Introduction to Image Processing via Neutrosophic Techniques

A. A. Salama¹, Florentin Smarandache² and Mohamed Eisa³

¹ Department of Mathematics and Computer Science, Faculty of Sciences, Port Said University, 23 December Street, Port Said 42522, Egypt. Email: drsalama44@gmail.com
² Department of Mathematics, University of New Mexico 705 Gurley Ave. Gallup, NM 87301, USA. Email: smarand@unm.edu
³ Computer Science Department, Port Said University, 42526 Port Said, Egypt. Email: mmmeisa@yahoo.com

Abstract. This paper is an attempt of proposing the processing approach of neutrosophic technique in image processing. As neutrosophic sets is a suitable tool to cope with imperfectly defined images, the properties, basic operations distance measure, entropy measures, of the neutrosophic sets method are presented here. In this paper we, introduce the distances between neutrosophic sets: the Hamming distance, the normalized Hamming distance, the Euclidean distance and normalized Euclidean distance. We will extend the concepts of distances to the case of neutrosophic hesitancy degree. Entropy plays an important role in image processing. In our further considerations on entropy for neutrosophic sets the concept of cardinality of a neutrosophic set will also be useful. Possible applications to image processing are touched upon.

Keywords: Neutrosophic sets; Hamming distance; Euclidean distance; Normalized Euclidean distance; Image processing.

1. Introduction

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. Smarandache [9, 10] and Salama et al [4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 45]. Entropy plays an important role in image processing. In this paper we, introduce the distances between neutrosophic sets: the Hamming distance. In this paper we, introduce the distances between neutrosophic sets: the Hamming distance, The normalized Hamming distance, the Euclidean distance and normalized Euclidean distance. We will extend the concepts of distances to the case of neutrosophic hesitancy degree. In our further considerations on entropy for neutrosophic sets the concept of cardinality of a neutrosophic set will also be useful.

2. Terminologies

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 11, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46] such as a neutrosophic set theory. We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [9, 10] and Salama et al. [4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 45]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where [0.0 , 1.0] is nonstandard unit interval. Salama et al. introduced the following:

Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $A = \{\mu_A(x), \sigma_A(x), \nu_A(x)\}$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-membership (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set $A$. Where
0 \leq \mu_A(x), \sigma_A(x), \nu_A(x) \leq 1^* \ and
0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^* . Smarandache
introduced the following: Let T, I, F be real standard or
nonstandard subsets of \left\{0, 1\right\}, with
Sup_T=t_{sup}, inf_T=t_{inf}
Sup_I=i_{sup}, inf_I=i_{inf}
Sup_F=f_{sup}, inf_F=f_{inf}

3. Distances Between Neutrosophic Sets

We will now extend the concepts of distances presented

Definition 3.1

Let \ A = \left\{ (\mu_A(x), \nu_A(x), \gamma_A(x), x \in X \right\} \ and
B = \left\{ (\mu_B(x), \nu_B(x), \gamma_B(x), x \in X \right\} \ in
X = \{x_1, x_2, x_3, \ldots, x_n\} \ then

i) The Hamming distance is equal to
d_{H}(A,B) = \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\gamma_A(x_i) - \gamma_B(x_i)|

ii) The Euclidean distance is equal to
e_{E}(A,B) = \sqrt{\sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)|^2 + |\nu_A(x_i) - \nu_B(x_i)|^2 + |\gamma_A(x_i) - \gamma_B(x_i)|^2)}

iii) The normalized Hamming distance is equal to
NH_{E}(A,B) = \frac{1}{n} \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\gamma_A(x_i) - \gamma_B(x_i)|

iv) The normalized Euclidean distance is equal to
NE_{E}(A,B) = \frac{1}{n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)|^2 + |\nu_A(x_i) - \nu_B(x_i)|^2 + |\gamma_A(x_i) - \gamma_B(x_i)|^2)

Example 3.1

Let us consider for simplicity degenerated
neutrosophic sets \ A, B, D, G, F \ in \ X = \{a\} . A full
description of each neutrosophic set i.e.
A = \left\{ (\mu_A(x), \nu_A(x), \gamma_A(x), a \in X \right\} , \ may \ be \ exemplified
by \ A = \left\{ (1,0,0), a \in X \right\} , \ B = \left\{ (0,1,0), a \in X \right\} ,
D = \left\{ (0,0,1), a \in X \right\} , \ G = \left\{ (0,5,0,5), a \in X \right\} ,
E = \left\{ (0.25,0.25,0.25,0.25), a \in X \right\} .

