Generalized closed sets and pre-closed sets via Bipolar single-valued neutrosophic Topological Spaces

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Abstract: The purpose of the paper is to introduce a new class of sets namely bipolar single-valued neutrosophic generalized closed sets and bipolar single-valued neutrosophic generalized pre-closed sets in bipolar single-valued neutrosophic topological spaces. Also we analysis the properties and its applications.

Keywords: Bipolar single-valued neutrosophic generalized closed sets, bipolar single-valued neutrosophic generalized pre-closed sets, and bipolar single-valued neutrosophic generalized pre-open sets, BSVN $T_{1/2}$ space, BSVN $pT_{1/2}$ space, BSVN $gpT_{1/2}$ space, BSVN $gpT_p$ space.

1. Introduction

Zadeh [37], the Father of the Fuzzy Logic who imported the fuzzy sets in 1965 where the Fuzzy logic feature the human decision making technique and it is a tool in research logical subject. The concept of fuzzy sets is to deal with contrasting types of uncertainties. Fuzzy topology was introduced by Chang [5] in 1967 after the introduction of fuzzy sets. In 1970, Levine [21] studied the generalized closed sets in general topology. In 1991, Binshahan [4] introduced and investigate the notion of fuzzy pre-open and fuzzy pre-closed sets. The concept of generalized fuzzy closed set was introduced by Balasubramanian and Sundaram [3]. Fukutake et al. [19] gave the generalized pre-closed fuzzy sets in fuzzy topological spaces.


Smarandache [31] introduced the neutrosophic set which is the base for the new mathematical theories. Neutrosophic set has the capability to induce classical sets, fuzzy set, Intuitionistic fuzzy sets. Introducing the components of the neutrosophic set are True (T), Indeterminacy (I), False (F) which represent the membership, indeterminacy, and non-membership values respectively. The notion of classical set, fuzzy set, interval-valued fuzzy set, Intuitionistic fuzzy, etc were generalized by the neutrosophic set. Neutrosophic topological spaces were presented by Salama et al. [30]. The concept of generalized closed sets and generalized pre-closed sets in neutrosophic Topological spaces were introduced by Wadei Al-Omeri et al.[33]. The neutrosophic pre-open and pre-closed sets in neutrosophic topology were extended by Venkateswara Rao et al.[32] who introduce
neutrosophic topological space and open sets, closed sets, semi-open and semi closed sets. Generalized neutrosophic closed sets was introduced and some of their characterizations were also discussed by Dhavaseelan and Jafari [17]. Many Researchers [6-15, 26] have studied Neutrosophic in different areas with applications and the results.

Deli et al.[16] developed bipolar neutrosophic sets and study their application in decision making problem. The notation of bipolar neutrosophic soft set was proposed by Mumtaz Ali et al.[27]. Single-valued neutrosophic sets (in sort, SVN) were proposed by Wang et al.[35] by simplifying the Neutrosophic set. Single-valued neutrosophic topological space was given by YL Liu and HL Yang [22] and discussed the relationships between single valued neutrosophic approximation spaces and single valued neutrosophic topological spaces. Many researchers have studied the applications of SVNSs as well as theory. Ye [36] proposed decision making based on correlation coefficients and weighted correlation coefficient of SVNSs and gave the application of proposed methods. Majumdar and Samant [23] studied distance, similarity and entropy of SVNSs from a theoretical aspect. Bipolar single-valued neutrosophic set was introduced by Mohana et al. [25] and also they give bipolar single-valued neutrosophic topological spaces.

In the paper, we introduce a new class of sets namely bipolar single-valued neutrosophic generalized closed sets and bipolar single-valued neutrosophic generalized pre-closed sets in bipolar single-valued neutrosophic topological spaces. Further we examine the interesting properties and some applications with counter examples.

2. Preliminaries

2.1 Definition [31]: Let a universe U of discourse. Then K=\{K(x), T(x), I(x), F(x)\} defined as a neutrosophic set where truth-membership function T, an indeterminacy-membership function I and a falsity-membership function F are real or non-standard elements of \([0, 1]^3\). No restriction on the sum of T(x), I(x) and F(x), so \(0 \leq \sup T(x) \leq \sup I(x) \leq \sup F(x) \leq 3\).

2.2 Definition [30]: A Neutrosophic topology [NT for short] is a non-empty set X is a family of Neutrosophic subsets in X satisfying the following axioms:

\begin{align*}
\text{(NT1):} & \quad 0, 1 \in \tau, \\
\text{(NT2):} & \quad G_i \in \tau, \text{ for any } G_i, G_j \in \tau, \\
\text{(NT3):} & \quad U \in \tau, \text{ for every } \{G_i : i \in J\} \subseteq \tau.
\end{align*}

The pair (X, \(\tau\)) is called a Neutrosophic topological space (NTS for short). The elements of \(\tau\) are called Neutrosophic open sets [NOS for short]. A complement C(A) of a NOS A in NTS (X, \(\tau\)) is called a Neutrosophic closed set [NCS for short] in X.

2.3 Definition: [30]: Let (X, \(\tau\)) be NTS and A = \{(x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X\} be a NS in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by NCl(A) = \{K : K is a NCS in X and A K\} NInt(A) = \{G : G is a NOS in X and G A\} It can be also shown that NCl(A) is NCS and NInt(A) is a NOS in X. a) A is NOS if and only if A = NInt(A), b) A is NCS if and only if A = NCl(A).

2.4 Definition: [34]: A Neutrosophic set A = \{(x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X\} in a NTS (X, \(\tau\)) is said to be

\begin{enumerate}
\item [(i)] Neutrosophic regular closed set (NRCS for short) if A = NCl(NInt(A)),
\item [(ii)] Neutrosophic regular open set (NROS for short) if A = NInt(NCl(A)),
\item [(iii)] Neutrosophic semi closed set (NSCS for short) if NInt(NCl(A)) \(\subseteq\) A,
\item [(iv)] Neutrosophic semi open set (NSOS for short) if A \(\subseteq\) NCl(NInt(A)),
\item [(v)] Neutrosophic pre closed set (NPSC for short) if NCl(NInt(A)) \(\subseteq\) A,
\item [(vi)] Neutrosophic pre-open set (NPOS for short) if A \(\subseteq\) NInt(NCl(A)),
\item [(vii)] Neutrosophic \(\alpha\)-closed set (NSCS for short) if NCl(NInt(NCl(A))) \(\subseteq\) A,
\item [(viii)] Neutrosophic \(\alpha\)-open set (NSOS for short) if A \(\subseteq\) NInt(NCl(NInt(A))).
\end{enumerate}
2.5 Definition: [33]: Let (X, τ) be NTS and A = {(x, µA(x), σA(x), νA(x)): x ∈ X} be a NS in X. Then the Neutrosophic pre closure and Neutrosophic pre interior of A are defined by NPCl(A) = {K : K is a NPCS in X and A ⊆ K}, NPInt(A) = {G : G is a NPOS in X and G ⊆ A}.

2.6 Definition: [28]: A Neutrosophic set A = {(x, µA(x), σA(x), νA(x)): x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized closed set (NGCS for short) if NPCl(A) U whenever A U and U is a NOS in (X, τ). A Neutrosophic set A of a NTS (X, τ) is called a Neutrosophic generalized open set (NGOS for short) if C(A) is a NGCS in (X, τ).

2.7 Definition: [33]: A Neutrosophic set A = {(x, µA(x), σA(x), νA(x)): x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic α- generalized closed set (NaGCS for short) if NaCl(A) U whenever A U and U is a NOS in (X, τ). A Neutrosophic set A of a NTS (X, τ) is called a Neutrosophic α- generalized open set (NaGOS for short) if C(A) is a NaGCS in (X, τ).

2.8 Definition: [24]: A Neutrosophic set A = {(x, µA(x), σA(x), νA(x)): x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic regular generalized closed set (NRGCS for short) if NCl(A) U whenever A U and U is a NROS in (X, τ). A Neutrosophic set A of a NTS (X, τ) is called a Neutrosophic regular generalized open set (NRGOS for short) if C(A) is a NRGCS in (X, τ).

2.9 Definition: [33]: A Neutrosophic set A = {(x, µA(x), σA(x), νA(x)): x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized pre closed set (NGPCS for short) if NPCl(A) U whenever A U and U is a NPOS in X. A Neutrosophic set A of a NTS (X, τ) is called a Neutrosophic generalized open set (NGPOS for short) if C(A) is a NGPCS in (X, τ).

2.10 Definition [35]: Let a universe X of discourse. Then ANSV = [〈x, T(x), I(x), F(x)〉: x ∈ X] defined as a single-valued neutrosophic set (SVNS in short) where truth-membership function T: X → [0, 1], an indeterminacy-membership function I: X → [0, 1] and a falsity-membership function F: X → [0, 1]. No restriction on the sum of T(x), I(x) and F(x), so 0 ≤ T(x) ≤ I(x) ≤ F(x) ≤ 3. \( \tilde{A} = \langle T, I, F \rangle \) is denoted as a single-valued neutrosophic number.

