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Games Based on Simplified Neutrosophic Multiplicative Soft Sets and Their Applications

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Abstract. In this paper, we firstly define simplified neutrosophic multiplicative soft sets by combining simplified neutrosophic multiplicative sets and soft sets. Meanwhile, we introduce some basic operations of simplified neutrosophic multiplicative soft sets and discuss their related properties. Later, we describe two person simplified neutrosophic multiplicative soft games, and give different types of solution models of these games which are simplified neutrosophic multiplicative soft saddle points, simplified neutrosophic multiplicative soft upper and lower values, simplified neutrosophic multiplicative soft dominated strategies and simplified neutrosophic multiplicative soft Nash equilibrium. Moreover, the solution models of two person simplified neutrosophic multiplicative soft games are applied to a real-world problem and supported by comparison analysis. Finally, the framework of n-person simplified neutrosophic multiplicative soft games is presented.

Keywords: Simplified neutrosophic multiplicative sets; Soft sets; Simplified neutrosophic multiplicative soft sets; Simplified neutrosophic multiplicative soft games

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1. **Introduction**

Almost all of the mathematical models proposed until the middle of the 19th century were not suitable for dealing with uncertainty and vagueness. In 1965, Zadeh [47] described the concept of fuzzy sets (FSs) that allow the representation of uncertainty in a mathematical way. While fuzzy sets were based on the truth-membership value of uncertainty, Atanassov [7] generalized the FSs by including the falsity-membership value, and thus proposed the idea of intuitionistic fuzzy sets (IFSs). In 1998, Smarandache [39] introduced the neutrosophic set (NS) to reflect the values of truth-membership, indeterminacy-membership and falsity-membership simultaneously. However, due to the difficulty in applying to real-world problems when the values of truth-membership, indeterminacy-membership and falsity-membership are real non-standard subsets of $]0^-, 1^+[$, Wang et al. [43] and Ye [46] derived single-valued neutrosophic set (SVNS) and simplified neutrosophic set (SNS) as specific descriptions of NSs, respectively. Recently, works on (single-valued/simplified) NS theory has been progressing rapidly and is presenting applications in a wide variety of fields, for instance; aggregation operators [16, 17, 19, 21] and information measures [3, 32] and various solution models for real-life problems [1–5, 42] of (single-valued/simplified) NS.

Although the FSs, IFSs and NSs are powerful mathematical models for dealing with uncertainties, these sets use the 0-1 scale, which is distributed symmetrically and uniformly. However, there are real-life problems that need to be scaled as unsymmetrically and non-uniformly. The grading system of universities can be given as the most obvious example of this situation [18]. In 1990, Saaty [36] proposed the 1-9 scale (or $\frac{1}{9} - 9$ scale) as an useful tool to deal with such problems that need to be scaled unsymmetrically and non-uniformly whilst assigning the variable grades. These different scales lead to the construction of multiplicative preference relation [37]. By inspired this idea, Xia et al. [45] demonstrated that the interval-valued fuzzy preference relations can be equivalent to the intuitionistic fuzzy preference relations, and then introduced the intuitionistic multiplicative sets (IMSs) and the intuitionistic multiplicative preference relations (IMPRs). Moreover, they present a comparison between 0.1-0.9 and $\frac{1}{9} - 9$ scales as in Table 1.

Table 1. The comparison between 0.1-0.9 and $\frac{1}{9} - 9$ scales [45]

$\frac{1}{9} - 9$ scale	0.1-0.9 scale	Meaning
$\frac{1}{9}$	0.1	Extremely not preferrred
$\frac{1}{7}$	0.2	Very strongly not preferrred
$\frac{1}{5}$	0.3	Strongly not preferrred
$\frac{1}{3}$	0.4	Moderately not preferrred
1	0.5	Equally preferrred
3	0.6	Moderately preferrred
5	0.7	Strongly preferrred
7	0.8	Very strongly preferrred
9	0.9	Extremely preferrred
Other values between $\frac{1}{9}$ and 9	Other values between 0 and 1	Intermediate values used to present compromise

The IMSs and IMPRs were studied widely [14,15,20,44]. In 2019, Köseoğlu et al. [26] proposed the idea of simplified neutrosophic multiplicative sets (SNMSs) generalizing the IMSs and studied the simplified neutrosophic multiplicative preference relations (SNMPRs).

In 1999, Molodtsov [28] initiated the theory of soft set (SS) which classifies objects according to parameters or attributes. Çağman and Enginoğlu [9] revisited the concept of soft set to make Molodtsov's soft set operations more functional. Many authors studied the theory [6, 8, 9, 27, 38, 41] and applications [22–25, 33–35] of soft sets. In 2016, Deli and Çağman [10] gave an application of soft sets in decision making based on game theory, and thus pioneered the idea of soft games. Moreover, Deli et al. [12] studied several expected impact functions and algorithms modelling games under the soft sets. In recent years, several game schemes based on the fuzzy soft sets, intuitionistic fuzzy soft sets and neutrosophic soft sets have been proposed [11, 29, 40]. The motivation of this paper is to propose a new extension of soft sets and revisit soft games from a different perspective. Relatedly, this paper introduces the simplified neutrosophic multiplicative soft sets (SNMSSs) fusing SNMSs and SSs, and proposes a new game framework based on the SNMSSs called simplified neutrosophic multiplicative soft game (SNM soft game).

The rest of this article is arranged as follows: Section 2 reviews some definitions and results related to the NSs, SNSs, SNMSs and SSs. Section 3 presents the concept of SNMSSs and their fundamental operations with structural properties. Section 4 is devoted to the four different solution methods of two person SNM soft games and and their efficiency in dealing with real-world issues. In Section 5, two person SNM soft games are extended to n-person SNM soft games. In Section 6, the concluding remarks are given.

2. **Preliminaries**

In this section, some basic concepts about the neutrosophic sets, simplified neutrosophic sets, simplified neutrosophic multiplicative sets and soft sets are given.

Let $\mathcal{H}$ be a space of points (object) with a generic element denoted by $\hbar$.

Definition 2.1. ([39]) A neutrosophic set (NS) $\mathfrak{N}$ in $\mathcal{H}$ is characterized by a truth-membership function $t_{\mathfrak{N}} : \mathcal{H} \to]0^-, 1^+[$, an indeterminacy-membership function $i_{\mathfrak{N}} : \mathcal{H} \to]0^-, 1^+[$, and a falsity-membership function $f_{\mathfrak{N}} : \mathcal{H} \to]0^-, 1^+[$. $t_{\mathfrak{N}}(\hbar)$, $i_{\mathfrak{N}}(\hbar)$ and $f_{\mathfrak{N}}(\hbar)$ are real standard or non-standard subsets of $]0^-, 1^+[$. There is no restriction on the sum of $t_{\mathfrak{N}}(\hbar)$, $i_{\mathfrak{N}}(\hbar)$ and $f_{\mathfrak{N}}(\hbar)$, so $0^- \leq \sup t_{\mathfrak{N}}(\hbar) + \sup i_{\mathfrak{N}}(\hbar) + \sup f_{\mathfrak{N}}(\hbar) \leq 3^+$ for $\hbar \in \mathcal{H}$.

However, Wang et al. [43] and Ye [46] stated the difficulty of employing the NSs of non-standard intervals in practice, and proposed the simplified neutrosophic sets.

Definition 2.2. ([46]) An NS $\mathfrak{N}$ is characterized by a truth-membership function $t_{\mathfrak{N}} : \mathcal{H} \to [0,1]$, an indeterminacy-membership function $i_{\mathfrak{N}} : \mathcal{H} \to [0,1]$, and a falsity-membership function $f_{\mathfrak{N}} : \mathcal{H} \to [0,1]$. $t_{\mathfrak{N}}(\hbar)$, $i_{\mathfrak{N}}(\hbar)$ and $f_{\mathfrak{N}}(\hbar)$ are singleton subintervals/subsets in the standard interval $[0,1]$, then it is said to be a simplified neutrosophic set (SNS) and described by

$$\mathfrak{N} = \{\langle \hbar, (t_{\mathfrak{N}}(\hbar), i_{\mathfrak{N}}(\hbar), f_{\mathfrak{N}}(\hbar))\rangle : \ \hbar \in \mathcal{H}\}. \tag{1}$$

This kind of NS is is termed to be a single-valued neutrosophic set (SVNS) by Wang et al. [43]. Throughout this paper, we will use the term "simplified neutrosophic set (SNS)".

Definition 2.3. ([26]) A simplified neutrosophic multiplicative set (SNMS) $\mathfrak{M}$ in $\mathcal{H}$ is defined as

$$\mathfrak{M} = \{\langle \hbar, (\rho_{\mathfrak{M}}(\hbar), \tau_{\mathfrak{M}}(\hbar), \sigma_{\mathfrak{M}}(\hbar))\rangle : \ \hbar \in \mathcal{H}\}, \tag{2}$$

which assigns to each element $\hbar$ a truth-membership information $\rho_{\mathfrak{M}}(\hbar)$, an indeterminacy-membership information $\tau_{\mathfrak{M}}(\hbar)$, and a falsity-membership information $\sigma_{\mathfrak{M}}(\hbar)$ with conditions

$$\frac{1}{9} \leq \rho_{\mathfrak{M}}(\hbar), \tau_{\mathfrak{M}}(\hbar), \sigma_{\mathfrak{M}}(\hbar) \leq 9 \text{ and } 0 < \rho_{\mathfrak{M}}(\hbar)\sigma_{\mathfrak{M}}(\hbar) \leq 1. \tag{3}$$

for each $\hbar \in \mathcal{H}$.

The set of all SNMSs in $\mathcal{H}$ is denoted by $\mathfrak{P}(\mathcal{H})$.

Definition 2.4. ([26]) Let $\mathfrak{M}$, $\mathfrak{M}_1$ and $\mathfrak{M}_2$ be the SNMSs. Then, some operational rules on SNMSs are given as follows.

