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Linear Diophantine Neutrosophic Sets and Their Properties

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Abstract: In 2019, Riaz et al. introduced the notion of linear Diophantine fuzzy set(LDFS) where there is an addition of reference parameters that help to address the issues that cannot be managed by the existing theories such as fuzzy sets(FSs), intuitionistic fuzzy sets(IFSs), Pythagorean fuzzy sets(PFSs), and q-rung orthopair fuzzy sets(q-ROFSs). But all these theories are not capable to describe indeterminacy that exists in numerous real-world problems. For this purpose, neutrosophic sets(NSs), single-valued neutrosophic sets(SVNSs), Pythagorean neutrosophic sets(PNSs) are introduced. In PNS, each object x in the universe is characterized by a dependent truth $\left(\mu_T(x)\right)$ and falsity $\left(\gamma_F(x)\right)$ membership values and indeterminacy $\left(\nu_I(x)\right)$ membership value with the restriction $0 \le \left(\mu_T(x)\right)^2 + \left(\gamma_F(x)\right)^2 + \left(\nu_I(x)\right)^2 \le 2$. If we consider a neutrosophic triplet as $\langle 0.9, 0.9, 0.9 \rangle$ then $0.9^2 + 0.9^2 + 0.9^2$ will give 2.43, which is > 2. Such a problem cannot be handled by the decision-makers under the Pythagorean neutrosophic environment. To take care of such an issue there is an urgency to develop another mathematical model. This lead to an introduction of linear Diophantine neutrosophic set(LDNS) as an extension of PNS. Thus, the main purpose of this paper is to introduce the LDNS model with an aid of reference parameters to ensure that through this new model the decision-makers can freely choose the neutrosophic membership values with an extended domain. Therefore, in a broad sense, the LDNSs are a new idea that removes the restrictions present in the existing concepts such as FSs, IFSs, PFSs, q-ROFSs, PNSs, LDFSs, etc. From example 3.1.1, it is quite visible that this new structure helps to classify the problem by changing the physical nature of reference parameters. Moreover, some basic properties and operations on LDNSs are investigated. We also define the score and accuracy function based on linear Diophantine neutrosophic number(LDNN). With the help of a novel linear Diophantine single-valued neutrosophic weighted arithmetic-geometric aggregation (LDSVNWAGA) operator, an algorithm has been developed for decision-making. Finally, the proposed algorithm has been successfully executed with the help of a numerical application.

Keywords: Neutrosophic set; Linear Diophantine neutrosophic set; Reference parameter; Decision-Making.

1. Introduction

Presently, in the real-world we are facing complicated problems that cannot be solved by the traditional mathematical tools. It is due to the involvement of uncertainty or vagueness in real-life situations. The crisp concept is no more valid to define ambiguity. A crisp set A can be characterized by a characteristic function χ_A and the values of χ_A corresponding to all the objects in A are either 0 or 1. Boolean algebra also useful to address the same situation. In mathematics, we find some linguistic terms such as "excellent", "beautiful", "intelligent" etc, which are subjective. To eradicate such a problem to some extent, Zadeh introduced the fuzzy set [1] in 1965 and fuzzy logic [2] in 1996. A fuzzy set is a significant mathematical tool to model vagueness or uncertainty in the data or information, that has been attracted the attention of many researchers across the globe in the last decades. A fuzzy set X be characterized by its membership function $\mu : X \rightarrow [0,1]$, which assigns a real value in the unit closed interval $[0,1]$ to each object of the universe. Thus, a fuzzy set is an extension of a crisp set whose boundary is blurred. The researchers have been studied fuzzy sets as problem-solving techniques in various fields including, engineering, computer science, medical science, social science, economics, environments, robotics, etc., having various uncertainties. Some significant works associated with fuzzy sets are studied in [3-7]. Later on, in 2010, Bustince [8] introduced an interval-valued fuzzy set (IVFS), where the membership function defined as $\mu : X \rightarrow \text{int}([0,1])$, $\text{int}([0,1])$ denotes the collection of all subsets of $[0,1]$. To define the incomplete information, Atanassov [9] introduced intuitionistic fuzzy set(IFS) as a direct extension of the fuzzy set by using the notion of membership degree (μ) and the non-membership degree (γ), where both the membership values belong to the interval $[0,1]$ with a restriction that their sum cannot exceed the unity and the hesitancy degree is calculated as $\pi = 1 - \mu - \gamma$. Bustince[10] defined vague sets are intuitionistic fuzzy sets, in [11], Garg et al. presented an improved possibility degree method to find the rank of intuitionistic fuzzy numbers(IFNs), Gou et al.[12] defined exponential operations for IFNs, Heilpern[13] proposed an application of fuzzy numbers(FNs), Nayagam et al.[14] defined ranking of IFNs, Szmidt et al.[15] gives an application of IFS, Wang et al.[16] proposed IFS and L-FS, Zeng et al.[17] presented multiattribute decision-making based on novel score function of intuitionistic fuzzy values and modified VIKOR method. If a decision-maker assigns an ordered pair $(0.65.0.55)$ to an alternative, then it is not an IFN, as $0.65+0.55 > 1$. To tackle such a case, Yager [18] introduced a Pythagorean fuzzy set(PFS) where the sum of the squares of Pythagorean fuzzy membership grades should not exceed unity. So, we have an enlarged space for PFSs as compared to IFSs. In

[19], Wan et al. introduced Pythagorean fuzzy number(PFN). PFSs have been further extended by introducing q-ROFSs[20-24]. Some novel works associated with PFSs and PFNs are proposed in [25-36]. In 2019, Jansi et al.[37] introduced correlation measure for Pythagorean neutrosophic sets where truthfulness and falseness are dependent components. Ajay et al.[38] introduced the Pythagorean neutrosophic fuzzy graphs.

In some real-life problems, the sum of the membership grade and non-membership grade to which an alternative satisfying an attribute provided by the decision-maker (DM) may be larger than 1 (e.g 0.8+0.7>1) and their sum of the squares is also larger than 1 (e.g $0.8^2+0.7^2>1$). Thus, IFS and PFS fail to hold in such situations. To overcome these deficiencies, the restrictions on membership and non-membership grades are altered to $0 \le \mu^q + \gamma^q \le 1$ in the case of q-rung orthopair fuzzy set(q-ROFS). Even for very large values of "q", we can deal with membership and non-membership grades independently to some extent. In some practical problems, when $\mu = \gamma = 1$, we obtain $1^q + 1^q \ge 1$, which contradicts the constraint of q-ROFS. It makes the MADM limited and affects the optimum decision. Linear Diophantine fuzzy set (LDFS) [39] can deal with such situations to some extent. LDFS provides a large number of applications to the MADM for such real-world problems. So, through the model of LDFS, we can deal with the intuitionistic, Pythagorean, and q-rung orthopair nature of attributes under the effect of reference parameters (α, β). For example, let (0.7+0.6>1), we can introduce reference parameters (α, β) such that $(\alpha)(0.7)+(\beta)(0.6)<1$, where (α, β) denotes the reference parameters concerning for to membership and non-membership grade respectively. Some recent works related to LDFS are given in [40-42].

