}_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.1}\right\}\right), \left((\mu_1, t, 1), \left\{\frac{m_1}{0.4}, \frac{m_2}{0.8}, \frac{m_3}{0.4}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_3, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.9}, \frac{m_3}{0.4}, \frac{m_4}{0.5}\right\}\right), \left((\mu_3, t, 1), \left\{\frac{m_1}{0.4}, \frac{m_2}{0.6}, \frac{m_3}{0.7}, \frac{m_4}{0.9}\right\}\right), \\ \left((\mu_1, u, 1), \left\{\frac{m_1}{0.7}, \frac{m_2}{0.5}, \frac{m_3}{0.6}, \frac{m_4}{0.3}\right\}\right), \left((\mu_3, u, 1), \left\{\frac{m_1}{0.7}, \frac{m_2}{0.3}, \frac{m_3}{0.5}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.1}, \frac{m_3}{0.3}, \frac{m_4}{0.5}\right\}\right), \left((\mu_1, t, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.9}, \frac{m_3}{0.6}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.4}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right), \left((\mu_3, t, 0), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.9}, \frac{m_3}{0.8}, \frac{m_4}{0.3}\right\}\right) \end{array} \right\}$$

which implies that $(\xi_1, A_1) \subseteq (\xi_2, A_2)$.

Definition 3.5. Two fuzzy hypersoft expert sets (ξ_1, A_1) and (ξ_2, A_2) over $\coprod$ are said to be equal if (ξ_1, A_1) is a hypersoft expert subset of (ξ_2, A_2) and (ξ_2, A_2) is a fuzzy hypersoft expert subset of (ξ_1, A_1).

Definition 3.6. Let $\mathcal{G}$ be a set as defined in definition 3.1 and $\mathcal{D}$, a set of experts. The NOT set of $\mathcal{H} = \mathcal{G} \times \mathcal{D} \times \mathbb{C}$ denoted by $\sim \mathcal{H}$, is defined by $\sim \mathcal{T} = \{(\sim g_i, d_j, c_k) \forall i, j, k\}$ where $\sim g_i$ is not g_i.

Definition 3.7. The complement of a fuzzy hypersoft expert set $(\xi, \mathbb{S})$, denoted by $(\xi, \mathbb{S})^c$, is defined by $(\xi, \mathbb{S})^c = (\xi^c, \sim \mathbb{S})$ while $\xi^c : \sim \mathbb{S} \to P(\coprod)$ is a mapping given by $\xi^c(\beta) = \coprod - \xi(\sim \beta)$, where $\beta \in \sim \mathbb{S}$.

Example 3.8. Taking complement of fuzzy hypersoft expert set determined in 3.2, we have

$$(\xi, \mathbb{S})^{\mathrm{c}} = \left\{ \begin{array}{l} \left((\sim \mu_1, s, 1), \left\{\frac{m_1}{0.8}, \frac{m_2}{0.3}, \frac{m_3}{0.5}, \frac{m_4}{0.9}\right\}\right), \left((\sim \mu_1, t, 1), \left\{\frac{m_1}{0.6}, \frac{m_2}{0.2}, \frac{m_3}{0.6}, \frac{m_4}{0.8}\right\}\right), \\ \left((\sim \mu_1, u, 1), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.5}, \frac{m_3}{0.4}, \frac{m_4}{0.7}\right\}\right), \left((\sim \mu_3, s, 1), \left\{\frac{m_1}{0.8}, \frac{m_2}{0.1}, \frac{m_3}{0.6}, \frac{m_4}{0.5}\right\}\right), \\ \left((\sim \mu_3, t, 1), \left\{\frac{m_1}{0.6}, \frac{m_2}{0.4}, \frac{m_3}{0.3}, \frac{m_4}{0.1}\right\}\right), \left((\sim \mu_3, u, 1), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.8}\right\}\right), \\ \left((\sim \mu_2, s, 1), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.6}, \frac{m_3}{0.3}, \frac{m_4}{0.7}\right\}\right), \left((\sim \mu_2, t, 1), \left\{\frac{m_1}{0.6}, \frac{m_2}{0.2}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right), \\ \left((\sim \mu_2, u, 1), \left\{\frac{m_1}{0.5}, \frac{m_2}{0.7}, \frac{m_3}{0.4}, \frac{m_4}{0.2}\right\}\right), \left((\sim \mu_1, s, 0), \left\{\frac{m_1}{0.7}, \frac{m_2}{0.3}, \frac{m_3}{0.6}, \frac{m_4}{0.9}\right\}\right), \\ \left((\sim \mu_1, t, 0), \left\{\frac{m_1}{0.9}, \frac{m_2}{0.1}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \left((\sim \mu_1, u, 0), \left\{\frac{m_1}{0.8}, \frac{m_2}{0.9}, \frac{m_3}{0.7}, \frac{m_4}{0.5}\right\}\right), \\ \left((\sim \mu_3, s, 0), \left\{\frac{m_1}{0.9}, \frac{m_2}{0.6}, \frac{m_3}{0.3}, \frac{m_4}{0.2}\right\}\right), \left((\sim \mu_3, t, 0), \left\{\frac{m_1}{0.8}, \frac{m_2}{0.1}, \frac{m_3}{0.2}, \frac{m_4}{0.7}\right\}\right), \\ \left((\sim (\mu_3, u, 0), \left\{\frac{m_1}{0.5}, \frac{m_2}{0.7}, \frac{m_3}{0.4}, \frac{m_4}{0.9}\right\}\right) \left((\sim \mu_2, s, 0), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.3}\right\}\right), \\ \left((\sim \mu_2, t, 0), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.8}, \frac{m_3}{0.1}, \frac{m_4}{0.4}\right\},\right), \left((\sim \mu_2, u, 0), \left\{\frac{m_1}{0.4}, \frac{m_2}{0.3}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right) \end{array} \right\}$$

Definition 3.9. An agree-fuzzy hypersoft expert set $(\xi, \mathbb{S})_{ag}$ over $\coprod$, is a fuzzy hypersoft expert subset of $(\xi, \mathbb{S})$ and is characterized as
$(\xi, \mathbb{S})_{ag} = \{\xi_{ag}(\beta) : \beta \in G \times D \times \{1\}\}$.