(a): $\mathfrak{M}_1 \subseteq \mathfrak{M}_2 \Leftrightarrow \rho_{\mathfrak{M}_1}(\hbar) \leq \rho_{\mathfrak{M}_2}(\hbar)$, $\tau_{\mathfrak{M}_1}(\hbar) \geq \tau_{\mathfrak{M}_2}(\hbar)$ and $\sigma_{\mathfrak{M}_1}(\hbar) \geq \sigma_{\mathfrak{M}_2}(\hbar)$ for all $\hbar \in \mathcal{H}$.
(b): $\mathfrak{M}_1 = \mathfrak{M}_2 \Leftrightarrow \rho_{\mathfrak{M}_1}(\hbar) = \rho_{\mathfrak{M}_2}(\hbar)$, $\tau_{\mathfrak{M}_1}(\hbar) = \tau_{\mathfrak{M}_2}(\hbar)$ and $\sigma_{\mathfrak{M}_1}(\hbar) = \sigma_{\mathfrak{M}_2}(\hbar)$ for all $\hbar \in \mathcal{H}$.
(c): $\mathfrak{M}^c = \{\langle \hbar, (\sigma_{\mathfrak{M}}(\hbar), \frac{1}{\tau_{\mathfrak{M}}(\hbar)}, \rho_{\mathfrak{M}}(\hbar)\}\rangle : \ \hbar \in \mathcal{H}\}$.
(d):

$$\mathfrak{M}_1 \cap \mathfrak{M}_2 = \left\{ \left\langle \hbar, \begin{pmatrix} \min\{\rho_{\mathfrak{M}_1}(\hbar), \rho_{\mathfrak{M}_2}(\hbar)\}, \\ \max\{\tau_{\mathfrak{M}_1}(\hbar), \tau_{\mathfrak{M}_2}(\hbar)\}, \\ \max\{\sigma_{\mathfrak{M}_1}(\hbar), \sigma_{\mathfrak{M}_2}(\hbar)\} \end{pmatrix} \right\rangle : \hbar \in \mathcal{H} \right\}.$$

(e):

$$\mathfrak{M}_1 \cup \mathfrak{M}_2 = \left\{ \left\langle \hbar, \begin{pmatrix} \max\{\rho_{\mathfrak{M}_1}(\hbar), \rho_{\mathfrak{M}_2}(\hbar)\}, \\ \min\{\tau_{\mathfrak{M}_1}(\hbar), \tau_{\mathfrak{M}_2}(\hbar)\}, \\ \min\{\sigma_{\mathfrak{M}_1}(\hbar), \sigma_{\mathfrak{M}_2}(\hbar)\} \end{pmatrix} \right\rangle : \hbar \in \mathcal{H} \right\}.$$

Definition 2.5. Let $\mathfrak{M}_1$ and $\mathfrak{M}_2$ be two SNMSs in $\mathcal{H}$. The cartesian product of $\mathfrak{M}_1$ and $\mathfrak{M}_2$, denoted by $\mathfrak{M}_1 \times \mathfrak{M}_2$, is an SNMS in $\mathcal{H} \times \mathcal{H}$ and defined as

$$\mathfrak{M}_1 \times \mathfrak{M}_2 = \left\{ \left\langle (\hbar, \hbar'), \begin{pmatrix} \min\{\rho_{\mathfrak{M}_1}(\hbar), \rho_{\mathfrak{M}_2}(\hbar')\}, \\ \max\{\tau_{\mathfrak{M}_1}(\hbar), \tau_{\mathfrak{M}_2}(\hbar')\}, \\ \max\{\sigma_{\mathfrak{M}_1}(\hbar), \sigma_{\mathfrak{M}_2}(\hbar')\} \end{pmatrix} \right\rangle : (\hbar, \hbar') \in \mathcal{H} \times \mathcal{H} \right\}.$$

In 1999, Molodtsov [28] introduced the notion of soft set as an effective mathematical model for dealing with uncertainty. In 2010, Çağman and Enginoğlu [9] revisited the concept of soft set to make Molodtsov's soft set operations more functional, and presented the following definition.

Definition 2.6. ([9, 28]) Let $\mathcal{H}$ be a set of alternatives, and $P(\mathcal{H})$ be a power set of $\mathcal{H}$. Also, let $\mathcal{S}$ be a set of parameters (or attributes) and $\mathcal{X} \subseteq \mathcal{S}$. The pair $\Gamma_{\mathcal{X}} = (\gamma_{\mathcal{X}}, \mathcal{S})$ is called a soft set (SS) over $\mathcal{H}$ and described as

$$\Gamma_{\mathcal{X}} = (\gamma_{\mathcal{X}}, \mathcal{S}) = \{(x, \gamma_{\mathcal{X}}(x)) : \ x \in \mathcal{S}, \ \gamma_{\mathcal{X}}(x) \in P(\mathcal{H})\}, \tag{4}$$

where $\gamma_{\mathcal{X}} : \mathcal{S} \to P(\mathcal{H})$, called an approximate function, such that $\gamma_{\mathcal{X}}(x) = \emptyset$ if $x \notin \mathcal{X}$.

3. Simplified Neutrosophic Multiplicative Soft Sets

In this section, we introduce the concept of simplified neutrosophic multiplicative soft set by combining SS and SNMS. Also, we study some simplified neutrosophic multiplicative soft set operations and their remarkable properties.

Definition 3.1. Let $\mathcal{H}$ be a set of alternatives, $\mathcal{S}$ be a set of parameters (or attributes) and $\mathcal{X} \subseteq \mathcal{S}$. Also, $\mathfrak{P}(\mathcal{H})$ denotes the set of all SNMSs in $\mathcal{H}$. The pair $\Theta_{\mathcal{X}} = (\theta_{\mathcal{X}}, \mathcal{S})$ is said to be a simplified neutrosophic multiplicative soft set (SNMSS) over $\mathcal{H}$ and described as

$$\Theta_{\mathcal{X}} = (\theta_{\mathcal{X}}, \mathcal{S}) = \{(x, \theta_{\mathcal{X}}(x)) : \ x \in \mathcal{S}, \ \theta_{\mathcal{X}}(x) \in \mathfrak{P}(\mathcal{H})\}, \tag{5}$$

where $\theta_{\mathcal{X}} : \mathcal{S} \to \mathfrak{P}(\mathcal{H})$, called an approximate function, such that $\theta_{\mathcal{X}}(x) = \emptyset$ if $x \notin \mathcal{X}$.

The set of all SNMSSs over $\mathcal{H}$ for the parameter set $\mathcal{S}$ is denoted by $\Im(\mathcal{H}, \mathcal{S})$.

Example 3.2. With the popularity of computers in daily life, more and more people prefer to buy well-equipped computers. Sometimes, instead of buying a new computer, the well-performing parts of poorly-equipped computer(s) are mounted to the other computer, and thus having a fairly well-equipped computer with more utility than the former. For this purpose, it is necessary to determine the performance of the main accessories (parts) for each computer, and the $\frac{1}{9} - 9$ scale is more suitable for this. Assume that $\mathcal{H} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5, \hbar_6\}$ is a set of computers. Also, the CPU (x_1), memory (x_2), hard disk (x_3), motherboard (x_4) and graphics card (x_5) are the main accessories (parts) whose the performance will be tested. By classifying the computers with the $\frac{1}{9} - 9$ scale according to the performance criteria of main accessories (parts) in $\mathcal{X} = \{x_1, x_2, x_3, x_4\}$, the following SNMSS is given.

$$\Theta_{\mathcal{X}} = \left\{ \begin{array}{l} (x_1, \{\langle \hbar_1, (3, 2, \frac{1}{6})\rangle, \langle \hbar_2, (1, \frac{1}{4}, 1)\rangle, \langle \hbar_3, (\frac{3}{7}, 2, 2)\rangle, \langle \hbar_4, (1, 1, 1)\rangle, \langle \hbar_5, (\frac{2}{5}, \frac{1}{9}, 1)\rangle, \langle \hbar_6, (8, 9, \frac{1}{9})\rangle\}) \\ (x_2, \{\langle \hbar_1, (\frac{1}{4}, \frac{3}{5}, \frac{1}{5})\rangle, \langle \hbar_2, (1, \frac{1}{9}, \frac{1}{9})\rangle, \langle \hbar_3, (\frac{2}{7}, 4, 2)\rangle, \langle \hbar_4, (\frac{1}{9}, 3, 5)\rangle, \langle \hbar_5, (4, 2, \frac{1}{4})\rangle, \langle \hbar_6, (\frac{7}{5}, 3, \frac{1}{5})\rangle\}) \\ (x_3, \{\langle \hbar_1, (2, 2, \frac{1}{2})\rangle, \langle \hbar_2, (\frac{1}{8}, \frac{1}{7}, \frac{1}{6})\rangle, \langle \hbar_3, (3, 1, \frac{2}{7})\rangle, \langle \hbar_4, (3, 4, \frac{1}{6})\rangle, \langle \hbar_5, (1, 4, 1)\rangle, \langle \hbar_6, (\frac{1}{9}, \frac{1}{2}, 9)\rangle\}) \\ (x_4, \{\langle \hbar_1, (2, \frac{1}{3}, \frac{1}{2})\rangle, \langle \hbar_2, (\frac{1}{6}, 1, 4)\rangle, \langle \hbar_3, (\frac{2}{9}, \frac{4}{5}, \frac{3}{2})\rangle, \langle \hbar_4, (\frac{1}{9}, 1, \frac{5}{6})\rangle, \langle \hbar_5, (3, 3, \frac{1}{4})\rangle, \langle \hbar_6, (2, \frac{1}{5}, \frac{1}{7})\rangle\}) \end{array} \right\}.$$

Here, $\langle \hbar_1, (3, 2, \frac{1}{6})\rangle \in \theta_{\mathcal{X}}(x_1)$ means that computer $\hbar_1$ has membership information of truth, indeterminacy and falsity as $(3, 2, \frac{1}{6})$ according to the performance of CPU. Other components can be interpreted similarly.

Definition 3.3. Let $\Theta_{\mathcal{X}}, \Theta_{\mathcal{Y}} \in \Im(\mathcal{H}, \mathcal{S})$.

- **(a):** $\Theta_{\mathcal{X}}$ is termed to be an SNMS subset of $\Theta_{\mathcal{Y}}$ if $\theta_{\mathcal{X}}(x) \subseteq \theta_{\mathcal{Y}}(x)$ for all $x \in S$. It is denoted by $\Theta_{\mathcal{X}} \widetilde{\subseteq} \Theta_{\mathcal{Y}}$.
- **(b):** The SNMSSs $\Theta_{\mathcal{X}}$ and $\Theta_{\mathcal{Y}}$ are equal if $\theta_{\mathcal{X}}(x) = \theta_{\mathcal{Y}}(x)$ for all $x \in S$. It is denoted by $\Theta_{\mathcal{X}} = \Theta_{\mathcal{Y}}$.
- **(c):** The complement of $\Theta_{\mathcal{X}}$, denoted by $\Theta_{\mathcal{X}}^{\tilde{c}}$, is an SNMSS defined by the approximate function $\theta_{\mathcal{X}}^{\tilde{c}} : \mathcal{S} \to \mathfrak{P}(\mathcal{H})$ such that

$$\theta_{\mathcal{X}^{\tilde{c}}}(x) = (\theta_{\mathcal{X}}(x))^c$$

for all $x \in \mathcal{S}$.
- **(d):** The intersection of $\Theta_{\mathcal{X}}$ and $\Theta_{\mathcal{Y}}$, denoted by $\Theta_{\mathcal{X}} \widetilde{\cap} \Theta_{\mathcal{Y}}$, is an SNMSS defined by the SNM approximate function $\theta_{\mathcal{X} \widetilde{\cap} \mathcal{Y}} : \mathcal{S} \to \mathfrak{P}(\mathcal{H})$ such that

$$\theta_{\mathcal{X} \widetilde{\cap} \mathcal{Y}}(x) = \theta_{\mathcal{X}}(x) \cap \theta_{\mathcal{Y}}(x)$$

for all $x \in \mathcal{S}$.