The term neutrosophy denotes the study of neutralities and it is proposed by Smarandache[43]. Neutrosophy can be treated as a branch of philosophy. If we consider $\langle A \rangle$ be an idea or proposition or an axiom or theorem then its opposite notion is denoted by $\langle \text{anti}A \rangle$ and for completeness property we consider another concept known as $\langle \text{nor } A \rangle$. But, some concepts are there which lie in between $\langle A \rangle$ and $\langle \text{anti}A \rangle$, they are denoted by $\langle \text{neut } A \rangle$.

So, realizing the importance of the study of neutrality, Smarandache[44] introduced a neutrosophic set(NS), as an extension of IFS. For technical use, Wang et al. [45] introduced a single-valued neutrosophic set(SVNS). Some recent research works associated with NSs are in the following: data development analysis for simplified NS is studied in [46]. Another data envelopment analysis under a triangular neutrosophic number environment has been done in [47]. In [48], Edalatpanah introduced the neutrosophic structured element. A triangular neutrosophic linear programming model is presented in [49]. Martin et al. [50] introduced the COVID-19 diagnostic model by using a new pilthogenic cognitive maps approach. Debnath[51] presented the neutrosophic statistical data to assess the knowledge, attitude, and symptoms of reproductive tract infection(RTI) among women in selected villages in India.

By using IFS, PFS, and q-ROFS we only define the incomplete information present in the data. But, in real life some information is there which is partially true and partially false i.e., they are indeterminate or inconsistent. To overcome such problems, the concept of the neutrosophic theory is very helpful. For the sake of computation, throughout the paper, we use SVNS instead of NS.

The main motivation behind presenting this paper is to extend the notion of LDFS to LDNS. Some MADM problems exist in real life which involves indeterminate attributes. To handle such problems we need a powerful tool to tackle. This leads to the introduction of LDNSs. Also, we have investigated some operations and properties based on LDNSs. Further, we have introduced an algorithm that can be applied successfully in solving real MADM problems with the help of a suitable example.

1.1 Novelty

There exists some real-world-based complex phenomenon that cannot be solved by using the existing fuzzy theories and their extensions. Such phenomenon can be tackled with addition of reference parameters that build a bridge between the existing theories and the physical world. For this purpose, we have introduced a novel concept known as linear Diophantine neutrosophic set(LDNS) to apply it in different MADM problems by categorizing the data using reference parameters. Therefore, the LDNS model surely provides a powerful mathematical tool for the further development of the neutrosophic theory. The objectives of the proposed study are discussed in the following manner:

- The PNS [37, 38] is developed to generalize the PFS[18] and the SVNS with dependent neutrosophic components. But, in some real-life situation, the sum of squares of a membership grade, non-membership grade, and indeterminacy grade to an attribute provided by a decision-maker may be > 2. Such problems cannot be described by FS, IFS, PFS, SVNS, PNS,q-ROFS, LDFS. To remove such inadequacy, the LDNS is introduced to deal with a large number of MADM problems by enlarging the domain with an aid of reference parameters.

  For better understanding, suppose the neutrosophic triplet of an attribute provided by the decision-maker is $\langle 0.8, 0.9, 0.9 \rangle$. The sum of their squares gives $2.26 > 2$. Corresponding to the neutrosophic triplet if we assign the grades of the reference parameters triplet as $\langle 0.5, 0.6, 0.7 \rangle$. Then, $0.8\times 0.5 + 0.9\times 0.6 + 0.9\times 0.7$ =1.57 < 2. It looks similar to the linear Diophantine equation $ax + by + cz = d$ which is a popular topic in number theory. So, the name of the proposed model is logical in this sense. Thus, by introducing LDNS, we fill the research gap.

- FS, IFS, NS, SVNS, PFS, q-ROFS, PNS cannot deal with parameters. So, by introducing reference parameters in LDNS, there is a huge scope for a decision-maker to address various types of MADM problems by changing the physical nature of the reference parameters.

- Define the linear Diophantine neutrosophic numbers (LDNNs) and study their properties.
- Define a new aggregate operator called LDSVNWAGA operator that helps to obtain the rank of the alternatives.
- Construction of a new algorithm for solving MADM problems by using the new aggregate operator.
- Justify the algorithm with the help of a numerical application based on real life.

1.2 Structure of the paper

The manuscript is organized in the following manner: Section 2 includes the basic definitions of FS, IFS, PFS, q-ROFS, PNS, LDFS which are useful to build the proposed study. Section 3 contains the definition of LDNNs and their properties. Section 4 contains the definitions of score function, accuracy function, and aggregate operator based on LDNNs. In Section 5, an algorithm is constructed for MADM problems. In Section 6, a numerical example is presented to justify the proposed algorithm. Section 7 contains a comparative study between the proposed and the existing theories. Conclusion and the future scope have been studied in Section 8.

2. Preliminaries

In this section, we review some basic definitions with examples that are very useful for the subsequent sections of this paper.

Definition 2.1 [1, 2, 6] Let X be an initial universe and $\mu_A : X \to [0,1]$ be the membership function. Then a fuzzy set A is defined by

$$A = \{(x, \mu_A(x)) : x \in X\}$$

$$= \sum \mu_A(x)/x \text{, when X is discrete}$$

$$= \int \mu_A(x)/x \text{, when X is continuous}$$

Here $\mu_A(x)$ denotes the degree of membership of x to the fuzzy set A. The value of the membership function $\mu_A(x)$ can be chosen by different experts may be different depending upon their experiences, perceptions, perspectives, etc. The collection of all fuzzy sets in X is denoted by I^X.

