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Generalized Neutrosophic Competition Graphs

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Abstract: The generalized neutrosophic graph is a generalization of the neutrosophic graph that represents a system perfectly. In this study, the concept of a neutrosophic digraph, generalized neutrosophic digraph and out-neighbourhood of a vertex of a generalized neutrosophic digraph is studied. The generalized neutrosophic competition graph and matrix representation are analyzed. Also, the minimal graph and competition number corresponding to generalized neutrosophic competition graph are defined with some properties. At last, an application in real life is discussed.

Keywords: Competition graph, neutrosophic graph, generalized neutrosophic competition graph, competition number.

1. Introduction

Graph theory is a significant part of applied mathematics, and it is applied as a tool for solving many problems in geometry, algebra, computer science, social networks [1] and optimization etc. Cohen (1968) introduced the concept of competition graph [2] with application in an ecosystem which was related to the competition among species in a food web. If two species have at least one common prey, then there is a competition between them. Let $\vec{G} = (V, \vec{E})$ be a digraph, which corresponds to a food web. A vertex $x \in V$ represents a species in the food web and an arc $(\overrightarrow{x,s}) \in \vec{E}$ means x preys on the species s. The competition graph $C(\vec{G})$ of a digraph $\vec{G}$ is an undirected graph $G = (V, E)$ which has same vertex set and has an edge between two distinct vertices $x, y \in V$ if there exists a vertex $s \in V$ and arcs $(\overrightarrow{x,s}), (\overrightarrow{y,s}) \in \vec{E}$.

Roberts et al. (1976,1978) studied that for any graph with isolated vertices is the competition graph [3, 4] and the minimum number of such vertices is called competition number. Opsut (1982) discussed the computation of competition number [5] of a graph. Kim et al. (1993,1995) introduced the p-competition graph [6] and also p-competition number [7]. Brigham et al. (1995) introduced $\emptyset - tolerance$ graph as a generalization of p-competition [8]. Cho and Kim (2005) studied competition number [9] of a graph having one hole. Li and Chang (2009) proposed about competition graph [10] with h holes. Factor and Merz introduced $(1,2)$ step competition graph [11] of a tournament and extended to $(1,2)$ $-$step competition graph.

In real life, it is full of imprecise data which motivated to define fuzzy graph [12] by Kaufman (1973) where all the vertices and edges of the graph have some degree of memberships. There are lots of research works on fuzzy graphs [13]. In 2006, Parvathi and Karunambigal introduced intuitionistic fuzzy graph [14] where all the vertices and edges of the graph have some degree of memberships and degree of non-memberships. The concepts of interval-valued fuzzy graphs [15] were introduced by Akram and Dubek (2011) where the membership values of vertices and edges are interval numbers. Even the representation of competition by competition does not show the characteristic properly. Considering in food web, species and prey are all fuzzy in nature, Samanta and Pal (2013) represent competition [16] in a more realistic way in fuzzy environment. After that, as a generalization of the fuzzy graph, Samanta and Sarkar (2016, 2018) proposed the generalized fuzzy graph [17] and generalized fuzzy competition graph [18] where the membership values of edges are functions of membership values of vertices. Pramanik et al. introduced fuzzy $\emptyset - tolerance$ competition graphs with the idea of fuzzy tolerance graphs [19].

Smarandache (1998) proposed the concept of a neutrosophic set [20] which has three components: the degree of truth membership, degree of falsity membership and degree of indeterminacy membership. The neutrosophic set is the generalization of fuzzy set [21] and intuitionistic fuzzy set [22].

The neutrosophic environment has several applications in real life including evaluation of the green supply chain management practices [23], evaluation Hospital medical care systems based on plithogenic sets [24], decision-making approach with quality function deployment for selecting supply chain sustainability metrics [25], intelligent medical decision support model based on soft computing and IoT [26], utilizing neutrosophic theory to solve transition difficulties of IoT-based enterprises [27], etc.

As a generalization of the fuzzy graph and intuitionistic fuzzy graph, Broumi et al. (2015) defined the single-valued neutrosophic graph [28]. The definition of a neutrosophic graph by Broumi et al. is different in the definition of neutrosophic graph [29] by Akram. Also, the presentation of competition [30] by neutrosophic graph was introduced by Akram and Siddique (2017). In that paper, the authors did not follow the same definition of Broumi. In these papers, there were restrictions on T, I, F values. To remove the restrictions on T, I, F values, Broumi et al. (2018) introduced the generalized neutrosophic graph [31] using the concept of generalized fuzzy graph. The concepts of generalized neutrosophic graph motivate us to introduce the generalized neutrosophic competition graph. There are few papers available for readers on neutrosophic graph theory [32-34].

The rest of the study is organized as follows. In the second section, the main problem definition is described. In section 3, the basic concepts related to the neutrosophic graph, neutrosophic directed graph, generalized neutrosophic graph, a generalized neutrosophic directed graph is discussed with example. In this section, the generalized neutrosophic competition graph is proposed and corresponding minimal graphs, competition number is studied. In section 4, a matrix representation of the generalized neutrosophic competition graph is proposed with a suitable example. In section 5, an application in economic growth is studied. In the last section, the conclusion of the proposed study and future directions is depicted.

A gist of contribution (Table 1) of authors is presented below.

Table 1. Contribution of authors to competition graphs

Authors	Year	Contributions
Cohen	1968	Introduced competition graph.
Kauffman	1973	Introduced fuzzy graphs
Smarandache	1998	Introduced the concepts of neutrosophic set
Parvathi and Karunambigal	2006	Introduced intuitionistic fuzzy graph
Samanta and Pal	2013	Introduced fuzzy competition graph
Broumi et al.	2015	Introduced neutrosophic graph
Samanta and Sarkar	2016	Introduced the generalized fuzzy graph
Akram and Siddique	2017	Introduced neutrosophic competition graph
Samanta and Sarkar	2018	Introduced representation of competition by a generalized fuzzy graph
Broumi et al.	2018	Introduced Generalized neutrosophic graph
Das et al.	This paper	Introduced generalized neutrosophic competition graph

2. Generalized neutrosophic competition graph

Definition 1.[28] A graph $G = (\mathrm{V}, E)$ where $E \subseteq V \times V$ is said to be neutrosophic graph if

i) there exist functions $\rho_T: V \to [0,1], \rho_F: V \to [0,1] and \rho_I: V \to [0,1]$ such that