2.11 Definition [22]: A Single-valued neutrosophic topology on a non-empty set U is a family τ of SVNSs in U that satisfies the following conditions:

(T1) \( \tilde{\varnothing}, \tilde{U} \in \tau \),

(T2) \( \tilde{A} \cap \tilde{B} \in \tau \) for any \( \tilde{A}, \tilde{B} \in \tau \),

(T3) \( \bigcup_{i \in I} \tilde{A}_i \in \tau \) for any \( \tilde{A}_i \in \tau \), i ∈ I, where I is an index set.

The pair (U, τ) is called Single valued neutrosophic topological space and each SVNS \( \tilde{A} \) in τ is referred to as a single valued neutrosophic open set in (U, τ). The complement of a single valued neutrosophic open set in (U, τ) is called a single valued neutrosophic closed set in (U, τ).

2.12 Definition [16]: In X, a bipolar neutrosophic set B is defined in the form B = 〈x, (T⁺(x), I⁺(x), F⁺(x)), (T⁻(x), I⁻(x), F⁻(x))〉: x ∈ X〉

Where T⁺, I⁺, F⁺: X → [1, 0] and T⁻, I⁻, F⁻: X → [-1, 0]. The positive membership degree denotes the truth membership T⁺(x), indeterminate membership I⁺(x) and false membership F⁺(x) of an element x ∈ X corresponding to the set A and the negative membership degree denotes the truth membership T⁻(x),
indeterminate membership I(x) and false membership F(x) of an element x ∈ X to some implicit counter-property corresponding to a bipolar neutrosophic set.

2.13 Definition [25]: A Bipolar Single-Valued Neutrosophic set (BSVN) S in X is defined in the form of

BSVN (S) = \{v, (T_{BSVN}^{+}(1), T_{BSVN}^{-}(1), I_{BSVN}^{+}(1), I_{BSVN}^{-}(1), F_{BSVN}^{+}(1), F_{BSVN}^{-}(1)), v ∈ X \}

where \( T_{BSVN}^{+}(1), T_{BSVN}^{-}(1), I_{BSVN}^{+}(1), I_{BSVN}^{-}(1), F_{BSVN}^{+}(1), F_{BSVN}^{-}(1) : X → [0,1] \) and \( T_{BSVN}^{+}(1), T_{BSVN}^{-}(1), I_{BSVN}^{+}(1), I_{BSVN}^{-}(1), F_{BSVN}^{+}(1), F_{BSVN}^{-}(1) : X → [-1,0] \). In this definition, there T_{BSVN}^{+} and T_{BSVN}^{-} are acceptable and unacceptable in past. Similarly I_{BSVN}^{+} and I_{BSVN}^{-} are acceptable and unacceptable in future. F_{BSVN}^{+} and F_{BSVN}^{-} are acceptable and unacceptable in present respectively.

2.14 Definition [25]: Let two bipolar single-valued neutrosophic sets BSVN_1(S) and BSVN_2(S) in X defined as

BSVN_1(S) = \{v, (T_{BSVN_1}^{+}(1), T_{BSVN_1}^{-}(1), I_{BSVN_1}^{+}(1), I_{BSVN_1}^{-}(1), F_{BSVN_1}^{+}(1), F_{BSVN_1}^{-}(1)), v ∈ X \}

and

BSVN_2(S) = \{v, (T_{BSVN_2}^{+}(1), T_{BSVN_2}^{-}(1), I_{BSVN_2}^{+}(1), I_{BSVN_2}^{-}(1), F_{BSVN_2}^{+}(1), F_{BSVN_2}^{-}(1)), v ∈ X \}. Then the operators are defined as follows:

(i) Complement

BSVN_1(S)^c = \{v, (-T_{BSVN_1}^{+}(1), -I_{BSVN_1}^{+}(1), -I_{BSVN_1}^{-}(1), -I_{BSVN_1}^{+}(1), -F_{BSVN_1}^{+}(1), -F_{BSVN_1}^{-}(1)), v ∈ X \}

(ii) Union of two BSVN

BSVN_1(S) \cup BSVN_2(S) = \{max(T_{BSVN_1}^{+}(1), T_{BSVN_2}^{+}(1)), min(I_{BSVN_1}^{+}(1), I_{BSVN_2}^{+}(1)), min(F_{BSVN_1}^{+}(1), F_{BSVN_2}^{+}(1))\}

BSVN_1(S) \cup BSVN_2(S) = \{max(T_{BSVN_1}^{-}(1), T_{BSVN_2}^{-}(1)), min(I_{BSVN_1}^{-}(1), I_{BSVN_2}^{-}(1)), min(F_{BSVN_1}^{-}(1), F_{BSVN_2}^{-}(1))\}

(iii) Intersection of two BSVN

BSVN_1(S) \cap BSVN_2(S) = \{min(T_{BSVN_1}^{+}(1), T_{BSVN_2}^{+}(1)), max(I_{BSVN_1}^{+}(1), I_{BSVN_2}^{+}(1)), max(F_{BSVN_1}^{+}(1), F_{BSVN_2}^{+}(1))\}

BSVN_1(S) \cap BSVN_2(S) = \{min(T_{BSVN_1}^{-}(1), T_{BSVN_2}^{-}(1)), max(I_{BSVN_1}^{-}(1), I_{BSVN_2}^{-}(1)), max(F_{BSVN_1}^{-}(1), F_{BSVN_2}^{-}(1))\}

2.15 Definition [25]: Let two bipolar single-valued neutrosophic sets be BSVN_1 and BSVN_2 in X defined as

BSVN_1(S) = \{v, (T_{BSVN_1}(1), I_{BSVN_1}(1), F_{BSVN_1}(1)), v ∈ X \}

and

BSVN_2(S) = \{v, (T_{BSVN_2}(1), I_{BSVN_2}(1), F_{BSVN_2}(1)), v ∈ X \}. Then BSVN_1(S) ⊆ BSVN_2(S) if and only if

T_{BSVN_1}(1) ≤ T_{BSVN_2}(1), I_{BSVN_1}(1) ≥ I_{BSVN_2}(1), F_{BSVN_1}(1) ≥ F_{BSVN_2}(1)

for all v ∈ X.

BSVN_2(S) ⊆ BSVN_1(S) if and only if

T_{BSVN_2}(1) ≤ T_{BSVN_1}(1), I_{BSVN_2}(1) ≥ I_{BSVN_1}(1), F_{BSVN_2}(1) ≥ F_{BSVN_1}(1)

for all v ∈ X.

2.16 Definition [25]: A bipolar single-valued neutrosophic topology (BSVNT) on a non-empty set X is a τ of BSVN sets satisfying the axioms

(i) \( 0_{BSVN}, 1_{BSVN} ∈ τ \)

(ii) \( S \cap S \in τ \) for any \( S \subseteq X \)

(iii) \( S \subseteq τ \) for any arbitrary family \( \{S_i : i ∈ J \} \in τ \)

The pair \((X, τ)\) is called BSVN topological space (BSVNTS). Any BSVN set in τ is called as BSVN open set (BSVNOS) in X. The complement \( S^c \) of BSVN set in BSVN topological space \((X, τ)\) is called a BSVN closed set (BSVNCs).
2.17 Definition [25]: Let \((X, \tau)\) be a BSVN topological space (BSVNTS) and BSVN \((S) = <\infty, (T_{BSVN}, I_{BSVN}, F_{BSVN}), (I_{BSVN}, I_{BSVN}, F_{BSVN}) : v \in X>\) be a BSVN set in \(X\). Then the closure and interior of \(S\) is defined as
\[
\text{Int}(S) = U \{F : \text{F is a BSVN open set (BSVNOs)} \text{ in } X \text{ and } F \subseteq S\}
\]
\[
\text{Cl}(S) = \cap \{F : \text{F is a BSVN closed set (BSVNCs)} \text{ in } X \text{ and } S \subseteq F\}.
\]
Here \(\text{cl}(S)\) is a BSVNCs and \(\text{Int}(S)\) is a BSVNOs in \(X\).

2.18 Proposition [25]: Let BSVNTS of \((X, \tau)\) and \(S,T\) be BSVNs' s in \(X\). Then the properties hold:

1. \(\text{Int}(S) \subseteq S\) and \(S \subseteq \text{Cl}(S)\)
2. \(S \subseteq T \Rightarrow \text{Int}(S) \subseteq \text{Int}(T)\)
3. \(\text{Int}(\text{Int}(S)) = \text{Int}(S)\)
4. \(\text{Int}(S \cap T) = \text{Int}(S) \cap \text{Int}(T)\)
5. \(\text{Int}((1_{BSVN}) = \text{Int}(0_{BSVN}) = S\)


For our convenience, we take \((I)\) as \(S = \{<x, (T_{BSVN}, I, F) > : x \in X\}\).

3.1 Definition: A BSVNs \(S\) of a BSVNTS \((X, \tau)\) is said to be bipolar single-valued neutrosophic generalized closed set (BSVNGCs) if BSVN \(\text{cl}(S)\) is a BSVNCs in \(X\) and \(\text{Int}(S)\) is a BSVNOs in \(X\).

