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Introduction to Topological Indices in Neutrosophic Graphs

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Abstract: Neutrosophic Graphs are graphs that follow three-valued logic. They may be considered a fuzzy graph, although in some cases, it is difficult to optimize and model them using fuzzy graphs. In this paper, the first and second Zagreb indices, the Harmonic index, the Randic′ index and the Connectivity index for these graphs are investigated and some of the theorems related to these indices are discussed and proven. These indices are also calculated for some specific types of Neutrosophic Graphs, such as regular Neutrosophic Graphs and regular complete Neutrosophic Graphs.

Keywords: Neutrosophic Graphs; Zagreb indices; Harmonic index; Randic′ index; Connectivity index

1. Introduction

Graph theory has many applications for modeling problems in various fields of computer science such as systems analysis, computer networks, transportation, operations research and economics. The vertices and edges of the graphs are used to represent objects and the relationships between them, respectively. Many of the optimization issues are caused by inaccurate information due to factors like lack of evidence, incomplete statistical data, and lack of sufficient information; this creates uncertainty in various issues. Classical Graphic Theory uses the basic concept of classical set theory, as proposed by Contour. In a classic graph, for each vertex or edge, there are two possibilities: either in the graph or not in the graph. Therefore, classical graphs cannot model uncertain optimization problems. Real-life issues are often unclear, making modeling by classical graphs difficult. Zadeh introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set. Atanassov [14] introduced the degree of nonmembership/falsehood (F) in 1983 and defined the intuitionistic fuzzy set. Smarandache [15] introduced the degree of indeterminacy/neutrality (I) as an independent component in 1995 and defined the neutrosophic set on three components (T, I, F) [4].

Fuzzy set [1] is a generalized version of the classical set in which objects have different membership degrees. A fuzzy set gives the degree of different members between zero and one. Much work has already been done on fuzzy graphs, including the calculation of various topology indices, indicators such as Zagreb index, Randic′, harmonic, and so on. However, there is another class of graphs that is a broad case of fuzzy graphs. In this type of graph, known as neutrosophic graphs, in addition to the degree of accuracy of each membership function, the degree of its membership is uncertain, as well as its inaccuracy. So in many cases, it may be more logical to use this model than graphs in real-world problems.

Since that neutrosophic graphs are more efficient than fuzzy graphs for modeling real problems. Therefore, in this paper, we try for the first time to calculate some topological indices for this type of graph.

2. Preliminaries

This section, provides some definitions and theorems needed.

Definition 1. [13] Let $G=(N,M)$ be a single-valued Neutrosophic graph, where N is a Neutrosophic set on V and, M is a Neutrosophic set on E, which satisfy the following

$$T_M(u,v) \leq min\big(T_N(u),T_N(v)\big),$$
$$I_M(u,v) \geq max\big(I_N(u),I_N(v)\big),$$
$$F_M(u,v) \geq max\big(F_N(u),F_N(v)\big),$$

Where u and v are two vertices of G, and $(u,v) \in E$ is an edge of G.

Definition 2. [2] Let $G=(N,M)$ be a Single-Valued Neutrosophic Graph and P is a path in G. P is a collection of different vertices, $v_0, v_1, v_2, \dots, v_n$ such that $\big(T_M(v_{i-1},v_i), I_M(v_{i-1},v_i), F_M(v_{i-1},v_i)\big) > 0$ for $0 \leq i \leq n$. P is a Neutrosophic cycle if $v_0 = v_n$ and $n \geq 3$.

Definition 3. [2] Suppose $G=(N,M)$ a single-valued Neutrosophic graph. G is a connected Single-Valued Neutrosophic Graph if there exists no isolated vertex in G. ($v \in V_G$ is isolated vertex, if there exists no incident edge to the vertex v.)

