Neutrosophic Sets and Systems, Vol. 51, 2022
University of New Mexico

Irresolute and its Contra Functions in Generalized Neutrosophic Topological Spaces

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Abstract: The intention to study the idea of Generalized Topological Spaces by means of Neutrosophic sets leads to develop this article. In this write up we launch new ideas on λ_N-Topological Spaces. We study some of its characteristics and behaviours of λ_N-α-irresolute function, λ_N-semi-irresolute function and λ_N-pre-irresolute function. Also we discuss the above for contra λ_N-irresolute functions and derived some relations between them.

2010 Mathematics Subject Classification: 54A05, 54B05, 54D10

Keywords: λ_N-α-irresolute function, λ_N-semi-irresolute function, λ_N-pre-irresolute function, contra λ_N-α-irresolute function, contra λ_N-semi-irresolute function, contra λ_N-pre-irresolute function.

1. Introduction

Zadeh [16] initiated fuzzy set theory in 1965 that deals with uncertainty in real life situations. Chang [2] designed fuzzy topology that gave a special note to the field of topology in 1968. Attanassov [1] in 1983, see the sights of intuitionistic fuzzy sets by considering both membership and non-membership of the elements. In 1997, Coker [4] worked on Intuitionistic fuzzy sets by extending the concepts of fuzziness and found a place for Intuitionistic fuzzy topological space.

Smarandache [5] to [7] & [14] introduced Neutrosophic set which is a generalization of fuzzy set and intuitionistic fuzzy set. This is a strong tool to discuss about the existence of incomplete, indeterminate and inconsistent information in the real life situation. Smarandache focused his observations en route for the degree of indeterminacy that directed into Neutrosophic Sets (NS). Soon after, Salama and Albowi [10] familiarized Neutrosophic Topological Spaces (NTS). Further, Salama, Smarandache and Valeri Kromov

[11] presented the continuous (Cts) functions in NTS. In [3], irresolute functions was introduced and analysed by Crossley and Hildebrand in Topological Spaces. Further, Vijaya [13] and Santhi [12] investigated the properties of λ-α-irresolute function and contra λ-α-irresolute function in Generalized Topological Spaces. In addition to that, properties of α-irresolute function and contra α-irresolute function in Nano Topological Spaces was look over by Yuvarani and et. al., [15]. By keeping all these works as a motivation, in 2020, Raksha Ben, Hari Siva Annam [8] & [9] contrived λ_N-Topological Space and deliberated its properties.

In this disquisition, we explore our perception of λ_N-α-irresolute function, λ_N-semi-irresolute function, λ_N-pre-irresolute function, contra λ_N-α-irresolute function, contra λ_N-semi-irresolute function, contra λ_N-pre-irresolute function and we have scrutinized about some of their basic properties. At every place the novel notions have been validated with apposite paradigms.

2. Prerequisites

2.1. Definition [10]

Let Ω be a non-empty fixed set. A NS, E = {⟨ ω, M_E(ω), I_E(ω), N_E(ω)⟩ : ω ∈ Ω} where M_E(ω), I_E (ω) and N_E (ω) represents the degree of membership, indeterminacy and non-membership functions respectively of every element ω ∈ Ω.

2.2. Remark [10]

A NS, E can be recognized as a structured triple E = {⟨ ω, M_E(ω), I_E(ω), N_E(ω)⟩ : ω ∈ Ω} in $]^-0, 1^+[$ on Ω.

2.3. Remark [10]

The NS, 0_N and 1_N in Ω is defined as

(P_1) 0_N = {⟨ ω, 0, 0, 1⟩ : ω ∈ Ω}

(P_2) 0_N = {⟨ ω, 0, 1, 1⟩ : ω ∈ Ω}

(P_3) 0_N = {⟨ ω, 0, 1, 0⟩ : ω ∈ Ω}

(P_4) 0_N = {⟨ ω, 0, 0, 0⟩ : ω ∈ Ω}

(P_5) 1_N = {⟨ ω, 1, 0, 0⟩ : ω ∈ Ω}

(P_6) 1_N = {⟨ ω, 1, 0, 1⟩ : ω ∈ Ω}

(P_7) 1_N = {⟨ ω, 1, 1, 0⟩ : ω ∈ Ω}

(P_8) 1_N = {⟨ ω, 1, 1, 1⟩ : ω ∈ Ω}

2.4. Definition [10]

If E = {⟨ M_E(ω), I_E(ω), N_E(ω)⟩ }, then the complement of E on Ω is

(P_9) E′ = {⟨ ω, 1 − M_E(ω), 1 - I_E(ω) and 1 - N_E(ω)⟩ : ω ∈ Ω}

(P_{10}) E′ = {⟨ ω, N_E(ω), I_E(ω) and M_E(ω)⟩ : ω ∈ Ω}

(P_{11}) E′ = {⟨ ω, N_E(ω), 1 - I_E(ω) and M_E(ω)⟩ : ω ∈ Ω}

2.5. Definition [10]

Let Ω be a non-empty set and let $E = \{\langle \omega, M_E(\omega), I_E(\omega), N_E(\omega)\rangle : \omega \in \Omega\}$ and $F = \{\langle \omega, M_F(\omega), I_F(\omega), N_F(\omega)\rangle : \omega \in \Omega\}$. Then

(i) $E \subseteq F \Rightarrow M_E(\omega) \leq M_F(\omega), I_E(\omega) \leq I_F(\omega), N_E(\omega) \geq N_F(\omega), \forall\ \omega \in \Omega$

(ii) $E \subseteq F \Rightarrow M_E(\omega) \leq M_F(\omega), I_E(\omega) \geq I_F(\omega), N_E(\omega) \geq N_F(\omega), \forall\ \omega \in \Omega$

