



Interval Valued, m-Polar and m-Polar Interval Valued Neutrosophic Hypersoft Sets

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Abstract: Decision making is a complex issue due to vague, imprecise and indeterminate environment specially, when attributes are more than one, and further bifurcated. To solve such type of problems, concept of neutrosophic hypersoft set (NHSS) was proposed [1]. The purpose of this paper is to provide the extension of NHSS into: Interval Valued, m-Polar and m-Polar interval valued Neutrosophic Hypersoft sets. The definitions of proposed extensions and mathematical operations are discussed in detail with suitable examples. Finally, concluded the present work with the future direction.

Keywords: MCDM, Uncertainty, Soft set (SS), Neutrosophic Soft set (NS's), Hypersoft set (HS's), Neutrosophic Hypersoft set (NHSS)

1. Introduction

The concept of membership was initiated by Zadeh [2] known as fuzzy set (F's). This, concept was extended by Atanassov [3] and known as intuitionistic fuzzy set (IF's) and this concept was extended by Smarandache [4] who proposed the theory of neutrosophic set (N's) with the addition of indeterminacy value along with membership, and non-membership values. The Hybrid within neutrosophic theory was suggested by [5], the hybrids consists of; single-valued neutrosophic set (SVNS), Interval-valued neutrosophic set (IVNS) [6], multi-valued neutrosophic set (MVNS) [7]. After these generalizations many researches related to SVNS have been conducted [8–17]. Broumi *et al.* [18] merged the concept of N's and multi-valued and proposed the new idea; known as; multi-valued interval neutrosophic set (MVINS). Many other developments within this structure has been discussed by [19-21]. One of the most important development in the field of fuzzy was made by Molodtsov [22] who provided the idea of soft set (SS), that is very useful to deal with uncertain and vague information. In recent years, the SS theory is extended to many other theories Firstly, Fuzzy soft set theory and its properties was developed by Cagman *et al.* [23]. The key role in these theories was made by Maji [24] who extended the theory of NS by combining with soft set, named as neutrosophic soft set (NSS). Within this set [25] introduced some basic definitions, operations, and decision-making approaches called as IVNSS. After this, these hybrids were extended to multi-valued neutrosophic soft set (MVNSS) by [26]. Some definitions, operations and applications of MCDM approach-based problems using MVNSS was introduced [27]. Utilizing this idea a few mathematicians have proposed their examination work in various scientific fields [28-37] and this idea is likewise utilized in advancing decision-making calculations [38-42].

Smarandache [43] generalized the concept of soft set (SS) to hypersoft set (HSS) by converting the function into multi-attribute function to deal with uncertainty along with all the hybrids like crisp, fuzzy, intuitionistic and neutrosophic. Saqlain *et al.* [44] proposed the aggregate operators and similarity measure [45] on NHSS. Also, Saqlain *et al.* [46] developed the generalization of TOPSIS for the NHSS, by using accuracy function they transformed the fuzzy neutrosophic numbers to crisp form.

The purpose of this paper is to overcome the uncertainty problem in more precise way by combing Interval-Valued Neutrosophic set IVNSS, m-Polar Neutrosophic set mpand m-Polar Interval-Valued Neutrosophic set with Hypersoft set.

The paper presentation is as follows. Section 2, provides the basic definitions and major relation with the extension Interval-Valued, multi-polar and multi-polar Interval-valued from NHSS are presented. Section 3, proposes the basic definitions along with: subset, null set and Universal set of each type. Finally, conclusion and future direction is presented in section 4.

1.1 Motivation

From the literature, it is found that Interval Valued, m-Polar and m-Polar Interval Valued Neutrosophic Hypersoft Set respectively has not yet been studied so far. This leads us to the present study.

2.Preliminaries

Definition 2.1: IVNSS [6]

Consider \mathbb{U} and \mathbb{E} be universal and set of attributes respectively and consider $\mathbb{A} \subseteq \mathbb{E}$. The mapping (F, \mathbb{A}) is called an IVNSS over \mathbb{U} and is given as;

$$F: \mathbb{A} \rightarrow \mathbb{P}(\mathbb{U}) \text{ and } (F, \mathbb{A}) = \{ \langle u, T(F(\mathbb{A})), I(F(\mathbb{A})), F(F(\mathbb{A})) \rangle, u \in \mathbb{U} \}$$

