



# Generalized neutrosophic $b$ -open sets in neutrosophic topological space

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**Abstract:** The purpose of the study is to introduce the notion of generalized neutrosophic  $b$ -open set in neutrosophic topological space. We define generalized neutrosophic  $b$ -open set, generalized neutrosophic  $b$ -interior, generalized neutrosophic  $b$ -closure and investigate some of their properties. By defining generalized neutrosophic  $b$ -open set, we prove some theorems on neutrosophic topological spaces. We also furnish some suitable examples.

**Keywords:** Neutrosophic set; neutrosophic  $b$ -open set; generalized neutrosophic  $b$ -open set; generalized neutrosophic  $b$ -interior; generalized neutrosophic  $b$ -closure

## 1. Introduction

Smarandache (1998) grounded the Neutrosophic Set (NS) in 1998. From then it became very popular and attracted many researchers' attention for theoretical and practical researches (Broumi et al., 2018; Khalid, 2020; Peng & Dai, 2018; Pramanik, 2013; 2016a; 2016b; 2020; Pramanik & Mallick, 2018; 2019; Pramanik & Mondal, 2016; Pramanik & Roy, 2014; Smarandache & Pramanik, 2016; 2018, Biswas, Pramanik & Giri, 2014; 2016a; 2016b; Dalapati et al., 2017; Dey, Pramanik, & Giri, 2016a; 2016b; Pramanik, Mallick, & Dasgupta, 2018; Mondal & Pramanik, 2015; Pramanik & Dalapati, 2018, Pramanik, Dey, & Smarandache, 2018; Pramanik, Mondal, & Smarandache, 2016a; 2016b).

Salama and Alblowi (2012a) grounded the "Neutrosophic Topological Space" (NTS). Salama and Alblowi (2012b) also presented generalized NS and generalized NTSs. Salama, Smarandache, & Alblowi (2014) studied the concept of neutrosophic crisp topological space. Arokiarani, Dhavaseelan, Jafari, and Parimala (2017) defined neutrosophic semi-open functions and established relation between them. Iswaraya and Bageerathi (2016) studied neutrosophic semi-closed set and neutrosophic semi-open set. Rao and Srinivasa (2017) introduced neutrosophic pre-open set and pre-closed set. Dhavaseelan and Jafari (2018) studied generalized neutrosophic closed sets. Pushpalatha and Nandhini (2019) defined the neutrosophic generalized closed sets in NTSs. Shanthi, Chandrasekar, Safina, and Begam (2018) presented the neutrosophic generalized semi closed sets in

NTSs. Ebenanjar, Immaculate, and Wilfred (2018) studied neutrosophic  $b$ -open sets in NTSs. Maheswari, Sathyabama, and Chandrasekar (2018) studied the neutrosophic generalized  $b$ -closed sets in NTSs.

**Research gap:** No investigation on neutrosophic generalized  $b$ -open set has been reported in the recent literature.

**Motivation:** In order to fill the research gap, we introduce neutrosophic generalized  $b$ -open set.

Remaining of the paper is designed as follows:

Section 2 recalls of NTS, neutrosophic  $b$ -closed sets and a theorem. Section 3 introduces neutrosophic generalized  $b$ -open set and proofs of some theorems on neutrosophic  $b$ -open sets. Section 4 presents concluding remarks.

## 2. Preliminaries and some properties

**Definition 2.1** Assume that  $(W, \tau)$  is an NTS. Then  $\chi$ , an NS over  $W$  is said to be a Neutrosophic  $b$ -Open ( $N$ - $b$ -open) set (Ebenanjar, Immaculate, & Wilfred, 2018) if and only if (iff)  $\chi \subseteq N_{int}(N_{cl}(\chi)) \cup N_{cl}(N_{int}(\chi))$ .

**Definition 2.2** In an NTS  $(W, \tau)$ , an NS  $\chi$  is said to be a Neutrosophic  $b$ -Closed ( $N$ - $b$ -closed) set (Ebenanjar, Immaculate, & Wilfred, 2018) iff  $\chi \supseteq N_{int}(N_{cl}(\chi)) \cap N_{cl}(N_{int}(\chi))$ .

**Remark 2.1** An NS  $\chi$  over  $W$  is said to be an  $N$ - $b$ -closed set (Ebenanjar, Immaculate, & Wilfred, 2018) in  $(W, \tau)$  iff  $\chi^c$  is a  $N$ - $b$ -open set in  $(W, \tau)$ .

