Neutrosophic Sets and Systems, Vol. 59, 2023
University of New Mexico

Generating Pythagoras Quadruples in Symbolic 2-Plithogenic Commutative Rings

Yaser Ahmad Alhasan[1,*], Abuobida Mohammed A. Alfahal[2], Raja Abdullah Abdulfatah[3]

[1]Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia
[2]Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia
[3]Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia

Emails:[1.] y.alhasan@psau.edu.sa; [2.]a.alfahal@psau.edu.sa;[3.] r.abdulfatah@psau.edu.sa ;
Corresponding author: y.alhasan@psau.edu.sa

Abstract:

This paper is dedicated to find a general algorithm for generating different solutions for Pythagoras non-linear Diophantine equation in four variables $x^2 + y^2 + z^2 = t^2$ in symbolic 2-plithogenic rings, which are known as Pythagoras quadruples.

Also, we present some examples about those quadruples in some finite symbolic 2-plithogenic rings.

Keywords: symbolic 2-plithogenic ring, Pythagoras quadruples, Diophantine equations

Introduction and Preliminaries

Symbolic n-plithogenic algebraic structures are a new generalization of classical algebraic structures, as they have serious algebraic properties to study.

In the previous literature, we can clearly note several algebraic studies that were interested in discovering the properties of these algebraic structures, for example we can find some applications of plithogenic structures in probability, ring theory, linear spaces, matrices, and equations [1-10].

zenodo.10031125/10.5281

Researchers have studied Pythagorean quadruples in the ring of ordinary algebraic numbers [11-14].

Several efficient algorithms for calculating these quadruples have been presented, as solutions to the corresponding Diophantine equation.

This has motivated us to study Pythagoras quadruples in the symbolic 2-plithogenic commutative case, where we find a general algorithm for generating different solutions for Pythagoras non-linear Diophantine equation in four variables $\boldsymbol{x^2+y^2+z^2=t^2}$ in symbolic 2-plithogenic rings.

Definition.

The symbolic 2-plithogenic ring of real numbers is defined as follows:

$$2-SP_R=\{t_0+t_1P_1+t_2P_2; t_i\in R, P_1\times P_2=P_2\times P_1=P_2, {P_1}^2={P_2}^2=P_2\}$$

The addition operation on $2-SP_R$ is defined as follows:

$$(t_0+t_1P_1+t_2P_2)+(t_0'+t_1'P_1+t_2'P_2)=(t_0+t_0')+(t_1+t_1')P_1+(t_2+t_2')P_2$$

The multiplication on $2-SP_R$ is defined as follows:

$$(t_0+t_1P_1+t_2P_2)(t_0'+t_1'P_1+t_2'P_2)$$
$$=t_0t_0'+(t_0t_1'+t_1t_0'+t_1t_1')P_1+(t_0t_2'+t_1t_2'+t_2t_2'+t_2t_0'+t_2t_1')P_2$$

Main Discussion

Definition.

Let $T=t_0+t_1P_1+t_2P_2, S=s_0+s_1P_1+s_2P_2, K=k_0+k_1P_1+k_2P_2, L=l_0+l_1P_1+l_2P_2$ be four symbolic 2-plithogenic elements of a symbolic 2-plithogenic commutative ring

$2-SP_R$, then (T,S,K,L) is called a symbolic 2-plithogenic Pythagoras quadruple if and only if $T^2+S^2+K^2=L^2$.

Theorem.

Let $T=t_0+t_1P_1+t_2P_2, S=s_0+s_1P_1+s_2P_2, K=k_0+k_1P_1+k_2P_2, L=l_0+l_1P_1+l_2P_2\in 2-SP_R$, then (T,S,K,L) is a symbolic 2-plithogenic Pythagoras quadruple if and only if:

$(t_0, s_0, k_0, l_0), (t_0 + t_1, s_0 + s_1, k_0 + k_1, l_0 + l_1), (t_0 + t_1 + t_2, s_0 + s_1 + s_2, k_0 + k_1 + k_2, l_0 + l_1 + l_2)$ are three Pythagoras quadruples in R.

