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A New Approach for Solving Bi-Level Multi-Objective Non-Linear Programming Model under Neutrosophic Environment

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Abstract
Multi-level programming problems (MLPPs) are considered very large decentralized decision problems, occur in hierarchical decision-making organizations where a decision maker (DM) is present at each decision-making level and is assigned the task of optimizing one or more objective functions. In this paper, a new computational algorithm using neutrosophic technique to solve bi-level multi-objective non-linear programming (BL-MONLP) problem is presented. Neutrosophic set theory is played an important role for dealing the inaccuracy and complexity of data found in solving real life problems. We compared also the performance of the optimal solution between fuzzy and neutrosophic optimization techniques through numerical example which has demonstrated the evolved algorithm.

Keywords: Multi-level programming with multiple objectives; Non-Linear programming problems; Fuzzy Programming; Neutrosophtic set; Satisfactory solution.

1. Introduction

In a hierarchical organization with several interacting decision-makers, multi-level programming problems (MLPPS), which are the primary mathematical optimization problems for representing large decentralized decision problems, commonly used in industry [1], agriculture [2], transport [3], public policy [4], finance [5], planning [6], municipal waste system [7] and supply chain management [8]. Decision makers make decisions in order from the top to the bottom level. The upper- level is a priority over the lower- level, but it still depends on reactions at the lower- level. Furthermore, the objective function is optimized by each decision-maker to the extent possible.

In particular, many authors have researched bi-level programming problems (BLPPs) [9-11] and tri-level programming problems (TLPPS) [12-13], noted that MLPPS was NP-hard and when the number of levels was greater than two, the decent method [14], the approach based on the conditions of Karush-Kuhn-Tucker [15], the cutting plane algorithm [16], the penalty function approach [17], the heuristics technique [18] and the vertex enumeration [19] are represented six main approaches for solving MLPPS.

There is a possibility that their methods lead to undesirable solutions due to the differences between fuzzy goals of objective functions and decision variable. An interactive fuzzy programming (FP) for MLPPS was presented to solve this situation with the elimination of the fuzzy goals of decision variables [20-21].

Although fuzzy set theory (FST) is very helpful for dealing the uncertainties, it does not resolve certain instances of uncertainty where it is difficult to use a special value to define the degree of membership. The intuitionistic fuzzy set (IFS) is considered an extension of FST to solve the non-member degree knowledge lake [22-23]. The membership grade and the non-membership grade in IFS [24] are attached to each variable in a collection, where the sum of these two grades is limited to less than or equal to one. For a specific element, the degree of non-belonging is equal to 1 minus the degree of belonging [25].

Moreover, some researchers have used IFS for different types of decision-making problems. IFS has been applied to the multi-attribute decision making model and methodology in recent years [26-27]. The problem of multi-objective optimization of reliability [28-30], the problems of transport [31-32], the problem of multi-level programming [33]. Although the development of FST and IFS still lacks a general framework in which indeterminate information cannot be handled to deal with all kinds of uncertainty in different areas. This problem is beyond the scope of FST and IFS, so dealing with a kind of infinite situation of unknown data is certainly a real problem.

The neutrosophic set studied by FST and IFS has recently been a generalized form of [34]. It offers a more general framework and a more suitable shape to solve the existing problem. Neutrosophic means neutral information, and the primary difference between fuzzy and intuitionist fuzzy logic is this neutral. The neutrosophic Set (NS) is created on a logical basis in which elements of the universe are presented in three degrees. That is, the degree of truth, the degree of indeterminacy and the degree of falsity, they are somewhere between [0,1]. It differs from the intuitionist fuzzy sets, where the uncertainty involved depends on the degree of belonging and non-belonging, here the uncertainty present, i.e., the component of indeterminacy is independent of the values of truth and falsity. Some emphasis has been built on optimization aspects since its inception by [34-36].

The main purpose of the solution proposed is to include a general framework to help deal with the impressions and uncertainties of the knowledge available. In addition, managing model memberships will produce the best compromise outcome that not only meets the desires of the decision-maker, but it also makes an undominated contribution to deal with experiences by considering the membership of truth, indeterminacy membership and falsity membership associated with satisfaction to some degree and dissatisfaction with objectives in finding the best compromise solution (BCS) respectively. It can also be done to cover a broad range of BCSs by interactively managing the membership functions. This is first time that the best of our experience is to broaden the principles of Zimmermann to solve the problem of BL-MONLP. This paper focuses on exploring the best compromise solution to the problem of bi-level multi-objective non-linear programming (BL-MONLP) under neutrosophic compromise programming approach (NS-CPA). The proposed NS-CPA is built by expanding the principles of Zimmermann [37] to the neutrosophic environment and a new neutrosophic BL-MONLP is provided by using three memberships to obtain the best compromise solution: membership of truth, membership of indeterminacy, and membership of falsity. NS-CPA is a modern way of dealing with unreliable, ambiguous, incomplete and contradictory data that is very common in science and engineering situations.

The paper is structured as follows: Section 2 outlines some fundamental principles applicable to the neutrosophic set; Section 3 presents the technique of neutrosophic optimization to solve the problem of bi-level multi-objective non-linear programming; Section 4 explains in a numerical example the new approach and compares this new strategy with the problem of fuzzy programming, we present the conclusion and future direction of research in section 5.