Definition 4. [2] Let $G=(N,M)$ be a Single-Valued Neutrosophic Graph, and $v \in V$ is vertex of G. The degree of vertex v is the sum of the truth membership values, the sum of the indeterminacy membership values, and the sum of the falsity membership values of all the edges that are adjacent to vertex v. And is denoted by $d(v)$, that

$$d(v) = \big(d_T(v), d_I(v), d_F(v)\big) = \left(\sum_{\substack{v\in V\\ v\neq u}} T_M(v,u), \sum_{\substack{v\in V\\ v\neq u}} I_M(v,u), \sum_{\substack{v\in V\\ v\neq u}} F_M(v,u)\right).$$

Definition 5. [2] Let $G=(N,M)$ be a Single-Valued Neutrosophic Graph, and the d_m–degree of any vertex v in G is denoted as $d_m(v)$ where

$$d_m(v) = \left(\sum_{u\neq v\in V} T_M^m(u,v), \sum_{u\neq v\in V} I_M^m(u,v), \sum_{u\neq v\in V} F_M^m(u,v)\right)$$

Here, the path $v = v_0 v_1\, v_2 \dots v_n = u$ is the shortest path between the vertices v and u, when the length of this path is m.

Definition 6. [2] Let $G=(N,M)$ be a Single-Valued Neutrosophic Graph, G is a regular neutrosophic graph if it satisfies the following,

$$\sum_{v\neq u} T_M(v,u) = c, \qquad \sum_{v\neq u} I_M(v,u) = c, \qquad \sum_{v\neq u} F_M(v,u) = c,$$

Where c is a constant value.

3. Topological Indices in Neutrosophic Graphs

In the section, we introduce Topological Indices in Neutrosophic Graphs and provide a number of examples. We define Zagreb indices, Harmonic index, and Randic′ index, and in finally Connectivity index on the neutrosophic graphs.

3.1. Zagreb index of First and Second Kind in Neutrosophic Graphs

Definition 8. Let $G=(N,M)$ be the Neutrosophic Graph whit non-empty vertex set. The first Zagreb index is denoted by $M(G)$ and defined as

$$M(G)=\sum_{i=1}^{n}(T_N(u_i),I_N(u_i),F_N(u_i))d_2(u_i), \qquad \forall\ u_i\in V.$$

Example 1. Consider the Neutrosophic Graph $G=(N,M)$ as shown in figure 1, with the vertex set $V=\{a,b,c\}$ such that $(T_N,I_N,F_N)(a)=(0.3,0.6,0.7)$, $(T_N,I_N,F_N)(b)=(0.3,0.5,0.6)$, and $(T_N,I_N,F_N)(c)=(0.4,0.5,0.6)$, The edge set contains $(T_M,I_M,F_M)(a,b)=(0.2,0.6,0.8)$, $(T_M,I_M,F_M)(b,c)=(0.2,0.6,0.7)$, and $(T_M,I_M,F_M)(a,c) = (02,0.8,0.9)$. We have,

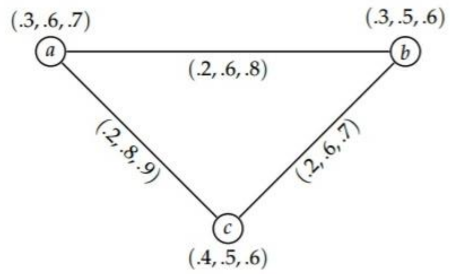

Figure 1. A neutrosophic graph with $V=\{a,b,c\}$

The first Zagreb index is

$$d(a)=(0.2+0.2,0.6+0.8,0.8+0.9)=(0.4,1.4,1.7),$$
$$d(b)=(0.2+0.2,0.6+0.6,0.8+0.7)=(0.4,1.2,1.5),$$
$$d(c)=(0.2+0.2,0.8+0.6,0.9+0.7)=(0.4,1.4,1.6).$$