Where $T(F(\mathbb{A})) \subseteq [0,1], I(F(\mathbb{A})) \subseteq [0,1]$ and $F(F(\mathbb{A})) \subseteq [0,1]$ are the intervals with side conditions $0 \leq \sup T(F(\mathbb{A})) + \sup I(F(\mathbb{A})) + \sup F(F(\mathbb{A})) \leq 3$. The terms $T(F(\mathbb{A})), I(F(\mathbb{A})), F(F(\mathbb{A}))$ represent the truthiness, indeterminacy and falsity of u to \mathbb{A} respectively. For our convenience,

we assume that $\mathbb{A} = \langle [T(F(\mathbb{A}))^-, T(F(\mathbb{A}))^+], [I(F(\mathbb{A}))^-, I(F(\mathbb{A}))^+], [F(F(\mathbb{A}))^-, F(F(\mathbb{A}))^+] \rangle$

where;

$$T(F(\mathbb{A})) = [T(F(\mathbb{A}))^-, T(F(\mathbb{A}))^+] \subseteq [0,1], I(F(\mathbb{A})) = [I(F(\mathbb{A}))^-, I(F(\mathbb{A}))^+] \subseteq [0,1] \text{ and}$$

$$F(F(\mathbb{A})) = [F(F(\mathbb{A}))^-, F(F(\mathbb{A}))^+] \subseteq [0,1].$$

Definition 2.2: m-Polar Neutrosophic Soft Set [26]

Consider \mathbb{U} and \mathbb{E} be universal and set of attributes respectively and consider $\mathbb{A} \subseteq \mathbb{E}$. The mapping (F, \mathbb{A}) is called an MVNSS over \mathbb{U} and is given as;

$$F: \mathbb{A} \rightarrow \mathbb{P}(\mathbb{U}) \text{ and } (F, \mathbb{A}) = \left\{ \frac{\langle T^x(F(\mathbb{A})), I^y(F(\mathbb{A})), F^z(F(\mathbb{A})) \rangle}{u}, u \in \mathbb{U} \right\}$$

Where $T(F(\mathbb{A})) \subseteq [0,1], I(F(\mathbb{A})) \subseteq [0,1]$ and $F(F(\mathbb{A})) \subseteq [0,1]$ are the multi-valued numbers and they are given as;

$$T^x(F(\mathbb{A})) = T^1(F(\mathbb{A})), T^2(F(\mathbb{A})) \dots T^x(F(\mathbb{A})),$$

$$\begin{aligned} \mathbb{I}^y(\mathbb{F}(\mathbb{A})) &= \mathbb{I}^1(\mathbb{F}(\mathbb{A})), \mathbb{I}^2(\mathbb{F}(\mathbb{A})) \dots \mathbb{I}^y(\mathbb{F}(\mathbb{A})), \\ \mathbb{F}^z(\mathbb{F}(\mathbb{A})) &= \mathbb{F}^1(\mathbb{F}(\mathbb{A})), \mathbb{F}^2(\mathbb{F}(\mathbb{A})) \dots \mathbb{F}^z(\mathbb{F}(\mathbb{A})). \end{aligned}$$

$\mathbb{T}(\mathbb{F}(\mathbb{A})), \mathbb{I}(\mathbb{F}(\mathbb{A})), \mathbb{F}(\mathbb{F}(\mathbb{A}))$ represent the truthiness, indeterminacy and falsity of u to \mathbb{A} respectively.

Definition 2.3: MVINSS [18]

Consider \mathbb{U} and \mathbb{E} be universal and set of attributes respectively and consider $\mathbb{A} \subseteq \mathbb{E}$. The mapping (\mathbb{F}, \mathbb{A}) is called an MVINSS over \mathbb{U} and is given as;

$$\mathbb{F}: \mathbb{A} \rightarrow \mathbb{P}(\mathbb{U}) \text{ and } (\mathbb{F}, \mathbb{A}) = \left\{ \frac{\langle \mathbb{T}^x(\mathbb{F}(\mathbb{A})), \mathbb{I}^y(\mathbb{F}(\mathbb{A})), \mathbb{F}^z(\mathbb{F}(\mathbb{A})) \rangle}{u}, u \in \mathbb{U} \right\}$$