Proof.

We have:

$T^2 = {t_0}^2 + [(t_0 + t_1)^2 - {t_0}^2]P_1 + [(t_0 + t_1 + t_2)^2 - (t_0 + t_1)^2]P_2,$

$S^2 = {s_0}^2 + [(s_0 + s_1)^2 - {s_0}^2]P_1 + [(s_0 + s_1 + s_2)^2 - (s_0 + s_1)^2]P_2,$

$K^2 = {k_0}^2 + [(k_0 + k_1)^2 - {k_0}^2]P_1 + [(k_0 + k_1 + k_2)^2 - (k_0 + k_1)^2]P_2,$

$L^2 = {l_0}^2 + [(l_0 + l_1)^2 - {l_0}^2]P_1 + [(l_0 + l_1 + l_2)^2 - (l_0 + l_1)^2]P_2,$

The equation $T^2 + S^2 + K^2 = L^2$ is equivalent to:

${t_0}^2 + {s_0}^2 + {k_0}^2 = {l_0}^2$ (1)

$(t_0 + t_1)^2 + (s_0 + s_1)^2 + (k_0 + k_1)^2 = (l_0 + l_1)^2$ (2)

$(t_0 + t_1 + t_2)^2 + (s_0 + s_1 + s_2)^2 + (k_0 + k_1 + k_2)^2 = (l_0 + l_1 + l_2)^2$ (3)

Thus, the proof holds.

Theorem.

Let $(t_0, s_0, k_0, l_0), (t_1, s_1, k_1, l_1), (t_2, s_2, k_2, l_2)$ be three Pythagoras quadruples in R, then the corresponding Pythagoras quadruple in $2 - SP_R$ is (T, S, K, L), where:

$T = t_0 + [t_1 - t_0]P_1 + [t_2 - t_1]P_2,$

$S = s_0 + [s_1 - s_0]P_1 + [s_2 - s_1]P_2,$

$K = k_0 + [k_1 - k_0]P_1 + [k_2 - k_1]P_2,$

$L = l_0 + [l_1 - l_0]P_1 + [l_2 - l_1]P_2.$

Proof.

We must compute $T^2 + S^2 + K^2$,

$T^2 + S^2 + K^2 = {t_0}^2 + ({t_1}^2 - {t_0}^2)P_1 + ({t_2}^2 - {t_1}^2)P_2 + {s_0}^2 + ({s_1}^2 - {s_0}^2)P_1 + ({s_2}^2 - {s_1}^2)P_2 + {k_0}^2 + ({k_1}^2 - {k_0}^2)P_1 + ({k_2}^2 - {k_1}^2)P_2 = ({t_0}^2 + {s_0}^2 + {k_0}^2) + ({t_1}^2 + {s_1}^2 + {k_1}^2 - {t_0}^2 - {s_0}^2 - {k_0}^2)P_1 + ({t_2}^2 + {s_2}^2 + {k_2}^2 - {t_1}^2 - {s_1}^2 - {k_1}^2)P_2 = {l_0}^2 + ({l_1}^2 - {l_0}^2)P_1 + ({l_2}^2 - {l_1}^2)P_2 = L^2.$

So that, the proof is complete.

Example.

We have $L_1 = (1, -1, i, 1), L_2 = (i, 1, -1, -1), L_3 = (-i, -1, 1, -1)$ are three Pythagoras quadruples in C.

The corresponding 2-plithogenic Pythagoras quadruple is (T, S, K, L), where:

$T = 1 + (-1 + i)P_1 - 2iP_2$

$S = -1 + 2P_1 - 2P_2$

$K = i + (-1 - i)P_1 + 2P_2$

$L = 1 - 2P_1 + 2P_2$

On the other hand, we have:

$T^2 = 1 - 2P_1,$

$S^2 = 1,$

$K^2 = -1 + 2P_1,$

$L^2 = 1 = T^2 + S^2 + K^2.$

Example.

