Neutrosophic Sets and Systems, Vol. 47, 2021
University of New Mexico

Generalized closed sets and pre-closed sets via Bipolar single-valued neutrosophic Topological Spaces

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Abstract: The purpose of the paper is to introduce a new class of sets namely bipolar single-valued neutrosophic generalized closed sets and bipolar single-valued neutrosophic generalized pre- closed sets in bipolar single-valued neutrosophic topological spaces. Also we analysis the properties and its applications.

Keywords: Bipolar single-valued neutrosophic generalized closed sets, bipolar single-valued neutrosophic generalized pre- closed sets, and bipolar single-valued neutrosophic generalized pre-open sets, BSVN $T_{1/2}$ space, BSVN $_pT_{1/2}$ space, BSVN $_{gp}T_{1/2}$ space, BSVN $_{gp}T_p$ space.

1. Introduction

Zadeh [37], the Father of the Fuzzy Logic who imported the fuzzy sets in 1965 where the Fuzzy logic feature the human decision making technique and it is a tool in research logical subject. The concept of fuzzy sets is to deal with contrasting types of uncertainties. Fuzzy topology was introduced by Chang [5] in 1967 after the introduction of fuzzy sets. In 1970, Levine [21] studied the generalized closed sets in general topology. In 1991, Binshahan [4] introduced and investigate the notion of fuzzy pre-open and fuzzy pre-closed sets. The concept of generalized fuzzy closed set was introduced by Balasubramanian and Sundaram [3]. Fukutake et al. [19] gave the generalized pre-closed fuzzy sets in fuzzy topological spaces.

In 1994, Zhang [38] introduced the notion of a bipolar fuzzy set. Azhagappan and Kamaraj [2] investigated bipolar valued fuzzy topological spaces. Bipolar fuzzy topological spaces were proposed by Kim J, Samanta S. K, Lim P. K, Lee J. G and Hur K[20]. An intuitionistic fuzzy set was introduced by Atanassov [1] in 1986 as the extension of Zadeh's Fuzzy Sets besides the degree of membership and degree of non-membership. Dogan Coker [18] who gave introduction to intuitionistic fuzzy topological spaces. Rajarajeswari and Senthil Kumar [29] introduced Generalized pre-closed sets in Intuitionistic fuzzy Topological spaces.

Smarandache [31] introduced the neutrosophic set which is the base for the new mathematical theories. Neutrosophic set has the capability to induce classical sets, fuzzy set, Intuitionistic fuzzy sets. Introducing the components of the neutrosophic set are True (T), Indeterminacy (I), False (F) which represent the membership, indeterminacy, and non-membership values respectively. The notion of classical set, fuzzy set, interval-valued fuzzy set, Intuitionistic fuzzy, etc were generalized by the neutrosophic set. Neutrosophic topological spaces were presented by Salama et al. [30]. The concept of generalized closed sets and generalized pre-closed sets in neutrosophic Topological spaces were introduced by Wadei Al-Omeri et al.[33]. The neutrosophic pre-open and pre-closed sets in neutrosophic topology were extended by Venkateswara Rao et al.[32] who introduce neutrosophic topological space and open sets, closed sets, semi-open and semi closed sets. Generalized neutrosophic closed sets was introduced and some of their characterizations were also discussed by Dhavaseelan and Jafari [17]. Many Researchers [6-15, 26] have studied Neutrosophic in different areas with applications and the results.

Deli et al.[16] developed bipolar neutrosophic sets and study their application in decision making problem. The notation of bipolar neutrosophic soft set was proposed by Mumtaz Ali et al.[27]. Single-valued neutrosophic sets (in sort, SVN) were proposed by Wang et al.[35] by simplifying the Neutrosophic set. Single-valued neutrosophic topological space was given by YL Liu and HL Yang [22] and discussed the relationships between single valued neutrosophic approximation spaces and single valued neutrosophic topological spaces. Many researchers have studied the applications of SVNSs as well as theory. Ye [36] proposed decision making based on correlation coefficients and weighted correlation coefficient of SVNSs, and gave the application of proposed methods. Majumdar and Samant [23] studied distance, similarity and entropy of SVNSs from a theoretical aspect. Bipolar single-valued neutrosophic set was introduced by Mohana et al. [25] and also they give bipolar single-valued neutrosophic topological spaces.

In the paper, we introduce a new class of sets namely bipolar single-valued neutrosophic generalized closed sets and bipolar single-valued neutrosophic generalized pre-closed sets in bipolar single-valued neutrosophic topological spaces. Further we examine the interesting properties and some applications with counter examples.

2. Preliminaries

2.1 Definition [31]: Let a universe U of discourse. Then $K=\{<x, T_K(x), I_K(x), F_K(x)>x\in X\}$ defined as a neutrosophic set where truth-membership function T_K, an indeterminacy-membership function I_K and a falsity-membership function F_K. T_K, I_K, F_K are real or non-standard elements of $]\,0^-, 1^+\,[$.
No restriction on the sum of $T_K(x)$, $I_K(x)$ and $F_K(x)$, so $0^- \leq \sup T_K(x) \leq \sup I_K(x) \leq \sup F_K(x) \leq 3^+$.

2.2 Definition [30]: A Neutrosophic topology [NT for short] is a non-empty set X is a family of Neutrosophic subsets in X satisfying the following axioms:
(NT_1) $0_N, 1_N \in \tau$,
(NT_2) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(NT3) $\cup G_i \in \tau$, for every $\{G_i : i \in J\} \subseteq \tau$.
The pair (X, τ) is called a Neutrosophic topological space (NTS for short). The elements of τ are called Neutrosophic open sets [NOS for short]. A complement C(A) of a NOS A in NTS (X, τ) is called a Neutrosophic closed set [NCS for short] in X.

2.3 Definition: [30]: Let (X, τ) be NTS and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ be a NS in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by NCl(A) = {K : K is a NCS in X and A K} NInt(A) = {G : G is a NOS in X and G A} It can be also shown that NCl(A) is NCS and NInt(A) is a NOS in X. a) A is NOS if and only if A = NInt(A), b) A is NCS if and only if A = NCl(A).

2.4 Definition: [34]: A Neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ in a NTS (X, τ) is said to be

(i) Neutrosophic regular closed set (NRCS for short) if A = NCl(NInt(A)),
(ii) Neutrosophic regular open set (NROS for short) if A = NInt(NCl(A)),
(iii) Neutrosophic semi closed set (NSCS for short) if NInt(NCl(A)) ⊆ A,
(iv) Neutrosophic semi open set (NSOS for short) if A ⊆ NCl(NInt(A)),
(v) Neutrosophic pre closed set (NPCS for short) if NCl(NInt(A)) ⊆ A,
(vi) Neutrosophic pre-open set (NPOS for short) if A ⊆ NInt(NCl(A)),
(vii) Neutrosophic α- closed set (NSCS for short) if NCl(NInt(NCl(A))) ⊆ A,
(viii) Neutrosophic α- open set (NSOS for short) if A ⊆ NInt(NCl(NInt(A))).

2.5 Definition: [33]: Let (X, τ) be NTS and A = {⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x ∈ X} be a NS in X. Then the Neutrosophic pre closure and Neutrosophic pre interior of A are defined by NPCl(A) = {K : K is a NPCS in X and A⊆ K}, NPInt(A) = {G : G is a NPOS in X and G⊆ A}.

2.6 Definition: [28] :A Neutrosophic set A = {⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized closed set (NGCS for short) if NCl(A) U whenever A U and U is a NOS in (X, τ). A Neutrosophic set A of a NTS (X, τ) is called a Neutrosophic generalized open set (NGOS for short) if C (A) is a NGCS in (X, τ).

2.7 Definition: [33]: A Neutrosophic set A = {⟨x, μA(x), σA(x), νA(x)⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic α- generalized closed set (NαGCS for short) if NαCl(A) U whenever A U and U is a NOS in (X, τ). A Neutrosophic set A of a NTS (X, τ) is called a Neutrosophic α- generalized open set (NαGOS for short) if C (A) is an NαGCS in (X, τ).

2.8 Definition: [24]: A Neutrosophic set A = {⟨x, μA(x), σA(x), νA(x)⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic regular generalized closed set (NRGCS for short) if NCl(A) U whenever A U and U is a NROS in (X, τ). A Neutrosophic set A of a NTS (X, τ) is called a Neutrosophic regular generalized open set (NRGOS for short) if C(A) is a NRGCS in (X, τ).