(e): The union of $\Theta_{\mathcal{X}}$ and $\Theta_{\mathcal{Y}}$, denoted by $\Theta_{\mathcal{X}}\widetilde{\cup}\Theta_{\mathcal{Y}}$, is an SNMSS defined by the SNM approximate function $\theta_{\mathcal{X}\widetilde{\cup}\mathcal{Y}} : \mathcal{S} \to \mathfrak{P}(\mathcal{H})$ such that

$$\theta_{\mathcal{X}\widetilde{\cup}\mathcal{Y}}(x) = \theta_{\mathcal{X}}(x) \cup \theta_{\mathcal{Y}}(x)$$

for all $x \in \mathcal{S}$.

Proposition 3.4. *Let $\Theta_{\mathcal{X}}, \Theta_{\mathcal{Y}}, \Theta_{\mathcal{Z}} \in \Im(\mathcal{H}, \mathcal{S})$. Then,*

(i): $\Theta_{\mathcal{X}}\alpha\Theta_{\mathcal{Y}}$ *and* $\Theta_{\mathcal{Y}}\alpha\Theta_{\mathcal{Z}} \Rightarrow \Theta_{\mathcal{X}}\alpha\Theta_{\mathcal{Z}}$ *for each* $\alpha \in \{\widetilde{\subseteq}, =\}$.
(ii): $\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{X}} = \Theta_{\mathcal{X}}$ *for each* $\beta \in \{\widetilde{\cap}, \widetilde{\cup}\}$.
(iii): $\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{Y}} = \Theta_{\mathcal{Y}}\beta\Theta_{\mathcal{X}}$ *for each* $\beta \in \{\widetilde{\cap}, \widetilde{\cup}\}$.
(iv): $\Theta_{\mathcal{X}}\beta(\Theta_{\mathcal{Y}}\beta\Theta_{\mathcal{Z}}) = (\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{Y}})\beta\Theta_{\mathcal{Z}}$ *for each* $\beta \in \{\widetilde{\cap}, \widetilde{\cup}\}$.
(v): $\Theta_{\mathcal{X}}\beta(\Theta_{\mathcal{Y}}\delta\Theta_{\mathcal{Z}}) = (\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{Y}})\delta(\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{Z}})$ *for each* $\beta, \delta \in \{\widetilde{\cap}, \widetilde{\cup}\}$.
(vi): $(\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{Y}})^{\widetilde{c}} = \Theta_{\mathcal{X}}^{\widetilde{c}}\delta\Theta_{\mathcal{Y}}^{\widetilde{c}}$ *for each* $\beta, \delta \in \{\widetilde{\cap}, \widetilde{\cup}\}$ *and* $\beta \neq \delta$.

Proof. The proofs are straightforward, so they are omitted. □

Definition 3.5. Let $\Theta_{\mathcal{X}}, \Theta_{\mathcal{Y}} \in \Im(\mathcal{H}, \mathcal{S})$.

(a): The And-product of $\Theta_{\mathcal{X}}$ and $\Theta_{\mathcal{Y}}$, denoted by $\Theta_{\mathcal{X}}\widetilde{\wedge}\Theta_{\mathcal{Y}}$, is an SNMSS defined by the SNM approximate function $\theta_{\mathcal{X}\widetilde{\wedge}\mathcal{Y}} : \mathcal{S} \times \mathcal{S} \to \mathfrak{P}(\mathcal{H})$ such that

$$\theta_{\mathcal{X}\widetilde{\wedge}\mathcal{Y}}(x, y) = \theta_{\mathcal{X}}(x) \cap \theta_{\mathcal{Y}}(y)$$

for all $(x, y) \in \mathcal{S} \times \mathcal{S}$.

(b): The Or-product of $\Theta_{\mathcal{X}}$ and $\Theta_{\mathcal{Y}}$, denoted by $\Theta_{\mathcal{X}}\widetilde{\vee}\Theta_{\mathcal{Y}}$, is an SNMSS defined by the SNM approximate function $\theta_{\mathcal{X}\widetilde{\vee}\mathcal{Y}} : \mathcal{S} \times \mathcal{S} \to \mathfrak{P}(\mathcal{H})$ such that

$$\theta_{\mathcal{X}\widetilde{\vee}\mathcal{Y}}(x, y) = \theta_{\mathcal{X}}(x) \cup \theta_{\mathcal{Y}}(y)$$

for all $(x, y) \in \mathcal{S} \times \mathcal{S}$.

(c): The cartesian product of $\Theta_{\mathcal{X}}$ and $\Theta_{\mathcal{Y}}$, denoted by $\Theta_{\mathcal{X}}\widetilde{\times}\Theta_{\mathcal{Y}}$, is an SNMSS defined by the SNM approximate function $\theta_{\mathcal{X}\widetilde{\times}\mathcal{Y}} : \mathcal{S} \times \mathcal{S} \to \mathfrak{P}(\mathcal{H} \times \mathcal{H})$ such that

$$\theta_{\mathcal{X}\widetilde{\times}\mathcal{Y}}(x, y) = \theta_{\mathcal{X}}(x) \times \theta_{\mathcal{Y}}(y)$$

for all $(x, y) \in \mathcal{S} \times \mathcal{S}$.

Proposition 3.6. *Let $\Theta_{\mathcal{X}}, \Theta_{\mathcal{Y}}, \Theta_{\mathcal{Z}}, \Theta_{\mathcal{T}}$ be the SNMSSs over $\mathcal{H}$. Then,*

(i): $\Theta_{\mathcal{X}}\alpha\Theta_{\mathcal{Y}}$ *and* $\Theta_{\mathcal{Z}}\alpha\Theta_{\mathcal{T}} \Rightarrow (\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{Z}})\alpha(\Theta_{\mathcal{Y}}\beta\Theta_{\mathcal{T}})$ *for each* $\alpha \in \{\widetilde{\subseteq}, =\}$ *and* $\beta \in \{\widetilde{\wedge}, \widetilde{\vee}, \widetilde{\times}\}$.
(ii): $\Theta_{\mathcal{X}}\alpha\Theta_{\mathcal{Y}} \Rightarrow (\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{Z}})\alpha(\Theta_{\mathcal{Y}}\beta\Theta_{\mathcal{Z}})$ *for each* $\alpha \in \{\widetilde{\subseteq}, =\}$ *and* $\beta \in \{\widetilde{\wedge}, \widetilde{\vee}, \widetilde{\times}\}$.
(iii): $\Theta_{\mathcal{X}}\beta(\Theta_{\mathcal{Y}}\beta\Theta_{\mathcal{Z}}) = (\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{Y}})\beta\Theta_{\mathcal{Z}}$ *for each* $\beta \in \{\widetilde{\wedge}, \widetilde{\vee}, \widetilde{\times}\}$.
(vi): $(\Theta_{\mathcal{X}}\beta\Theta_{\mathcal{Y}})^{\widetilde{c}} = \Theta_{\mathcal{X}}^{\widetilde{c}}\delta\Theta_{\mathcal{Y}}^{\widetilde{c}}$ *for each* $\beta, \delta \in \{\widetilde{\wedge}, \widetilde{\vee}\}$ *and* $\beta \neq \delta$.

Proof. The proofs are straightforward, hence they are omitted. □

The emerged SNMSS operations are generalized for the family of SNMSSs as follows.

Definition 3.7. Let $\Theta_{\mathcal{X}_p}$ be the SNMSS for each $p \in I = \{1, 2, ..., q\}$.

(a): The intersection of SNMSSs $\Theta_{\mathcal{X}_p}$ $(p = 1, 2, ..., q)$, denoted by $\widetilde{\bigcap}_{p\in I}\Theta_{\mathcal{X}_p}$, is an SNMSS defined by the SNM approximate function $\theta_{\widetilde{\bigcap}_{p\in I}\mathcal{X}_p} : \mathcal{S} \to \mathfrak{P}(\mathcal{H})$ such that

$$\theta_{\widetilde{\bigcap}_{p\in I}\mathcal{X}_p}(x) = \bigcap_{p\in I} \theta_{\mathcal{X}_p}(x)$$

for all $x \in \mathcal{S}$.

(b): The union of SNMSSs $\Theta_{\mathcal{X}_p}$ $(p = 1, 2, ..., q)$, denoted by $\widetilde{\bigcup}_{p\in I}\Theta_{\mathcal{X}_p}$, is an SNMSS defined by the SNM approximate function $\theta_{\widetilde{\bigcup}_{p\in I}\mathcal{X}_p} : \mathcal{S} \to \mathfrak{P}(\mathcal{H})$ such that

$$\theta_{\widetilde{\bigcup}_{p\in I}\mathcal{X}_p}(x) = \bigcup_{p\in I} \theta_{\mathcal{X}_p}(x)$$

for all $x \in \mathcal{S}$.

(c): The And-product of SNMSSs $\Theta_{\mathcal{X}_p}$ $(p = 1, 2, ..., q)$, denoted by $\widetilde{\bigwedge}_{p\in I}\Theta_{\mathcal{X}_p}$, is an SNMSS defined by the SNM approximate function $\theta_{\widetilde{\bigwedge}_{p\in I}\mathcal{X}_p} : \prod_{p\in I} \mathcal{S} \to \mathfrak{P}(\mathcal{H})$ such that

$$\theta_{\widetilde{\bigwedge}_{p\in I}\mathcal{X}_p}((x^p)_{p\in I}) = \bigcap_{p\in I} \theta_{\mathcal{X}_p}(x^p)$$

for all $(x^p)_{p\in I} = (x^1, x^2, ..., x^q) \in \prod_{p\in I} \mathcal{S} = \mathcal{S}^q$.

(d): The Or-product of SNMSSs $\Theta_{\mathcal{X}_p}$ $(p = 1, 2, ..., q)$, denoted by $\widetilde{\bigvee}_{p\in I}\Theta_{\mathcal{X}_p}$, is an SNMSS defined by the SNM approximate function $\theta_{\widetilde{\bigvee}_{p\in I}\mathcal{X}_p} : \prod_{p\in I} \mathcal{S} \to \mathfrak{P}(\mathcal{H})$ such that

$$\theta_{\widetilde{\bigvee}_{p\in I}\mathcal{X}_p}((x^p)_{p\in I}) = \bigcup_{p\in I} \theta_{\mathcal{X}_p}(x^p)$$

for all $(x^p)_{p\in I} = (x^1, x^2, ..., x^q) \in \prod_{p\in I} \mathcal{S} = \mathcal{S}^q$.

(e): The cartesian product of SNMSSs $\Theta_{\mathcal{X}_p}$ $(p = 1, 2, ..., q)$, denoted by $\widetilde{\prod}_{p\in I}\Theta_{\mathcal{X}_p}$, is an SNMSS defined by the SNM approximate function $\theta_{\widetilde{\prod}_{p\in I}\mathcal{X}_p} : \prod_{p\in I} \mathcal{S} \to \mathfrak{P}(\mathcal{H}^q)$ such that

$$\theta_{\widetilde{\prod}_{p\in I}\mathcal{X}_p}((x^p)_{p\in I}) = \prod_{p\in I} \theta_{\mathcal{X}_p}(x^p)$$

for all $(x^p)_{p\in I} = (x^1, x^2, ..., x^q) \in \prod_{p\in I} \mathcal{S} = \mathcal{S}^q$.