$0 \leq \rho_T(v_i) + \rho_F(v_i) + \rho_I(v_i) \leq 3$ for all $v_i \in V$ $(i = 1,2,3, \dots, n)$

where $\rho_T(v_i)$, $\rho_F(v_i)$, $\rho_I(v_i)$ denote the degree of true membership, degree of falsity membership and degree of indeterminacy membership of the vertex $v_i \in V$ respectively.

ii) there exist functions $\mu_T: E \to [0,1], \mu_F: E \to [0,1]$ and $\mu_I: E \to [0,1]$ such that

$$\mu_T(v_i, v_j) \leq \min\ [\rho_T(v_i),\ \rho_T(v_j)]$$

$\mu_F(v_i, v_j) \geq max[\rho_F(v_i),\ \rho_F(v_j)]$

$\mu_I(v_i, v_j) \geq max[\rho_I(v_i),\ \rho_I(v_j)]$

and $0 \leq \mu_T(v_i, v_j) + \mu_F(v_i, v_j) + \mu_I(v_i, v_j) \leq 3$ for all $(v_i, v_j) \in E$

where $\mu_T(v_i, v_j)$, $\mu_F(v_i, v_j)$, $\mu_I(v_i, v_j)$ denote the degree of true membership, degree of falsity membership and degree of indeterminacy membership of the edge $(v_i, v_j) \in E$ respectively.

Definition 2.[31] A graph $G = (\mathrm{V}, E)$ where $E \subseteq V \times V$ is said to be generalized neutrosophic graph if there exist functions

$$\rho_T: V \to [0,1], \rho_F: V \to [0,1] and \rho_I: V \to [0,1],$$

$$\mu_T: E \to [0,1], \mu_F: E \to [0,1] \ and\ \mu_I: E \to [0,1]$$

$$\phi_T: E_T \to [0,1], \phi_F: E_F \to [0,1] \ and\ \phi_I: E_I \to [0,1]$$

such that

$0 \leq \rho_T(v_i) + \rho_F(v_i) + \rho_I(v_i) \leq 3$ for all $v_i \in V$ $(i = 1,2,3, \dots, n)$

and

$$\mu_T(v_i, v_j) = \phi_T(\rho_T(v_i),\ \rho_T(v_j))$$
$$\mu_F(v_i, v_j) = \phi_F(\rho_F(v_i),\ \rho_F(v_j))$$
$$\mu_I(v_i, v_j) = \phi_I(\rho_I(v_i),\ \rho_I(v_j))$$

where $E_T = \{(\rho_T(v_i),\ \rho_T(v_j)) : \mu_T(v_i, v_j) \geq 0\}$, $E_F = \{(\rho_F(v_i),\ \rho_F(v_j)) : \mu_F(v_i, v_j) \geq 0\}$, $E_I = \{(\rho_I(v_i),\ \rho_I(v_j)) : \mu_I(v_i, v_j) \geq 0\}$ and $\rho_T(v_i),\ \rho_F(v_i),\ \rho_I(v_i)$ denote the degree of true membership, the degree of falsity membership, the indeterminacy membership of vertex $v_i \in V$ respectively and $\mu_T(v_i, v_j),\ \mu_F(v_i, v_j),\ \mu_I(v_i, v_j)$ denote the degree of true membership, the degree of falsity membership and the degree of indeterminacy membership of edge$(v_i, v_j) \in E$ respectively.

Definition 3. A graph $\vec{G} = (V, \vec{E})$ where $\vec{E} \subseteq V \times V$ is said to be neutrosophic digraph if

i) there exist functions $\rho_T : V \to [0,1], \rho_F : V \to [0,1]$ and $\rho_I : V \to [0,1]$ such that $0 \leq \rho_T(v_i) + \rho_F(v_i) + \rho_I(v_i) \leq 3$ for all $v_i \in V$ $(i = 1,2,3, \ldots, n)$
where $\rho_T(v_i),\ \rho_F(v_i),\ \rho_I(v_i)$ denote the degree of true membership, degree of falsity membership and degree of indeterminacy membership of the vertex v_i respectively.

ii) there exist functions $\mu_T : \vec{E} \to [0,1], \mu_F : \vec{E} \to [0,1]$ and $\mu_I : \vec{E} \to [0,1]$such that

$$\mu_T(\overrightarrow{v_i, v_j}) \leq \min\ [\rho_T(v_i),\ \rho_T(v_j)]$$
$$\mu_F(\overrightarrow{v_i, v_j}) \geq max[\rho_F(v_i),\ \rho_F(v_j)]$$
$$\mu_I(\overrightarrow{v_i, v_j}) \geq max[\ \rho_I(v_i),\ \rho_I(v_j)]$$

and $0 \leq \mu_T(\overrightarrow{v_i, v_j}) + \mu_F(\overrightarrow{v_i, v_j}) + \mu_I(\overrightarrow{v_i, v_j}) \leq 3$ for all $(v_i, v_j) \in E$
where $\mu_T(\overrightarrow{v_i, v_j}),\ \mu_F(\overrightarrow{v_i, v_j}),\ \mu_I(\overrightarrow{v_i, v_j})$ denote the degree of true membership, degree of falsity membership and degree of indeterminacy membership of the edge $(\overrightarrow{v_i, v_j}) \in \vec{E}$ respectively.

Example 1. Consider a graph (Fig.1) $\vec{G} = (V, \vec{E})$ where $V = \{v_1, v_2, v_3, v_4\}$ and $\vec{E} = \{(\overrightarrow{v_1, v_2}), (\overrightarrow{v_1, v_3}), (\overrightarrow{v_2, v_3}), (\overrightarrow{v_3, v_4})\}$. The membership values of vertices (Table 2) and edges (Table 3) and the corresponding graph are given following.

