Interval Valued Neutrosophic Soft Topological Spaces

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Abstract. In this paper we introduce the concept of interval valued neutrosophic soft topological space together with interval valued neutrosophic soft finer and interval valued neutrosophic soft coarser topology. We also define interval valued neutrosophic interior and closure of an interval valued neutrosophic soft set. Some theorems and examples are cited. Interval valued neutrosophic soft subspace topology are studied. Some examples and theorems regarding this concept are presented.

Keywords: Soft set, interval valued neutrosophic set, interval valued neutrosophic soft set, interval valued neutrosophic soft topological space.

1 Introduction


In this paper we form a topological structure on interval valued neutrosophic soft sets and establish some properties of interval valued neutrosophic soft topological space with supporting proofs and examples.

2 Preliminaries

In this section we recall some basic notions relevant to soft sets, interval-valued neutrosophic sets and interval-valued neutrosophic soft sets.

Definition 2.1: [9] Let $U$ be an initial universe and $E$ be a set of parameters. Let $P(U)$ denotes the power set of $U$ and $A \subseteq E$. Then the pair $(f, A)$ is called a soft set over $U$, where $f$ is a mapping given by $f : A \rightarrow P(U)$.

Definition 2.2: [13] A neutrosophic set $A$ on the universe of discourse $U$ is defined as $A = \{ (x, \mu_A(x), \gamma_A(x), \delta_A(x) : x \in U \}$, where $\mu_A, \gamma_A, \delta_A : U \rightarrow [0,1]$ are functions such that the condition: $\forall x \in U, \ 0 \leq \mu_A(x) + \gamma_A(x) + \delta_A(x) \leq 3$ is satisfied.

Here $\mu_A(x), \gamma_A(x), \delta_A(x)$ represent the truth-membership, indeterminacy-membership and falsity-membership respectively of the element $x \in U$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $[0,1]$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $[0,1]$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.

Definition 2.3: [14] An interval valued neutrosophic set $A$ on the universe of discourse $U$ is defined as $A = \{ (x, [\mu_A(x), \gamma_A(x), \delta_A(x) : x \in U \}$, where $\mu_A, \gamma_A, \delta_A : U \rightarrow \text{Int}[0,1]$ are functions such that the...
condition:
\[ \forall x \in U, \quad 0 \leq \sup_{\mu_{f(a)}}(x) + \sup_{\gamma_{f(a)}}(x) + \sup_{\delta_{f(a)}}(x) \leq 3 \] is satisfied.

In real life applications it is difficult to use interval valued neutrosophic set with interval-value from real standard or non-standard subset of \( \text{Int}([0,1]) \).

Hence we consider the interval valued neutrosophic set which takes the interval-value from the subset of \( \text{Int}([0,1]) \) (where \( \text{Int}([0,1]) \) denotes the set of all closed sub intervals of \([0,1]\)). The set of all interval valued neutrosophic sets on \( U \) is denoted by IVNS(U).

**Definition 2.4:** [6] Let \( U \) be an universe set, \( E \) be a set of parameters and \( A \subseteq E \). Let IVNS\((U)\) denotes the set of all interval valued neutrosophic sets of \( U \). Then the pair \( (f, A) \) is called an interval valued neutrosophic soft set (IVNSs in short) over \( U \), where \( f \) is a mapping given by \( f : A \rightarrow \text{IVNS}(U) \). The collection of all interval valued neutrosophic soft sets over \( U \) is denoted by IVNS\((U)\).

**Definition 2.5:** [6] Let \( U \) be a universe set and \( E \) be a set of parameters. Let \((f, A), (g, B) \in \text{IVNS}(U)\), where \( f : A \rightarrow \text{IVNS}(U) \) is defined by
\[ f(a) = \{ (x, \mu_{f(a)}, \gamma_{f(a)}, \delta_{f(a)}): x \in U \} \]
and \( g : B \rightarrow \text{IVNS}(U) \) is defined by
\[ g(b) = \{ (x, \mu_{g(b)}, \gamma_{g(b)}, \delta_{g(b)}): x \in U \} \]
where \( \mu_{f(a)}(x), \gamma_{f(a)}(x), \delta_{f(a)}(x), \mu_{g(b)}(x), \gamma_{g(b)}(x), \delta_{g(b)}(x) \in \text{Int}([0,1]) \)
for \( x \in U \). Then
(i) \( (f, A) \) is called interval valued neutrosophic subset of \( (g, B) \) (denoted by \( (f, A) \subseteq (g, B) \)) if \( A \subseteq B \) and \( \mu_{f(a)}(x) \leq \mu_{g(b)}(x), \gamma_{f(a)}(x) \geq \gamma_{g(b)}(x), \delta_{f(a)}(x) \geq \delta_{g(b)}(x) \) for \( x \in U \).