3.2 Definition: Let \(0_{BSVN}\) and \(1_{BSVN}\) be BSVNS in \(X\) defined as \(0_{BSVN} = \{0, 0, 0, 0, 0, 0, 0, 0\} < x \in X >\) is said to be Null or Empty bipolar single-valued neutrosophic set.

3.3 Example: Let \(X = \{p, q\}\) and
\[
S = \{< p, (0, 0.5, 0.1, -0.2, -0.4, -0.3) >, < q, (0, 2, 1, 0, 1, 0, 1, 0, 1, 2, 1, 0, 2, 0, 2) > \}
\]
\[
T = \{< p, (0, 0.4, 0.4, 0.1, -0.1, -0.5, -0.4) >, < q, (0, 3, 0.7, 0.1, -0.3, -0.6, -0.9) > \}
\]

Then \(\tau = \{0_{BSVN}, 1_{BSVN}, S, T\}\) is a BSVNT on \(X\). The BSVNs
\[
R = \{< p, (0, 0.2, 0.3, 0.7, -0.8, -0.1, -0.3) >, < q, (0, 3, 0.8, 0.3, -0.4, -0.1, -0.4) > \}
\]
is BSVNGCs in \(X\).

3.4 Definition: A BSVNs \(S = \{<x, (T_{BSVN}, I_{BSVN}, F_{BSVN}), (I_{BSVN}, I_{BSVN}, F_{BSVN}) : x \in X>\} \text{ in BSVNTS }\)

\((X, \tau)\) is said to be

1. Bipolar single-valued neutrosophic semi closed set (BSVNSScs) if BSVN \(\text{Int}(S) \subseteq S\),
2. Bipolar single-valued neutrosophic semi open set (BSVNSOs) if \(S \subseteq \text{BSVNC}(\text{BSVN Int}(S))\),
(3) Bipolar single-valued neutrosophic pre-closed set (BSVNPCs) if $\text{BSVN cl} (\text{BSVN int}(S)) \subseteq S$,
(4) Bipolar single-valued neutrosophic pre-open set (BSVNPOs) if $S \subseteq \text{BSVN int}(\text{BSVN cl}(S))$,
(5) Bipolar single-valued neutrosophic $\alpha$-closed set (BSVN $\alpha$Cs) if $\text{BSVN cl}(\text{BSVN int}(\text{BSVN cl}(S))) \subseteq S$,
(6) Bipolar single-valued neutrosophic $\alpha$-open set (BSVN $\alpha$Os) if $S \subseteq \text{BSVN int}(\text{BSVN cl}(\text{BSVN cl}(S)))$,
(7) Bipolar single-valued neutrosophic semi pre-closed set (BSVNSPCs) if $\text{BSVN int}(\text{BSVN cl}(\text{BSVN int}(S))) \subseteq S$,
(8) Bipolar single-valued neutrosophic semi pre-open set (BSVNSPOs) if $S \subseteq \text{BSVN cl}(\text{BSVN int}(\text{BSVN cl}(S)))$,
(9) Bipolar single-valued neutrosophic regular open set (BSVNROs) if $S = \text{BSVN int}(\text{BSVN cl}(S))$,
(10) Bipolar single-valued neutrosophic regular closed set (BSVNRCs) if $S = \text{BSVN cl}(\text{BSVN int}(S))$.

3.5 Definition: Let $(X, \tau)$ be BSVNTS and $S$ be BSVNs in $X$. Then the bipolar single-valued neutrosophic generalized interior and bipolar single-valued neutrosophic generalized closure are denoted by

1. $\text{BSVNG int}(S) = \bigcup \{G / G \text{ is a BSVNGOs in } X \text{ and } G \subseteq S\}$
2. $\text{BSVNG cl}(S) = \bigcap \{K / K \text{ is a BSVNGCs in } X \text{ and } S \subseteq K\}$

3.6 Definition: Let $(X, \tau)$ be any BSVNTS and let $S$ and $T$ be BSVNs in $X$. Then the bipolar single-valued neutrosophic generalized closure operator satisfies the properties:

1. $S \subseteq \text{BSVN cl}(S)$
2. $\text{BSVN int}(S) \subseteq S$
3. $S \subseteq T \Rightarrow \text{BSVN cl}(S) \subseteq \text{BSVN cl}(T)$
4. $S \subseteq T \Rightarrow \text{BSVN int}(S) \subseteq \text{BSVN int}(T)$
5. $\text{BSVN cl}(S \cup T) = \text{BSVN cl}(S) \cup \text{BSVN cl}(T)$
6. $\text{BSVN int}(S \cap T) = \text{BSVN int}(S) \cap \text{BSVN int}(T)$
7. $(\text{BSVN cl}(S))^{c} = \text{BSVN int}(S^{c})$
8. $(\text{BSVN cl}(S))^{c} = \text{BSVN int}(S^{c})$

Proof:

1. $\text{BSVN cl}(S) = \bigcap \{K / K \text{ is a BSVNGCs in } X \text{ and } S \subseteq K\}. \text{ Thus } S \subseteq \text{BSVN cl}(S)$.
2. $\text{BSVNG int}(S) = \bigcup \{G / G \text{ is a BSVNGOs in } X \text{ and } G \subseteq S\}. \text{ Thus } \text{BSVN int}(S) \subseteq S$.
3. $\text{BSVN cl}(T) = \bigcap \{K / K \text{ is a BSVNGCs in } X \text{ and } T \subseteq K\},$
   $\quad \supseteq \bigcap \{K / K \text{ is a BSVNGCs in } X \text{ and } S \subseteq K\},$
   $\quad \supseteq \text{BSVN cl}(S). \text{ Thus } \text{BSVN cl}(S) \subseteq \text{BSVN cl}(T)$.
4. $\text{BSVN int}(T) = \bigcup \{G / G \text{ is a BSVNGOs in } X \text{ and } G \subseteq T\},$
   $\quad \supseteq \bigcup \{G / G \text{ is a BSVNGOs in } X \text{ and } G \subseteq S\},$
   $\quad \supseteq \text{BSVN int}(S). \text{ Thus } \text{BSVN int}(S) \subseteq \text{BSVN int}(T)$.
5. $\text{BSVN cl}(S \cup T) = \bigcap \{K / K \text{ is a BSVNGCs in } X \text{ and } S \cup T \subseteq K\},$
   $\quad (\bigcap \{K / K \text{ is a BSVNGCs in } X \text{ and } S \subseteq K\}) \cup (\bigcap \{K / K \text{ is a BSVNGCs in } X \text{ and } T \subseteq K\}),$
   $\quad = \text{BSVN cl}(S) \cup \text{BSVN cl}(T). \text{ Thus } \text{BSVN cl}(S \cup T) = \text{BSVN cl}(S) \cup \text{BSVN cl}(T)$.
6. \( \text{BSVN \, int} (S \cap T) = \bigcup \{ G \mid G \text{ is a BSVNGOs in } X \text{ and } G \subseteq S \cap T \}, \)
   \( (\bigcup \{ G \mid G \text{ is a BSVNGOs in } X \text{ and } G \subseteq S \}) \cap (\bigcup \{ G \mid G \text{ is a BSVNGOs in } X \text{ and } G \subseteq T \}) = \text{BSVN \, int}(S) \cap \text{BSVN \, int}(S). \)
   Thus BSVN \, int\((S \cap T) = \text{BSVN \, int}(S) \cap \text{BSVN \, int}(S). \)

7. (BSVN \, cl(S)) = \bigcap \{ K \mid K \text{ is a BSVNGCs in } X \text{ and } S \subseteq K \},
   (BSVN \, cl(S))^c = \bigcap \{ K^c \mid K^c \text{ is a BSVNGCs in } X \text{ and } S^c \subseteq K \},
   = \text{BSVN \, int}(S)^c. \text{ Thus } (BSVN \, cl(S))^c = \text{BSVN \, int}(S^c). \)

8. BSVNG \, int (S) = \bigcup \{ G \mid G \text{ is a BSVNGOs in } X \text{ and } G \subseteq S \},

9. (BSVN \, cl(S))^c = \bigcap \{ G \mid G \text{ is a BSVNGOs in } X \text{ and } G^c \subseteq S \} = \text{BSVN \, int}(S^c)
   Thus (BSVN \, cl(S))^c = \text{BSVN \, int}(S^c). \)

3.6 Definition: Let \((X, \tau)\) be a BSVNTS and \(S\) be a BSVNs in \(X\). The bipolar single-valued neutrosophic pre interior of \(S\) and denoted by BSVN \, pint (S) and bipolar single-valued neutrosophic pre-closure of \(S\) is denoted by BSVN \, pcl (S).
   (1) BSVN \, pint (S) = \bigcup \{ G \mid G \text{ is a BSVNPOs in } X \text{ and } G \subseteq S \}
   (2) BSVN \, pcl (S) = \bigcap \{ K \mid K \text{ is a BSVNPCs in } X \text{ and } S \subseteq K \}

3.7 Result 3.21: Let \(A\) be BSVNs of a BSVNTS \((X, \tau)\), then
   (1). BSVN \, pcl (S) = S \bigcup \text{BSVN \, cl } (\text{BSVN \, int} (S)),
   (2). BSVN \, pint (S) = S \bigcap \text{BSVN \, int} (\text{BSVN \, cl} (S)).