2.6. Definition [10]

Let Ω be a non-empty set and $E = \{\langle \omega, M_E(\omega), I_E(\omega), N_E(\omega)\rangle : \omega \in \Omega\}$, $F = \{\langle \omega, M_F(\omega), I_F(\omega), N_F(\omega)\rangle : \omega \in \Omega\}$ are NSs. Then,

(P_{12}) $E \cap F = \langle \omega, M_E(\omega) \wedge M_F(\omega), I_E(\omega) \vee I_F(\omega), N_E(\omega) \vee N_F(\omega)\rangle$

(P_{13}) $E \cap F = \langle \omega, M_E(\omega) \wedge M_F(\omega), I_E(\omega) \wedge I_F(\omega), N_E(\omega) \vee N_F(\omega)\rangle$

(P_{14}) $E \cup F = \langle \omega, M_E(\omega) \vee M_F(\omega), I_E(\omega) \wedge I_F(\omega), N_E(\omega) \wedge N_F(\omega)\rangle$

(P_{15}) $E \cup F = \langle \omega, M_E(\omega) \vee M_F(\omega), I_E(\omega) \vee I_F(\omega), N_E(\omega) \wedge N_F(\omega)\rangle$

2.7. Definition [9]

Let $\Omega \neq \phi$. A family of Neutrosophic subsets of Ω is λ_N-topology if it satisfies

(Δ_1) $0_N \in \lambda_N$ (Δ_2) $E_1 \cup E_2 \in \lambda_N$ for any $E_1, E_2 \in \lambda_N$.

2.8. Remark [9]

Members of λ_N-topology are λ_N-Open Sets (λ_N-OS) and their complements are λ_N-Closed Sets (λ_N-CS).

2.9. Definition [9]

Let (Ω, λ_N) be a λ_N-TS and $E = \{\langle \omega, M_E(\omega), I_E(\omega), N_E(\omega)\rangle\}$ be a NS in Ω. Then

λ_N-Closure (E) = $\bigcap\{F: E \subseteq F, F \text{ is } \lambda_N\text{-CS}\}$

λ_N-Interior (E) = $\bigcup\{G: G \subseteq E, G \text{ is } \lambda_N\text{-OS}\}$

2.10. Definition [8]

A NS, E in λ_N-TS is said to be

(i) λ_N-Semi-Open Set (λ_N-SOS) if $E \subseteq \lambda_N\text{-Cl}(\lambda_N\text{-Int}(E))$,

(ii) λ_N-Pre-Open Set (λ_N-POS) if $E \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(E))$,

(iii) λ_N-α-Open Set (λ_N-αOS) if $E \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))$.

2.11. Lemma [8]

Every λ_N- αOS is λ_N-SOS and λ_N-POS.

2.12. Definition [8]

Let the function $h: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$ is defined to be λ_N-Cts (resp. λ_N-SCts, λ_N-PCts, λ_N-αCts) if the inverse image of λ_N-CS in (Ω_2, τ_2) is a λ_N-CS (resp. λ_N-SCS, λ_N-PCS, λ_N-αCS) in (Ω_1, τ_1).

3. λ_N-Irresolute Functions

3.1. Definition

Let (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. Then $h: \Omega_1 \rightarrow \Omega_2$ is said to be a λ_N-α-irresolute function (resp. λ_N-semi-irresolute, λ_N-pre-irresolute) if the inverse image of every λ_N-αOS (resp. λ_N-SOS, λ_N-POS) in (Ω_2, τ_2) is an λ_N-αOS (resp. λ_N-SOS, λ_N-POS) in (Ω_1, τ_1).

3.2. Example

Let h: (Ω_1, τ_1) → (Ω_2, τ_2) be defined as h(p) = s and h(q) = r, where Ω_1 = {p, q} and Ω_2 = {r, s}, τ_1 = {0_N, A, B}, τ_2 = {0_N, C, D}.

(i) A = ⟨ (0.2, 0.8, 0.9), (0.1, 0.7, 0.8) ⟩, B = ⟨ (0.3, 0.5, 0.6), (0.4, 0.6, 0.7) ⟩,
C = ⟨ (0.1, 0.7, 0.8), (0.2, 0.8, 0.9) ⟩, D = ⟨ (0.4, 0.6, 0.7), (0.3, 0.5, 0.6) ⟩,
G = ⟨ (0.3, 0.7, 0.8), (0.2, 0.6, 0.7) ⟩, H = ⟨ (0.2, 0.6, 0.7), (0.3, 0.7, 0.8) ⟩.

Here {0_N, A, B, G} and {0_N, C, D, H} are λ_N-αOS of (Ω_1, τ_1) and (Ω_2, τ_2) respectively. Hence, h is a λ_N-α-irresolute function.

(ii) A = ⟨ (0.3, 0.7, 0.8), (0.2, 0.6, 0.8) ⟩, B = ⟨ (0.4, 0.6, 0.7), (0.5, 0.5, 0.6) ⟩,
C = ⟨ (0.5, 0.5, 0.6), (0.4, 0.6, 0.7) >, D = ⟨ (0.2, 0.6, 0.8), (0.3, 0.7, 0.8) ⟩,
G = ⟨ (0.3, 0.7, 0.8), (0.4, 0.5, 0.7) >, H = ⟨ (0.4, 0.5, 0.7), (0.3, 0.7, 0.8) ⟩.

Here {0_N, A, B, G} and {0_N, C, D, H} are λ_N-SOS of (Ω_1, τ_1) and (Ω_2, τ_2) respectively. Therefore, h is a λ_N-semi-irresolute function.