(ii) Their union, denoted by \( (f, A) \cup (g, B) = (h, C) \) (say), is an interval valued neutrosophic soft set over \( U \), where \( C = A \cup B \) and for \( e \in C \), \( h : C \rightarrow \text{IVNS}(U) \) is defined by
\[ h(e) = \{ (x, \mu_{h(e)}, \gamma_{h(e)}, \delta_{h(e)}): x \in U \} \]
where \( h(e) = \{ (x, \mu_{h(e)}, \gamma_{h(e)}, \delta_{h(e)}): x \in U \} \) for \( x \in U \) and \( e \in C \).

(iii) Their intersection, denoted by \( (f, A) \cap (g, B) = (h, C) \) (say), is an interval valued neutrosophic soft set of over \( U \), where \( C = A \cap B \) and for \( e \in C \), \( h : C \rightarrow \text{IVNS}(U) \) is defined by
\[ h(e) = \{ (x, \mu_{h(e)}, \gamma_{h(e)}, \delta_{h(e)}): x \in U \} \] where \( h(e) = \{ (x, \mu_{h(e)}, \gamma_{h(e)}, \delta_{h(e)}): x \in U \} \) for \( x \in U \) and \( e \in C \).

(iv) The complement of \( (f, A) \), denoted by \( (f, A)^c \) is an interval valued neutrosophic soft set over \( U \) and is defined as \( (f, A)^c = (f^c, \overline{A}) \) where
\[ f^c = \{ x, \mu_{f(a)}, [1 - \sup_{\gamma_{f(a)}}(x), 1 - \inf_{\delta_{f(a)}}(x), \mu_{a}(x)] : x \in U \} \]
for \( a \in A \).

**Definition 2.6:**[5,6] An IVNSs \( (f, A) \) over the universe \( U \) is said to be universe IVNSs with respect to \( A \) if
\[ \mu_{f(a)}(x) = [1,1], \quad \gamma_{f(a)}(x) = [0,0], \quad \delta_{f(a)}(x) = [0,0] \]
\( \forall x \in U, \forall a \in A \). It is denoted by \( I \).
**Definition 2.7:** An IVNSs \((f, A)\) over the universe \(U\) is said to be null IVNSs with respect to \(A\) if \(\mu_{f(a)}(x) = [0,0]\), \(y_{f(a)}(x) = [1,1]\), \(\delta_{f(a)}(x) = [1,1]\) \(\forall x \in U, \forall a \in A\). It is denoted by \(\phi\).

### 3 Interval Valued Neutrosophic Soft Topological Spaces

In this section, we give the definition of interval valued neutrosophic soft topological spaces with some examples and results. We also define discrete and indiscrete interval valued neutrosophic soft topological space along with interval valued neutrosophic soft finer and coarser topology.

Let \(U\) be an universe set, \(E\) be the set of parameters, \(\varphi(U)\) be the set of all subsets of \(U\), \(IVNs(U)\) be the set of all interval valued neutrosophic sets in \(U\) and \(IVSNs(U;E)\) be the family of all interval valued neutrosophic soft sets over \(U\) via parameters in \(E\).