2. Prerequisite Mathematics

Definition-1 (Fuzzy set) [37]

Le X be fixed set. The fuzzy set A of X is defined the set of an object that has the form $\widetilde{A}=\{(x,\mu_A(x)),\ x\in X\}$ where the function $\mu_A(x): X\to[0,1]$ determines the element's $x\in X$ true membership to the set A.

Definition -2 (Intuitionistic fuzzy set) [22]

Let X be fixed set. The set of the form

$\widetilde{A}^{i} = \left\{< X, \mu_A(x),\ \upsilon_A^{\upsilon}(x) > \middle|\ x \in X \right\}$ is defined an intuitionistic fuzzy set (IFS) in X where $\mu_A(x): X \to [0,1]$ and $\upsilon_A(x): X \to [0,1]$ are called the Truth-membership and Falsity-membership respectively, for every element of $x \in X, 0 \le \mu_A(x) + \upsilon_A(x) \le 1.$

Definition -3 (Neutrosophic set) [34]

Let X be a set of objects and defined as $x \in X$. A neutrosophic set $\widetilde{A}^{n}$ in X is defined by a truth-membership function $\mu_A(x)$, an indeterminacy membership function $\sigma_A(x)$ and a falsity-membership function $\upsilon_A(x)$, having the form:

$$\widetilde{A}^{n} = \left\{< X,\ \mu_A(x), \sigma_A(x),\ \upsilon_A(x) > \middle|\ x \in X \right\}.$$

$\mu_A(x), \sigma_A(x)$ and $\upsilon_A(x)$ are real standard or non-standard elements of $\left]0^-, 0^+\right[$, that is: $\mu_A(x): X \to \left]0^-, 1^+\right[, \sigma_A(x): X \to \left]0^-, 1^+\right[, \upsilon_A(x): X \to \left]0^-, 1^+\right[$

There is no limit to the sum of $\mu_A(x), \sigma_A(x)$ *and* $\upsilon_A(x)$, so

$$^{-}0 \le \sup\ \mu_A(x) + \sup \sigma_A(x) + \sup\ \upsilon_A(x) \le 3^{+}$$

Definition -4 (Complement) [38]

A single valued neutrosophic set A is called complement, denoted by C(A) if is defined by

$$\mu_{C(A)}(x) = \upsilon_A(x),\ \sigma_{C(A)}(x) = 1 - \sigma_A(x),\ \upsilon_{C(A)}(x) = \mu_A(x),$$

for all *x's* in X.

Definition -5 (Union) [38]

A single valued neutrosophic set C is union of two single valued neutrosophic sets A and B, written as C = AUB, having the functions of truth-membership, indeterminacy- membership and falsity-membership defined by:

$$\mu_{C(A)}(x) = \max(\mu_A(x), \mu_B(x))$$
$$\sigma_{C(A)}(x) = \max(\sigma_A(x), \sigma_B(x))$$
$$\upsilon_{C(A)}(x) = \min(\upsilon_A(x), \upsilon_B(x)) \quad \text{forall } x \text{ in } X$$

Definition-6 (Intersection) [38]

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C written as $C = A \cap B$, having the functions of truth-membership, indeterminacy-membership and falsity-membership defined by:

$$\mu_{C(A)}(x) = \min\left(\mu_A(x), \mu_B(x)\right) = \min\left(\sigma_A(x), \sigma_B(x)\right)$$
$$\upsilon_{C(A)}(x) = \max\left(\upsilon_{(A)}(x),\ \upsilon_B(x)\right)$$

Here, we note that single valued neutrosophic sets satisfy the most characteristics as the classic set, fuzzy set and intuitionistic fuzzy set by the concept of complement, union and intersection of single valued neutrosophic sets. The Fuzzy collection does not satisfy the middle exclusion principle.

3. Neutrosophic technique to solve bi-level multi-objective non-linear programming (NS-BLMONLP) Problem.

3.1. Problem Formulation

In the problem of bi-level multi-objective non-linear programming (BL-MONLP), multiple decision-makers (DMs) exist at each decision-making level and are known to optimize one or more objective functions as bi-level multi-leader and / or multi-follower decision problem. The BL-MONLP problem's mathematical formulation can be written as:

$$\min_{x_1, x_2}\ F_k(x) = \left(F_1(x),\ F_2(x), ..., F_m(x)\right) \qquad \text{(The upper-level (UL))} \qquad (1)$$

where x_2 solves:

$$\min_{x_2} F_k(x) = \left(F_{m+1}(x),\, F_{m+2}(x), ..., F_N(x)\right) \qquad \text{(The lower-level (LL))} \tag{2}$$

s.t .

$$G_j = \left\{ \mathrm{x} \;\middle|\; g_j(x) \le b_j \quad , \quad j = 1,2,...., J \right\} \tag{3}$$

where $x_i = \{x_{ir}\}$, $i = 1,2$ and r = 1, 2, ..., n_i , $x_i \in R^{n_i}$, $(x_1, x_2) \subset R^{n=n_1+n_2}$ and $F(x)$ are the decision variables and the objective functions of the UL and LL respectively. G is the feasible set of problem form [(1) – (3)].