(iii) A = ⟨ (0.3, 0.8, 0.9), (0.4, 0.7, 0.6) ⟩, B = ⟨ (0.4, 0.6, 0.7), (0.5, 0.6, 0.6) ⟩,
C = ⟨ (0.5, 0.6, 0.6), (0.4, 0.6, 0.7) ⟩, D = ⟨ (0.4, 0.7, 0.6), (0.3, 0.8, 0.9) ⟩,
G = ⟨ (0.2, 0.9, 0.9), (0.3, 0.8, 0.9) ⟩, H = ⟨ (0.3, 0.7, 0.8), (0.5, 0.5, 0.6) ⟩,
I = ⟨ (0.3, 0.8, 0.9), (0.2, 0.9, 0.9) ⟩, J = ⟨ (0.5, 0.5, 0.6), (0.3, 0.7, 0.8) ⟩.

Here {0_N, A, B, G, H} and {0_N, C, D, I, J} are λ_N-POS of (Ω_1, τ_1) and (Ω_2, τ_2) respectively and so h is a λ_N-pre-irresolute function.

3.3. Theorem

Let (Ω, τ) be a λ_N-TS and E $\subseteq$ Ω. Then E is λ_N-αOS iff it is λ_N-SOS and λ_N-POS.

Proof:

If E is λ_N-αOS, then by Lemma 2.11, E is λ_N-SOS and λ_N-POS. Conversely if E is λ_N-SOS and λ_N-POS, then E ⊆ λ_N-Cl(λ_N-Int(E)) and E ⊆ λ_N-Int(λ_N-Cl(E)). Therefore λ_N-Int(λ_N-Cl(E)) ⊆ λ_N-Int(λ_N-Cl(λ_N-Cl(λ_N-Int(E)))) = λ_N-Int(λ_N-Cl(λ_N-Int(E))). That is λ_N-Int(λ_N-Cl(E)) ⊆ λ_N-Int(λ_N-Cl(λ_N-Int(E))). Also E ⊆ λ_N-Int(λ_N-Cl(E)) ⊆ λ_N-Int(λ_N-Cl(λ_N-Int(E))) implies E ⊆ λ_N-Int(λ_N-Cl(λ_N-Int(E))). Thus E is λ_N-αOS.

3.4. Theorem

Let h: $\Omega_1 \to \Omega_2$ be a function, where (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. Then the succeeding are equivalent.

(i) h is λ_N-α-irresolute.

(ii) h^{-1}(E) is λ_N-αCS in (Ω_1, τ_1), for every λ_N-αCS E in (Ω_2, τ_2).

(iii) h(λ_N-αCl(E)) $\subseteq$ λ_N-αCl(h(E)) $\forall$ E $\subseteq \Omega_1$.

(iv) λ_N-αCl(h^{-1}(E)) $\subseteq$ h^{-1}(λ_N-αCl(E)) $\forall$ E $\subseteq \Omega_2$.

(v) h^{-1}(λ_N-αInt(E)) $\subseteq$ λ_N-αInt(h^{-1}(E)) $\forall$ E $\subseteq \Omega_2$.

(vi) h is λ_N-α-irresolute for every $\omega \in$ (Ω_1, τ_1).

Proof

(i) implies (ii) It is obvious.

(ii) implies (iii) Let E $\subseteq$ Ω_1. In that case, λ_N-αCl(h(E)) is a λ_N-αCS of (Ω_2, τ_2). By (ii), h^{-1}(λ_N-αCl(h(E))) is a λ_N-αCS in (Ω_1, τ_1), and λ_N-αCl(E) $\subseteq$ λ_N-αCl(h^{-1}h(E)) $\subseteq$ λ_N-αCl(h^{-1}(λ_N-αCl(h(E)))) = h^{-1}(λ_N-αCl(h(E))). So h(λ_N-αCl(E)) $\subseteq$ λ_N-αCl(h(E)).

(iii) implies (iv) Let E $\subseteq$ Ω_2. By (iii), h(λ_N-αCl(h^{-1}(E))) $\subseteq$ λ_N-αCl(hh^{-1}(E)) $\subseteq$ λ_N-αCl(E). So λ_N-αCl(h^{-1}(E)) $\subseteq$ h^{-1}(λ_N-αCl(E)).

(iv) implies (v) Let E $\subseteq$ Ω_2. By (iv), h^{-1}(λ_N-αCl(Ω_2-E)) $\supseteq$ λ_N-αCl(h^{-1}(Ω_2-E)) = λ_N-αCl(Ω_1- h^{-1}(E)). Since Ω_1–λ_N-αCl(Ω_1–E) = λ_N-αInt(E), subsequently h^{-1}(λ_N-αInt(E)) = h^{-1}(Ω_2-λ_N-αCl(Ω_2-E)) = Ω_1 - h^{-1}(λ_N-αCl(Ω_2-E)) $\subseteq$ Ω_1 - λ_N-αCl(Ω_1 – h^{-1}(E)) = λ_N-αInt(h^{-1}(E)).

(v) implies (vi) Let E be any λ_N-αOS of (Ω_2, τ_2), subsequently E = λ_N-αInt(E). By (v), h^{-1}(E) = h^{-1}(λ_N-αInt(E)) $\subseteq$ λ_N-αInt(h^{-1}(E)) $\subseteq$ h^{-1}(E). So, h^{-1}(E) = λ_N-αInt(h^{-1}(E)). Thus, h^{-1}(E) is a λ_N-αOS of (Ω_1, τ_1). Therefore, h is λ_N-α-irresolute.