4. **Two Person Simplified Neutrosophic Multiplicative Soft Games and Their Applications**

4.1. *Two Person Simplified Neutrosophic Multiplicative Soft Games*

In this part, we create two person simplified neutrosophic multiplicative soft games with simplified neutrosophic multiplicative soft payoffs. Moreover, we propose the solution models for the simplified neutrosophic multiplicative soft games. For some fundamental notions (such as game, strategy, payoff, saddle point, Nash equilibrium) on game theory, we refer to [13, 30, 31].

In the following, we revisit some concepts and results on game theory given in [13,30,31] and thus adapt them to the simplified neutrosophic multiplicative soft games (SNM soft games) by using SNMSSs.

Definition 4.1. Let $\mathcal{S}$ be a set of strategies and $\mathcal{X}, \mathcal{Y} \subseteq \mathcal{S}$. A choice of behaviour in an SNM soft game is called an action. Each element of $\mathcal{X} \times \mathcal{Y}$ is called action pair. That is, $\mathcal{X} \times \mathcal{Y}$ is the set of available actions.

Definition 4.2. Let $\mathcal{H}$ be a set of alternatives and $\mathfrak{P}(\mathcal{H})$ be set of all SNMSs in $\mathcal{H}$. Also, $\mathcal{S}$ be a set of strategies and $\mathcal{X}, \mathcal{Y} \subseteq \mathcal{S}$. Then, a set-valued function

$$\theta_{\mathcal{X}\times\mathcal{Y}} : \mathcal{X} \times \mathcal{Y} \to \mathfrak{P}(\mathcal{H}) \tag{6}$$

is said to be a simplified neutrosophic multiplicative soft payoff function (SNM soft payoff function). For each $(x, y) \in \mathcal{X} \times \mathcal{Y}$, the value $\theta_{\mathcal{X}\times\mathcal{Y}}(x, y)$ is named a simplified neutrosophic multiplicative soft payoff (SNM soft payoff).

Definition 4.3. Let $\mathcal{X} \times \mathcal{Y}$ be a set of action pairs. Then, an action $(x^*, y^*) \in \mathcal{X} \times \mathcal{Y}$ is said to be an optimal action if

$$\theta_{\mathcal{X}\times\mathcal{Y}}(x, y) \subseteq \theta_{\mathcal{X}\times\mathcal{Y}}(x^*, y^*) \tag{7}$$

for all $(x, y) \in \mathcal{X} \times \mathcal{Y}$.

Definition 4.4. Let $\mathcal{X} \times \mathcal{Y}$ be a set of action pairs and $(x_i, y_j), (x_k, y_l) \in \mathcal{X} \times \mathcal{Y}$.

- **(a):** If $\theta_{\mathcal{X}\times\mathcal{Y}}(x_k, y_l) \subset \theta_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j)$ then it can be said that a player strictly prefers action pair (x_i, y_j) over action pair (x_k, y_l),
- **(b):** If $\theta_{\mathcal{X}\times\mathcal{Y}}(x_k, y_l) = \theta_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j)$ then it can be said that a player is indifferent between the action pairs (x_i, y_j) and (x_k, y_l),
- **(c):** If $\theta_{\mathcal{X}\times\mathcal{Y}}(x_k, y_l) \subseteq \theta_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j)$ then it can be said that a player either prefers action pair (x_i, y_j) to action pair (x_k, y_l) or is indifferent between the action pairs (x_i, y_j) and (x_k, y_l).

Definition 4.5. Let $\theta^r_{\mathcal{X}\times\mathcal{Y}}$ be an SNM soft payoff for Player r and $(x_i, y_j), (x_k, y_l) \in \mathcal{X} \times \mathcal{Y}$. Then, Player r is named rational if the player's SNM soft payoff satisfies the following properties.

- **(1):** Either $\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_k, y_l) \subseteq \theta^r_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j)$ or $\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_k, y_l) \supseteq \theta^r_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j)$.
- **(2):** If $\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_k, y_l) \subseteq \theta^r_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j)$ and $\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_k, y_l) \supseteq \theta^r_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j)$, then $\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_k, y_l) = \theta^r_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j)$.

Definition 4.6. Let $\mathcal{X}$ and $\mathcal{Y}$ be the sets of strategies of Player 1 and Player 2, respectively. Also, $\theta^r_{\mathcal{X}\times\mathcal{Y}} : \mathcal{X} \times \mathcal{Y} \to \mathfrak{P}(\mathcal{H})$ is an SNM soft payoff function for Player r $(r = 1, 2)$. Then, for each Player r, a two person simplified neutrosophic multiplicative soft game (tpSNM soft game) is defined by an SNMSS over $\mathcal{H}$ as

$$\Theta^r_{\mathcal{X}\times\mathcal{Y}} = \{((x, y), \theta^r_{\mathcal{X}\times\mathcal{Y}}(x, y)) : (x, y) \in \mathcal{X} \times \mathcal{Y},\ \theta^r_{\mathcal{X}\times\mathcal{Y}}(x, y) \in \mathfrak{P}(\mathcal{H})\} \tag{8}$$

where $\theta^r_{\mathcal{X}\times\mathcal{Y}}(x, y) = \{\langle \hbar, (\rho^r_{\mathcal{X}\times\mathcal{Y}}(x, y), \tau^r_{\mathcal{X}\times\mathcal{Y}}(x, y), \sigma^r_{\mathcal{X}\times\mathcal{Y}}(x, y))\rangle : \hbar \in \mathcal{H}\}$ and for the triplet $(\rho^r_{\mathcal{X}\times\mathcal{Y}}(x, y), \tau^r_{\mathcal{X}\times\mathcal{Y}}(x, y), \sigma^r_{\mathcal{X}\times\mathcal{Y}}(x, y))$, 1^{st} component is truth-membership information, 2^{nd} component is indeterminacy-membership information and 3^{rd} component is falsity-membership information of $\hbar \in \mathcal{H}$ with respect to the action pair (x, y) for Player r.

The tpSNM soft game is played as follows. At a certain time Player 1 selects a strategy $x_i \in \mathcal{X}$, simultaneously Player 2 selects a strategy $y_i \in \mathcal{Y}$ and once this done each Player r $(r = 1, 2)$ receives the SNM soft payoff $\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j)$.
If $\mathcal{X} = \{x_1, x_2, ..., x_t\}$ and $\mathcal{Y} = \{y_1, y_2, ..., y_v\}$ then the SNM soft payoffs of $\Theta^r_{\mathcal{X}\times\mathcal{Y}}$ can be represented in the following form (see Table 2). To illustrate the tpSNM soft game, we present the following example.

Table 2. Two person simplified neutrosophic multiplicative soft game

$\Theta^r_{\mathcal{X}\times\mathcal{Y}}$	y_1	y_2	.	.	.	y_v
x_1	$\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_1, y_1)$	$\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_1, y_2)$	.	.	.	$\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_1, y_v)$
x_2	$\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_2, y_1)$	$\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_2, y_2)$	.	.	.	$\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_2, y_v)$
.	.	.	.			.
.	.	.		.		.
.	.	.			.	.
x_t	$\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_t, y_1)$	$\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_t, y_2)$	.	.	.	$\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_t, y_v)$

Example 4.7. Let $\mathcal{H} = \{\hbar_1, \hbar_2\}$ be a set of alternatives and $\mathcal{S} = \{x_1, x_2, x_3, x_4, x_5\}$ be a set of strategies. Assume that $\mathcal{X} = \{x_1, x_3\}$ and $\mathcal{Y} = \{x_1, x_4, x_5\}$ are the sets of the strategies Player 1 and Player 2, respectively.
If Player 1 creates the following tpSNM soft game

$$\Theta^1_{\mathcal{X}\times\mathcal{Y}} = \left\{ \begin{array}{l} ((x_1, x_1), \{\langle \hbar_1, (7, \frac{1}{3}, \frac{1}{8})\rangle, \langle \hbar_2, (\frac{1}{2}, 5, 2)\rangle\}), ((x_1, x_4), \{\langle \hbar_1, (5, 5, \frac{1}{7})\rangle, \langle \hbar_2, (1, 1, 1)\rangle\}), \\ ((x_1, x_5), \{\langle \hbar_1, (\frac{1}{2}, 3, 1)\rangle, \langle \hbar_2, (9, 3, \frac{1}{9})\rangle\}), ((x_3, x_1), \{\langle \hbar_1, (3, \frac{4}{3}, \frac{1}{4})\rangle, \langle \hbar_2, (\frac{1}{3}, 3, 1)\rangle\}), \\ ((x_3, x_4), \{\langle \hbar_1, (7, \frac{1}{9}, \frac{1}{9})\rangle, \langle \hbar_2, (3, 3, \frac{1}{3})\rangle\}), ((x_3, x_5), \{\langle \hbar_1, (\frac{1}{4}, \frac{1}{4}, \frac{1}{4})\rangle, \langle \hbar_2, (2, 5, \frac{1}{9})\rangle\}) \end{array} \right\}.$$

the SNM soft payoffs of the game can be illustrated as in Table 3.

Table 3. The tpSNM soft payoffs of Player 1

$\Theta^1_{\mathcal{X}\times\mathcal{Y}}$	x_1	x_4	x_5
x_1	$\{\langle \hbar_1, (7, \frac{1}{3}, \frac{1}{8})\rangle, \langle \hbar_2, (\frac{1}{2}, 5, 2)\rangle\}$	$\{\langle \hbar_1, (5, 5, \frac{1}{7})\rangle, \langle \hbar_2, (1, 1, 1)\rangle\}$	$\{\langle \hbar_1, (\frac{1}{2}, 3, 1)\rangle, \langle \hbar_2, (9, 3, \frac{1}{9})\rangle\}$
x_3	$\{\langle \hbar_1, (3, \frac{4}{3}, \frac{1}{4})\rangle, \langle \hbar_2, (\frac{1}{3}, 3, 1)\rangle\}$	$\{\langle \hbar_1, (7, \frac{1}{9}, \frac{1}{9})\rangle, \langle \hbar_2, (3, 3, \frac{1}{3})\rangle\}$	$\{\langle \hbar_1, (\frac{1}{4}, \frac{1}{4}, \frac{1}{4})\rangle, \langle \hbar_2, (2, 5, \frac{1}{9})\rangle\}$