3.2. Neutrosophic Optimization Technique for solving MONLP problem.

Consider the following MONLP problem represented by problem from (1) or (2) and (3). For all objectives as $F_k(x)$, $k = 1,2,...,m,$ find firstly the best values ℓ_k (minimum values) and the worst values u_k (maximum values). So, ℓ_k can be calculated by:

$$\ell_k = \min_{x \in G} F_k(x) \tag{4}$$

If the feasible set G is bounded, then u_k can be derived by:

$$u_k = \max_{x \in G} F_k(x) \tag{5}$$

Otherwise, let the solutions of (4) are $\bar{x}_k$ then u_k are calculated by:

$$u_k = \max_k F_k(\bar{x}_k), \tag{6}$$

Now, a mapping $\mu_k : x \to [0,1]$ is known as the membership function and the acceptable degrees of decision makers for a solution can be expressed. It is possible to state a membership function as:

$$\mu_k(F_k) = \begin{cases} 1 & , \quad F_k(x) \le \ell_k \\ \dfrac{u_k - F_k}{u_k - \ell_k} & , \quad \ell_k \le F_k(x) \le u_k \\ 0 & , \quad F_k(x) \ge u_k \end{cases} \tag{7}$$

Here, we present a new approach under the set of constraints centered on a neutrosophic set to solve the MONLP problem. A creating information into indeterminacy treatment is introduced by the neutrosophic approach (NSA), which is described in the main optimization problem as the aim of maximizing the degree of truth (T) at the same time and minimizing the degrees of falsity (F) and indeterminacy (I)of a neutrosophic decision set (D_s).

Generally, we describe what is called a combination of neutrosophic objectives and constraints as:

$$D_s = \left(\bigcap_{k=1}^{m} Z_k \right) \bigcap \left(\bigcap_{j=1}^{J} C_j \right) = \left\{ \left(x, T_{D_S}(x),\, I_{D_S}(x), F_{D_S}(x) \right) \right\} \tag{8}$$

where, $T_{D_S}(x)$: the function of truth membership, $I_{D_S}(x)$: the function of indeterminacy membership and $F_{D_S}(x)$: the function of falsity membership of neutrosophic decision set D_S which is defined as:

$$T_{D_s}(x)=\min\begin{Bmatrix} T_{z_1}(x) & , & T_{z_2}(x) & , & \dots & , & T_{z_k}(x); \\ T_{c_1}(x) & , & T_{c_2}(x) & , & \dots & , & T_{c_j}(x) \end{Bmatrix} \text{ for all } x\in X \tag{9}$$

$$I_{D_s}(x)=\max\begin{Bmatrix} I_{z_1}(x) & , & I_{z_2}(x) & , & \dots & , & I_{z_k}(x); \\ I_{c_1}(x) & , & I_{c_2}(x) & , & \dots & , & I_{c_j}(x) \end{Bmatrix} \text{ for all } x\in X \tag{10}$$

$$F_{D_s}(x)=\max\begin{Bmatrix} F_{z_1}(x) & , & F_{z_2}(x) & , & \dots & , & F_{z_k}(x); \\ F_{c_1}(x) & , & F_{c_2}(x) & , & \dots & , & F_{c_j}(x) \end{Bmatrix} \text{ for all } x\in X \tag{11}$$

To formulate the membership functions for NS-MONLP problem, we evaluate firstly the lower (ℓ_k) and upper (u_k) bounds for each objective function by using (4) and (6), then the bounds for NS can be determined as:

$$u_k^T=u_k \text{ and } l_k^T=\ell_k \text{ , for truth membership} \left(T_k(F_k)\right) \tag{12}$$

$$u_k^I=l_k^T+s_k\left(u_k^T-\ell_k^T\right) \text{and } l_k^I=l_k^T \text{ , for indeterminacy membership} \left(I_k(F_k)\right) \tag{13}$$

$$u_k^F=u_k^T \text{ and } l_k^F=l_k^T+t_k\left(u_k^T-l_k^T\right), \text{ for falsity membership} \left(F_k(F_k)\right) \tag{14}$$

where t_k and s_k real numbers are predetermined in (0,1).

We can define now the membership functions (7) according to the above bounds as:

$$T_k(F_k(x))=\begin{cases} 1 & ,\text{if} \quad F_k(x)<l_k^T \\ \dfrac{u_k^T-F_k(x)}{u_k^T-\ell_k^T} & ,\text{if} \quad l_k^T\le F_k(x)\le u_k^T \\ 0 & ,\text{if} \quad F_k(x)>u_k^T \end{cases} \tag{15}$$

$$I_k(F_k(x))=\begin{cases} 0 & ,\text{if} \quad F_k(x)<l_k^I \\ \dfrac{F_k(x)-l_k^I}{u_k^I-l_k^I} & ,\text{if} \quad l_k^I\le F_k(x)\le u_k^I \\ 1 & ,\text{if} \quad F_k(x)>u_k^I \end{cases} \tag{16}$$

$$F_k(F_k(x))=\begin{cases} 0 & ,\text{if} \quad F_k(x)<l_k^F \\ \dfrac{F_k-l_k^F}{u_k^F-l_k^F} & ,\text{if} \quad l_k^F\le F_k(x)\le u_k^F \\ 1 & ,\text{if} \quad F_k(x)>u_k^F \end{cases} \tag{17}$$

The problem with NS-MONLP can be stated as:

$$\max \min_{k=1,2,\dots,m} T_k(F_k(x)) \tag{18}$$

$$\min \max_{k=1,2,\dots,m} I_k(F_k(x)) \tag{19}$$

$$\min \max_{k=1,2,\dots,m} F_k(F_k(x)) \tag{20}$$

s.t .