Let us calculate four distances between the above
neutrosophic sets using i), ii), iii) and iv) formulas ,

\begin{center}
\begin{tikzpicture}
\filldraw[fill=black!20] (0,0) circle (2pt) node [below] {A(1,0,0)};
\filldraw[fill=black!20] (0,1) circle (2pt) node [right] {B(0,1,0)};
\filldraw[fill=black!20] (0,0) circle (2pt) node [below] {G};
\filldraw[fill=black!20] (0,0) circle (2pt) node [right] {E};
\filldraw[fill=white] (0,0) circle (2pt) node [above] {D(0,0,1)};
\end{tikzpicture}
\end{center}

(Fig.1) A geometrical interpretation of the neutrosophic
considered in Example 5.1. 
We obtain \[ e_{N_{E}}(A,D) = \frac{1}{2} , \quad e_{N_{E}}(B,D) = \frac{1}{2} , \]
\[ e_{N_{E}}(A,B) = \frac{1}{2} , \quad e_{N_{E}}(A,G) = \frac{1}{4} , \quad e_{N_{E}}(B,G) = \frac{1}{2} . \]
\[ e_{N_{E}}(E,G) = \frac{1}{2} , \quad e_{N_{E}}(D,G) = \frac{1}{4} , \quad NE_{N_{E}}(A,B) = 1, \]
\[ NE_{N_{E}}(A,D) = 1, \quad NE_{N_{E}}(B,D) = 1, \quad NE_{N_{E}}(A,G) = \frac{1}{2} , \]
\[ NE_{N_{E}}(B,G) = \frac{1}{2} , \quad NE_{N_{E}}(E,G) = \sqrt{\frac{3}{4}} , \quad a \]

From the above results the triangle ABD (Fig.1) has edges equal to \[ \sqrt{2} \] and
\[ e_{N_{E}}(A,D) = e_{N_{E}}(B,D) = e_{N_{E}}(A,B) = \frac{1}{2} \] and
\[ NE_{N_{E}}(A,B) = NE_{N_{E}}(A,D) = NE_{N_{E}}(B,D) = 2NE_{N_{E}}(A,G) = 2NE_{N_{E}}(B,G) = 1, \] and \[ NE_{N_{E}}(E,G) \] is
equal to half of the height of triangle with all edges equal
to \[ \sqrt{2} \] multiplied by, \[ \frac{1}{\sqrt{2}} \] i.e. \[ \sqrt{\frac{3}{4}} . \]

Example 3.2

Let us consider the following neutrosophic sets A and B in \ X = \{a, b, c, d, e\} ,
A = \left\{ (0.5,0.0,0.5), (0.20,0.6,0.2), (0.0,0.2,0.5), (0.5,0.2,0.6), (1,0) \right\} ,
B = \left\{ (0.0,0.2,0.0,0.5), (0.3,0.2,0.5), (0.0,0.0,0.5), (0.9,0.0,0.1), (0,0,0) \right\} .

Then
d_{N_{E}}(A,B) = 3 , \quad NH_{N_{E}}(A,B) = 0.43 , \quad e_{N_{E}}(A,B) = 1.49 \quad \text{and} \quad NE_{N_{E}}(A,B) = 0.55 .

Remark 3.1

Clearly these distances satisfy the conditions of metric space.

Remark 3.2

It is easy to notice that for formulas i), ii), iii) and
iv) the following is valid:
a) \[ 0 \leq d_{N_{E}}(A,B) \leq n \]
b) \[ 0 \leq NH_{N_{E}}(A,B) \leq 1 \]
c) \[ 0 \leq e_{N_{E}}(A,B) \leq \sqrt{n} \]
d) \[ 0 \leq NE_{N_{E}}(A,B) \leq 1 . \]

This representation of a neutrosophic set (Fig. 2) will be
a point of departure for neutrosophic crisp distances, and
entropy of neutrosophic sets.
4. Hesitancy Degree and Cardinality for Neutrosophic Sets

We will now extend the concepts of distances to the case of neutrosophic hesitancy degree. By taking into account the four parameters characterization of neutrosophic sets $i.e.$ $A = \{ \mu_A(x), \nu_A(x), \gamma_A(x), \pi_A(x) \mid x \in X \}$

**Definition 4.1**

Let $A = \{ \mu_A(x), \nu_A(x), \gamma_A(x), \pi_A(x) \mid x \in X \}$ and $B = \{ \mu_B(x), \nu_B(x), \gamma_B(x), x \in X \}$ on $X = \{x_1, x_2, x_3, \ldots, x_n\}$

For a neutrosophic set $A = \{ \mu_A(x), \nu_A(x), \gamma_A(x), \pi_A(x) \mid x \in X \}$ in $X$, we call $\pi_A(x) = 3 - \mu_A(x) - \nu_A(x) - \gamma_A(x)$ the neutrosophic index of $x$ in $A$. It is a hesitancy degree of $x$ to $A$ it is obvious that $0 \leq \pi_A(x) \leq 3$.

