Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making

Muhammad Saeed¹, Muhammad Saqlain¹*, Asad Mehmood², Khushbakht Naseer³ and Sonia Yaqoob⁴

¹Department of Mathematics, University of Management and Technology, Lahore, Pakistan; muhammad.saeed@umt.edu.pk
¹*Department of Mathematics, Lahore Garrison University, Lahore, Pakistan; msaqlain@lgu.edu.pk
²Department of Mathematics, University of Management and Technology, Lahore, Pakistan; a.asadkhan.khi@gmail.com
³Department of Mathematics, University of Management and Technology, Lahore, Pakistan; khushbakht433@gmail.com
⁴Department of Mathematics, University of Management and Technology, Lahore, Pakistan; soniayaqoob1122@gmail.com

Abstract. A Similarity measure for Neutrosophic function performs a fundamental role in tackling the problems that include blurred and hazed information but is not able to handle the fuzziness and vagueness of the problems which have numerous information. The objective of this research paper is to generalize neutrosophic soft set to the multi-polar neutrosophic soft set (mNS set), aggregation operators and their properties on mNS sets. It also discusses the distance-based similarity measures that rely on between two mNS sets. It explains with the help of examples that the intended similarity measures of mNS sets are applicable in the field of medical diagnosis and decision-making problem for selection of lecturer in universities. Eventually, this proposed method is concluded as an algorithm in the application.

Keywords: mNS Set; Operators on mNS; Properties; Distance and Similarity Measure; Medical Diagnosis; Decision-Making

1. Introduction

Zadeh [1] introduced fuzzy sets as an additional classical conception of set. The theory of fuzzy set can be widely utilized in those domains where the information is deficient or incomplete like in bio-informatics fuzzy set logics, the members in a set are allowed to have a moderate assessment of membership, this is explained by the help of membership function admired in real unit interim [0,1]. Fuzzy sets have been derived from crisp sets, because the different aspects of functions of crisp sets are extraordinary occasions of degree functions of...
fuzzy sets if the later contains the values of fuzzy relations like 0 or 1 used in different areas like clustering (Bezdek [16]), linguistics (Cock [17]), and decision-making (Deli [18]) are significant theories of L-relations when the unit interval L is [0,1]. Further than intuitionistic fuzzy set was being proposed by Atanassov [2] which was an extension of Zadeh’s conviction and it was in itself adjunct the classical conviction of a set. The Intuitionistic fuzzy set was only able to grasp insufficient particulars and facts, not unspecified particulars and facts. Neutrosophy was proposed by Florentin [19]. It is a limb of philosophy that scrutinizes the origination, essence, and range of neutralities along with interconnection with various ideational spectra. Neutrosophic set is a powerful general authorized substructure which deducts the theory of fuzzy sets and intuitionistic fuzzy set. The Soft set supposition is induction of fuzzy set supposition that was being presented by Molodtsov [3] to trade with unpredictability in a parametric model. He designated strains and troubles that were in mathematical representations and tackle the complications by proposing a soft set theory. Maji [4,23] expanded the soft set scheme into a fuzzy soft set and neutrosophic soft set theory. Feng [20] further explored decision-making that was supported by fuzzy soft sets.

Bipolar fuzzy sets and its connections were further introduced by Zhang [5], then Chen [6] proposed the notion of multi-polar fuzzy sets that was an abstraction of bipolar fuzzy sets. It was being manifested that bipolar fuzzy sets were isomorphic mathematical conceptions. Further than Wang and Liu [22] proposed decision-making on the multi-polar neutrosophic numbers. Deli [24] studied the neutrosophic soft multi-set theory with the multi distinct universe.


Smarandache [28] generalized the soft set to hypersoft set by converting the function into a multi-decision function. Aggregate operators, similarity measure and a TOPSIS technique is introduced by [29-34] in his work. Application of fuzzy numbers in mobile selection in metros like Lahore is proposed by [35]. TOPSIS technique of MCDM can also be used for the prediction of games, and it’s applied in FIFA 2018 by [36], prediction of games is a very
complex topic and this game is also predicted by [37]. Abdel-Basset [38-41] has published a set of articles on Medical disease diagnosis based on neutrosophic environment.

1.1. Motivation

A huge number of articles is published in neutrosophic field, and as well as this theory is applied and extended in different branches such as Decision Making. The extension of neutrosophic environment to $m$-polar Neuetrosophic Soft set is totally new. Here, a set of questions arises that how $m$NS set can be represented? what is the purpose of $m$-polar structure? and how $m$-polar structures can be utilized in medical diagnosis and in decision making problems? From this point of view, $m$NS structure is good choice to get better results to the problems in decision making.

1.2. Paper Presentation

This article visualizes new concept $m$-polar Neutrosophic Soft Set as an extension of Neuetrosophic Soft Set.

- Basic operators such as union, intersection on $m$NS set are introduced
- Properties related to operators on $m$NS structure
- Distance measure of different types are introduced between any two $m$NS sets
- A case study of two applications are concluded with an algorithm in the field of medical diagnosis and decision-making problem.

2. Preliminaries

This section studies some basic definitions related to this article.

2.1. Neutrosophic Set

**Definition 2.1** [19] Let $Z$ be a universal set. A neutrosophic set $X$ is defined as :

$$X = \{ z, (T_X(z), I_X(z), F_X(z)) : z \in Z \},$$

where,

$$T_X(z), I_X(z), F_X(z) \in [0,1]$$

$$0 \leq T_X(z) + I_X(z) + F_X(z) \leq 3$$

for all $z \in Z$

2.2. Multi-Polar Neutrosophic Set

**Definition 2.2** [22] An $mN$ set on a universal set $Z$ is a mapping

$$X = \left( (s_1 \circ T_X(z), s_2 \circ T_X(z), \cdots, s_m \circ T_X(z)), (s_1 \circ I_X(z), s_2 \circ I_X(z), \cdots, s_m \circ I_X(z)), (s_1 \circ F_X(z), s_2 \circ F_X(z), \cdots, s_m \circ F_X(z)) \right) : Z \to ([0,1]^m, [0,1]^m, [0,1]^m)$$
where \( i-th \) mapping is defined as
\[
\begin{align*}
s_i \circ T_X &: [0, 1]^m \to [0, 1] \\
s_i \circ I_X &: [0, 1]^m \to [0, 1] \\
s_i \circ F_X &: [0, 1]^m \to [0, 1]
\end{align*}
\]
and
\[
0 \leq s_i \circ T_X(z) + s_i \circ I_X + s_i \circ F_X \leq 3
\]
for all \( i = 1, 2, \cdots, m \) and \( z \in Z \)

2.3. Soft Set

\textbf{Definition 2.3} [3] Let \( Z \) be a universal set and \( E \) be the set of attributes of elements in \( Z \). Take \( X \) to be a subset of \( E \) then a function \( F \) defined as
\[
F : X \to P(Z),
\]
then a pair \((F, X)\) is called a soft set over \( Z \) such as
\[
(F, X) = \{e, F(e) : e \in X, F(e) \in P(Z)\}
\]

2.4. Not Set

\textbf{Definition 2.4} [23] The Not Set of set of parameters \( E = \{e_1, e_2, \cdots, e_q\} \), denoted by \( \neg E \) is defined as
\[
\neg E = \{-e_1, -e_2, \cdots, -e_q\}
\]
where \( -e_j \) means not \( e_j \) for all \( j = 1, 2, \cdots, q \).