Let us explain any component of this game. If Player 1 chooses x_1 and Player 2 chooses x_4 then the value of game will be an SNM soft payoff $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_1, x_4) = \{\langle \hbar_1, (5, 5, \frac{1}{7})\rangle, \langle \hbar_2, (1, 1, 1)\rangle\}$. Then, Player 1 wins the set (of alternatives) $\{\langle \hbar_1, (5, 5, \frac{1}{7})\rangle, \langle \hbar_2, (1, 1, 1)\rangle\}$ and Player 2 loses the same set.
Similarly, if Player 2 creates the following tpSNM soft game

$$\Theta^2_{\mathcal{X}\times\mathcal{Y}} = \left\{ \begin{array}{l} ((x_1, x_1), \{\langle \hbar_1, (8, \frac{1}{8}, \frac{1}{8})\rangle, \langle \hbar_2, (7, 9, \frac{1}{9})\rangle\}), ((x_1, x_4), \{\langle \hbar_1, (\frac{1}{4}, 5, 4)\rangle, \langle \hbar_2, (2, \frac{1}{4}, \frac{1}{6})\rangle\}), \\ ((x_1, x_5), \{\langle \hbar_1, (3, \frac{1}{2}, \frac{1}{5})\rangle, \langle \hbar_2, (4, \frac{1}{5}, \frac{1}{5})\rangle\}), ((x_3, x_1), \{\langle \hbar_1, (3, \frac{1}{4}, \frac{1}{3})\rangle, \langle \hbar_2, (8, 8, \frac{1}{9})\rangle\}), \\ ((x_3, x_4), \{\langle \hbar_1, (1, 5, 1)\rangle, \langle \hbar_2, (4, 3, \frac{1}{7})\rangle\}), ((x_3, x_5), \{\langle \hbar_1, (2, 2, \frac{1}{4})\rangle, \langle \hbar_2, (1, \frac{1}{4}, \frac{1}{9})\rangle\}) \end{array} \right\}.$$

TABLE 4. The tpSNM soft payoffs of Player 2

$\Theta^2_{\mathcal{X}\times\mathcal{Y}}$	x_1	x_4	x_5
x_1	$\{\langle\hbar_1,(8,\frac{1}{8},\frac{1}{8})\rangle,\langle\hbar_2,(7,9,\frac{1}{9})\rangle\}$	$\{\langle\hbar_1,(\frac{1}{4},5,4)\rangle,\langle\hbar_2,(2,\frac{1}{4},\frac{1}{6})\rangle\}$	$\{\langle\hbar_1,(3,\frac{1}{2},\frac{1}{5})\rangle,\langle\hbar_2,(4,\frac{1}{5},\frac{1}{5})\rangle\}$
x_3	$\{\langle\hbar_1,(3,\frac{1}{4},\frac{1}{3})\rangle,\langle\hbar_2,(8,8,\frac{1}{9})\rangle\}$	$\{\langle\hbar_1,(1,5,1)\rangle,\langle\hbar_2,(4,3,\frac{1}{7})\rangle\}$	$\{\langle\hbar_1,(2,2,\frac{1}{4})\rangle,\langle\hbar_2,(1,\frac{1}{4},\frac{1}{9})\rangle\}$

the SNM soft payoffs of the game can be tabularized as in Table 4.
The component (x_1,x_4) in this game (or the component in first row-second column of Table 4) can be interpreted that the value of game will be an SNM soft payoff $\theta^2_{\mathcal{X}\times\mathcal{Y}}(x_1,x_4) = \{\langle\hbar_1,(\frac{1}{4},5,4)\rangle,\langle\hbar_2,(2,\frac{1}{4},\frac{1}{6})\rangle\}$ when Player 1 chooses x_1 and Player 2 chooses x_4. Then, Player 1 wins the set (of alternatives) $\{\langle\hbar_1,(\frac{1}{4},5,4)\rangle,\langle\hbar_2,(2,\frac{1}{4},\frac{1}{6})\rangle\}$ and Player 2 loses the same set.

Definition 4.8. Let $\theta^r_{\mathcal{X}\times\mathcal{Y}}$ be an SNM soft payoff function of a tpSNM soft game $\Theta^r_{\mathcal{X}\times\mathcal{Y}}$. If the following properties are satisfied

(1):

$$\widetilde{\bigcup}_{i=1}^{t}\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)=\left\{\left\langle \hbar,\begin{pmatrix}\max\limits_{i\in\{1,2,..,t\}}\{\rho^r_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\},\\ \min\limits_{i\in\{1,2,..,t\}}\{\tau^r_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\},\\ \min\limits_{i\in\{1,2,..,t\}}\{\sigma^r_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\}\end{pmatrix}\right\rangle : \hbar\in\mathcal{H}\right\}=\theta^r_{\mathcal{X}\times\mathcal{Y}}(x,y),$$

(2):

$$\widetilde{\bigcap}_{j=1}^{v}\theta^r_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)=\left\{\left\langle \hbar,\begin{pmatrix}\min\limits_{j\in\{1,2,..,v\}}\{\rho^r_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\},\\ \max\limits_{j\in\{1,2,..,v\}}\{\tau^r_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\},\\ \max\limits_{j\in\{1,2,..,v\}}\{\sigma^r_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\}\end{pmatrix}\right\rangle : \hbar\in\mathcal{H}\right\}=\theta^r_{\mathcal{X}\times\mathcal{Y}}(x,y),$$

then $\theta^r_{\mathcal{X}\times\mathcal{Y}}(x,y)$ is named a simplified neutrosophic multiplicative soft saddle point value (SNM soft saddle point value) and (x,y) is called an SNM soft saddle point of Player r in the tpSNM soft game.

Note that if (x,y) is an SNM soft saddle point of a tpSNM soft game $\Theta^1_{\mathcal{X}\times\mathcal{Y}}$ then Player 1 can win at least by selecting the strategy $x\in\mathcal{X}$ and Player 2 can keep her/his loss to at most $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x,y)$ by selecting the strategy $y\in\mathcal{Y}$. Hence the tpSNM soft saddle point is a value of the tpSNM soft game.

Example 4.9. Let $\mathcal{H}=\{\hbar_1,\hbar_2,\hbar_3,\hbar_4\}$ be a set of alternatives and $\mathcal{X}=\{x_1,x_2,x_3\}$ and $\mathcal{Y}=\{y_1,y_2\}$ be the sets of the strategies Player 1 and Player 2, respectively. Then, tpSNM soft game of Player 1 is presented as in Table 5.

TABLE 5. The tpSNM soft game $\Theta^1_{\mathcal{X}\times\mathcal{Y}}$ of Player 1

$\Theta^1_{\mathcal{X}\times\mathcal{Y}}$	y_1	y_2
x_1	$\{\langle\hbar_1,(5,4,\frac{1}{6})\rangle,\langle\hbar_2,(\frac{1}{2},1,1)\rangle,\langle\hbar_3,(3,\frac{1}{7},\frac{1}{9})\rangle,\langle\hbar_4,(\frac{1}{5},4,3)\rangle\}$	$\{\langle\hbar_1,(3,4,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{3})\rangle,\langle\hbar_3,(\frac{1}{8},\frac{1}{2},\frac{1}{2})\rangle,\langle\hbar_4,(1,4,1)\rangle\}$
x_2	$\{\langle\hbar_1,(5,2,\frac{1}{8})\rangle,\langle\hbar_2,(\frac{1}{2},1,\frac{1}{5})\rangle,\langle\hbar_3,(3,\frac{1}{9},\frac{1}{9})\rangle,\langle\hbar_4,(4,\frac{1}{5},\frac{1}{4})\rangle\}$	$\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(3,\frac{1}{4},\frac{1}{3})\rangle\}$
x_3	$\{\langle\hbar_1,(2,2,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{4},4,\frac{1}{5})\rangle,\langle\hbar_3,(3,\frac{1}{9},\frac{1}{3})\rangle,\langle\hbar_4,(\frac{1}{5},\frac{1}{2},3)\rangle\}$	$\{\langle\hbar_1,(\frac{1}{9},3,\frac{1}{3})\rangle,\langle\hbar_2,(\frac{1}{8},4,\frac{1}{3})\rangle,\langle\hbar_3,(\frac{1}{4},5,\frac{1}{3})\rangle,\langle\hbar_4,(1,4,1)\rangle\}$

Then, we have
$\bigcup_{i=1}^{3}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i,y_1)=\{\langle\hbar_1,(5,2,\frac{1}{8})\rangle,\langle\hbar_2,(\frac{1}{2},1,\frac{1}{5})\rangle,\langle\hbar_3,(3,\frac{1}{9},\frac{1}{9})\rangle,\langle\hbar_4,(4,\frac{1}{5},\frac{1}{4})\rangle\}$,
$\bigcup_{i=1}^{3}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i,y_2)=\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(3,\frac{1}{4},\frac{1}{3})\rangle\}$,
and
$\bigcap_{j=1}^{2}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_1,y_j)=\{\langle\hbar_1,(3,4,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,1)\rangle,\langle\hbar_3,(\frac{1}{8},\frac{1}{2},\frac{1}{2})\rangle,\langle\hbar_4,(\frac{1}{5},4,3)\rangle\}$,
$\bigcap_{j=1}^{2}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2,y_j)=\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(3,\frac{1}{4},\frac{1}{3})\rangle\}$,
$\bigcap_{j=1}^{2}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_3,y_j)=\{\langle\hbar_1,(\frac{1}{9},3,\frac{1}{3})\rangle,\langle\hbar_2,(\frac{1}{8},4,\frac{1}{3})\rangle,\langle\hbar_3,(\frac{1}{4},5,\frac{1}{3})\rangle,\langle\hbar_4,(\frac{1}{5},4,1)\rangle\}$.
Since $\bigcup_{i=1}^{3}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i,y_2)=\bigcap_{j=1}^{2}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2,y_j)=\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2,y_2)$, we say that $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2,y_2)=\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(3,\frac{1}{4},\frac{1}{3})\rangle\}$ is an SNM soft saddle point value of the tpSNM soft game. Hence, the value of the tpSNM soft game is $\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(3,\frac{1}{4},\frac{1}{3})\rangle\}$.

Note that every tpSNM soft game has not an SNM soft saddle point value. For instance, in Example 4.9, if $\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(5,5,\frac{1}{5})\rangle\}$ is taken instead of $\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(3,\frac{1}{4},\frac{1}{3})\rangle\}$ in the SNM soft payoff $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2,y_2)$ then this tpSNM soft game has not an SNM soft saddle point value. If the saddle point cannot found for a tpSNM soft game then simplified neutrosophic multiplicative soft upper value and simplified neutrosophic multiplicative soft lower value of tpSNM soft game may be used. These concepts are given in the following definition.

Definition 4.10. Let $\Theta_{\mathcal{X}\times\mathcal{Y}}$ be a tpSNM soft game with its SNM soft payoff function $\theta_{\mathcal{X}\times\mathcal{Y}}$, where $\mathcal{X}=\{x_i:i=1,2,...,t\}$ and $\mathcal{Y}=\{y_j:j=1,2,...,t\}$. Then,

(a): SNM soft upper value of the tpSNM soft game, symbolized by V_U, is defined by

$$V_U=\widetilde{\bigcap}_{j=1}^{v}(\widetilde{\bigcup}_{i=1}^{t}(\theta_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j))) \tag{9}$$

(b): SNM soft lower value of the tpSNM soft game, symbolized by V_L, is defined by

$$V_L=\widetilde{\bigcup}_{i=1}^{t}(\widetilde{\bigcap}_{j=1}^{v}(\theta_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j))) \tag{10}$$

(c): If the SNM soft upper value and SNM soft lower value of the tpSNM soft game are equal then these are called value of the tpSNM soft game, symbolized by V. That is, $V=V_U=V_L$.