Now, we have

$$d_2(a)=(0.04+0.04,0.36+0.64,0.64+0.81)=(0.08,1,1.45),$$
$$d_2(b)=(0.04+0.04,0.36+0.36,0.64+0.49)=(0.08,0.72,1.13),$$
$$d_2(c)=(0.04+0.04,0.64+0.36,0.81+0.49)=(0.08,1,1.3).$$

$$M(G)=\sum_{i=1}^{4}(T_N(u_i),I_N(u_i),F_N(u_i))d_2(u_i)$$

$$=(0.3,0.6,0.7)(0.08,1,1.45)+(0.3,0.5,0.6)(0.08,0.72,1.13)$$
$$+(0.4,0.5,0.6)(0.08,1.1.3)$$
$$=(0.024+0.6+1.015)+(0.024+0.36+0.678)+(0.032+0.5+0.78)=4.013.$$

Definition 9. The second Zagreb index is denoted by $M^*(G)$ and defined as

$$M^*(G)=\frac{1}{2}\sum[(T_N(u_i),I_N(u_i),F_N(u_i))d(u_i)][(T_N(v_j),I_N(v_j),F_N(v_j))d(v_j)], \quad \forall i\neq j\ and\ (u_i,v_j)\in E.$$

Example 2. If G is the same Neutrosophic Graph as example 1, we have

$$M^*(G) = \frac{1}{2}[(0.3, 0.6, 0.7).(0.4, 1.4, 1.7) \times (0.3, 0.5, 0.6).(0.4, 1.2, 1.5) + (0.3, 0.6, 0.7).(0.4, 1.4, 1.7)$$
$$\times (0.4, 0.5, 0.6).(0.4, 1.4, 1.6) + (0.3, 0.5, 0.6).(0.4, 1.2, 1.5)$$
$$\times (0.4, 0.5, 0.6).(0.4, 1.4, 1.6)]$$
$$= \frac{1}{2}[(0.12 + 0.84 + 1.19) \times (0.12 + 0.6 + 0.9) + (0.12 + 0.84 + 1.19)$$
$$\times (0.16 + 0.7 + 0.96) + (0.12 + 0.6 + 0.9) \times (0.16 + 0.7 + 0.96)$$
$$= \frac{1}{2}[(2.15)(1.62) + (2.15)(1.82) + (1.62)(1.82)] = \frac{1}{2}(10.3444) = 5.1722.$$

Note 1. As we have seen, the value of $M^*(G)$ is less than the value of $M(G)$, and this is always the case.

Theorem 1. Let G is the Neutrosophic Graph and H is the Neutrosophic sub graph of G such that $H = G - u$ then $M(H) < M(G)$ and $M^*(H) < M^*(G)$.

Proof. Given that by omitting a vertex of G, a positive value, the sum is lost, so the proof is obvious.

□

Theorem 2. Let G be the regular neutrosophic graph. Then, we have

$$M(G) = c^2 \times \sum_{i=1}^{n} (T_N(u_i) + I_N(u_i) + F_N(u_i)), \qquad \forall\ u_i \in V.$$

Where $\sum_{v \neq u} T_M(v, u) = c$, $\sum_{v \neq u} I_M(v, u) = c$, $\sum_{v \neq u} F_M(v, u) = c$.

Proof. Given the degree of definition of each vertex,

$$d(v) = \big(d_T(v), d_I(v), d_F(v)\big) = \left(\sum_{\substack{v \in V \\ v \neq u}} T_M(v, u), \sum_{\substack{v \in V \\ v \neq u}} I_M(v, u), \sum_{\substack{v \in V \\ v \neq u}} F_M(v, u)\right).$$

On the other hand, for regular neutrosophic graphs, we know that

$$\sum_{v \neq u} T_M(v, u) = c, \qquad \sum_{v \neq u} I_M(v, u) = c, \qquad \sum_{v \neq u} F_M(v, u) = c,$$

Therefore

$$d(v) = \big(d_T(v), d_I(v), d_F(v)\big) = (c, c, c).$$

Now, by embedding the formula in the first Zagreb index, we will get the desired result. The proof is complete.