Example 2.1.1 Let $X = \{x_1, x_2, x_3, x_4, x_5\}$ be a collection of beautiful students, and then the fuzzy set A associated with X is defined by a decision-maker (DM) as

$$A = \{(x_1, 0.5), (x_2, 0.6), (x_3, 1.0), (x_4, 0.0), (x_5, 0.3)\}$$

If all the membership values in A are either 0 or 1, then A reduces to a crisp set. So, a crisp set is a particular class of a fuzzy set.

Definition 2.2 [9] An intuitionistic fuzzy set(IFS) A over the universe X is defined as

$A=\{(x,\mu_A(x),\gamma_A(x)):x\in X\}$ such that $0\le\mu_A(x)+\gamma_A(x)\le 1$, $\forall x\in X$ where $\mu_A:X\to[0,1]$ and $\gamma_A:X\to[0,1]$ denote the membership function and the non-membership function, respectively. However, the hesitancy degree is given by $\pi_A(x)=1-\mu_A(x)-\gamma_A(x), \forall x\in X$.

Definition 2.3 [18, 26, 36] A Pythagorean fuzzy set(PFS) P over the universe X is defined by $P=\{(x,\mu_P(x),\gamma_P(x)):x\in X\}$ where $\mu_P,\gamma_P:X\to[0,1]$ with the restriction $0\le(\mu_P(x))^2+(\gamma_P(x))^2\le 1.$

Hence PFSs have a wide range of space of application as compared to IFSs.

The degree of hesitancy may be computed as $I_P(x)=\sqrt{1-(\mu_P(x))^2-(\gamma_P(x))^2}$

Definition 2.4 [20, 21] Let $\omega=\{\xi_1,\xi_2,....,\xi_n\}$ be a finite universal set, then a q-ROFS, Q in ω can be defined as follows:

$Q=\{\langle\xi,\mu_Q(\xi),\gamma_Q(\xi)\rangle:\xi\in\omega\}$ where $\mu_Q,\gamma_Q:\omega\to[0,1]$ with the condition $0\le(\mu_Q(\xi))^q+(\gamma_Q(\xi))^q\le 1,\ q\ge 1,\ \forall\xi\in\omega$.

The value $\pi_Q(\xi)=\sqrt{1-(\mu_Q(\xi))^q-(\gamma_Q(\xi))^q}$ is called the degree of indeterminacy of Q in ω.

Also, $0\le\pi_Q(\xi)\le 1, \forall\xi\in\omega$.

Definition 2.5 [39] Let Q be the non-empty reference set. An LDFS £D on Q is an object of the form:

£D= $\{(\varsigma,\langle\mu_D(\varsigma),\gamma_D(\varsigma)\rangle,\langle\alpha,\beta\rangle):\varsigma\in Q\}$ where, $\mu_D(\varsigma),\gamma_D(\varsigma),\alpha,\beta\in[0,1]$ are membership, non-membership and reference parameters with the following conditions:

$0\le\alpha\mu_D(\varsigma)+\beta\gamma_D(\varsigma)\le 1, \forall\varsigma\in Q$ and $0\le\alpha+\beta\le 1$. These reference parameters can help in defining or classifying a particular system. The hesitation part can be evaluated as:

$\xi\pi_D=1-\alpha\mu_D(\varsigma)-\beta\gamma_D(\varsigma)$ where ξ is the reference parameters related to the degree of hesitancy.

Definition 2.6 [37, 38] Let X be a non-empty universal set. A Pythagorean neutrosophic set with T and F are dependent neutrosophic components A over X is an object of the form $\{\langle x,T_A(x),I_A(x),F_A(x)\rangle:x\in X\}$, where $T_A(x),I_A(x),F_A(x)\in[0,1]$, $0\le T_A^2(x)+I_A^2(x)+F_A^2(x)\le 2$, for all $x\in X$. Here,

$T_A(x), I_A(x)$ and $F_A(x)$ respectively denote the degree of truth membership, degree of indeterminacy membership, and the degree of falsity membership.

3. Linear Diophantine Neutrosophic Set(LDNS)

Definition 3.1

Let Q be the non-empty reference set. A LDNS $£_{ND}$ on $\mathbb{Q}$ is an object of the form:

$£_{ND}= \left\{\left(\varsigma,\left\langle \mu_{ND}(\varsigma),\nu_{ND}(\varsigma),\gamma_{ND}(\varsigma)\right\rangle,\left\langle \alpha,\delta,\beta\right\rangle\right):\varsigma\in Q\right\}$ where, $\mu_{ND}(\varsigma),\nu_{ND}(\varsigma),\gamma_{ND}(\varsigma)$, $\alpha,\delta,\beta \in[0,1]$ are truth-membership, indeterminacy-membership, falsity-membership, and their reference parameters respectively with the following conditions:

$0\le\alpha\mu_{ND}(\varsigma)+\delta\nu_{ND}(\varsigma)+\beta\gamma_{ND}(\varsigma)\le 2$, $\forall\varsigma\in Q$ and $0\le\alpha+\delta+\beta\le 2$. These reference parameters can help in defining or classifying a particular system. The hesitation part can be evaluated as:

$\xi\pi_{ND}=2-\left(\alpha\mu_{ND}(\varsigma)+\delta\nu_{ND}(\varsigma)+\beta\gamma_{ND}(\varsigma)\right)$ where ξ is the reference parameter related to the degree of indeterminacy. Simply $\Lambda=\left(\left\langle \mu_{ND},\nu_{ND},\gamma_{ND}\right\rangle,\left\langle \alpha,\delta,\beta\right\rangle\right)$ is called linear Diophantine neutrosophic number(LDNN) with $0\le\alpha\mu_{ND}+\delta\nu_{ND}+\beta\gamma_{ND}\le 2$ and $0\le\alpha+\delta+\beta\le 2$.

Since the proposed model looks similar to the well-known linear Diophantine equation $ax+by+cz=d$ in the number theory, so LDNS is the most suitable name for the proposed model. The proposed model enhances the existing methodologies and the decision-maker (DM) can choose the grades with more liberty as compared to the other existing theories. This structure also categorizes the problem by changing the physical sense of reference alternatives in MADM.

Example 3.1.1 Chemical bonding can be described as a force that binds two or more atoms together to form molecules or ionic compounds. Chemical bonds form because the overall energy of the bonded atoms is less than the atoms have separately. Atoms form bonds to attain a noble gas configuration. There are two main types of bonds such as ionic bonds and covalent bonds. Covalent bonds are dived into polar and non-polar covalent bonds. Some atoms have high electro negativity (e.g. fluorine), some have low electro negativity (e.g. cesium) and some are neutral (e.g. carbon) in nature.