Example 3.10. Finding agree-fuzzy hypersoft expert set determined in 3.2, we get

$$(\xi,\mathbb{S})=\left\{\begin{array}{l}\left((\mu_1,s,1),\left\{\frac{m_1}{0.2},\frac{m_2}{0.7},\frac{m_3}{0.5},\frac{m_4}{0.1}\right\}\right),\left((\mu_1,t,1),\left\{\frac{m_1}{0.4},\frac{m_2}{0.8},\frac{m_3}{0.4},\frac{m_4}{0.2}\right\}\right),\\ \left((\mu_1,u,1),\left\{\frac{m_1}{0.7},\frac{m_2}{0.5},\frac{m_3}{0.6},\frac{m_4}{0.3}\right\}\right),\left((\mu_3,s,1),\left\{\frac{m_1}{0.2},\frac{m_2}{0.9},\frac{m_3}{0.4},\frac{m_4}{0.5}\right\}\right),\\ \left((\mu_3,t,1),\left\{\frac{m_1}{0.4},\frac{m_2}{0.6},\frac{m_3}{0.7},\frac{m_4}{0.9}\right\}\right),\left((\mu_3,u,1),\left\{\frac{m_1}{0.7},\frac{m_2}{0.3},\frac{m_3}{0.5},\frac{m_4}{0.2}\right\}\right),\\ \left((\mu_2,s,1),\left\{\frac{m_1}{0.9},\frac{m_2}{0.4},\frac{m_3}{0.7},\frac{m_4}{0.3}\right\}\right),\left((\mu_2,u,1),\left\{\frac{m_1}{0.5},\frac{m_2}{0.3},\frac{m_3}{0.6},\frac{m_4}{0.8}\right\}\right)\end{array}\right\}$$

Definition 3.11. A disagree- fuzzy hypersoft expert set $(\xi,\mathbb{S})_{dag}$ over $\coprod$, is a fuzzy hypersoft expert subset of $(\xi,\mathbb{S})$ and is characterized as
$(\xi,\mathbb{S})_{dag}=\{\xi_{dag}(\beta):\beta\in\mathbb{G}\times\mathcal{D}\times\{0\}\}$.

Example 3.12. Getting disagree-fuzzy hypersoft expert set determined in 3.2,

$$(\xi,\mathbb{S})=\left\{\begin{array}{l}\left((\mu_1,s,0),\left\{\frac{m_1}{0.3},\frac{m_2}{0.2},\frac{m_3}{0.4},\frac{m_4}{0.1}\right\}\right),\left((\mu_1,t,0),\left\{\frac{m_1}{0.1},\frac{m_2}{0.9},\frac{m_3}{0.6},\frac{m_4}{0.2}\right\}\right),\\ \left((\mu_1,u,0),\left\{\frac{m_1}{0.2},\frac{m_2}{0.1},\frac{m_3}{0.3},\frac{m_4}{0.5}\right\}\right),\left((\mu_3,s,0),\left\{\frac{m_1}{0.1},\frac{m_2}{0.4},\frac{m_3}{0.7},\frac{m_4}{0.8}\right\}\right),\\ \left((\mu_3,t,0),\left\{\frac{m_1}{0.2},\frac{m_2}{0.9},\frac{m_3}{0.8},\frac{m_4}{0.3}\right\}\right),\left((\mu_3,u,0),\left\{\frac{m_1}{0.5},\frac{m_2}{0.3},\frac{m_3}{0.6},\frac{m_4}{0.1}\right\}\right)\\ \left((\mu_2,s,0),\left\{\frac{m_1}{0.8},\frac{m_2}{0.3},\frac{m_3}{0.5},\frac{m_4}{0.7}\right\}\right),\left((\mu_2,t,0),\left\{\frac{m_1}{0.7},\frac{m_2}{0.2},\frac{m_3}{0.9},\frac{m_4}{0.4}\right\}\right),\\ \left((\mu_2,u,0),\left\{\frac{m_1}{0.6},\frac{m_2}{0.7},\frac{m_3}{0.3},\frac{m_4}{0.2}\right\}\right)\end{array}\right\}$$

Proposition 3.13. *If $(\xi,\mathbb{S})$ is a fuzzy hypersoft expert set over $\coprod$, then*

(1). $((\xi,\mathbb{S})^c)^c=(\xi,\mathbb{S})$
(2). $(\xi,\mathbb{S})^c_{ag}=(\xi,\mathbb{S})_{dag}$
(3). $(\xi,\mathbb{S})^c_{dag}=(\xi,\mathbb{S})_{ag}$

Definition 3.14. The union of $(\xi_1,\mathbb{S})$ and (ξ_2,R) over $\coprod$ is (ξ_3,L) with $L=\mathbb{S}\cup R$, defined as

$$\xi_3(\beta)=\begin{cases}\xi_1(\beta) & ;\beta\in\mathbb{S}-R\\ \xi_2(\beta) & ;\beta\in\mathbb{R}-S\\ \xi_1(\beta)\cup\xi_2(\beta) & ;\beta\in\mathbb{S}\cap R\end{cases}$$