Consider the following three Pythagoras quadruples in Z_2:

$L_1 = (0,0,0,0), L_2 = (1,1,1,1), L_3 = (1,1,0,0)$

For every triple $(L_i, L_j, L_s); 1 \leq i, j, s \leq 3$, we can get a symbolic 2-plithogenic pythagoras quadruple.

We will find some symbolic 2-plithogenic Pythagoras quadruple in $2 - SP_{Z_2}$.

Let us write the following quadruples:

$$\begin{cases} Y_1 = 0 \\ \acute{Y}_1 = 0 \\ {Y_1}'' = 0 \\ {Y_1}''' = 0 \end{cases}$$

$$\begin{cases} Y_2 = P_2 \\ \acute{Y}_2 = P_2 \\ {Y_2}'' = P_2 \\ {Y_2}''' = P_2 \end{cases}$$

$$\begin{cases} Y_3 = 0 \\ \acute{Y}_3 = 0 \\ {Y_3}'' = P_2 \\ {Y_3}''' = P_2 \end{cases}$$

$$\begin{cases} Y_4 = P_1 + P_2 \\ \acute{Y}_4 = P_1 + P_2 \\ Y_4'' = P_1 + P_2 \\ Y_4''' = P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_5 = P_1 + P_2 \\ \acute{Y}_5 = P_1 + P_2 \\ Y_5'' = 0 \\ Y_5''' = 0 \end{cases}$$

$$\begin{cases} Y_6 = 0 \\ \acute{Y}_6 = 0 \\ Y_6'' = P_1 + P_2 \\ Y_6''' = P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_7 = 1 + P_1 \\ \acute{Y}_7 = 1 + P_1 \\ Y_7'' = 1 + P_1 \\ Y_7''' = 1 + P_1 \end{cases}$$

$$\begin{cases} Y_8 = 1 + P_1 \\ \acute{Y}_8 = 1 + P_1 \\ Y_8'' = 0 \\ Y_8''' = 0 \end{cases}$$

$$\begin{cases} Y_9 = 0 \\ \acute{Y}_9 = 0 \\ Y_9'' = 1 + P_1 \\ Y_9''' = 1 + P_1 \end{cases}$$

$$\begin{cases} Y_{10} = 1 \\ \acute{Y}_{10} = 1 \\ Y_{10}'' = 1 \\ Y_{10}''' = 1 \end{cases}$$

$$\begin{cases} Y_{11} = 1 + P_2 \\ \acute{Y}_{11} = 1 + P_2 \\ Y_{11}'' = 1 + P_2 \\ Y_{11}''' = 1 + P_2 \end{cases}$$

$$\begin{cases} Y_{12} = 1 + P_2 \\ \acute{Y}_{12} = 1 + P_2 \\ Y_{12}'' = 1 \\ Y_{12}''' = 1 \end{cases}$$

$$\begin{cases} Y_{13} = 1 + P_1 + P_2 \\ \acute{Y}_{13} = 1 + P_1 + P_2 \\ Y_{13}'' = 1 + P_1 + P_2 \\ Y_{13}''' = 1 + P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{14} = 1 \\ Y_{14}' = 1 \\ Y_{14}'' = 1 + P_1 + P_2 \\ Y_{14}''' = 1 + P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{15} = 1 + P_1 + P_2 \\ Y_{15}' = 1 + P_1 + P_2 \\ Y_{15}'' = 1 \\ Y_{15}''' = 1 \end{cases}$$

$$\begin{cases} Y_{16} = P_1 \\ Y_{16}' = P_1 \\ Y_{16}'' = P_1 \\ Y_{16}''' = P_1 \end{cases}$$

$$\begin{cases} Y_{17} = 1 \\ Y_{17}' = 1 \\ Y_{17}'' = P_1 \\ Y_{17}''' = P_1 \end{cases}$$