Where $\mathbb{T}(\mathbb{F}(\mathbb{A})) \subseteq [0,1], \mathbb{I}(\mathbb{F}(\mathbb{A})) \subseteq [0,1]$ and $\mathbb{F}(\mathbb{F}(\mathbb{A})) \subseteq [0,1]$ are the multi-valued numbers and they are given as;

$$\mathbb{T}^x(\mathbb{F}(\mathbb{A})) = \left[\mathbb{T}^1(\mathbb{F}(\mathbb{A}))^-, \mathbb{T}^1(\mathbb{F}(\mathbb{A}))^+ \right], \left[\mathbb{T}^2(\mathbb{F}(\mathbb{A}))^-, \mathbb{T}^2(\mathbb{F}(\mathbb{A}))^+ \right] \dots \left[\mathbb{T}^x(\mathbb{F}(\mathbb{A}))^-, \mathbb{T}^x(\mathbb{F}(\mathbb{A}))^+ \right]$$

$$\mathbb{I}^y(\mathbb{F}(\mathbb{A})) = \left[\mathbb{I}^1(\mathbb{F}(\mathbb{A}))^-, \mathbb{I}^1(\mathbb{F}(\mathbb{A}))^+ \right], \left[\mathbb{I}^2(\mathbb{F}(\mathbb{A}))^-, \mathbb{I}^2(\mathbb{F}(\mathbb{A}))^+ \right] \dots \left[\mathbb{I}^y(\mathbb{F}(\mathbb{A}))^-, \mathbb{I}^y(\mathbb{F}(\mathbb{A}))^+ \right],$$

$$\mathbb{F}^z(\mathbb{F}(\mathbb{A})) = \left[\mathbb{F}^1(\mathbb{F}(\mathbb{A}))^-, \mathbb{F}^1(\mathbb{F}(\mathbb{A}))^+ \right], \left[\mathbb{F}^3(\mathbb{F}(\mathbb{A}))^-, \mathbb{F}^3(\mathbb{F}(\mathbb{A}))^+ \right] \dots \left[\mathbb{F}^z(\mathbb{F}(\mathbb{A}))^-, \mathbb{F}^z(\mathbb{F}(\mathbb{A}))^+ \right].$$

$\mathbb{T}(\mathbb{F}(\mathbb{A})), \mathbb{I}(\mathbb{F}(\mathbb{A})), \mathbb{F}(\mathbb{F}(\mathbb{A}))$ represent the truthiness, indeterminacy and falsity of u to \mathbb{A} respectively.

Definition 2.4: Neutrosophic Hypersoft Set [44]

Let $\mathbb{U} = \{u^1, u^2, \dots, u^a\}$ and $\mathbb{P}(\mathbb{U})$ be the universal set and power set of universal set respectively, also consider $\mathbb{L}_1, \mathbb{L}_2, \dots, \mathbb{L}_\ell$ for $\ell \geq 1, \ell$ well defined attributes, and corresponding attributive values are the set $\mathbb{L}_1^a, \mathbb{L}_2^b, \dots, \mathbb{L}_\ell^z$ and their relation $\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z$ where $a, b, c, \dots, z = 1, 2, \dots, n$ then the pair $(\mathbb{F}, \mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z)$ is said to be Neutrosophic Hypersoft set over \mathbb{U} where $\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z) \rightarrow \mathbb{P}(\mathbb{U})$ and it is define as

$$\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z) \rightarrow \mathbb{P}(\mathbb{U}) \text{ and}$$

$$\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z) = \{ \langle u, \mathbb{T}_\ell(u), \mathbb{I}_\ell(u), \mathbb{F}_\ell(u) \rangle \mid u \in \mathbb{U}, \ell \in (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z) \} \text{ where } \mathbb{T}, \mathbb{I}, \mathbb{F} \text{ represent the truthiness, indeterminacy and falsity of } u \text{ to } \mathbb{A} \text{ respectively such that } \mathbb{T}, \mathbb{I}, \mathbb{F}: \mathbb{U} \rightarrow [0,1] \text{ also } 0 \leq \mathbb{T}_\ell(u) + \mathbb{I}_\ell(u) + \mathbb{F}_\ell(u) \leq 3.$$

3. Calculations

In this section, NHSS is extended into the following:

Notions: Following abbreviation will be used throughout the article,

- Interval-valued Neutrosophic Hypersoft Set (IVNHSS)
- m-Polar Neutrosophic Hypersoft Set (m-Polar NHSS)
- m-Polar Interval-valued Neutrosophic Hypersoft Set (m-Polar IVNHSS)

Example 1: (Following formulation and assumptions will be considered throughout as an example)