(i) implies (vi) Let h be λ_N-α-irresolute, $\omega \in$ (Ω_1, τ_1) and any λ_N-αOS E of (Ω_2, τ_2), $\ni$ h(ω) $\subseteq$ E. Then $\omega \in h^{-1}$(E) = λ_N-αInt(h^{-1}(E)). Let F = h^{-1}(E) followed by F is a λ_N-αOS of (Ω_1, τ_1) and so h(F) = hh^{-1}(E) $\subseteq$ E. Thus, h is λ_N-α-irresolute for each $\omega \in$ (Ω_1, τ_1).

(vi) implies (i) Let E be a λ_N-αOS of (Ω_2, τ_2), $\omega \in h^{-1}$(E). Then h(ω) $\in$ E. By hypothesis there exists a λ_N-αOS F of (Ω_1, τ_1) $\ni$ $\omega \in$ F and h(F) $\subseteq$ E. Thus $\omega \in$ F $\subseteq$ h^{-1}(h(F)) $\subseteq$ h^{-1}(E) and ω $\in$ F = λ_N-αInt(F) $\subseteq$ λ_N-αInt(h^{-1}(E)) $\Rightarrow$ h^{-1}(E) $\subseteq$ λ_N-αInt(h^{-1}(E)). Hence h^{-1}(E) = λ_N-αInt(f^{-1}(E)). Thus, h is λ_N-α-irresolute.

3.5. Theorem

Let h: $\Omega_1 \rightarrow \Omega_2$ be a bijective function, where (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. Then h is λ_N-α-irresolute iff λ_N-αInt(h(E)) $\subseteq$ h(λ_N-αInt(E)) $\forall$ E $\subseteq$ Ω_1.

Proof

Let E $\subseteq$ Ω_1. By Theorem 3.4 and since h is bijective, h^{-1}(λ_N-αInt(h(E))) $\subseteq$ λ_N-αInt(h^{-1}(h(E))) = λ_N-αInt(E). So, hh^{-1}(λ_N-αInt(h(E))) $\subseteq$ h(λ_N-αInt(E)). Consequently λ_N-αInt(h(E)) $\subseteq$ h(λ_N-αInt(E)).

Conversely, let E be a λ_N-αOS of (Ω_2, τ_2). Then E = λ_N-αInt(E). By hypothesis, h(λ_N-αInt(h^{-1}(E))) $\supseteq$ λ_N-αInt(h(h^{-1}(E))) = λ_N-αInt(E) = E implies h^{-1}h(λ_N-αInt(h^{-1}(E))) $\supseteq$ h^{-1}(E). Since h is bijective, λ_N-αInt(h^{-1}(E)) = h^{-1}h(λ_N-αInt(h^{-1}(E))) $\supseteq$ h^{-1}(E).

Hence h^{-1}(E) = λ_N-αInt(h^{-1}(E)). So h^{-1}(E) is λ_N-αOS of (Ω_1, τ_1). Thus, h is λ_N-α-irresolute.

3.6. Lemma

Let (Ω, τ) be a λ_N-TS and E $\subset$ Ω. Then λ_N-αInt(E) = E $\bigcap$ λ_N-Int((λ_N-Cl(λ_N-Int(E))), λ_N-αCl(E) = E $\bigcup$ λ_N-Cl(λ_N-Int(λ_N-Cl(E))).

3.7. Lemma

Let (Ω, τ) be a λ_N-TS, then

(i) λ_N-αCl(E) $\subseteq$ λ_N-Cl(E) $\forall$ E $\subseteq$ Ω.

(ii) λ_N-Cl(E) = λ_N-αCl(E) $\forall$ E $\subseteq$ Ω where E is λ_N-αOS.

Proof

(i) Let E $\subseteq$ Ω. Since λ_N-Int(E) $\subseteq \lambda_N$-αInt(E), U–λ_N-Int(E) $\supseteq$ U-λ_N-αInt(E). Hence λ_N-αCl(E) $\subseteq$ λ_N-Cl(E).

(ii) Let E be any λ_N-αOS of (Ω, τ), then E $\subseteq$ λ_N-Int(λ_N-Cl(λ_N-Int(E))). Then λ_N-Cl(E) $\subseteq$ λ_N-Cl(λ_N-Int(λ_N-Cl(λ_N-Int(E)))) = λ_N-Cl(λ_N-Int(E)) $\subseteq$ λ_N-Cl(λ_N-Int(λ_N-Cl(E))). So, λ_N-Cl(E) $\subseteq$ E $\bigcup$ λ_N-Cl(λ_N-Int(λ_N-Cl(E))). By Lemma 3.6, λ_N-Cl(E) $\subseteq$ λ_N-αCl(E). By (i), λ_N-αCl(E) $\subseteq$ λ_N-Cl(E), therefore λ_N-Cl(E) = λ_N-αCl(E).

3.8. Theorem

Let h: $\Omega_1 \to \Omega_2$ be a λ_N-α-irresolute function, where (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. Then λ_N-Cl(h^{-1}(E)) $\subseteq$ h^{-1}(λ_N-Cl(E)) for every λ_N-OS E of Ω_2.

Proof

Let E be any λ_N-OS of Ω_2. Since h is λ_N-α-irresolute and by Lemma 3.7, λ_N-αCl(h^{-1}(E)) = λ_N-Cl(h^{-1}(E)). By Theorem 3.4, λ_N-αCl(h^{-1}(E)) $\subseteq$ h^{-1}(λ_N-αCl(E)) and by Lemma 3.7, h^{-1}(λ_N-αCl(E)) $\subseteq$ h^{-1}(λ_N-Cl(E)). Then λ_N-αCl(h^{-1}(E)) $\subseteq$ h^{-1}(λ_N-Cl(E)). Therefore λ_N-Cl(h^{-1}(E)) $\subseteq$ h^{-1}(λ_N-Cl(E)).

