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# Fuzzy Neutrosophic Strongly Alpha Generalized Closed Sets in Fuzzy Neutrosophic Topological spaces

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**Abstract:** In this paper, we will define a new set called fuzzy neutrosophic strongly alpha generalized closed set, so we will prove some theorems related to this concept. After that, we will give some interesting properties were investigated and referred to some results related to the new definitions by theorems, propositions to get some relationships among fuzzy neutrosophic strongly alpha generalized closed sets, fuzzy neutrosophic closed sets, fuzzy neutrosophic regular closed sets, fuzzy neutrosophic alpha closed sets, fuzzy neutrosophic alpha generalized closed sets and fuzzy neutrosophic pre closed sets which are compared with necessary examples based of fuzzy neutrosophic topological spaces.

**Keywords:** Fuzzy neutrosophic set, fuzzy neutrosophic topological space, fuzzy neutrosophic strongly alpha generalized closed set.

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## 1. Introduction

The concept of fuzzy set "FS"was introduced by Lotfi Zadeh in 1965 [1], then Chang depended the fuzzy set to introduce the concept of fuzzy topological space "FTS" in 1968 [7]. After that the concept of fuzzy set was developed into the concept of intutionistic fuzzy set "IFS" by Atanassov in 1983 [4-6], the intutionistic fuzzy set gives a degree of membership and a degree of non-membership functions. Cokor in 1997 [7] relied on intutionistic fuzzy set to introduced the concept of intutionistic fuzzy topological space."IFTS". In 2005 Smaradache [23] study the concept of neutrosophic set. "NS". After that and as developed the term of neutrosophic set, Salama has studied neutrosophic topological space "NTS"and many of its applications [18-21]. In 2013 Arockiarani Sumathi and Martina Jency [2] introduced the concept of fuzzy neutrosophic set as generalizes the concept of fuzzy set and intutionistic fuzzy set. where each element had three associated defining functions on the universe of discourse X, namely the membership function (T), indeterminacy function (I), the non-membership function (F) that is added an indeterminacy degree between the degree of membership and the degree of non- membership. In 2012 Salama and Alblowi defined fuzzy neutrosophic topological space [18].

In the present work, we will generalized the concept of strongly alpha generalized closed set in fuzzy neutrosophic topological spaces which was studied by Santhi and Sakthivel in 2011 [22] via intuitinistic topological spaces and generalizing our works in 2018 [ 9,10 ], the new set will called fuzzy neutrosophic strongly alpha generalized closed set in fuzzy neutrosophic topological spaces.

Finally, there are many application of neutrosophic sets in many fields so we can enhance our work, we will try in the future to applied this work in different fields such as many authors applications see [11] and [13-17].

**2. Preliminaries:**

In this section, we will define some basic definitions and some operations which are useful in our present study.

**Definition 2.1 [18]:** Let X be a non-empty fixed set. The fuzzy neutrosophic set (FNS, for short), $\eta_N$ is an object having the form $\eta_N = \{< x, \mu_{\eta N}(x), \sigma_{\eta N}(x), \nu_{\eta N}(x) >: x \in X\}$ where the functions $\mu_{\eta N}$, $\sigma_{\eta N}$, $\nu_{\eta N}$: $X \to [0, 1]$ denote the degree of membership function (namely $\mu_{\eta N}(x)$), the degree of indeterminacy function (namely $\sigma_{\eta N}(x)$) and the degree of non-membership (namely $\nu_{\eta N}(x)$) respectively of each element $x \in X$ to the set $\eta_N$ and $0 \le \mu_{\eta N}(x) + \sigma_{\eta N}(x) + \nu_{\eta N}(x) \le 3$, for each $x \in X$.

**Remark 2.2 [18]:** FNS $\eta_N = \{< x, \mu_{\eta N}(x), \sigma_{\eta N}(x), \nu_{\eta N}(x) >: x \in X\}$ can be identified to an ordered triple $< x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N} >$ in [0, 1] on X.