**Definition 3.1:** Let \((\zeta_A, E)\) be an element of \(IVNSs(U;E)\), \(\varphi(\zeta_A, E)\) be the collection of all interval valued neutrosophic soft subsets of \((\zeta_A, E)\). A sub family \(\tau\) of \(\varphi(\zeta_A, E)\) is called an interval valued neutrosophic soft topology (in short IVNS-topology) on \((\zeta_A, E)\) if the following axioms are satisfied:

(i) \(\{\phi_{x,e}(\zeta_A, E), (\zeta_A, E) \in \tau\}\)
(ii) \(\{f_1, E : k \in K\} \subseteq \tau \Rightarrow \bigcup_{k \in K} (f_1, E) \in \tau\)
(iii) If \((g_1, E), (h_1, E) \in \tau\) then \((g_1, E) \cap (h_1, E) \in \tau\)

The triplet \((\zeta_A, E, \tau)\) is called interval valued neutrosophic soft topological space (in short IVNS-topological space) over \((\zeta_A, E)\). The members of \(\tau\) are called \(\tau\) -open IVNS sets (or simply open sets). Here \(\phi_{x,e} : A \rightarrow IVNS(U)\) is defined as \(\phi_{x,e}(\zeta_A, E) = \{x \in [0,1], [1,1] : x \in U\} \forall e \in A\).

**Example 3.2:** Let \(U = \{u_1, u_2, u_3\}, E = \{e_1, e_2, e_3\}\), \(A = \{e_1, e_2, e_3\}\). The tabular representation of \((\zeta_A, E)\) given by

<table>
<thead>
<tr>
<th>(U)</th>
<th>(e_1)</th>
<th>(e_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1)</td>
<td>([.5,.8], [.3,.5], [.2,.7])</td>
<td>([.4,.7], [.2,.3], [.1,.3])</td>
</tr>
<tr>
<td>(u_2)</td>
<td>([.4,.7], [.3,.4], [.1,.2])</td>
<td>([.6,.9], [.1,.2], [.1,.2])</td>
</tr>
<tr>
<td>(u_3)</td>
<td>([.5,.1], [.0,.1], [.3,.6])</td>
<td>([.6,.8], [.2,.4], [.1,.3])</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(U)</th>
<th>(c_1)</th>
<th>(c_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1)</td>
<td>([.3,.9], [.0,.1], [.0,.2])</td>
<td>([.4,.8], [.1,.2], [.0,.5])</td>
</tr>
<tr>
<td>(u_2)</td>
<td>([.3,.9], [.1,.3], [.2,.4])</td>
<td>([.4,.9], [.1,.3], [.2,.4])</td>
</tr>
</tbody>
</table>

**Table 1:** Tabular representation of \((\zeta_A, E)\)

The tabular representation of \((f_{A_1}, E)\) is given by

<table>
<thead>
<tr>
<th>(U)</th>
<th>(c_1)</th>
<th>(c_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1)</td>
<td>([.0,.1], [.1,.1])</td>
<td>([.0,.1], [.1,.1])</td>
</tr>
<tr>
<td>(u_2)</td>
<td>([.0,.1], [.1,.1])</td>
<td>([.0,.1], [.1,.1])</td>
</tr>
<tr>
<td>(u_3)</td>
<td>([.0,.1], [.1,.1])</td>
<td>([.0,.1], [.1,.1])</td>
</tr>
</tbody>
</table>

**Table 2:** Tabular representation of \((f_{A_1}, E)\)

The tabular representation of \((f_{A_2}, E)\) is given by

<table>
<thead>
<tr>
<th>(U)</th>
<th>(c_1)</th>
<th>(c_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1)</td>
<td>([.1,.7], [.4,.8], [.3,.1])</td>
<td>([.1,.3], [.4,.6], [.2,.6])</td>
</tr>
<tr>
<td>(u_2)</td>
<td>([.1,.3], [.6,.7], [.2,.8])</td>
<td>([.0,.5], [.5,.8], [.4,.1])</td>
</tr>
<tr>
<td>(u_3)</td>
<td>([.4,.8], [.6,.7], [.6,.9])</td>
<td>([.0,.3], [.4,.7], [.2,.8])</td>
</tr>
</tbody>
</table>

**Table 3:** Tabular representation of \((f_{A_2}, E)\)

Let \((f_{A_1}, E) = (f_{A_1}, E) \cap (f_{A_2}, E)\) then the tabular representation of \((f_{A_1}, E)\) is given by