In 2018, Ebenanjar, Immaculate, and Wilfred (2018) studied the concept of  $N$ - $b$ -open set in NTS but they did not check whether the union or intersection of two  $N$ - $b$ -open sets ( $N$ - $b$ -closed sets) is again an  $N$ - $b$ -open set ( $N$ - $b$ -closed set) or not. In this paper we show some results on the intersection and union of neutrosophic  $b$ -closed sets.

**Theorem 2.1** The intersection of any two  $N$ - $b$ -closed sets is again an  $N$ - $b$ -closed set.

**Proof.** Assume that  $E, F$  be any two  $N$ - $b$ -closed sets in an NTS  $(W, \tau)$ . Then we have

$$E \supseteq N_{int}(N_{cl}(E)) \cap N_{cl}(N_{int}(E)) \tag{1}$$

$$\text{and } F \supseteq N_{int}(N_{cl}(F)) \cap N_{cl}(N_{int}(F)) \tag{2}$$

For any two NSs  $E$  and  $F$  We know that  $E \cap F \subseteq E$  and  $E \cap F \subseteq F$ .

$$\text{Now } E \cap F \subseteq E \Rightarrow N_{int}(E \cap F) \subseteq N_{int}(E) \Rightarrow N_{cl}(N_{int}(E \cap F)) \subseteq N_{cl}(N_{int}(E)) \tag{3}$$

$$E \cap F \subseteq E \Rightarrow N_{cl}(E \cap F) \subseteq N_{cl}(E) \Rightarrow N_{int}(N_{cl}(E \cap F)) \subseteq N_{int}(N_{cl}(E)) \tag{4}$$

$$E \cap F \subseteq F \Rightarrow N_{int}(E \cap F) \subseteq N_{int}(F) \Rightarrow N_{cl}(N_{int}(E \cap F)) \subseteq N_{cl}(N_{int}(F)) \tag{5}$$

$$E \cap F \subseteq F \Rightarrow N_{cl}(E \cap F) \subseteq N_{cl}(F) \Rightarrow N_{int}(N_{cl}(E \cap F)) \subseteq N_{int}(N_{cl}(F)) \tag{6}$$

From (1) and (2) we have,

$$\begin{aligned} E \cap F &\supseteq N_{int}(N_{cl}(E)) \cap N_{cl}(N_{int}(E)) \cap N_{int}(N_{cl}(F)) \cap N_{cl}(N_{int}(F)) \\ &\supseteq N_{int}(N_{cl}(E \cap F)) \cap N_{cl}(N_{int}(E \cap F)) \cap N_{int}(N_{cl}(E \cap F)) \cap N_{cl}(N_{int}(E \cap F)) \\ &\hspace{15em} [\text{ by eqs (3), (4), (5) \& (6)}] \\ &= N_{int}(N_{cl}(E \cap F)) \cap N_{cl}(N_{int}(E \cap F)) \end{aligned}$$

$$\Rightarrow E \cap F \supseteq N_{cl}(N_{int}(E \cap F)) \cap N_{int}(N_{cl}(E \cap F)).$$

Therefore  $E \cap F$  is an  $N$ - $b$ -closed set.

Hence the intersection of any two N-*b*-closed sets is again an N-*b*-closed set.

**Remark 2.2:** The union of any two N-*b*-closed sets may not be an N-*b*-closed set. This is proved as follows:

**Example 2.1:** Assume that  $W = \{p_1, p_2\}$  and  $\tau = \{0_N, 1_N, \{(p_1, 0.5, 0.2, 0.4), (p_2, 0.6, 0.1, 0.3)\}, \{(p_1, 0.3, 0.5, 0.6), (p_2, 0.4, 0.4, 0.5)\}\}$  be the family of some NSs over  $W$ . Then  $\tau$  is an NT on  $W$ . Now it can be verified that  $E = \{(a, 0.6, 0.5, 0.6), (b, 0.5, 0.6, 0.7)\}$ ,  $F = \{(a, 1, 0, 1), (b, 0.9, 0.1, 0.1)\}$  are two N-*b*-closed sets in  $(W, \tau)$ . But their union  $E \cup F = \{(a, 1, 0, 0.6), (b, 0.9, 0.1, 0.1)\}$  is not an N-*b*-closed set.