2.5. Neutrosophic Soft Sets

\textbf{Definition 2.5} [23] A Neutrosophic Soft set \((\omega, X)\) over a universal set \( Z \) is a mapping from \( X \) to \( P(Z) \) and defined as
\[
(\omega, X) = \Omega_X = \{(e, (z, T_X(e)(z), I_X(e)(z), F_X(e)(z)) : z \in Z, e \in E)\}
\]
where, \( P(Z) \) denotes collection of all neutrosophic subsets of \( Z \). Each of \( T_X(e), I_X(e) \) and \( F_X(e) \) is a mapping from \( Z \) to interval \([0, 1]\) and
\[
0 \leq T_X(e)(z) + I_X(e)(z) + F_X(e)(z) \leq 3
\]
for all \( e \in E \) and \( z \in Z \)

2.6. Multi-Polar Neutrosophic Soft set

\textbf{Definition 2.6} [24] Let \( Z \) be a universal set, \( E \) be a set of attributes and \( X \subseteq E \).
Define \( \omega : X \to mN^Z \) where \( mN^Z \) is the collection of all \( mN \) subsets of set \( Z \). Then \((\omega, X)\) is called an mNS set over \( Z \) as follows
\[
\Omega_X = (\omega, X) = \{e, \omega_X(e) : e \in E, \omega_X(e) \in mN^Z\}
\]
3. Operations on \( mN \) Soft Sets

This section discusses some operators on \( mNS \) sets.

3.1. Multi-Polar Neutrosophic Soft subset

**Definition 3.1** Let \( Z \) be a universal set, \( X \) and \( Y \) are subsets of a set of attributes \( E \). A set \( \Omega_X \) is an \( mNS \) subset of \( \Psi_Y \) denoted by \( \Omega_X \subseteq \Psi_Y \) if

(i) \( X \subseteq Y \)
(ii) \( \omega_X(e) \subseteq \psi_Y(e) \) i.e.

\[
s_i \circ T_X(e)(z) \leq s_i \circ T_Y(e)(z), s_i \circ I_X(e)(z) \leq s_i \circ I_Y(e)(z) \quad \text{and} \quad s_i \circ F_X(e)(z) \geq s_i \circ F_Y(e)(z)
\]

for all \( i = 1, 2, \cdots, m; e \in E \) and \( z \in Z \)

**Example 3.1** Let \( Z = \{z_1, z_2\} \) be a universal set and \( E = \{e_1, e_2, e_3, \} \) be a set of attributes. \( X = \{e_1, e_2\}, Y = \{e_1, e_2\} \subseteq E \). Let \( \Omega_X \) and \( \Psi_Y \) be two \( 3-\text{NS} \) set defined as:

\[
\Omega_X = \{e_1, (z_1, (0.7, 0.5, 0.5), (0.2, 0.2, 0.3), (0.3, 0.5, 0.3)), (z_2, (0.3, 0.4, 0.5), (0.3, 0.2, 0.4), (0.3, 0.5, 0.8)),
\]
\[
e_2, (z_1, (0.3, 0.5, 0.6), (0.1, 0.4, 0.2), (0.7, 0.5, 0.3)), (z_2, (0.7, 0.2, 0.7), (0.3, 0.4, 0.4), (0.2, 0.7, 0.4))\}
\]

\[
\Psi_Y = \{e_1, (z_1, (0.8, 0.8, 0.7), (0.4, 0.5, 0.4), (0.3, 0.3, 0.2)), (z_2, (0.4, 0.7, 0.4), (0.6, 0.4, 0.5), (0.3, 0.3, 0.6)),
\]
\[
e_2, (z_1, (0.7, 0.8, 0.8), (0.3, 0.8, 0.2), (0.3, 0.4, 0.3)), (z_2, (0.8, 0.5, 0.8), (0.5, 0.8, 0.4), (0.2, 0.5, 0.3))\}\]

this implies \( \Omega_X \subseteq \Psi_Y \)

3.2. Equal Multi-Polar Neutrosophic Soft set

**Definition 3.2** Let \( \Omega_X \) and \( \Psi_Y \) be two \( mNS \) set over a universal set \( Z \), where \( X \) and \( Y \) are subsets of sets of attributes \( E \). Then two \( mNS \) sets \( \Omega_X \) and \( \Psi_Y \) are said to be equal denoted as \( \Omega_X \equiv \Psi_Y \) if and only if \( \Omega_X \subseteq \Psi_Y \) and \( \Psi_Y \subseteq \Omega_X \)

3.3. Relative Null Multi-Polar Neutrosophic Soft set

**Definition 3.3** An \( mNS \) set over the universal set \( Z \) is said to be relative empty or relative null \( mNS \) set concerning the set of attributes \( X \subseteq E \), denoted by \( \Phi_X \) if

\[
\omega_X(e) = \{z, s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z) : z \in Z\}
\]

and

\[
0 \leq s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z) \leq 3
\]

for all \( i = 1, 2, \cdots, m; e \in E \) and \( z \in Z \)
that is
\[ \tilde{\Phi}_X = \{ e, (z, ((0,0,\cdots,0),(0,0,\cdots,0),(1,1,\cdots,1))) : z \in Z, e \in X \} \]

3.4. Relative Whole Multi-Polar Neutrosophic Soft set

**Definition 3.4** An \( m \)NS set over the universal set \( Z \) is said to be relative whole \( m \)NS set concerning the set of attributes \( X \subseteq E \), denoted by \( \check{Z}_X \) if
\[
\begin{align*}
& s_i \circ T_X(e)(z) = 1 \\
& s_i \circ I_X(e)(z) = 1 \\
& s_i \circ F_X(e)(z) = 0
\end{align*}
\]
for all \( i = 1, 2, \cdots, m \); \( e \in X \) and \( z \in Z \)

that is
\[ \check{Z}_X = \{ e, (z, ((1,1,\cdots,1),(1,1,\cdots,1),(0,0,\cdots,0))) : z \in Z, e \in X \} \]

3.5. Absolute Multi-Polar Neutrosophic Soft set

**Definition 3.5** An \( m \)NS set over the universal set \( Z \) is said to be an absolute \( m \)NS set concerning the set of attributes \( E \), denoted by \( \check{Z}_E \) if
\[
\begin{align*}
& s_i \circ T_X(e)(z) = 1 \\
& s_i \circ I_X(e)(z) = 1 \\
& s_i \circ F_X(e)(z) = 0
\end{align*}
\]
for all \( i = 1, 2, \cdots, m \); \( e \in E \) and \( z \in Z \)

that is
\[ \check{Z}_E = \{ e, (z, ((1,1,\cdots,1),(1,1,\cdots,1),(0,0,\cdots,0))) : z \in Z, e \in E \} \]

Example 3.2 Let \( Z = \{ z_1, z_2 \} \) be universal set and \( E = \{ e_1, e_2, e_3 \} \) is set of attributes, if \( X = \{ e_1, e_2 \} \subseteq E \), a 3–NS set \( \Omega_X \) such that
\[
\Omega_X = \{ e_1, (z_1, (0,0,0), (0,0,0), (1,1,1)), (z_2, (0,0,0), (0,0,0), (1,1,1)), \\
e_2, (z_1, (0,0,0), (0,0,0), (1,1,1)), (z_2, (0,0,0), (0,0,0), (1,1,1)) \} = \tilde{\Phi}_X
\]

then \( \Omega_X \) is a relative null 3–NS set \( \tilde{\Phi}_X \).