3.8 Definition: Let \(S\) be BSVNs of a BSVNTS \((X, \tau)\). Then the bipolar single-valued neutrosophic semi interior of \(S\) (BSVN \, sint \((S))\) and bipolar single-valued neutrosophic semi closure of \(S\) (BSVN \, scl \((S))\) are defined by
   (1) BSVN \, sint (S) = \bigcup \{ G \mid G \text{ is a BSVNSOs in } X \text{ and } G \subseteq S \}
   (2) BSVN \, scl (S) = \bigcap \{ K \mid K \text{ is a BSVNSCs in } X \text{ and } S \subseteq K \}

3.9 Result: Let \(S\) be BSVNs of a BSVNTS \((X, \tau)\), then
   (1) BSVN \, scl (S) = S \bigcup \text{BSVN \, int } (\text{BSVN \, cl} (BSVN \, int (S))),
   (2). BSVN \, sint (S) = S \bigcap \text{BSVN \, cl} (\text{BSVN \, int} (S)).

3.10 Definition: Let \(S\) be BSVNs of a BSVNTS \((X, \tau)\). Then the bipolar single-valued neutrosophic alpha interior of \(S\) (BSVN \, aint (S)) and bipolar single-valued neutrosophic alpha closure of \(S\) (BSVN \, acl \((S))\) is defined by
   (1) BSVN \, aint (S) = \bigcup \{ G \mid G \text{ is a BSVN\,aOs in } X \text{ and } G \subseteq S \}
   (2) BSVN \, acl (S) = \bigcap \{ K \mid K \text{ is a BSVN\,aCs in } X \text{ and } S \subseteq K \}

3.11 Result: Let \(S\) be BSVNs of a BSVNTS \((X, \tau)\), then
   (1) BSVN \, acl (S) = S \bigcup \text{BSVN \, cl } (\text{BSVN \, cl } (BSVN \, int (BSVN \, cl \,(S)))),
   (2) BSVN \, aint (S) = S \bigcap \text{BSVN \, int } (\text{BSVN \, cl } (BSVN \, cl \,(BSVN \, int \,(S)))).
3.12 Definition: Let $A$ be BSVNs of a BSVNTS $(X, \tau)$. Then the bipolar single-valued neutrosophic semi-pre interior of $S$ (BSVN spint $(S)$) and bipolar single-valued neutrosophic semi-pre closure of $S$ (BSVN spcl $(S)$) are defined by

1. $\text{BSVN spint } (S) = \bigcup \{G / G \text{ is a BSVNSPOs in } X \text{ and } G \subseteq S\}$
2. $\text{BSVN spcl } (S) = \bigcap \{K / K \text{ is a BSVNSPCs in } X \text{ and } S \subseteq K\}$

3.13 Definition: A BSVNs $S$ of a BSVNTS $(X, \tau)$ is said to be bipolar single-valued neutrosophic generalized semi closed set (BSVNGSCS) if $\text{BSVN scl } (S) \subseteq U$ whenever $S \subseteq U$ and $U$ is BSVNOs in $X$.

3.14 Definition: A BSVNs $S$ of a BSVNTS $(X, \tau)$ is said to be bipolar single-valued neutrosophic alpha generalized closed set (BSVN $\alpha$GCS) if $\text{BSVN cl } (S) \subseteq U$ whenever $S \subseteq U$ and $U$ is BSVNOs in $X$.

3.15 Definition: A BSVNs $S$ of a BSVNTS $(X, \tau)$ is said to be bipolar single-valued neutrosophic generalized semi-pre closed set (BSVNGSPCs) if $\text{BSVN spcl } (S) \subseteq U$ whenever $S \subseteq U$ and $U$ is BSVNOs in $X$.

3.16 Definition: Let $\{A_i : i \in J\}$ be an arbitrary family of BSVNs in $X$. Then

1. $\bigcap S_i = \{<x, \min (T_{S_i}^+(x)), \max (I_{S_i}^+(x)), \max (F_{S_i}^+(x)), \min (T_{S_i}^-(x)), \max (I_{S_i}^-(x)), \max (F_{S_i}^-(x))>\}$
2. $\bigcup A_i = \{<x, \max (T_{S_i}^+(x)), \min (I_{S_i}^+(x)), \min (F_{S_i}^+(x)), \max (T_{S_i}^-(x)), \min (I_{S_i}^-(x)), \min (F_{S_i}^-(x))>\}$

3.18 Corollary: Let $S, T, M$ and $N$ be bipolar single-valued neutrosophic set in $X$. Then

1. $S \subseteq T$ and $M \subseteq N \Rightarrow S \cup M \subseteq T \cup N$ and $S \cap M \subseteq T \cap N$
2. $S \subseteq T$ and $S \subseteq M \Rightarrow S \subseteq T \cap M$
3. $S \subseteq M$ and $T \subseteq M \Rightarrow S \cup T \subseteq M$
4. $S \subseteq T$ and $T \subseteq M \Rightarrow S \subseteq M$
5. $(S \cup T)^c = S^c \cap T^c$
6. $(S \cap T)^c = S^c \cup T^c$
7. $S \subseteq T \Rightarrow T^c \subseteq S^c$
8. $(S^c)^c = S$
9. $0^c_{BSVN} = 1_{BSVN}$
10. $1^c_{BSVN} = 0_{BSVN}$
Proof: The proof is obvious.

3.19 Theorem: Every bipolar single-valued neutrosophic closed set is bipolar single-valued neutrosophic generalized closed set.

Proof. Let S be BSVNCs in X. Suppose U is BSVNOs in X, such that \( S \subseteq U \). Then \( \text{BSVN cl}(S) = S \subseteq U \). Hence S is BSVNGCs in X.

3.20 Remark: The converse of the above theorem is not true which is shown in the example.

3.21 Example: Let \( X = \{p, q\} \) and \( S = \{< p, (0.3,0.5,0.1,-0.2,-0.4,-0.3) >, < q, (0.2,0.8,0.2,-0.4,-0.6,-0.9) >\} \) and \( T = \{< p, (0.4,0.4,0.1,-0.1,-0.5,-0.4) >, < q, (0.3,0.7,0.1,-0.3,-0.6,-0.9) >\} \). Then \( \tau = \{0_{BSVN}, 1_{BSVN}, S, T\} \) is a BSVNT on X. The BSVNs \( R = \{< p, (0.2,0.3,0.7,-0.8,-0.1,-0.3) >, < q, (0.3,0.8,0.3,-0.4,-0.1,-0.4) >\} \) is BSVNGCs in X but not BSVNCs in X.

3.22 Remark: The Intersection of two BSVNGCs is need not be true. Shown in the following example.

3.23 Example: Let \( X = \{p, q\} \) and \( S = \{< p, (0.3,0.5,0.1,-0.2,-0.4,-0.3) >, < q, (0.2,0.8,0.2,-0.4,-0.6,-0.9) >\} \) and \( T = \{< p, (0.4,0.4,0.1,-0.1,-0.5,-0.4) >, < q, (0.3,0.7,0.1,-0.3,-0.6,-0.9) >\} \). Then \( \tau = \{0_{BSVN}, 1_{BSVN}, S, T\} \) is a BSVNT on X. The BSVNs \( R = \{< p, (0.2,0.3,0.7,-0.8,-0.1,-0.3) >, < q, (0.3,0.8,0.3,-0.4,-0.1,-0.4) >\} \) and \( V = \{< p, (0.6,0.6,0.9,-0.9,-0.5,-0.6) >, < q, (0.7,0.3,0.9,-0.7,-0.1,-0.1) >\} \) are BSVNGCs in X but \( R \cap V \) is not BSVNGCs in X.

3.24 Proposition: Let \( (X, \tau) \) be BSVNTS. If S is a bipolar single-valued neutrosophic generalized closed set and \( S \subseteq T \subseteq \text{BSVN cl}(S) \) then T is bipolar single-valued neutrosophic generalized closed set.

Proof: Let G be a bipolar single-valued neutrosophic open set in \( (X, \tau) \), such that \( T \subseteq G \). Since \( S \subseteq T \), \( S \subseteq G \). Now S is a bipolar single-valued neutrosophic generalized closed set and \( \text{BSVN cl}(S) \subseteq G \). But \( \text{BSVN cl}(T) \subseteq \text{BSVN cl}(S) \subseteq G \). \( \text{BSVN cl}(T) \subseteq G \). Hence T is a bipolar single-valued neutrosophic generalized closed set.

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3.25 Proposition: Let $(X, \tau)$ be BSVNTS and a BSVNs $S$ is a bipolar single-valued neutrosophic generalized open if and only if $T \subseteq BSVN \text{ int}(S)$ whenever $T$ is bipolar single-valued neutrosophic closed set and $T \subseteq S$.