**Definition 2.3 [18]:** Let X be a non-empty set and the FNSs $\eta_N$ and $\gamma_N$ be in the form:

$\eta_N = \{< x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N} >: x \in X\}$ and $\gamma_N = \{< x, \mu_{\gamma N}, \sigma_{\gamma N}, \nu_{\gamma N} >: x \in X\}$ on X then:

**i.** $\eta_N \subseteq \gamma_N$ iff $\mu_{\eta N} \le \mu_{\gamma N}$, $\sigma_{\eta N} \le \sigma_{\gamma N}$ and $\nu_{\eta N} \ge \nu_{\gamma N}$.

ii. $\eta_N = \gamma_N$ iff $\eta_N \subseteq \gamma_N$ and $\gamma_N \subseteq \eta_N$,

iii. $1_N - \eta_N = \{< x, \nu_{\eta N}, 1 - \sigma_{\eta N}, \mu_{\eta N} >: x \in X\}$,

**iv.** $\eta_N \cup \gamma_N = \{< x, Max(\mu_{\eta N}, \mu_{\gamma N}), Max(\sigma_{\eta N}, \sigma_{\gamma N}, Min(\nu_{\eta N}, \nu_{\gamma N}) >: x \in X\}$,

**v.** $\eta_N \cap \gamma_N = \{< x, Min(\mu_{\eta N}, \mu_{\gamma N}), Min(\sigma_{\eta N}, \sigma_{\gamma N}), Max(\nu_{\eta N}, \nu_{\gamma N}) >: x \in X\}$,

**vi.** $0_N = < x, 0, 0, 1>$ and $1_N = <x, 1, 1, 0 >$.

**Definition 2.4 [18]:** "Fuzzy neutrosophic topology (FNT, for short) on a non-empty set X is a family $\tau_N$ of fuzzy neutrosophic subsets in X satisfying the following axioms.

**i.** $0_N, 1_N \in \tau_N$,

**ii.** $\eta_{N1} \cap \eta_{N2} \in \tau_N$ for any $\eta_{N1}, \eta_{N2} \in \tau_N$,

**iii.** $\cup \eta_{Ni} \in \tau_N$, $\forall \{\eta_{Ni}: i \in J\} \subseteq \tau_N$."

In this case the pair $(X, \tau_N)$ is called fuzzy neutrosophic topological space (FNTS, for short). The elements of $\tau_N$ are called fuzzy neutrosophic open set (FNOS, for short). The complement of FNOS in the FNTS $(X, \tau_N)$ is called fuzzy neutrosophic closed set (FNCS, for short).

**Definition 2.5 [18]:** Let (X, $\tau_N$) be FNTS and $\eta_N$ = < x, $\mu_{\eta N}$, $\sigma_{\eta N}$, $\nu_{\eta N}$ > be FNS in X. Then the fuzzy neutrosophic closure of $\eta_N$ (FNCL, for short) and fuzzy neutrosophic interior of $\eta_N$ (FNIn, for short) are defined by:

$$FNCL(\eta_N) = \cap \{C_N: C_N \text{ is FNCS in X and } \eta_N \subseteq C_N \},$$

$$FNIn (\eta_N) = \cup \{O_N: O_N \text{ is FNOS in X and } O_N \subseteq \eta_N \}.$$

We know, FNCL($\eta_N$) is FNCS and FNIn ($\eta_N$) is FNOS in X. Further,

**i.** $\eta_N$ is FNCS in X iff FNCL ($\eta_N$) = $\eta_N$,

**ii.** $\eta_N$ is FNOS in X iff FNIn ($\eta_N$) = $\eta_N$.

**Proposition 2.6 [25]:** Let (X, $\tau_N$) is FNTS and $\eta_N$, $\gamma_N$ are FNSs in X. Then the following properties hold:

**i.** FNIn ($\eta_N$) $\subseteq \eta_N$ and $\eta_N \subseteq$ FNCL($\eta_N$),

**ii.** $\eta_N \subseteq \gamma_N \Longrightarrow$ FNIn ($\eta_N$) $\subseteq$ FNIn ($\gamma_N$) and $\eta_N \subseteq \gamma_N \Longrightarrow$ FNCL($\eta_N$) $\subseteq$ FNCL($\gamma_N$),