If \( Y = \{ e_1, e_3 \} \subseteq E \), a 3–NS set \( \Psi_Y \) such that
\[
\Psi_Y = \{ e_1, (z_1, (1,1,1), (1,1,1), (0,0,0)), (z_2, (1,1,1), (1,1,1), (0,0,0)), \\
e_3, (z_1, (1,1,1), (1,1,1), (0,0,0)), (z_2, (1,1,1), (1,1,1), (0,0,0)) \} = \check{Z}_Y
\]

then \( \Psi_Y \) is a relative whole 3–NS set \( \check{Z}_Y \).
if \( W = E = \{e_1, e_2, e_3\} \), a 3-NS set \( \Lambda_W \) such that
\[
\Lambda_W = \{e_1, (z_1, (1, 1, 1), (1, 1, 1), (0, 0, 0)), (z_2, (1, 1, 1), (1, 1, 1), (0, 0, 0)),
\]
\[
e_2, (z_1, (1, 1, 1), (1, 1, 1), (0, 0, 0)), (z_2, (1, 1, 1), (1, 1, 1), (0, 0, 0)),
\]
\[
e_3, (z_1, (1, 1, 1), (1, 1, 1), (0, 0, 0)), (z_2, (1, 1, 1), (1, 1, 1), (0, 0, 0)) \} = \tilde{Z}_E
\]
then \( \Lambda_W \) is an absolute 3-NS set \( \tilde{Z}_E \).

**Proposition 3.1** Let \( Z \) be a universal set, \( E \) a set of attributes, \( X, Y, W \subseteq E \). If \( \Omega_X, \Psi_Y \) and \( \Lambda_W \) are mNS sets over \( Z \), then

(i) \( \Omega_X \subseteq \tilde{Z}_X \)

(ii) \( \Phi_X \subseteq \Omega_X \)

(iii) \( \Omega_X \subseteq \Omega_X \)

(iv) \( \Omega_X \subseteq \Psi_Y \) and \( \Psi_Y \subseteq \Lambda_W \), then \( \Omega_X \subseteq \Lambda_W \)

(v) \( \Omega_X \not \subseteq \Psi_Y \) and \( \Psi_Y \not \subseteq \Lambda_W \), then \( \Omega_X \not \not \subseteq \Lambda_W \)

**3.6. Complement of mN Soft Set**

**Definition 3.6** The complement of an mNS set \( \Omega_X \) over a universal set \( Z \) with respect to the set of attributes \( X \subseteq E \), denoted by \( \Omega_X^c = (\omega^c, X) \) where \( \omega^c : \neg X \rightarrow mNS^Z \) is a mapping given as
\[
\omega^c(e) = \{z, ((s_i \circ T_X^c(e))(z) = s_i \circ F_X(e)(z)), (s_i \circ I_X^c(e))(z) = (1, 1, \cdots, 1) - s_i \circ I_X(e)(z)),
\]
\[
(s_i \circ F_X^c(e)(z) = s_i \circ T_X(e)(z)) \}
\]
for all \( i = 1, 2, \cdots, m, -e \in \neg X \) and \( z \in Z \)

**3.7. Relative Complement of mN Soft Set**

**Definition 3.7** The relative complement of an mNS set \( \Omega_X \) over a universal set \( Z \) with respect to the set of attributes \( X \subseteq E \), denoted by \( \Omega_X^r = (\omega^r, X) \) where \( \omega^r : X \rightarrow mNS^Z \) is a mapping given as
\[
\omega^r(e) = \{z, ((s_i \circ T_X^r(e))(z) = s_i \circ F_X(e)(z)), (s_i \circ I_X^r(e))(z) = (1, 1, \cdots, 1) - s_i \circ I_X(e)(z)),
\]
\[
(s_i \circ F_X^r(e)(z) = s_i \circ T_X(e)(z)) \}
\]
for all \( i = 1, 2, \cdots, m; e \in X \) and \( z \in Z \)

**Example 3.3** Let \( Z = \{z_1, z_2\} \) be universal set and \( E = \{e_1, e_2, e_3\} \) is set of attributes, if \( X = \{e_1, e_2\} \subseteq E \), A 3-NS set \( \Omega_X \) such that
\[
\Omega_X = \{e_1, (z_1, (0.4, 0.4, 0.4), (0.2, 0.4, 0.5), (0.5, 0.3, 0.7)), (z_2, (0.5, 0.7, 0.5), (0.6, 0.5, 0.3), (0.5, 0.7, 0.3)),
\]
\[
e_2, (z_1, (0.7, 0.4, 0.6), (0.8, 0.6, 0.4), (0.2, 0.6, 0.7)), (z_2, (0.5, 0.7, 0.6), (0.8, 0.4, 0.7), (0.9, 0.3, 0.6)) \}
\]
Then,
\[
\Omega_X^c = \{\neg e_1, (z_1, (0.5, 0.3, 0.7), (0.8, 0.6, 0.4), (0.4, 0.4, 0.6)), (z_2, (0.5, 0.7, 0.3), (0.4, 0.5, 0.7), (0.5, 0.7, 0.5)),
\]
\[
\neg e_2, (z_1, (0.2, 0.6, 0.7), (0.2, 0.4, 0.6), (0.7, 0.4, 0.6)), (z_2, (0.9, 0.3, 0.6), (0.2, 0.6, 0.3), (0.5, 0.7, 0.6)) \}
\]

Saeed et al., Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making
\[ \Omega_X = \{ e_1, (z_1, (0.5, 0.3, 0.7), (0.8, 0.6, 0.4), (0.4, 0.4, 0.6)), (z_2, (0.5, 0.7, 0.3), (0.4, 0.5, 0.7), (0.5, 0.7, 0.5)), e_2, (z_1, (0.2, 0.6, 0.7), (0.2, 0.4, 0.6), (0.7, 0.4, 0.6)), (z_2, (0.9, 0.3, 0.6), (0.2, 0.6, 0.3), (0.5, 0.7, 0.6)) \} \]