Table 2. Membership values of vertices of a graph (Fig.1)

	v_1	v_2	v_3	v_4
ρ_T	0.4	0.3	0.5	0.3
ρ_F	0.3	0.1	0.6	0.4
ρ_I	0.2	0.4	0.4	0.6

Table 3. membership values of edges of a graph (Fig.1)

	$(\overrightarrow{v_1, v_2})$	$(\overrightarrow{v_1, v_3})$	$(\overrightarrow{v_2, v_3})$	$(\overrightarrow{v_3, v_4})$
μ_T	0.3	0.3	0.2	0.3
μ_F	0.4	0.6	0.6	0.6
μ_I	0.4	0.5	0.5	0.6

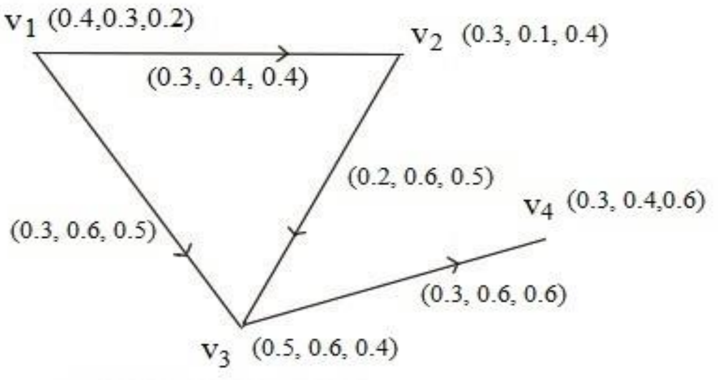

Figure.1. A neutrosophic digraph

Definition 4. A graph $\overrightarrow{G'}$ = (V,$\vec{E}$) where $\vec{E} \subseteq V \times V$ is said to be generalized neutrosophic digraph if there exist functions

$$\rho_T: V \to [0,1], \rho_F: V \to [0,1] and \rho_I: V \to [0,1],$$

$$\mu_T: \vec{E} \to [0,1], \mu_F: \vec{E} \to [0,1] \; and \; \mu_I: \vec{E} \to [0,1]$$
$$\phi_T: E_T \to [0,1], \phi_F: E_F \to [0,1] \; and \; \phi_I: E_I \to [0,1]$$

such that

$$0 \leq \rho_T(v_i) + \rho_F(v_i) + \rho_I(v_i) \leq 3 \text{ for all } v_i \in V \; (i = 1,2,3, \ldots., n)$$

and

$$\mu_T(\overrightarrow{v_i, v_j}) = \phi_T(\rho_T(v_i), \rho_T(v_j))$$
$$\mu_F(\overrightarrow{v_i, v_j}) = \phi_F(\rho_F(v_i), \rho_F(v_j))$$
$$\mu_I(\overrightarrow{v_i, v_j}) = \phi_I(\rho_I(v_i), \rho_I(v_j))$$

where $E_T = \{(\rho_T(v_i), \rho_T(v_j)): \mu_T(v_i, v_j) \geq 0\}$, $E_F = \{(\rho_F(v_i), \rho_F(v_j)): \mu_F(v_i, v_j) \geq 0\}$, $E_I = \{(\rho_I(v_i), \rho_I(v_j)): \mu_I(v_i, v_j) \geq 0\}$ and $\rho_T(v_i)$, $\rho_F(v_i)$, $\rho_I(v_i)$ denote the degree of true membership, the degree of falsity membership, the indeterminacy membership of vertex $v_i \in V$ respectively and $\mu_T(\overrightarrow{v_i, v_j})$, $\mu_F(\overrightarrow{v_i, v_j})$, $\mu_I(\overrightarrow{v_i, v_j})$ denote the degree of true membership, the degree of falsity membership and the degree of indeterminacy membership of edge$\overrightarrow{(v_i, v_j)} \in \vec{E}$ respectively.

Example 2. Consider a graph (Fig.2)$\vec{G} = (V, \vec{E})$ where $V = \{v_1, v_2, v_3, v_4\}$ and $\vec{E} = \{(\overrightarrow{v_1, v_2}), (\overrightarrow{v_1, v_3}), (\overrightarrow{v_4, v_1}), (\overrightarrow{v_3, v_2})\}$.

Consider the membership values of vertices (Table 4) are given below:

Table 4. Membership values of vertices of a graph (Fig.2)

	v_1	v_2	v_3	v_4
ρ_T	0.5	0.6	0.2	0.7
ρ_F	0.4	0.5	0.4	0.3
ρ_I	0.3	0.6	0.7	0.4

Consider the membership values of edges (Table 5) as

$$\mu_T(m,n) = \max\{m,n\} = \mu_F(m,n) = \mu_I(m,n)$$

Table 5. Membership values of edges of a graph (Fig.2)

	$(\overrightarrow{v_1,v_2})$	$(\overrightarrow{v_1,v_3})$	$(\overrightarrow{v_4,v_1})$	$(\overrightarrow{v_3,v_2})$
μ_T	0.3	0.3	0.2	0.3
μ_F	0.4	0.6	0.6	0.6
μ_I	0.4	0.5	0.5	0.6

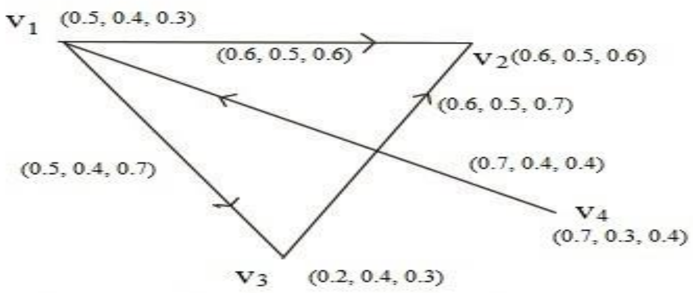

Figure 2. A generalized neutrosophic digraph

Definition 5. Let $\overrightarrow{G'} = (V,\vec{E})$ be a generalized neutrosophic digraph. Then out-neighbourhood $N^+(v_i)$ of a vertex $v_i \in V$ is given by

$$N^+(v_i) = \{v_j, (\mu_T(\overrightarrow{v_i,v_j}), \mu_F(\overrightarrow{v_i,v_j}), \mu_I(\overrightarrow{v_i,v_j})) : (\overrightarrow{v_i,v_j}) \in \vec{E}\}$$

where $\mu_T(\overrightarrow{v_i,v_j}), \mu_F(\overrightarrow{v_i,v_j}), \mu_I(\overrightarrow{v_i,v_j})$ denote the degree of true membership, the degree of falsity membership and indeterminacy membership of edge $(\overrightarrow{v_i,v_j}) \in \vec{E}$.

Example 3. Consider a GN digraph (Fig.3) $\vec{G} = (V,\vec{E})$ where $V = \{v_1, v_2, v_3, v_4\}$ and $\vec{E} = \{(\overrightarrow{v_1,v_2}), (\overrightarrow{v_1,v_3}), (\overrightarrow{v_1,v_4}), (\overrightarrow{v_2,v_3}), (\overrightarrow{v_3,v_4})\}$.