2.9 Definition: [33]: A Neutrosophic set A = {⟨x, μA(x), σA(x), νA(x)⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized pre closed set (NGPCS for short) if NPCl(A) U whenever A U and U is a NOS in (X, τ). A Neutrosophic set A of a NTS (X, τ) is called a Neutrosophic generalized pre-open set (NGPOS for short) if C(A) is a NGPCS in (X, τ).

2.10 Definition [35]: Let a universe X of discourse. Then A_{NS}={<x,T_A(x),I_A(x)F_A(x)>x∈X} defined as a single-valued neutrosophic set(SVNS in short) where truth-membership function T_A:X→[0,1],an indeterminacy-membership function I_A: X→[0,1] and a falsity-membership function F_A: X → [0,1].No restriction on the sum of T_A(x), I_A(x) and F_A(x), so 0≤sup T_A(x) ≤sup I_A(x) ≤supF_A(x) ≤3. $\tilde{A}$=<T, I, F> is denoted as a single-valued neutrosophic number.

2.11 Definition [22]: A Single-valued neutrosophic topology on a non-empty set U is a family τ of SVNSs in U that satisfies the following conditions:

(T_1) $\tilde{\phi}$, $\tilde{U}$ ∈ τ,

(T_2) $\tilde{A} \cap \tilde{B}$ ∈ τ for any $\tilde{A}$, $\tilde{B}$ ∈ τ,

(T_3) $\cup_{i\in I} \tilde{A}_i$ ∈ τ for any $\tilde{A}_i$ ∈ τ, i∈ I, where I is an index set

The pair (U,τ) is called Single valued neutrosophic topological space and each SVNS $\tilde{A}$ in τ is referred to as a single valued neutrosophic open set in (U, τ). The complement of a single valued neutrosophic open set in (U, τ) is called a single valued neutrosophic closed set in (U, τ).

2.12 Definition [16]: In X, a bipolar neutrosophic set B is defined in the form
B=<x, (T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x)):x∈X>
Where T^+, I^+, F^+: X→ [1, 0] and T^- , I^- , F^- : X [-1, 0].The positive membership degree denotes the truth membership T^+(x), indeterminate membership I^+ (x) and false membership F^+ (x) of an element x∈X corresponding to the set A and the negative membership degree denotes the truth membership T^-(x),

indeterminate membership $I^-(x)$ and false membership $F^-(x)$ of an element x∈ X to some implicit counter-property corresponding to a bipolar neutrosophic set .

2.13 Definition [25]: A Bipolar Single-Valued Neutrosophic set (BSVNs) S in X is defined in the form of BSVN (S) =<v,(T_{BSVN}^+,T_{BSVN}^-),(I_{BSVN}^+,I_{BSVN}^-),(F_{BSVN}^+,F_{BSVN}^-):v∈X> → (I)
where (T_{BSVN}^+,I_{BSVN}^+,F_{BSVN}^+):X→[0,1] and (T_{BSVN}^-,I_{BSVN}^-,F_{BSVN}^-):X→ [-1,0] .In this definition, there T_{BSVN}^+ and T_{BSVN}^- are acceptable and unacceptable in past. Similarly I_{BSVN}^+ and I_{BSVN}^- are acceptable and unacceptable in future. F_{BSVN}^+ and F_{BSVN}^- are acceptable and unacceptable in present respectively.

2.14 Definition [25]: Let two bipolar single-valued neutrosophic sets $BSVN_1(S)$ and $BSVN_2(S)$ in X defined as
$BSVN_1$ (S)=<v,($T_{BSVN}^+(1)$,$T_{BSVN}^-(1)$),($I_{BSVN}^+(1)$,$I_{BSVN}^-(1)$),($F_{BSVN}^+(1)$,$F_{BSVN}^-(1)$):v∈X> and
$BSVN_2$(S)=<v,($T_{BSVN}^+(2)$,$T_{BSVN}^-(2)$),($I_{BSVN}^+(2)$,$I_{BSVN}^-(2)$),($F_{BSVN}^+(2)$,$F_{BSVN}^-(2)$):v∈X>. Then the operators are defined as follows:

(i) Complement

$BSVN^c(S)$ = {< v,(1-T_{BSVN}^+), (-1-T_{BSVN}^-), (1-I_{BSVN}^+), (-1-I_{BSVN}^-), (1-F_{BSVN}^+), (-1-F_{BSVN}^-): v∈X >}

(ii) Union of two BSVN

$BSVN_1(S)UBSVN_2(S)$=

$$\left\langle \begin{matrix} \max(T^+_{BSVN}(1),T^+_{BSVN}(2)),\min(I^+_{BSVN}(1),I^+_{BSVN}(2)),\min(F^+_{BSVN}(1),F^+_{BSVN}(2)) \\ \max(T^-_{BSVN}(1),T^-_{BSVN}(2)),\min(I^-_{BSVN}(1),I^-_{BSVN}(2)),\min(F^-_{BSVN}(1),F^-_{BSVN}(2)) \end{matrix} \right\rangle$$

(iii) Intersection of two BSVN

$BSVN_1(S)∩BSVN_2(S)$=

$$\left\langle \begin{matrix} \min(T^+_{BSVN}(1),T^+_{BSVN}(2)),\max(I^+_{BSVN}(1),I^+_{BSVN}(2)),\max(F^+_{BSVN}(1),F^+_{BSVN}(2)) \\ \min(T^-_{BSVN}(1),T^-_{BSVN}(2)),\max(I^-_{BSVN}(1),I^-_{BSVN}(2)),\max(F^-_{BSVN}(1),F^-_{BSVN}(2)) \end{matrix} \right\rangle$$

2.15 Definition [25]: Let two bipolar single-valued neutrosophic sets be $BSVN_1$ and $BSVN_2$ in X defined as
$BSVN_1$(S)=<v,($T_{BSVN}^+(1)$,$T_{BSVN}^-(1)$),($I_{BSVN}^+(1)$,$I_{BSVN}^-(1)$),($F_{BSVN}^+(1)$,$F_{BSVN}^-(1)$):v∈X> and
$BSVN_2$(S)=<v, ($T_{BSVN}^+(2)$,$T_{BSVN}^-(2)$),($I_{BSVN}^+(2)$,$I_{BSVN}^-(2)$), ($F_{BSVN}^+(2)$,$F_{BSVN}^-(2)$):v∈X>.

(i) Then $S_1 \subseteq S_2$ if and only if

$T_{BSVN}^+(1) \le T_{BSVN}^+(2)$, $I_{BSVN}^+(1) \ge I_{BSVN}^+(2)$, $F_{BSVN}^+(1) \ge F_{BSVN}^+(2)$,
$T_{BSVN}^-(1) \le T_{BSVN}^-(2)$, $I_{BSVN}^-(1) \ge I_{BSVN}^-(2)$, $F_{BSVN}^-(1) \ge F_{BSVN}^-(2)$ for all v∈X.

(ii) Then $S_1=S_2$ if and only if

$T_{BSVN}^+(1) = T_{BSVN}^+(2)$, $I_{BSVN}^+(1) = I_{BSVN}^+(2)$, $F_{BSVN}^+(1) = F_{BSVN}^+(2)$,
$T_{BSVN}^-(1) = T_{BSVN}^-(2)$, $I_{BSVN}^-(1) = I_{BSVN}^-(2)$, $F_{BSVN}^-(1) = F_{BSVN}^-(2)$ for all v∈X.

2.16 Definition [25]: A bipolar single-valued neutrosophic topology (BSVNT) on a non-empty set X is a τ of BSVN sets satisfying the axioms

(i) 0_{BSVN} , 1_{BSVN} ∈ τ

(ii) $S_1 \cap S_2 \in \tau$ for any $S_1,S_2 \in \tau$

(iii) $\cup S_i \in \tau$ for any arbitrary family {S_i :i∈ j} ∈ τ

The pair (X,τ) is called BSVN topological space(BSVNTS). Any BSVN set in τ is called as BSVN open set(BSVNOs) in X. The complement S^c of BSVN set in BSVN topological space (X, τ) is called a BSVN closed set(BSVNCs).

2.17 Definition [25]: Let (X,τ) be a BSVN topological space (BSVNTS) and
BSVN (S) =<v, (T_{BSVN^+},T_{BSVN^-}), (I_{BSVN^+},I_{BSVN^-}),(F_{BSVN^+},F_{BSVN^-}):v∈X> be a BSVN set in X. Then the closure and interior of A is defined as

Int (S) = U {F: F is a BSVN open set (BSVNOs) in X and F⊆S}

Cl (S) = ∩ {F: F is a BSVN closed set (BSVNCs) in X and S⊆F}.