Let Q= $\{\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6\}$ be a collection of atoms having different electro negativity and by combining two or more of them, molecule is formed. If we consider the reference or control parameters as:

α =polar covalent bond, δ =ionic bond and β =non-polar covalent bond

Then its LDNS is given in Table 1

Alternatives	LDNSs
η_1	$(\langle 0.871, 0.563, 0.643\rangle, \langle 0.321, 0.564, 0.456\rangle)$
η_2	$(\langle 0.862, 0.573, 0.776\rangle, \langle 0.354, 0.567, 0.786\rangle)$
η_3	$(\langle 0.578, 0.654, 0.456\rangle, \langle 0.567, 0.865, 0.546\rangle)$
η_4	$(\langle 0.525, 0.943, 0.654\rangle, \langle 0.324, 0.456, 0.567\rangle)$
η_5	$(\langle 0.675, 0.765, 0.845\rangle, \langle 0.865, 0.467, 0.656\rangle)$
η_6	$(\langle 0.456, 0.678, 0.897\rangle, \langle 0.564, 0.867, 0.567\rangle)$

Table 1. LDNS for Molecule

Definition 3.2

A LDNS on Q of the form ${}^1£_{ND}$ = $\left\{(\varsigma, \langle 1,0,0\rangle, \langle 1,0,0\rangle) : \varsigma \in Q\right\}$ is called absolute LDNS and

${}^0£_{ND}$=$\left\{(\varsigma, \langle 0,1,1\rangle, \langle 0,1,1\rangle) : \varsigma \in Q\right\}$ is called empty or void LDNS.

Now, we define some operations on LDNNs associated with LDNSs

Definition 3.3

Let $\Lambda_Q = \left(\left\langle {}^Q\mu_{ND}, {}^Q\nu_{ND}, {}^Q\gamma_{ND}\right\rangle, \left\langle {}^Q\alpha, {}^Q\delta, {}^Q\beta\right\rangle\right)$, where $Q \in \omega$ be an assembling of LDNNs , then

(i) $\Lambda^c{}_Q = \left(\left\langle {}^Q\gamma_{ND}, {}^Q\nu_{ND}, {}^Q\mu_{ND}\right\rangle, \left\langle {}^Q\beta, {}^Q\delta, {}^Q\alpha\right\rangle\right)$

(ii) $\Lambda_1 = \Lambda_2 \Leftrightarrow {}^1\mu_{ND} = {}^2\mu_{ND}, {}^1\nu_{ND} = {}^2\nu_{ND}, {}^1\gamma_{ND} = {}^2\gamma_{ND}, {}^1\alpha = {}^2\alpha, {}^1\delta = {}^2\delta, {}^1\beta = {}^2\beta$

(iii) $\Lambda_1 \subseteq \Lambda_2 \Leftrightarrow {}^1\mu_{ND} \le {}^2\mu_{ND}, {}^1\nu_{ND} \ge {}^2\nu_{ND}, {}^1\gamma_{ND} \ge {}^2\gamma_{ND}, {}^1\alpha \le {}^2\alpha, {}^1\delta \ge {}^2\delta, {}^1\beta \ge {}^2\beta$

(iv) $$\bigcup_{Q\in\omega} \Lambda_Q = \left(\left\langle \sup_{Q\in\omega} {}^Q\mu_{ND}, \inf_{Q\in\omega} {}^Q\nu_{ND}, \inf_{Q\in\omega} {}^Q\gamma_{ND}\right\rangle, \left\langle \sup_{Q\in\omega} {}^Q\alpha_{ND}, \inf_{Q\in\omega} {}^Q\delta_{ND}, \inf_{Q\in\omega} {}^Q\beta_{ND}\right\rangle\right)$$

(V) $$\bigcap_{Q\in\omega} \Lambda_Q = \left(\left\langle \inf_{Q\in\omega} {}^Q\mu_{ND}, \sup_{Q\in\omega} {}^Q\nu_{ND}, \sup_{Q\in\omega} {}^Q\gamma_{ND}\right\rangle, \left\langle \inf_{Q\in\omega} {}^Q\alpha_{ND}, \sup_{Q\in\omega} {}^Q\delta_{ND}, \sup_{Q\in\omega} {}^Q\beta_{ND}\right\rangle\right)$$

(vi)

$$\Lambda_1 \oplus \Lambda_2 = \left(\left\langle {}^1\mu_{ND} + {}^2\mu_{ND} - {}^1\mu_{ND}{}^2\mu_{ND}, {}^1\nu_{ND}{}^2\nu_{ND}, {}^1\gamma_{ND}{}^2\gamma_{ND} \right\rangle, \left\langle {}^1\alpha + {}^2\alpha - {}^1\alpha{}^2\alpha, {}^1\delta^2\delta, {}^1\beta^2\beta \right\rangle \right)$$

(vii) $$\Lambda_1 \otimes \Lambda_2 = \begin{pmatrix} \left\langle {}^1\mu_{ND}{}^2\mu_{ND}, {}^1\nu_{ND} + {}^2\nu_{ND} - {}^1\nu_{ND}{}^2\nu_{ND}, {}^1\gamma_{ND} + {}^2\gamma_{ND} - {}^1\gamma_{ND}{}^2\gamma_{ND} \right\rangle, \\ \left\langle {}^1\alpha^2\alpha, {}^1\delta + {}^2\delta - {}^1\delta^2\delta, {}^1\beta + {}^2\beta - {}^1\beta^2\beta \right\rangle \end{pmatrix}$$

(viii) $$\lambda\Lambda_1 = \left(\left\langle 1 - \left(1 - {}^1\mu_{ND}\right)^\lambda, {}^1\nu_{ND}^\lambda, {}^1\gamma_{ND}^\lambda \right\rangle, \left\langle 1 - \left(1 - {}^1\alpha_{ND}\right)^\lambda, {}^1\delta_{ND}^\lambda, {}^1\beta_{ND}^\lambda \right\rangle \right), \lambda > 0$$

(ix)

$$\Lambda^\lambda{}_1 = \left(\left\langle {}^1\mu_{ND}^\lambda, 1 - \left(1 - {}^1\nu_{ND}\right)^\lambda, 1 - \left(1 - {}^1\gamma_{ND}\right)^\lambda \right\rangle, \left\langle {}^1\alpha_{ND}^\lambda, 1 - \left(1 - {}^1\delta_{ND}\right)^\lambda, 1 - \left(1 - {}^1\beta_{ND}\right)^\lambda \right\rangle \right), \lambda > 0$$

It is to be noted that LDNNs don't obey De Morgan's laws. It is one of the drawbacks of using LDNNs.