Example 4.11. Let us consider Table 5 in Example 4.9. Then, we have that the SNM soft upper value V_U and SNM soft lower value V_L are equal, i.e.,

$$V_U=V_L=\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(3,\frac{1}{4},\frac{1}{3})\rangle\}.$$

Therefore, we can say that the value of the tpSNM soft game is $V=V_U=V_L$.
On the other hand, for the SNM soft payoff $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2,y_2)$ in Table 5, if

$$\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(3,\frac{1}{4},\frac{1}{3})\rangle\}$$

is replaced by $\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(5,5,\frac{1}{5})\rangle\}$ then we calculate the SNM soft upper value V_U and the SNM soft lower value V_L as

$$V_U=\widetilde{\bigcap}_{j=1}^{2}(\widetilde{\bigcup}_{i=1}^{3}(\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)))=\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(4,4,\frac{1}{4})\rangle\},$$

$$V_L=\widetilde{\bigcup}_{i=1}^{3}(\widetilde{\bigcap}_{j=1}^{2}(\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)))=\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(4,4,\frac{1}{4})\rangle\}.$$

Thus, since $V_U = V_L$, we deduce that the value of the tpSNM soft game $\Theta^1_{\mathcal{X}\times\mathcal{Y}}$ is $\{\langle\hbar_1,(3,3,\frac{1}{5})\rangle,\langle\hbar_2,(\frac{1}{3},3,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_4,(4,4,\frac{1}{4})\rangle\}$.

Theorem 4.12. *Let V_U and V_L be the values of SNM soft upper and SNM soft lower of a tpSNM soft game, respectively. Then,*

$$V_L\subseteq V_U. \tag{11}$$

Proof. Suppose that V_U and V_L are the SNM soft upper and lower values a tpSNM soft game, respectively. Also, $\mathcal{X}=\{x_i : i=1,2,...,t\}$ and $\mathcal{Y}=\{y_j : j=1,2,...,t\}$ are sets of strategies for Player 1 and Player 2, respectively. Then, we calculate

$$\begin{aligned}
V_L=\widetilde{\bigcup}_{i=1}^{t}(\widetilde{\bigcap}_{j=1}^{v}(\theta_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j))) &= \left\{\left\langle \hbar,\begin{pmatrix}\max\limits_{i\in\{1,2,...,t\}}(\min\limits_{j\in\{1,2,...,v\}}\{\rho_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\}),\\ \min\limits_{i\in\{1,2,...,t\}}(\max\limits_{j\in\{1,2,...,v\}}\{\tau_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\}),\\ \min\limits_{i\in\{1,2,...,t\}}(\max\limits_{j\in\{1,2,...,v\}}\{\sigma_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\})\end{pmatrix}\right\rangle : \hbar\in\mathcal{H}\right\}\\
&\subseteq \left\{\left\langle \hbar,\begin{pmatrix}\min\limits_{j\in\{1,2,...,v\}}\{\rho_{\mathcal{X}\times\mathcal{Y}}(x_{i_{p_1}},y_j)\}),\\ \max\limits_{j\in\{1,2,...,v\}}\{\tau_{\mathcal{X}\times\mathcal{Y}}(x_{i_{p_2}},y_j)\}),\\ \max\limits_{j\in\{1,2,...,v\}}\{\sigma_{\mathcal{X}\times\mathcal{Y}}(x_{i_{p_3}},y_j)\})\end{pmatrix}\right\rangle : \hbar\in\mathcal{H}\right\}\\
&\subseteq \left\{\left\langle \hbar,\begin{pmatrix}\rho_{\mathcal{X}\times\mathcal{Y}}(x_{i_{p_1}},y_{j_{q_1}}),\\ \tau_{\mathcal{X}\times\mathcal{Y}}(x_{i_{p_2}},y_{j_{q_2}}),\\ \sigma_{\mathcal{X}\times\mathcal{Y}}(x_{i_{p_3}},y_{j_{q_3}})\end{pmatrix}\right\rangle : \hbar\in\mathcal{H}\right\}\\
&\subseteq \left\{\left\langle \hbar,\begin{pmatrix}\max\limits_{i\in\{1,2,...,t\}}\{\rho_{\mathcal{X}\times\mathcal{Y}}(x_i,x_{j_{q_1}})\}),\\ \min\limits_{i\in\{1,2,...,t\}}\{\tau_{\mathcal{X}\times\mathcal{Y}}(x_i,x_{j_{q_2}})\}),\\ \min\limits_{i\in\{1,2,...,t\}}\{\sigma_{\mathcal{X}\times\mathcal{Y}}(x_i,x_{j_{q_3}})\})\end{pmatrix}\right\rangle : \hbar\in\mathcal{H}\right\}\\
&= \left\{\left\langle \hbar,\begin{pmatrix}\min\limits_{j\in\{1,2,...,v\}}(\max\limits_{i\in\{1,2,...,t\}}\{\rho_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\}),\\ \max\limits_{j\in\{1,2,...,v\}}(\min\limits_{i\in\{1,2,...,t\}}\{\tau_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\}),\\ \max\limits_{j\in\{1,2,...,v\}}(\min\limits_{i\in\{1,2,...,t\}}\{\sigma_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)\})\end{pmatrix}\right\rangle : \hbar\in\mathcal{H}\right\}\\
&= \widetilde{\bigcap}_{j=1}^{v}(\widetilde{\bigcup}_{i=1}^{t}(\theta_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j)))
\end{aligned}$$

where $i_{p_1},i_{p_2},i_{p_3}\in\{1,2,...,t\}$ and $j_{p_1},j_{p_2},j_{p_3}\in\{1,2,...,v\}$. Hence, we have $V_L\subseteq V_U$. □

Example 4.13. For the SNM soft payoff $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2, y_2)$ in Table 5, we take

$$\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2, y_2) = \{\langle \hbar_1, (3, 3, \frac{1}{5})\rangle, \langle \hbar_2, (\frac{1}{3}, 3, \frac{1}{4})\rangle, \langle \hbar_3, (2, \frac{1}{3}, \frac{1}{5})\rangle, \langle \hbar_4, (\frac{1}{6}, 5, \frac{1}{6})\rangle\}.$$

Then, we obtain the SNM soft upper value V_U and the SNM soft lower value V_L as

$$V_U = \widetilde{\bigcap}_{j=1}^{2}(\widetilde{\bigcup}_{i=1}^{3}(\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j))) = \{\langle \hbar_1, (3, 3, \frac{1}{5})\rangle, \langle \hbar_2, (\frac{1}{3}, 3, \frac{1}{4})\rangle, \langle \hbar_3, (2, \frac{1}{3}, \frac{1}{5})\rangle, \langle \hbar_4, (1, 4, \frac{1}{4})\rangle\}$$

and

$$V_L = \widetilde{\bigcup}_{i=1}^{3}(\widetilde{\bigcap}_{j=1}^{2}(\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i, y_j))) = \{\langle \hbar_1, (3, 3, \frac{1}{5})\rangle, \langle \hbar_2, (\frac{1}{3}, 3, \frac{1}{4})\rangle, \langle \hbar_3, (2, \frac{1}{3}, \frac{1}{5})\rangle, \langle \hbar_4, (\frac{1}{5}, 4, \frac{1}{4})\rangle\}.$$

It is clear that $V_L \subseteq V_U$.

Theorem 4.14. *Let $\theta_{\mathcal{X}\times\mathcal{Y}}(x, y)$ be an SNM soft saddle point value, and V_U and V_L be the values of SNM soft upper and SNM soft lower of a tpSNM soft game, respectively. Then,*

$$V_L \subseteq \theta_{\mathcal{X}\times\mathcal{Y}}(x, y) \subseteq V_U. \tag{12}$$

Proof. It can be demonstrated using techniques similar to those in the proof of Theorem 4.12. □

Corollary 4.15. *Let (x, y) be an SNM soft saddle point, and V_U and V_L be the values of SNM soft upper and SNM soft lower of a tpSNM soft game, respectively. If $V_U = V_L = V$ then $\theta_{\mathcal{X}\times\mathcal{Y}}(x, y)$ is exactly V.*

Example 4.16. Consider the SNM soft saddle point value in Example 4.9, and SNM soft upper value V_U and SNM soft upper value V_L in Example 4.11. It is obvious that the SNM soft saddle point value $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2, y_2)$ is exactly $V = V_U = V_L$.

Note that in every tpSNM soft game, the SNM soft upper value V_U and SNM soft lower value V_L cannot be equals. If $V_U \neq V_L$ in a tpSNM soft game then we achieve the solution of game by using the following simplified neutrosophic multiplicative soft dominated strategy (SNM soft dominated strategy).

Definition 4.17. Let $\Theta_{\mathcal{X}\times\mathcal{Y}}$ be a tpSNM soft game with its SNM soft payoff function $\theta_{\mathcal{X}\times\mathcal{Y}}$. Then,

- **(a):** a strategy $x_i \in \mathcal{X}$ is termed to be an SNM soft dominated to another strategy $x_k \in \mathcal{X}$ if $\theta_{\mathcal{X}\times\mathcal{Y}}(x_k, y) \subseteq \theta_{\mathcal{X}\times\mathcal{Y}}(x_i, y)$ for all $y \in \mathcal{Y}$,
- **(b):** a strategy $y_j \in \mathcal{Y}$ is termed to be an SNM soft dominated to another strategy $y_l \in \mathcal{Y}$ if $\theta_{\mathcal{X}\times\mathcal{Y}}(x, y_j) \subseteq \theta_{\mathcal{X}\times\mathcal{Y}}(x, y_l)$ for all $x \in \mathcal{X}$.

By using the SNM soft dominated strategy, tpSNM soft games may be reduced by deleting columns and rows, which are obviously bad for the player of game. This process of eliminating SNM soft dominated strategies sometimes leads us to a solution of a tpSNM soft game. This method of solving tpSNM soft game is named a simplified neutrosophic multiplicative soft elimination method (SNM soft elimination method).

Now, let us solve the following tpSNM soft game by using the SNM soft elimination method.

Example 4.18. We consider Table 5 in Example 4.9. Since $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_1, y_j) \subseteq \theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2, y_j)$ and $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_3, y_j) \subseteq \theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2, y_j)$ for all $y_j \in \mathcal{Y}$, we can say that the strategy x_2 dominates to the strategies x_1 and x_3. That is, the first row and third row are deleted from Table 5, and so Table 6 are created.