Example 3.15. Taking into consideration the concept of example 3.2, consider the following two sets

$$A_1=\left\{(\mu_1,s,1),(\mu_3,s,0),(\mu_1,t,1),(\mu_3,t,1),(\mu_3,t,0),(\mu_1,u,0),(\mu_3,u,1)\right\}$$

$$A_2=\left\{(\mu_1,s,1),(\mu_3,s,0),(\mu_3,s,1),(\mu_1,t,1),(\mu_3,t,1),(\mu_1,u,1),(\mu_3,t,0),(\mu_1,u,0),(\mu_3,u,1),(\mu_1,t,0)\right\}$$

Suppose (ξ_1,A_1) and (ξ_2,A_2) over $\coprod$ are two fuzzy hypersoft expert sets such that

$$(\xi_1,\mathrm{A}_1)=\left\{\begin{array}{l}\left((\mu_1,s,1),\left\{\frac{m_1}{0.1},\frac{m_2}{0.6},\frac{m_3}{0.4},\frac{m_4}{0.1}\right\}\right),\left((\mu_1,t,1),\left\{\frac{m_1}{0.3},\frac{m_2}{0.6},\frac{m_3}{0.2},\frac{m_4}{0.1}\right\}\right),\\ \left((\mu_3,t,1),\left\{\frac{m_1}{0.2},\frac{m_2}{0.5},\frac{m_3}{0.6},\frac{m_4}{0.8}\right\}\right),\left((\mu_3,u,1),\left\{\frac{m_1}{0.6},\frac{m_2}{0.2},\frac{m_3}{0.4},\frac{m_4}{0.1}\right\}\right),\\ \left((\mu_1,u,0),\left\{\frac{m_1}{0.1},\frac{m_2}{0.1},\frac{m_3}{0.2},\frac{m_4}{0.4}\right\}\right),\left((\mu_3,s,0),\left\{\frac{m_1}{0.1},\frac{m_2}{0.3},\frac{m_3}{0.6},\frac{m_4}{0.7}\right\}\right),\\ \left((\mu_3,t,0)\left\{\frac{m_1}{0.1},\frac{m_2}{0.8},\frac{m_3}{0.7},\frac{m_4}{0.2}\right\}\right)\end{array}\right\}$$

$$(\xi_2, \mathrm{A}_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.1}\right\}\right), \left((\mu_1, t, 1), \left\{\frac{m_1}{0.4}, \frac{m_2}{0.8}, \frac{m_3}{0.4}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_3, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.9}, \frac{m_3}{0.4}, \frac{m_4}{0.5}\right\}\right), \left((\mu_3, t, 1), \left\{\frac{m_1}{0.4}, \frac{m_2}{0.6}, \frac{m_3}{0.7}, \frac{m_4}{0.9}\right\}\right), \\ \left((\mu_1, u, 1), \left\{\frac{m_1}{0.7}, \frac{m_2}{0.5}, \frac{m_3}{0.6}, \frac{m_4}{0.3}\right\}\right), \left((\mu_3, u, 1), \left\{\frac{m_1}{0.7}, \frac{m_2}{0.3}, \frac{m_3}{0.5}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.1}, \frac{m_3}{0.3}, \frac{m_4}{0.5}\right\}\right), \left((\mu_1, t, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.9}, \frac{m_3}{0.6}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.4}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right), \left((\mu_3, t, 0), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.9}, \frac{m_3}{0.8}, \frac{m_4}{0.3}\right\}\right) \end{array} \right\}$$

Then $(\xi_1, A_1) \cup (\xi_2, A_2) = (\xi_3, A_3)$

$$(\xi_2, \mathrm{A}_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.1}\right\}\right), \left((\mu_1, t, 1), \left\{\frac{m_1}{0.4}, \frac{m_2}{0.8}, \frac{m_3}{0.4}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_3, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.9}, \frac{m_3}{0.4}, \frac{m_4}{0.5}\right\}\right), \left((\mu_3, t, 1), \left\{\frac{m_1}{0.4}, \frac{m_2}{0.6}, \frac{m_3}{0.7}, \frac{m_4}{0.9}\right\}\right), \\ \left((\mu_1, u, 1), \left\{\frac{m_1}{0.7}, \frac{m_2}{0.5}, \frac{m_3}{0.6}, \frac{m_4}{0.3}\right\}\right), \left((\mu_3, u, 1), \left\{\frac{m_1}{0.7}, \frac{m_2}{0.3}, \frac{m_3}{0.5}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.1}, \frac{m_3}{0.3}, \frac{m_4}{0.5}\right\}\right), \left((\mu_1, t, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.9}, \frac{m_3}{0.6}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.4}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right), \left((\mu_3, t, 0), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.9}, \frac{m_3}{0.8}, \frac{m_4}{0.3}\right\}\right) \end{array} \right\}$$

Proposition 3.16. *If* (ξ_1, A_1),(ξ_2, A_2) *and* (ξ_3, A_3) *are three fuzzy hypersoft expert sets over* $\coprod$, *then*

(1). $(\xi_1, A_1) \cup (\xi_2, A_2) = (\xi_2, A_2) \cup (\xi_1, A_1)$

(2). $((\xi_1, A_1) \cup (\xi_2, A_2)) \cup (\xi_3, A_3) = (\xi_1, A_1) \cup ((\xi_2, A_2) \cup (\xi_3, A_3))$

Definition 3.17. The intersection of $(\xi_1, \mathbb{S})$ and (ξ_2, R) over $\coprod$ is (ξ_3, L) with $L = \mathbb{S} \cap R$, defined as

$$\xi_3(\beta) = \left\{ \begin{array}{cl} \xi_1(\beta) & ; \beta \in \mathbb{S} - R \\ \xi_2(\beta) & ; \beta \in R - S \\ \xi_1(\beta) \cap \xi_2(\beta) & ; \beta \in \mathbb{S} \cap R \end{array} \right.$$