**iii.** FNIn (FNIn ($\eta_N$)) = FNIn ($\eta_N$) and FNCL(FNCL($\eta_N$)) = FNCL($\eta_N$),

**iv.** FNIn ($\eta_N \cap \gamma_N$) = FNIn ($\eta_N$) $\cap$ FNIn ($\gamma_N$) and FNCL($\eta_N \cup \gamma_N$) = FNCL($\eta_N$) $\cup$FNCL($\gamma_N$),

**v.** FNIn ($1_N$) =$1_N$ and FNCL($1_N$) = $1_N$,

**vi.** FNIn ($0_N$) = $0_N$ and FNCL($0_N$) = $0_N$.

**Definition 2.7 [9]:** FNS $\eta_N$ in FNTS (X, $\tau_N$) is called:

i. Fuzzy neutrosophic regular closed set (FNRCS, for short) if $\eta_N$ = FNCL(FNIn ($\eta_N$)).

ii. Fuzzy neutrosophic pre closed set (FNPCS, for short) if FNCL(FNIn ($\eta_N$)) $\subseteq \eta_N$.

iii. Fuzzy neutrosophic α closed set (FNαCS, for short) if FNCL(FNIn(FNCL($\eta_N$))) $\subseteq \eta_N$.

**Definition 2.8 [10]:** Let (X, $\tau_N$) be FNTS and $\eta_N$ = < x, $\mu_{\eta N}$, $\sigma_{\eta N}$, $\nu_{\eta N}$ > be FNS in X. Then the fuzzy neutrosophic alpha closure of $\eta_N$ (FNαCL, for short) and fuzzy neutrosophic alpha interior of $\eta_N$ (FNαIn,
for short) are defined by:

$$FN\alpha CL(\eta_N) = \cap \{C_N: C_N \text{ is FN}\alpha\text{CS in X and } \eta_N \subseteq C_N \},$$

$$FN\alpha In (\eta_N) = \cup \{O_N: O_N \text{ is FN}\alpha\text{OS in X and } O_N \subseteq \eta_N\}.$$

We know, FNαCL($\eta_N$) is FNαCLS and FNαIn ($\eta_N$) is FNαOS in X. Further,

**i.** $\eta_N$ is FNαCS in X iff FNαCL($\eta_N$) = $\eta_N$,

**ii.** $\eta_N$ is FNαOS in X iff FNαIn ($\eta_N$) = $\eta_N$.

**Definition 2.9 [9,10]:** Fuzzy neutrosophic sub set $\eta_N$ of FNTS (X, $\tau_N$) is called:

**i.** fuzzy neutrosophic generalized closed set (FNGCS, for short ) if FNCL($\eta_N$) $\subseteq U_N$ wherever, $\eta_N \subseteq U_N$ and $U_N$ is FNOS in X. And $\eta_N$ is said to be fuzzy neutrosophic generalized open set (FNGOS, for short) if the complement $1_N$- $\eta_N$ is FNGCS set in (X, $\tau_N$).

**ii.** fuzzy neutrosophic alpha generalized closed set (FNαGCS, for short ) if FNαCL($\eta_N$) $\subseteq$ $U_N$ wherever, $\eta_N \subseteq U_N$ and $U_N$ is FNOS in X. And $\eta_N$ is said to be fuzzy neutrosophic alpha generalized open set (FNαGOS, for short) if the complement $1_N$- $\eta_N$ is FNαGCS set in (X, $\tau_N$).

## 3. Fuzzy Neutrosophic Strongly Alpha Generalized Closed Sets in Fuzzy Neutrosophic Topological Spaces.

Now, we will introduce the concept of fuzzy neutrosophic strongly alpha generalized closed set in fuzzy neutrosophic topological spaces.

**Definition 3.1:** Fuzzy neutrosophic subset $\eta_N$ of FNTS (X, $\tau_N$) is called fuzzy neutrosophic strongly alpha generalized closed set (FNSαGCS, for short ) if FNαCL ($\eta_N$) ⊆ $U_N$ wherever, $\eta_N \subseteq U_N$ and $U_N$ is FNGOS in X.