Proof: Let $S$ be a bipolar single-valued neutrosophic generalized open set and $T$ be a bipolar single-valued neutrosophic closed set such that $T \subseteq S$. Now $T \subseteq S \Rightarrow S' \subseteq T'$ and $S'$ is a bipolar single-valued neutrosophic generalized closed set implies that $BSVN \text{ cl}(S') \subseteq T'$. (i.e) $T = (T')' \subseteq (BSVN \text{ cl}(S'))'$. But $(BSVN \text{ cl}(S'))' = BSVN \text{ int}(S)$. Thus $T \subseteq BSVN \text{ int}(S)$.

Conversely, suppose that $S$ be a bipolar single-valued neutrosophic set, such that $T \subseteq BSVN \text{ int}(S)$ whenever $T$ is bipolar single-valued neutrosophic closed and $T \subseteq S$. Let $S' \subseteq T$ whenever $T$ is bipolar single-valued neutrosophic open. Now $S' \subseteq T \Rightarrow T' \subseteq S$. Hence by the assumption, $T' \subseteq BSVN \text{ int}(S)$. (i.e) $(BSVN \text{ int}(S))' \subseteq T$. But $(BSVN \text{ int}(S))' = BSVN \text{ cl}(S')$. Hence $(BSVN \text{ int}(S))' \subseteq BSVN \text{ cl}(S')$ . (i.e) $S'$ is bipolar single-valued neutrosophic generalized closed set. Therefore, $S$ is bipolar single-valued neutrosophic generalized open set. Hence proved.

3.26 Proposition: If $BSVN \text{ int}(S) \subseteq T \subseteq S$ and if $S$ is bipolar single-valued neutrosophic generalized open set then $T$ is also bipolar single-valued neutrosophic generalized open set.

Proof: Now $S' \subseteq T' \subseteq (BSVN \text{ int}(S))' = BSVN \text{ cl}(S')$. As $S$ is a bipolar single-valued neutrosophic generalized open, $S'$ is bipolar single-valued neutrosophic generalized closed set. Then by the proposition 3.24, $T$ is bipolar single-valued neutrosophic generalized open set. Hence Proved.

4. Bipolar Single-Valued Neutrosophic Generalized Pre-Closed Set

4.1 Definition: A BSVNs $S$ is said to be bipolar single-valued neutrosophic generalized pre-closed set (BSVNGPCs) in $(X, \tau)$ if $BSVN \text{ pcl}(S) \subseteq U$ whenever $S \subseteq U$ and $U$ is BSVNOs in $X$. The family of all BSVNGPC's of a BSVNTS $(X, \tau)$ is denoted by BSVNGPC $(X)$.

4.2 Example: Let $X = \{p, q\}$ and

$$S = \left\{ < p, (0.1,-0.7),(0.3,-0.8),(0.5, -0.1) >, < q, (0.2,0.4,0.6,-0.8,-0.2,-0.4) > \right\} \quad \text{and} \quad T = \left\{ < p, (0.1,0.2,0.3,-0.7,-0.9,-0.9) >, < q, (0.4,0.3,0.6,-0.1,-0.3,-0.5) > \right\}$$

Then $\tau = \{0_{BSVN}, 1_{BSVN}, S, T\}$ is a BSVN on $X$. The BSVNs

$$R = \left\{ < p, (0.8,0.7,0.8,-0.7,-0.2,-0.3) >, < q, (0.1,0.7,0.9,-0.2,-0.7,-0.2) > \right\}$$

is BSVNGPCs in $X$.

4.3 Theorem:

1. Every bipolar single-valued neutrosophic closed set is bipolar single-valued neutrosophic generalized pre-closed set.
2. Every bipolar single-valued neutrosophic generalized closed set is bipolar single-valued neutrosophic generalized pre-closed set.
(3) Every bipolar single-valued neutrosophic $\alpha$ closed set is bipolar single-valued neutrosophic generalized pre-closed set.

(4) Every bipolar single-valued neutrosophic regular closed set is bipolar single-valued neutrosophic generalized pre-closed set.

(5) Every bipolar single-valued neutrosophic pre-closed set is bipolar single-valued neutrosophic generalized pre-closed set.

(6) Every bipolar single-valued neutrosophic $\alpha$ generalized closed set is bipolar single-valued neutrosophic generalized pre-closed set.

(7) Every bipolar single-valued neutrosophic generalized pre-closed set is bipolar single-valued neutrosophic semi-pre closed set.

(8) Every bipolar single-valued neutrosophic generalized pre-closed set is bipolar single-valued neutrosophic generalized semi-pre closed set.

Proof. (1) Let $S$ be BSVNCs in $X$ and let $S \subseteq U$ and $U$ be BSVNOs in $X$. Since $BSVN \text{ pcl}(S) \subseteq BSVN \text{ cl}(S)$ and $S$ is BSVNCs in $X$, $BSVN \text{ pcl}(S) \subseteq BSVN \text{ cl}(S) = S \subseteq U$. Therefore $S$ is BSVNGPCs in $X$.

(2) Let $S$ be BSVNGCs in $X$ and let $S \subseteq U$ and $U$ is BSVNOs in $(X, \tau)$. Since $BSVN \text{ pcl}(S) \subseteq BSVN \text{ cl}(S)$ and by hypothesis, $BSVN \text{ pcl}(S) \subseteq U$. Therefore $S$ is BSVNGPCs in $X$.

(3) Let $S$ be BSVN $\alpha$CS in $X$ and let $S \subseteq U$ and $U$ be BSVNOs in $X$. By hypothesis, $BSVN \text{ cl}(BSVN \text{ int}(BSVN \text{ cl}(S))) \subseteq S$. Since $S \subseteq BSVN \text{ cl}(S)$;

$BSVN \text{ cl}(BSVN \text{ int}(S)) \subseteq BSVN \text{ cl}(BSVN \text{ int}(BSVN \text{ cl}(S))) \subseteq S$. Hence $BSVN \text{ pcl}(S) \subseteq S \subseteq U$. Therefore $S$ is BSVNGPCs in $X$.

(4) Let $S$ be a BSVNRCs in $X$. By Definition $S = BSVN \text{ cl}(BSVN \text{ int}(S))$. This implies $BSVN \text{ cl}(S) = BSVN \text{ cl}(BSVN \text{ int}(S))$. Therefore $BSVN \text{ cl}(S) = S$. (i.e) $S$ is BSVNCs in $X$. $S$ is BSVNGPCs in $X$.

(5) Let $S$ be BSVNPCs in $X$ and let $S \subseteq U$ and $U$ is BSVNOs in $X$. By Definition, $BSVN \text{ cl}(BSVN \text{ int}(S)) \subseteq S$. This implies $BSVN \text{ pcl}(S) = S \cup BSVN \text{ cl}(BSVN \text{ int}(S)) \subseteq S$. Therefore $BSVN \text{ pcl}(S) \subseteq U$. Hence $S$ is BSVNGPCs in $X$.

(6) Let $S$ be BSVN $\alpha$GCs in $X$ and let $S \subseteq U$ and $U$ is BSVNOs in $(X, \tau)$. By Result 3.11, $S \cup BSVN \text{ cl}(BSVN \text{ int}(BSVN \text{ cl}(S))) \subseteq U$. This implies $BSVN \text{ cl}(BSVN \text{ int}(BSVN \text{ cl}(S))) \subseteq U$ and $BSVN \text{ cl}(BSVN \text{ int}(S)) \subseteq U$. Thus $BSVN \text{ pcl}(S) = S \cup BSVN \text{ cl}(BSVN \text{ int}(S)) \subseteq U$. Hence $S$ is BSVNGPCs in $X$.

(7) Let $S$ be BSVNGPCs in $X$, this implies $BSVN \text{ pcl}(S) \subseteq U$ whenever $S \subseteq U$ and $U$ is BSVNOs in $X$. By hypothesis $BSVN \text{ cl}(BSVN \text{ int}(S)) \subseteq S$. Therefore $BSVN \text{ int}(BSVN \text{ cl}(BSVN \text{ int}(S))) \subseteq BSVN \text{ int}(S) \subseteq S$. Therefore $BSVN \text{ int}(BSVN \text{ cl}(BSVN \text{ int}(S))) \subseteq S$. Hence $S$ is BSVNSPCs in $X$. 

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(8) Let s be BSVN\textsuperscript{GPc}s in X and let S \subseteq U and U is BSVN\textsuperscript{O}s in X. By hypothesis BSVN cl (BSVN int (S)) \subseteq S \subseteq U. Therefore BSVN int (BSVN cl (BSVN int (S))) \subseteq BSVN int (S) \subseteq U. This implies BSVN spcl (S) \subseteq U whenever S \subseteq U and U is BSVN\textsuperscript{O}s in X. Therefore S is BSVN\textsuperscript{GPc}s in X.