Example 3.3.1 Let

$\Lambda_1 = \left(\langle 0.55, 0.65, 0.84 \rangle, \langle 0.56, 0.64, 0.46 \rangle \right)$ and $\Lambda_2 = \left(\langle 0.65, 0.45, 0.54 \rangle, \langle 0.66, 0.34, 0.36 \rangle \right)$

be two LDNNs. Then, we obtain the following results:

$(\Lambda_1)^c = \left(\langle 0.84, 0.65, 0.55 \rangle, \langle 0.46, 0.64, 0.56 \rangle \right)$ and $(\Lambda_2)^c = \left(\langle 0.54, 0.45, 0.65 \rangle, \langle 0.36, 0.34, 0.66 \rangle \right)$

Here, $\Lambda_1 \subseteq \Lambda_2$ (By definition 3.3)

Now,

$\Lambda_1 \bigcup \Lambda_2 = \left(\langle 0.65, 0.45, 0.54 \rangle, \langle 0.66, 0.34, 0.36 \rangle \right) = \Lambda_2$ and $\Lambda_1 \bigcap \Lambda_2 = \Lambda_1$

$$\Lambda_1 \oplus \Lambda_2 = \left(\langle 0.8425, 0.2925, 0.4536 \rangle, \langle 0.8504, 0.2176, 0.1656 \rangle \right)$$

$$\Lambda_1 \otimes \Lambda_2 = \left(\langle 0.3575, 0.8075, 0.9264 \rangle, \langle 0.3696, 0.7624, 0.6544 \rangle \right)$$

For λ =0.4, $\lambda\Lambda_1 = \left(\langle 0.273, 0.841, 0.932 \rangle, \langle 0.279, 0.836, 0.732 \rangle \right)$

For λ =0.2, $\Lambda^\lambda{}_1 = \left(\langle 0.887, 0.189, 0.315 \rangle, \langle 0.89, 0.184, 0.115 \rangle \right)$

Proposition 3.4 Let Λ_1, Λ_2 and Λ_3 be three LDNNs then we have the following results:

(i) If $\Lambda_1 \subseteq \Lambda_2$ and $\Lambda_2 \subseteq \Lambda_3 \Rightarrow \Lambda_1 \subseteq \Lambda_3$ (Transitivity)

(ii) $\Lambda_1 \bigcup \Lambda_2 = \Lambda_2 \bigcup \Lambda_1$ and $\Lambda_1 \bigcap \Lambda_2 = \Lambda_2 \bigcap \Lambda_1$ (commutativity)

(iii) $\Lambda_1 \cup (\Lambda_2 \cup \Lambda_3) = (\Lambda_1 \cup \Lambda_2) \cup \Lambda_3$ and $\Lambda_1 \cap (\Lambda_2 \cap \Lambda_3) = (\Lambda_1 \cap \Lambda_2) \cap \Lambda_3$ (Associativity)

(iv) $\Lambda_1 \cap (\Lambda_2 \cup \Lambda_3) = (\Lambda_1 \cap \Lambda_2) \cup (\Lambda_1 \cap \Lambda_3)$ and $\Lambda_1 \cup (\Lambda_2 \cap \Lambda_3) = (\Lambda_1 \cup \Lambda_2) \cap (\Lambda_1 \cup \Lambda_3)$

(Distributivity)

Proof. All proofs are straightforward.

4. Linear Diophantine single-valued neutrosophic weighted arithmetic and geometric aggregation(LDSVNWAGA) operator

In this section, we describe the score and accuracy function for the comparative analysis in MADM of LDNNs. The notion of score and accuracy function of neutrosophic numbers proposed by Smarandache in [52]. However, hybrid arithmetic and geometric aggregation operators of single-valued neutrosophic numbers are proposed in [53].

Definition 4.1

Let $\Lambda_Q = \left(\langle {}^Q\mu_{ND}, {}^Q\nu_{ND}, {}^Q\gamma_{ND} \rangle, \langle {}^Q\alpha, {}^Q\delta, {}^Q\beta \rangle \right)$ be a LDNN, then the score function(SF) on Λ_Q can be defined by the mapping $\varphi : LDNN(Q) \to [0,1]$ and given by

$$\varphi(\Lambda_Q) = \varphi_{\Lambda_Q} = \frac{1}{3}\left[\left(2 + {}^Q\mu_{ND} - {}^Q\nu_{ND} - {}^Q\gamma_{ND}\right) + \left(2 + {}^Q\alpha - {}^Q\delta - {}^Q\beta\right) \right]$$

where $LDNN(Q)$ is an assembling of $LDNNs$ on Q.

Definition 4.2

Let $\Lambda_Q = \left(\langle {}^Q\mu_{ND}, {}^Q\nu_{ND}, {}^Q\gamma_{ND} \rangle, \langle {}^Q\alpha, {}^Q\delta, {}^Q\beta \rangle \right)$ be a LDNN, then the accuracy function(AF) on Λ_Q can be defined by the mapping $\psi : LDNN(Q) \to [-1,1]$ and given by

$$\psi(\Lambda_Q) = \psi_{\Lambda_Q} = \frac{1}{3}\left[\left({}^Q\mu_{ND} - {}^Q\gamma_{ND}\right) + \left({}^Q\alpha - {}^Q\beta\right) \right]$$

Definition 4.3

Let Λ_{Q_1} and Λ_{Q_2} be two LDNNs, then on the context of SF and AF we can compare the two LDNNs as follows:

(i) If $\varphi_{\Lambda_{Q_1}} < \varphi_{\Lambda_{Q_2}}$, then $\Lambda_{Q_1} < \Lambda_{Q_2}$

(ii) If $\varphi_{\Lambda_{Q_1}} > \varphi_{\Lambda_{Q_2}}$, then $\Lambda_{Q_1} > \Lambda_{Q_2}$

(iii) If $\varphi_{\Lambda_{Q_1}} = \varphi_{\Lambda_{Q_2}}$ then,

(a) If $\psi_{\Lambda_{Q_1}} < \psi_{\Lambda_{Q_2}}$ then $\Lambda_{Q_1} < \Lambda_{Q_2}$

(b) If $\psi_{\Lambda_{Q_1}} > \psi_{\Lambda_{Q_2}}$ then $\Lambda_{Q_1} > \Lambda_{Q_2}$