**Definition 2.3** Assume that  $(W, \tau)$  is an NTS and  $\chi$  is an NS over  $W$ . Then the Neutrosophic *b*-Closure ( $N_{bcl}$ ) and Neutrosophic *b*-Interior ( $N_{bint}$ ) (Ebenanjar, Immaculate & Wilfred, 2018) of  $\chi$  are defined by

$$N_{bcl}(\chi) = \cap \{ \psi : \psi \text{ is an N-}b\text{-closed set in } (W, \tau) \text{ and } \chi \subseteq \psi \};$$

$$N_{bint}(\chi) = \cup \{ \xi : \xi \text{ is an N-}b\text{-open set in } (W, \tau) \text{ and } \xi \subseteq \chi \}.$$

**Remark 2.3** Clearly  $N_{bint}(\chi)$  is the largest N-*b*-open set (Ebenanjar, Immaculate, & Wilfred, 2018) in  $(W, \tau)$  which is contained in  $\chi$  and  $N_{bcl}(\chi)$  is the smallest N-*b*-closed set in  $(W, \tau)$  which contains  $\chi$ .

**Definition 2.4** Assume that  $(W, \tau)$  is an NTS. A neutrosophic subset  $E$  of  $(W, \tau)$  is said to be a Neutrosophic Generalized Closed Set (NGCS) (Dhavaseelan & Jafari, 2018) if  $N_{cl}(E) \subseteq F$  whenever  $E \subseteq F$  and  $F$  is an NOS. A subset  $K$  of  $(W, \tau)$  is called Neutrosophic Generalized Open Set (NGOS) iff  $K^c$  is an NGCS in  $(W, \tau)$ .

### 3. Generalized neutrosophic *b*-open set

**Definition 3.1** Assume that  $(W, \tau)$  is an NTS. An NS  $G$  over  $W$  is called a Generalized Neutrosophic *b*-Open (g-N-*b*-open) set if  $\exists$  an N-*b*-closed set  $H$  (except  $1_N$ ) with  $G \subseteq H$  such that  $G \subseteq N_{int}(H)$ . A neutrosophic subset  $K$  in  $(W, \tau)$  is called a Generalized Neutrosophic *b*-Closed (g-N-*b*-closed) set iff  $K^c$  is a g-N-*b*-open set in  $(W, \tau)$ .

**Example 3.1** Assume that  $W = \{p_1, p_2\}$  and  $\tau = \{0_N, 1_N, \{(p_1, 0.5, 0.6, 0.7), (p_2, 0.6, 0.7, 0.8)\}, \{(p_1, 0.6, 0.5, 0.6), (p_2, 0.7, 0.6, 0.7)\}\}$  are the collection of some NSs over  $W$ . Then  $(W, \tau)$  is clearly an NTS. Here  $K = \{(p_1, 0.6, 0.7, 0.8), (p_2, 0.5, 0.8, 0.8)\}$  is a g-N-*b*-open set, because there exists an N-*b*-closed set  $G = \{(p_1, 0.7, 0.3, 0.4), (p_2, 0.8, 0.3, 0.4)\}$  in  $(W, \tau)$  with  $K \subseteq G$  such that  $K \subseteq N_{int}(G)$ .

**Proposition 3.1** In an NTS  $(W, \tau)$ ,  $0_N$  is a g-N-*b*-open set but  $1_N$  is not a g-N-*b*-open set.

**Proof.** Assume that  $(W, \tau)$  is an NTS. Since a Neutrosophic Open Set (NOS) is an N-*b*-open set, so  $1_N$  is an N-*b*-open set. Therefore,  $0_N$  is an N-*b*-closed set (since it is the complement of N-*b*-open set  $1_N$ ). Now  $0_N \subseteq 0_N$  and  $0_N \subseteq N_{int}(0_N) = 0_N$ .

Thus there exist an N-*b*-closed set  $0_N$  (except  $1_N$ ) with  $0_N \subseteq 0_N$  such that  $0_N \subseteq N_{int}(0_N)$ . Hence  $0_N$  is a g-N-*b*-open set in  $(W, \tau)$ .

But in case of NS  $1_N$ , we cannot find any neutrosophic *b*-closed set  $H$  (except  $1_N$ ) with  $1_N \subseteq H$  such that  $1_N \subseteq N_{int}(H)$ . Hence  $1_N$  is not a g-N-*b*-open set in  $(W, \tau)$ .

**Proposition 3.2** Assume that  $\psi$  is a g-N-*b*-open set in an NTS  $(W, \tau)$ . Then, every NS contained in  $\psi$  is a g-N-*b*-open set.