<table>
<thead>
<tr>
<th>(U)</th>
<th>(c_1)</th>
<th>(c_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1)</td>
<td>([.3,.9], [.5,.7], [.4,.9])</td>
<td>([.2,.3], [.4,.5], [.7,.9])</td>
</tr>
<tr>
<td>(u_2)</td>
<td>([.3,.5], [.4,.8], [.1,.4])</td>
<td>([.4,.6], [.3,.5], [.2,.5])</td>
</tr>
<tr>
<td>(u_3)</td>
<td>([.3,.9], [.1,.2], [.6,.7])</td>
<td>([.5,.7], [.6,.7], [.3,.4])</td>
</tr>
</tbody>
</table>

**Table 4:** Tabular representation of \((f_{A_1}, E)\)
Definition 3.4: Let $\xi$ denotes the family of all IVNS-subsets of $(\zeta_A, E)$. Then we observe that $\xi$ satisfies all the axioms of topology on $(\zeta_A, E)$. This topology is called discrete interval valued neutrosophic soft topology and the triplet $(\zeta_A, E, \xi)$ is called discrete interval valued neutrosophic soft topological space (or simply discrete IVNS-topological space).

Theorem 3.5: Let $\{\tau_i : i \in I\}$ be any collection of IVNS-topology on $(\zeta_A, E)$. Then their intersection $\bigcap_{i \in I} \tau_i$ is also a IVNS-topology on $(\zeta_A, E)$.

Proof: (i) Since $(\phi_{\xi}, E), (\zeta_A, E) \in \tau_i$ for each $i \in I$. Hence $(\phi_{\xi}, E), (\zeta_A, E) \in \bigcap_{i \in I} \tau_i$.

(ii) Let $(f_i^A, E) : k \in K$ be an arbitrary family of interval valued neutrosophic soft sets where $(f_i^A, E) \in \bigcap_{i \in I} \tau_i$ for each $k \in K$. Then for each $i \in I$, $(f_i^A, E) \in \tau_i$ for $k \in K$ and since for each $i \in I$, $\tau_i$ is an IVNS-topology, therefore $\bigcup_{k \in K} (f_i^A, E) \in \tau_i$ for each $i \in I$.

Hence $\bigcup_{k \in K} (f_i^A, E) \in \bigcap_{i \in I} \tau_i$.

(iii) Let $(f_i^A, E), (f_i^A, E) \in \bigcap_{i \in I} \tau_i$, then $(f_i^A, E), (f_i^A, E) \in \tau_i$ for each $i \in I$. Since for each $i \in I$, $\tau_i$ is an IVNS-topology, therefore $(f_i^A, E) \cap (f_i^A, E) \in \tau_i$ for each $i \in I$. Hence $(f_i^A, E) \cap (f_i^A, E) \in \bigcap_{i \in I} \tau_i$.

Thus $\bigcap_{i \in I} \tau_i$ satisfies all the axioms of topology. Hence $\bigcap_{i \in I} \tau_i$ forms a IVNS-topology. But union of IVNS-topologies need not be a IVNS-topology. Let us show this with the following example.

Example 3.6: In example 3.2, the sub families $\tau_1 = \{(\phi_{\xi}, E), (\zeta_A, E), (f_i^A, E)\}$ and $\tau_2 = \{(\phi_{\xi}, E), (\zeta_A, E), (f_i^A, E)\}$ are IVNS-topologies in $(\zeta_A, E)$. But their union $\tau_1 \cup \tau_2 = \{(\phi_{\xi}, E), (\zeta_A, E), (f_i^A, E), (f_i^A, E)\}$ is not a IVNS-topology in $(\zeta_A, E)$.

Definition 3.7: Let $(\zeta_A, E, \tau)$ be an IVNS-topological space over $(\zeta_A, E)$. An interval valued neutrosophic soft
subset \((f_A, E)\) of \((\xi_A, E)\) is called interval valued neutrosophic soft closed set (in short IVNS-closed set) if its complement \((f_A, E)^c\) is a member of \(\tau\).