$$\begin{cases} Y_{26} = P_1 \\ Y_{26}' = P_1 \\ Y_{26}'' = 1 \\ Y_{26}''' = 1 \end{cases}$$

$$\begin{cases} Y_{27} = 1 \\ Y_{27}' = 1 \\ Y_{27}'' = 0 \\ Y_{27}''' = 0 \end{cases}$$

$$\begin{cases} Y_{32} = 1 \\ Y_{32}' = 1 \\ Y_{32}'' = P_2 \\ Y_{32}''' = P_2 \end{cases}$$

$$\begin{cases} Y_{33} = 1 + P_2 \\ Y_{33}' = 1 + P_2 \\ Y_{33}'' = P_2 \\ Y_{33}''' = P_2 \end{cases}$$

$$\begin{cases} Y_{35} = 1 \\ Y_{35}' = 1 \\ Y_{35}'' = P_1 + P_2 \\ Y_{35}''' = P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{36} = 1 + P_1 + P_2 \\ Y_{36}' = 1 + P_1 + P_2 \\ Y_{36}'' = P_1 + P_2 \\ Y_{36}''' = P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{37} = P_1 \\ \acute{Y}_{37} = P_1 \\ Y_{37}'' = 0 \\ Y_{37}''' = 0 \end{cases}$$

$$\begin{cases} Y_{38} = 1 \\ \acute{Y}_{38} = 1 \\ Y_{38}'' = 1 + P_1 \\ Y_{38}''' = 1 + P_1 \end{cases}$$

$$\begin{cases} Y_{39} = P_1 \\ \acute{Y}_{39} = P_1 \\ Y_{39}'' = 1 + P_1 \\ Y_{39}''' = 1 + P_1 \end{cases}$$

$$\begin{cases} Y_{40} = 0 \\ \acute{Y}_{40} = 0 \\ Y_{40}'' = 1 \\ Y_{40}''' = 1 \end{cases}$$

$$\begin{cases} Y_{44} = 0 \\ \acute{Y}_{44} = 0 \\ Y_{44}'' = 1 + P_2 \\ Y_{44}''' = 1 + P_2 \end{cases}$$

$$\begin{cases} Y_{45} = P_2 \\ \acute{Y}_{45} = P_2 \\ Y_{45}'' = 1 \\ Y_{45}''' = 1 \end{cases}$$

$$\begin{cases} Y_{46} = P_2 \\ \acute{Y}_{46} = P_2 \\ Y_{46}'' = 1 + P_2 \\ Y_{46}''' = 1 + P_2 \end{cases}$$

$$\begin{cases} Y_{47} = 0 \\ \acute{Y}_{47} = 0 \\ Y_{47}'' = 1 + P_1 + P_2 \\ Y_{47}''' = 1 + P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{48} = P_1 + P_2 \\ \acute{Y}_{48} = P_1 + P_2 \\ Y_{48}'' = 1 \\ Y_{48}''' = 1 \end{cases}$$

$$\begin{cases} Y_{49} = P_1 + P_2 \\ \acute{Y}_{49} = P_1 + P_2 \\ Y_{49}'' = 1 + P_1 + P_2 \\ Y_{49}''' = 1 + P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{50} = 0 \\ \acute{Y}_{50} = 0 \\ Y_{50}'' = P_1 \\ Y_{50}''' = P_1 \end{cases}$$

$$\begin{cases} Y_{51} = 1 + P_1 \\ \acute{Y}_{51} = 1 + P_1 \\ Y_{51}'' = 1 \\ Y_{51}''' = 1 \end{cases}$$

$$\begin{cases} Y_{52} = 1 + P_1 \\ \acute{Y}_{52} = 1 + P_1 \\ Y_{52}'' = P_1 \\ Y_{52}''' = P_1 \end{cases}$$