Here cl(S) is a BSVNCs and int (S) is a BSVNOs in X.

(a) S is a BSVNCs in X iff cl (S) =S.
(b) S is a BSVNOs in X iff int (S) =S.

2.18 Proposition [25]: Let BSVNTS of (X,τ) and S,T be BSVNs's in X. Then the properties hold:

i. int (S) ⊆ S and S ⊆ cl(S)
ii. S⊆ T ⇒int(S) ⊆ int(T)
   S⊆ T ⇒cl(S) ⊆ cl(T)
iii. int(int(S))=int(S)
   cl(cl(S))=cl(S)
iv. int(S∩T)=int(S)∩int(T)
   cl(SUT)=cl(S)Ucl(T)
v. int(1_{BSVN})=1_{BSVN}
   cl(0_{BSVN})=0_{BSVN}

3. Bipolar Single-Valued Neutrosophic Generalized Closed Sets

For our convenience, we take (I) as S = {<x, (T_S^+ (x), I_S^+(x), F_S^+(x), T_S^-(x), I_S^-(x), F_S^-(x))>: x ∈X}.

3.1 Definition: A BSVNs S of a BSVNTS (X, τ) is said to be bipolar single-valued neutrosophic generalized closed set (BSVNGCs) if BSVN cl (S) ⊆ U whenever S ⊆ U and U is BSVNOs in X.

3.2 Definition: Let 0_{BSVN} and 1_{BSVN} be BSVNS in X defined as

0_{BSVN} = {<x, 0, 1, 1, -1, 0, 0: x ∈ X >} is said to be Null or Empty bipolar single-valued neutrosophic set.

1_{BSVN} = {<x, 1, 0, 0, 0, -1, -1: x ∈ X >} is said to be Absolute or Unit bipolar single-valued neutrosophic set.

3.3 Example: Let X ={p, q} and

S=$\begin{Bmatrix} < p,(0.3,0.5,0.1,-0.2,-0.4,-0.3) > \\ < q,(0.2,0.8,0.2,-0.4,-0.6,-0.9) > \end{Bmatrix}$ T=$\begin{Bmatrix} < p,(0.4,0.4,0.1,-0.1,-0.5,-0.4) > \\ < q,(0.3,0.7,0.1,-0.3,-0.6,-0.9) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R=$\begin{Bmatrix} < p,(0.2,0.3,0.7,-0.8,-0.1,-0.3) > \\ < q,(0.3,0.8,0.3,-0.4,-0.1,-0.4) > \end{Bmatrix}$ is BSVNGCs in X.

3.4 Definition: A BSVNs S = {<x, (T_S^+ (x), I_S^+ (x), F_S^+ (x), T_S^- (x), I_S^- (x), F_S^- (x))>: x ∈ X} in BSVNTS

(X, τ) is said to be

(1) Bipolar single-valued neutrosophic semi closed set (BSVNSCs) if BSVN int (BSVN cl (S)) ⊆ S,
(2) Bipolar single-valued neutrosophic semi open set (BSVNSOs) if S ⊆ BSVN cl (BSVN int (S)),

(3) Bipolar single-valued neutrosophic pre-closed set (BSVNPCs) if BSVN cl (BSVN int (S)) $\subseteq$ S,

(4) Bipolar single-valued neutrosophic pre-open set (BSVNPOs) if S $\subseteq$ BSVN int (BSVN cl (S)),

(5) Bipolar single-valued neutrosophic α-closed set (BSVN αCs) if
BSVN cl (BSVN int (BSVN cl (S))) $\subseteq$ S,

(6) Bipolar single-valued neutrosophic α-open set (BSVN αOs) if
S $\subseteq$ BSVN int (BSVN cl (BSVN int (S))),

(7) Bipolar single-valued neutrosophic semi pre-closed set (BSVNSPCs) if
BSVN int (BSVN cl (BSVN int (S))) $\subseteq$ S,

(8) Bipolar single-valued neutrosophic semi pre-open set (BSVNSPOs) if
S $\subseteq$ BSVN cl (BSVN int (BSVN cl (S))),

(9) Bipolar single-valued neutrosophic regular open set (BSVNROs) if S = BSVN int (BSVN cl (S)),

(10) Bipolar single-valued neutrosophic regular closed set (BSVNRCs) if S = BSVN cl (BSVN int (S)).

3.5 Definition: Let (X, τ) be BSVNTS and S be BSVNs in X. Then the bipolar single-valued neutrosophic generalized interior and bipolar single-valued neutrosophic generalized closure are denoted by

(1) BSVNG int (S) = $\cup$ {G / G is a BSVNGOs in X and G $\subseteq$ S}

(2) BSVNG cl (S) = $\cap$ {K / K is a BSVNGCs in X and S $\subseteq$ K}

3.6 Definition: Let (X, τ) be any BSVNTS and let S and T be BSVNs in X. Then the bipolar single-valued neutrosophic generalized closure operator satisfies the properties:

1. S $\subseteq$ BSVN cl(S)
2. BSVN int(S) $\subseteq$ S
3. S $\subseteq$ T $\Rightarrow$ BSVN cl(S) $\subseteq$ BSVN cl(T)
4. S $\subseteq$ T $\Rightarrow$ BSVN int(S) $\subseteq$ BSVN int(T)
5. BSVN cl(S $\cup$ T)= BSVN cl(S) $\cup$ BSVN cl(T)
6. BSVN int(S $\cap$ T)= BSVN int(S) $\cap$ BSVN int(S)
7. (BSVN cl(S))^c = BSVN int(S^c)
8. (BSVN cl(S))^c = BSVN int(S^c)

Proof:

1. BSVN cl(S) = $\cap$ {K / K is a BSVNGCs in X and S $\subseteq$ K}. Thus S $\subseteq$ BSVN cl(S).
2. BSVNG int (S) = $\cup$ {G / G is a BSVNGOs in X and G $\subseteq$ S}. Thus BSVN int(S) $\subseteq$ S.
3. BSVN cl(T) = $\cap$ {K / K is a BSVNGCs in X and T $\subseteq$ K},
   $\supseteq$ $\cap$ {K / K is a BSVNGCs in X and S $\subseteq$ K},
   $\supseteq$ BSVN cl(S). Thus BSVN cl(S) $\subseteq$ BSVN cl(T).
4. BSVN int (T) = $\cup$ {G / G is a BSVNGOs in X and G $\subseteq$ T},
   $\supseteq$ $\cup$ {G / G is a BSVNGOs in X and G $\subseteq$ S},
   $\supseteq$ BSVN int (S). Thus BSVN int (S) $\subseteq$ BSVN int (T).
5. BSVN cl(S $\cup$ T)= $\cap$ {K / K is a BSVNGCs in X and S $\cup$ T $\subseteq$ K},
   ($\cap$ {K / K is a BSVNGCs in X and S $\subseteq$ K}) $\cup$ ($\cap$ {K / K is a BSVNGCs in X and T $\subseteq$ K}),
   = BSVN cl(S) $\cup$ BSVN cl(T). Thus BSVN cl(S $\cup$ T)= BSVN cl(S) $\cup$ BSVN cl(T).

6. BSVN int($S\cap T$)= $\cup$ {G / G is a BSVNGOs in X and G$\subseteq$ $S\cap T$ },
   ($\cup$ {G / G is a BSVNGOs in X and G$\subseteq$ S}) $\cap$($\cup$ {G / G is a BSVNGOs in X and G$\subseteq$T}),
   = BSVN int(S) $\cap$BSVN int(S). Thus BSVN int($S\cap T$)= BSVN int(S) $\cap$BSVN int(S).
7. (BSVN cl(S)) = $\cap${K / K is a BSVNGCs in X and S$\subseteq$K},
   (BSVN cl(S))^c = $\cap${K^c / K^c is a BSVNGCs in X and $S^c\subseteq K^c$},
   = BSVN int(S^c). Thus (BSVN cl(S))^c = BSVN int(S^c).
8. BSVNG int (S) = $\cup$ {G / G is a BSVNGOs in X and G$\subseteq$S},
9. (BSVNG int (S))^c = $\cup$ {G / G is a BSVNGOs in X and $G^c\subseteq S^c$}= BSVN int(S^c)
   Thus (BSVN cl(S))^c = BSVN int(S^c).

3.6 Definition: Let (X, τ) be a BSVNTS and S be a BSVNs in X. The bipolar single-valued neutrosophic pre interior of S and denoted by BSVN pint (S) and bipolar single-valued neutrosophic pre-closure of S is denoted by BSVN pcl (S).