**Example 3.8:** Let us consider example 3.2, then the IVNS-closed sets in \((\xi_A, E, \tau)\) are

\[
\begin{array}{c|ccc}
U & e_1 & e_2 & \vdots \\
\hline
u_1 & ([1.2], [5.7], [5.8]) & ([4.3], [7.8], [4.7]) & \vdots \\
u_2 & ([1.2], [6.7], [4.7]) & ([4.1], [8.9], [6.9]) & \vdots \\
u_3 & ([3.6], [9.1], [5.1]) & ([1.3], [6.8], [6.8]) & \vdots \\
\end{array}
\]

\[
\begin{array}{c|ccc}
U & e_1 & e_2 & \vdots \\
\hline
c_3 & ([0.2], [9.1], [3.9]) & ([0.5], [8.9], [4.8]) & ([2.4], [7.9], [4.9]) \\
\end{array}
\]

are the IVNS-closed sets in \((\xi_A, E, \tau)\).

**Theorem 3.9:** Let \((\xi_A, E, \tau)\) be an IVNS-topological space over \((\xi_A, E)\). Then

1. \((\phi_A, E)^c\), \((\xi_A, E)^c\) are IVNS-closed sets.
2. Arbitrary intersection of IVNS-closed sets is IVNS-closed set.
3. Finite union of IVNS-closed sets is IVNS-closed set.

**Proof:**

1. Since \((\phi_A, E), (\xi_A, E) \in \tau\), therefore \((\phi_A, E)^c\), \((\xi_A, E)^c\) are IVNS-closed sets.

2. Let \(\left\{f_A, E\right\} : k \in K\) be an arbitrary family of IVNS-closed sets in \((\xi_A, E, \tau)\) and let \((f_A, E) = \bigcap_{i=k} f_A, E)\).
Now \((f, E) = \left( \bigcap_{i=1}^{n} (f_i, E) \right) = \bigcup_{i=1}^{n} (f_i, E)\) and \((f, E) \in \tau\) for each \(k \in K\), so \( \bigcup_{i=1}^{n} (f_i, E) \in \tau\). Hence \((f, E) \in \tau\).

Thus \((f, E)\) is IVNS-closed set.

3. Let \(\{(f_i, E): i = 1, 2, 3, \ldots, n\}\) be a family of IVNS-closed sets in \((\zeta, E, \tau)\) and let \((g, E) = \bigcap_{i=1}^{n} (f_i, E)\).

Now \((g, E) = \left( \bigcup_{i=1}^{n} (f_i, E) \right) = \bigcap_{i=1}^{n} (f_i, E)\) and \((f_i, E) \in \tau\) for \(i = 1, 2, 3, \ldots, n\), so \(\bigcap_{i=1}^{n} (f_i, E) \in \tau\). Hence \((g, E) \in \tau\). Thus \((g, E)\) is IVNS-closed set.

**Definition 3.10:** Let \((\zeta, E, \tau_1)\) and \((\zeta, E, \tau_2)\) be two IVNS-topological spaces over \((\zeta, E)\). If each \((f, E) \in \tau_2\) implies \((f, E) \in \tau_1\), then \(\tau_1\) is called interval valued neutrosophic soft finer topology than \(\tau_2\) and \(\tau_2\) is called interval valued neutrosophic soft coarser topology than \(\tau_1\).

**Example 3.11:** In example 3.2 and 3.6, \(\tau_1\) is interval valued neutrosophic soft finer topology than \(\tau_3\) and \(\tau_3\) is called interval valued neutrosophic soft coarser topology than \(\tau_1\).

**Definition 3.12:** Let \((\zeta, E, \tau)\) be a IVNS-topological space over \((\zeta, E)\) and \(\beta\) be a subfamily of \(\tau\). If every element of \(\tau\) can be express as the arbitrary interval valued neutrosophic soft union of some elements of \(\beta\), then \(\beta\) is called an interval valued neutrosophic soft basis for the IVNS-topology \(\tau\).

**Example 3.13:** In example 3.2, for the IVNS-topology \(\tau = \{ (\phi_0, E), (\zeta, E), (f_1, E), (f_2, E), (f_3, E) \}\), the subfamily \(\beta = \{ (\phi_0, E), (\zeta, E), (f_1, E), (f_2, E), (f_3, E) \}\) of \(\beta(\zeta, E)\) is a interval valued neutrosophic soft basis for the IVNS-topology \(\tau_1\).

4 Some Properties of Interval Valued Neutrosophic Soft Topological Spaces

In this section some properties of interval valued neutrosophic soft topological spaces are introduced. Some results on IVNSSInt and IVNSSCl are also introduced.