**Example 3.2:** Let X= {a, b} define FNS $\eta_N$ in X as follows:
$\eta_N$ =<x, (0.2$_{(a)}$, 0.3$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.8$_{(a)}$,0.7$_{(b)}$) >, where the family $\tau_N$ ={$0_N$, $1_N$, $\eta_N$ }.
If we take, $\psi_N$ = <x, (0.8$_{(a)}$, 0.7$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.1$_{(a)}$,0$_{(b)}$) >.
And, $U_N$ = $1_N$ where $U_N$ is FNGOS such that, $\psi_N \subseteq U_N$. Then, FNαCL($\psi_N$) = $1_N$. So, FNαCL($\psi_N$) ⊆ $U_N$.
Hence, $\psi_N$ is FNSαGCS.

**Theorem 3.3:** For any FNSs, the following statements are true in general:

i. Every FNOS is FNGOS.
ii. Every FNCS is FNαCS.
iii. Every FNCS is FNSαGCS.
iv. Every FNRCS is FNSαGCS.
v. Every FNαCS is FNSαGCS.
vi. Every FNαGCS is FNSαGCS.
**vii.** Every FNRCS is FNCS.
**viii.** Every FNαCS is FNαGCS.

**Proof:**.
i. Let $\eta_N$ = <x, $\mu_{\eta N}$, $\sigma_{\eta N}$, $\nu_{\eta N}$ > be FNOS in the FNTS (X,$\tau_N$).
Then **by Definition 2.5 ii** we get, FNIn ($\eta_N$) = $\eta_N$.
Now, let $U_N$ is FNCS such that, $U_N \subseteq \eta_N$. Therefore, FNIn ($\eta_N$) = $\eta_N \supseteq U_N$.
Hence, $\eta_N$ is FNGOS in (X,$\tau_N$) .

ii. Let $\eta_N$ = <x, $\mu_{\eta N}$, $\sigma_{\eta N}$, $\nu_{\eta N}$ > be FNCLS in the FNTS (X, $\tau_N$).
Then by **Definition 2.5 (i)** we get, FN<u>CL</u>($\eta_N$) = $\eta_N$……(1).
And **by Proposition 2.6 i** we get, FNIn ($\eta_N$) ⊆ $\eta_N$.

So we get, FNIn (FN$\underline{CL}$($\eta_N$)) ⊆ $\eta_N$
This implies FN$\underline{CL}$(FNIn (FNCL($\eta_N$))) ⊆ FNCL($\eta_N$).
So **by (1)** we get, FNCL(FNIn (FNCL($\eta_N$))) ⊆ $\eta_N$.
Hence, $\eta_N$ is FNαCS in (X, $\tau_N$).

iii. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNCS in FNTS (X,$\tau_N$).
Then by **Definition 2.5 (i)** we get, FNCL($\eta_N$) = $\eta_N$. Now, let $U_N$ be FNGOS such that, $\eta_N$ ⊆ $U_N$.
Since, FNαCL($\eta_N$) ⊆FNCL($\eta_N$) **by Definition 2.5 and Definition 2.8.**
So we get, FNαCL($\eta_N$) ⊆ FNCL($\eta_N$) = $\eta_N$ ⊆ $U_N$.
Hence, $\eta_N$ is FNSαGCS in (X,$\tau_N$) .

iv. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNRCS in the FNTS (X, $\tau_N$).
Then, FNCL(FNIn ($\eta_N$)) = $\eta_N$…….(1).
This implies, FNCL(FNIn ($\eta_N$)) = FNCL($\eta_N$)…….(2).
Now, let $U_N$ be FNGOS such that, $\eta_N$ ⊆ $U_N$.
From **(1) and (2)** we get, FNCL($\eta_N$) = $\eta_N$.
That $\eta_N$ is FNCS in X.
So **by iii** we get, FNαCL($\eta_N$) ⊆ FNCL($\eta_N$) = $\eta_N$ ⊆ $U_N$.
Hence, $\eta_N$ is FNSαGCS in (X,$\tau_N$) .

v. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNαCLOS in the FNTS (X, $\tau_N$).
Then **by Definition 2.8 i** we get, FNαCL ($\eta_N$) = $\eta_N$.
Now, let $U_N$ be FNGOS such that, $\eta_N$ ⊆ $U_N$. So, FNαCL ($\eta_N$) = $\eta_N$ ⊆ $U_N$.
Hence, $\eta_N$ is FNSαGCS in (X, $\tau_N$).