**Proof.** Assume that  $\psi$  be a g-N- $b$ -open set in an NTS  $(W, \tau)$  and  $\xi$  be any arbitrary NS over  $W$  which is contained in  $\psi$ . Since  $\psi$  is a g-N- $b$ -open set, so there exists an N- $b$ -closed set  $\eta$  (except  $1_N$ ) with  $\psi \subseteq \eta$  such that  $\psi \subseteq N_{int}(\eta)$ .

Now  $\xi$  is contained in  $A$ , so

$$\begin{aligned} &\xi \subseteq \psi \\ \Rightarrow &\xi \subseteq \psi \subseteq \eta \ \& \ \xi \subseteq \psi \subseteq N_{int}(\eta). \end{aligned}$$

Therefore there exists an N- $b$ -closed set  $\eta$  (except  $1_N$ ) with  $\xi \subseteq \eta$  such that  $\xi \subseteq N_{int}(\eta)$ . Hence  $\xi$  is a g-N- $b$ -open set. Thus each NS contained in  $\psi$  is again a g-N- $b$ -open set in  $(W, \tau)$ .

**Definition 3.2** Assume that  $(W, \tau)$  is an NTS and  $\psi$  be an NS over  $W$ . Then the Generalized Neutrosophic  $b$ -Interior ( $g-N_{bint}$ ) and Generalized Neutrosophic  $b$ - Closure ( $g-N_{bcl}$ ) of  $\psi$  are defined by

$$\begin{aligned} g-N_{bint}(\psi) &= \cup \{ \xi : \xi \text{ is a g-N-}b\text{-open set and } \xi \subseteq \psi \}; \\ g-N_{bcl}(\psi) &= \cap \{ \eta : \eta \text{ is a g-N-}b\text{-closed set and } \psi \subseteq \eta \}. \end{aligned}$$

**Theorem 3.1** Assume that  $(W, \tau)$  is an NTS. Then each neutrosophic open subset of  $(W, \tau)$  is a g-N- $b$ -open set.

**Proof.** Assume that  $\psi$  be an arbitrary NOS in an NTS  $(W, \tau)$ . So  $\psi = N_{int}(\psi)$ . Since each neutrosophic closed set is an N- $b$ -closed set so  $N_{cl}(\psi)$  is an N- $b$ -closed set. Also we know that  $\psi \subseteq N_{cl}(\psi)$ .

$$\begin{aligned} \text{Now } &\psi \subseteq N_{cl}(\psi) \\ \Rightarrow &N_{int}(\psi) \subseteq N_{int}(N_{cl}(\psi)) \\ \Rightarrow &\psi = N_{int}(\psi) \subseteq N_{int}(N_{cl}(\psi)) \\ \Rightarrow &\psi \subseteq N_{int}(N_{cl}(\psi)) \end{aligned}$$

Therefore there exists an N- $b$ -closed set  $N_{cl}(\psi)$  with  $\psi \subseteq N_{cl}(\psi)$  such that  $\psi \subseteq N_{int}(N_{cl}(\psi))$ . Hence  $\psi$  is a g-N- $b$ -open set in  $(W, \tau)$ . Thus each neutrosophic open subset of  $(W, \tau)$  is again a g-N- $b$ -open set.

**Remark 3.1** The converse of the theorem 3.1 is not true. This can be shown by the example 3.2.

**Example 3.2** In example 3.1, it can be easily seen that  $K = \{(a, 0.6, 0.7, 0.8), (b, 0.5, 0.8, 0.8)\}$  is a g-N- $b$ -open set in  $(W, \tau)$  but it is not an NOS.

**Theorem 3.2** Assume that  $(W, \tau)$  is an NTS. Then each Neutrosophic Pre-Open Set (NPOS) in  $(W, \tau)$  is a g-N- $b$ -open set.

**Proof.** Assume that  $(W, \tau)$  is an NTS and  $\psi$  is an NPOS. Then  $\psi \subseteq N_{int}(N_{cl}(\psi))$ . Since for any NS  $\psi$ ,  $N_{cl}(\psi)$  is an N- $b$ -closed set and  $\psi \subseteq N_{cl}(\psi)$ . Therefore there exists an N- $b$ -closed set  $N_{cl}(\psi)$  with  $\psi \subseteq N_{cl}(\psi)$  such that  $\psi \subseteq N_{int}(N_{cl}(\psi))$ . Hence  $\psi$  is a g-N- $b$ -open set in  $(W, \tau)$ . Thus each NPOS in  $(W, \tau)$  is again a g-N- $b$ -open set.