(c) If $\psi_{\Lambda_{Q_1}} = \psi_{\Lambda_{Q_2}}$ then $\Lambda_{Q_1} \approx \Lambda_{Q_2}$

Definition 4.4

Let $\Lambda_{ND_\tau} = \left\{ \left(\left\langle {}^{\tau}\mu_{ND}, {}^{\tau}\nu_{ND}, {}^{\tau}\gamma_{ND} \right\rangle, \left\langle {}^{\tau}\alpha, {}^{\tau}\delta, {}^{\tau}\beta \right\rangle \right) : \tau = 1, 2, ..., n \right\}$ be an assembling of LDNNs on the reference set ¥ and $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weight vector with $\sum_{\tau=1}^{n} \omega_\tau = 1$, then the linear Diophantine single-valued neutrosophic weighted Arithmetic geometric aggregation $(LDSVNWAGA)$ operator defined as

$$LDSVNWAGA\left(\Lambda_{ND_1}, \Lambda_{ND_2}, ..., \Lambda_{ND_n}\right) =$$

$$\left\langle \Sigma\left(\left(1 - \prod_{j=1}^{n}\left(1 - {}^{j}\mu_{ND}\right)^{\omega_j}\right)^{j\alpha} \left(\coprod_{j=1}^{n} {}^{j}\mu^{\omega_j}{}_{ND} \right)^{(1-j\alpha)} \right), \Sigma\left(1 - \left(1 - \prod_{j=1}^{n}\left({}^{j}\nu_{ND}\right)^{\omega_j}\right)^{j_\delta} \left(\prod_{j=1}^{n}\left(1 - {}^{j}\nu_{ND}\right)^{\omega_j} \right)^{(1-j_\delta)} \right), \Sigma\left(1 - \left(1 - \prod_{j=1}^{n}\left({}^{j}\gamma_{ND}\right)^{\omega_j}\right)^{j_\beta} \left(\prod_{j=1}^{n}\left(1 - {}^{j}\gamma_{ND}\right)^{\omega_j} \right)^{(1-j_\beta)} \right) \right\rangle$$

5. An algorithmic approach

For mathematical modeling, we construct an algorithm that is based on LDNNs. The steps of the algorithm are given in the following:

Algorithm:

Step1: Input the opinion of the expert's $\wp_l \ (l = 1, 2, ..., n)$ in the form of LDNNs for each attribute.

Step2: Input the weight vector of the experts.

Step3: Calculate the aggregate value of each attribute by using *LDSVNWAGA* operator proposed in definition 4.4

Step4: Find the total weight of the aggregate value of each alternative.

Step5: Rank the weight in ascending order and choose the attribute having the highest weight.

If more than one attributes having the same weight then we repeat all the previous steps by reassessing the expert's opinion.

6. Numerical Example

In this section, we cite an example of the real world that helps to realize the importance of LDNNs in real decision-making problems. We consider the following example:

Suppose that Mr. Advik, together want to invest their money in any one of investment plans belong to the set given by

$$\zeta = \{\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5\}$$

Where

ζ_1 =Monthly Income Plan(MIP)

ζ_2 =Mutual Fund(MF)

ζ_3 =Public Provident Fund(PPF)

ζ_4 =Life Insurance Plan(LIP)

ζ_5 =Unit Linked Insurance Plan(ULIP)

According to the performance of the above investment plans, there associated three risk factors, they are denoted by the set of three reference parameters given by

$\xi = \{\alpha, \beta, \gamma\}$, where α =low-risk investment, β =medium-risk investment, and γ =high-risk investment.

To choose the best investment scheme influenced by three risk factors, Mr. Advik takes the advice of three experts(decision-makers) denoted by the set

$\omega = \{\omega_1, \omega_2, \omega_3\}$.

The set of LDNNs of the set of attributes of the three experts are shown below in the form of the following tables:

Alternatives	LDNSs
ζ_1	$(\langle 0.8, 0.9, 0.7\rangle, \langle 0.7, 0.8, 0.6\rangle)$
ζ_2	$(\langle 0.5, 0.6, 0.8\rangle, \langle 0.9, 0.7, 0.8\rangle)$
ζ_3	$(\langle 0.7, 0.6, 0.9\rangle, \langle 0.5, 0.8, 0.6\rangle)$
ζ_4	$(\langle 0.7, 0.6, 0.4\rangle, \langle 0.3, 0.9, 0.6\rangle)$
ζ_5	$(\langle 0.9, 0.8, 0.6\rangle, \langle 0.8, 0.6, 0.7\rangle)$

Table2. LDNS for ω_1

Alternatives	LDNSs
ζ_1	$(\langle 0.6, 0.8, 0.8\rangle, \langle 0.9, 0.5, 0.8\rangle)$
ζ_2	$(\langle 0.4, 0.3, 0.7\rangle, \langle 0.8, 0.9, 0.6\rangle)$

ζ_3	$(\langle 0.9, 0.7, 0.8\rangle, \langle 0.7, 0.9, 0.7\rangle)$
ζ_4	$(\langle 0.7, 0.8, 0.7\rangle, \langle 0.8, 0.6, 0.8\rangle)$
ζ_5	$(\langle 0.9, 0.7, 0.4\rangle, \langle 0.5, 0.8, 0.6\rangle)$

Table3. LDNS for ω_2

Alternatives	LDNSs
ζ_1	$(\langle 0.7, 0.6, 0.7\rangle, \langle 0.7, 0.5, 0.8\rangle)$
ζ_2	$(\langle 0.8, 0.7, 0.6\rangle, \langle 0.7, 0.6, 0.5\rangle)$
ζ_3	$(\langle 0.8, 0.7, 0.9\rangle, \langle 0.6, 0.7, 0.8\rangle)$
ζ_4	$(\langle 0.8, 0.7, 0.6\rangle, \langle 0.6, 0.8, 0.7\rangle)$
ζ_5	$(\langle 0.8, 0.6, 0.6\rangle, \langle 0.7, 0.5, 0.8\rangle)$

Table4. LDNS for ω_3

According to the experience of the experts, we consider the weight vector as

$\omega = (0.3,\ 0.4,\ 0.3)$

Now, the aggregate value of each alternative, by using *LDSVNWAGA* operator is given by:

$$LDSVNWAGA(\zeta_1) = (2.0938, 0.9499, 1.3212)$$

$$LDSVNWAGA(\zeta_2) = (1.7335, 0.9318, 1.031)$$

$$LDSVNWAGA(\zeta_3) = (2.4582, 1.3853, 1.2407)$$

$$LDSVNWAGA(\zeta_4) = (2.0399, 1.3804, 1.0146)$$

$$LDSVNWAGA(\zeta_5) = (2.6224, 1.031, 0.9207)$$

Next, we calculate the total weight of the aggregate values of all the alternatives given by,