□

Theorem 3. Let G be the regular neutrosophic graph. Then, we have

$$M^*(G) = \frac{1}{2}(c^2) \sum [T_N(u_i) + I_N(u_i) + F_N(u_i)][T_N(v_j) + I_N(v_j) + F_N(v_j)],$$
$$\forall i \neq j \ and\ (u_i, v_j) \in E,$$

Where $\sum_{v \neq u} T_M(v, u) = c$, $\sum_{v \neq u} I_M(v, u) = c$, $\sum_{v \neq u} F_M(v, u) = c$.

Proof. Assume G is a regular neutrosophic graph, using the second Zagreb index formula for G, we have $\forall\ i \neq j\ and\ (u_i, v_j) \in E$,

$$M^*(G) = \frac{1}{2}\sum[(T_N(u_i), I_N(u_i), F_N(u_i))d(u_i)][(T_N(v_j), I_N(v_j), F_N(v_j))d(v_j)]$$
$$= \frac{1}{2}\sum[(T_N(u_i), I_N(u_i), F_N(u_i))d\big(d_T(u_i), d_I(u_i), d_F(u_i)\big)]$$
$$\times [(T_N(v_j), I_N(v_j), F_N(v_j))d\left(d_T(v_j), d_I(v_j), d_F(v_j)\right)]$$
$$= \frac{1}{2}\sum[(T_N(u_i), I_N(u_i), F_N(u_i)).(c,c,c)][(T_N(v_j), I_N(v_j), F_N(v_j)).(c,c,c)]$$
$$= \frac{1}{2}\sum[c.T_N(u_i) + c.I_N(u_i) + c.F_N(u_i)][c.T_N(v_j) + c.I_N(v_j) + c.F_N(v_j)]$$
$$= \frac{1}{2}\sum c[T_N(u_i) + I_N(u_i) + F_N(u_i)].c[T_N(v_j) + I_N(v_j) + F_N(v_j)]$$
$$= \frac{1}{2}c^2\sum[T_N(u_i) + I_N(u_i) + F_N(u_i)][T_N(v_j) + I_N(v_j) + F_N(v_j)].$$

The desired result was obtained.

□

3.2. Harmonic index in Neutrosophic Graphs

Definition 10. The Harmonic index of Neutrosophic Graph G is defined as

$$H(G) = \sum \frac{1}{(T_N(u_i), I_N(u_i), F_N(u_i))d(u_i) + (T_N(v_j), I_N(v_j), F_N(v_j))d(v_j)}, \quad \forall i \neq j \text{ and } (u_i, v_j) \in E.$$

Example 3. We have the previous example,

$$H(G) = \frac{1}{(0.3, 0.6, 0.7)(0.4, 1.4, 1.7) + (0.3, 0.5, 0.6)(0.4, 1.2, 1.5)}$$
$$+ \frac{1}{(0.3, 0.6, 0.7)(0.4, 1.4, 1.7) + (0.4, 0.5, 0.6)(0.4, 1.4, 1.6)}$$
$$+ \frac{1}{(0.3, 0.5, 0.6)(0.4, 1.2, 1.5) + (0.4, 0.5, 0.6)(0.4, 1.4, 1.6)}$$
$$= \frac{1}{2.15 + 1.62} + \frac{1}{2.15 + 1.82} + \frac{1}{1.62 + 1.82} = \frac{1}{3.77} + \frac{1}{3.97} + \frac{1}{3.44} = 0.8078.$$

3.3. Randic′ index in Neutrosophic Graphs

Definition 11. The Randic′ index of Neutrosophic Graph G is defined as

$$R(G) = \sum((T_N(u_i), I_N(u_i), F_N(u_i))d(u_i)(T_N(v_j), I_N(v_j), F_N(v_j))d(v_j))^{\frac{-1}{2}}, \quad \forall i \neq j \text{ and } (u_i, v_j) \in E.$$