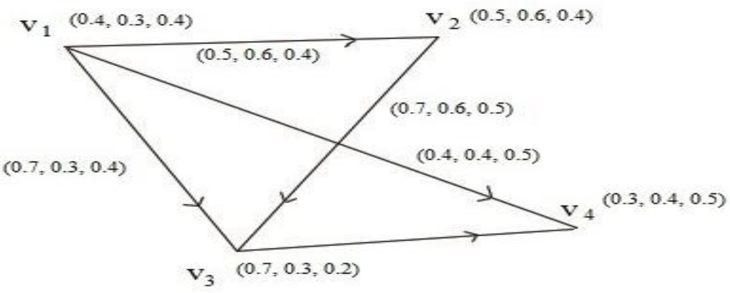

Fig.3. A generalized neutrosophic digraph

$$N^+(v_1) = \{(v_2, (0.5, 0.6, 0.4)), (v_3, (0.7, 0.3, 0.4)), (v_4, (0.4, 0.4, 0.5))\}$$

$$N^+(v_2) = \{(v_3, (0.7,0.6,0.5))\} \; , \; N^+(v_3) = \{(v_4, (0.7,0.4,0.5))\}, \quad N^+(v_4) = \emptyset.$$

Definition 6. Let $\overrightarrow{G'} = (V, \vec{E})$ be a generalized neutrosophic digraph. Then the generalized neutrosophic competition graph $C(\vec{G}')$ of $\vec{G} = (V, \vec{E})$ is a generalized neutrosophic graph which has the same vertex set V and has a neutrosophic edge between u, v if and only if $N^+(u) \cap N^+(v) \neq \emptyset$ and there exist sets $S_1 = \{\gamma_u^T, u \in V\}$, $S_2 = \{\gamma_u^F, u \in V\}$, $S_3 = \{\gamma_u^I, u \in V\}$ and functions $\phi_1: S_1 \times S_1 \to [0,1]$, $\phi_2: S_2 \times S_2 \to [0,1]$, $\phi_3: S_3 \times S_3 \to [0,1]$ such that edge-membership value of an edge $(u, v) \in E'$ is $(\mu_T(u,v),\ \mu_F(u,v),\ \mu_I(u,v))$ where

$$\mu_T(u,v) = \phi_1(\gamma_u^T, \gamma_v^T)$$
$$\mu_F(u,v) = \phi_2(\gamma_u^F, \gamma_v^F)$$
$$\mu_I(u,v) = \phi_3(\gamma_u^I, \gamma_v^I)$$
$$\gamma_u^T = \min\{\mu_T(\overrightarrow{u,w}), \forall w \in N^+(u) \cap N^+(v)\}, \gamma_v^T = \min\{\mu_T(\overrightarrow{u,w}), \forall w \in N^+(u) \cap N^+(v)\},$$
$$\gamma_u^F = \max\{\mu_F(\overrightarrow{u,w}), \forall w \in N^+(u) \cap N^+(v)\},\ \gamma_v^F = \max\{\mu_F(\overrightarrow{u,w}), \forall w \in N^+(u) \cap N^+(v)\},$$
$$\gamma_u^I = \max\{\mu_I(u,w), \forall w \in N^+(u) \cap N^+(v)\},\ \gamma_u^I = \min\{\mu_I(v,w), \forall w \in N^+(u) \cap N^+(v)\}.$$

Example 4. Consider a GN digraph(Fig.3) $G = (V, \vec{E})$ where $V = \{v_1, v_2, v_3, v_4\}$ and $\vec{E} = \{(\overrightarrow{v_1, v_2}), (\overrightarrow{v_1, v_3}), (\overrightarrow{v_1, v_4}), (\overrightarrow{v_2, v_3}), (\overrightarrow{v_3, v_4})\}$.

Then the corresponding competition graph (Fig.4) with membership values of edges (Table 6) is

Table 6. Membership values of edges a graph (Fig.4)

	(v_1, v_2)	(v_1, v_3)
μ_T	0.7	0.4
μ_F	0.3	0.3
μ_I	0.4	0.2

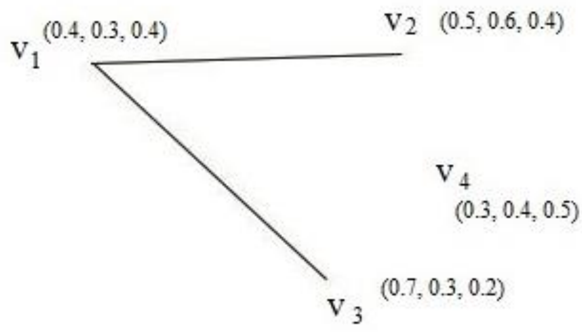

Figure 4. A generalized neutrosophic competition graph of a graph (Fig.3)

Theorem 1. Let G be a generalized neutrosophic graph. Then there exists a generalized neutrosophic digraph $\overrightarrow{G'}$ such that $C(\overrightarrow{G'}) = G$.

Proof. Let $G = (V, E)$ be a generalized neutrosophic graph and (x,y) be an edge in G. Now, a generalized neutrosophic digraph $\overrightarrow{G'}$ is to be constructed such that $C(\overrightarrow{G'}) = G$.

Let $x^{'}, y' \in \overrightarrow{G'}$ be the corresponding vertices of $x, y \in G$. Then we can draw two directed edges from vertices $x^{'}, y$ to a vertex $z^{'} \in \overrightarrow{G^{'}}$ such that $z^{'} \in N^{+}(x') \cap N^{+}(y')$. Similarly, we can do for all vertices and edges of G and hence $C\left(\overrightarrow{G'}\right) = G$.

Definition 7. Let G be a generalized neutrosophic graph. Minimal graph, $\overrightarrow{G'}$ of G is a generalized neutrosophic digraph such that $C\left(\overrightarrow{G'}\right) = G$ and $\overrightarrow{G'}$ has the minimum number of edges i.e. if there exists another graph G'' with $C\left(\overrightarrow{G''}\right) = G$, then number of edges of $\overrightarrow{G''}$ is greater than or equal to the number of edges of $\overrightarrow{G'}$.

Consider a generalized neutrosophic graph. If it is assumed as a generalized neutrosophic competition graph, then our task is to find the corresponding generalized neutrosophic digraph. Then there are a lot of graphs for a single generalized neutrosophic competition graph. We will consider the graph with a minimum number of edges.

Theorem 2. Let G be a generalised neutrosophic connected graph whose underlying graph is a complete graph with n vertices. Then the number of edges in a minimal graph of G is equal to 2n, n ≥ 3.