$$\begin{cases} Y_{53} = P_1 \\ \acute{Y}_{53} = P_1 \\ Y_{53}'' = P_1 + P_2 \\ Y_{53}''' = P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{54} = P_1 + P_2 \\ \acute{Y}_{54} = P_1 + P_2 \\ Y_{54}'' = P_1 + P_2 \\ Y_{54}''' = P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{55} = P_1 \\ \acute{Y}_{55} = P_1 \\ Y_{55}'' = P_2 \\ Y_{55}''' = P_2 \end{cases}$$

$$\begin{cases} Y_{56} = P_1 + P_2 \\ \acute{Y}_{56} = P_1 + P_2 \\ Y_{56}'' = P_2 \\ Y_{56}''' = P_2 \end{cases}$$

$$\begin{cases} Y_{57} = P_2 \\ \acute{Y}_{57} = P_2 \\ Y_{57}'' = P_1 \\ Y_{57}''' = P_1 \end{cases}$$

$$\begin{cases} Y_{58} = P_2 \\ \acute{Y}_{58} = P_2 \\ Y_{58}'' = P_1 + P_2 \\ Y_{58}''' = P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{59} = 1 + P_1 + P_2 \\ \acute{Y}_{59} = 1 + P_1 + P_2 \\ Y_{59}'' = 1 + P_1 \\ Y_{59}''' = 1 + P_1 \end{cases}$$

$$\begin{cases} Y_{60} = 1 + P_1 \\ Y_{60}' = 1 + P_1 \\ Y_{60}'' = P_2 \\ Y_{60}''' = P_2 \end{cases}$$

$$\begin{cases} Y_{61} = 1 + P_1 + P_2 \\ Y_{61}' = 1 + P_1 + P_2 \\ Y_{61}'' = P_2 \\ Y_{61}''' = P_2 \end{cases}$$

$$\begin{cases} Y_{62} = 1 + P_1 \\ Y_{62}' = 1 + P_1 \\ Y_{62}'' = P_2 \\ Y_{62}''' = P_2 \end{cases}$$

$$\begin{cases} Y_{63} = P_2 \\ Y_{63}' = P_2 \\ Y_{63}'' = 1 + P_2 \\ Y_{63}''' = 1 + P_2 \end{cases}$$

$$\begin{cases} Y_{64} = P_2 \\ Y_{64}' = P_2 \\ Y_{64}'' = 1 \\ Y_{64}''' = 1 \end{cases}$$

$$\begin{cases} Y_{65} = 1 + P_2 \\ Y_{65}' = 1 + P_2 \\ Y_{65}'' = 1 \\ Y_{65}''' = 1 \end{cases}$$

$$\begin{cases} Y_{66} = 1 + P_2 \\ Y_{66}' = 1 + P_2 \\ Y_{66}'' = 1 + P_1 + P_2 \\ Y_{66}''' = 1 + P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{67} = 1 + P_2 \\ Y_{67}' = 1 + P_2 \\ Y_{67}'' = P_1 + P_2 \\ Y_{67}''' = P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{68} = 1 + P_2 \\ Y_{68}' = 1 + P_2 \\ Y_{68}'' = 1 \\ Y_{68}''' = 1 \end{cases}$$

$$\begin{cases} Y_{69} = 1 + P_1 \\ Y_{69}' = 1 + P_1 \\ Y_{69}'' = P_1 \\ Y_{69}''' = P_1 \end{cases}$$

$$\begin{cases} Y_{70} = 1 + P_1 + P_2 \\ Y_{70}' = 1 + P_1 + P_2 \\ Y_{70}'' = P_1 \\ Y_{70}''' = P_1 \end{cases}$$

$$\begin{cases} Y_{71} = P_1 + P_2 \\ Y_{71}' = P_1 + P_2 \\ Y_{71}'' = 1 \\ Y_{71}''' = 1 \end{cases}$$