vi. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNαGCS in the FNTS (X, $\tau_N$).
Then, FNαCL ($\eta_N$) ⊆ $U_N$, $\eta_N$ ⊆ $U_N$ and $U_N$ be FNOS, so **by i** we get , FNOS be FNGOS in (X, $\tau_N$).
Therefore, FNαCL ($\eta_N$) ⊆ $U_N$, $\eta_N$ ⊆ $U_N$ and $U_N$ be FNGOS. Hence, $\eta_N$ is FNSαGCS in (X, $\tau_N$).

vii. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNαCS in the FNTS (X, $\tau_N$). Then, FNαCL ($\eta_N$) = $\eta_N$.
Now, let $U_N$ be FNOS such that, $\eta_N$ ⊆ $U_N$, so, FNαCL ($\eta_N$) = $\eta_N$ ⊆ $U_N$.
Hence, $\eta_N$ is FNαGCS in (X, $\tau_N$).

**Remark 3.4:** The convers of **Theorem 3.3** is not true and this can be clarified in the following examples.

**Example 3.5:**

**i.** Let X= {a, b} define FNS $\eta_N$ in X as follows:

$\eta_N$ = <x, ($0.5_{(a)}$, $0.7_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.5_{(a)}$,$0.2_{(b)}$) >.

The family $\tau_N$ = {$0_N$, $1_N$, $\eta_N$ } be FNT.

If we take, $\psi_N$ = <x, ($0.1_{(a)}$, $0.6_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.9_{(a)}$,$0.3_{(b)}$) >.

And let, $U_N = 0_N$, where $U_N$ be FNCS such that, $U_N \subseteq \psi_N$.

Then, FNIn ($\psi_N$) = <x, ($0_{(a)}$, $0_{(b)}$), ($0_{(a)}$, $0_{(b)}$), ($1_{(a)}$,$1_{(b)}$) > ⊆ <x, ($0.1_{(a)}$, $0.6_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.9_{(a)}$,$0.3_{(b)}$) > such that, ($0_{(a)}$, $0_{(b)}$) ≤ ($0.1_{(a)}$, $0.6_{(b)}$), ($0_{(a)}$, $0_{(b)}$) ≤ ($0.5_{(a)}$, $0.5_{(b)}$) and ($1_{(a)}$,$1_{(b)}$) ≥ ($0.9_{(a)}$,$0.3_{(b)}$) = $0_N$. So, FNIn ($\psi_N$) ⊇ $U_N$. Hence, $\psi_N$ is FNGOS but, not FNOS.

Since $\psi_N \notin \tau_N$.

**ii.** Let X={a } define the FNSs $\eta_N$ and $\gamma_N$ in X as follows:

$\eta_N$ = <x, ($0.5_{(a)}$), ($0.4_{(a)}$), ($0.7_{(a)}$) >, $\gamma_N$ = <x, ($0.4_{(a)}$), ($0.1_{(a)}$), ($0.8_{(a)}$) >.

The family $\tau_N$ ={$0_N$, $1_N$, $\eta_N$, $\gamma_N$ } be FNT.

If we take, $\psi_N$ = <x, ($0.8_{(a)}$), ($0.6_{(a)}$), ($0.5_{(a)}$) >.

Then, FNCL($\psi_N$) = <x, ($0.8_{(a)}$), ($0.9_{(a)}$), ($0.4_{(a)}$) >. And, FNIn (FNCL($\psi_N$)) = <x, ($0.5_{(a)}$), ($0.4_{(a)}$), ($0.7_{(a)}$) >. So, FNCL(FNIn (FNCL($\psi_N$))) = <x, ($0.7_{(a)}$), ($0.6_{(a)}$), ($0.5_{(a)}$) >.

Therefore, <x, ($0.7_{(a)}$), ($0.6_{(a)}$), ($0.5_{(a)}$) > ⊆ $\psi_N$.

Hence, $\psi_N$ is FNαCS but not FNCS. Since $\psi_N \notin 1_N$-$\tau_N$.

**iii.** Take **Example 3.2.** Then, $\psi_N$ is FNSαGCS but, not FNCS.

Since, $\psi_N \notin 1_N$- $\tau_N$.