$\varpi(\zeta_1)$=4.3649

$\varpi(\zeta_2)$=3.6963

$\varpi(\zeta_3)$=5.0842

$\varpi(\zeta_4)$=4.4349

$\varpi(\zeta_5)$=4.5741

The rank of the total weight in ascending order is given by

$$\varpi(\zeta_2) \prec \varpi(\zeta_1) \prec \varpi(\zeta_4) \prec \varpi(\zeta_5) \prec \varpi(\zeta_3)$$

From the ascending order of the rank, we observe that ζ_3 has the highest value. Thus, we conclude that Mr. Advik will select Public Provident Fund to invest his money and earn the maximum return in the future.

Thus, by using the reference parameters in LDNS, we can handle another particular class of neutrosophic data.

7. Comparison Analysis of LDNS model with the existing models in the literature

Types of set	Uncertainty	Falsity	Indeterminacy	Hesitancy	Parametrization
FS[1]	✓	×	×	×	×
IFS[9]	✓	✓	×	✓	×
PFS[18]	✓	✓	×	✓	×
q-ROFS[20]	✓	✓	×	✓	×
SVNS[45]	✓	✓	✓	×	×
PNS[37,38]	✓	✓	✓	✓	×
LDFS[39]	✓	✓	×	✓	✓
LDNS(Proposed)	✓	✓	✓	✓	✓

Table5. Comparison analysis of LDNS model with the existing models in the literature

8. Conclusion and Future Scope

In this work, we have introduced the notion of LDNS which can be viewed as an extension of FS, IFS, PFS, q-ROFS, PNS, etc. LDNS is a new structure that deals with uncertainty and indeterminacy with the support of reference parameters. The LDNS model can transform the problem related to the physical world into numerical form due to its parametric nature. Therefore, it provides more flexibility to handle uncertainty as compared to the existing theories. We have discussed some properties of LDNSs. For comparison of LDNNs, we have defined score and accuracy functions. Moreover, we have introduced *LDSVNWAGA* operator for solving MADM problems with the help of an algorithm. We have presented an illustrative example to give a practical application of the proposed method. Finally, we have presented the comparative analysis of the proposed model and the existing models which gives a clear picture to the researchers of the importance of this study and it will surely motivate them to enrich the present study by introducing many other important theories and results associated to LDNNs and apply them in various practical applications.

In the future, we hope that there is a huge scope for the researchers and the policymakers(decision-makers) to explore several practical real-world applications related to topics based on linear Diophantine interval neutrosophic set(LDINS), linear Diophantine neutrosophic rough set(LDNRS), linear Diophantine neutrosophic graph(LDNG), linguistic linear Diophantine neutrosophic set(LLDNS), linear Diophantine hesitant neutrosophic set(LDHNS). The proposed study may be further extended by introducing TOPSIS, VIKOR, AHP, aggregate operators, several distance-based similarity measures.

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References

1. Zadeh, L. A. (1965). Information and control. *Fuzzy sets*, *8*, 338-353.

2. Klir, G. J., & Yuan, B. (Eds.). (1996). *Fuzzy sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers by Lotfi A Zadeh* (Vol.6). World Scientific.

3. Anoop, M. B., Rao, K. B., & Rao, T. V. S. R. A. (2002). Application of fuzzy sets for estimating service life of reinforced concrete structural members in corrosive environments. *Engineering structures*, *24*, 1229-1242.

4. Liang, T. F., & Cheng, H. W. (2009). Application of fuzzy sets tomanufacturing/distribution planning decisions with multi-product and multi-time period in supply chains. *Expert systems with applications*, *36*, 3367-3377.

5. Riaz, M., & Tehrim, S. T. (2021). A robust extension of VIKOR method for bipolar fuzzy sets using connection numbers of SPA theory based metric spaces. *Artificial Intelligence Review*, *54*, 561-591.

6. Zadeh, L. A. (1977). Fuzzy sets and their application to pattern classification and clustering analysis. *In Classification and clustering*, 251-299. Academic press.

7. Zhu, B., Xu, Z., & Xia, M. (2012). Dual hesitant fuzzy sets. *Journal of Applied Mathematics, 2012*. https://doi.org/10.1155/2012/879629.

8. Bustince, H. (2010). Interval-valued fuzzy sets in soft computing. *International Journal of Computational Intelligence Systems*, *3*, 215-222.

9. Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems 20*, 87-96.

10. Bustince, H., & Burillo, P. (1996). Vague sets are intuitionistic fuzzy sets. *Fuzzy sets and systems*, *79*, 403-405.

11. Garg, H., & Kumar, K. (2019). Improved possibility degree method for ranking intuitionistic fuzzy numbers and their application in multi attribute decision-making. *Granular Computing*, *4*, 237-247.

12. Gou, X., & Xu, Z. (2017). Exponential operations for intuitionistic fuzzy numbers and interval numbers in multi-attribute decision making. *Fuzzy Optimization and Decision Making*, *16*, 183-204.

13. Heilpern, S. (1997). Representation and application of fuzzy numbers. *Fuzzy sets and Systems*, *91*, 259-268.

14. Nayagam, V. L. G., Venkateshwari, G., & Sivaraman, G. (2008). Ranking of intuitionistic fuzzy numbers. *In 2008 IEEE International Conference on Fuzzy Systems (IEEE World Congress on Computational Intelligence)*, 1971-1974. IEEE.

15. Szmidt, E., & Kacprzyk, J.(2001). Intuitionistic fuzzy sets in some medical applications. *In 2001 International conference on computational intelligence*, 148-151. Springer, Berlin, Heidelberg.

16. Wang, G. J., & He, Y. Y. (2000). Intuitionistic fuzzy sets and L-fuzzy sets. *Fuzzy Sets and Systems*, *110*, 271-274.

17. Zeng, S., Chen, S. M., & Kuo, L. W. (2019). Multi attribute decision making based on novel score function of intuitionistic fuzzy values and modified VIKOR method. *InformationSciences*, *488*, 76-92.