$$x\in G \tag{21}$$

Problem from [(18) – (21)] can be taken the following form as:

$$\max \alpha \tag{22}$$
$$\min \gamma \tag{23}$$
$$\min \beta \tag{24}$$
s.t .
$$T_k(F_k(x)) \ge \alpha \ , \ I_k(F_k(x)) \le \gamma \ , \ F_k(F_k(x)) \le \beta, \tag{25}$$
$$* \ x \in G(x), x \ge 0, \tag{26}$$
$$\alpha + \gamma + \beta \le 3 \ , \ \alpha \ge \beta \ , \ \alpha \ge \gamma \ , \ \alpha, \gamma, \beta \in [0,1]. \tag{27}$$

Problem from [(22) – (27)] can be simplified to non-linear programming (NLP) problem by using neutrosophic model as:

$$\max \ \alpha - \gamma - \beta \tag{28}$$
s.t .
$$F_k(x) + \left(u_k^T - l_k^T\right)\alpha \ \le u_k^T \ , \tag{29}$$
$$F_k(x) - \left(u_k^I - l_k^I\right)\gamma \ \le l_k^I \ , \tag{30}$$
$$F_k(x) - \left(u_k^F - l_k^F\right)\beta \ \le l_k^F \ , \tag{31}$$
$$x \in G(x) \ \ , \ \ x \ge 0, \tag{32}$$
$$\alpha + \gamma + \beta \le 3 \ , \ \alpha \ge \beta \ , \ \alpha \ge \gamma \ , \ \alpha, \gamma, \beta \in [0,1] \tag{33}$$

3.3. Algorithm (ALG (1)) for solving NS-ULMONLP problem

In this section, a new approach which discussed above is simplified to find the neutrosophic optimal solution of UL-MONLP problem.

Step1: Solve each UL-MONLP problem form (1) and (3) individually as a single objective NLP problem. Let $x_k, k = 1,2,...,m,$ be the respective optimal solution for the k^th^ different objective $F_k(x), k = 1,2,...,m$ and calculate each objective value for each these k^th^ optimal solutions. If two optimal solutions are different and far from the set of optimal solution and has the different bound values, then go to step2. Otherwise, if all $F_k(x)$ have the same solution, $x_1 = x_2 = ... = x_k$, then choose one of them as the optimal compromise solution and go to step 6.

Step 2: Find lower and upper bounds for all objectives by using (4) and (6).

Step 3: Calculate the NS bounds for each objective $F_k(x)$ to find the lower bound l_k and the upper bound $u_k, k = 1,2,...,m.$ for truth membership $(T_k(F_k))$, indeterminacy membership $(I_k(F_k))$, and falsity membership $(F_k(F_k))$, of objectives as (12), (13) and (14).

Step 4: Use step (3) to construct the membership functions as (15), (16) and (17).

Step 5: Define and solve NS for UL-MONLP problem as in [(28)-(33)] to get the best compromise solution. Also, find the values of the $F_k(x), k = 1,...,m$ .at the best solution.

Step 6: stop.

3.4. Algorithm (ALG (2)) for solving NS-LLMONLP problem

As in the previous section considers LL-MONLP problem is (2) and (3). The procedures for using the neutrosophic technique for solving LL-MONLP problem to get the NS optimal solution can be summarized as:

Step1: Solve the problem LL-MONLP (2) and (3) as a single non-linear (NL) objective problem k times, $k = m+1,..., N,$ for each problem by taking one of the goals at a time and ignoring the others subject to the set of constraints G in order to obtain the set of solutions $x_k, k = m+1,..., N,$ and calculate the values of each objectives at x_k.

Step2: If all $F_k(x), k = m+1,..., N,$ have the same solutions, then select and stop one of them as the optimal compromise solution. Otherwise, proceed to step 3.

Step3: Calculate the NS boundaries for each objective $F_k(x), k = m+1,..., N,$ in order to find the lower l_k and upper $u_k, k = m+1,..., N,$ boundaries for $T_k(F_k), I_k(F_k)$ and $F_k(F_k)$ as in (12), (13) and (14).

Step 4: Use step (3) to construct the membership functions as (15), (16) and (17).

Step5: Solve NS for LL-MONLP problem as [(28) – (33)] to find the best compromise solution. Also, find the values of $F_k(x), k = m+1,..., N,$ at the best solution.

3.5. Algorithm (ALG (3)) for solving NS-BLMONLP problem

To solve a bi-level multi-objective non-linear programming problem with a linear membership function by neutrosophic methodology, the following algorithm steps are used to find an optimal compromise solution.

Step 1: Substitute by optimal compromise solution x_k^U of NS-ULMONLP problem in $F_k, k = m+1,..., N,$ and it is denoted by $\hat{u}_k, k = m+1,..., N.$ Also, substituting by optimal compromise solution x_k^L of NS-LLMONLP problem in $F_k, k = 1,..., m,$ to obtain $u'_k, k = 1,..., m.$

Step 2: Use the optimal decision x^U of NS-ULMONLP problem as a control factor for NS-LLMONLP problem. It is not practical, so we find some tolerance that gives the NS-LLMONLP problem a feasible region to search for his/ her optimal solution. The range of the decision variable *x* should be found around x^U with its maximum tolerances *k*.