**iv.** Take **Example 3.2**. Then $\psi_N$ is FNSαGCS but, not FNRCS.

Since, FNIn ($\psi_N$) = <x, ($0.2_{(a)}$, $0.3_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.8_{(a)}$,$0.7_{(b)}$) > and

FNCL(FNIn ($\psi_N$)) = <x, ($0.8_{(a)}$, $0.7_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.2_{(a)}$,$0.3_{(b)}$) > ≠ $\psi_N$.

**v.** Let X={a, b } define the FNSs $\eta_N$ and $\gamma_N$ in X as follows:

$\eta_N$ = <x, ($0.4_{(a)}$, $0.2_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.6_{(a)}$,$0.7_{(b)}$) >,

$\gamma_N$ = <x, ($0.8_{(a)}$, $0.8_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.2_{(a)}$,$0.2_{(b)}$) >.

The family $\tau_N$ ={$0_N$, $1_N$, $\eta_N$, $\gamma_N$ } be FNT.

Now if, $\psi_N$ = <x, ($0.6_{(a)}$, $0.7_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.4_{(a)}$,$0.3_{(b)}$) >.

By **Theorem 3.3** **i**. If $U_N$ is FNOS then is FNGOS.

So, $U_N = \gamma_N$ where, $U_N$ be FNGOS such that, $\psi_N \subseteq U_N$.

By **Theorem 3.3** **ii**. Every FNCS is FNαCS.

Then, $FN\alpha CL(\psi_N) = 1_N - \eta_N$. Therefore $FN\alpha CL(\psi_N) \subseteq U_N$.

Hence, $\psi_N$ is FNSαGCS but, not FNαCS.

Since, $FNCL(\psi_N) = 1_N - \eta_N$, $FNIn\ (FNCL(\psi_N)) = \eta_N$ and

$FNCL(FNIn\ (FNCL(\psi_N))) = 1_N - \eta_N \not\subseteq \psi_N$.

**vi.** Let X={a} define the FNSs $\eta_N$ and $\gamma_N$ in X as follows:

$\eta_N = <x, (0.5_{(a)}), (0.5_{(a)}), (0.5_{(a)})>$, $\gamma_N = <x, (0.5_{(a)}), (0_{(a)}), (1_{(a)})>$.

The family $\tau_N = \{0_N, 1_N, \eta_N, \gamma_N\}$ be FNT.

Now if, $\psi_N = <x, (0.6_{(a)}), (0.6_{(a)}), (0.6_{(a)})>$.

Let $U_N = <x, (1_{(a)}), (1_{(a)}), (0.4_{(a)})>$ be FNGOS such that, $\psi_N \subseteq U_N$.

Then, $FN\alpha CL(\psi_N) = 1_N - \gamma_N$. So $FN\alpha CL(\psi_N) \subseteq U_N$.

Hence, $\psi_N$ is FNSαGCS but, not FNαGCS.

Since, $U_N$ is FNGOS but not FNOS.

**vii.** Let X={a } define the FNSs $\eta_N$ and $\gamma_N$ in X as follows:

$\eta_N = <x, (0.5_{(a)}), (0.5_{(a)}), (0.7_{(a)})>$, $\gamma_N = <x, (0.4_{(a)}), (0_{(a)}), (1_{(a)})>$.

The family $\tau_N = \{0_N, 1_N, \eta_N, \gamma_N\}$ be FNT.

Now if, $\psi_N = <x, (1_{(a)}), (1_{(a)}), (0.4_{(a)})>$.

Then, $\psi_N$ is FNCS. Since $\psi_N \in 1_N - \tau_N$ but, not FNRCS.

Since $FNIn\ (\psi_N) = <x, (0.5_{(a)}), (0.5_{(a)}), (0.7_{(a)})>$ and

$FNCL(FNIn\ (\psi_N)) = <x, (0.7_{(a)}), (0.5_{(a)}), (0.5_{(a)})> \neq \psi_N$.

**viii.** Let X={a} define the FNSs $\eta_N$ and $\gamma_N$ in X as follows:

$\eta_N = <x, (0.5_{(a)}), (0.5_{(a)}), (0.6_{(a)})>$, $\gamma_N = <x, (0.5_{(a)}), (0_{(a)}), (1_{(a)})>$.