Proof. Let $G = (V, E)$ be a connected generalized neutrosophic graph whose underlying graph is a complete graph of n vertices so that each vertex of G is connected with each other. Let u, v be two adjacent vertices in G and u_1, v_1 be the corresponding vertices in the minimal graph $\overrightarrow{G}^{'}$. Consider a generalised neutrosophic directed graph $\vec{G}_1^{'}$in such a way that every vertex of $\vec{G}$ other than u_1 has only out-neighbourhood as u_1. Thus $\vec{G}_1^{'}$ has $(n-1)$ edges. Similarly, a generalised neutrosophic directed graph $\vec{G}_2^{'}$ is considered for v_1 and hence $\vec{G}_2^{'}$ has $(n-1)$ edges. Now, consider a generalised neutrosophic directed graph $\vec{G}_3^{'}$ with only edges $(\overrightarrow{u_1, w_1}), (\overrightarrow{v_1, w_1})$. Thus $\overrightarrow{G}^{'} = \vec{G}_1^{'} \cup \vec{G}_2^{'} \cup \vec{G}_3^{'}$. The number of edges in $\overrightarrow{G}^{'}$ is $(n-1) + (n-1) + 2 = 2n$.

Definition 8. Scores of an edge (u, v) between two vertices in a generalized neutrosophic graph is given by $s(u, v) = [2\mu_T(1 - \mu_F) + \mu_I]/3$ where μ_T, μ_F and μ_I are the degree of truth membership, degree of falsity membership and degree of indeterminacy membership of the edge (u, v) respectively.

Example 5. Consider a GN graph (Fig.5) $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4\}$ and
$E = \{(v_1, v_2), (v_1, v_4), (v_2, v_3), (v_3, v_4), (v_2, v_4)\}$.

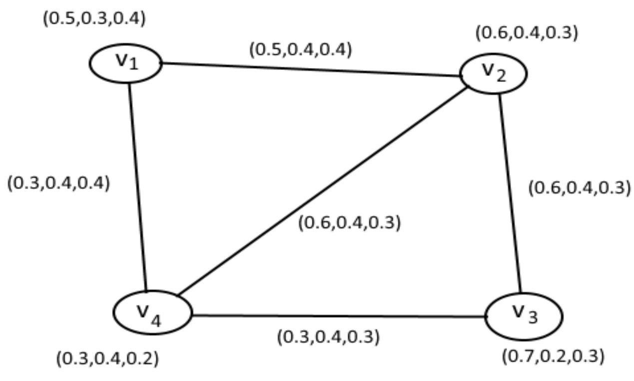

Figure 5. An example of a generalized neutrosophic graph

The score of the edge (v_3, v_4) is 0.42. Similarly, the scores of all edges should be found.

Definition 9. In a generalized neutrosophic graph, a vertex u with adjacent vertices $v_1, v_2, \ldots., v_k$ is said to be isolated if $s(u, v_i) = 0$ for $i = 1,2,3 \ldots.., k$.

Note1. If $\mu_F = 1, \mu_I = 0$, then score = 0 and if $\mu_T = 0 = \mu_I$ then score = 0.

Example 6. Consider a GN graph (Fig.6) $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4\}$ and

$$E = \{(v_1, v_2), (v_1, v_3), (v_2, v_3), (v_2, v_4)\}$$

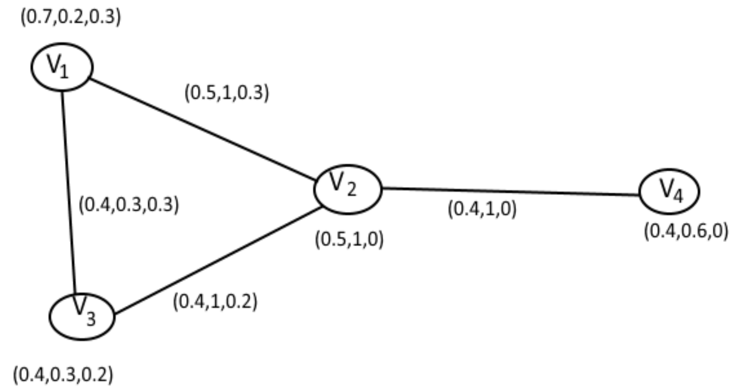

Figure 6. An example of a generalized neutrosophic graph with isolated vertex

The adjacent vertex of v_4 is v_2 and the score of the edge(v_2, v_4) is 0, so v_4 is an isolated vertex.

Definition 10. A cycle of length ≥ 4 in a generalized neutrosophic graph is called a hole if all the edges of this cycle have a non-zero score.

Example 7. Consider the graph in example 5, $v_1 - v_2 - v_3 - v_4 - v_1$is a cycle of length 4 and all the of the cycle have non-zero score and hence it is a hole.

Definition 11. The smallest number of the isolated vertex in a generalized neighbourhood graph is called competition number. It is denoted by $k_N(G)$.

Lemma 1. If a crisp graph has one hole, then its completion number is at most 2. But the Competition number of a generalized neutrosophic graph with exactly one hole may be greater than two. Let us consider a graph (Fig.7) with exactly one hole with competition number 2.

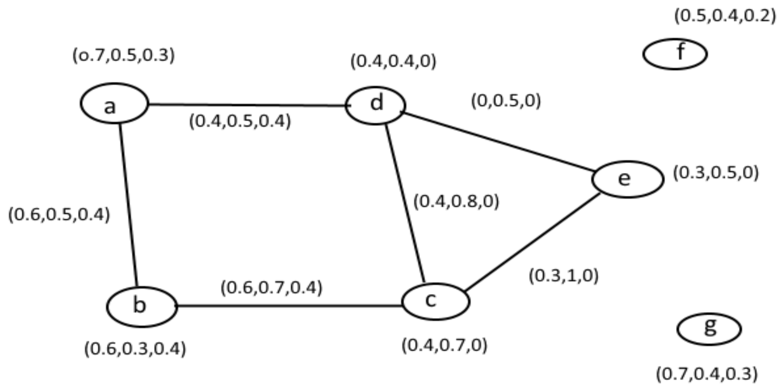

Figure 7. Generalized neutrosophic graph with competition number 2.