**Definition 4.2**

Let $A = \{ \mu_A(x), \nu_A(x), \gamma_A(x), \pi_A(x) \mid x \in X \}$ and $B = \{ \mu_B(x), \nu_B(x), \gamma_B(x), x \in X \}$ in $X = \{x_1, x_2, x_3, \ldots, x_n\}$ then

i) The Hamming distance is equal to

$$d_{\text{ham}}(A, B) = \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\gamma_A(x_i) - \gamma_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|$$

. Taking into account that

$$\pi_A(x_i) = 3 - \mu_A(x_i) - \nu_A(x_i) - \gamma_A(x_i)$$

and

$$\pi_B(x_i) = 3 - \mu_B(x_i) - \nu_B(x_i) - \gamma_B(x_i)$$

we have

$$|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\gamma_A(x_i) - \gamma_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|$$

.ii) The Euclidean distance is equal to

$$d_{\text{eucl}}(A, B) = \sum_{i=1}^{n} [\mu_A(x_i) - \mu_B(x_i)]^2 + [\nu_A(x_i) - \nu_B(x_i)]^2 + [\gamma_A(x_i) - \gamma_B(x_i)]^2 + [\pi_A(x_i) - \pi_B(x_i)]^2$$

we have

$$|\pi_A(x_i) - \pi_B(x_i)|^2 = (-\mu_A(x_i) - \nu_A(x_i) - \gamma_A(x_i) + \mu_B(x_i) + \nu_B(x_i) + \gamma_B(x_i))^2$$

$$= (\mu_B(x_i) - \mu_A(x_i) + \nu_B(x_i) - \nu_A(x_i) + \gamma_B(x_i) - \gamma_A(x_i))^2$$

$$+ 2(\mu_B(x_i) - \mu_A(x_i))(\nu_B(x_i) - \nu_A(x_i))$$

$$+ (\gamma_B(x_i) - \gamma_A(x_i))^2$$

iii) The normalized Hamming distance is equal to

$$NH_{im}(A, B) = \frac{1}{2n} \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\gamma_A(x_i) - \gamma_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|$$

iv) The normalized Euclidean distance is equal to

$$NE_{im}(A, B) = \frac{1}{2n} \sum_{i=1}^{n} [\mu_A(x_i) - \mu_B(x_i)]^2 + [\nu_A(x_i) - \nu_B(x_i)]^2 + [\gamma_A(x_i) - \gamma_B(x_i)]^2 + [\pi_A(x_i) - \pi_B(x_i)]^2$$

5.2 Remark

It is easy to notice that for formulas i), ii), iii) and the following is valid:

a) $0 \leq d_{\text{im}}(A, B) \leq 2n$

b) $0 \leq NH_{im}(A, B) \leq 2$

c) $0 \leq e_{\text{im}}(A, B) \leq \sqrt{2n}$

d) $0 \leq NE_{im}(A, B) \leq \sqrt{2}$.

5. From Images to Neutrosophic Sets, and Entropy

Given the definitions of the previous section several possible contributions are discussed. Neutrosophic sets may be used to solve some of the problems of data causes problems in the classification of pixels. Hesitancy in images originates from various factors, which in their majority are due to the inherent weaknesses of the acquisition and the imaging mechanisms. Limitations of the acquisition chain, such as the quantization noise, the suppression of the dynamic range, or the nonlinear behavior of the mapping system, aect our certainty on deciding whether a pixel is “gray” or “edgy” and therefore introduce a degree of hesitancy associated with the corresponding pixel. Therefore, hesitancy should encapsulate the aforementioned sources of indeterminacy that characterize digital images. Defining the membership component of the A-NS that describes the brightness of pixels in an image, is a more straightforward task that can be carried out in a similar manner as in traditional fuzzy image processing systems. In the presented heuristic framework, we consider the membership value of a gray level $g$ to be its normalized
intensity level; that is \( \mu_A(g) = \frac{g}{L-1} \) where \( g \in \{0, \ldots, L-1\} \). It should be mentioned that any other method for calculating \( \mu_A(g) \) can also be applied.

In the image is \( A \) being (\( x, y \)) the coordinates of each pixel and the \( g(x, y) \) be the gray level of the pixel \((x, y)\) implies \( 0 \leq g(x, y) \leq L-1 \). Each image pixel is associated with four numerical values:

- A value representing the membership \( \mu_A(x) \), obtained by means of membership function associated with the set that represents the expert’s knowledge of the image.
- A value representing the indeterminacy \( \nu_A(x) \), obtained by means of the indeterminacy function associated with the set that represents the ignorance of the expert’s decision.
- A value representing the non-membership \( \gamma_A(x) \), obtained by means of the non-membership function associated with the set that represents the ignorance of the expert’s decision.
- A value representing the hesitation measure \( \pi_A(x) \), obtained by means of the indeterminacy function associated with the set that represents the indeterminacy of the expert’s decision.