The family $\tau_N = \{0_N, 1_N, \eta_N, \gamma_N\}$ be FNT.

Now if, $\psi_N = <x, (0.6_{(a)}), (0.6_{(a)}), (0.6_{(a)})>$.

Let, $U_N = 1_N$ be FNOS such that, $\psi_N \subseteq U_N$.

Then, $FNCL(\psi_N) = <x, (1_{(a)}), (1_{(a)}), (0.5_{(a)})>$ and $FNCL(\psi_N) \subseteq U_N$.

Hence, $\psi_N$ is FNαGCS but, not FNαCS.

Since, $FNCL(\psi_N) = <x, (1_{(a)}), (1_{(a)}), (0.5_{(a)})>$, $FNIn\ (FNCL(\psi_N)) = <x, (0.5_{(a)}), (0.5_{(a)}), (0.6_{(a)})>$ and

$FNCL(FNIn\ (FNCL(\psi_N))) = <x, (0.6_{(a)}), (0.5_{(a)}), (0.5_{(a)})> \not\subseteq \psi_N$.

**Remark 3.6: i.** The relation between FNPCS and FNSαGCS is independent and this can be clarified in the next example.

ii. The intersection of two FNSαGCS is not FNSαGCS in general and we explained it in the next example.

**Example 3.7:**

**i. (1)** Let X= {a, b} define FNS $\eta_N$ in X as follows:

$\eta_N$ = <x, ($0.5_{(a)}$, $0.5_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.4_{(a)}$,$0.5_{(b)}$) >.

The family $\tau_N$ = {$0_N$, $1_N$, $\eta_N$ } be FNT.

Now if, $\psi_N$ = <x, ($0.5_{(a)}$, $0.4_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.6_{(a)}$,$0.5_{(b)}$) >.

Then, FNIn ($\psi_N$) = $0_N$ and FNCL(FNIn ($\psi_N$)) = $0_N$. So, FNCL(FNIn ($\psi_N$)) $\subseteq$ $\psi_N$.

Hence, $\psi_N$ is FNPCS but, not FNSαGCS. Since

Let, $U_N = \eta_N$, where $U_N$ be FNGOS such that, $\psi_N \subseteq U_N$. Then, FNαCL($\psi_N$) = $1_N$. So FNαCL($\psi_N$) $\nsubseteq$ $U_N$.

**(2)** Let X={a, b} define the FNSs $\eta_N$ and $\gamma_N$ in X as follows:

$\eta_N$ = <x, ($0.5_{(a)}$, $0.2_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.5_{(a)}$,$0.7_{(b)}$) >,

$\gamma_N$ =<x, ($0.8_{(a)}$, $0.8_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.2_{(a)}$,$0.2_{(b)}$) >.

The family $\tau_N$ ={$0_N$, $1_N$, $\eta_N$, $\gamma_N$ }be FNT.

Now if, $\psi_N$ = <x, ($0.5_{(a)}$, $0.7_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.5_{(a)}$,$0.3_{(b)}$) >.

Let, $U_N = \gamma_N$, where $U_N$ be FNGOS such that, $\psi_N \subseteq U_N$.

Then, FNαCL($\psi_N$) = <x, ($0.5_{(a)}$, $0.7_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.5_{(a)}$,$0.2_{(b)}$) > $\subseteq$ $U_N$.

Hence, $\psi_N$ is FNSαGCS but, not FNPCS.

Since, FNIn ($\psi_N$) = $\eta_N$ and FNCL(FNIn ($\psi_N$)) = <x, ($0.5_{(a)}$, $0.7_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.5_{(a)}$,$0.2_{(b)}$) >.

So, FNCL(FNIn ($\psi_N$)) $\nsubseteq$ $\psi_N$.