$$\begin{cases} Y_{72} = P_1 \\ Y_{72}' = P_1 \\ Y_{72}'' = 1 + P_1 + P_2 \\ Y_{72}''' = 1 + P_1 + P_2 \end{cases}$$

$$\begin{cases} Y_{73} = P_1 + P_2 \\ Y_{73}' = P_1 + P_2 \\ Y_{73}'' = 1 + P_2 \\ Y_{73}''' = 1 + P_2 \end{cases}$$

$$\begin{cases} Y_{74} = P_1 \\ Y_{74}' = P_1 \\ Y_{74}'' = 1 + P_2 \\ Y_{74}''' = 1 + P_2 \end{cases}$$

$$\begin{cases} Y_{75} = 1 + P_1 \\ Y_{75}' = 1 + P_1 \\ Y_{75}'' = 1 + P_2 \\ Y_{75}''' = 1 + P_2 \end{cases}$$

$$\begin{cases} Y_{76} = 1 + P_1 + P_2 \\ Y_{76}' = 1 + P_1 + P_2 \\ Y_{76}'' = 1 + P_2 \\ Y_{76}''' = 1 + P_2 \end{cases}$$

$$\begin{cases} Y_{77} = 1 \\ Y_{77}' = 1 \\ Y_{77}'' = 1 + P_2 \\ Y_{77}''' = 1 + P_2 \end{cases}$$

Conclusion.

In this paper, we have studied Pythagoras quadruples in symbolic 2-plithogenic commutative rings, where necessary and sufficient conditions for a symbolic 2-plithogenic quadruple (x, y, z, t) to be a Pythagoras quadruple.

Also, we have presented some related examples that explain how to find 2-plithogenic quadruples from classical quadruples.

Acknowledgments " This study is supported via funding from Prince sattam bin Abdulaziz University project number (PSAU/2023/R/1445) ".

References

[1] M. B. Zeina, N. Altounji, M. Abobala, and Y. Karmouta, “Introduction to Symbolic 2-Plithogenic Probability Theory,” *Galoitica: Journal of Mathematical Structures and Applications*, vol. 7, no. 1, 2023.

[2] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.

[3] Nader Mahmoud Taffach , Ahmed Hatip., "A Review on Symbolic 2-Plithogenic Algebraic Structures " Galoitica Journal Of Mathematical Structures and Applications, Vol.5, 2023.

[4] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.

[5] Ben Othman, K., "On Some Algorithms For Solving Symbolic 3-Plithogenic Equations", Neoma Journal Of Mathematics and Computer Science, 2023.

[6] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Concept Of Symbolic 7-Plithogenic Real Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.

[7] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Symbolic 8-Plithogenic Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.

[8]Mohamed, M., "Modified Approach for Optimization of Real-Life Transportation Environment: Suggested Modifications", American Journal of Business and Operations research, 2021.

[9] Nabeeh, N., Alshaimaa, A., and Tantawy, A., "A Neutrosophic Proposed Model For Evaluation Blockchain Technology in Secure Enterprise Distributed Applications", Journal of Cypersecurity and Information Management, 2023.

[10]Abualkishik, A., Almajed, R., Thompson, W., "Improving The Performance of Fog-assisted Internet of Things Networks Using Bipolar Trapezoidal Neutrosophic Sets", Journal of Wireless and Ad Hoc Communication, 2023.

[11] R. A. Trivedi, S. A. Bhanotar, Pythagorean Triplets - Views, Analysis and Classification, IIOSR J. Math., 11 (2015).

[12] J. Rukavicka, Dickson's method for generating Pythagorean triples revisited, Eur. J. Pure Appl.Math., 6 (2013).

[13] T.Roy and F J. Sonia, A Direct Method To Generate Pythagorean Triples And Its Generalization To Pythagorean Quadruples And n-tuples, , Jadavpur University, India (2010)

[14] Paul Oliverio, " Self-Generating Pythagorean Quadruples And TV-Tuples", Los Angeles, (1993)

Received:1/5/2023, Accepted: 5/10/2023