3.9. Theorem

Let (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs and h: $\Omega_1 \to \Omega_2$ is λ_N-semi-irresolute iff h^{-1}(E) is λ_N-SCS in Ω_1, $\forall$ λ_N-SCS E of Ω_2.

Proof

If h is λ_N-semi-irresolute, then for every λ_N-SOS F of Ω_2, h^{-1}(F) is λ_N-SOS in Ω_1. If E is any λ_N-SCS of Ω_2, then Ω_2 – E is λ_N-SOS. As a consequence, h^{-1}(Ω_2 – E) is λ_N-SOS but h^{-1}(Ω_2 – E) = Ω_1–h^{-1}(E) so that h^{-1}(E) is λ_N-SCS in Ω_1.

Conversely, if, for all λ_N-SCS E of Ω_2, h^{-1}(E) is λ_N-SCS in Ω_1 and if F is any λ_N-SOS of Ω_2, then Ω_2–F is λ_N-SCS. Also h^{-1}(Ω_2–F) = Ω_1–h^{-1}(F) is λ_N-SCS in Ω_1. Accordingly h^{-1}(F) is λ_N-SOS in Ω_1. As a result, h is λ_N-semi-irresolute.

3.10. Theorem

If h_1: (Ω_1, τ_1) $\to$ (Ω_2, τ_2) is λ_N-semi-irresolute and h_2: (Ω_2, τ_2) $\to$ (Ω_3, τ_3) is λ_N-semi-irresolute, then $h_2 \circ h_1$: (Ω_1, τ_1) $\to$ (Ω_3, τ_3) is λ_N-semi-irresolute.

Proof

If E $\subseteq$ Ω_3 is λ_N-SOS, then h_2^{-1}(E) is λ_N-SOS in Ω_2 because h_2 is λ_N-semi-irresolute. Consequently since h_1 is λ_N-semi-irresolute, h_1^{-1}(h_2^{-1}(E)) = ($h_2 \circ h_1$)$^{-1}$(E) is λ_N-SOS in Ω_1. Hence $h_2 \circ h_1$ is λ_N-semi-irresolute.

3.11. Example ($h_2 \circ h_1$ is λ_N-semi-irresolute $\not\Rightarrow$ h_1 & h_2 is λ_N-semi-irresolute)

Let h_1: (Ω_1, τ_1) $\to$ (Ω_2, τ_2) be defined by h_1(p) = s, h_1(q) = r and h_2: (Ω_2, τ_2) $\to$ (Ω_3, τ_3) be defined by h_2(r) = u and h_2(s) = v where Ω_1 = {p, q}, Ω_2 = {r, s} and Ω_3 = {u, v}. Let τ_1 = {0_N, A, B},

τ_2 = {0_N, C, D} and τ_3 = {0_N, E, F}. Now, {0_N, A, B, G}, {0_N, C, D, H} and {0_N, E, F, I} are λ_N-SOS of (Ω_1, τ_1), (Ω_2, τ_2) and (Ω_3, τ_3) respectively, where

A = ⟨ (0.3, 0.7, 0.8), (0.2, 0.6, 0.8) ⟩, B = ⟨ (0.4, 0.6, 0.7), (0.5, 0.5, 0.6) ⟩,

C = ⟨ (0.8, 0.4, 0.2), (0.8, 0.3, 0.3) ⟩, D = ⟨ (0.6, 0.5, 0.5), (0.7, 0.4, 0.4) ⟩,

E = ⟨ (0.2, 0.6, 0.8), (0.3, 0.7, 0.8) ⟩, F = ⟨ (0.5, 0.5, 0.6), (0.4, 0.6, 0.7) ⟩,

G = ⟨ (0.3, 0.7, 0.8), (0.4, 0.5, 0.7) ⟩, H = ⟨ (0.7, 0.5, 0.4), (0.8, 0.3, 0.3) ⟩,

I = ⟨ (0.4, 0.5, 0.7), (0.3, 0.7, 0.8) ⟩.

Here, $h_2 o h_1$: $\Omega_1 \rightarrow \Omega_3$ defined by $h_2 \circ h_1$ (p) = v and $h_2 \circ h_1$(q) = u is λ_N-semi-irresolute, but h_1 and h_2 are not λ_N-semi-irresolute.

3.12. Corollary

Let (Ω_1, τ_1), (Ω_2, τ_2) and (Ω_3, τ_3) be λ_N-TSs. If h_1: $\Omega_1 \rightarrow \Omega_2$ and h_2: $\Omega_2 \rightarrow \Omega_3$ are λ_N-α-irresolute then $h_2 \circ h_1$: $\Omega_1 \rightarrow \Omega_3$ is λ_N-α-irresolute.

Proof

Let E is λ_N-αOS in (Ω_3, τ_3). Since h_2 is λ_N-α-irresolute, h_2^{-1}(E) is λ_N-αOS in (Ω_2, τ_2). Also since h_1 is λ_N-α-irresolute, h_1^{-1} (h_2^{-1}(E)) = $(h_2 \circ h_1)^{-1}$(E) is λ_N-αOS in (Ω_1, τ_1). Therefore $h_2 \circ h_1$ is λ_N-α-irresolute.

3.13. Corollary

If h_1: (Ω_1, τ_1) $\rightarrow$ (Ω_2, τ_2) is λ_N-α-irresolute (resp. λ_N-semi-irresolute, λ_N-pre-irresolute) and h_2: (Ω_2, τ_2) $\rightarrow$ (Ω_3, τ_3) is λ_N-αCts (resp. λ_N-SCts, λ_N-PCts) then $h_2 \circ h_1$: (Ω_1, τ_1) $\rightarrow$ (Ω_3, τ_3) is λ_N-αCts (resp. λ_N-SCts, λ_N-PCts).