(1) BSVN pint (S) = $\cup$ {G / G is a BSVNPOs in X and G$\subseteq$S}

(2) BSVN pcl (S) = $\cap${K / K is a BSVNPCs in X and S$\subseteq$K}

3.7 Result 3.21: Let A be BSVNs of a BSVNTS (X, τ), then

(1). BSVN pcl (S) = S $\bigcup$BSVN cl (BSVN int (S)),

(2). BSVN pint (S) = S $\bigcap$ BSVN int (BSVN cl (S)).

3.8 Definition: Let S be BSVNs of a BSVNTS (X, τ). Then the bipolar single-valued neutrosophic semi interior of S (BSVN sint (S)) and bipolar single-valued neutrosophic semi closure of S (BSVN scl (S)) are defined by

(1) BSVN sint (S) = $\cup$ {G / G is a BSVNSOs in X and G$\subseteq$S}

(2) BSVN scl (S) = $\cap${K / K is a BSVNSCs in X and S$\subseteq$K}

3.9 Result: Let S be BSVNs of a BSVNTS (X, τ), then

(1) BSVN scl (S) = S$\bigcup$ BSVN int (BSVN cl (S)),

(2). BSVN sint (S) = S $\bigcap$ BSVN cl (BSVN int (S)).

3.10 Definition: Let S be BSVNs of a BSVNTS (X, τ). Then the bipolar single-valued neutrosophic alpha interior of S (BSVN αint (S) and bipolar single-valued neutrosophic alpha closure of S (BSVN αcl (S)) is defined by

(1) BSVN αint (S) = $\cup$ {G / G is a BSVNαOs in X and G$\subseteq$S}

(2) BSVN αcl (S) = $\cap${K / K is a BSVNαCs in X and S$\subseteq$K}

3.11 Result: Let S be BSVNs of a BSVNTS (X, τ), then

(1) BSVN αcl (S) = S $\bigcup$ BSVN cl (BSVN int (BSVN cl (S))),

(2) BSVN αint (S) = S $\bigcap$BSVN int (BSVN cl (BSVN int (S))).

3.12 Definition: Let A be BSVNs of a BSVNTS (X, τ). Then the bipolar single-valued neutrosophic semi-pre interior of S (BSVN spint (S)) and bipolar single-valued neutrosophic semi-pre closure of S (BSVN spcl (S)) are defined by

(1) BSVN spint (S) = $\cup$ {G / G is a BSVNSPOs in X and G $\subseteq$ S}

(2) BSVN spcl (S) = $\cap$ {K / K is a BSVNSPCs in X and S $\subseteq$ K}

3.13 Definition: A BSVNs S of a BSVNTS (X, τ) is said to be bipolar single-valued neutrosophic generalized semi closed set (BSVNGSCS) if BSVN scl (S) $\subseteq$ U whenever S $\subseteq$ U and U is BSVNOs in X.

3.14 Definition: A BSVNs S of a BSVNTS (X, τ) is said to be bipolar single-valued neutrosophic alpha generalized closed set (BSVN αGCS) if BSVN αcl (S) $\subseteq$ U whenever S $\subseteq$ U and U is BSVNOs in X.

3.15 Definition: A BSVNs S of a BSVNTS (X, τ) is said to be bipolar single-valued neutrosophic generalized semi-pre closed set (BSVNGSPCs) if BSVN spcl (S) $\subseteq$ U whenever S $\subseteq$ U and U is BSVNOs in X.

3.16 Definition: Let {A_i : i $\in$ J} be an arbitrary family of BSVNs in X. Then

(1). $\bigcap S_i$= {<x, min ($T^+_{S_i}$ (x)), max ($I^+_{S_i}$ (x)), max ($F^+_{S_i}$ (x)),

min ($T^-_{S_i}$ (x)), max ($I^-_{S_i}$ (x)), max ($F^-_{S_i}$ (x))>}

(2). $\bigcup A_i$= {<x, max ($T^+_{S_i}$ (x)), min ($I^+_{S_i}$ (x)), min ($F^+_{S_i}$ (x),

max ($T^-_{S_i}$ (x)),min ($I^-_{S_i}$ (x)), min ($F^-_{S_i}$ (x))>}

3.18 Corollary: Let S, T, M and N be bipolar single-valued neutrosophic set in X. Then

(1) S $\subseteq$ T and M $\subseteq$ N $\Rightarrow$ S $\bigcup$ M $\subseteq$ T $\bigcup$ N and S $\bigcap$ M $\subseteq$ T $\bigcap$ N

(2) S $\subseteq$ T and S $\subseteq$ M $\Rightarrow$ S $\subseteq$ T $\bigcap$ M

(3) S $\subseteq$ M and T $\subseteq$ M $\Rightarrow$ S $\bigcup$ T $\subseteq$ M

(4) S $\subseteq$ T and T $\subseteq$ M $\Rightarrow$ S $\subseteq$ M

(5) $(S \bigcup T)^c = S^c \bigcap T^c$

(6) $(S \bigcap T)^c = S^c \bigcup T^c$

(7) S $\subseteq$ T $\Rightarrow$ $T^c \subseteq S^c$

(8) $(S^c)^c = S$

(9) $0^C_{BSVN} = 1_{BSVN}$

(10) $1^C_{BSVN} = 0_{BSVN}$

Proof: The proof is obvious.

3.19 Theorem: Every bipolar single-valued neutrosophic closed set is bipolar single-valued neutrosophic generalized closed set.

Proof. Let S be BSVNCs in X. Suppose U is BSVNOs in X, such that S $\subseteq$ U. Then BSVN cl (S) = S $\subseteq$ U. Hence S is BSVNGCs in X.

3.20 Remark: The converse of the above theorem is not true which is shown in the example.

3.21 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.3,0.5,0.1,-0.2,-0.4,-0.3) > \\ < q,(0.2,0.8,0.2,-0.4,-0.6,-0.9) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.4,0.4,0.1,-0.1,-0.5,-0.4) > \\ < q,(0.3,0.7,0.1,-0.3,-0.6,-0.9) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.2,0.3,0.7,-0.8,-0.1,-0.3) > \\ < q,(0.3,0.8,0.3,-0.4,-0.1,-0.4) > \end{Bmatrix}$ is BSVNGCs in X but not BSVNCs in X.

3.22 Remark: The Intersection of two BSVNGCs is need not be true. Shown in the following example.

3.23 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.3,0.5,0.1,-0.2,-0.4,-0.3) > \\ < q,(0.2,0.8,0.2,-0.4,-0.6,-0.9) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.4,0.4,0.1,-0.1,-0.5,-0.4) > \\ < q,(0.3,0.7,0.1,-0.3,-0.6,-0.9) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.2,0.3,0.7,-0.8,-0.1,-0.3) > \\ < q,(0.3,0.8,0.3,-0.4,-0.1,-0.4) > \end{Bmatrix}$ V= $\begin{Bmatrix} < p,(0.6,0.6,0.9,-0.9,-0.5,-0.6) > \\ < q,(0.7,0.3,0.9,-0.7,-0.1,-0.1) > \end{Bmatrix}$ are BSVNGCs in X but R∩V is not BSVNGCs in X.

3.24 Proposition: Let (X, τ) be BSVNTS. If S is a bipolar single-valued neutrosophic generalized closed set and S $\subseteq$ T $\subseteq$ BSVN cl (S) then T is bipolar single-valued neutrosophic generalized closed set.

Proof: Let G be a bipolar single-valued neutrosophic open set in (X, τ), such that T $\subseteq$ G. Since S $\subseteq$ T, S $\subseteq$ G. Now S is a bipolar single-valued neutrosophic generalized closed set and BSVN cl (S) $\subseteq$ G. But BSVN cl (T) $\subseteq$ BSVN cl (S). Since BSVN cl (T) $\subseteq$ BSVN cl (S) $\subseteq$ G. BSVN cl (T) $\subseteq$ G.

Hence T is a bipolar single-valued neutrosophic generalized closed set.

3.25 Proposition: Let (X, τ) be BSVNTS and a BSVNs S is a bipolar single-valued neutrosophic generalized open if and only if T $\subseteq$BSVN int (S) whenever T is bipolar single-valued neutrosophic closed set and T$\subseteq$S.

Proof: Let S is a bipolar single-valued neutrosophic generalized open set and T be a bipolar single-valued neutrosophic closed set, such that T $\subseteq$S. Now T $\subseteq$ S$\Rightarrow$$S^c$ $\subseteq$$T^c$ and S^c is a bipolar single-valued neutrosophic generalized closed set implies that BSVN cl (S^c) $\subseteq$$T^c$. (i.e) T=$(T^c)^c$$\subseteq$(BSVN cl($S^c$))^c. But (BSVN cl ($S^c$))^c = BSVN int (S). Thus T$\subseteq$BSVN int (S).