**Theorem 3.3** If  $\psi$  is both NOS and Neutrosophic Semi-Open Set (NSOS) in an NTS  $(W, \tau)$  then it is a g-N- $b$ -open set.

**Proof.** Assume that  $(W, \tau)$  is an NTS and  $\psi$  is both NSOS and NOS. Since  $\psi$  is an NOS, so  $\psi = N_{int}(\psi)$ . Again since  $\psi$  is an NSOS, so  $\psi \subseteq N_{cl}(N_{int}(\psi))$ . It can be verified that  $N_{cl}(N_{int}(\psi))$  is an N- $b$ -closed set (since it is an NCS).

$$\begin{aligned} \text{Now } \psi &\subseteq N_{cl}(N_{int}(\psi)) \\ \Rightarrow N_{int}(\psi) &\subseteq N_{int}(N_{cl}(N_{int}(\psi))) && [\text{ since } \psi \subseteq \delta \Rightarrow N_{int}(\psi) \subseteq N_{int}(\delta) ] \\ \Rightarrow \psi = N_{int}(\psi) &\subseteq N_{int}(N_{cl}(N_{int}(\psi))) && [\text{ since } \psi = N_{int}(\psi) ] \\ \Rightarrow \psi &\subseteq N_{int}(N_{cl}(N_{int}(\psi))) \end{aligned}$$

Therefore there exists an N- $b$ -closed set  $N_{cl}(N_{int}(\psi))$  with  $\psi \subseteq N_{cl}(N_{int}(\psi))$  in  $(W, \tau)$  such that  $\psi \subseteq N_{int}(N_{cl}(N_{int}(\psi)))$ . Hence  $\psi$  is a g-N- $b$ -open set.

**Theorem 3.4** Assume that  $(W, \tau)$  is an NTS and  $\psi$  is both neutrosophic  $\alpha$ -open and neutrosophic open set. Then  $\psi$  is again a g-N- $b$ -open set.

**Proof.** Assume that  $\psi$  is an arbitrary NS which is both neutrosophic  $\alpha$ -open set and NOS. Since  $\psi$  is an NOS so  $\psi = N_{int}(\psi)$ . Again since  $\psi$  is a neutrosophic  $\alpha$ -open set, so  $\psi \subseteq N_{int}(N_{cl}(N_{int}(\psi)))$ . Hence, it is clear that  $N_{cl}(N_{int}(\psi))$  is an N- $b$ -closed set (since it is an NCS) in  $(W, \tau)$ .

$$\begin{aligned} \text{Now } \psi &= N_{int}(\psi) \\ \Rightarrow \psi = N_{int}(\psi) &\subseteq N_{cl}(N_{int}(\psi)) \\ \Rightarrow \psi &\subseteq N_{cl}(N_{int}(\psi)) \end{aligned}$$

Therefore there exists an N- $b$ -closed set  $N_{cl}(N_{int}(\psi))$  with  $\psi \subseteq N_{cl}(N_{int}(\psi))$  such that  $\psi \subseteq N_{int}(N_{cl}(N_{int}(\psi)))$ . Hence  $\psi$  is a generalized N- $b$ -open set in  $(W, \tau)$ .

**Theorem 3.5** The intersection of any two g-N- $b$ -open sets in an NTS  $(W, \tau)$  is again a g-N- $b$ -open set.

**Proof.** Let  $\psi$  and  $\xi$  be any two g-N- $b$ -open sets in an NTS  $(W, \tau)$ . Then there exist two N- $b$ -closed sets  $K, L$  with  $\psi \subseteq K$ ,  $\xi \subseteq L$  such that  $\psi \subseteq N_{int}(K)$  and  $\xi \subseteq N_{int}(L)$ .

$$\text{Here } \psi \cap \xi \subseteq K \cap L.$$

We know that the intersection of two N- $b$ -closed sets is again an N- $b$ -closed set. So  $K \cap L$  is an N- $b$ -closed set in  $(W, \tau)$ .

$$\begin{aligned} \text{Now } \psi \cap \xi &\subseteq N_{int}(K) \cap N_{int}(L) [\text{ since } \psi \subseteq N_{int}(K), \xi \subseteq N_{int}(L)] \\ &= N_{int}(K \cap L) \\ \Rightarrow \psi \cap \xi &\subseteq N_{int}(K \cap L). \end{aligned}$$

Therefore there exists an N- $b$ -closed set  $K \cap L$  with  $\psi \cap \xi \subseteq K \cap L$  such that  $\psi \cap \xi \subseteq N_{int}(K \cap L)$ . Hence  $\psi \cap \xi$  is a g-N- $b$ -open set in  $(W, \tau)$ . Thus the intersection of any two g-N- $b$ -open sets in  $(W, \tau)$  is again a g-N- $b$ -open set.