**Proposition 3.2** Let \( \Omega_X \) be an mNS set over a universal set \( Z \). Then

(i) \((\Omega_X^c)^c = \Omega_X \)

(ii) \((\Omega_X^r)^r = \Omega_X \)

(iii) \(\bar{\Omega_X} = \bar{\Phi} \)

(iv) \(\bar{\Phi} = \bar{\Phi} \)

3.8. **Union of Two mN Soft Sets**

**Definition 3.8** Let \( Z \) be a universal set and \( X \) and \( Y \) are subsets of the set of attributes \( E \). The set \( \Omega_X \) and \( \Psi_Y \) are two mNS sets. Let \( W = X \cup Y \), then the union of \( \Omega_X \) and \( \Psi_Y \) is an mNS set denoted by \( \Omega_X \cup \Psi_Y \) and defined as for all \( e \in W \)

\[
\Omega_X \cup \Psi_Y = \left\{ \begin{array}{ll}
\omega_X(e), & e \in X \setminus Y; \\
\psi_Y(e), & e \in Y \setminus X; \\
\omega_X(e) \cup \psi_Y(e), & e \in X \cap Y.
\end{array} \right.
\]

where,

\[
\omega_X(e) \cup \psi_Y(e) = (\max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)), \max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), \\
\min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)))
\]

for all \( i = 1, 2, \cdots, m; e \in W \) and \( z \in Z \)

3.9. **Intersection of Two mN Soft Sets**

**Definition 3.9** Let \( Z \) be a universal set and \( X \) and \( Y \) are subsets of the set of attributes \( E \). The set \( \Omega_X \) and \( \Psi_Y \) are two mNS sets. Let \( W = X \cap Y \), then the intersection of \( \Omega_X \) and \( \Psi_Y \) is an mNS set denoted by \( \Omega_X \cap \Psi_Y \) and defined as for all \( e \in W \)

\[
\Omega_X \cap \Psi_Y = \omega_X(e) \cap \psi_Y(e)
\]

where,

\[
\omega_X(e) \cap \psi_Y(e) = (\min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)), \min(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), \\
\max(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)))
\]

for all \( i = 1, 2, \cdots, m; e \in W \) and \( z \in Z \)

3.10. **Restricted Union of Two mN Soft Sets**

**Definition 3.10** Let \( Z \) be a universal set and \( X \) and \( Y \) are subsets of the set of attributes \( E \). The set \( \Omega_X \) and \( \Psi_Y \) are two mNS sets. Let \( W = X \cap Y \), then the Restricted union of \( \Omega_X \) and \( \Psi_Y \) is an mNS set denoted by \( \Omega_X \cup_R \Psi_Y \) and defined as for all \( e \in W \)

\[
\Omega_X \cup_R \Psi_Y = \omega_X(e) \cup \psi_Y(e)
\]
3.11. **Extended Intersection of Two mN Soft Sets**

**Definition 3.11** Let \( Z \) be a universal set and \( X \) and \( Y \) are subsets of the set of attributes \( E \). The set \( \Omega_X \) and \( \Psi_Y \) are two mNS sets. Let \( W = X \cup Y \), then the Extended intersection of \( \Omega_X \) and \( \Psi_Y \) is an mNS set denoted by \( \Omega_X \cap_e \Psi_Y \) and defined as for all \( e \in W \)

\[
\Omega_X \cap_e \Psi_Y = \begin{cases} 
\omega_X(e), & e \in X \setminus Y; \\
\psi_Y(e), & e \in Y \setminus X; \\
\omega_X(e) \cap \psi_Y(e), & e \in X \cap Y. 
\end{cases}
\]

3.12. **OR-operator of Two mN Soft Sets**

**Definition 3.12** Let \( Z \) be a universal set and \( X \) and \( Y \) are subsets of the set of attributes \( E \). The set \( \Omega_X \) and \( \Psi_Y \) are two mNS sets, then the OR-operator of \( \Omega_X \) and \( \Psi_Y \) is an mNS set denoted by \( \Omega_X \lor \Psi_Y \) and defined as \( \Omega_X \lor \Psi_Y = \Lambda_{X \times Y} \) where

\[
\lambda_{X \times Y}(x, y) = \omega_X(x) \lor \psi_Y(y)
\]

for all \( (x, y) \in X \times Y \)

3.13. **AND-operator of Two mN Soft Sets**

**Definition 3.13** Let \( Z \) be a universal set and \( X \) and \( Y \) are subsets of the set of attributes \( E \). The set \( \Omega_X \) and \( \Psi_Y \) are two mNS sets, then the AND-operator of \( \Omega_X \) and \( \Psi_Y \) is an mNS set denoted by \( \Omega_X \land \Psi_Y \) and defined as \( \Omega_X \land \Psi_Y = \Lambda_{X \times Y} \) where

\[
\lambda_{X \times Y}(x, y) = \omega_X(x) \land \psi_Y(y)
\]

for all \( (x, y) \in X \times Y \)

**Example 3.4** Let \( Z = \{z_1, z_2\} \) be a universal set and \( E = \{e_1, e_2, e_3\} \) be a set of attributes. Let \( X = \{e_1, e_2\}, Y = \{e_2, e_3\} \subseteq E \). Let \( \Omega_X \) and \( \Psi_Y \) be two 3–N soft set defined as.

\[
\Omega_X = \{e_1, (z_1, (0.7, 0.5, 0.5), (0.4, 0.5, 0.4), (0.3, 0.5, 0.3)), (z_2, (0.3, 0.4, 0.5), (0.6, 0.4, 0.5), (0.3, 0.5, 0.8)), \\
e_2, (z_1, (0.3, 0.5, 0.6), (0.3, 0.8, 0.2), (0.7, 0.5, 0.3)), (z_2, (0.7, 0.2, 0.7), (0.5, 0.8, 0.4), (0.2, 0.7, 0.4))\}
\]

\[
\Psi_Y = \{e_2, (z_1, (0.3, 0.5, 0.7), (0.2, 0.6, 0.3), (0.3, 0.4, 0.6)), (z_2, (0.4, 0.6, 0.7), (0.3, 0.5, 0.7), (0.4, 0.7, 0.3)), \\
e_3, (z_1, (0.4, 0.7, 0.4), (0.7, 0.5, 0.3), (0.8, 0.5, 0.7)), (z_2, (0.2, 0.7, 0.4), (0.3, 0.8, 0.9), (0.2, 0.1, 0.6))\}
\]