Conversely, suppose that S be a bipolar single-valued neutrosophic set, such that T$\subseteq$BSVN int(S) whenever T is bipolar single-valued neutrosophic closed and T$\subseteq$S. Let $S^c$$\subseteq$T whenever T is bipolar single-valued neutrosophic open. Now $S^c$$\subseteq$T $\Rightarrow$$T^c$ $\subseteq$S. Hence by the assumption, T^c $\subseteq$ BSVN int (S). (i.e) (BSVN int (S))^c $\subseteq$ T. But (BSVN int (S))^c = BSVN cl (S^c). Hence (BSVN int (S))^c $\subseteq$BSVN cl (S^c) . (i.e) S^c is bipolar single-valued neutrosophic generalized closed set. Therefore, S is bipolar single-valued neutrosophic generalized open set. Hence proved.

3.26 Proposition: If BSVN int (S) $\subseteq$T $\subseteq$S and if S is bipolar single-valued neutrosophic generalized open set then T is also bipolar single-valued neutrosophic generalized open set.

Proof: Now S^c $\subseteq$ $T^c$$\subseteq$ (BSVN int (S))^c= BSVN cl (S^c). As S is a bipolar single-valued neutrosophic generalized open, S^c is bipolar single-valued neutrosophic generalized closed set. Then by the proposition 3.24, T is bipolar single-valued neutrosophic generalized open set. Hence Proved.

4. Bipolar Single-Valued Neutrosophic Generalized Pre-Closed Set

4.1 Definition: A BSVNs S is said to be bipolar single-valued neutrosophic generalized pre-closed set (BSVNGPCs) in (X, τ) if BSVN pcl (S) $\subseteq$ U whenever S $\subseteq$ U and U is BSVNOs in X. The family of all BSVNGPCs's of a BSVNTS (X, τ) is denoted by BSVNGPC (X).

4.2 Example: Let X ={p, q} and

$$S=\begin{Bmatrix} < p,(0.1,-0.7),(0.3,-0.8),(0.5,-0.1) > \\ < q,(0.2,0.4,0.6,-0.8,-0.2,-0.4) > \end{Bmatrix} \quad T=\begin{Bmatrix} < p,(0.1,0.2,0.3,-0.7,-0.9,-0.9) > \\ < q,(0.4,0.3,0.6,-0.1,-0.3,-0.5) > \end{Bmatrix}$$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

$$R=\begin{Bmatrix} < p,(0.8,0.7,0.8,-0.7,-0.2,-0.3) > \\ < q,(0.1,0.7,0.9,-0.2,-0.7,-0.2) > \end{Bmatrix}$$ is BSVNGPCs in X.

4.3 Theorem:

(1) Every bipolar single-valued neutrosophic closed set is bipolar single-valued neutrosophic generalized pre-closed set.

(2) Every bipolar single-valued neutrosophic generalized closed set is bipolar single-valued neutrosophic generalized pre-closed set.

(3) Every bipolar single-valued neutrosophicα closed set is bipolar single-valued neutrosophic generalized pre-closed set.
(4) Every bipolar single-valued neutrosophic regular closed set is bipolar single-valued neutrosophic generalized pre-closed set.
(5) Every bipolar single-valued neutrosophic pre-closed set is bipolar single-valued neutrosophic generalized pre-closed set.
(6) Every bipolar single-valued neutrosophicα generalized closed set is bipolar single-valued neutrosophic generalized pre-closed set.
(7) Every bipolar single-valued neutrosophic generalized pre-closed set is bipolar single-valued neutrosophic semi-pre closed set.
(8) Every bipolar single-valued neutrosophic generalized pre-closed set is bipolar single-valued neutrosophic generalized semi-pre closed set.

Proof. (1) Let S be BSVNCs in X and let S $\subseteq$U and U be BSVNOs in X. Since BSVN pcl (S) $\subseteq$ BSVN cl (S) and S is BSVNCs in X, BSVN pcl (S) $\subseteq$ BSVN cl (S) = S $\subseteq$ U. Therefore S is BSVNGPCs in X.

(2) Let S be BSVNGCs in X and let S $\subseteq$ U and U is BSVNOs in (X, τ). Since
BSVN pcl (S) $\subseteq$ BSVN cl (S) and by hypothesis, BSVN pcl (S) $\subseteq$ U. Therefore S is BSVNGPCs in X.

(3) Let S be BSVN αCS in X and let S $\subseteq$ U and U be BSVNOs in X. By hypothesis,
BSVN cl (BSVN int (BSVN cl (S))) $\subseteq$ S. Since S$\subseteq$ BSVN cl (S);
BSVN cl (BSVN int (S)) $\subseteq$ BSVN cl (BSVN int (BSVN cl (S))) $\subseteq$ S. Hence BSVN pcl (S) $\subseteq$ S $\subseteq$ U. Therefore S is BSVNGPCs in X.

(4) Let S be a BSVNRCs in X. By Definition S = BSVN cl (BSVN int (S)). This implies
BSVN cl (S) = BSVN cl (BSVN int (S)). Therefore BSVN cl (S) = S. (i.e) S is BSVNCs in X. S is BSVNGPCs in X.

(5) Let S be BSVNPCs in X and let S $\subseteq$ U and U is BSVNOs in X. By Definition, BSVN cl (BSVN int (S)) $\subseteq$ S. This implies BSVN pcl (S) = S $\bigcup$ BSVN cl (BSVN int (S)) $\subseteq$ S. Therefore
BSVN pcl (S) $\subseteq$ U. Hence S is BSVNGPCs in X.

(6) Let S be BSVN αGCs in X and let S $\subseteq$ U and U is BSVNOs in (X, τ). By Result 3.11,
S$\bigcup$ BSVN cl (BSVN int (BSVN cl (S))) $\subseteq$U. This implies BSVN cl (BSVN int (BSVN cl (S))) $\subseteq$ U and BSVN cl (BSVN int (S)) $\subseteq$ U. Thus BSVN pcl (S) = S$\bigcup$ BSVN cl (BSVN int (S)) $\subseteq$U. Hence S is BSVNGPCs in X.
(7) Let S be BSVNGPCs in X , this implies BSVN pcl (S) $\subseteq$ U whenever S $\subseteq$ U and U is BSVNOs in X. By hypothesis BSVN cl (BSVN int (S)) $\subseteq$ S. Therefore
BSVN int (BSVN cl (BSVN int (S))) $\subseteq$ BSVN int (S) $\subseteq$ S. Therefore
BSVN int (BSVN cl (BSVN int (S))) $\subseteq$S. Hence S is BSVNSPCs in X.

(8) Let s be BSVNGPCsin X and let S $\subseteq$ U and U is BSVNOs in X. By hypothesis BSVN cl (BSVN int (S)) $\subseteq$ S $\subseteq$ U. Therefore BSVN int (BSVN cl (BSVN int (S))) $\subseteq$ BSVN int (S) $\subseteq$ U. This implies BSVN spcl (S) $\subseteq$ U whenever S $\subseteq$ U and U is BSVNOs in X. Therefore S is BSVNGSPCs in X.

4.4 Remark: The converse of the above theorem 4.3 (1-8) is not true which is shown in the example.

4.5 Example:

(1) Let X ={p, q} and

S= $\left\{\begin{array}{l} < p,(0.1,0.3,0.5,-0.7,-0.8,-0.1) > \\ < q,(0.2,0.4,0.6,-0.8,-0.2,-0.4) > \end{array}\right\}$ T= $\left\{\begin{array}{l} < p,(0.1,0.2,0.3,-0.7,-0.9,-0.9) > \\ < q,(0.4,0.3,0.6,-0.1,-0.3,-0.5) > \end{array}\right\}$

Then τ= {0BSVN, 1BSVN, S, T} is a BSVNT on X. The BSVNs

R= $\left\{\begin{array}{l} < p,(0.8,0.7,0.8,-0.7,-0.2,-0.3) > \\ < q,(0.1,0.7,0.9,-0.2,-0.7,-0.2) > \end{array}\right\}$ is BSVNGPCs in X but not BSVNCs in X.