Example 3. For above example, by simple calculations, it is easy to see that

$$R(G) = \frac{1}{\sqrt{(0.3, 0.6, 0.7).(0.4, 1.4, 1.7) \times (0.3, 0.5, 0.6).(0.4, 1.2, 1.5)}}$$
$$+ \frac{1}{\sqrt{(0.3, 0.6, 0.7).(0.4, 1.4, 1.7) \times (0.4, 0.5, 0.6).(0.4, 1.4, 1.6)}}$$
$$+ \frac{1}{\sqrt{(0.3, 0.5, 0.6).(0.4, 1.2, 1.5) \times (0.4, 0.5, 0.6).(0.4, 1.4, 1.6)}}$$
$$= \frac{1}{\sqrt{2.15 \times 1.62}} + \frac{1}{\sqrt{2.15 \times 1.82}} + \frac{1}{\sqrt{1.62 \times 1.82}} = 1.6237.$$

3.4. Connectivity index in Neutrosophic Graphs

Connectivity index is an important parameter in the graph. Using it, we can study and study some of the features of graph models.

Definition 12. Let $G = (N, M)$ be the Neutrosophic Graph. The connectivity index of G is defined by

$$CI(G) = \sum_{u_i, v_j \in V} (T_N(u_i), I_N(u_i), F_N(u_i))(T_N(v_j), I_N(v_j), F_N(v_j)) \times CONN_G(u_i, v_j).$$

Where $CONN_G(u_i, v_j)$ is the strength of connectedness between u_i and v_j.

Definition 13. The strength of connectedness between u_i and v_j is defined as

$$CONN_P(u_i, v_j) = \left(\min_{e \in P_{u_i v_j}} T_M(e), \max_{e \in P_{u_i v_j}} I_M(e), \max_{e \in P_{u_i v_j}} F_M(e) \right),$$

Where $P_{u_i v_j}$ is the path between u_i and v_j.

$$|CONN_P(u_i, v_j)| = 2\left(\min_{e \in P_{u_i v_j}} T_M(e) \right) - \left(\max_{e \in P_{u_i v_j}} I_M(e) \right) - \left(\max_{e \in P_{u_i v_j}} F_M(e) \right),$$

Then

$$CONN_G(u_i, v_j) = \max_P \{ |CONN_P(u_i, v_j)| \}.$$

Example 4. For example, in the above figure, the strength of connectedness between:
a and b from the direct path $P_1 = ab$ is

$$CONN_{P_1}(a, b) = M_{ab} = (0.2, 0.6, 0.8),$$

From path $P_2 = acb$ is

$$CONN_{P_2}(a, b) = (min\{0.2, 0.2\}, \quad max\{0.8, 0.6\}, \quad max\{0.9, 0.7\}) = (0.2, 0.8, 0.9);$$

a and c from the direct path $P_1 = ac$ is

$$CONN_{P_1}(a, c) = M_{ac} = (0.2, 0.8, 0.9),$$

From path $P_2 = abc$ is

$$CONN_{P_2}(a, c) = (min\{0.2, 0.2\}, \quad max\{0.6, 0.6\}, \quad max\{0.8, 0.7\}) = (0.2, 0.6, 0.8);$$

b and c from the direct path $P_1 = bc$ is

$$CONN_{P_1}(b, c) = M_{bc} = (0.2, 0.6, 0.7),$$

From path $P_2 = bac$ is

$$CONN_{P_2}(b, c) = (min\{0.2, 0.2\}, \quad max\{0.6, 0.8\}, \quad max\{0.8, 0.9\}) = (0.2, 0.8, 0.9).$$

Then, we have for a and b

$$|CONN_{P_1}(a, b)| = 2 \times (0.2) - 0.6 - 0.8 = -1,$$
$$|CONN_{P_2}(a, b)| = 2 \times (0.2) - 0.8 - 0.9 = -1.3.$$

For a and c,

$$|CONN_{P_1}(a, c)| = 2 \times (0.2) - 0.8 - 0.9 = -1.3,$$
$$|CONN_{P_2}(a, c)| = 2 \times (0.2) - 0.6 - 0.8 = -1.$$