Let \mathbb{U} be the set of different schools nominated for best school given as;

$$\mathbb{U} = \{S^1, S^2, S^3, S^4, S^5\}$$

also consider the set of attributes as;

Assumptions:

$A_1 = \text{Teaching standards}, A_2 = \text{Organization}, A_3 = \text{Ongoing Evaluation}, A_4 = \text{Goals}$

And their respective attributes are given as

$A_1^a = \text{Teaching standards} = \{\text{high, mediocre, low}\}$

$A_2^b = \text{Organization} = \{\text{good, average, poor}\}$

$A_3^c = \text{Ongoing Evaluation} = \{\text{yes, no}\}$

$A_4^d = \text{Goals} = \{\text{effective, committed, upto date}\}$

Formulation:

$$F: A_1^a \times A_2^b \times A_3^c \times A_4^d \rightarrow \mathbb{P}(U)$$

Let's assume $F(\text{high, average, yes, effective}) = \{S^1, S^5\}$

Then one of, **Neutrosophic Hypersoft set NHSS** of above assumed relation is

$$\begin{aligned} &F(\text{high, average, yes, effective}) \\ &= \{S^1, (\text{high } \{0.9, 0.3, 0.1\}, \text{average } \{0.8, 0.2, 0.4\}, \text{yes } \{0.4, 0.9, 0.6\}, \text{effective } \{0.6, 0.4, 0.5\}) >, \\ &< S^5(\text{high } \{0.5, 0.3, 0.8\}, \text{average } \{0.6, 0.1, 0.2\}, \text{yes } \{0.6, 0.4, 0.7\}, \text{effective } \{0.4, 0.5, 0.3\}) >\} \end{aligned}$$

In this Example,

Case 1: Substituting attributive values as; interval values in the form of neutrosophic, call it, IVNHSS.

Case 2: Substituting attributive values as; m-neutrosophic values, call it, m-polar NHSS.

Case 3: Substituting attributive values as; m-neutrosophic interval values, call it, IVNHSS.

3.1 Interval-valued Neutrosophic Hypersoft Set (IVNHSS)

Definition 3.1.1 IVNHSS

Let $U = \{u^1, u^2, \dots, u^a\}$ and $\mathbb{P}(U)$ be the universal set and power set of universal set respectively, also consider L_1, L_2, \dots, L_b for $b \geq 1, b$ well defined attributes, and corresponding attributive values are the set $L_1^a, L_2^b, \dots, L_b^z$ and their relation $L_1^a \times L_2^b \times \dots \times L_b^z$ where $a, b, c, \dots, z = 1, 2, \dots, n$ then the pair $(F, L_1^a \times L_2^b \times \dots \times L_b^z)$ is said to be Interval-Valued Neutrosophic Hypersoft set IVNHSS, over U where $F: (L_1^a \times L_2^b \times \dots \times L_b^z) \rightarrow \mathbb{P}(U)$ and it is define as

$$F: (L_1^a \times L_2^b \times \dots \times L_b^z) \rightarrow \mathbb{P}(U)$$

And, $F: (L_1^a \times L_2^b \times \dots \times L_b^z) = \{< u, T_\ell(u), I_\ell(u), F_\ell(u) > \mid u \in U, \ell \in (L_1^a \times L_2^b \times \dots \times L_b^z)\}$

Where $T_\ell(u) \subseteq [0,1], I_\ell(u) \subseteq [0,1]$ and $F_\ell(u) \subseteq [0,1]$ are the interval numbers and $0 \leq \sup T_\ell(u) + \sup I_\ell(u) + \sup F_\ell(u) \leq 3$. The intervals $T_\ell(u), I_\ell(u), F_\ell(u)$ represent the truthiness, indeterminacy and falsity of u to A respectively. For convenience, we assume that:

$$A = \{ < [(T_\ell(u))^- , (T_\ell(u))^+], [(I_\ell(u))^- , (I_\ell(u))^+], [(F_\ell(u))^- , (F_\ell(u))^+] > \text{ where}$$

$$T_\ell(u) = [(T_\ell(u))^- , (T_\ell(u))^+] \subseteq [0,1], I_\ell(u) = [(I_\ell(u))^- , (I_\ell(u))^+] \subseteq [0,1] \text{ and}$$

$$F_\ell(u) = [(F_\ell(u))^- , (F_\ell(u))^+] \subseteq [0,1].$$

Example:

$$F: A_1^a \times A_2^b \times A_3^c \times A_4^d \rightarrow \mathbb{P}(U)$$

Let's assume $F(\text{high, average, yes, effective}) = \{S^1, S^5\}$

Then Interval-Valued Neutrosophic Hypersoft set of above assumed relation is

$$\mathbb{F}(\text{high}, \text{average}, \text{yes}, \text{effective}) = \{ \langle \mathbb{S}^1, \begin{pmatrix} \text{high} < [0.4, 0.9], [0.3, 0.5], [0.1, 0.7] >, \\ \text{average} < [0.3, 0.7], [0.3, 0.4], [0.01, 0.17] >, \\ \text{yes} < [0.41, 0.49], [0.03, 0.15], [0.18, 0.28] >, \\ \text{effective} < [0.12, 0.54], [0.23, 0.75], [0.51, 0.81] > \end{pmatrix} \rangle, \right. \\ \left. (\text{high}, \text{average}, \text{yes}, \text{effective}) = \{ \langle \mathbb{S}^5, \begin{pmatrix} \text{high} < [0.6, 0.86], [0.53, 0.65], [0.71, 0.89] >, \\ \text{average} < [0.3, 0.4], [0.31, 0.55], [0.01, 0.03] >, \\ \text{yes} < [0.83, 0.9], [0.23, 0.59], [0.05, 0.09] >, \\ \text{effective} < [0.23, 0.58], [0.03, 0.3], [0.01, 0.1] > \end{pmatrix} \rangle \} \right.$$

Definition 3.1.2 Subset of IVNHSS

Let $\psi_i, \omega_i \in \text{IVNHSS}(\mathbb{U})$. Then, ψ_i for all $x \in \mathbb{F}$ is an IVNHSS subset of ω_i , denoted by $\psi_i \subseteq \omega_i$. If $\psi_i(x) \subseteq \omega_i(x)$ for all $x \in \mathbb{F}$.

Definition 3.1.3 Empty IVNHSS

Let $\psi_i \in \text{IVNHSS}(\mathbb{U})$. If $\psi_i = \emptyset$ for all $x \in \mathbb{F}$ then \mathbb{N} is called an empty IVNHSS, denoted by $\hat{\phi}$.

Definition 3.1.4 Universal Set of IVNHSS

Let $\psi_i \in \text{IVNHSS}(\mathbb{U})$. If $\psi_i = \mathbb{F}$ for all $x \in \mathbb{F}$ then \mathbb{N} is called universal set of IVNHSS, denoted by $\hat{\kappa}$.

3.2 m-Polar Neutrosophic Hypersoft Set (m-Polar NHSS)

Definition 3.2.1 m-Polar NHSS

Let $\mathbb{U} = \{u^1, u^2, \dots, u^a\}$ and $\mathbb{P}(\mathbb{U})$ be the universal set and power set of universal set respectively, also consider $\mathbb{L}_1, \mathbb{L}_2, \dots, \mathbb{L}_b$ for $b \geq 1$, b well defined attributes, and corresponding attributive values are the set $\mathbb{L}_1^a, \mathbb{L}_2^b, \dots, \mathbb{L}_b^z$ and their relation $\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_b^z$ where $a, b, c, \dots, z = 1, 2, \dots, n$ then the pair $(\mathbb{F}, \mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_b^z)$ is said to be m-Polar Neutrosophic Hypersoft set m-Polar NHSS, over \mathbb{U} where $\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_b^z) \rightarrow \mathbb{P}(\mathbb{U})$ and it is define as

$$\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_b^z) \rightarrow \mathbb{P}(\mathbb{U})$$

And,

$$\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_b^z) = \left\{ \langle u, \mathbb{T}_\ell^i(u), \mathbb{I}_\ell^j(u), \mathbb{F}_\ell^k(u) \rangle \mid u \in \mathbb{U}, \ell \in (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_b^z) : \right\} \text{ Also} \\ \mathbf{0} \leq \sum_{i=1}^p \mathbb{T}_\ell^i(u) \leq \mathbf{1}, \quad \mathbf{0} \leq \sum_{j=1}^q \mathbb{I}_\ell^j(u) \leq \mathbf{1}, \quad \mathbf{0} \leq \sum_{k=1}^r \mathbb{F}_\ell^k(u) \leq \mathbf{1}$$