(2) Let X ={p, q} and

S= $\left\{\begin{array}{l} < p,(0.1,0.3,0.2,-0.3,-0.4,-0.6) > \\ < q,(0.2,0.4,0.5,-0.1,-0.1,-0.3) > \end{array}\right\}$ T= $\left\{\begin{array}{l} < p,(0.1,0.3,0.4,-0.4,-0.1,-0.4) > \\ < q,(0.2,0.5,0.6,-0.3,-0.1,-0.1) > \end{array}\right\}$

Then τ= {0BSVN, 1BSVN, S, T} is a BSVNT on X. The BSVNs

R= $\left\{\begin{array}{l} < p,(0.1,0.5,0.4,-0.5,-0.3,-0.1) > \\ < q,(0.1,0.6,0.5,-0.2,-0.1,-0.3) > \end{array}\right\}$ is BSVNGPCs in X but not BSVNGCs in X.

(3) Let X ={p, q} and

S= $\left\{\begin{array}{l} < p,(0.5,0.4,0.1,-0.6,-0.5,-0.4) > \\ < q,(0.5,0.1,0.1,-0.3,-0.1,-0.2) > \end{array}\right\}$ T= $\left\{\begin{array}{l} < p,(0.4,0.5,0.3,-0.6,-0.3,-0.1) > \\ < q,(0.2,0.3,0.6,-0.4,-0.2,-0.1) > \end{array}\right\}$

Then τ= {0BSVN, 1BSVN, S, T} is a BSVNT on X. The BSVNs

R= $\left\{\begin{array}{l} < p,(0.1,0.5,0.5,-0.7,-0.3,-0.4) > \\ < q,(0.5,0.9,0.2,-0.3,-0.2,-0.1) > \end{array}\right\}$ is BSVNGPCs in X but not BSVNαCs in X.

(4) Let X ={p, q} and

S= $\left\{\begin{array}{l} < p,(0.1,0.3,0.2,-0.3,-0.4,-0.6) > \\ < q,(0.2,0.4,0.5,-0.1,-0.1,-0.3) > \end{array}\right\}$ T= $\left\{\begin{array}{l} < p,(0.1,0.3,0.4,-0.4,-0.1,-0.4) > \\ < q,(0.2,0.5,0.6,-0.3,-0.1,-0.1) > \end{array}\right\}$

Then τ= {0BSVN, 1BSVN, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.1,0.5,0.4,-0.5,-0.3,-0.1) > \\ < q,(0.1,0.6,0.5,-0.2,-0.1,-0.3) > \end{Bmatrix}$ is BSVNGPCs in X but not BSVNRCs in X.

(5) Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.5,0.4,0.3,-0.6,-0.4,-0.2) > \\ < q,(0.2,0.5,0.1,-0.5,-0.3,-0.1) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.6,0.2,0.1,-0.5,-0.6,-0.8) > \\ < q,(0.3,0.1,0.1,-0.4,-0.4,-0.3) > \end{Bmatrix}$

Then τ= {0BSVN, 1BSVN, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.5,0.3,0.2,-0.1,-0.7,-0.3) > \\ < q,(0.2,0.4,0.1,-0.3,-0.4,-0.1) > \end{Bmatrix}$ is BSVNGPCs in X but not BSVNPCs in X.

(6) Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.1,0.3,0.6,-0.2,-0.4,-0.5) > \\ < q,(0.2,0.4,0.5,-0.1,-0.9,-0.5) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.1,0.3,0.6,-0.7,-0.3,-0.2) > \\ < q,(0.2,0.6,0.7,-0.8,-0.4,-0.1) > \end{Bmatrix}$

Then τ= {0BSVN, 1BSVN, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.1,0.4,0.7,-0.8,-0.2,-0.1) > \\ < q,(0.2,0.7,0.7,-0.9,-0.1,-0.1) > \end{Bmatrix}$ is BSVNGPCs in X but not BSVN αGCs in X.

(7) Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.1,0.5,0.5,-0.5,-0.1,-0.4) > \\ < q,(0.3,0.5,0.6,-0.7,-0.1,-0.2) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.3,0.2,0.3,-0.2,-0.3,-0.5) > \\ < q,(0.4,0.3,0.1,-0.1,-0.4,-0.5) > \end{Bmatrix}$

Then τ= {0BSVN, 1BSVN, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.1,0.3,0.5,-0.4,-0.1,-0.5) > \\ < q,(0.4,0.3,0.1,-0.1,-0.2,-0.3) > \end{Bmatrix}$ is BSVNSPCs in X but not BSVNGPCs in X.

(8) Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.4,0.7,0.4,-0.5,-0.4,-0.2) > \\ < q,(0.3,0.2,0.4,-0.3,-0.1,-0.1) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.3,0.8,0.8,-0.7,-0.3,-0.1) > \\ < q,(0.2,0.3,0.7,-0.4,-0.1,-0.1) > \end{Bmatrix}$

Then τ= {0BSVN, 1BSVN, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.3,0.8,0.5,-0.6,-0.3,-0.2) > \\ < q,(0.2,0.3,0.7,-0.3,-0.1,-0.1) > \end{Bmatrix}$ is BSVNGSPCs in X but not BSVNGPCs in X.

4.6 Proposition: BSVNSCs and BSVNGPCs are independent to each other which are shown in the example.

4.7 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.5,0.4,0.2,-0.1,-0.2,-0.7) > \\ < q,(0.7,0.6,0.3,-0.6,-0.1,-0.5) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.8,0.3,0.1,-0.1,-0.3,-0.8) > \\ < q,(0.8,0.2,0.3,-0.4,-0.5,-0.6) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.2,0.5,0.3,-0.1,-0.1,-0.7) > \\ < q,(0.6,0.7,0.4,-0.7,-0.1,-0.2) > \end{Bmatrix}$ is BSVNGPCs in X but not BSVNSCs in X.

4.8 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.1,0.7,0.6,-0.8,-0.2,-0.5) > \\ < q,(0.3,0.7,0.7,-0.8,-0.2,-0.2) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.9,0.5,0.5,-0.3,-0.4,-0.7) > \\ < q,(0.5,0.5,0.3,-0.2,-0.7,-0.8) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.2,0.6,0.5,-0.7,-0.3,-0.5) > \\ < q,(0.3,0.7,0.7,-0.8,-0.2,-0.2) > \end{Bmatrix}$ is BSVNSCs in X but not BSVNGPCs in X.

4.9 Proposition: BSVNGSCs and BSVNGPCs are independent to each other which are shown in the example.

4.10 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.9,0.5,0.6,-0.3,-0.8,-0.5) > \\ < q,(0.9,0.1,0.3,-0.2,-0.6,-0.6) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.8,0.7,0.7,-0.4,-0.8,-0.5) > \\ < q,(0.8,0.8,0.7,-0.3,-0.5,-0.4) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.9,0.8,0.7,-0.6,-0.5,-0.4) > \\ < q,(0.3,0.2,0.3,-0.2,-0.3,-0.4) > \end{Bmatrix}$ is BSVNGPCs in X but not BSVNGSCs in X.

4.11 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.1,0.6,0.9,-0.9,-0.1,-0.1) > \\ < q,(0.2,0.8,0.9,-0.8,-0.3,-0.2) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.2,0.5,0.8,-0.8,-0.1,-0.2) > \\ < q,(0.4,0.7,0.8,-0.8,-0.3,-0.4) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.2,0.5,0.8,-0.8,-0.1,-0.2) > \\ < q,(0.4,0.7,0.8,-0.8,-0.3,-0.4) > \end{Bmatrix}$ is BSVNGSCs in X but not BSVNGPCs in X.

Figure 1: The Diagram represents the implication of the above theorem 4.3.