Example 3.18. Taking into consideration the concept of example 3.2, consider the following two sets

$$A_1 = \Big\{ (\mu_1, s, 1), (\mu_3, s, 0), (\mu_1, t, 1), (\mu_3, t, 1), (\mu_3, t, 0), (\mu_1, u, 0), (\mu_3, u, 1) \Big\}$$

$$A_2 = \Big\{ (\mu_1, s, 1), (\mu_3, s, 0), (\mu_3, s, 1), (\mu_1, t, 1), (\mu_3, t, 1), (\mu_1, t, 0), (\mu_3, t, 0), (\mu_1, u, 0), (\mu_3, u, 1), , (\mu_1, u, 1) \Big\}$$

Suppose (ξ_1, A_1) and (ξ_2, A_2) over $\coprod$ are two fuzzy hypersoft expert sets such that

$$(\xi_1, \mathrm{A}_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.6}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \left((\mu_1, t, 1), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.6}, \frac{m_3}{0.2}, \frac{m_4}{0.1}\right\}\right), \\ \left((\mu_3, t, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.5}, \frac{m_3}{0.6}, \frac{m_4}{0.8}\right\}\right), \left((\mu_3, u, 1), \left\{\frac{m_1}{0.6}, \frac{m_2}{0.2}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.1}, \frac{m_3}{0.2}, \frac{m_4}{0.4}\right\}\right), \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.3}, \frac{m_3}{0.6}, \frac{m_4}{0.7}\right\}\right), \\ \left((\mu_3, t, 0) \left\{\frac{m_1}{0.1}, \frac{m_2}{0.8}, \frac{m_3}{0.7}, \frac{m_4}{0.2}\right\}\right) \end{array} \right\}$$

$$(\xi_2, \mathrm{A}_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.1}\right\}\right), \left((\mu_1, t, 1), \left\{\frac{m_1}{0.4}, \frac{m_2}{0.8}, \frac{m_3}{0.4}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_3, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.9}, \frac{m_3}{0.4}, \frac{m_4}{0.5}\right\}\right), \left((\mu_3, t, 1), \left\{\frac{m_1}{0.4}, \frac{m_2}{0.6}, \frac{m_3}{0.7}, \frac{m_4}{0.9}\right\}\right), \\ \left((\mu_1, u, 1), \left\{\frac{m_1}{0.7}, \frac{m_2}{0.5}, \frac{m_3}{0.6}, \frac{m_4}{0.3}\right\}\right), \left((\mu_3, u, 1), \left\{\frac{m_1}{0.7}, \frac{m_2}{0.3}, \frac{m_3}{0.5}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.1}, \frac{m_3}{0.3}, \frac{m_4}{0.5}\right\}\right), \left((\mu_1, t, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.9}, \frac{m_3}{0.6}, \frac{m_4}{0.2}\right\}\right), \\ \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.4}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right), \left((\mu_3, t, 0), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.9}, \frac{m_3}{0.8}, \frac{m_4}{0.3}\right\}\right) \end{array} \right\}$$

Then $(\xi_1, A_1) \cap (\xi_2, A_2) = (\xi_3, A_3)$

$$(\xi_3, \mathrm{A}_3) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.6}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \left((\mu_1, t, 1), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.6}, \frac{m_3}{0.2}, \frac{m_4}{0.1}\right\}\right), \\ \left((\mu_3, t, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.5}, \frac{m_3}{0.4}, \frac{m_4}{0.5}\right\}\right), \left((\mu_3, u, 1), \left\{\frac{m_1}{0.6}, \frac{m_2}{0.2}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \\ \left((\mu_1, u, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.1}, \frac{m_3}{0.2}, \frac{m_4}{0.4}\right\}\right), \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.3}, \frac{m_3}{0.6}, \frac{m_4}{0.7}\right\}\right), \\ \left((\mu_3, t, 0) \left\{\frac{m_1}{0.1}, \frac{m_2}{0.8}, \frac{m_3}{0.7}, \frac{m_4}{0.2}\right\}\right) \end{array} \right\}$$

Proposition 3.19. *If* (ξ_1, A_1),(ξ_2, A_2) *and* (ξ_3, A_3) *are three fuzzy hypersoft expert sets over* $\coprod$*, then*

(1). $(\xi_1, A_1) \cap (\xi_2, A_2) = (\xi_2, A_2) \cap (\xi_1, A_1)$

(2). $((\xi_1, A_1) \cap (\xi_2, A_2)) \cap (\xi_3, A_3) = (\xi_1, A_1) \cap ((\xi_2, A_2) \cap (\xi_3, A_3))$

Proposition 3.20. *If* (ξ_1, A_1),(ξ_2, A_2) *and* (ξ_3, A_3) *are three fuzzy hypersoft expert sets over* $\coprod$*, then*

(1). $(\xi_1, A_1) \cup ((\xi_2, A_2) \cap (\xi_3, A_3)) = ((\xi_1, A_1) \cup ((\xi_2, A_2)) \cap ((\xi_1, A_1) \cup (\xi_3, A_3))$

(2). $(\xi_1, A_1) \cap ((\xi_2, A_2) \cup (\xi_3, A_3)) = ((\xi_1, A_1) \cap ((\xi_2, A_2)) \cup ((\xi_1, A_1) \cap (\xi_3, A_3))$

Definition 3.21. If (ξ_1, A_1) and (ξ_2, A_2) are two fuzzy hypersoft expert sets over $\coprod$ then (ξ_1, A_1) AND (ξ_2, A_2) denoted by $(\xi_1, A_1) \wedge (\xi_2, A_2)$ is defined by
$(\xi_1, A_1) \wedge (\xi_2, A_2) = (\xi_3, A_1 \times A_2)$,
while $\xi_3(\beta, \gamma) = \xi_1(\beta) \cap \xi_2(\gamma), \forall (\beta, \gamma) \in A_1 \times A_2$.