Then,

\[
\Omega_X \lor \Psi_Y = \{e_1, (z_1, (0.7, 0.5, 0.5), (0.4, 0.5, 0.4), (0.3, 0.5, 0.3)), (z_2, (0.3, 0.4, 0.5), (0.6, 0.4, 0.5), (0.3, 0.5, 0.8)), \\
e_2, (z_1, (0.3, 0.5, 0.7), (0.3, 0.8, 0.3), (0.3, 0.4, 0.3)), (z_2, (0.7, 0.6, 0.7), (0.5, 0.8, 0.7), (0.2, 0.7, 0.3)), \\
e_3, (z_1, (0.4, 0.7, 0.4), (0.7, 0.5, 0.3), (0.8, 0.5, 0.7)), (z_2, (0.2, 0.7, 0.4), (0.3, 0.8, 0.9), (0.2, 0.1, 0.6))\}
\]

\[
\Omega_X \land \Psi_Y = \{e_2, (z_1, (0.3, 0.5, 0.6), (0.2, 0.6, 0.2), (0.7, 0.5, 0.6)), (z_2, (0.4, 0.2, 0.7), (0.3, 0.5, 0.4), (0.4, 0.7, 0.4))\}
\]

\[
\Omega_X \cup \Psi_Y = \{e_2, (z_1, (0.3, 0.5, 0.7), (0.3, 0.8, 0.3), (0.3, 0.4, 0.3)), (z_2, (0.7, 0.6, 0.7), (0.5, 0.8, 0.7), (0.2, 0.7, 0.3))\}
\]

Saeed et al., Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making
4. Properties of mNS Set Operators

In this section, we define some properties of mNS set operators that satisfied among mNS sets. We also give proof of some of them, while others can also be proved. Let $\Omega_X$, $\Psi_Y$ and $\Lambda_W$ be three mNS sets over universal set $Z$ with respect to parameter set $E$ where $X$, $Y$ and $W$ are subsets of $E$. The approximation functions of $\Omega_X$, $\Psi_Y$ and $\Lambda_W$ are defined as

\[
\omega_X(e) = \{ (z, s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z)) : z \in Z, e \in X \} \\
\psi_Y(e) = \{ (z, s_i \circ T_Y(e)(z), s_i \circ I_Y(e)(z), s_i \circ F_Y(e)(z)) : z \in Z, e \in Y \} \\
\lambda_W(e) = \{ (z, s_i \circ T_W(e)(z), s_i \circ I_W(e)(z), s_i \circ F_W(e)(z)) : z \in Z, e \in W \}
\]

for all $i = 1, 2, \cdots, m$

4.1. Idempotent properties

(i) $\Omega_X \cup \Omega_X = \Omega_X = \Omega_X \cup_R \Omega_X$  
(ii) $\Omega_X \cap \Omega_X = \Omega_X = \Omega_X \cap_\varepsilon \Omega_X$

4.2. Identity Properties

(i) $\Omega_X \cup \Phi_X = \Omega_X = \Omega_X \cup_R \Phi_X$  
(ii) $\Omega_X \cap Z_X = \Omega_X = \Omega_X \cap_\varepsilon Z_X$

4.3. Domination Properties

(i) $\Omega_X \cup \tilde{Z}_X = \tilde{Z}_X = \Omega_X \cup_R \tilde{Z}_X$  
(ii) $\Omega_X \cap \tilde{\Phi}_X = \tilde{\Phi}_X = \Omega_X \cap_\varepsilon \tilde{\Phi}_X$

4.4. Complementation Properties

(i) $\tilde{Z}_X^c = \tilde{\Phi}_X = \tilde{Z}_X$  
(ii) $\tilde{\Phi}_X^c = \tilde{Z}_X = \tilde{\Phi}_X^c$
4.5. **Double Complementation Property**

(i) \((\Omega^c_X)^c = \Omega_X = (\Omega_X)^c\)

4.6. **Absorption Properties**

(i) \(\Omega_X \cup (\Omega_X \cap \Psi_Y) = \Omega_X\)
(ii) \(\Omega_X \cap (\Omega_X \cup \Psi_Y) = \Omega_X\)
(iii) \(\Omega_X \cup_R (\Omega_X \cap_e \Psi_Y) = \Omega_X\)
(iv) \(\Omega_X \cap_e (\Omega_X \cup_R \Psi_Y) = \Omega_X\)

**Remark 4.1**

(i) Union \(\tilde{U}\) and extended intersection \(\cap_e\) do not absorb over each other among \(m\)NS sets
(ii) Restricted Union \(\cup_R\) and intersection \(\tilde{\cap}\) do not absorb over each other among \(m\)NS sets

4.7. **Commutative Properties**

(i) \(\Omega_X \cup \Psi_Y = \Psi_Y \cup \Omega_X\)
(ii) \(\Omega_X \cup_R \Psi_Y = \Psi_Y \cup_R \Omega_X\)
(iii) \(\Omega_X \cap \Psi_Y = \Psi_Y \cap \Omega_X\)
(iv) \(\Omega_X \cap_e \Psi_Y = \Psi_Y \cap_e \Omega_X\)

**Remark 4.2**

(i) OR-operator \(\lor\) and AND-operator \(\land\) do not commute among \(m\)NS sets

4.8. **Associative Properties**

(i) \(\Omega_X \cup (\Psi_Y \cup \Lambda_W) = (\Omega_X \cup \Psi_Y) \cup \Lambda_W\)
(ii) \(\Omega_X \cap (\Psi_Y \cap \Lambda_W) = (\Omega_X \cap \Psi_Y) \cap \Lambda_W\)
(iii) \(\Omega_X \cup_R (\Psi_Y \cup_R \Lambda_W) = (\Omega_X \cup_R \Psi_Y) \cup_R \Lambda_W\)
(iv) \(\Omega_X \cap_e (\Psi_Y \cap_e \Lambda_W) = (\Omega_X \cap_e \Psi_Y) \cap_e \Lambda_W\)
(v) \(\Omega_X \lor (\Psi_Y \lor \Lambda_W) = (\Omega_X \lor \Psi_Y) \lor \Lambda_W\)
(vi) \(\Omega_X \land (\Psi_Y \land \Lambda_W) = (\Omega_X \land \Psi_Y) \land \Lambda_W\)

**Proof**

\[
\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cup \lambda_Y(e)) = \max\{s_i \circ T_X(e)(z), \max(s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z))\}, \\
\quad \max\{s_i \circ I_X(e)(z), \max(s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z))\}, \\
\quad \min\{s_i \circ F_X(e)(z), \min(s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z))\}
\]

for all \(i = 1, 2, \cdots, m; e \in X \cup (Y \cup Z) = (X \cup Y) \cup Z\) and \(z \in Z\)

\[
\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cup \lambda_Y(e)) = \max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z)), \\
\quad \max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z)), \min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z))
\]

for all \(i = 1, 2, \cdots, m; e \in X \cup (Y \cup Z) = (X \cup Y) \cup Z\) and \(z \in Z\)
\[ \Rightarrow \omega_X(e) \cup (\psi_Y(e) \cup \lambda_Y(e)) = \max\{s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z)\}, \]
\[ \max\{s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z)\}, \]
\[ \min\{s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z)\} \]
for all \( i = 1, 2, \ldots, m; e \in X \cup (Y \cup Z) = (X \cup Y) \cup Z \]
\[ \Rightarrow \omega_X(e) \cup (\psi_Y(e) \cup \lambda_Y(e)) = (\omega_X(e) \cup \psi_Y(e)) \cup \lambda_Y(e) \]
for all \( i = 1, 2, \ldots, m; e \in X \cup (Y \cup Z) = (X \cup Y) \cup Z \]
\[ \Rightarrow \Omega_X \Uparrow (\Psi_Y \Uparrow \Lambda_W) = (\Omega_X \Uparrow \Psi_Y) \Uparrow \Lambda_W \]