4.4 Remark: The converse of the above theorem 4.3 (1-8) is not true which is shown in the example.

4.5 Example:

(1) Let X = [p, q] and

\[
S = \begin{Bmatrix} < p, (0.1,0.3,0.5; -0.7, -0.8, -0.1) > \end{Bmatrix} \quad T = \begin{Bmatrix} < p, (0.1,0.2,0.3; -0.7, -0.9, -0.9) > \end{Bmatrix}
\]

Then \( \tau = \{0_{\text{BSVN}}, 1_{\text{BSVN}}, S, T\} \) is a BSVNT on X. The BSVNs

\[
R = \begin{Bmatrix} < p, (0.8,0.7,0.8; -0.7, -0.2, -0.3) > \end{Bmatrix} \quad \text{is BSVN\textsuperscript{GPc}s in X but not BSVN\textsuperscript{C}s in X.}
\]

(2) Let X = [p, q] and

\[
S = \begin{Bmatrix} < p, (0.1,0.3,0.2; -0.3, -0.4, -0.6) > \end{Bmatrix} \quad T = \begin{Bmatrix} < p, (0.1,0.3,0.4; -0.4, -0.1, -0.4) > \end{Bmatrix}
\]

Then \( \tau = \{0_{\text{BSVN}}, 1_{\text{BSVN}}, S, T\} \) is a BSVNT on X. The BSVNs

\[
R = \begin{Bmatrix} < p, (0.1,0.6,0.5; -0.2, -0.1, -0.3) > \end{Bmatrix} \quad \text{is BSVN\textsuperscript{GPc}s in X but not BSVN\textsuperscript{C}s in X.}
\]

(3) Let X = [p, q] and

\[
S = \begin{Bmatrix} < p, (0.5,0.4,0.1; -0.6, -0.5, -0.4) > \end{Bmatrix} \quad T = \begin{Bmatrix} < p, (0.4,0.5,0.3; -0.6, -0.3, -0.1) > \end{Bmatrix}
\]

Then \( \tau = \{0_{\text{BSVN}}, 1_{\text{BSVN}}, S, T\} \) is a BSVNT on X. The BSVNs

\[
R = \begin{Bmatrix} < p, (0.5,0.9,0.2; -0.3, -0.2, -0.1) > \end{Bmatrix} \quad \text{is BSVN\textsuperscript{GPc}s in X but not BSVN\textsuperscript{C}s in X.}
\]

(4) Let X = [p, q] and

\[
S = \begin{Bmatrix} < p, (0.1,0.3,0.2; -0.3, -0.4, -0.6) > \end{Bmatrix} \quad T = \begin{Bmatrix} < p, (0.1,0.3,0.4; -0.4, -0.1, -0.4) > \end{Bmatrix}
\]

\[
R = \begin{Bmatrix} < p, (0.2,0.4,0.5; -0.1, -0.1, -0.3) > \end{Bmatrix} \quad \text{is BSVN\textsuperscript{GPc}s in X but not BSVN\textsuperscript{C}s in X.}
\]

\[
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\]
Then $\tau = \{0_{BSVN}, 1_{BSVN}, S, T\}$ is a BSVNT on $X$. The BSVNs
\[
\begin{align*}
R &= \{< p, (0.1,0.5,0.4,0.5,0.3,0.1), > < q, (0.1,0.6,0.5,0.2,0.1,0.3), > \}
\end{align*}
\]
is BSVNGPCs in $X$ but not BSVNRCs in $X$.

(5) Let $X = [p, q]$ and
\[
\begin{align*}
S &= \{< p, (0.5,0.4,0.3,0.6,0.4,0.2), > < q, (0.2,0.5,0.1,0.5,0.3,0.1), > \}
T &= \{< p, (0.6,0.2,0.1,0.5,0.6,0.8), > < q, (0.3,0.1,0.1,0.4,0.4,0.3), > \}
\end{align*}
\]
Then $\tau = \{0_{BSVN}, 1_{BSVN}, S, T\}$ is a BSVNT on $X$. The BSVNs
\[
\begin{align*}
R &= \{< p, (0.5,0.3,0.2,0.1,0.7,0.3), > < q, (0.2,0.4,0.1,0.3,0.4,0.1), > \}
\end{align*}
\]
is BSVNGPCs in $X$ but not BSVNPCs in $X$.

(6) Let $X = [p, q]$ and
\[
\begin{align*}
S &= \{< p, (0.1,0.3,0.6,0.2,0.4,0.5), > < q, (0.2,0.4,0.5,0.1,0.9,0.5), > \}
T &= \{< p, (0.1,0.3,0.6,0.7,0.3,0.2), > < q, (0.2,0.6,0.7,0.8,0.4,0.1), > \}
\end{align*}
\]
Then $\tau = \{0_{BSVN}, 1_{BSVN}, S, T\}$ is a BSVNT on $X$. The BSVNs
\[
\begin{align*}
R &= \{< p, (0.1,0.4,0.7,0.8,0.2,0.1), > < q, (0.2,0.7,0.7,0.9,0.1,0.1), > \}
\end{align*}
\]
is BSVNGPCs in $X$ but not BSVN $\alpha$GCs in $X$.

(7) Let $X = [p, q]$ and
\[
\begin{align*}
S &= \{< p, (0.1,0.5,0.5,0.2,0.4), > < q, (0.3,0.5,0.6,0.7,0.1,0.2), > \}
T &= \{< p, (0.3,0.2,0.3,0.2,0.3,0.5), > < q, (0.4,0.3,0.1,0.1,0.4,0.5), > \}
\end{align*}
\]
Then $\tau = \{0_{BSVN}, 1_{BSVN}, S, T\}$ is a BSVNT on $X$. The BSVNs
\[
\begin{align*}
R &= \{< p, (0.1,0.3,0.5,0.4,0.1,0.5), > < q, (0.4,0.3,0.1,0.1,0.2,0.3), > \}
\end{align*}
\]
is BSVNSPCs in $X$ but not BSVNGPCs in $X$.

(8) Let $X = [p, q]$ and
\[
\begin{align*}
S &= \{< p, (0.4,0.7,0.4,0.5,0.4,0.2), > < q, (0.3,0.2,0.4,0.3,0.1,0.1), > \}
T &= \{< p, (0.3,0.8,0.8,0.7,0.3,0.1), > < q, (0.2,0.3,0.7,0.4,0.1,0.1), > \}
\end{align*}
\]
Then $\tau = \{0_{BSVN}, 1_{BSVN}, S, T\}$ is a BSVNT on $X$. The BSVNs
4.6 Proposition: BSVNSCs and BSVNGPCs are independent to each other which are shown in the example.

4.7 Example: Let $X=\{p, q\}$ and

$\mathbb{S}=\{<p, (0.3,0.8,0.5,0.6,0.3,0.2)> <q, (0.2,0.3,0.7,0.3,0.1,0.1)>\}$

is BSVNGPCs in $X$ but not BSVNGPCs in $X.$

4.8 Example: Let $X=\{p, q\}$ and

$\mathbb{S}=\{<p, (0.5,0.4,0.2,0.1,0.2,0.7)> <q, (0.7,0.6,0.3,0.6,0.1,0.5)>\}$

Then $\tau=\{0_{\text{BSVN}}, 1_{\text{BSVN}}, S, T\}$ is a BSVNT on $X.$ The BSVNs

$\mathbb{R}=\{<p, (0.2,0.5,0.3,0.1,0.1,0.7)> <q, (0.6,0.7,0.4,0.7,0.1,0.2)>\}$

is BSVNGPCs in $X$ but not BSVNSCs in $X.$

4.9 Proposition: BSVNGSCs and BSVNGPCs are independent to each other which are shown in the example.

4.10 Example: Let $X=\{p, q\}$ and

$\mathbb{S}=\{<p, (0.1,0.7,0.6,0.8,0.2,0.5)> <q, (0.3,0.7,0.7,0.8,0.2,0.2)>\}$

Then $\tau=\{0_{\text{BSVN}}, 1_{\text{BSVN}}, S, T\}$ is a BSVNT on $X.$ The BSVNs

$\mathbb{R}=\{<p, (0.2,0.6,0.5,0.7,0.3,0.5)> <q, (0.3,0.7,0.7,0.8,0.2,0.2)>\}$

is BSVNSCs in $X$ but not BSVNGPCs in $X.$
4.11 Example: Let \( X = \{p, q\} \) and
\[
S = \left\{ < p, (0.1, 0.6, 0.9, -0.9, -0.1, -0.1) >, < q, (0.2, 0.8, 0.9, -0.8, -0.3, -0.2) > \right\}
\]
\[
T = \left\{ < p, (0.2, 0.5, 0.8, -0.8, -0.1, -0.2) >, < q, (0.4, 0.7, 0.8, -0.8, -0.3, -0.4) > \right\}
\]
Then \( \tau = \{0_{BSVN}, 1_{BSVN}, S, T\} \) is a BSVNT on \( X \). The BSVNs
\[
R = \left\{ < p, (0.2, 0.5, 0.8, -0.8, -0.1, -0.2) >, < q, (0.4, 0.7, 0.8, -0.8, -0.3, -0.4) > \right\}
\]
is BSVNGSCs in \( X \) but not BSVNGPCs in \( X \).

Figure 1: The Diagram represents the implication of the above theorem 4.3.
1. BSVNGPCs 2. BSVNCs 3. BSVNGCs 4. BSVNαCs 5. BSVNαGCs 6. BSVNRCs
7. BSVNPCs 8. BSVNSPCs 9. BSVNGSPCs 10. BSVNSCs 11. BSVNGSCs.