**Theorem 3.6** The union of two g-N- $b$ -open sets is a g-N- $b$ -open set if one is contained in the other.

**Proof.** Let  $\psi, \xi$  are any two g-N- $b$ -open sets in  $(W, \tau)$  such that  $\psi \subseteq \xi$ . Since  $\psi$  and  $\xi$  are g-N- $b$ -open sets, so there exist two N- $b$ -closed sets  $G_1, G_2$  with  $\psi \subseteq G_1$  and  $\xi \subseteq G_2$  such that  $\psi \subseteq N_{int}(G_1)$  and  $\xi \subseteq N_{int}(G_2)$ .

$$\begin{aligned} \text{Now } \psi \cup \xi &\subseteq \xi [\text{ since } \psi \subseteq \xi ] \\ &\subseteq G_2 \\ \Rightarrow \psi \cup \xi &\subseteq G_2 \end{aligned}$$

Again  $\psi \cup \xi \subseteq \xi \subseteq N_{int}(G_2)$ , where  $G_2$  is an N- $b$ -closed set in  $(X, \tau)$ .

Therefore there exists an N-*b*-closed set  $G_2$  with  $\psi \cup \xi \subseteq G_2$  in  $(X, \tau)$  such that  $\psi \cup \xi \subseteq N_{\text{int}}(G_2)$ .

Hence the union of two g-N-*b*-open sets is again a g-N-open set if one is contained in the other.

**Definition 3.3** An NS  $\chi$  is called a g-N-*b*-open set relative to an NS  $\psi$  if there exists an N-*b*-closed set  $\xi$  with  $\chi \subseteq \psi \cap \xi$  such that  $\chi \subseteq N_{\text{int}}(\psi \cap \xi)$ .

**Theorem 3.7** Assume that  $(W, \tau)$  is an NTS. If  $\xi$  is a g-N-*b*-open set relative to  $\psi$  and  $\psi$  is a g-N-*b*-open set relative to  $\chi$  then  $\xi$  is a g-N-*b*-open set relative to  $\chi$ .

**Proof.** Since  $\xi$  is a g-N-*b*-open set relative to  $\psi$  so there exists an N-*b*-closed set  $K$  with  $\xi \subseteq \psi \cap K$  such that  $\xi \subseteq N_{\text{int}}(\psi \cap K)$ . Similarly, since  $\psi$  is a g-N-*b*-open set relative to  $\chi$  then there exists an N-*b*-closed set  $L$  with  $\psi \subseteq \chi \cap L$  such that  $\psi \subseteq N_{\text{int}}(\chi \cap L)$ .

We know that the intersection of two N-*b*-closed sets is again an N-*b*-closed set. So  $K \cap L$  is an N-*b*-closed set.

$$\begin{aligned} \text{Now } \xi &\subseteq \psi \cap K \subseteq \chi \cap L \cap K \\ &= \chi \cap (L \cap K) \\ &= \chi \cap G, \text{ where } G = K \cap L \text{ is an N-}b\text{-closed set.} \end{aligned}$$

$$\begin{aligned} \text{Again } \xi &\subseteq N_{\text{int}}(\psi \cap K) \\ &\subseteq N_{\text{int}}(\chi \cap G). \end{aligned}$$

Therefore there exists an N-*b*-closed set  $G$  with  $\xi \subseteq \chi \cap G$  such that  $\xi \subseteq N_{\text{int}}(\chi \cap G)$

Hence  $\xi$  is a g-N-*b*-open relative to  $\chi$ .

#### 4. Conclusion

In this article, we introduce generalized neutrosophic *b*-open set, generalized neutrosophic *b*-interior, generalized neutrosophic *b*-closure and investigate some of their properties. By defining generalized neutrosophic *b*-open set, we prove some theorems on NTSs and few illustrative examples are provided. In the future, we hope that based on these notions in NTSs, many new investigations can be carried out.

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