Let an image \( A \) of size \( M \times N \) pixels having \( L \) gray levels ranging between 0 and \( L-1 \). The image in the neutrosophic domain is considered as an array of neutrosophic singletons. Here, each element denoted the degree of the membership, indeterminacy and non-membership according to a pixel with respect to an image considered. An image \( A \) in neutrosophic set is \( A = \{ \mu_A(g), \nu_A(g), \gamma_A(g) \} \), \( g \in \{0, \ldots, L-1\} \)

where \( \mu_A(g), \nu_A(g), \gamma_A(g) \) denote the degrees of membership indeterminacy and non-membership of the \((i, j)-th\) pixel to the set \( A \) associated with an image property

\[
\mu_A(g) = \frac{g - g_{\min}}{g - g_{\max}} \quad \text{where} \quad g_{\min} \quad \text{and} \quad g_{\max} \quad \text{are the minimum and the maximum gray levels of the image.}
\]

Entropy plays an important role in image processing. In our further considerations on entropy for neutrosophic sets the concept of cardinality of a neutrosophic set will also be useful.

Definition 5.1

Let \( A = \{ (\mu_A(x), \nu_A(x), \gamma_A(x)), x \in X \} \) a neutrosophic set in \( X \), first, we define two cardinalities of a neutrosophic set

- The least (sure) cardinality of \( A \) is equal to so is called sigma-count, and is called here the

\[
\min \sum \text{cont}(A) = \sum_{i=1}^{n} \mu_A(x_i) + \sum_{i=1}^{n} \nu_A(x_i)
\]

- The biggest cardinality of \( A \), which is possible due to \( \pi_A(x) \) is equal to

\[
\max \sum \text{cont}(A) = \sum_{i=1}^{n} (\mu_A(x_i) + \pi_A(x_i)) + \sum_{i=1}^{n} \nu_A(x_i) + \pi_A(x_i)
\]

and, clearly for \( A^c \) we have

\[
\min \sum \text{cont}(A^c) = \sum_{i=1}^{n} \nu_A(x_i) + \sum_{i=1}^{n} \pi_A(x_i)
\]

\[
\max \sum \text{cont}(A^c) = \sum_{i=1}^{n} \gamma_A(x_i) + \sum_{i=1}^{n} \nu_A(x_i) + \pi_A(x_i)
\]

. Then the cardinality of neutrosophic set is defined as the interval

\[
\text{Card}(A) = \left[ \min \sum \text{Cont}(A), \max \sum \text{Cont}(A) \right]
\]

Definition 5.2

An entropy on \( NS(X) \) is a real-valued functional \( E : NS(X) \rightarrow [0,1] \), satisfying the following axiomatic requirements:

- \( E(A) = 0 \) iff \( A \) is a neutrosophic crisp set; that is \( \mu_A(x_i) = 0 \) or \( \mu_A(x_i) = 1 \) for all \( x_i \in X \).
- \( E_2: E(A) = 1 \) iff \( \mu_A(x_i) = \nu_A(x_i) = \gamma_A(x_i) \) for all \( x_i \in X \) that is \( A = A^c \).
- \( E_3: E(A) \leq E(B) \) if \( A \) refine \( B \); i.e. \( A \leq B \).

- \( E_4: E(A) = E(A^c) \)

Where a neutrosophic entropy measure be define as

\[
E(A) = \frac{1}{n} \sum_{i=1}^{n} \max \text{Count}(\bar{A}_i \cup A_i^c)
\]

\( n = \text{Cardinal}(X) \) and \( A_i \) denotes the single-element \( A \)-NS corresponding to the \( i \)th element of the universe \( X \) and is described as

\[
A_i = \{ (\mu_A(x_i), \nu_A(x_i), \gamma_A(x_i)), x_i \in X \}
\]

In other words, \( A_i \) is the \( i \)th “component” of \( A \). Moreover, \( \max \text{Count}(A) \) denotes the biggest cardinality of \( A \) and is given by:

\[
\max \sum \text{cont}(A) = \sum_{i=1}^{n} (\mu_A(x_i) + \pi_A(x_i)) + \sum_{i=1}^{n} \nu_A(x_i) + \pi_A(x_i)
\]

Conclusion

Some of the properties of the neutrosophic sets, Distance measures, Hesitancy Degree, Cardinality and Entropy measures are briefed in this paper. These measures can be used effectively in image processing and pattern recognition. The future work will cover the application of these measures.
References

[23] A. A. Salama, Mohamed Abdelfattah, Mohamed Eisa, Distances, Hesitancy Degree


Received: July 29, 2014. Accepted: August 19, 2014.