It may be noted that scores of edges $(\overrightarrow{a, b})$,$(\overrightarrow{b, c})$, $(\overrightarrow{c, d})$and $(\overrightarrow{d, a})$ are non-zero as per definition of the hole. But the score of $(\overrightarrow{d, e})$ and $(\overrightarrow{c, e})$ may be zero. Hence e is an isolated vertex. Thus competition number is 3.

Definition 12. A neutrosophic graph is said to be a neutrosophic chordal graph if all the holes have a chord with score > 0.

Example 10. Consider the graph in example 5, $v_1 - v_2 - v_3 - v_4 - v_1$are only a hole and the edge (v_2, v_4) is a chord with a non-zero score, then the graph is a neutrosophic chordal graph.

Lemma 2. The competition number of a neutrosophic chordal graph with pendant vertex be greater than 1. In the neutrosophic chordal graph (Fig.8) given below, since the vertex e is isolated, then the competition number is greater than 2.

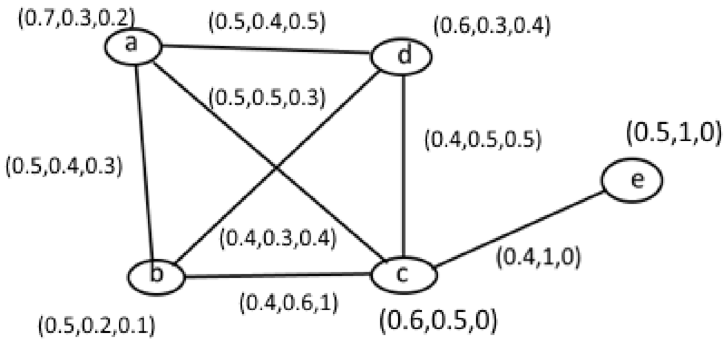

Figure 8. Neutrosophic chordal graph

3. Matrix representation of GNCG

It is one kind of adjacency matrix of the GNCG. The entries of the matrix are calculated as follows:

Step-1: Let us consider a generalized neutrosophic digraph (GNDG).

Step-2: Find the pair of vertices $u_i, v_i (i = 1,2, \ldots, m)$ such that there exist edges $(\overrightarrow{u_i, x_k})$, $(\overrightarrow{v_i, x_l})$ for $(k, l = 1,2, \ldots, p)$ with $N^+(u_i)$ and $N^+(v_i)$.

Step-3: Find the set $N^+(u_i) \cap N^+(v_i) = \{x_n, n = 1,2, \ldots, q\}, say.$

Step-4: let $\gamma_u^T = \min \{\mu_T(\overrightarrow{u_i, x_1}), \mu_T(\overrightarrow{u_i, x_2}), \ldots, \mu_T(\overrightarrow{u_i, x_q})\}$

$$\gamma_v^T = \min \{\mu_T(\overrightarrow{v_i, x_1}), \quad \mu_T(\overrightarrow{v_i, x_2}), \ldots, \mu_T(\overrightarrow{v_i, x_q})\}$$

$$\gamma_u^F = \max \{\mu_F(\overrightarrow{u_i, x_1}), \mu_F(\overrightarrow{u_i, x_2}), \ldots, \mu_F(\overrightarrow{u_i, x_q})\}$$

$$\gamma_v^F = \max \{\mu_F(\overrightarrow{v_i, x_1}), \mu_F(\overrightarrow{v_i, x_2}), \ldots, \mu_F(\overrightarrow{v_i, x_q})\}$$

$$\gamma_u^I = \min \{\mu_I(\overrightarrow{u_i, x_1}), \mu_I(\overrightarrow{u_i, x_2}), \ldots, \mu_I(\overrightarrow{u_i, x_q})\}$$

$$\gamma_v^I = \max \{\mu_I(\overrightarrow{v_i, x_1}), \mu_I(\overrightarrow{v_i, x_2}), \ldots, \mu_I(\overrightarrow{v_i, x_q})\}.$$

Step-5: Find the degree of true membership, degree of falsity membership and degree of indeterminacy membership by the following formula

$$\mu_T(u, v) = \varphi_1(\gamma_u^T, \gamma_v^T),$$

$$\mu_F(u, v) = \varphi_2(\gamma_u^F, \gamma_v^F),$$

$$\mu_I(u, v) = \varphi_3(\gamma_u^I, \gamma_v^I)$$

For simplification, one function φ may be used in place of $\varphi_1, \varphi_2, \varphi_3$.

Step-6: the competition matrix is a square matrix. Its order equal to the number of vertices. Its entries are given below.

$$a_{ij} = \begin{cases} (\varphi_1(\gamma_i^T, \gamma_j^T), \varphi_2(\gamma_i^F, \gamma_j^F), \varphi_3(\gamma_i^I, \gamma_j^I)) & if\ there\ is\ an\ edge\ between\ vertex\ i\ and\ j \\ (0,0,0), & if\ there\ is\ no\ edge\ between\ vertex\ i\ and\ j. \end{cases}$$

Example 11. An example of matrix representation is presented with all steps.

Step -1: Consider a GNDG (Fig.9)$\overrightarrow{G'} = (V, \vec{E})$. The membership values of vertices and edges are given in the graph (Fig.)

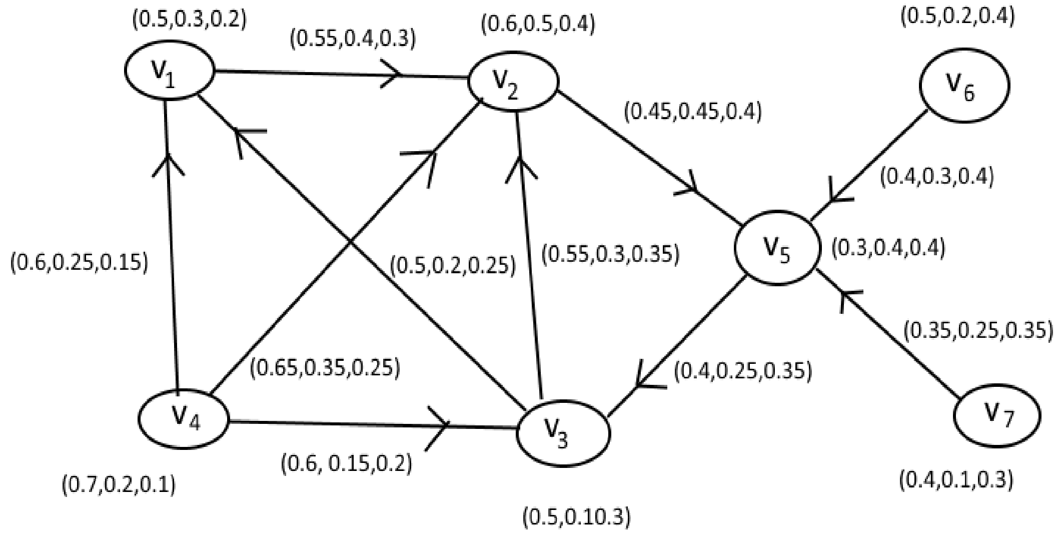

Figure 9. A generalized neutrosophic graph with seven vertices

Step-2:$N^+(v_1) = \{v_2\} N^+(v_2) = \{v_5\}\ N^+(v_3) = \{v_2, v_1\}$

$N^+(v_4) = \{v_1, v_3\} N^+(v_5) = \{v_3\} N^+(v_6) = \{v_5\} N^+(v_7) = \{v_5\}$.