Example 3.22. Taking into consideration the concept of example 3.2, let two sets

$$A_1 = \left\{ (\mu_1, s, 1), (\mu_1, t, 1), (\mu_3, s, 0) \right\}$$

$$A_2 = \left\{ (\mu_1, s, 1), (\mu_3, s, 0) \right\}$$

Suppose (ξ_1, A_1) and (ξ_2, A_2) over $\coprod$ are two fuzzy hypersoft expert sets such that

$$(\xi_1, \mathrm{A}_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.6}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \left((\mu_1, t, 1), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.6}, \frac{m_3}{0.2}, \frac{m_4}{0.1}\right\}\right), \\ , \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.3}, \frac{m_3}{0.6}, \frac{m_4}{0.7}\right\}\right), \end{array} \right\}$$

$$(\xi_2, \mathrm{A}_2) = \left\{ \left((\mu_1, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.1}\right\}\right), \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.4}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right), \right\}$$

Then $(\xi_3, A_3) \wedge (\xi_2, A_2) = (\xi_3, A_1 \times A_2)$,

$$(\xi_3, \mathrm{A}_1 \times \mathrm{A}_2) = \left\{ \begin{array}{l} \left(((\mu_1, s, 1), (\mu_1, s, 1)), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.1}\right\}\right), \\ \left(((\mu_1, t, 1), (\mu_1, s, 1)), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.1}\right\}\right), \\ \left(((\mu_1, t, 1), (\mu_3, s, 0)), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.6}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right), \\ \left(((\mu_1, s, 1), (\mu_3, s, 0)), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.6}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right), \\ \left(((\mu_3, s, 0), (\mu_1, s, 1)), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.7}, \frac{m_3}{0.6}, \frac{m_4}{0.7}\right\}\right), \\ \left(((\mu_3, s, 0), (\mu_3, s, 0)), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.4}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right) \end{array} \right\}$$

Definition 3.23. If (ξ_1, A_1) and (ξ_2, A_2) are two fuzzy hypersoft expert sets over $\coprod$ then (ξ_1, A_1) OR (ξ_2, A_2) denoted by $(\xi_1, A_1) \vee (\xi_2, A_2)$ is defined by
$(\xi_1, A_1) \vee (\xi_2, A_2) = (\xi_3, A_1 \times A_2)$,
while $\xi_3(\beta, \gamma) = \xi_1(\beta) \cup \xi_2(\gamma), \forall (\beta, \gamma) \in A_1 \times A_2$.

Example 3.24. Taking into consideration the concept of example 3.2, suppose the following sets

$$A_1 = \Big\{ (\mu_1, s, 1), (\mu_1, t, 1), (\mu_3, s, 0) \Big\}$$
$$A_2 = \Big\{ (\mu_1, s, 1), (\mu_3, s, 0) \Big\}$$

Suppose (ξ_1, A_1) and (ξ_2, A_2) over $\coprod$ are two fuzzy hypersoft expert sets such that

$$(\xi_1, \mathrm{A}_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.6}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \left((\mu_1, t, 1), \left\{\frac{m_1}{0.3}, \frac{m_2}{0.6}, \frac{m_3}{0.2}, \frac{m_4}{0.1}\right\}\right), \\ , \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.3}, \frac{m_3}{0.6}, \frac{m_4}{0.7}\right\}\right), \end{array} \right\}$$

$$(\xi_2, \mathrm{A}_2) = \left\{ \left((\mu_1, s, 1), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.7}, \frac{m_3}{0.5}, \frac{m_4}{0.1}\right\}\right), \left((\mu_3, s, 0), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.4}, \frac{m_3}{0.7}, \frac{m_4}{0.8}\right\}\right), \right\}$$

Then $(\xi_3, A_3) \wedge (\xi_2, A_2) = (\xi_3, A_1 \times A_2)$,

$$(\xi_3, \mathrm{A}_1 \times \mathrm{A}_2) = \left\{ \begin{array}{l} \left(((\mu_1, s, 1), (\mu_1, s, 1)), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.6}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \\ \left(((\mu_1, t, 1), (\mu_1, s, 1)), \left\{\frac{m_1}{0.2}, \frac{m_2}{0.6}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \\ \left(((\mu_1, t, 1), (\mu_3, s, 0)), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.4}, \frac{m_3}{0.2}, \frac{m_4}{0.1}\right\}\right), \\ \left(((\mu_1, s, 1), (\mu_3, s, 0)), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.4}, \frac{m_3}{0.4}, \frac{m_4}{0.1}\right\}\right), \\ \left(((\mu_3, s, 0), (\mu_1, s, 1)), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.3}, \frac{m_3}{0.5}, \frac{m_4}{0.1}\right\}\right), \\ \left(((\mu_3, s, 0), (\mu_3, s, 0)), \left\{\frac{m_1}{0.1}, \frac{m_2}{0.3}, \frac{m_3}{0.6}, \frac{m_4}{0.7}\right\}\right) \end{array} \right\}$$

Proposition 3.25. *If* (ξ_1, A_1),(ξ_2, A_2) *and* (ξ_3, A_3) *are three fuzzy hypersoft expert sets over* $\coprod$, *then*

(1). $((\xi_1, A_1) \wedge (\xi_2, A_2))^c = ((\xi_1, A_1))^c \vee ((\xi_2, A_2))^c$
(2). $((\xi_1, A_1) \vee (\xi_2, A_2))^c = ((\xi_1, A_1))^c \wedge ((\xi_2, A_2))^c$

Proposition 3.26. *If* (ξ_1, A_1),(ξ_2, A_2) *and* (ξ_3, A_3) *are three fuzzy hypersoft expert sets over* $\coprod$, *then*