For b and c,

$$|CONN_{P_1}(b, c)| = 2 \times (0.2) - 0.6 - 0.7 = -0.9,$$
$$|CONN_{P_2}(b, c)| = 2 \times (0.2) - 0.8 - 0.9 = -1.3.$$

Since we have

$$CONN_G(a,b) = -1; \quad CONN_G(a,c) = -1; \quad CONN_G(b,c) = -0.9.$$

Then $CI(G)$ is calculated as follows

$$\begin{aligned} CI(G) = \sum_{u_i,v_j \in V} &(T_N(u_i), I_N(u_i), F_N(u_i))(T_N(v_j), I_N(v_j), F_N(v_j)) \times CONN_G(u_i, v_j) \\ &= (0.3, 0.6, 0.7).(0.3, 0.5, 0.6) \times (-1) + (0.3, 0.6, 0.7).(0.4, 0.5, 0.6) \times (-1) \\ &+ (0.3, 0.5, 0.6).(0.4, 0.5, 0.6) \times (-0.9) \\ &= (0.09 + 0.3 + 0.42)(-1) + (0.12 + 0.3 + 0.42)(-1) + (0.12 + 0.25 + 0.36)(-0.9) \\ &= (0.81)(-1) + (0.84)(-1) + (0.73)(-0.9) = -2.307. \end{aligned}$$

The connectivity index of G is equal -2.307, which the negative sing indicates the high level of false and indeterminacy information in the problem.

Theorem 4. Let G and H be the two Neutrosophic Graphs are isomorphic, then the topological indices values of two Neutrosophic Graphs are equal.

Proof. To prove, let $G = (V_G, N_G, M_G)$ and $H = (V_H, N_H, M_H)$ be isomorphic Neutrosophic Graphs. Hence there is an identity function $\mu_N: N_G(u) \to N_H(u^*)$, for all $u \in V_G$ there exist $u^* \in V_H$ as well as $\mu_M: M_G(u,v) \to M_H(u^*, v^*)$, then each vertex of G corresponds to an vertex in H, with the same membership value and the same edges. Hence, the Neutrosophic graph structure may differ but collection of vertices and edges are same gives the equal topological indices value.
□

Theorem 5. Let $G = (V_G, N_G, M_G)$, is a neutrosophic Graph and H is the neutrosophic sub graph of G, Such that H is made by removing edge $uv \in M_G$ from G. Then, we have, $CI(H) < CI(G)$ iff uv is a bridge.

Proof. To prove the first side of the theorem we consider two cases:

Case 1. Let uv be an edge with all three components having the least value, Therefore the edge uv will have no effect on the result. Then we have $CI(H) = CI(G)$.

Case 2. Now suppose that uv is an edge that has maximum components, so they will have an effect on $CONN_G(u,v)$. Therefore, by removing edge uv, the value of $CONN_G(u,v)$ will decrease, then we have $CI(H) < CI(G)$. Since the bridge is called the edge that has its deletion reducing the $CONN_G(u,v)$, However, uv is a bridge.

Conversely, given that uv is a bridge. According to the definition of bridge we have, for the edge uv, $CONN_G(u,v) > CONN_{G-uv}(u,v)$, So we conclude that, $CI(H) < CI(G)$.
□

4. Applications

Fuzzy set theory and intuitionistic fuzzy set theory are useful models for modelling problems in real life. But they may not be sufficient in modelling of indeterminate and inconsistent information encountered in real word. In cases where our information is incomplete or part of our information is incompatible with each other, depending on the features of the neutrosophic graphs, we can use them for modeling. However, neutrosophic graphs have many application in real life. For example, social network model, detection of a safe root for an Airline journey and military problems are application neutrosophic graph theory [4]. Note that to many applications that neutrosophic graphs have, obtaining topological indices can be a way to compare the different problems that are modeled by neutrosophic graphs. For example, by obtaining different indicators for the two social networks Telegram and Whatsapp, we can analyze some of the features of the network and their impact.

To see more applications of the neutrosophic graphs, you can refer to [5-12].