18. Yager, R. (2013). Pythagorean fuzzy subsets. *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, 57-61.

19. Wan, S. P., Jin, Z., & Dong, J. Y. (2020). A new order relation for Pythagorean fuzzy numbers and application to multi-attribute group decision making. *Knowledge and Information Systems*, *62*, 751-785.

20. Ali, M. I. (2018). Another view on q-rung orthopair fuzzy sets. *International Journal of Intelligent Systems*, *33*, 2139-2153.

21. Ali, Z., & Mahmood, T. (2020). Maclaurin symmetric mean operators and their applications in the environment of complex q-rung orthopair fuzzy sets. *Computational andApplied Mathematics*, *39*, 1-27.

22. Liu, P., & Wang, P. (2018). Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. *International Journal of Intelligent Systems*, *33*, 259-280.

23. Liu, P., & Wang, P. (2018). Multiple-attribute decision-making based on Archimedean Bonferroni operators of q-rung orthopair fuzzy numbers. *IEEE Transactions on Fuzzy systems*, *27*, 834-848.

24. Yager, R. R. (2016). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, *25*, 1222-1230.

25. Çalık, A. (2021). A novel Pythagorean fuzzy AHP and fuzzy TOPSIS methodology for green supplier selection in the Industry 4.0 era. *Soft Computing*, *25*, 2253-2265.

26. Ejegwa, P. A. (2019). Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition. *Complex & Intelligent Systems*, *5*, 165-175.

27. Garg, H. (2016). A new generalized Pythagorean fuzzy information aggregation using Einstein operations and its application to decision making. *International Journal of Intelligent Systems*, *31*, 886-920.

28. Garg, H. (2017). Generalized Pythagorean fuzzy geometric aggregation operators using Einstein t-norm and t-conorm for multicriteria decision-making process. *International Journal of Intelligent Systems*, *32*, 597-630.

29. Garg, H. (2018). A linear programming method based on an improved score function for interval-valued Pythagorean fuzzy numbers and its application to decision-making. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, *26*, 67-80.

30. Garg, H. (2018). Linguistic Pythagorean fuzzy sets and its applications in multiattribute decision-making process. *International Journal of Intelligent Systems*, *33*, 1234-1263.

31. Garg, H. (2018). Hesitant Pythagorean fuzzy sets and their aggregation operators in multiple attribute decision-making. *International Journal for Uncertainty Quantification*, *8*, 267-289.

32. Garg, H. (2019). New logarithmic operational laws and their aggregation operators for Pythagorean fuzzy set and their applications. *International Journal of Intelligent Systems*, *34*, 82-106.

33. Liang, D., Zhang, Y., Xu, Z., & Darko, A. P. (2018). Pythagorean fuzzy Bonferroni mean aggregation operator and its accelerative calculating algorithm with the multithreading. *International Journal of Intelligent Systems*, *33*, 615-633.

34. Naeem, K., Riaz, M., & Afzal, D. (2019). Pythagorean m-polar fuzzy sets and TOPSIS method for the selection of advertisement mode. *Journal of Intelligent & Fuzzy Systems*, *37*, 8441-8458.

35. Xian, S., Yin, Y., Fu, M., & Yu, F. (2018). A ranking function based on principal-value Pythagorean fuzzy set in multicriteria decision making. *International Journal of Intelligent Systems*, *33*, 1717-1730.

36. Yager, R. R. (2013). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, *22*, 958-965.

37. Jansi, R., Mohana, K., & Smarandache, F. (2019). Correlation measure for Pythagorean neutrosophic sets with T and F. *Neutrosophic Sets and Systems, 30*, 202-212.

38. Ajay, D., & Chell*amani, P.* (2020). Pythagorean neutrosophic fuzzy graphs. *International Journal of Neutrosophic Science, 11,* 108-114.

39. Riaz, M., & Hashmi, M. R. (2019). Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems. *Journal of Intelligent and Fuzzy Systems*, *37,* 5417-5439.

40. Kamacı, H. (2021). Linear Diophantine fuzzy algebraic structures. *Journal of Ambient Intelligence and Humanized Computing.* http://doi.org/10.1007/s12652-020-02826-x.

41. Riaz, M., Hashmi, M. R., Kalsoom, H., Pamucar, D., & Chu, Y. M. (2020). Linear Diophantine fuzzy soft rough sets for the selection of sustainable material handling equipment. *Symmetry, 12*, 1215. doi:10.3390/sym12081215.

42. Riaz, M., Hashmi, M. R., Pamucar, D., & Chu, Y. M. (2021). Spherical linear Diophantine fuzzy sets with modeling uncertainties in MCDM. *Computer Modeling in Engineering and Sciences, 126*, 1125-1164.

43. Smarandache, F. (1999). A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic. *American Research Press, Rehoboth*, 1-141.

44. Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy set. *International journal of pure and applied mathematics*, *24*, 287-297.

45. Wang, H., Smarandache, F., Zhang, Y., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Technical Sciences and Applied Mathematics,* 10-14.

46. Edalatpanah, S. A., & Smarandache, F. (2019). Data envelopment analysis for simplified neutrosophic sets. *Neutrosophic Sets and Systems*, *29*, 215-226.

47. Edalatpanah, S. A. (2020). Data envelopment analysis based on triangular neutrosophic numbers. *CAAI Transactions on Intelligence Technology*, *5*, 94-98.

48. Edalatpanah, S. A. (2020). *Neutrosophic structured element. Expert Systems, 37, e12542.* https://doi.org/10.1111/exsy.12542.

49. Edalatpanah, S. A. (2020). A direct model for triangular neutrosophic linear programming. *International journal of neutrosophic science, 1*, 19-28.

50. Martin, N., Priya, R., & Smarandache, F. (2021). New Plithogenic sub cognitive maps approach with mediating effects of factors in COVID-19 diagnostic model. *Journal of Fuzzy Extension and Applications*, *2*, 1-15. doi: 10.22105/jfea.2020.250164.1015.

51. Debnath, S. Neutrosophication of statistical data in a study to assess the knowledge, attitude and symptoms on reproductive tract infection among women. *Journal of Fuzzy Extension and Applications*, *2*, 33-40. doi: 10.22105/jfea.2021.272508.1073.

52. Smarandache, F. (2020). The score, accuracy, and certainty functions determine a total order on the set of neutrosophic triplets (T, I, F). *Neutrosophic Sets and Systems, 38,* 1-14.

53. Lu, Z., & Ye, J. (2017). Single-valued neutrosophic hybrid arithmetic and geometric aggregation operators and their decision-making method. *Information*, *8*, 84. doi:10.3390/info803.

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