**Definition 4.1:** Let \((\zeta, E, \tau)\) be a IVNS-topological space and let \((f, E) \in IVNSS(U; E)\). The interval valued neutrosophic soft interior and closer of \((f, E)\) is denoted by \(IVNSSInt(f, E)\) and \(IVNSSCl(f, E)\) are defined as

\[ IVNSSInt(f, E) = \bigcup \{(g, E) \in \tau : (g, E) \subseteq (f, E)\} \]

and \(IVNSSCl(f, E) = \bigcap \{(g, E) \in \tau : (f, E) \subseteq (g, E)\}\) respectively.

**Example 4.2:** Let us consider example 3.2 and take an IVNSS \((f, E)\) as

<table>
<thead>
<tr>
<th>U</th>
<th>(e_1)</th>
<th>(e_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1 ([1.2,8],[3.6],[2,8])</td>
<td>([2,4],[4,6],[2,4])</td>
<td></td>
</tr>
<tr>
<td>u_2 ([1,6],[4,5],[2,7])</td>
<td>([2,6],[5,7],[1,7])</td>
<td></td>
</tr>
<tr>
<td>u_3 ([5,8],[5,6],[5,8])</td>
<td>([1,4],[4,6],[1,5])</td>
<td></td>
</tr>
</tbody>
</table>

Table 13: Tabular representation of \((f, E)\)

Now \(IVNSSInt(f, E) = \{f, E\}\) and \(IVNSSCl(f, E) = \{f, E\}\).

**Theorem 4.3:** Let \((\zeta, E, \tau)\) be a IVNS-topological space and \((f, E) \in IVNSS(U; E)\), \(\{g, E\} \in IVNSS(U; E)\) then the following properties hold

1. \(IVNSSInt(f, E) \subseteq (f, E)\)
2. \(\{g, E\} \subseteq (g, E) \Rightarrow IVNSSInt(f, E) \subseteq IVNSSInt(g, E)\)
3. \(IVNSSInt(f, E) \in \tau\)
4. \((f, E) \in \tau \Rightarrow IVNSSInt(f, E) = (f, E)\)
5. \(IVNSSInt(IVNSSInt(f, E)) = IVNSSInt(f, E)\)
6. \(IVNSSInt(\phi_0, E) = \phi_0, IVNSSInt(U, E) = U\)

**Proof:**

1. Straight forward.
2. \((f, E) \subseteq (g, E)\) implies all the IVNS-open sets contained in \((f, E)\) also contained in \((g, E)\). i.e.

\[ \{(f, E) \in \tau : (f, E) \subseteq (g, E)\} \subseteq \{(g, E) \in \tau : (g, E) \subseteq (g, E)\} \]
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6. Similar to 5.
Equality does not hold in theorem 4.4 (2), (4). Let us show this by an example.

Example 4.6: Let \( U = \{u_1, u_2\} \), \( E = \{e_1, e_2, e_3\} \), \( A = \{e_4, e_5\} \). The tabular representation of \((\xi_A, E)\) is given by

\[
\begin{array}{ccc}
U & e_1 & e_2 \\
\downarrow & \downarrow & \downarrow \\
(0,0.0) & (0.5,1.1) & (1,1)
\end{array}
\]

Now \((g_A, E)\) be an \(IVNS\)-topological space on \((\xi_A, E)\). Let us now take two interval valued neutrosophic soft sets \((g_A, E)\) and \((h_A, E)\) as

\[
\begin{array}{ccc}
U & e_1 & e_2 \\
\downarrow & \downarrow & \downarrow \\
(0.0,0.1) & (1,1) & (1,1)
\end{array}
\]

By theorem 4.4 (5),
\[
IVNSCl((g_A, E)) = IVNSCl((h_A, E)) = \{\phi_A, E\} = (\xi_A, E).
\]

Similarly \(IVNScl(h_A, E)\) is \((\xi_A, E)\).

Therefore
\[
IVNSCl((g_A, E)) \cap IVNScl(h_A, E) = (\xi_A, E) \cap (\xi_A, E) = (\xi_A, E)
\]

Also
\[
IVNSCl((g_A, E)) \cap IVNScl(h_A, E) = IVNSCl((g_A, E)) \cap IVNScl(h_A, E)
\]

Thus
\[
IVNSCl((f_A, E)) \cap IVNSCl(g_A, E) \neq IVNSCl((f_A, E)) \cup IVNSCl(g_A, E).
\]

Therefore equality does not hold in (4).