Step 3: Formulate the following membership function which specify x^U as:

$$T_X(x) = \begin{cases} \dfrac{X-(x^U-k)}{k} & , \quad \text{if} \quad x^U - k \le X \le x^U, \\ \dfrac{(x^U+k)-X}{k} & , \quad \text{if} \quad x^U \le X \le x^U + k \\ 0 & , \quad \text{otherwise} \end{cases} \tag{34}$$

$$I_X(x)=\begin{cases}\dfrac{X-(x^U-k)}{k} & , \text{ if } \quad x^U-k\le X\le x^U\\ \dfrac{(x^U+k)-X}{k} & , \text{ if } \quad x^U\le X\le x^U+k\\ 0 & , \quad \text{otherwise}\end{cases} \tag{35}$$

$$F_X(x)=\begin{cases}\dfrac{X-(x^U-k)}{k} & , \text{ if } \quad x^U-k\le X\le x^U\\ \dfrac{(x^U+k)-X}{k} & , \text{ if } \quad x^U\le X\le x^U+k\\ 0 & , \quad \text{otherwise}\end{cases} \tag{36}$$

where x^U is the most preferred solution [39].

Step 4: Construct the membership functions of NS-ULMONLP problem at all F_k where $l_k \le F_k \le u'_k$, $u'_k = F_k(x^L), k=1,2,...m,$ as:

$$T_k(F_k)=\begin{cases}1 & , \text{ if } \quad F_k(x)<l_k^T\\ \dfrac{u_k'^T-F_k(x)}{u_k'^T-l_k^T} & , \text{ if } \quad l_k^T\le F_k(x)\le u_k'^T\\ 0 & , \text{ if } \quad F_k(x)>u_k'^T\end{cases} \tag{37}$$

$$I_k(F_k)=\begin{cases}0 & , \text{ if } \quad F_k(x)<l_k^I\\ \dfrac{F_k(x)-l_k^I}{u_k'^I-l_k^I} & , \text{ if } \quad l_k^I\le F_k(x)\le u_k'^I\\ 1 & , \text{ if } \quad F_k(x)>u_k'^I\end{cases} \tag{38}$$

$$F_k(F_k)=\begin{cases}0 & , \text{ if } \quad F_k(x)<l_k^F\\ \dfrac{F_k(x)-l_k^F}{u_k'^F-l_k^F} & , \text{ if } \quad l_k^F\le F_k(x)\le u_k'^F\\ 1 & , \text{ if } \quad F_k(x)>u_k'^F\end{cases} \tag{39}$$

Step 5: Define the membership functions of NS-LLMONLP problem for all goals F_k where $l_k \le F_k \le \hat{u}_k$, $\hat{u}_k = F_k(x^U), k=m+1,..., N,$ as:

$$T_k(F_k)=\begin{cases}1 & , \text{ if } \quad F_k(x)<l_k^T\\ \dfrac{\hat{u}_k^T-F_k(x)}{\hat{u}_k^T-l_k^T} & , \text{ if } \quad l_k^T\le F_k(x)\le \hat{u}_k^T\\ 0 & , \text{ if } \quad F_k(x)>\hat{u}_k^T\end{cases} \tag{40}$$

$$I_k(F_k)=\begin{cases} 0 & , \text{ if } \quad F_k(x)<l_k^I \\ \dfrac{F_k(x)-l_k^I}{\hat{u}_k^I-l_k^I} & , \text{ if } \quad l_k^I \le F_k(x) \le \hat{u}_k^I \\ 1 & , \text{ if } \quad F_k(x)>\hat{u}_k^I \end{cases} \tag{41}$$

$$F_k(F_k)=\begin{cases} 0 & , \text{ if } \quad F_k(x)<l_k^F \\ \dfrac{F_k(x)-l_k^F}{\hat{u}_k^F-l_k^F} & , \text{ if } \quad l_k^F \le F_k(x) \le \hat{u}_k^F \\ 1 & , \text{ if } \quad F_k(x)>\hat{u}_k^F \end{cases} \tag{42}$$

Step 6: Generate a suitable solution that is also a pareto optimal solution for both decision makers (DMs) with an overall satisfaction by solving the following Tchebycheff problem [40] which is considered NS-BLMONLP problem as:

$$Max\ \alpha-\gamma-\beta \tag{43}$$

s.t .

$$\begin{aligned} &\left[X-(x^u-k)\right]/K \ge \alpha \text{ I}, \qquad \left[(x^U+k)-X\right]/K \ge \alpha \text{ I}, \\ &\left[X-(x^U-k)\right]/K \le \gamma \text{ I}, \qquad \left[(x^U+k)-X\right]/K \ge \gamma \text{ I}, \\ &\left[X-(x^U-k)\right]/K \le \beta \text{ I}, \qquad \left[(x^U+k)-k\right]/K \le \beta \text{ I}, \end{aligned} \tag{44}$$

$$\begin{aligned} &F_k(x)+\left(u_k'^T-l_k^T\right)\alpha \le u_k'^T, \quad k=1,2,\ldots,m, \\ &F_k(x)-\left(u_k'^I-l_k^I\right)\gamma \le l_k^I, \quad k=1,2,\ldots,m, \\ &F_k(x)-\left(u_k'^F-l_k^F\right)\beta \le l_k^F, \quad k=1,2,\ldots,m, \\ &F_k(x)+\left(\hat{u}_k^T-l_k^T\right)\alpha \le \hat{u}_k^T, \quad k=m+1,\ldots,N, \\ &F_k(x)-\left(\hat{u}_k^I-l_k^I\right)\gamma \le l_k^I, \quad k=m+1,\ldots,N, \\ &F_k(x)-\left(\hat{u}_k^F-l_k^F\right)\beta \le l_k^F, \quad k=m+1,\ldots,N, \end{aligned} \tag{45}$$

$$x \in G \quad , \quad x \ge 0, \tag{46}$$

$$\alpha+\beta+\gamma \le 3,\ \alpha \ge \beta,\ \alpha \ge \gamma,\ \alpha,\beta,\gamma \in [0,1] \tag{47}$$

where α, β and γ are the overall satisfaction and I, with all elements = 1 and the same direction as x, is the column vector.