Where $\mathbb{T}_\ell^i(u) \subseteq [0, 1], \mathbb{I}_\ell^j(u) \subseteq [0, 1]$ and $\mathbb{F}_\ell^k(u) \subseteq [0, 1]$ are the numbers and

$$\mathbf{0} \leq \sum_{i=1}^p \mathbb{T}_\ell^i(u) + \sum_{j=1}^q \mathbb{I}_\ell^j(u) + \sum_{k=1}^r \mathbb{F}_\ell^k(u) \leq \mathbf{3}$$

For our convenience, we assume that

$$\mathbb{T}_\ell^i(u) = \mathbb{T}_{\ell_1^1}(u), \mathbb{T}_{\ell_2^2}(u), \mathbb{T}_{\ell_3^3}(u), \dots, \mathbb{T}_{\ell_p^p}(u) \\ \mathbb{I}_\ell^j(u) = \mathbb{I}_{\ell_1^1}(u), \mathbb{I}_{\ell_2^2}(u), \mathbb{I}_{\ell_3^3}(u), \dots, \mathbb{I}_{\ell_q^q}(u) \\ \mathbb{F}_\ell^k(u) = \mathbb{F}_{\ell_1^1}(u), \mathbb{F}_{\ell_2^2}(u), \mathbb{F}_{\ell_3^3}(u), \dots, \mathbb{F}_{\ell_r^r}(u).$$

Example:

$$\mathbb{F}: \mathbb{A}_1^a \times \mathbb{A}_2^b \times \mathbb{A}_3^c \times \mathbb{A}_4^d \rightarrow \mathbb{P}(\mathbb{U})$$

Let's assume $\mathbb{F}(\text{high, average, yes, effective}) = \{\mathbb{S}^1, \mathbb{S}^5\}$

Then in Neutrosophic Hypersoft set of above assumed relation is,

$\mathbb{F}(\text{high, average, yes, effective})$

$$= \left\langle \mathbb{S}^1, \left(\begin{array}{l} \text{high} < (0.01,0.003,0.1,0.023,0.07), (0.092,0.073,0.08,0.2,0.4), (0.2,0.017,0.06,0.13,0.3) >, \\ \text{average} < (0.2,0.1,0.5,0.019,0.051), (0.21,0.14,0.27,0.009,0.1), (0.113,0.35,0.25,0.12,0.03) >, \\ \text{yes} < (0.12,0.13,0.14,0.15,0.39), (0.17,0.20,0.24,0.15,0.1), (0.2,0.1,0.5,0.019,0.051) >, \\ \text{effective} < (0.2,0.017,0.06,0.13,0.3), (0.12,0.025,0.07,0.22,0.074), (0.01,0.003,0.1,0.023,0.07) > \end{array} \right) \right\rangle$$

$\mathbb{F}(\text{high, average, yes, effective})$

$$= \left\langle \mathbb{S}^5, \left(\begin{array}{l} \text{high} < (0.09,0.08,0.7,0.026,0.05), (0.04,0.03,0.02,0.1,0.09), (0.32,0.51,0.06,0.03,0.12) >, \\ \text{average} < (0.12,0.13,0.14,0.15,0.39), (0.17,0.20,0.24,0.15,0.1), (0.2,0.1,0.5,0.019,0.051) >, \\ \text{yes} < (0.09,0.08,0.7,0.026,0.05), (0.04,0.03,0.02,0.1,0.09), (0.32,0.51,0.06,0.03,0.12) >, \\ \text{effective} < (0.12,0.13,0.14,0.15,0.39), (0.12,0.025,0.07,0.22,0.074), (0.12,0.025,0.07,0.22,0.4) > \end{array} \right) \right\rangle$$

Definition 3.2.2 Subset of m-Polar NHSS

Let $\zeta_i, \delta_i \in m - \text{Polar NHSS}(\mathbb{U})$. Then, $\zeta_i \forall x \in \mathbb{F}$ is an m-Polar NHSS subset of δ_i , represented by $\zeta_i \subseteq \delta_i$. If $\zeta_i(x) \subseteq \delta_i(x)$ for all $x \in \mathbb{F}$.

Definition 3.2.3 Empty m-Polar NHSS

Let $\zeta_i \in m - \text{Polar NHSS}(\mathbb{U})$. If $\zeta_i = \emptyset \forall x \in \mathbb{F}$ then \mathbb{N} is said to be an empty m-polar NHSS, represented by $\tilde{\emptyset}$.