Step-3: $N^+(v_1) \cap N^+(v_2) = \emptyset$, $N^+(v_1) \cap N^+(v_3) = \{v_2\}$, $N^+(v_1) \cap N^+(v_4) = \{v_2\}$,

$N^+(v_1) \cap N^+(v_5) = \emptyset$, $N^+(v_1) \cap N^+(v_6) = \emptyset$, $N^+(v_1) \cap N^+(v_7) = \emptyset$,

$N^+(v_2) \cap N^+(v_3) = \emptyset$, $N^+(v_2) \cap N^+(v_4) = \emptyset$,$N^+(v_2) \cap N^+(v_5) = \emptyset$,

$N^+(v_2) \cap N^+(v_6) = \{v_5\}$,$N^+(v_2) \cap N^+(v_7) = \{v_5\}$, , $N^+(v_3) \cap N^+(v_4) = \{v_1\}$,

$N^+(v_3) \cap N^+(v_5) = \emptyset$,$N^+(v_3) \cap N^+(v_6) = \emptyset$, $N^+(v_3) \cap N^+(v_7) = \emptyset$,

$N^+(v_4) \cap N^+(v_5) = \{v_3\}$,$N^+(v_4) \cap N^+(v_6) = \emptyset$, $N^+(v_4) \cap N^+(v_7) = \emptyset$,

$N^+(v_5) \cap N^+(v_6) = \emptyset$, $N^+(v_5) \cap N^+(v_7) = \emptyset$, $N^+(v_6) \cap N^+(v_7) = \{v_5\}$,

Step-4:

$$\gamma_{12}^T = 0.55, \quad \gamma_{12}^F = 0.4, \quad \gamma_{12}^I = 0.3$$
$$\gamma_{32}^T = 0.55, \quad \gamma_{32}^F = 0.3, \quad \gamma_{32}^I = 0.35$$
$$\gamma_{42}^T = 0.65, \quad \gamma_{42}^F = 0.35, \quad \gamma_{42}^I = 0.25$$
$$\gamma_{25}^T = 0.45, \quad \gamma_{25}^F = 0.45, \quad \gamma_{25}^I = 0.4$$
$$\gamma_{65}^T = 0.4, \quad \gamma_{65}^F = 0.3, \quad \gamma_{65}^T = 0.4$$
$$\gamma_{75}^T = 0.35, \quad \gamma_{75}^F = 0.25, \quad \gamma_{75}^I = 0.35$$
$$\gamma_{31}^T = 0.5, \quad \gamma_{31}^F = 0.2, \quad \gamma_{31}^I = 0.25$$

$$\gamma_{41}^{T} = 0.6, \quad \gamma_{41}^{F} = 0.25, \quad \gamma_{41}^{I} = 0.15$$
$$\gamma_{43}^{T} = 0.6, \quad \gamma_{43}^{F} = 0.15, \quad \gamma_{43}^{I} = 0.2$$
$$\gamma_{53}^{T} = 0.4, \quad \gamma_{53}^{F} = 0.25, \quad \gamma_{53}^{I} = 0.35$$

Step-5:

$$\mu_{13}^{T} = 0, \quad \mu_{13}^{F} = 0.1, \quad \mu_{13}^{I} = 0.05$$
$$\mu_{14}^{T} = 0.1, \quad \mu_{14}^{F} = 0.05, \quad \mu_{13}^{I} = 0.05$$
$$\mu_{34}^{T} = 0.1, \quad \mu_{34}^{F} = 0.05, \quad \mu_{34}^{I} = 0.1$$
$$\mu_{45}^{T} = 0.2, \quad \mu_{45}^{F} = 0.1, \quad \mu_{45}^{I} = 0.15$$
$$\mu_{26}^{T} = 0.05, \quad \mu_{26}^{F} = 0.15, \quad \mu_{26}^{I} = 0$$
$$\mu_{27}^{T} = 0.1, \quad \mu_{27}^{F} = 0.2, \quad \mu_{27}^{I} = 0.05$$
$$\mu_{67}^{T} = 0.05, \quad \mu_{67}^{F} = 0.05, \quad \mu_{67}^{I} = 0.05$$

Step-6: the corresponding matrix is

$$\begin{pmatrix} - & (0,0,0) & (0,0.1,0.05) & (0.1,0.05,0.05) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & - & (0,0,0) & (0,0,0) & (0,0,0) & (0.05,0.15,0) & (0.1,0.2,0.05) \\ (0,0.1,0.05) & (0,0,0) & - & (0.1,0.05,0.1) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0.1,0.05,0.05) & (0,0,0) & (0.1,0.05,0.1) & - & (0.2,0.1,0.15) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) & (0.2,0.1,0.15) & - & (0,0,0) & (0,0,0) \\ (0,0,0) & (0.05,0.15,0) & (0,0,0) & (0,0,0) & (0,0,0) & - & (0.05,0.05,0.05) \\ (0,0,0) & (0.1,0.2,0.05) & (0,0,0) & (0,0,0) & (0,0,0) & (0.05,0.05,0.05) & - \end{pmatrix}$$

4. An application in economic competition

Like competitions in the ecosystem, there are many competitions running in real life. In this study, the competition in economic growth among the countries (Fig.10) are presented in the neutrosophic environment. We consider two factors: GDP and GPI. Gross Domestic Product (GDP) of a country is the total market value of all goods and services produced in a specific time period in the country. The Global Peaceful Index (GPI) of a country is the value of peacefulness in the country relative to global.