Proof

Let E is λ_N-OS in (Ω_3, τ_3). Since h_2 is λ_N-αCts (resp. λ_N-SCts, λ_N-PCts), h_2^{-1}(E) is λ_N-αOS (resp. λ_N-SOS, λ_N-POS) in (Ω_2, τ_2). Also since h_1 is λ_N-α-irresolute (resp. λ_N-semi-irresolute, λ_N-pre-irresolute), h_1^{-1} (h_2^{-1}(E)) = $(h_2 \circ h_1)^{-1}$(E) is λ_N-αOS (resp. λ_N-SOS, λ_N-POS) in (Ω_1, τ_1). Therefore $h_2 \circ h_1$ is λ_N-αCts (resp. λ_N-SCts, λ_N-PCts).

3.14. Theorem

Let (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. If h: $\Omega_1 \rightarrow \Omega_2$ is λ_N-semi-irresolute and λ_N-pre-irresolute then h is λ_N-α-irresolute.

Proof

Let E is λ_N-αOS in (Ω_2, τ_2), then by Theorem 3.3, E is λ_N-SOS and λ_N-POS. Since h is λ_N-semi-irresolute and λ_N-pre-irresolute, h^{-1}(E) is λ_N-SOS and λ_N-POS. Therefore h^{-1}(E) is λ_N-αOS. Hence h is λ_N-α-irresolute.

3.15. Theorem

Let (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. A function h: $\Omega_1 \rightarrow \Omega_2$ is λ_N-αCts iff it is λ_N-SCts and λ_N-PCts.

Proof

It is clear from Theorem 3.3.

4. Contra λ_N-Irresolute Functions

4.1. Definition

Let (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. Then h: $\Omega_1 \rightarrow \Omega_2$ is said to be contra λ_N-α-irresolute (resp. contra λ_N-semi-irresolute, contra λ_N-pre-irresolute) if the inverse image of every λ_N-αOS (resp. λ_N-SOS, λ_N-POS) in (Ω_2, τ_2) is a λ_N-αCS (resp. λ_N-SCS, λ_N-PCS) in (Ω_1, τ_1).

4.2. Example

(i) Let h: $(\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$ be defined as h(s) = u and h(t) = v, where Ω_1 = {s, t} and Ω_2 = {u, v}, τ_1 = {0_N, A, B}, τ_2 = {0_N, C, D}.

A = ⟨ (0.2, 0.8, 0.9), (0.1, 0.7, 0.8) ⟩, B = ⟨ (0.3, 0.5, 0.6), (0.4, 0.6, 0.7) ⟩,

C = ⟨ (0.8, 0.3, 0.1), (0.9, 0.2, 0.2) ⟩, D = ⟨ (0.7, 0.4, 0.4), (0.6, 0.5, 0.3) ⟩,

G = ⟨ (0.3, 0.7, 0.8), (0.2, 0.6, 0.7) ⟩, H = ⟨ (0.7, 0.4, 0.2), (0.8, 0.3, 0.3) ⟩.

Here, {A′, B′, G′, 1_N} are λ_N-αCS of (Ω_1, τ_1) and {0_N, C, D, H} are λ_N-αOS of (Ω_2, τ_2). Consequently, h is contra λ_N-α-irresolute function.

(ii) Let h: $(\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$ be defined as h(p) = v, h(q) = w and h(r) = u, where Ω_1 = {p, q, r} and Ω_2 = {u, v, w}, τ_1 = {0_N, A, B}, τ_2 = {0_N, C, D}.

A = ⟨ (0.2, 0.6, 0.8), (0.1, 0.7, 0.9), (0.2, 0.8, 0.9) ⟩, B = ⟨ (0.3, 0.4, 0.7), (0.2, 0.5, 0.8), (0.4, 0.6, 0.7) ⟩,

C = ⟨ (0.9, 0.3, 0.1), (0.9, 0.2, 0.2), (0.8, 0.4, 0.2) ⟩, D = ⟨ (0.8, 0.5, 0.2), (0.7, 0.4, 0.4), (0.7, 0.6, 0.3) ⟩,

G = ⟨ (0.3, 0.5, 0.7), (0.2, 0.6, 0.9), (0.3, 0.7, 0.8) ⟩, H = ⟨ (0.9, 0.4, 0.2), (0.8, 0.3, 0.3), (0.7, 0.5, 0.3) ⟩.

Here, {A′, B′, G′, 1_N} are λ_N-SCS of (Ω_1, τ_1) and {0_N, C, D, H} are λ_N-SOS of (Ω_2, τ_2). Hence h is contra λ_N-semi-irresolute function.

(iii) Let h : $(\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$ be defined as h(p) = w, h(q) = u and h(r) = v, where Ω_1 = {p, q, r} and Ω_2 = {u, v, w}, τ_1 = {0_N, A, B}, τ_2 = {0_N, C, D}.

A = ⟨ (0.2, 0.7, 0.7), (0.3, 0.7, 0.8), (0.1, 0.8, 0.8) ⟩, B = ⟨ (0.3, 0.7, 0.6), (0.4, 0.6, 0.7), (0.2, 0.7, 0.8) ⟩,

C = ⟨ (0.9, 0.1, 0.1), (0.8, 0.2, 0.2), (0.8, 0.3, 0.2) ⟩, D = ⟨ (0.8, 0.3, 0.2), (0.6, 0.3, 0.3), (0.7, 0.4, 0.4) ⟩,

G = ⟨ (0.2, 0.8, 0.8), (0.2, 0.7, 0.8), (0.1, 0.9, 0.9) ⟩, H = ⟨ 0.8, 0.2, 0.1), (0.7, 0.3, 0.2), (0.8, 0.3, 0.3) ⟩.