Definition 3.2.4 Universal Set of m-Polar NHSS

Let $\zeta_i \in m - \text{Polar NHSS}(\mathbb{U})$. If $\zeta_i = \hat{\mathbb{F}} \forall x \in \mathbb{F}$ then \mathbb{N} is called universal set of m-Polar NHSS, represented by $\hat{\sigma}$.

3.3 m-Polar Interval-Valued Neutrosophic Hypersoft Set (m-Polar IVNHSS)

Definition 3.3.1: m-Polar IVNHSS)

Let $\mathbb{U} = \{u^1, u^2, \dots, u^a\}$ and $\mathbb{P}(\mathbb{U})$ be the universal set and power set of universal set respectively, also consider $\mathbb{L}_1, \mathbb{L}_2, \dots, \mathbb{L}_\ell$ for $\ell \geq 1$, ℓ well defined attributes, and corresponding attributive values are the set $\mathbb{L}_1^a, \mathbb{L}_2^b, \dots, \mathbb{L}_\ell^z$ and their relation $\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z$ where $a, b, c, \dots, z = 1, 2, \dots, n$ then the pair $(\mathbb{F}, \mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z)$ is said to be Interval-Valued Neutrosophic Hypersoft set IVNHSS, over \mathbb{U} where $\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z) \rightarrow \mathbb{P}(\mathbb{U})$ and it is define as

$$\mathbb{F}: (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z) \rightarrow \mathbb{P}(\mathbb{U})$$

$$\mathbb{F} =: \left\{ \left\langle u, \mathbb{T}_\ell^x(u), \mathbb{I}_\ell^y(u), \mathbb{F}_\ell^z(u) \right\rangle \mid u \in \mathbb{U}, \ell \in (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \dots \times \mathbb{L}_\ell^z) : \right\}$$

$$x, y, z = 1, 2, 3, \dots$$

Where,

$$\mathbb{T}_\ell^x(u) = [(\mathbb{T}_\ell^x(u))^- , (\mathbb{T}_\ell^x(u))^+] \subseteq [0,1]$$

$$\mathbb{I}_\ell^y(u) = [(\mathbb{I}_\ell^y(u))^- , (\mathbb{I}_\ell^y(u))^+] \subseteq [0,1]$$

$$\mathbb{F}_\ell^z(u) = [(\mathbb{F}_\ell^z(u))^- , (\mathbb{F}_\ell^z(u))^+] \subseteq [0,1]$$

Also

$$0 \leq \sum_{x=1}^s \text{Sup} \{T_\ell^x(\mathbf{u})\} \leq 1, \quad 0 \leq \sum_{y=1}^t \text{Sup}\{I_\ell^y(\mathbf{u})\} \leq 1, \quad 0 \leq \sum_{z=1}^v \{F_\ell^z(\mathbf{u})\} \leq 1$$

And,

$$0 \leq \sum_{x=1}^s \text{Sup} \{T_\ell^x(\mathbf{u})\} + \sum_{y=1}^t \text{Sup}\{I_\ell^y(\mathbf{u})\} + \sum_{z=1}^v \{F_\ell^z(\mathbf{u})\} \leq 3$$

For our convenience, we assume that:

$$T_\ell^x(\mathbf{u}) = \langle [(T_\ell^1(\mathbf{u}))^-, (T_\ell^1(\mathbf{u}))^+], [(T_\ell^2(\mathbf{u}))^-, (T_\ell^2(\mathbf{u}))^+], \dots, [(T_\ell^s(\mathbf{u}))^-, (T_\ell^s(\mathbf{u}))^+] \rangle$$

$$I_\ell^y(\mathbf{u}) = \langle [(I_\ell^1(\mathbf{u}))^-, (I_\ell^1(\mathbf{u}))^+], [(I_\ell^2(\mathbf{u}))^-, (I_\ell^2(\mathbf{u}))^+], \dots, [(I_\ell^t(\mathbf{u}))^-, (I_\ell^t(\mathbf{u}))^+] \rangle$$

$$F_\ell^z(\mathbf{u}) = \langle [(F_\ell^1(\mathbf{u}))^-, (F_\ell^1(\mathbf{u}))^+], [(F_\ell^2(\mathbf{u}))^-, (F_\ell^2(\mathbf{u}))^+], \dots, [(F_\ell^v(\mathbf{u}))^-, (F_\ell^v(\mathbf{u}))^+] \rangle$$

Example:

$$F: A_1^a \times A_2^b \times A_3^c \times A_4^d \rightarrow \mathbb{P}(U)$$

Let's assume $F(\text{high, average, yes, effective}) = \{S^1, S^5\}$

Then m-Polar Interval-Valued Neutrosophic Hypersoft set of above assumed relation is