4. Numerical Example

An example in [41] is used to explain and comparison of optimal solutions by fuzzy programming (FP) for BL-MOGPP and NS for BL-MONLP problem.

Let us consider BL-MONLP problem as:

(UL-MONLP): $\min\limits_{x_1} F_1(x)=20\ x_1^{-1}x_2^{-3}x_3^{-5}+60\ x_1^{-1}\ x_2^{-1}$

$$\min_{x_1} F_2(x) = 50\, x_1^{-1} x_2^{-2} x_3^{-2} + 60\, x_1^{3}\, x_2^{-2} x_3^{-3}$$
$$\text{s.t.}$$
$$G_1(x) = x_1 x_2 x_3^2 + x_2 x_3 \le 3,$$
$$x = (x_1, x_2, x_3) > 0$$

where x_2 and x_3 solve LL-MONLP problem as:

(LL-MONLP): $$\min_{x_2, x_3} F_3(x) = x_1^{-2} + 0.25\, x_2^2\, x_3^{-1},$$
$$\min_{x_2, x_3} F_4(x) = 2x_1^{-1} x_2^{-1} x_3^{-1} + 2x_1 x_2$$
$$\text{s.t.}$$
$$G_2(x) = \frac{3}{4} x_1^2 x_2^{-2} + \frac{3}{8} x_2 x_3^2 \le 1,$$
$$x = (x_1, x_2, x_3) > 0.$$

First: For UL- MONLP problem

$$\ell_1 = \min_{x_1 \in G_1} F_1(x_1) = 20.666 \text{ for } x_1 = (1.952,\quad 4.462,\quad 0.384)$$
$$\ell_2 = \min_{x_2 \in G_1} F_2(x_2) = 18.207 \text{ for } x_2 = (0.471,\quad 11.442,\quad 0.236)$$
$$u_1 = \max F_1(x_2) = F_1(0.471,\ 11.442,\quad 0.236) = 49.891$$
$$u_2 = \max F_2(x_1) = F_2(1.952,\quad 4.462,\quad 0.384) = 404.016$$
$$\therefore\ 20.666 \le F_1 \le 49.891 \text{ and } 18.207 \le F_2 \le 404.016$$

Here the upper and lower bounds for NS can be calculates from [(12)-(14)] as:

For F1: $l_1^T = 20.666,\ u_1^T = 49.891,\ l_1^I = 20.666,$
$$u_1^I = 20.666 + 29.25 s_1,\ l_1^F = 20.666 + 29.225 t_1,\ u_1^F = 49.891.$$

For F2: $l_2^T = 18.207,\ u_2^T = 404.016,\ l_2^I = 18.207,$
$$u_2^I = 18.207 + 385.809 s_1,\ l_2^F = 18.207 + 385.809 t_1,\ u_2^F = 404.016.$$

Neutrosophic optimization method for UL-MONLP problem can be constituted from [(28) – (33)] as:

$$\max \alpha_1 - \gamma_1 - \beta_1$$
$$\text{s.t.}$$
$$\left(20x_1^{-1} x_2^{-3} x_3^{-5} + 60\, x_1^{-1} x_2^{-1}\right) + 29.225\, \alpha_1 \le 49.891,$$
$$\left(20x_1^{-1} x_2^{-3} x_3^{-5} + 60\, x_1^{-1} x_2^{-1}\right) - 29.225\, s_1 \gamma_1 \le 20.666,$$
$$\left(20x_1^{-1} x_2^{-3} x_3^{-5} + 60\, x_1^{-1} x_2^{-1}\right) - (29.225 - 29.225\, t_1)\beta_1 \le 20.666 + 29.225 t_1$$

$$\left(50x_1^{-1}x_2^{-2}x_3^{-2}+60x_1^{3}x_2^{-2}x_3^{-3}\right)+385.809\alpha_1 \le 404.016\,,$$
$$\left(50x_1^{-1}x_2^{-2}x_3^{-2}+60x_1^{3}x_2^{-2}x_3^{-3}\right)-385.809\, s_1\gamma_1 \le 18.207\,,$$
$$\left(50x_1^{-1}x_2^{-2}x_3^{-2}+60x_1^{3}x_2^{-2}x_3^{-3}\right)-(385.809-385.809t_1)\ \beta_1 \le$$
$$18.207+385.809t_1,$$
$$x_1\, x_2\, x_3^2+x_2x_3 \le 3\ ,\ (x_1,x_2,x_3)>0,$$
$$\alpha_1+\gamma_1+\beta_1 \le 3\quad,\quad \alpha_1 \ge \beta_1\ ,\ \alpha_1 \ge \gamma_1 \text{ and } \alpha_1,\gamma_1,\beta_1 \in [0,1].$$

The solution of the above problem can be given as the comparison of optimal solutions between the solution by using fuzzy programming problem (FPP) and neutrosophic technique (NS) as:

Optimization technique	$x^U=\left(x_1^U,x_2^U,x_3^U\right)$	$F^U=\left(F_1^U,F_2^U\right)$	$\alpha_1^*,\gamma_1^*,\beta_1^*$	Sum of optimal objective values
FPP	$x_1^U=1.072595$ $x_2^U=5.0157517$ $x_3^U=0.4053974$	$F_1^U=23.25882$ $F_2^U=52.44017$	$\alpha^*=0.91127$	0.91127
NS	$x_1^U=1.07258$ $x_2^U=5.157534$ $x_3^U=0.4053977$	$F_1^U=23.259$ $F_2^U=5244$	$\alpha_1^*=0.911275$ $\gamma_1^*=0.088724$ $\beta_1^*=0.0000$	0.82255

and $s_1 = 1.000$, $t_1 = 0.4622$

Second: for LL-MONLP problem

$$\ell_3=\min_{x_3\in G_2} F_3(x_3)=1.172\quad,\quad x_3=(1.24,\ \ 1.27,\ \ 0.775)$$
$$\ell_4=\min_{x_4\in G_2} F_4(x_4)=3.504\quad,\ x_4=(0.623,\ \ 1.206,\ \ 1.33)$$
$$u_3=\max\ F_3(x_4)=2.852\ \ ,\ u_4=\max\ \ F_4\ (x_3)=4.789$$
$$\therefore 1.172\le F_3\le 2.852\ \ ,3.504\le\ \ F_4\le 4.789$$

For F3: $l_3^T=1.172\quad,\ \ u_3^T=2.852\ ,\ l_3^I=1.172,$

$$u_3^I=1.172+1.68s_2\,,\ l_3^F=3.504+1.285\,t_2\ \ ,\ u_3^F=2.852$$

For F4: $l_4^T=3.504,\ u_4^T=4.789,\ l_4^I=3.504, u_4^I=3.504+1.285\,s_2,$

$$l_4^F=3.504+1.285\,t_2\ \ ,\ u_4^F=4.789$$

Neturosophic optimization method for LL-MONLP problem from [(28)–(33)] can be simplified as:

$$\max\ \alpha_2 - \gamma_2 - \beta_2$$
s.t
$$x_1^{-2} + 0.25\, x_2^2\ x_3^{-1} + 1.68\,\alpha_2 \le 2.852$$
$$x_1^{-2} + 0.25\, x_2^2\ x_3^{-1} - 1.68\, s_2\gamma_2 \le 1.172$$
$$x_1^{-2} + 0.25\, x_2^2\ x_3^{-1} - (1.68 - 1.68 t_2)\,\beta_2 \le 1.172 + 1.68\, t_2,$$
$$2x_1^{-1}x_2^{-1}\ x_3^{-1} + 2x_1x_2 + 1.285\,\alpha_2 \le 4.789$$
$$2x_1^{-1}x_2^{-1}\ x_3^{-1} + 2x_1x_2 - 1.285 s_2\,\gamma_2 \le 3.504$$
$$2x_1^{-1}x_2^{-1}\ x_3^{-1} + 2x_1x_2 - (1.285 - 1.285\, t_2)\,\beta_2 \le 3.504 + 1.285 t_2,$$
$$\frac{3}{4}\, x_1^2 x_2^{-2} + \frac{3}{8} x_2 x_3^2 \le 1 \quad , \quad (x_1, x_2, x_3) > 0,$$
$$\alpha_2 + \gamma_2 + \beta_2 \le 3\ ,\ \alpha_2 \ge \beta_2\ ,\ \alpha_2 \ge \gamma_2\ \ and\ \alpha_2, \gamma_2, \beta_2 \in [0,1].$$

The comparison of optimal solution between FPP and NS technique can be summarized as:

Optimal technique	$x^L = \left(x_1^L, x_2^L, x_3^L\right)$	$F^L = \left(F_3^L, F_4^L\right)$	$\alpha_2^*, \gamma_2^*, \beta_2^*$	Sum of optimal objective values
FPP	$x_1^L = 0.8932686$ $x_2^L = 1.135477$ $x_3^L = 1.121792$	$F_3^L = 1.5405769$ $F_4^L = 3.786322$	$\beta = 0.780441$	0.91127
NS	$x_1^L = 0.8932$ $x_2^L = 1.1355$ $x_3^L = 1.1219$	$F_3^L = 1.54$ $F_4^L = 3.786$	$\alpha_2 = 0.7805$ $\gamma_2 = 0.2195$ $\beta_2 = 0.000$	0.82255

and $s_2 = 1.000$, $t_2 = 0.242$

Third: Substituting by optimal compromise solution $\left(x_1^L, x_2^L, x_3^L\right)$ in F_1 and F_2 respectively, we get:

$32.259 < F_1 < 67.764$ and $52.44 < F_2 < 54.25$

We are also substituted by optimum compromise solution $\left(x_1^U, x_2^U, x_3^U\right)$ in F_3 and F_4, respectively we get:

$1.54 < F_3 < 17.237$ and $3.786 < F_4 < 11.956$

Application of steps algorithm (3) and assuming the control decision is x_1^U with tolerance k = 1 of NS-UL-MONLP problem. The NS-BLMONLP problem from [(43) – (47)] can be generated as:

$$
\begin{aligned}
&\max\ \alpha_3 - \gamma_3, -\beta_3\\
&\text{s.t.}\\
&x_1 - 0.07 \ge \alpha_3 \quad , \quad 2.07 - x_1 \ge \alpha_3,\\
&x_1 - 0.07 \le \gamma_3 \quad , \quad 2.07 - x_1 \le \gamma_3,\\
&x_1 - 0.07 \le \beta_3 \quad , \quad 2.07 - x_1 \le \beta_3,\\
&20x_1^{-1}x_2^{-3}x_3^{-5} + 60\ x_1^{-1}x_2^{-1} + 44.505\alpha_3 \le 67.764,\\
&50x_1^{-1}x_2^{-2}x_3^{-2} + 60\ x_1^{3}x_2^{-2}x_3^{-3} + 1.81\alpha_3 \le 54.25\ ,\\
&x_1^{-2} + 0.25\ x_2^{2}x_3^{-1} + 15.733\,\alpha_3 \le 17.273 \qquad ,\\
&2x_1^{-1}x_2^{-1}x_3^{-1} + 2\ x_1x_2 + 8.17\ \alpha_3 \le 11.956 \qquad ,\\
&20x_1^{-1}x_2^{-3}x_3^{-5} + 60\ x_1^{-1}x_2^{-1} - 44.505s_3\gamma_3 \le 23.259,\\
&50x_1^{-1}x_2^{-2}x_3^{-2} + 60\ x_1^{3}x_2^{-2}x_3^{-3} - 1.81s_3\,\gamma_3 \le 52.44,\\
&x_1^{-2} + 0.25\ x_2^{2}x_3^{-1} - 15.733s_3\,\gamma_3 \le 1.54,\\
&2x_1^{-1}x_2^{-1}x_3^{-1} + 2\ x_1x_2 - 8.17s_3\,\gamma_3 \le 3.786,\\
&20x_1^{-1}x_2^{-3}x_3^{-5} + 60\ x_1^{-1}x_2^{-1} - (44.505 - 44.505t_3)\,\beta_3 \le 23.259 + 44.505t_3,\\
&50x_1^{-1}x_2^{-2}x_3^{-2} + 60\ x_1^{3}x_2^{-2}x_3^{-3} - (1.81 - 1.81t_3)\,\beta_3 \le 52.44 + 1.81t_3,\\
&x_1^{-2} + 0.25\ x_2^{2}x_3^{-1} - (15.733 - 15.733\,t_3)\beta_3 \le 1.54 + 15.733t_3,\\
&2x_1^{-1}x_2^{-1}x_3^{-1} + 2x_1x_2 - (8.17 - 8.17t_3)\,\beta_3 \le 3.786 + 8.17t_3,\\
&x_1x_2x_3^{2} + x_1x_2 \le 3,\\
&\frac{3}{4}x_1^{2}x_2^{-2} + \frac{3}{8}x_2x_3^{2} \le 1, \quad (x_1, x_2, x_3) > 0,\\
&\alpha_3 + \gamma_3 + \beta_3 \le 3, \quad \alpha_3 \ge \beta_3\ ,\ \alpha_3 \ge \gamma_3 \ \text{ and } \alpha_3, \gamma_3, \beta_3 \in [0,1].
\end{aligned}
$$

The comparison of optimal solutions between FPP and NS-technique can be summarized as:

Optimization technique	$x^* = (x_1^*, x_2^*, x_3^*)$	$F^* = (F_1^*, F_2^*, F_3^*, F_4^*)$	$\alpha_3^*, \gamma_3^*, \beta_3^*$	Sum of optimal objective values
FPP	$x_1^* = 1.071753$ $x_2^* = 2.114856$ $x_3^* = 0.7749304$	$F_1^* = 33.9266984$ $F_2^* = 53.86580161$ $F_3^* = 2.318923741$ $F_4^* = 5.665864$	$\delta = 0.77$	0.77
NS	$x_1^* = 1.071751$ $x_2^* = 2.114812$	$F_1^* = 33.9267$ $F_2^* = 538658$ $F_3^* = 2.3189$ $F_4^* = 5.6659$	$\alpha_3^* = 0.7691842$ $\gamma_3^* = 0.230815$ $\beta_3^* = 0.000$	0.538368

and $s_3 = 1.000$, $t_3 = 0.287599$

Here, we demonstrate that the technique of neutrousophic optimization results better than the problem of fuzzy programming.

5. Conclusions and research directions

Neutrosophic set theory considers an important role in resolving the inaccuracy and uncertainty of data in solving real-life problems. The known methods as fuzzy theory, not sufficient in many bi-level programming situations for dealing with these situations in which indeterminacy is necessarily involved. This paper introduces a new neutrosophic compromise programming strategy (NS-CPA) to resolve the problem of bi-level multi-objective non-linear programming (NS-BLMONLP) under fuzziness.

At the same time, this method is characterized by maximizing the degree of truth (satisfaction), minimizing the degrees of both falsity (dissatisfaction) and indeterminacy (satisfaction to some extent) of neutrosophic decision-making. The analysis of the results obtained for the problem undertaken clearly indicates that neutrosophic optimization is superior to fuzzy optimization. To apply the steps of the NS-CPA, a numerical problem is solved.

We hope that in the neutrosophic setting, the bi-level non-linear programming technique can open a new avenue of study for future neutrsophic researchers. In addition, we agree that the propose neutrosophic compromise programming solution can be useful in addressing multi-objective geometric programming, Multi-objective decentralized bi-level non-linear programming, multi-objective decentralized multi-level non-linear programming, multi-objective non-linear programming problems based on priority, real-world decision-making problems such as agriculture, production of bio-fuels, selection of portfolios, transport.

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