1. BSVNGPCs 2.BSVNCs 3.BSVNGCs 4.BSVNαCs 5.BSVNαGCs 6.BSVNRCs 7.BSVNPCs 8.BSVNSPCs 9.BSVNGSPCs 10.BSVNSCs 11.BSVNGSCs.

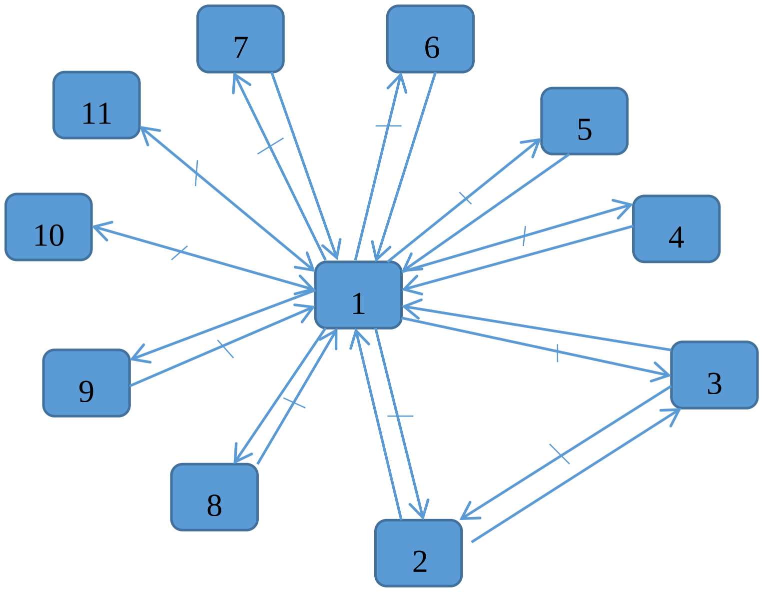

Figure 1

4.12 Remark: The union of any two BSVNGPCs's is not BSVNGPCs in general as seen in the following example.

4.13 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.1,0.5,0.5,-0.7,-0.3,-0.4) > \\ < q,(0.5,0.9,0.8,-0.3,-0.2,-0.1) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.4,0.5,0.4,-0.5,-0.3,-0.6) > \\ < q,(0.7,0.6,0.5,-0.2,-0.4,-0.3) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.2,0.5,0.4,-0.6,-0.2,-0.6) > \\ < q,(0.7,0.9,0.8,-0.9,-0.2,-0.2) > \end{Bmatrix}$, V= $\begin{Bmatrix} < p,(0.1,0.6,0.7,-0.8,-0.3,-0.3) > \\ < q,(0.4,0.9,0.9,-0.3,-0.2,-0.1) > \end{Bmatrix}$ are BSVNGPCs in X but R $\bigcup$ V is not BSVNGPCs in X.

5. Bipolar Single-Valued Neutrosophic generalized Pre-Open Set

5.1 Definition: A BSVNs S is said to be bipolar single-valued neutrosophic generalized pre-open set (BSVNGPOs) in (X, τ) if the complement S^c is BSVNGPCs in (X, τ). The family of all BSVNGPOs's of BSVNTS (X, τ) is denoted by BSVNGPO (X).

5.2 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.1,0.5,0.8,-0.9,-0.4,-0.2) > \\ < q,(0.2,0.6,0.7,-0.9,-0.3,-0.4) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.3,0.3,0.8,-0.2,-0.5,-0.3) > \\ < q,(0.4,0.5,0.5,-0.1,-0.4,-0.4) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.7,0.5,0.3,-0.2,-0.5,-0.7) > \\ < q,(0.7,0.4,0.3,-0.1,-0.7,-0.6) > \end{Bmatrix}$ is BSVNGPOs in X.

5.3 Theorem: For any BSVNTS (X, τ), we have the following results.

(1). Every BSVNOs is BSVNGPOs.

(2). Every BSVNROs is BSVNGPOs.

(3). Every BSVN αOs is BSVNGPOs.

(4). Every BSVNPOs is BSVNGPOs.

5.4 Remark: The converse of the above theorem need not be true which can be seen from the following examples.

5.5 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.1,0.5,0.8,-0.9,-0.4,-0.2) > \\ < q,(0.2,0.6,0.7,-0.9,-0.3,-0.4) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.3,0.3,0.8,-0.2,-0.5,-0.3) > \\ < q,(0.4,0.5,0.5,-0.1,-0.4,-0.4) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.7,0.5,0.3,-0.2,-0.5,-0.7) > \\ < q,(0.7,0.4,0.3,-0.1,-0.7,-0.6) > \end{Bmatrix}$ is BSVNGPOs in X but not BSVNOs in X.

5.6 Example: Let X ={p, q} and

S= $\left\{\begin{matrix} < p,(0.5,0.4,0.5,-0.7,-0.2,-0.3) > \\ < q,(0.2,0.4,0.5,-0.3,-0.1,-0.4) > \end{matrix}\right\}$ T= $\left\{\begin{matrix} < p,(0.5,0.3,0.4,-0.5,-0.3,-0.5) > \\ < q,(0.5,0.3,0.4,-0.2,-0.1,-0.5) > \end{matrix}\right\}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\left\{\begin{matrix} < p,(0.5,0.5,0.5,-0.3,-0.9,-0.4) > \\ < q,(0.8,0.5,0.4,-0.7,-0.9,-0.5) > \end{matrix}\right\}$ is BSVNGPOs in X but not BSVNROs in X.

5.7 Example: Let X ={p, q} and

S= $\left\{\begin{matrix} < p,(0.5,0.4,0.5,-0.7,-0.2,-0.3) > \\ < q,(0.2,0.4,0.5,-0.3,-0.1,-0.4) > \end{matrix}\right\}$ T= $\left\{\begin{matrix} < p,(0.5,0.3,0.4,-0.5,-0.3,-0.5) > \\ < q,(0.5,0.3,0.4,-0.2,-0.1,-0.5) > \end{matrix}\right\}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\left\{\begin{matrix} < p,(0.5,0.5,0.5,-0.3,-0.9,-0.4) > \\ < q,(0.8,0.5,0.4,-0.7,-0.9,-0.5) > \end{matrix}\right\}$ is BSVNGPOs in X but not BSVNαOs in X.

5.8 Example: Let X ={p, q} and

S= $\left\{\begin{matrix} < p,(0.7,0.6,0.5,-0.8,-0.9,-0.7) > \\ < q,(0.4,0.6,0.7,-0.8,-0.9,-0.8) > \end{matrix}\right\}$ T= $\left\{\begin{matrix} < p,(0.7,0.8,0.6,-0.9,-0.9,-0.6) > \\ < q,(0.3,0.7,0.8,-0.9,-0.9,-0.7) > \end{matrix}\right\}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\left\{\begin{matrix} < p,(0.3,0.3,0.5,-0.3,-0.1,-0.3) > \\ < q,(0.5,0.5,0.3,-0.2,-0.1,-0.1) > \end{matrix}\right\}$ is BSVNGPOs in X but not BSVNPOs in X.

5.9 Remark: The intersection of any two BSVNGPOs's is not BSVNGPOs in general and it is shown in the following example.

5.10 Example: Let X ={p, q} and

S= $\left\{\begin{matrix} < p,(0.8,0.4,0.3,-0.1,-0.3,-0.5) > \\ < q,(0.5,0.4,0.3,-0.8,-0.5,-0.6) > \end{matrix}\right\}$ T= $\left\{\begin{matrix} < p,(0.4,0.6,0.7,-0.9,-0.2,-0.4) > \\ < q,(0.4,0.5,0.4,-0.9,-0.4,-0.5) > \end{matrix}\right\}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\left\{\begin{matrix} < p,(0.7,0.3,0.3,-0.1,-0.2,-0.8) > \\ < q,(0.7,0.4,0.3,-0.1,-0.7,-0.7) > \end{matrix}\right\}$, V= $\left\{\begin{matrix} < p,(0.6,0.4,0.1,-0.1,-0.2,-0.9) > \\ < q,(0.9,0.2,0.6,-0.1,-0.6,-0.7) > \end{matrix}\right\}$ are BSVNGPOs in X but R∩V is not BSVNGPOs in X.

5.11 Theorem: Let (X, τ) be BSVNTS. If S ∈ BSVNGPO(X) then V ⊆ BSVN int (BSVN cl (S)) whenever V ⊆ S and V is BSVNCs in X.

Proof. Let S ∈ BSVNGPO(X). Then S^c is BSVNGPCs in X. Therefore BSVN pcl (S^c) ⊆ U whenever S^c ⊆ U and U is BSVNOS in X. That is BSVN cl (BSVN int (S^c)) ⊆ U. This implies U^c ⊆ BSVN int (BSVN cl (S)) whenever U^c ⊆ S and U^c is BSVNCs in X. Replacing U^c by V , we get V ⊆ BSVN int (BSVN cl (S)) whenever V ⊆ S and V is BSVNCs in X.

5.12 Theorem: Let (X, τ) be BSVNTS. Then for every S ∈ BSVNGPO(X) and for every T ∈ BSVNs(X), BSVN pint (S) ⊆ T ⊆ S implies T ⊆ BSVNGPO(X).

Proof. By hypothesis S^c ⊆ T^c ⊆ (BSVN pint (S))^c. Let T^c ⊆ U and U be BSVNOs. Since S^c ⊆ T^c, S^c ⊆ U. But S^c is BSVNGPCs, BSVN pcl (S^c) ⊆ U. Also T^c ⊆ (BSVN pint (S))^c = BSVN pcl (S^c). Therefore BSVN pcl (T^c) ⊆ BSVN pcl (S^c) ⊆ U. Hence T^c is BSVNGPCs. Which implies T is BSVNGPOs of X.