**Proof(ii)**

\[ \Rightarrow \omega_X(e) \cap (\psi_Y(e) \cap \lambda_Y(e)) = \min\{s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z)\}, \]
\[ \min\{s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z)\}, \]
\[ \max\{s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z)\} \]
for all \( i = 1, 2, \ldots, m; e \in X \cap (Y \cap Z) = (X \cap Y) \cap Z \]
\[ \Rightarrow \omega_X(e) \cap (\psi_Y(e) \cap \lambda_Y(e)) = \min\{s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z)\}, \]
\[ \min\{s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z)\}, \]
\[ \max\{s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z)\} \]
for all \( i = 1, 2, \ldots, m; e \in X \cap (Y \cap Z) = (X \cap Y) \cap Z \]
\[ \Rightarrow \omega_X(e) \cap (\psi_Y(e) \cap \lambda_Y(e)) = (\omega_X(e) \cup \psi_Y(e)) \cup \lambda_Y(e) \]
for all \( i = 1, 2, \ldots, m; e \in X \cap (Y \cap Z) = (X \cap Y) \cap Z \]
\[ \Rightarrow \Omega_X \Uparrow (\Psi_Y \Uparrow \Lambda_W) = (\Omega_X \Uparrow \Psi_Y) \Uparrow \Lambda_W \]

Similarly, others associative properties also satisfy equality.

4.9. **Distributive Properties**

(i) \( \Omega_X \Uparrow (\Psi_Y \Uparrow \Lambda_W) = (\Omega_X \Uparrow \Psi_Y) \Uparrow (\Omega_X \Uparrow \Lambda_W) \)

(ii) \( \Omega_X \Uparrow (\Psi_Y \Uparrow \Lambda_W) = (\Omega_X \Uparrow \Psi_Y) \Uparrow (\Omega_X \Uparrow \Lambda_W) \)

(iii) \( \Omega_X \cup_R (\Psi_Y \cap \epsilon \Lambda_W) = (\Omega_X \cup_R \Psi_Y) \cap \epsilon (\Omega_X \cup_R \Lambda_W) \)

(iv) \( \Omega_X \cap \epsilon (\Psi_Y \cap \epsilon \Lambda_W) = (\Omega_X \cap \epsilon \Psi_Y) \cup_R (\Omega_X \cap \epsilon \Lambda_W) \)

(v) \( \Omega_X \cup_R (\Psi_Y \cap \epsilon \Lambda_W) = (\Omega_X \cup_R \Psi_Y) \Uparrow (\Omega_X \cup_R \Lambda_W) \)

(vi) \( \Omega_X \Uparrow (\Psi_Y \cup_R \Lambda_W) = (\Omega_X \Uparrow \Psi_Y) \cup_R (\Omega_X \Uparrow \Lambda_W) \)

**Proof(i)**

\[ \Rightarrow \omega_X(e) \cup (\psi_Y(e) \cap \lambda_W(e)) = \max\{s_i \circ T_X(e)(z), min(s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z)\}, \]
\[ \max\{s_i \circ I_X(e)(z), min(s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z)\}, \]
\[ \min\{s_i \circ T_X(e)(z), max(s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z)\} \]
for all $i = 1, 2, \ldots, m$; $e \in X \cup (Y \cap W) = (X \cup Y) \cap (X \cup W)$ and $z \in Z$

$$
\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cap \lambda_W(e) =
\min\{\max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)), \max(s_i \circ T_X(e)(z), s_i \circ T_W(e)(z))\},
\min\{\max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), \max(s_i \circ I_X(e)(z), s_i \circ I_W(e)(z))\},
\max\{\min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)), \min(s_i \circ F_X(e)(z), s_i \circ F_W(e)(z))\}
$$
for all $i = 1, 2, \ldots, m$; $e \in X \cup (Y \cap W) = (X \cup Y) \cap (X \cup W)$ and $z \in Z$

$$
\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cap \lambda_W(e) = (\omega_X(e) \cup \psi_Y(e)) \cap (\omega_X(e) \cup \lambda_W(e))
$$
for all $i = 1, 2, \ldots, m$; $e \in X \cup (Y \cap W) = (X \cup Y) \cap (X \cup W)$ and $z \in Z$

$$
\Rightarrow \Omega_X \wedge (\psi_Y \wedge \lambda_W) = (\Omega_X \wedge \psi_Y) \wedge (\Omega_X \wedge \lambda_W)
$$

**Proof (ii)**

$$
\Rightarrow \omega_X(e) \cap (\psi_Y(e) \cup \lambda_W(e) = \min\{\max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z))\},
\min\{\max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z))\},
\max\{\min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z))\}
$$
for all $i = 1, 2, \ldots, m$; $e \in X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)$ and $z \in Z$

$$
\Rightarrow \omega_X(e) \cap (\psi_Y(e) \cup \lambda_W(e) = (\omega_X(e) \cap \psi_Y(e)) \cup (\omega_X(e) \cap \lambda_W(e))
$$
for all $i = 1, 2, \ldots, m$; $e \in X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)$ and $z \in Z$

$$
\Rightarrow \Omega_X \wedge (\psi_Y \wedge \lambda_W) = (\Omega_X \wedge \psi_Y) \wedge (\Omega_X \wedge \lambda_W)
$$

Similarly, others distributive properties also satisfy equality.

**Remark 4.3**

(i) Union $\wedge$ and extended intersection $\wedge$ do not distribute over each other among $m$NS sets

(ii) OR-operator $\vee$ and AND-operator $\wedge$ do not distribute over each other among $m$NS sets

(iii) Restricted union $\cup_R$ distribute over union $\vee$ but converse does not hold true

(iv) Intersection $\wedge$ distribute over extended intersection $\wedge$ but converse does not hold true

**Counter-Example 4.1**

Let $\Omega_X = \{e_2, (z_1, (0.5, 0.6), (0.3, 0.2), (0.9, 0.7)), (z_2, (0.3, 0.6), (0.4, 0.7), (0.5, 0.8))\}$;

$\Psi_Y = \{e_1, (z_1(0.3, 0.2), (0.4, 0.6), (0.1, 0.8)), (z_2, (0.6, 0.4), (0.6, 0.8), (0.8, 0.8))\}$ and

$\Lambda_W = \{e_2, (z_1, (0.7, 0.2), (0.3, 0.5), (0.2, 0.1)), (z_2, (0.6, 0.5), (0.3, 0.6), (0.5, 0.4))\}$ be three