(1). $((\xi_1, A_1) \wedge (\xi_2, A_2)) \wedge (\xi_3, A_3) = (\xi_1, A_1) \wedge ((\xi_2, A_2) \wedge (\xi_3, A_3))$
(2). $((\xi_1, A_1) \vee (\xi_2, A_2)) \vee (\xi_3, A_3) = (\xi_1, A_1) \vee ((\xi_2, A_2) \vee (\xi_3, A_3))$
(3). $(\xi_1, A_1) \vee ((\xi_2, A_2) \wedge (\xi_3, A_3) = ((\xi_1, A_1) \vee ((\xi_2, A_2)) \wedge ((\xi_1, A_1) \vee (\xi_3, A_3))$
(4). $(\xi_1, A_1) \wedge ((\xi_2, A_2) \vee (\xi_3, A_3)) = ((\xi_1, A_1) \wedge ((\xi_2, A_2)) \vee ((\xi_1, A_1) \wedge (\xi_3, A_3))$

4. An Applications to Fuzzy Hypersoft expert set

In this section, an application of fuzzy hypersoft expert set theory in a decision making problem, is presented.

Statement of the problem

Mr. John wants to purchase a mobile from a mobile market for his personal use. He takes help from his some friends (Stephen, Thomas and Umar) who have expertise in mobile purchase.

Proposed Algorithm

The following algorithm is adopted for this selection (purchase).

(1). Construct fuzzy hypersoft soft expert set (ξ, K),

(2). Determine an Agree-fuzzy hypersoft expert set and a Disagree-fuzzy hypersoft expert set,

(3). Compute $d_i = \sum_i c_{ij}$ for Agree-fuzzy hypersoft expert set,

(4). Determine $f_i = \sum_i c_{ij}$ for Disagree-fuzzy hypersoft expert set,

(5). Determine $g_j = d_j - f_j$ for Agree-fuzzy hypersoft expert set,

(6). Compute n, for which $p_n =$ max p_j for Agree-fuzzy hypersoft expert set,

Step-1

Let eight categories of mobile are there which form the universe of discourse $\Omega = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$ and X $= \{E_1 = Stephen, E_2 = Thomas, E_3 = Umar\}$ be a set of experts for this purchase. The following are the attribute-valued sets for prescribed attributes:

$L_1 = Brand = \{l_1, l_2\}$

$L_2 = Price = \{l_3, l_4\}$

$L_3 = Colour = \{l_5, l_6\}$

$L_4 = Memory = \{l_7, l_8\}$

$L_5 = Resolution = \{l_9, l_{10}\}$

and then

$L = L_1 \times L_2 \times L_3 \times L_4 \times L_5$

$$L = \left\{ \begin{array}{l} (l_1, l_3, l_5, l_7, l_9), (l_1, l_3, l_5, l_7, l_{10}), (l_1, l_3, l_5, l_8, l_9), (l_1, l_3, l_5, l_8, l_{10}), (l_1, l_3, l_6, l_7, l_9), \\ (l_1, l_3, l_6, l_7, l_{10}), (l_1, l_3, l_6, l_8, l_9), (l_1, l_3, l_6, l_8, l_{10}), (l_1, l_4, l_5, l_7, l_9), (l_1, l_4, l_5, l_7, l_{10}), \\ (l_1, l_4, l_5, l_8, l_9), (l_1, l_4, l_5, l_8, l_{10}), (l_1, l_4, l_6, l_7, l_9), (l_1, l_4, l_6, l_7, l_{10}), (l_1, l_4, l_6, l_8, l_9), \\ (l_1, l_4, l_6, l_8, l_{10}), (l_2, l_3, l_5, l_7, l_9), (l_2, l_3, l_5, l_7, l_{10}), (l_2, l_3, l_5, l_8, l_9), (l_2, l_3, l_5, l_8, l_{10}), \\ (l_2, l_3, l_6, l_7, l_9), (l_2, l_3, l_6, l_7, l_{10}), (l_2, l_3, l_6, l_8, l_9), (l_2, l_3, l_6, l_8, l_{10}), (l_2, l_4, l_5, l_7, l_9), \\ (l_2, l_4, l_5, l_7, l_{10}), (l_2, l_4, l_5, l_8, l_9), (l_2, l_4, l_5, l_8, l_{10}), (l_2, l_4, l_6, l_7, l_9), (l_2, l_4, l_6, l_7, l_{10}), \\ (l_2, l_4, l_6, l_8, l_9), (l_2, l_4, l_6, l_8, l_{10}) \end{array} \right\}$$

and now take $K \subseteq H$ as

$$K = \left\{ \begin{array}{l} k_1 = (l_1, l_3, l_5, l_7, l_9), k_2 = (l_1, l_3, l_6, l_7, l_{10}), k_3 = (l_1, l_4, l_6, l_8, l_9), \\ k_4 = (l_2, l_3, l_6, l_8, l_9), k_5 = (l_2, l_4, l_6, l_7, l_{10}) \end{array} \right\}$$