5.13 Theorem: A BSVNs S of BSVNTS (X, τ) is BSVNGPOs if and only if F ⊆ BSVN pint (S) whenever F is BSVNCs and F ⊆ S.

Proof. <u>Necessity:</u> Suppose S is BSVNGPOs in X. Let F be BSVNCs and F ⊆ S. Then F^c is BSVNOs in X such that S^c ⊆ F^c. Since S^c is BSVNGPCs, we have BSVN pcl (S^c) ⊆ F^c. Hence (BSVN pint (S))^c ⊆ F^c. Therefore F ⊆ BSVN pint (S).

<u>Suffciency:</u> Let S be BSVNs of X and let F ⊆ BSVN pint (S) whenever F is BSVNCs and F ⊆ S. Then S^c ⊆ F^c and F^c is BSVNOs. By hypothesis, (BSVN pint (S))^c ⊆ F^c. This implies BSVN pcl (S^c) ⊆ F^c. Therefore S^c is BSVNGPCs of X. Hence S is BSVNGPOs of X.

5.14 Corollary: A BSVNs S of a BSVNTS (X, τ) is BSVNGPOs if and only if F ⊆ BSVN int (BSVN cl (S)) whenever F is BSVNCS and F ⊆ S.

Proof. <u>Necessity:</u> Suppose S is BSVNGPOs in X. Let F be BSVNCs and F ⊆ S. Then F^c is BSVNOs in X such that S^c ⊆ F^c. Since S^c is BSVNGPCs, we have BSVN pcl (S^c) ⊆ F^c. Therefore BSVN cl (BSVN int (S^c)) ⊆ F^c. Hence (BSVN int (BSVN cl (S)))^c ⊆ F^c. This implies F ⊆ BSVN int (BSVN cl (S)).

<u>Suffciency:</u> Let S be BSVNs of X and let F ⊆ BSVN int (BSVN cl (S)) whenever F is BSVNCs and F ⊆ S. Then S^c ⊆ F^c and F^c is BSVNOs. By hypothesis, (BSVN int (BSVN cl (S)))^c ⊆ F^c. Hence BSVN cl (BSVN int (S^c)) ⊆ F^c, which implies BSVN pcl (S^c) ⊆ F^c. Hence S is BSVNGPOs of X.

5.15 Theorem: For a BSVNs S, S is BSVNOs and BSVNGPCs in X if and only if S is BSVNROs in X.

Proof. <u>Necessity:</u> Let S be BSVNOs and BSVNGPCs in X. Then BSVN pcl (S) ⊆ S. This implies BSVN cl (BSVN int (S)) ⊆ S. Since S is BSVNOs, it is BSVNPOs. Hence S ⊆ BSVN int (BSVN cl (S)). Therefore S = BSVN int (BSVN cl (S)). Hence S is BSVNROs in X.

Suffciency: Let S be BSVNROs in X. Therefore S = BSVN int (BSVN cl (S)). Let S $\subseteq$ U and U is BSVNOs in X. This implies BSVN pcl (S) $\subseteq$ S. Hence S is BSVNGPCs in X.

6. Applications Of Bipolar Single-Valued Neutrosophic generalized Pre-Closed Sets

6.1 Definition: A BSVNTS (X, τ) is said to be bipolar single-valued neutrosophic $T_{1/2}$ space

(BSVN $T_{1/2}$ space) if every BSVNGCs in X is BSVNCs in X.

6.2 Definition: A BSVNTS (X, τ) is said to be bipolar single-valued neutrosophic ${}_{p}T_{1/2}$ space

(BSVN ${}_{p}T_{1/2}$ space) if every BSVNPCs in X is BSVNCs in X.

6.3 Definition: A BSVNTS (X, τ) is said to be bipolar single-valued neutrosophic ${}_{gp}T_{1/2}$ space

(BSVN ${}_{gp}T_{1/2}$ space) if every BSVNGPCs in X is BSVNCs in X.

6.4 Definition: A BSVNTS (X, τ) is said to be a bipolar single-valued neutrosophic ${}_{gp}T_{p}$ space

(BSVN ${}_{gp}T_{p}$ space) if every BSVNGPCs in X is BSVNPCs in X.

6.5 Theorem: Every BSVN $T_{1/2}$ space is BSVN ${}_{gp}T_{p}$ space.

Proof. Let X is BSVN $T_{1/2}$ space and let S be BSVNGCs in X, we know that every BSVNGCs is BSVNGPCs; hence S is BSVNGPCs in X. By hypothesis S is BSVNCs in X. Since every BSVNCs is BSVNPCs, S is BSVNPCs in X. Hence X is BSVN ${}_{gp}T_{p}$ space.

6.6 Remark: The converse of the above theorem is not true which is shown in the example.

6.7 Example: Let X ={p, q} and

$$S=\begin{Bmatrix} < p,(0.1,0.3,0.5,-0.3,-0.5,-0.1) > \\ < q,(0.2,0.4,0.6,-0.4,-0.6,-0.3) > \end{Bmatrix} \quad T=\begin{Bmatrix} < p,(0.3,0.2,0.4,-0.2,-0.6,-0.3) > \\ < q,(0.3,0.3,0.5,-0.2,-0.7,-0.3) > \end{Bmatrix}$$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.3,0.4,0.5,-0.6,-0.6,-0.3) > \\ < q,(0.2,0.4,0.3,-0.3,-0.1,-0.2) > \end{Bmatrix}$. Then (X, τ) is BSVN $_{gp}$ T $_{p}$ space. But not BSVN T $_{1/2}$ space. Since R is BSVNGCs but not BSVNCs in X.

6.8 Theorem: Every BSVN $_{gp}$ T $_{1/2}$ space is BSVN $_{gp}$ T $_{p}$ space.

Proof. Let X is BSVN $_{gp}$ T $_{p}$ space and let S be BSVNGPCs in X. By hypothesis S is BSVNCs in X. Since every BSVNCs is BSVNPCs, S is BSVNPCs in X. Hence X is BSVN $_{gp}$ T $_{p}$ space.

6.9 Remark: The converse of the above theorem is not true which is shown in the example.

6.10 Example: Let X ={p, q} and

S= $\begin{Bmatrix} < p,(0.2,0.4,0.7,-0.5,-0.3,-0.4) > \\ < q,(0.6,0.9,0.8,-0.7,-0.1,-0.2) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.3,0.3,0.6,-0.3,-0.4,-0.5) > \\ < q,(0.7,0.8,0.7,-0.6,-0.1,-0.3) > \end{Bmatrix}$

Then τ= {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT on X. The BSVNs

R= $\begin{Bmatrix} < p,(0.1,0.5,0.2,-0.7,-0.3,-0.6) > \\ < q,(0.4,0.8,0.9,-0.2,-0.4,-0.5) > \end{Bmatrix}$. Then (X, τ) is BSVN $_{gp}$ T $_{p}$ space. But not $_{gp}$ T $_{1/2}$ space.

Since R is BSVNGPCs but not BSVNCs in X.

6.11 Theorem: Let (X, τ) be BSVNTS and X is BSVN $_{gp}$ T $_{1/2}$ space then,

(1). Any union of BSVNGPCs's is BSVNGPCs.
(2). Any intersection of BSVNGPOs's is BSVNGPOS.
Proof.

(1). Let $\{A_i\}_{i \in J}$ is a collection of BSVNGPCs's in BSVN $_{gp}$ T $_{1/2}$ space (X, τ). Therefore every BSVNGPCs is BSVNCs. But the union of BSVNCs is BSVNCs. Hence the union of BSVNGPCs is BSVNGPCs in X.
(2). Take complement of (1) to prove.

6.12 Theorem: A BSVNTS X is BSVN $_{gp}$ T $_{1/2}$ space if and only if BSVNGPO(X) = BSVNPO(X).

Proof. Necessity: Let S be BSVNGPOs in X, then S^c is BSVNGPCs in X. By hypothesis S^c is BSVNGPCs in X. Therefore S is BSVNPOs in X. Hence BSVNGPO(X) = BSVNPO(X).

Sufficiency: Let S be BSVNGPCs in X. Then S^c is BSVNGPOs in X. By hypothesis S^c is BSVNGPOs in X. Therefore S is BSVNPCs in X. Hence X is BSVN $_{gp}T_{1/2}$ space.

7. Conclusions

We introduced a new class of sets namely bipolar single-valued neutrosophic generalized closed sets and bipolar single-valued neutrosophic generalized pre- closed sets in bipolar single-valued neutrosophic topological spaces. We also analyzed the properties and its applications with some examples.

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Received: Aug 5, 2021. Accepted: Dec 1, 2021