4.12 Remark: The union of any two BSVNGPCs’s is not BSVNGPCs in general as seen in the following example.

4.13 Example: Let \( X = \{p, q\} \) and
\[
S = \left\{ < p, (0.1, 0.5, 0.9, -0.7, -0.3, -0.4) >, < q, (0.5, 0.9, 0.8, -0.3, -0.2, -0.1) > \right\}
\]
\[
T = \left\{ < p, (0.4, 0.5, 0.4, -0.5, -0.3, -0.6) >, < q, (0.7, 0.6, 0.5, -0.2, -0.4, -0.3) > \right\}
\]
Then \( \tau = \{0_{BSVN}, 1_{BSVN}, S, T\} \) is a BSVNT on \( X \). The BSVNs
5. Bipolar Single-Valued Neutrosophic Generalized Pre-Open Set

5.1 Definition: A BSVNs \( S \) is said to be bipolar single-valued neutrosophic generalized pre-open set (BSVNPGOs) in \((X, \tau)\) if the complement \( S^c \) is BSVNGPCs in \((X, \tau)\). The family of all BSVNGPOs’s of BSVNTS \((X, \tau)\) is denoted by BSVNGPO \((X)\).

5.2 Example: Let \( X = \{p, q\} \) and

\[
S = \begin{cases} 
< p, (0.1, 0.5, 0.8, -0.9, -0.4, -0.2) > \\
< q, (0.2, 0.6, 0.7, -0.9, -0.3, -0.4) >
\end{cases}
\]

\[
T = \begin{cases} 
< p, (0.3, 0.3, 0.8, -0.2, -0.5, -0.3) > \\
< q, (0.4, 0.5, 0.5, -0.1, -0.4, -0.4) >
\end{cases}
\]

Then \( \tau = \{0_{BSVN}, 1_{BSVN}, S, T\} \) is a BSVNT on \( X \). The BSVNs

\[
R = \begin{cases} 
< p, (0.7, 0.5, 0.3, -0.2, -0.5, -0.7) > \\
< q, (0.7, 0.4, 0.3, -0.1, -0.7, -0.6) >
\end{cases}
\]

is BSVNGPOs in \( X \).

5.3 Theorem: For any BSVNTS \((X, \tau)\), we have the following results.

(1). Every BSVNOs is BSVNGPOs.
(2). Every BSVNROs is BSVNGPOs.
(3). Every BSVN \( \alpha \)Os is BSVNGPOs.
(4). Every BSVNPOs is BSVNGPOs.

5.4 Remark: The converse of the above theorem need not be true which can be seen from the following examples.

5.5 Example: Let \( X = \{p, q\} \) and

\[
S = \begin{cases} 
< p, (0.1, 0.5, 0.8, -0.9, -0.4, -0.2) > \\
< q, (0.2, 0.6, 0.7, -0.9, -0.3, -0.4) >
\end{cases}
\]

\[
T = \begin{cases} 
< p, (0.3, 0.3, 0.8, -0.2, -0.5, -0.3) > \\
< q, (0.4, 0.5, 0.5, -0.1, -0.4, -0.4) >
\end{cases}
\]

Then \( \tau = \{0_{BSVN}, 1_{BSVN}, S, T\} \) is a BSVNT on \( X \). The BSVNs

\[
R = \begin{cases} 
< p, (0.7, 0.5, 0.3, -0.2, -0.5, -0.7) > \\
< q, (0.7, 0.4, 0.3, -0.1, -0.7, -0.6) >
\end{cases}
\]

is BSVNGPOs in \( X \) but not BSVNOs in \( X \).

5.6 Example: Let \( X = \{p, q\} \) and
Then \( \tau = \{0, 1\} \) is a BSVNT on \( X \). The BSVNs

\[
S = \left\{ \begin{array}{l}
< p, (0.5,0.4,0.5,0.7,0.2,0.3) > \\
< q, (0.2,0.4,0.5,0.3,0.1,0.4) > 
\end{array} \right\}
\]
\[
T = \left\{ \begin{array}{l}
< p, (0.5,0.3,0.4,0.5,0.3,0.5) > \\
< q, (0.5,0.3,0.4,0.2,0.1,0.5) > 
\end{array} \right\}
\]

5.7 Example: Let \( X = \{p, q\} \) and

\[
S = \left\{ \begin{array}{l}
< p, (0.5,0.4,0.5,0.7,0.2,0.3) > \\
< q, (0.2,0.4,0.5,0.3,0.1,0.4) > 
\end{array} \right\}
\]
\[
T = \left\{ \begin{array}{l}
< p, (0.5,0.3,0.4,0.5,0.3,0.5) > \\
< q, (0.5,0.3,0.4,0.2,0.1,0.5) > 
\end{array} \right\}
\]

Then \( \tau = \{0, 1\} \) is a BSVNT on \( X \). The BSVNs

\[
R = \left\{ \begin{array}{l}
< p, (0.5,0.5,0.5,0.3,0.9,0.4) > \\
< q, (0.8,0.5,0.4,0.7,0.9,0.5) > 
\end{array} \right\}
\]

is BSVNGPOs in \( X \) but not BSVNROs in \( X \).

5.8 Example: Let \( X = \{p, q\} \) and

\[
S = \left\{ \begin{array}{l}
< p, (0.7,0.6,0.5,0.8,0.9,0.7) > \\
< q, (0.4,0.6,0.7,0.8,0.9,0.8) > 
\end{array} \right\}
\]
\[
T = \left\{ \begin{array}{l}
< p, (0.7,0.8,0.6,0.9,0.9,0.6) > \\
< q, (0.3,0.7,0.8,0.9,0.9,0.7) > 
\end{array} \right\}
\]

Then \( \tau = \{0, 1\} \) is a BSVNT on \( X \). The BSVNs

\[
R = \left\{ \begin{array}{l}
< p, (0.3,0.3,0.5,0.3,0.1,0.3) > \\
< q, (0.5,0.5,0.3,0.2,0.1,0.1) > 
\end{array} \right\}
\]

is BSVNGPOs in \( X \) but not BSVNPOs in \( X \).

5.9 Remark: The intersection of any two BSVNGPOs’s is not BSVNGPOs in general and it is shown in the following example.

5.10 Example: Let \( X = \{p, q\} \) and

\[
S = \left\{ \begin{array}{l}
< p, (0.8,0.4,0.3,0.1,0.3,0.5) > \\
< q, (0.5,0.4,0.3,0.8,0.5,0.6) > 
\end{array} \right\}
\]
\[
T = \left\{ \begin{array}{l}
< p, (0.4,0.6,0.7,0.9,0.2,0.4) > \\
< q, (0.4,0.5,0.4,0.9,0.4,0.5) > 
\end{array} \right\}
\]

Then \( \tau = \{0, 1\} \) is a BSVNT on \( X \). The BSVNs
Let $(X, \tau)$ be BSVNTS. If $S \in BSVNGPO(X)$ then $V \subseteq BSVN \text{ int} (BSVN \text{ cl} (S))$ whenever $V \subseteq S$ and $V$ is BSVNCs in $X$.

5.11 Theorem: Suppose $S$ is BSVNOs and BSVNGPCs in $X$. Then $S$ is BSVNGPCs, we have BSVN pcl $(S^c) \subseteq F^c$. Hence $(BSVN \text{ pcl} (S))^c \subseteq F^c$. Therefore $F \subseteq BSVN \text{ pcl} (S)$.

5.12 Theorem: Let $(X, \tau)$ be BSVNTS. Then for every $S \in BSVNGPO(X)$ and for every $T \in BSVNs(X)$, $BSVN \text{ pint} (S) \subseteq T \subseteq S$ implies $T \subseteq BSVNGPO(X)$.

5.13 Theorem: A BSVNs $S$ of BSVNTS $(X, \tau)$ is BSVNGPOs if and only if $F \subseteq BSVN \text{ pint} (S)$ whenever $F$ is BSVNCs and $F \subseteq S$.

5.14 Corollary: A BSVNs $S$ of a BSVNTS $(X, \tau)$ is BSVNGPOs if and only if $F \subseteq BSVN \text{ int} (BSVN \text{ cl} (S))$ whenever $F$ is BSVNCs and $F \subseteq S$.