Here, {A′, B′, G′, 1_N} are λ_N-PCS of (Ω_1, τ_1) and {0_N, C, D, H} are λ_N-POS of (Ω_2, τ_2). That's why h is contra λ_N-pre-irresolute function.

4.3. Theorem

Let (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. Then h: $\Omega_1 \rightarrow \Omega_2$ is contra λ_N-α-irresolute iff for every λ_N-αCS E of Ω_2, $h^{-1}(E)$ is λ_N-αOS in Ω_1.

Proof

If h is contra λ_N-α-irresolute, then for each λ_N-αOS F of Ω_2, $h^{-1}(F)$ is λ_N-αCS in Ω_1. If E is any λ_N-αCS of Ω_2, then $\Omega_2 - E$ is λ_N-αOS. Thus $h^{-1}(\Omega_2 - E)$ is λ_N-αCS but $h^{-1}(\Omega_2 - E) = \Omega_1 - h^{-1}(E)$ so that $h^{-1}(E)$ is λ_N-αOS in Ω_1.

Conversely, if, for all λ_N-αCS E of Ω_2, $h^{-1}(E)$ is λ_N-αOS in Ω_1 and if F is any λ_N-αOS of Ω_2, then $\Omega_2 - F$ is λ_N-αCS. Also, $h^{-1}(\Omega_2 - F) = \Omega_1 - h^{-1}(F)$ is λ_N-αOS. Thus $h^{-1}(F)$ is λ_N-αCS in Ω_1. Hence h is contra λ_N-α-irresolute.

4.4. Corollary

Let (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. Then h: $\Omega_1 \to \Omega_2$ is contra λ_N-semi-irresolute (contra λ_N-pre-irresolute) iff for every λ_N-SCS (λ_N-PCS) E of Ω_2, $h^{-1}(E)$ is λ_N-SOS (λ_N-POS) in Ω_1.

Proof

If h is contra λ_N-semi-irresolute (contra λ_N-pre-irresolute), then for each λ_N-SOS (λ_N-POS) F of Ω_2, $h^{-1}(F)$ is λ_N-SCS (λ_N-PCS) in Ω_1. If E is any λ_N-SCS (λ_N-PCS) of Ω_2, then $\Omega_2 - E$ is λ_N-SOS (λ_N-POS). Thus $h^{-1}(\Omega_2 - E)$ is λ_N-SCS (λ_N-PCS) but $h^{-1}(\Omega_2 - E) = \Omega_1 - h^{-1}(E)$ so that $h^{-1}(E)$ is λ_N-SOS (λ_N-POS) in Ω_1.

Conversely, if, for all λ_N-SCS (λ_N-PCS) E of Ω_2, $h^{-1}(E)$ is λ_N-SOS (λ_N-POS) in Ω_1 and if F is any λ_N-SOS (λ_N-POS) of Ω_2, then $\Omega_2 - F$ is λ_N-SCS (λ_N-PCS). Also, $h^{-1}(\Omega_2 - F) = \Omega_1 - h^{-1}(F)$ is λ_N-SOS (λ_N-POS). Thus $h^{-1}(F)$ is λ_N-SCS (λ_N-PCS) in Ω_1. Hence h is contra λ_N-semi-irresolute (contra λ_N-pre-irresolute).

4.5. Theorem

Let (Ω_1, τ_1), (Ω_2, τ_2) and (Ω_3, τ_3) be λ_N-TSs. If h_1: $\Omega_1 \to \Omega_2$ and h_2: $\Omega_2 \to \Omega_3$ are contra λ_N-semi-irresolute functions, then $h_2 \circ h_1$: $\Omega_1 \to \Omega_3$ is λ_N-semi-irresolute.

Proof

If $E \subseteq Z$ is λ_N-SOS, then $h_2^{-1}(E)$ is λ_N-SCS in Ω_2 because h_2 is contra λ_N-semi-irresolute. Consequently, since h_1 is contra λ_N-semi-irresolute, $h_1^{-1}(h_2^{-1}(E)) = (h_2 \circ h_1)^{-1}(E)$ is λ_N-SOS in Ω_1. Hence $h_2 o h_1$ is λ_N-semi-irresolute.

4.6. Example ($h_2 \circ h_1$ is λ_N-semi-irresolute $\not\Rightarrow$ h_1 & h_2 is contra λ_N-semi-irresolute)

Let h_1: $(\Omega_1, \tau_1) \to (\Omega_2, \tau_2)$ be defined by $h_1(l) = q$, $h_1(m) = r$, $h_1(n) = p$ and h_2: $(\Omega_2, \tau_2) \to (\Omega_3, \tau_3)$ be defined by $h_2(p) = v$, $h_2(q) = w$ and $h_2(r) = u$ where $\Omega_1 = \{l, m, n\}$, $\Omega_2 = \{p, q, r\}$ and $\Omega_3 = \{u, v, w\}$. Let $\tau_1 = \{0_N, A, B\}$, $\tau_2 = \{0_N, C, D\}$ and $\tau_3 = \{0_N, E, F\}$. Here, $\{0_N, A, B, G\}$, $\{0_N, C, D, H\}$ and $\{0_N, E, F, I\}$ are λ_N-SOS of (Ω_1, τ_1), (Ω_2, τ_2) and (Ω_3, τ_3) where

A = $\langle$ (0.2, 0.6, 0.8), (0.1, 0.7, 0.9), (0.2, 0.8, 0.9) $\rangle$, B = $\langle$ (0.3, 0.4, 0.7), (0.2, 0.5, 0.8), (0.4, 0.6, 0.7) $\rangle$,