and

$$(\xi, \mathrm{K}) = \left\{ \begin{array}{l} \left((k_1,E_1,1),\left\{\frac{c_1}{0.3},\frac{c_2}{0.8},\frac{c_4}{0.5},\frac{c_7}{0.2},\frac{c_8}{0.5}\right\}\right),\left((k_1,E_2,1),\left\{\frac{c_1}{0.8},\frac{c_4}{0.7},\frac{c_5}{0.8},\frac{c_8}{0.2}\right\}\right), \\ \left((k_1,E_3,1),\left\{\frac{c_1}{0.7},\frac{c_3}{0.8},\frac{c_4}{0.6},\frac{c_5}{0.1},\frac{c_6}{0.5},\frac{c_7}{0.3},\frac{c_8}{0.2}\right\}\right),\left((k_2,E_1,1),\left\{\frac{c_1}{0.3},\frac{c_3}{0.6},\frac{c_5}{0.2},\frac{c_8}{0.1}\right\}\right), \\ \left((k_2,E_2,1),\left\{\frac{c_1}{0.3},\frac{c_3}{0.8},\frac{c_4}{0.6},\frac{c_5}{0.1},\frac{c_6}{0.9},\frac{c_8}{0.1}\right\}\right),\left((k_2,E_3,1),\left\{\frac{c_1}{0.2},\frac{c_2}{0.5},\frac{c_4}{0.1},\frac{c_5}{0.1},\frac{c_6}{0.7},\frac{c_8}{0.9}\right\}\right), \\ \left((k_3,E_1,1),\left\{\frac{c_1}{0.2},\frac{c_4}{0.8},\frac{c_5}{0.1},\frac{c_7}{0.9}\right\}\right),\left((k_3,E_2,1),\left\{\frac{c_1}{0.3},\frac{c_2}{0.7},\frac{c_5}{0.8},\frac{c_8}{0.1}\right\}\right), \\ \left((k_3,E_3,1),\left\{\frac{c_1}{0.5},\frac{c_3}{0.7},\frac{c_5}{0.7},\frac{c_8}{0.1}\right\}\right),\left((k_4,E_1,1),\left\{\frac{c_1}{0.4},\frac{c_7}{0.7},\frac{c_8}{0.8}\right\}\right), \\ \left((k_4,E_2,1),\left\{\frac{c_1}{0.5},\frac{c_4}{0.8},\frac{c_5}{0.9},\frac{c_8}{0.1}\right\}\right),\left((k_4,E_3,1),\left\{\frac{c_1}{0.3},\frac{c_6}{0.7},\frac{c_7}{0.9},\frac{c_8}{0.7}\right\}\right), \\ \left((k_5,E_1,1),\left\{\frac{c_1}{0.2},\frac{c_3}{0.7},\frac{c_4}{0.3},\frac{c_5}{0.6},\frac{c_7}{0.1},\frac{c_8}{0.8}\right\}\right),\left((k_5,E_2,1),\left\{\frac{c_1}{0.1},\frac{c_4}{0.8},\frac{c_5}{0.8},\frac{c_6}{0.2},\frac{c_8}{0.3}\right\}\right), \\ \left((k_5,E_3,1),\left\{\frac{c_1}{0.2},\frac{c_3}{0.7},\frac{c_4}{0.3},\frac{c_5}{0.9},\frac{c_6}{0.4},\frac{c_7}{0.1},\frac{c_8}{0.8}\right\}\right),\left((k_1,E_1,0),\left\{\frac{c_1}{0.3},\frac{c_6}{0.7},\frac{c_7}{0.4},\frac{c_8}{0.7}\right\}\right), \\ \left((k_1,E_2,0),\left\{\frac{c_2}{0.4},\frac{c_3}{0.8},\frac{c_6}{0.7},\frac{c_7}{0.1},\frac{c_8}{0.8}\right\}\right),\left((k_1,E_3,0),\left\{\frac{c_1}{0.2},\frac{c_5}{0.3}\right\}\right), \\ \left((k_2,E_1,0),\left\{\frac{c_1}{0.1},\frac{c_2}{0.7},\frac{c_4}{0.3},\frac{c_5}{0.4},\frac{c_6}{0.5}\right\}\right),\left((k_2,E_2,0),\left\{\frac{c_1}{0.3},\frac{c_7}{0.2}\right\}\right), \\ \left((k_2,E_3,0),\left\{\frac{c_1}{0.4},\frac{c_3}{0.5},\frac{c_4}{0.1},\frac{c_5}{0.8},\frac{c_6}{0.9},\frac{c_7}{0.2}\right\}\right),\left((k_3,E_1,0),\left\{\frac{c_1}{0.6},\frac{c_2}{0.7},\frac{c_6}{0.8},\frac{c_8}{0.9}\right\}\right), \\ \left((k_3,E_2,0),\left\{\frac{c_3}{0.2},\frac{c_4}{0.4},\frac{c_6}{0.5},\frac{c_7}{0.6}\right\}\right),\left((k_3,E_3,0),\left\{\frac{c_1}{0.2},\frac{c_3}{0.7},\frac{c_4}{0.8},\frac{c_5}{0.1},\frac{c_7}{0.9}\right\}\right), \\ \left((k_4,E_1,0),\left\{\frac{c_1}{0.2},\frac{c_2}{0.6},\frac{c_3}{0.7},\frac{c_4}{0.9},\frac{c_5}{0.1},\frac{c_7}{0.4}\right\}\right),\left((k_4,E_2,0),\left\{\frac{c_2}{0.3},\frac{c_3}{0.7},\frac{c_6}{0.5},\frac{c_7}{0.1}\right\}\right), \\ \left((k_4,E_3,0),\left\{\frac{c_1}{0.4},\frac{c_3}{0.7},\frac{c_4}{0.8},\frac{c_5}{0.1}\right\}\right),\left((k_5,E_1,0),\left\{\frac{c_1}{0.6},\frac{c_6}{0.7}\right\}\right), \\ \left((k_5,E_2,0),\left\{\frac{c_1}{0.7},\frac{c_2}{0.5},\frac{c_6}{0.1},\frac{c_7}{0.2}\right\}\right),\left((k_5,E_3,0),\left\{\frac{c_1}{0.2},\frac{c_4}{0.3},\frac{c_6}{0.1}\right\}\right), \end{array} \right\}$$

is a fuzzy hypersoft expert set.