5.15 Theorem: For a BSVNs $S$, $S$ is BSVNOs and BSVNGPCs in $X$ if and only if $S$ is BSVNROs in $X$.

\[ R = \{ < p, (0.7, 0.3, 0.3, -0.1, -0.2, -0.8) > \}, \quad V = \{ < q, (0.7, 0.4, 0.3, -0.1, -0.7, -0.7) > \} \]

BSVNGPOs in $X$ but $R \cap V$ is not BSVNGPOs in $X$.

\[ \text{Proof.} \]

Let $S$ be BSVNOs and BSVNGPCs in $X$. Then $S$ is BSVNGPCs, we have BSVN pcl $(S^c) \subseteq F^c$. Hence $(BSVN \text{ pcl} (S))^c \subseteq F^c$. Therefore $F \subseteq BSVN \text{ pcl} (S)$.

\[ \text{Suppose } S \text{ is BSVNGPOs in } X \text{ and let } F \subseteq BSVN \text{ pint} (S) \text{ whenever } F \text{ is BSVNCs and } F \subseteq S. \text{ Then } S^c \subseteq F^c \text{ and } F^c \text{ is BSVNOs. By hypothesis, } (BSVN \text{ pint} (S))^c \subseteq F^c. \text{ This implies BSVN pcl } (S^c) \subseteq F^c. \text{ Therefore } S^c \text{ is BSVNGPCs of } X. \text{ Hence } S \text{ is BSVNGPOs of } X. \]

\[ \text{Proof.} \text{ Necessity: } \text{Suppose } S \text{ is BSVNGPOs in } X. \text{ Let } F \text{ be BSVNCs and } F \subseteq S. \text{ Then } F^c \text{ is BSVNOs in } X \text{ such that } S^c \subseteq F^c. \text{ Since } S^c \text{ is BSVNGPCs, we have BSVN pcl } (S^c) \subseteq F^c. \text{ Hence } (BSVN \text{ pcl} (S))^c \subseteq F^c. \text{ This implies } F \subseteq BSVN \text{ pcl} (S). \]

\[ \text{Suppose } S \text{ be BSVNs of } X \text{ and let } F \subseteq BSVN \text{ pint} (S) \text{ whenever } F \text{ is BSVNCs and } F \subseteq S. \text{ Then } S^c \subseteq F^c \text{ and } F^c \text{ is BSVNOs. By hypothesis, } (BSVN \text{ pint} (S))^c \subseteq F^c. \text{ This implies BSVN pcl } (S^c) \subseteq F^c. \text{ Therefore } S^c \text{ is BSVNGPCs of } X. \text{ Hence } S \text{ is BSVNGPOs of } X. \]

\[ \text{Proof.} \text{ Necessity: } \text{Suppose } S \text{ is BSVNGPOs in } X. \text{ Let } F \text{ be BSVNCs and } F \subseteq S. \text{ Then } F^c \text{ is BSVNOs in } X \text{ such that } S^c \subseteq F^c. \text{ Since } S^c \text{ is BSVNGPCs, we have BSVN pcl } (S^c) \subseteq F^c. \text{ Therefore } BSVN \text{ cl } (BSVN \text{ int} (S^c)) \subseteq F^c. \text{ Hence } (BSVN \text{ cl} (BSVN \text{ int} (S^c)))^c \subseteq F^c. \text{ This implies } F \subseteq BSVN \text{ cl} (BSVN \text{ int} (S)). \]

\[ \text{Suppose } S \text{ be BSVNs of } X \text{ and let } F \subseteq BSVN \text{ int} (BSVN \text{ cl} (S)) \text{ whenever } F \text{ is BSVNCs and } F \subseteq S. \text{ Then } S^c \subseteq F^c \text{ and } F^c \text{ is BSVNOs. By hypothesis, } (BSVN \text{ int} (BSVN \text{ cl} (S)))^c \subseteq F^c. \text{ Hence } BSVN \text{ cl } (BSVN \text{ int} (S)) \subseteq F^c, \text{ which implies BSVN pcl } (S^c) \subseteq F^c. \text{ Hence } S \text{ is BSVNGPOs of } X. \]

\[ \text{Proof.} \text{ Necessity: } \text{Let } S \text{ be BSVNOs and BSVNGPCs in } X. \text{ Then BSVN pcl } (S) \subseteq S. \text{ This implies BSVN cl } (BSVN \text{ int } (S)) \subseteq S. \text{ Since } S \text{ is BSVNOs, it is BSVNPOs. Hence } S \subseteq BSVN \text{ int } (BSVN \text{ cl } (S)). \text{ Therefore } S = BSVN \text{ int } (BSVN \text{ cl } (S)). \text{ Hence } S \text{ is BSVNROs in } X. \]
Sufficiency: Let $S$ be BSVNROs in $X$. Therefore $S = \text{BSVN int} (\text{BSVN cl} (S))$. Let $S \subseteq U$ and $U$ is BSVNOs in $X$. This implies BSVN pcl $(S) \subseteq S$. Hence $S$ is BSVNGPCs in $X$.

6. Applications Of Bipolar Single-Valued Neutrosophic generalized Pre-Closed Sets

6.1 Definition: A BSVNTS $(X, \tau)$ is said to be bipolar single-valued neutrosophic $T_{1/2}$ space (BSVN $T_{1/2}$ space) if every BSVNGCs in $X$ is BSVNCs in $X$.

6.2 Definition: A BSVNTS $(X, \tau)$ is said to be bipolar single-valued neutrosophic $p$ $T_{1/2}$ space (BSVN $p$ $T_{1/2}$ space) if every BSVNPCs in $X$ is BSVNCs in $X$.

6.3 Definition: A BSVNTS $(X, \tau)$ is said to be bipolar single-valued neutrosophic $sp$ $T_{1/2}$ space (BSVN $sp$ $T_{1/2}$ space) if every BSVNGPCs in $X$ is BSVNCs in $X$.

6.4 Definition: A BSVNTS $(X, \tau)$ is said to be a bipolar single-valued neutrosophic $sp$ $p$ space (BSVN $sp$ $p$ space) if every BSVNGPCs in $X$ is BSVNPCs in $X$.

6.5 Theorem: Every BSVN $T_{1/2}$ space is BSVN $sp$ $p$ space.

Proof. Let $X$ be BSVN $T_{1/2}$ space and let $S$ be BSVNGCs in $X$, we know that every BSVNGCs is BSVNGPCs; hence $S$ is BSVNGPCs in $X$. By hypothesis $S$ is BSVNCs in $X$. Since every BSVNCs is BSVNPCs, $S$ is BSVNPCs in $X$. Hence $X$ is BSVN $sp$ $p$ space.

6.6 Remark: The converse of the above theorem is not true which is shown in the example.

6.7 Example: Let $X = \{p, q\}$ and

$S = \{< p, (0.1,0.3,0.5,0.3,0.5,0.1) > < q, (0.2,0.4,0.6,0.4,0.6,0.3) > \}$

$T = \{< p, (0.3,0.2,0.4,0.2,0.6,0.3) > < q, (0.3,0.3,0.5,0.2,0.7,0.3) > \}$

Then $\tau = \{0_{BSVN}, 1_{BSVN}, S, T\}$ is a BSVNT on $X$. The BSVNs

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6.8 Theorem: Every BSVN \( s_p T \frac{1}{2} \) space is BSVN \( s_p T \) space.

Proof. Let \( X \) be BSVN \( s_p T \) space and let \( S \) be BSVNGPCs in \( X \). By hypothesis \( S \) is BSVNCs in \( X \).

Since every BSVNCs is BSVNPCs, \( S \) is BSVNPCs in \( X \). Hence \( X \) is BSVN \( s_p T \) space.

6.9 Remark: The converse of the above theorem is not true which is shown in the example.

6.10 Example: Let \( X = \{p, q\} \) and

\[
R= \left\{ \begin{array}{l}
< p, (0.3,0.4,0.5,-0.6,-0.6,-0.3) > \\
< q, (0.2,0.4,0.3,-0.3,-0.1,-0.2) > 
\end{array} \right. 
\]

Then \( (X, \tau) \) is BSVN \( s_p T \frac{1}{2} \) space. But not BSVN \( s_p T \frac{1}{2} \) space.

Since \( R \) is BSVNGPCs but not BSVNCs in \( X \).

6.11 Theorem: Let \( (X, \tau) \) be BSVNTS and \( X \) is BSVN \( s_p T \frac{1}{2} \) space then,

(1). Any union of BSVNGPCs’s is BSVNGPCs.

(2). Any intersection of BSVNGPOs’s is BSVNGPOS.

Proof.

(1). Let \( \{A_i\}_{i=1} \) is a collection of BSVNGPCs’s in BSVN \( s_p T \frac{1}{2} \) space \( (X, \tau) \). Therefore every BSVNGPCs is BSVNCs. But the union of BSVNCs is BSVNCs. Hence the union of BSVNGPCs is BSVNGPCs in \( X \).

(2). Take complement of (1) to prove.

6.12 Theorem: A BSVNTS \( X \) is BSVN \( s_p T \frac{1}{2} \) space if and only if BSVNPO(X) = BSVNPO(X).

Proof. Necessity: Let \( S \) be BSVNGPOs in \( X \), then \( S^c \) is BSVNGPCs in \( X \). By hypothesis \( S^c \) is BSVNGPCs in \( X \). Therefore \( S \) is BSVNPCs in \( X \). Hence BSVNPO(X) = BSVNPO(X).
**Sufficiency:** Let $S$ be BSVNGPCs in $X$. Then $S^c$ is BSVNGPOs in $X$. By hypothesis $S^c$ is BSVNGPOs in $X$. Therefore $S$ is BSVNPCs in $X$. Hence $X$ is BSVN$_{sp}$, $T_{1/2}$ space.

7. Conclusions

We introduced a new class of sets namely bipolar single-valued neutrosophic generalized closed sets and bipolar single-valued neutrosophic generalized pre-closed sets in bipolar single-valued neutrosophic topological spaces. We also analyzed the properties and its applications with some examples.

References


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