C = $\langle$ 0.2, 0.8, 0.9), (0.2, 0.6, 0.8), (0.1, 0.7, 0.9) $\rangle$, D = $\langle$ 0.4, 0.6, 0.7), (0.3, 0.4, 0.7), (0.2, 0.5, 0.8) $\rangle$,

E = $\langle$ (0.1, 0.7, 0.9), (0.2, 0.8, 0.9), (0.2, 0.6, 0.8) $\rangle$, F = $\langle$ (0.2, 0.5, 0.8), (0.4, 0.6, 0.7), (0.3, 0.4, 0.7) $\rangle$,

G = $\langle$ (0.3, 0.5, 0.7), (0.2, 0.6, 0.9), (0.3, 0.7, 0.8) $\rangle$, H = $\langle$ (0.3, 0.7, 0.8), (0.3, 0.5, 0.7), (0.2, 0.6, 0.9) $\rangle$,

I = $\langle$ (0.2, 0.6, 0.9), (0.3, 0.7, 0.8), (0.3, 0.5, 0.7) $\rangle$.

Here, $h_2 \circ h_1$: $\Omega_1 \to \Omega_3$ which is defined by $h_2 \circ h_1(l) = w$, $h_2 \circ h_1(m) = u$ and $h_2 \circ h_1(n) = v$ is λ_N-semi-irresolute , but h_1 and h_2 are not contra λ_N-semi-irresolute.

4.7. Corollary

Let (Ω_1, τ_1), (Ω_2, τ_2) and (Ω_3, τ_3) be λ_N-TSs. If h_1: $\Omega_1 \to \Omega_2$ and h_2: $\Omega_2 \to \Omega_3$ are contra λ_N-α-irresolute (contra λ_N-pre-irresolute) functions, then $h_2 \circ h_1$: $\Omega_1 \to \Omega_3$ is a λ_N-α-irresolute (λ_N-pre-irresolute) function.

4.8. Theorem

Let (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. If h: $\Omega_1 \rightarrow \Omega_2$ is contra λ_N-α-irresolute, then it is contra λ_N-αCts.

Proof

Let E be any λ_N-OS in Ω_2. Then E is λ_N-αOS in Ω_2. Since h is contra λ_N-α-irresolute, $h^{-1}(E)$ is a λ_N-αCS in Ω_1. It shows that h is contra λ_N-αCts function.

4.9. Theorem

Let (Ω_1, τ_1), (Ω_2, τ_2) and (Ω_3, τ_3) be λ_N-TSs. If h_1: $\Omega_1 \rightarrow \Omega_2$ is contra λ_N-α-irresolute and h_2: $\Omega_2 \rightarrow \Omega_3$ is contra λ_N-αCts, then $h_2 \circ h_1$: $\Omega_1 \rightarrow \Omega_3$ is λ_N-αCts.

Proof

Let $E \subseteq \Omega_3$ is λ_N-OS. Since h_2 is contra λ_N-αCts, $h_2^{-1}(E)$ is λ_N-αCS in Ω_2. Consequently, since h_1 is contra λ_N-α-irresolute, $h_1^{-1}(h_2^{-1}(E)) = (h_2 \circ h_1)^{-1}(E)$ is λ_N-αOS in Ω_1, by Theorem 4.3. Hence $h_2 \circ h_1$ is λ_N-αCts.

4.10. Corollary

Let (Ω_1, τ_1), (Ω_2, τ_2) and (Ω_3, τ_3) be λ_N-TSs, and h_1: $\Omega_1 \rightarrow \Omega_2$ and h_2: $\Omega_2 \rightarrow \Omega_3$ be two functions. Then if h_1 is contra λ_N-semi-irresolute (contra λ_N-pre-irresolute) and h_2 is contra λ_N-SCts (contra λ_N-PCts), then $h_2 \circ h_1$: $\Omega_1 \rightarrow \Omega_3$ is λ_N-SCts (λ_N-PCts).

4.11. Theorem

Let (Ω_1, τ_1) and (Ω_2, τ_2) be λ_N-TSs. If h: $\Omega_1 \rightarrow \Omega_2$ is contra λ_N-semi-irresolute and contra λ_N-pre-irresolute, then h is contra λ_N-α-irresolute.

Proof

Let E is λ_N-αOS in (Ω_2, τ_2), then by Theorem 3.3, E is λ_N-SOS and λ_N-POS. Since h is contra λ_N-semi-irresolute and contra λ_N-pre-irresolute, $h^{-1}(E)$ is λ_N-SCS and λ_N-PCS. Therefore $h^{-1}(E)$ is λ_N-αCS. Hence h is contra λ_N-α-irresolute.

5. Conclusion

In this confab, we instigated λ_N-α-irresolute function, λ_N-semi-irresolute function and λ_N-pre-irresolute function on λ_N-TS. Subsequently, we have analyzed its various properties. Followed by this, the new postulations of contra λ_N-α-irresolute function, contra λ_N-semi-irresolute function and contra λ_N-pre-irresolute function were put forth on λ_N-TS and their features were probed along with illustrations.

λ_N-TS idea can be further developed and extended in the actual life applications such as medical field, robotics, machine learning, neural networks, natural image sensing, speech recognition, and so on.

In future, it provokes to apply these perceptions in further extensions of λ_N-TS such as almost continuity and its unique characteristics in G_N-TSs along with some separation axioms related to G_N-TSs. Also, this concept may be extended to Intuitionistic Fuzzy and Neutrosophic Fixed Point Theory.

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Received: July 25, 2022. Accepted: September 22, 2022