**ii.** Let X= {a, b} define FNS $\eta_N$ in X as follows: $\eta_N$ = <x, ($0.5_{(a)}$, $0_{(b)}$), ($0.5_{(a)}$, $0.5_{(b)}$), ($0.1_{(a)}$,$1_{(b)}$) >.
The family $\tau_N$ = {$0_N$, $1_N$, $\eta_N$ } be FNT.
Now if, $\psi_{N1}$ = <x, ($0.2_{(a)}$, $1_{(b)}$), ($1_{(a)}$, $1_{(b)}$), ($0.7_{(a)}$,$0_{(b)}$) > and $\psi_{N2}$ = <x, ($0.6_{(a)}$, $0_{(b)}$), ($1_{(a)}$, $1_{(b)}$), ($0.3_{(a)}$,$1_{(b)}$) > are FNSαGCS. But, $\psi_{N1} \cap \psi_{N2}$ = <x, ($0.2_{(a)}$, $0_{(b)}$), ($1_{(a)}$, $1_{(b)}$), ($0.7_{(a)}$,$1_{(b)}$)>.
Now let, $U_N = \eta_N$, where $U_N$ be FNGOS such that, $\psi_{N1} \cap \psi_{N2} \subseteq U_N$. Then, FNαCL($\psi_{N1} \cap \psi_{N2}$) = $1_N \nsubseteq$ $U_N$.
Hence, $\psi_{N1} \cap \psi_{N2}$ is not FNSαGCS.

**Remark 3.8:** The next diagram explains the relationships among different sets in the FNTS and the convers is not true in general.

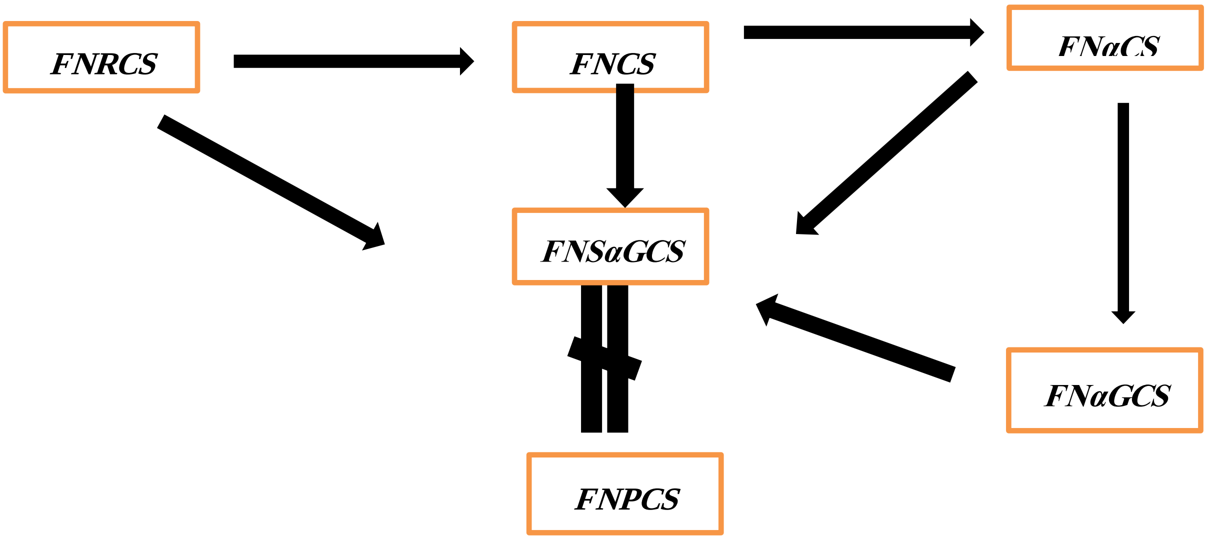


**Diagram 3.1**

## 5. Conclusions

In this present paper, we have defined new class of neutrosophic generalized closed sets called, fuzzy neutrosophic strongly alpha generalized closed set in fuzzy neutrosophic topological spaces. Many results have been discussed with some properties. Further, we giving some theorems, propositions and provided some useful examples where such properties failed to be preserved in order to get the relations between fuzzy neutrosophic strongly alpha generalized closed set and existing fuzzy neutrosophic closed sets in fuzzy neutrosophic topological spaces . We think, our studied class of sets belongs to the new class of fuzzy neutrosophic sets which is useful not only in the deepening of our understanding of some special features of the well-known notions of fuzzy neutrosophic topology but also useful in neutrosophic control theory.

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