2-NS sets over the universal set $Z = \{z_1, z_2\}$ with respect to set of attributes $E = \{e_1, e_2\}$, then
\[ \Omega_X \cap (\Psi_Y \cup \Lambda_W) = \{ e_1, (z_1(0.3, 0.2), (0.4, 0.6), (0.1, 0.8)), (z_2, (0.6, 0.4), (0.6, 0.8), (0.8, 0.8)), \]
\[ e_2, (z_1(0.5, 0.2), (0.3, 0.2), (0.9, 0.7)), (z_2, (0.3, 0.5), (0.3, 0.6), (0.5, 0.8)) \}\]
and
\[ (\Omega_X \cap \Psi_Y) \cup (\Omega_X \cap \Lambda_W) = \{ e_1, (z_1(0.3, 0.2), (0.4, 0.6), (0.1, 0.8)), (z_2, (0.6, 0.4), (0.6, 0.8), (0.8, 0.8)), \]
\[ e_2, (z_1(0.5, 0.6), (0.3, 0.2), (0.9, 0.7)), (z_2, (0.3, 0.6), (0.4, 0.7), (0.5, 0.8)) \}

Hence, \( \Omega_X \cap (\Psi_Y \cup \Lambda_W) \neq (\Omega_X \cap \Psi_Y) \cup (\Omega_X \cap \Lambda_W) \)

4.10. De Morgan’s Properties

(i) \( (\Omega_X \cup_R \Psi_Y)^c = \Omega_X^c \cap \Psi_Y^c \)

(ii) \( (\Omega_X \cap_R \Psi_Y)^c = \Omega_X^c \cup_R \Psi_Y^c \)

(iii) \( (\Omega_X \wedge \Psi_Y)^c = \Omega_X^c \vee (\Psi_Y^c) \)

(iv) \( (\Omega_X \vee \Psi_Y)^c = \Omega_X^c \wedge \Psi_Y^c \)

(v) \( (\Omega_X \cup \Psi_Y)^c = \Omega_X^c \cap \Psi_Y^c \)

(vi) \( (\Omega_X \cap \Psi_Y)^c = \Omega_X^c \cup \Psi_Y^c \)

(vii) \( (\Omega_X \cup_R \Psi_Y)^c = \Omega_X^c \cap_R \Psi_Y^c \)

(viii) \( (\Omega_X \cap_R \Psi_Y)^c = \Omega_X^c \cup_R \Psi_Y^c \)

(ix) \( (\Omega_X \wedge \Psi_Y)^c = \Omega_X^c \vee \Psi_Y^c \)

(x) \( (\Omega_X \vee \Psi_Y)^c = \Omega_X^c \wedge \Psi_Y^c \)

(xi) \( (\Omega_X \cup \Psi_Y)^c = \Omega_X^c \cap \Psi_Y^c \)

(xii) \( (\Omega_X \cap \Psi_Y)^c = \Omega_X^c \cup \Psi_Y^c \)

\[ \Rightarrow (\omega_X(e) \cup \psi_Y(e))^c = \max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)), \]
\[ \max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), \min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)))^c \]
\[ \text{for all } i = 1, 2, \cdots, m; e \in X \cap Y \text{ and } z \in Z \]

\[ \Rightarrow (\omega_X(e) \cup \psi_Y(e))^c = \min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)), \]
\[ (1, 1, \cdots, 1) - \max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), \max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)) \]
\[ \text{for all } i = 1, 2, \cdots, m; e \in X \cap Y \text{ and } z \in Z \]

\[ \Rightarrow (\omega_X(e) \cup \psi_Y(e))^c = \min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)), \]
\[ \min\{(1, 1, \cdots, 1) - s_i \circ I_X(e)(z), (1, 1, \cdots, 1) - s_i \circ I_Y(e)(z))\}, \max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)) \]
\[ \text{for all } i = 1, 2, \cdots, m; e \in X \cap Y \text{ and } z \in Z \]

\[ \Rightarrow (\omega_X(e) \cup \psi_Y(e))^c = [s_i \circ F_X(e)(z), (1, 1, \cdots, 1) - s_i \circ I_X(e)(z), s_i \circ T_X(e)(z)] \]
\[ \cap [s_i \circ F_Y(e)(z)), (1, 1, \cdots, 1) - s_i \circ I_Y(e)(z)), s_i \circ T_Y(e)(z))] \]
\[ \text{for all } i = 1, 2, \cdots, m; e \in X \cap Y \text{ and } z \in Z \]

\[ \Rightarrow (\omega_X(e) \cup \psi_Y(e))^c = [s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z)]^c \]
\[ \cap [s_i \circ T_Y(e)(z)), s_i \circ I_Y(e)(z)), s_i \circ F_Y(e)(z))]^c \]
\[ \text{for all } i = 1, 2, \cdots, m; e \in X \cap Y \text{ and } z \in Z \]
\[ \Rightarrow (\omega_X(e) \cup \psi_Y(e))^r = \omega_X^r(e) \cap \psi_Y^r(e) \]
for all \( i = 1, 2, \cdots, m; \ e \in X \cap Y \) and \( z \in Z \)

\[ \Rightarrow (\Omega_X \cup_R \Psi_Y)^r = (\Omega_X)^r \cap (\Psi_Y)^r \]

**Proof (ii)**

\[ \Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = \min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)), \]
\[ \min(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), \max(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z))]^r \]
for all \( i = 1, 2, \cdots, m; \ e \in X \cap Y \) and \( z \in Z \)

\[ \Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = \max(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)), \]
\[ (1, 1, \cdots, 1) - \min(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), \min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)) \]
for all \( i = 1, 2, \cdots, m; \ e \in X \cap Y \) and \( z \in Z \)

\[ \Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = \max\{1, 1, \cdots, 1\} - s_i \circ I_X(e)(z), (1, 1, \cdots, 1) - s_i \circ I_Y(e)(z)\}
\[ \min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)) \]
for all \( i = 1, 2, \cdots, m; \ e \in X \cap Y \) and \( z \in Z \)

\[ \Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = [s_i \circ T_X(e)(z), (1, 1, \cdots, 1) - s_i \circ I_X(e)(z), s_i \circ T_X(e)(z)] \]
\[ \cup [s_i \circ F_Y(e)(z), (1, 1, \cdots, 1) - s_i \circ I_Y(e)(z), s_i \circ T_Y(e)(z))] \]
for all \( i = 1, 2, \cdots, m; \ e \in X \cap Y \) and \( z \in Z \)

\[ \Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = [s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z)]^r \]
\[ \cup [s_i \circ T_Y(e)(z), s_i \circ I_Y(e)(z), s_i \circ F_Y(e)(z))]^r \]
for all \( i = 1, 2, \cdots, m; \ e \in X \cap Y \) and \( z \in Z \)

\[ \Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = \omega_X^r(e) \cup \psi_Y^r(e) \]
for all \( i = 1, 2, \cdots, m; \ e \in X \cap Y \) and \( z \in Z \)

\[ \Rightarrow (\Omega_X \cap_P \Psi_Y)^r = (\Omega_X)^r \cup_R (\Psi_Y)^r \]

Similarly, all the other De Morgan’s properties can be proved in the same way.