5 Interval Valued Neutrosophic Soft Subspace Topology

In this section we introduce the concept of interval valued neutrosophic soft subspace topology along with some examples and results.

Theorem 5.1: Let \((\xi_A, E, \tau)\) be an IVNS-topological space on \((\xi_A, E)\) and \((f_A, E) \in \varphi(\xi_A, E)\). Then the collection \(\tau_{(f_A, E)} = \{(f_A, E) \cap (g_A, E) : (g_A, E) \in \tau\}\) is an IVNS-topology on \((\xi_A, E)\).

Proof:
(i) Since \((\phi_A, E), (\xi_A, E) \in \tau\), therefore
\[
(f_A, E) \cap (\phi_A, E) = (f_A, E) \cap (\xi_A, E) \in \tau_{(f_A, E)}
\]

(ii) Let \((f_A, E) \in \tau_{(f_A, E)} \forall k \in K\). Then
\[
(f_A, E) \cap (g_A, E) \in \tau_{(f_A, E)}
\]

(iii) Let \((f_A, E), (f_A, E) \in \tau_{(f_A, E)} \)

Now \((g_A, E) \cup (h_A, E) = (f_A, E) \cup (f_A, E)

\[
IVNSInt((g_A, E) \cup (h_A, E)) = IVNSInt((f_A, E) \cup (f_A, E)) = (f_A, E)
\]

Also \(IVNSInt(g_A, E) = (\phi_A, E), IVNSInt(h_A, E) = (\phi_A, E)\)

\[
IVNSInt((f_A, E) \cup (g_A, E)) \neq IVNSInt((f_A, E) \cup IVNSInt(g_A, E).
\]

Therefore equality does not hold for (2).
Now \((f_\ast,E) \cap (f_\ast,E) = \left((f_\ast,E) \cap (g_\ast,E)\right) \cap \left((f_\ast,E) \cap (g_\ast,E)\right)\)

\[= (f_\ast,E) \cap \left(g_\ast,E\right) \cap \left(g_\ast,E\right) \in \tau_{(f_\ast,E)}\]

(since \((g_\ast,E) \cap (g_\ast,E) \in \tau\) as \((g_\ast,E), (g_\ast,E) \in \tau\).

**Definition 5.2:** Let \((\zeta_\ast,E,\tau)\) be an IVNS-topological space on \((\zeta_\ast,E)\) and \((f_\ast,E) \in \phi(\zeta_\ast,E)\). Then the IVNS-topology \(\tau_{(f_\ast,E)} = \{ (f_\ast,E) \cap (g_\ast,E) : (g_\ast,E) \in \tau \}\)

is called interval valued neutrosophic soft subspace topology and \((f_\ast,E, \tau_{(f_\ast,E)})\) is called interval valued neutrosophic soft subspace of \((\zeta_\ast,E,\tau)\).

**Example 5.3:** Let us consider the IVNS-topology \(\tau_{(f_\ast,E)} = \{ (f_\ast,E) \cap (g_\ast,E) : (g_\ast,E) \in \tau \}\)

as in example 3.2 and an IVNSS \((f_\ast,E)\):

<table>
<thead>
<tr>
<th>U</th>
<th>e_1</th>
<th>e_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>(0.4,0.6,0.7,0.3)</td>
<td>(0.5,0.7,0.4,0.6,0.3)</td>
</tr>
<tr>
<td>u_2</td>
<td>(0.3,0.5,0.7,0.2)</td>
<td>(0.6,0.8,0.4,0.5,0.2)</td>
</tr>
<tr>
<td>u_3</td>
<td>(0.5,0.7,0.3,0.4,0.3)</td>
<td>(0.4,0.5,0.7,0.4,0.3)</td>
</tr>
</tbody>
</table>

Table 19: Tabular representation of \((f_\ast,E)\)

Then \((\phi_\ast,E) = (f_\ast,E) \cap (\phi_\ast,E)\):