$F(\text{high, average, yes, effective})$

$$= \left\langle S^1, \left(\begin{array}{l} \text{high} < ([0.01,0.03], [0.1,0.23], [0.07,0.5]), ([0.072,0.073], [0.08,0.2], [0.14,0.32]), ([0.12,0.017], [0.06,0.13], [0.3,0.4]) >, \\ \text{average} < ([0.2,0.21], [0.15,0.19], [0.02,0.03]), ([0.11,0.14], [0.27,0.39], [0.1,0.11]), ([0.113,0.35], [0.11,0.12], [0, 0.3]) >, \\ \text{yes} < ([0.12,0.13], [0.14,0.15], [0.39,0.4]), ([0.17,0.20], [0.14,0.15], [0.1,0.2]), ([0.1,0.11], [0.15,0.19], [0.01,0.09]) >, \\ \text{effective} < ([0.15,0.17], [0.06,0.13], [0.3,0.31]), ([0.12,0.25], [0.07,0.22], [0.07,0.09]), ([0.01,0.03], [0.1,0.23], [0.17,0.19]) > \end{array} \right) \right\rangle$$

$F(\text{high, average, yes, effective})$

$$= \left\langle S^5, \left(\begin{array}{l} \text{high} < ([0.01,0.03], [0.1,0.23], [0.07,0.5]), ([0.072,0.073], [0.08,0.2], [0.14,0.32]), ([0.12,0.017], [0.06,0.13], [0.3,0.4]) >, \\ \text{average} < ([0.2,0.21], [0.15,0.19], [0.02,0.03]), ([0.11,0.14], [0.27,0.39], [0.1,0.11]), ([0.113,0.35], [0.11,0.12], [0, 0.3]) >, \\ \text{yes} < ([0.12,0.13], [0.14,0.15], [0.39,0.4]), ([0.17,0.20], [0.14,0.15], [0.1,0.2]), ([0.1,0.11], [0.15,0.19], [0.01,0.09]) >, \\ \text{effective} < ([0.15,0.17], [0.06,0.13], [0.3,0.31]), ([0.12,0.25], [0.07,0.22], [0.07,0.09]), ([0.01,0.03], [0.1,0.23], [0.17,0.19]) > \end{array} \right) \right\rangle$$

Definition 3.3.2 Subset of m-Polar IVNHSS

Let $\omega_i, \beta_i \in m - \text{Polar IVNHSS}(U)$. Then, $\omega_i \forall x \in F$ is an m-Polar IVNHSS subset of β_i , represented by $\omega_i \subseteq \beta_i$. If $\omega_i(x) \subseteq \beta_i(x)$ for all $x \in F$.

Definition 3.3.3 Empty IVNHSS

Let $\omega_i \in m - \text{Polar IVNHSS}(U)$. If $\omega_i = \phi \forall x \in F$ then \mathbb{N} is said to be an empty m-Polar IVNHSS, represented by $\bar{\phi}$.

Definition 3.3.4 Universal Set of IVNHSS

Let $\omega_i \in m - \text{Polar IVNHSS}(U)$. If $\omega_i = \hat{F} \forall x \in F$ then \mathbb{N} is called universal set of m-Polar IVNHSS, represented by $\hat{\Omega}$.

4. Conclusions

In this paper, the concept of Interval Valued NHSS, m-Polar NHSS and m-Polar interval-valued NHSS are proposed. The proposed sets have several significant features. Firstly, they emphasize the hesitant, indeterminate and uncertainty and can be used more practical to solve decision-making

problem. Secondly, some basic types of the proposed sets such as; universal set, empty set and subset of each type is defined.

- Since this study has not yet been studied yet, comparative study cannot be done with the existing methods.
- Further, this proposed can be applied immensely in various fields of research. In future, the present work may be extended to other special types of neutrosophic set like neutrosophic rough set etc.
- The sets which are proposed in this paper can be applied in solving supply chain, time series forecasting and decision-making problem such as partner selection, wastewater treatment selection and renewable energy selection, by defining the following:
 - the aggregate operators,
 - distance measures,
 - matrix theory and
 - Algorithms.

Acknowledgments:

Funding: This research has no funding.

Conflicts of Interest: The authors declare no conflict of interest.

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Received: April 19,2020. Accepted: September 30, 2020