Step-2

Table 1 presents an Agree-fuzzy hypersoft expert set and table 2 presents a Disagree-fuzzy hypersoft expert set respectively, such that if $c_i \in \xi_1(\beta)$ then $c_{ij} \in [0,1]$ otherwise $c_{ij} = \times = 0$, and if

$$c_i \in \xi_0(\beta)$$

then $c_{ij} \in [0,1]$ otherwise $c_{ij} = \times = 0$ where c_{ij} are the entries in tables 1 and 2.

Step-(3-6)

Table 3 presents $d_i = \sum_i c_{ij}$ for Agree-fuzzy hypersoft expert set, $f_i = \sum_i c_{ij}$ for Disagree-fuzzy hypersoft expert set, $g_j = d_j - f_j$ for Agree-fuzzy hypersoft expert set, and n, for which $p_n =$ max p_j for Agree-fuzzy hypersoft expert set.

Decision

As g_5 is maximum, so category c_5 is preferred to be best for purchase.

5. Conclusions

Insufficiency of soft set, fuzzy soft set, soft expert set and fuzzy soft expert set for multi-attribute function (attribute-valued sets) is addressed with the development and characterization of novel hybrid structure i.e. fuzzy hypersoft expert set, in this study. Moreover

(1) The fundamentals of fuzzy hypersoft expert set (FHSE-Set) are established and the basic properties of FHSE-Set like subset, superset, equal sets, not set, agree FHSE-Set and disagree FHSE-Set are described with examples.

TABLE 1. Agree-fuzzy hypersoft expert set

C	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
(k_1, E_1)	0.3	0.8	×	0.5	×	×	0.2	0.5
(k_1, E_2)	0.8	×	×	0.7	0.8	×	×	0.2
(k_1, E_3)	0.7	×	0.8	0.6	0.1	0.5	0.3	0.2
(k_2, E_1)	0.3	×	0.6	×	0.2	×	×	0.1
(k_2, E_2)	0.3	×	0.8	0.6	0.1	0.9	×	0.1
(k_2, E_3)	0.2	0.5	×	0.1	0.1	0.7	×	0.9
(k_3, E_1)	0.2	×	×	0.8	0.1	×	0.9	×
(k_3, E_2)	0.3	0.7	×	×	0.8	×	×	0.1
(k_3, E_3)	0.5	×	0.7	×	0.7	×	×	0.1
(k_4, E_1)	0.4	×	×	×	×	×	0.7	0.8
(k_4, E_2)	0.5	×	×	0.8	0.9	×	×	0.1
(k_4, E_3)	0.3	×	×	×	×	0.7	0.9	0.7
(k_5, E_1)	0.2	×	0.7	0.3	0.6	×	0.1	0.8
(k_5, E_2)	0.1	×	×	0.8	0.8	0.2	×	0.3
(k_5, E_3)	0.2	×	0.7	0.3	0.9	0.4	0.1	0.8
$d_j = \sum_i c_{ij}$	5.3	2.0	4.3	5.5	5.9	3.4	3.2	5.7
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8

TABLE 2. Disagree-fuzzy hypersoft expert set

C	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
(k_1, E_1)	0.3	×	×	×	×	0.7	0.4	0.7
(k_1, E_2)	×	0.4	0.8	×	×	0.7	0.1	0.8
(k_1, E_3)	0.2	×	×	×	0.3	×	×	×
(k_2, E_1)	0.1	0.7	×	0.3	0.4	0.5	×	×
(k_2, E_2)	0.3	×	×	×	×	×	0.2	×
(k_2, E_3)	0.4	×	0.5	0.1	0.8	0.9	0.2	×
(k_3, E_1)	0.6	0.7	×	×	×	0.8	×	0.9
(k_3, E_2)	×	×	0.2	0.4	×	0.5	0.6	×
(k_3, E_3)	0.2	×	0.7	0.8	0.1	×	0.9	×
(k_4, E_1)	0.2	0.6	0.7	0.9	0.1	×	0.4	×
(k_4, E_2)	×	0.3	0.7	×	×	0.5	0.1	×
(k_4, E_3)	0.4	×	0.7	0.8	0.1	×	×	×
(k_5, E_1)	0.6	×	×	×	×	0.7	×	×
(k_5, E_2)	0.7	0.5	×	×	×	0.1	0.2	×
(k_5, E_3)	0.2	×	×	0.3	×	0.1	×	×
$f_i = \sum_i c_{ij}$	4.2	3.2	4.3	3.6	1.8	5.5	3.1	2.4
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8

TABLE 3. Optimal

$d_i = \sum_i c_{ij}$	$f_i = \sum_i c_{ij}$	$g_j = d_j - f_j$
$d_1 = 5.3$	$f_1 = 4.2$	$g_1 = 1.1$
$d_2 = 2.0$	$f_2 = 3.2$	$g_2 = -1.0$
$d_3 = 4.3$	$f_3 = 4.3$	$g_3 = 0.0$
$d_4 = 5.5$	$f_4 = 3.6$	$g_4 = 1.9$
$d_5 = 5.9$	$f_5 = 1.8$	$g_5 = 4.1$
$d_6 = 3.4$	$f_6 = 5.5$	$g_6 = -2.1$
$d_7 = 3.2$	$f_7 = 3.1$	$g_7 = 1.0$
$d_8 = 5.7$	$f_8 = 2.4$	$g_8 = 3.3$

(2) The essential set-theoretic operations on FHSE-Set like complement, union, intersection, OR and AND operations are established and some laws such as commutative, associative and De Morgan are presented with suitable examples.
(3) A decision-making application regarding the best selection of a certain product is presented with the help of proposed algorithm.
(4) A daily life based example is discussed for the understanding of decision making process.
(5) Future work may include the extension of the presented work for other hypersoft-like hybrids i.e. intuitionistic fuzzy set, interval-valued fuzzy set, pythagorean fuzzy set, neutrosophic set etc.

Conflicts of Interest:

The authors declare no conflict of interest.

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