TABLE 6. The reduced tpSNM soft game $\Theta^1_{\mathcal{X}\times\mathcal{Y}}$ for dominated strategy x_i

$\Theta^1_{\mathcal{X}\times\mathcal{Y}}$	y_1	y_2
x_2	$\{\langle\hbar_1, (5, 2, \frac{1}{8})\rangle, \langle\hbar_2, (\frac{1}{2}, 1, \frac{1}{5})\rangle, \langle\hbar_3, (3, \frac{1}{9}, \frac{1}{9})\rangle, \langle\hbar_4, (4, \frac{1}{5}, \frac{1}{4})\rangle\}$	$\{\langle\hbar_1, (3, 3, \frac{1}{5})\rangle, \langle\hbar_2, (\frac{1}{3}, 3, \frac{1}{4})\rangle, \langle\hbar_3, (2, \frac{1}{3}, \frac{1}{5})\rangle, \langle\hbar_4, (3, \frac{1}{4}, \frac{1}{3})\rangle\}$

Now, we consider Table 6. Since $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2, y_2) \subseteq \theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2, y_1)$ for all $x_2 \in \mathcal{X}$, we can say that the strategy y_1 is dominated by the strategy y_2. Player 1 has SNM soft dominated strategy y_2 so that the strategy y_1 is eliminated. Thus, we delete the first column from Table 6 and present Table 7.

TABLE 7. The reduced tpSNM soft game $\Theta^1_{\mathcal{X}\times\mathcal{Y}}$ for dominated strategies x_i and y_j

$\Theta^1_{\mathcal{X}\times\mathcal{Y}}$	y_2
x_2	$\{\langle\hbar_1, (3, 3, \frac{1}{5})\rangle, \langle\hbar_2, (\frac{1}{3}, 3, \frac{1}{4})\rangle, \langle\hbar_3, (2, \frac{1}{3}, \frac{1}{5})\rangle, \langle\hbar_4, (3, \frac{1}{4}, \frac{1}{3})\rangle\}$

Consequently, the solution using tpSNM soft elimination method is (x_2, y_2), that is, the value of tpSNM soft game is $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2, y_2) = \{\langle\hbar_1, (3, 3, \frac{1}{5})\rangle, \langle\hbar_2, (\frac{1}{3}, 3, \frac{1}{4})\rangle, \langle\hbar_3, (2, \frac{1}{3}, \frac{1}{5})\rangle, \langle\hbar_4, (3, \frac{1}{4}, \frac{1}{3})\rangle\}$.

Note that the tpSNM soft elimination method cannot achieve the solutions for some tpSNM soft games that do not have an SNM soft dominated strategies. In such cases, we can utilize simplified neutrosophic multiplicative soft Nash equilibrium (SNM soft Nash equilibrium) described in the following.

Definition 4.19. Let $\Theta^r_{\mathcal{X}\times\mathcal{Y}}$ be a tpSNM soft game with its SNM soft payoff function $\theta^r_{\mathcal{X}\times\mathcal{Y}}$ $(r = 1, 2)$. If the following properties are satisfied then $(x^*, y^*) \in \mathcal{X} \times \mathcal{Y}$ is called an SNM soft Nash equilibrium of a tpSNM soft game.

- **(1):** $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i, y^*) \subseteq \theta^1_{\mathcal{X}\times\mathcal{Y}}(x^*, y^*)$ for all $x_i \in \mathcal{X}$.
- **(2):** $\theta^2_{\mathcal{X}\times\mathcal{Y}}(x^*, y_j) \subseteq \theta^2_{\mathcal{X}\times\mathcal{Y}}(x^*, y^*)$ for all $y_j \in \mathcal{Y}$.

Note that if $(x^*, y^*) \in \mathcal{X}\times\mathcal{Y}$ is an SNM soft Nash equilibrium of a tpSNM soft game, then Player 1 can win at least $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x^*, y^*)$ by selecting strategy $x^* \in \mathcal{X}$, and Player 2 can win at least $\theta^2_{\mathcal{X}\times\mathcal{Y}}(x^*, y^*)$ by selecting strategy $y^* \in \mathcal{Y}$. Therefore, the SNM soft Nash equilibrium is an optimal action for tpSNM soft game, and so $\theta^r_{\mathcal{X}\times\mathcal{Y}}(x^*, y^*)$ is the solution of the tpSNM soft game for Player r $(r = 1, 2)$.

Example 4.20. Assume that the tpSNM soft games of Player 1 and Player 2 are given as in Tables 8 and 9, respectively.

Each of tpSNM soft games $\Theta^1_{\mathcal{X}\times\mathcal{Y}}$ and $\Theta^2_{\mathcal{X}\times\mathcal{Y}}$ has not an SNM soft saddle point value and $V_U \neq V_L$. Also, it is obvious that the tpSNM soft elimination method cannot be used for the solutions of these tpSNM soft games.

TABLE 8. The tpSNM soft game of Player 1

$\Theta^1_{\mathcal{X}\times\mathcal{Y}}$	y_1	y_2
x_1	$\{\langle\hbar_1,(3,\frac{1}{9},\frac{1}{3})\rangle,\langle\hbar_2,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_3,(2,2,\frac{1}{5})\rangle,\langle\hbar_4,(\frac{1}{4},4,\frac{1}{5})\rangle\}$	$\{\langle\hbar_1,(\frac{1}{4},5,\frac{1}{3})\rangle,\langle\hbar_2,(1,4,1)\rangle,\langle\hbar_3,(\frac{1}{9},3,\frac{1}{3})\rangle,\langle\hbar_4,(\frac{1}{8},4,\frac{1}{3})\rangle\}$
x_2	$\{\langle\hbar_1,(3,\frac{1}{7},\frac{1}{9})\rangle,\langle\hbar_2,(\frac{1}{5},4,3)\rangle,\langle\hbar_3,(5,4,\frac{1}{6})\rangle,\langle\hbar_4,(\frac{1}{2},1,1)\rangle\}$	$\{\langle\hbar_1,(\frac{1}{8},\frac{1}{2},\frac{1}{2})\rangle,\langle\hbar_2,(1,4,1)\rangle,\langle\hbar_3,(3,4,\frac{1}{5})\rangle,\langle\hbar_4,(\frac{1}{3},3,\frac{1}{3})\rangle\}$
x_3	$\{\langle\hbar_1,(3,\frac{1}{7},\frac{1}{7})\rangle,\langle\hbar_2,(2,\frac{1}{5},\frac{1}{4})\rangle,\langle\hbar_3,(5,2,\frac{1}{8})\rangle,\langle\hbar_4,(\frac{1}{2},1,\frac{1}{5})\rangle\}$	$\{\langle\hbar_1,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_2,(3,\frac{1}{4},\frac{1}{3})\rangle,\langle\hbar_3,(3,3,\frac{1}{5})\rangle,\langle\hbar_4,(\frac{1}{3},3,\frac{1}{4})\rangle\}$

TABLE 9. The tpSNM soft game of Player 2

$\Theta^2_{\mathcal{X}\times\mathcal{Y}}$	y_1	y_2
x_1	$\{\langle\hbar_1,(4,\frac{1}{4},\frac{1}{9})\rangle,\langle\hbar_2,(\frac{1}{5},4,3)\rangle,\langle\hbar_3,(\frac{1}{4},4,\frac{1}{5})\rangle,\langle\hbar_4,(5,2,\frac{1}{8})\rangle\}$	$\{\langle\hbar_1,(3,\frac{1}{9},\frac{1}{4})\rangle,\langle\hbar_2,(4,2,\frac{1}{4})\rangle,\langle\hbar_3,(2,\frac{1}{2},\frac{1}{2})\rangle,\langle\hbar_4,(2,\frac{1}{2},\frac{1}{4})\rangle\}$
x_2	$\{\langle\hbar_1,(\frac{1}{2},2,1)\rangle,\langle\hbar_2,(2,\frac{1}{4},\frac{1}{4})\rangle,\langle\hbar_3,(1,\frac{1}{2},\frac{1}{3})\rangle,\langle\hbar_4,(2,\frac{1}{4},\frac{1}{2})\rangle\}$	$\{\langle\hbar_1,(1,1,\frac{1}{2})\rangle,\langle\hbar_2,(3,2,\frac{1}{4})\rangle,\langle\hbar_3,(3,\frac{1}{2},\frac{1}{3})\rangle,\langle\hbar_4,(2,\frac{1}{2},\frac{1}{4})\rangle\}$
x_3	$\{\langle\hbar_1,(2,\frac{1}{4},\frac{1}{2})\rangle,\langle\hbar_2,(5,1,\frac{1}{6})\rangle,\langle\hbar_3,(1,1,1)\rangle,\langle\hbar_4,(\frac{1}{2},1,1)\rangle\}$	$\{\langle\hbar_1,(4,\frac{1}{5},\frac{1}{5})\rangle,\langle\hbar_2,(5,1,\frac{1}{9})\rangle,\langle\hbar_3,(3,\frac{1}{3},\frac{1}{3})\rangle,\langle\hbar_4,(2,\frac{1}{2},\frac{1}{4})\rangle\}$

From Tables 8 and 9, we have

(1): $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i,y_2)\subseteq\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_3,y_2)$ for all $x_i\in\mathcal{X}$.

(2): $\theta^2_{\mathcal{X}\times\mathcal{Y}}(x_3,y_j)\subseteq\theta^2_{\mathcal{X}\times\mathcal{Y}}(x_3,y_2)$ for all $y_j\in\mathcal{Y}$.

Then, $(x_3,y_2)\in\mathcal{X}\times\mathcal{Y}$ is an SNM soft Nash equilibrium. Hence,

$$\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_3,y_2)=\{\langle\hbar_1,(2,\frac{1}{3},\frac{1}{5})\rangle,\langle\hbar_2,(3,\frac{1}{4},\frac{1}{3})\rangle,\langle\hbar_3,(3,3,\frac{1}{5})\rangle,\langle\hbar_4,(\frac{1}{3},3,\frac{1}{4})\rangle\} \tag{13}$$

and

$$\theta^2_{\mathcal{X}\times\mathcal{Y}}(x_3,y_2)=\{\langle\hbar_1,(4,\frac{1}{5},\frac{1}{5})\rangle,\langle\hbar_2,(5,1,\frac{1}{9})\rangle,\langle\hbar_3,(3,\frac{1}{3},\frac{1}{3})\rangle,\langle\hbar_4,(2,\frac{1}{2},\frac{1}{4})\rangle\} \tag{14}$$

are the solutions of the above tpSNM soft games for Player 1 and Player 2, respectively.

4.2. *Applications of Two Person Simplified Neutrosophic Multiplicative Soft Games*

This part presents an example to illustrate the solution procedures (SNM soft saddle point method and SNM soft elimination method) of a tpSNM soft game and also gives comparison implementations.

Example 4.21. Assuming that the demand for beverages in the market is essentially the same, Beverage Company I (Player 1) and Beverage Company II (Player 2) want to increase their market share. These companies have a set of different beverages as $\mathcal{H}=\{\hbar_1=\mathit{coke},\hbar_2=\mathit{lemonade},\hbar_3=\mathit{concentrated\ drink}\}$. To achieve their goal, they come up with three alternative marketing strategies: reducing-price (x_1), advertising investment (x_2) and lagnappe (x_3).
Suppose that Beverage Company I (Player 1) chooses the strategies x_1, x_2 and x_3, i.e., $\mathcal{X}=\{x_1,x_2,x_3\}$, and Beverage Company II (Player 2) chooses the strategies x_1 and x_2, i.e., $\mathcal{Y}=\{x_1,x_2\}$. Due to the vagueness and indeterminacy of information, Beverage Company I and II can use simplified neutrosophic multiplicative values to represent the payoff for any one of the marketing strategies. The SNM soft game of Beverage Company I are considered in Table 10.