Here we refer to one of the applications of the connectivity index for an example of [4].

4.1. Optimal flight path for weather emergency landing

In this application, we use the concept of rough neutrosophic digraph for decision-making in real-life problems [4]. There, provided a formula for obtaining the desired result, and after performing the calculations, reached the desired result.

Now, using the connectivity index for different paths, it is possible to predict the optimal path for flying in weather emergency landing.

Suppose $V = \{Chicago(CH), Beijing(BJ), Lahore(LH), Paris(PA), Istanbul(IS)\}$, be the set of cities under consideration and R an equivalence relation on V, where equivalence classes represent cities having same characteristics.

Assume that a flight Boeing 747 of Pakistan International Airways (PIA) travels to these cities. In case of bad weather, the flight will be directed to the city with good weather condition among the cities under consideration.

Let

$$N = \{CH, 0.1, 0.2, 0.8), (BJ, 0.9, 0.7, 0.5), (LH, 0.8, 0.4, 0.3), (PA, 0.6, 0.5, 0.4), (IS, 0.2, 0.4, 0.6)\},$$

And

$$M = \{\big((BJ, CH), 0.1, 0.1, 0.3\big), \big((LH, CH), 0.1, 0.2, 0.3\big), \big((BJ, LH), 0.1, 0.3, 0.2\big),$$
$$\big((IS, BJ), 0.2, 0.1, 0.1\big), \big((PA, BJ), 0.1, 0.1, 0.4\big), \big((PA, LH), 0.2, 0.2, 0.3\big)\}.$$

Now, we obtain the connectivity index for all paths.

The direct path $BJ \_ CH$

$$CONN_P(BJ, CH) = 2(0.1) - 0.1 - 0.3 = -0.2,$$

The direct path $BJ \_ LH$

$$CONN_P(BJ, LH) = 2(0.1) - 0.3 - 0.2 = -0.3,$$

The direct path $LH \_ CH$

$$CONN_P(LH, CH) = 2(0.1) - 0.2 - 0.3 = -0.3,$$

The direct path $IS \_ BJ$

$$CONN_P(IS, BJ) = 2(0.2) - 0.1 - 0.1 = 0.2,$$

The direct path $PA \_ BJ$

$$CONN_P(BJ, CH) = 2(0.1) - 0.1 - 0.4 = -0.3,$$

The direct path $PA \_ LH$

$$CONN_P(PA, LH) = 2(0.2) - 0.2 - 0.3 = -0.1.$$

Hence, as expected from [4], the weather condition between Beijing and Istanbul is good, and Boeing 747 can use this path in case of weather emergency. We were able to achieve the desired result with much shorter calculations. Also, if needed, we can calculate the connectivity index for indirect paths and finally for neutrosophic graph.

For connectivity index of G we have,

$$CI(G) = \sum_{u_i,v_j \in V} (T_N(u_i), I_N(u_i), F_N(u_i))(T_N(v_j), I_N(v_j), F_N(v_j)) \times CONN_G(u_i, v_j)$$

$$= (0.63)(-0.2) + (0.76)(-0.3) + (0.4)(-0.3) + (1.09)(-0.3) + (0.8)(-0.1) + (0.76)(0.2) + (0.5)(-0.3) + (0.58)(-0.2) + (0.8)(-0.5) + (0.48)(-0.3) + (0.58)(-0.4) + (0.48)(-0.5)$$

$$= -0.126 - 0.228 - 0.12 - 0.327 - 0.08 + 0.152 - 0.15 - 0.116 - 0.4 - 0.144 - 0.232 - 0.24 = -1.783.$$

As you can see, the negative numerical connectivity index was obtained, which means that our incorrect information was less than our correct information.

Conclusion

In this paper, for the first time, some topological indices for neutrosophic graphs are defined. This topic has a lot of work to do, and it can also be used for its results on various issues related to this category of graphs. In the rest of our research and in future articles, we will address more of these theorems and their applications.

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