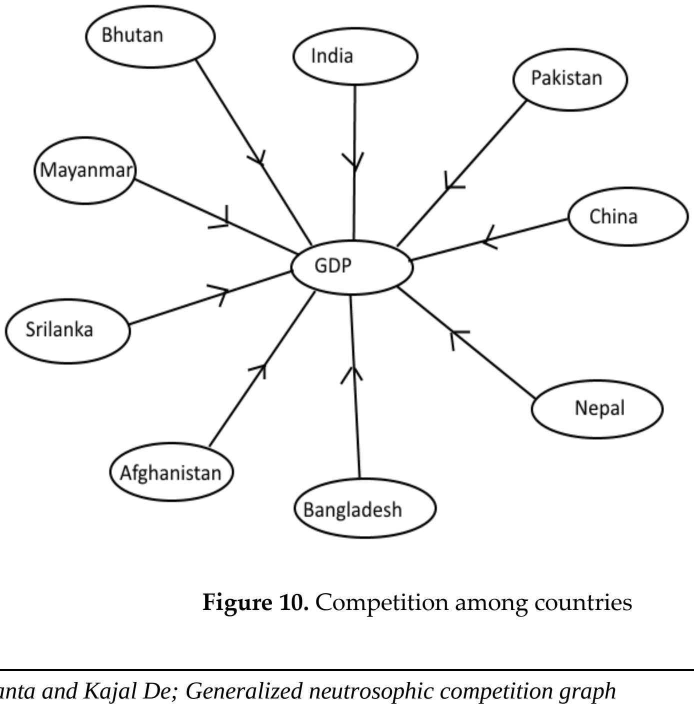

Figure 10. Competition among countries

The GDP growth is taken as the degree of truth membership, GPI is taken as the degree of falsity memberships. The uncertainty causes like flood, elections etc. may be taken as the degree of indeterminacy membership. The data of GDP growth and GPI are collected from internet. The country of India with neighbours countries are competing with each other to become more strong. Since all countries are competing, so the corresponding competition graph is a complete graph.

The membership values of countries (nodes) are given in the tabular form (Table 7, Table 8) and the membership values of edges are calculated by the following formula and are represented by a matrix.

$$\mu_T(u,v) = 1 - |\sigma_T^u - \sigma_T^v|,$$

$$\mu_F(u,v) = 1 - |\sigma_F^u - \sigma_F^v|,$$

$$\mu_I(u,v) = 0$$

Table 7. Countries with GDP and GPI values

SL. No.	Country	GDP	GPI
1	India	7.257	2.605
2	Pakistan	2.905	3.072
3	China	6.267	2.217
4	Nepal	6.536	2.003
5	Bangladesh	7.289	2.128
6	Bhutan	4.816	1.506
7	Myanmar	6.448	2.393
8	Afganistan	3	3.574
9	Srilanka	3.5	1.986

Table 8. Countries with their normalized values of GDP and GPI.

Sl. No.	Country	N GDP	1/GPI	N GPI	N GDP~ N GPI
1	India	0.996	0.38	0.576	0.42
2	Pakistan	0.399	0.33	0.5	0.101
3	China	0.86	0.45	0.682	0.178
4	Nepal	0.897	0.5	0.758	0.139
5	Bangaladesh	1	0.47	0.712	0.288
6	Bhutan	0.661	0.66	1	0.339
7	Mayanmar	0.885	0.42	0.636	0.249
8	Afganistan	0.412	0.28	0.424	0.012
9	Srilanka	0.48	0.5	0.758	0.278

The competition among countries is given above by the matrix form.

$$\begin{pmatrix}
(1,1,0) & (0.924,0.681,0) & (0.864,0.758,0) & (0.901,0.719,0) & (0.996,0.868,0) & (0.665,0.919,0) & (0.889,0.829,0) & (0.416,0.592,0) & (0.484,0.858,0) \\
(0.403,0.681,0) & (1,1,0) & (0.539,0.923,0) & (0.502,0.962,0) & (0.399,0.813,0) & (0.738,0.762,0) & (0.514,0.852,0) & (0.987,0.911,0) & (0.919,0.823,0) \\
(0.864,0.758,0) & (0.894,0.923,0) & (1,1,0) & (0.963,0.961,0) & (0.86,0.89,0) & (0.801,0.839,0) & (0.975,0.929,0) & (0.552,0.834,0) & (0.62,0.9,0) \\
(0.901,0.719,0) & (0.818,0.962,0) & (0.963,0.961,0) & (1,1,0) & (0.897,0.851,0) & (0.764,0.8,0) & (0.988,0.89,0) & (0.515,0.873,0) & (0.583,0.861,0) \\
(0.996,0.868,0) & (0.864,0.813,0) & (0.86,0.89,0) & (0.897,0.851,0) & (1,1,0) & (0.661,0.949,0) & (0.885,0.961,0) & (0.412,0.724,0) & (0.48,0.99,0) \\
(0.665,0.919,0) & (0.576,0.762,0) & (0.801,0.839,0) & (0.764,0.8,0) & (0.661,0.949,0) & (1,1,0) & (0.776,0.91,0) & (0.751,0.673,0) & (0.819,0.939,0) \\
(0.889,0.829,0) & (0.94,0.852,0) & (0.975,0.929,0) & (0.988,0.89,0) & (0.885,0.961,0) & (0.776,0.91,0) & (1,1,0) & (0.527,0.763,0) & (0.595,0.971,0) \\
(0.416,0.592,0) & (0.848,0.911,0) & (0.552,0.834,0) & (0.515,0.873,0) & (0.412,0.724,0) & (0.751,0.673,0) & (0.527,0.763,0) & (1,1,0) & (0.932,0.734,0) \\
(0.004,0.858,0) & (0.424,0.823,0) & (0.62,0.9,0) & (0.583,0.861,0) & (0.48,0.99,0) & (0.819,0.939,0) & (0.595,0.971,0) & (0.932,0.734,0) & (1,1,0)
\end{pmatrix}$$

Conclusion

This study presents the generalization of neutrosophic competition graph where edge restrictions are withdrawn. A representation of GNCG is presented by a square matrix. Also, the minimal graph and competition number are introduced. A real-life application is presented and discussed by the GNCG. In this application, true membership value is taken as GDP, the gross domestic product of countries, and falsity is taken as complement of of GPI, Global Peace Index of such countries. These parameters may be taken differently to capture the competitions among countries. This representation will be helpful to perceive real-life competitions. This study assumed only one step competition. In future, n-step neutrosophic competition graph and several other related notions will be studied. This study will be the backbone of that.

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