Table 10. The tpSNM soft game of Beverage Company I

$\Theta^1_{\mathcal{X}\times\mathcal{Y}}$	x_1	x_2
x_1	$\{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,\frac{1}{5},\frac{1}{9})\rangle\}$	$\{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,1,\frac{1}{8})\rangle\}$
x_2	$\{\langle\hbar_1,(4,2,\frac{1}{4})\rangle,\langle\hbar_2,(1,4,\frac{1}{3})\rangle,\langle\hbar_3,(3,\frac{1}{5},\frac{1}{3})\rangle\}$	$\{\langle\hbar_1,(3,2,\frac{1}{4})\rangle,\langle\hbar_2,(\frac{1}{5},3,\frac{1}{2})\rangle,\langle\hbar_3,(4,1,\frac{1}{5})\rangle\}$
x_3	$\{\langle\hbar_1,(1,\frac{1}{2},\frac{1}{9})\rangle,\langle\hbar_2,(2,4,\frac{1}{2})\rangle,\langle\hbar_3,(\frac{1}{9},\frac{1}{9},9)\rangle\}$	$\{\langle\hbar_1,(2,\frac{1}{2},\frac{1}{3})\rangle,\langle\hbar_2,(2,2,\frac{1}{2})\rangle,\langle\hbar_3,(\frac{1}{5},1,4)\rangle\}$

In Table 10, we can explain the action pair (x_1, x_2), if Beverage Company I (Player 1) selects the strategy reducing-price (x_1) when Beverage Company II (Player 2) selects the strategy advertising investment (x_2) then the SNM soft payoff of Beverage Company I is a set $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_1,x_2) = \{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,1,\frac{1}{8})\rangle\}$. In such case, Beverage Company I increases sale of $\{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,1,\frac{1}{8})\rangle\}$ and Beverage Company II decreases sale of $\{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,1,\frac{1}{8})\rangle\}$.
Now, we ready to solve this tpSNM soft game.
It is easily seen from Table 10 that the strategy x_1 dominates to the strategy x_2 since $\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_2,x_j) \subseteq \theta^1_{\mathcal{X}\times\mathcal{Y}}(x_1,x_j)$ for all $x_j \in \mathcal{Y}$. That is, the second row is dominated by the first row. Deleting the second row from Table 10, we obtain Table 11.

Table 11. The reduced tpSNM soft game of Beverage Company I

$\Theta^1_{\mathcal{X}\times\mathcal{Y}}$	x_1	x_2
x_1	$\{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,\frac{1}{5},\frac{1}{9})\rangle\}$	$\{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,1,\frac{1}{8})\rangle\}$
x_3	$\{\langle\hbar_1,(1,\frac{1}{2},\frac{1}{9})\rangle,\langle\hbar_2,(2,4,\frac{1}{2})\rangle,\langle\hbar_3,(\frac{1}{9},\frac{1}{9},9)\rangle\}$	$\{\langle\hbar_1,(2,\frac{1}{2},\frac{1}{3})\rangle,\langle\hbar_2,(2,2,\frac{1}{2})\rangle,\langle\hbar_3,(\frac{1}{5},1,4)\rangle\}$

In Table 11, there is no another SNM soft dominated strategy. Now, we try to find the SNM soft saddle point value by using the SNM soft saddle point method.
$\widetilde{\bigcup}_{i\in\{1,3\}}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i,x_1) = \{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{9})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,\frac{1}{9},\frac{1}{9})\rangle\}$,
$\widetilde{\bigcup}_{i\in\{1,3\}}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i,x_2) = \{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,1,\frac{1}{8})\rangle\}$,
and
$\widetilde{\bigcap}_{j\in\{1,2\}}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_1,x_j) = \{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,1,\frac{1}{8})\rangle\}$,
$\widetilde{\bigcap}_{j\in\{1,2\}}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_3,x_j) = \{\langle\hbar_1,(1,\frac{1}{2},\frac{1}{3})\rangle,\langle\hbar_2,(2,4,\frac{1}{2})\rangle,\langle\hbar_3,(\frac{1}{9},1,9)\rangle\}$.
Since $\widetilde{\bigcup}_{i\in\{1,3\}}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_i,x_2) = \widetilde{\bigcap}_{j\in\{1,2\}}\theta^1_{\mathcal{X}\times\mathcal{Y}}(x_1,x_j) = \theta^1_{\mathcal{X}\times\mathcal{Y}}(x_1,x_2)$, the optimal strategy of the game is (x_1,x_2). Hence, the value of tpSNM soft game is $\{\langle\hbar_1,(4,\frac{1}{2},\frac{1}{6})\rangle,\langle\hbar_2,(2,2,\frac{1}{5})\rangle,\langle\hbar_3,(7,1,\frac{1}{8})\rangle\}$.

Comparison and Discussion: In 2016, Deli and Çağman [10] published a seminal paper on soft games and thus took the first step to the application of soft sets in decision making based on game theory. Now, we consider the application (Table 10) in Section 4 of [10]. If the calculations are made by respectively corresponding to $\theta_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j) = \langle u,(9,\frac{1}{9},\frac{1}{9})\rangle$ and $\theta_{\mathcal{X}\times\mathcal{Y}}(x_i,y_j) = \langle u,(\frac{1}{9},9,9)\rangle$ when $u \in f_{S_1}(x_i,y_j)$ and $u \notin f_{S_1}(x_i,y_j)$, then we obtain that the optimal strategy of game (described in [10]) is (x_3,y_3) and the value of game is $\{\langle u_1,(9,\frac{1}{9},\frac{1}{9})\rangle,\langle u_2,(9,\frac{1}{9},\frac{1}{9})\rangle,\langle u_3,(9,\frac{1}{9},\frac{1}{9})\rangle,\langle u_4,(\frac{1}{9},9,9)\rangle,\langle u_5,(\frac{1}{9},9,9)\rangle,\langle u_6,(\frac{1}{9},9,9)\rangle,\langle u_7,(\frac{1}{9},9,9)\rangle,\langle u_8,(\frac{1}{9},9,9)\rangle\}$.

Thus, it is obvious that similar results are obtained. Also, the applications of fuzzy soft games can be adapted by deriving new comparison methods between $0-1$ and $\frac{1}{9}-9$ scales similar to matches between $0-1$ and $\frac{1}{9}-9$ scales given in Table 1 in the Introduction section. The tpSNM soft games proposed in this study use the $\frac{1}{9}-9$ scale instead of the $0-1$ scale used for fuzzy (intuitionistic fuzzy/neutrosophic) soft games, and therefore may be advantageous in some cases. Consequently, we can say that the tpSNM soft games present the solutions to the soft games where alternatives are evaluated with truth, indeterminacy, falsity values scaled between $\frac{1}{9}-9$ with respect to the strategies.

5. n-Person Simplified Neutrosophic Multiplicative Soft Games

In this section, we introduce some fundamental concepts of n-person simplified neutrosophic multiplicative soft games.

In many stages of the real-world, the SNM soft games can also be played between more than two players. To propose the solution procedures for these games, we describe n-person SNM soft games by extending the tpSNM soft games as follows.

From now on, $\prod\limits_{r=1}^{n} \mathcal{X}_r = \mathcal{X}_1 \times \mathcal{X}_2 \times ... \times \mathcal{X}_n$.

Definition 5.1. Let $\mathcal{S}$ be a set of strategies and $\mathcal{X}_1, \mathcal{X}_2, ..., \mathcal{X}_n \subseteq \mathcal{S}$ where $\mathcal{X}_r$ is the set of strategies of Player r $(r = 1, 2, ..., n)$. Then, for each Player r, an n-person SNM soft game (npSNM soft game) can be defined by an SNMSS over $\mathcal{H}$ as follows.

$$\Theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r} = \{((x_1, x_2, ..., x_n), \theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}(x_1, x_2, ..., x_n)) : (x_1, x_2, ..., x_n) \in \prod_{r=1}^{n} \mathcal{X}_r, \theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}(x_1, x_2, ..., x_n) \in \mathfrak{P}(\mathcal{H})\}$$

where $\theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}$ is a SNM soft payoff function of Player r.

The npSNM soft game is played as below: at a certain Player 1 selects a strategy $x_1 \in \mathcal{X}_1$ and simultaneously each Player r $(r = 1, 2, ..., s)$ selects a strategy $x_r \in \mathcal{X}_r$ and once this is done each Player r receives the SNM soft payoff $\theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}(x_1, x_2, ..., x_n)$.

Definition 5.2. Let $\Theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}$ be an npSNM soft game with its SNM soft payoff function $\theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}$ for $r = 1, 2, ..., n$. Then, a strategy $x_r \in \mathcal{X}_r$ is said to be an SNM soft dominated to another strategy $x \in \mathcal{X}_r$, if

$$\theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}(x_1, x_2, ..., x_{r-1}, x, x_{r+1}, ..., x_n) \subseteq \theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}(x_1, x_2, ..., x_{r-1}, x_r, x_{r+1}, ..., x_n)$$

for each $x_q \in \mathcal{X}_q$ of Player q $(q = 1, 2, ..., r-1, r+1, ..., n)$, respectively.

Definition 5.3. Let $\theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}$ be an SNM soft payoff function of an npSNM soft game $\Theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}$. If for each Player r $(r = 1, 2, ..., n)$ the following property are provided

$$\theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}(x_1^*, x_2^*, ..., x_{r-1}^*, x, x_{r+1}^*, ..., x_n^*) \subseteq \theta^r_{\prod\limits_{r=1}^{n} \mathcal{X}_r}(x_1^*, x_2^*, ..., x_{r-1}^*, x_r^*, x_{r+1}^*, ..., x_n^*)$$

for each $x \in \mathcal{X}_r$, then $(x_1^*, x_2^*, ..., x_n^*) \in \prod\limits_{r=1}^{n} \mathcal{X}_r$ is termed to be an npSNM soft Nash equilibrium of an npSNM soft game.

6. Conclusions

In this paper, the concept of SNMSS was introduced and their fundamental operations such as intersection, union, complement, And-product, Or-product and cartesian product were presented. The desirable properties of the emerged operations of SNMSSs were investigated in detail. By using SNMSS operations, the fundamentals of SNM soft games were studied. The proposed SNM soft game schemes were illustrated by an example regarding the strategy problem. In the near future, it is expected that the approach of SNMSS will advance in several directions such as new operations, measures of similarity, distance and entropy, correlation coefficients, algebraic and topological structures, and thus contribute to many research areas both theoretically and practically. By applying SNM soft games to problems in different fields, their success in practice may be illustrated.

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