<table>
<thead>
<tr>
<th>U</th>
<th>e_1</th>
<th>e_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>(0.0,0.1,0.1,0.1)</td>
<td>(0.0,0.1,0.1,0.1)</td>
</tr>
<tr>
<td>u_2</td>
<td>(0.0,0.1,0.1,0.1)</td>
<td>(0.0,0.1,0.1,0.1)</td>
</tr>
<tr>
<td>u_3</td>
<td>(0.0,0.1,0.1,0.1)</td>
<td>(0.0,0.1,0.1,0.1)</td>
</tr>
</tbody>
</table>

Table 20: Tabular representation of \((\phi_\ast,E)\)

\((g_\ast,E) = (f_\ast,E) \cap (f_\ast,E)\):

<table>
<thead>
<tr>
<th>U</th>
<th>e_1</th>
<th>e_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>(0.4,0.5,0.6,0.2)</td>
<td>(0.4,0.5,0.6,0.2)</td>
</tr>
<tr>
<td>u_2</td>
<td>(0.2,0.3,0.5,0.2)</td>
<td>(0.5,0.7,0.4,0.5,0.2)</td>
</tr>
<tr>
<td>u_3</td>
<td>(0.3,0.5,0.7,0.3,0.4)</td>
<td>(0.4,0.5,0.7,0.4,0.3)</td>
</tr>
</tbody>
</table>

Table 21: Tabular representation of \((g_\ast,E)\)

Then \((g_\ast,E) = (f_\ast,E) \cap (f_\ast,E)\):

<table>
<thead>
<tr>
<th>U</th>
<th>e_1</th>
<th>e_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>(0.4,0.5,0.6,0.2)</td>
<td>(0.4,0.5,0.6,0.2)</td>
</tr>
<tr>
<td>u_2</td>
<td>(0.2,0.3,0.5,0.2)</td>
<td>(0.5,0.7,0.4,0.5,0.2)</td>
</tr>
<tr>
<td>u_3</td>
<td>(0.3,0.5,0.7,0.3,0.4)</td>
<td>(0.4,0.5,0.7,0.4,0.3)</td>
</tr>
</tbody>
</table>

Table 22: Tabular representation of \((g_\ast,E)\)

Then \(\tau_{(g_\ast,E)} = \{ (\phi_\ast,E) \cap (g_\ast,E) : (g_\ast,E) \in \tau \}\)

is an interval valued neutrosophic soft subspace.
topology for $\tau_i$ and $(f_A, E, \tau_i)$ is called interval valued neutrosophic soft subspace of $(\zeta_A, E, \tau_i)$.

**Theorem 5.4:** Let $(\zeta_A, E, \tau)$ be an IVNS-topological space on $(\zeta_A, E)$, $B$ be an IVNS-basis for $\tau$ and $(f_A, E) \in \wp(\zeta_A, E)$. Then the family $B_{(f_A, E)} = \{ (f_A, E) \cap (g_A, E) : (g_A, E) \in B \}$ is an IVNS-basis for subspace topology $\tau(f_A, E)$.

**Proof:** Let $(h_A, E) \in \tau(f_A, E)$ be arbitrary, then there exists an IVNSS $(g_A, E) \in \tau$ such that $(h_A, E) = (f_A, E) \cap (g_A, E)$. Since $B$ is a basis for $\tau$, therefore there exists a sub collection $\{ (\zeta_A, E) : i \in I \}$ of $B$ such that $(g_A, E) = \cup_{i \in I} (\zeta_A, E)$.

Now $(h_A, E) = (f_A, E) \cap (g_A, E) = \cup_{i \in I} (\zeta_A, E) = \cup_{i \in I} (f_A, E) \cap (\zeta_A, E)$.

Since $(f_A, E) \cap (\zeta_A, E) \in B_{(f_A, E)}$, therefore $B_{(f_A, E)}$ is an IVNS-basis for the subspace topology $\tau(f_A, E)$.

**Conclusion**

In this paper we introduce the concept of interval valued neutrosophic soft topology. Some basic theorem and properties of the above concept are also studied. IVN interior and IVN closer of an interval valued neutrosophic soft set are also defined. Interval valued neutrosophic soft subspace topology is also studied.

In future there will be more research work in this concept, taking the basic definitions and results from this article.

**References**


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