4.11. **Exclusion and Contradiction Properties**

The Exclusion and Contradiction Properties among \( mNS \) set do not hold, we show it by a counter-example

(i) \( \Omega_X \cup \Omega_X^r \neq \tilde{Z}_X \neq \Omega_X \cup_R \Omega_X^c \)
(ii) \( \Omega_X \cup \Omega_X^c \neq \tilde{Z}_X \neq \Omega_X \cup_R \Omega_X^c \)
(iii) \( \Omega_X \cap \Omega_X^r \neq \Phi_X \neq \Omega_X \cap_R \Omega_X^c \)
(iv) \( \Omega_X \cap \Omega_X^c \neq \Phi_X \neq \Omega_X \cap_R \Omega_X^c \)

**Counter-Example 4.2**

Saeed et al., Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making
Let $\Omega_X = \{e_1, (z_1, (0.5, 0.6), (0.3, 0.2), (0.9, 0.3)), (z_2, (0.3, 0.6), (0.4, 0.7), (0.5, 0.8))\}$ be a 2-NS set over universal set $Z = \{z_1, z_2\}$ with respect to the set of attributes $X \subseteq E$, relative complement of 2-NS set $\Omega_X$ will be

$$\Omega_X^c = \{e_1, (z_1, (0.9, 0.3), (0.7, 0.8), (0.5, 0.6)), (z_2, (0.5, 0.8), (0.6, 0.3), (0.3, 0.6))\},$$

then $\Omega_X \cup \Omega_X^c = \{e_1, (z_1, (0.9, 0.6), (0.7, 0.8), (0.5, 0.3)), (z_2, (0.5, 0.8), (0.6, 0.7), (0.3, 0.6))\} \neq \bar{Z}_X$

and $\Omega_X \cap \Omega_X^c = \{e_1, (z_1, (0.5, 0.3), (0.3, 0.2), (0.9, 0.6)), (z_2, (0.3, 0.6), (0.4, 0.3), (0.5, 0.8))\} \neq \bar{X}$

Similarly, others can also be proved by counter-example.

5. Distances and Similarity Measure

In this section we define distances and similarity measure formulas for $m$NS set as follows:

5.1. Distances

**Definition 4.1** Let $Z = \{z_1, z_2, ..., z_n\}$ be a universal set, $E = \{e_1, e_2, ..., e_q\}$ be a set of attributes and $X, Y \in E$. Let $\Omega_X, \Psi_Y$ are two mNS sets over $Z$ with their $mN$ approximate mapping

$$\omega_X(e_j) = \{(z, s_i \circ T_X(e_j)(z_k), s_i \circ I_X(e_j)(z_k), s_i \circ F_X(e_j)(z_k))\}$$

$$\psi_Y(e_j) = \{(z, s_i \circ T_Y(e_j)(z_k), s_i \circ I_Y(e_j)(z_k), s_i \circ F_Y(e_j)(z_k))\}$$

for all $i = 1, 2, ..., m; j = 1, 2, ..., q$ and $k = 1, 2, ..., n$ respectively, then the distance measure between $\Omega_X$ and $\Psi_Y$ is defined as

(1) Hamming distance:

$$d_H(\Omega_X, \Psi_Y) = \frac{1}{3mq} \sum_{i=1}^{m} \sum_{j=1}^{q} \sum_{k=1}^{n} \{(s_i \circ T_X(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k)) |$$

$$+ (s_i \circ I_X(e_j)(z_k) - s_i \circ I_Y(e_j)(z_k)) |$$

$$+ (s_i \circ F_X(e_j)(z_k) - s_i \circ F_Y(e_j)(z_k))\}$$

(2) Normalized Hamming distance:

$$d_{NH}(\Omega_X, \Psi_Y) = \frac{1}{3mqn} \sum_{i=1}^{m} \sum_{j=1}^{q} \sum_{k=1}^{n} \{(s_i \circ T_X(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k)) |$$

$$+ (s_i \circ I_X(e_j)(z_k) - s_i \circ I_Y(e_j)(z_k)) |$$

$$+ (s_i \circ F_X(e_j)(z_k) - s_i \circ F_Y(e_j)(z_k))\}$$

(3) Euclidean distance:

$$d_E(\Omega_X, \Psi_Y) = \frac{1}{3mqn} \sum_{i=1}^{m} \sum_{j=1}^{q} \sum_{k=1}^{n} \{(s_i \circ T_X(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k))^2$$

$$+ (s_i \circ I_X(e_j)(z_k) - s_i \circ I_Y(e_j)(z_k))^2$$

$$+ (s_i \circ F_X(e_j)(z_k) - s_i \circ F_Y(e_j)(z_k))^2\}^{\frac{1}{2}}$$

Saeed et al., Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making
(4) Normalized Euclidean distance:

\[
\begin{align*}
 d_{NE}(\Omega_X, \Psi_Y) &= \left\{ \frac{1}{3mqn} \sum_{i=1}^{m} \sum_{j=1}^{q} \sum_{k=1}^{n} (\left| s_i \circ T_X(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k) \right|)^2 \\
 &\quad + (s_i \circ I_X(e_j)(z_k) - s_i \circ I_Y(e_j)(z_k))^2 \\
 &\quad + (s_i \circ F_X(e_j)(z_k) - s_i \circ F_Y(e_j)(z_k))^2 \right\}^{\frac{1}{2}}
\end{align*}
\]

Theorem 5.1 The distance measures between \( \Omega_X \) and \( \Psi_Y \) satisfy the following inequality

1. \( d_H(\Omega_X, \Psi_Y) \leq n \)
2. \( d_{NH}(\Omega_X, \Psi_Y) \leq 1 \)
3. \( d_E(\Omega_X, \Psi_Y) \leq \sqrt{n} \)
4. \( d_{NE}(\Omega_X, \Psi_Y) \leq 1 \)

Theorem 5.2

The distance mappings \( d_H, d_{NH}, d_E \) and \( d_{NE} \) are defined from \( mN^Z \to R^+ \) are metric

Proof

Let \( \Omega_X = (\omega, X), \Psi_Y = (\psi, Y) \) and \( \Lambda_W = (\lambda, W) \) be three mNS sets over \( Z \), then

1. \( d_H(\Omega_X, \Psi_Y) \geq 0 \)
2. Suppose \( d_H(\Omega_X, \Psi_Y) = 0 \)

\[
\iff \frac{1}{3mqn} \sum_{i=1}^{m} \sum_{j=1}^{q} \sum_{k=1}^{n} (\left| s_i \circ T_X(e_j)(z_k) - p_i \circ T_Y(e_j)(z_k) \right| + \left| s_i \circ I_X(e_j)(z_k) - p_i \circ I_Y(e_j)(z_k) \right| + \left| s_i \circ F_X(e_j)(z_k) - p_i \circ F_Y(e_j)(z_k) \right|) = 0
\]

for all \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, q \) and \( k = 1, 2, \ldots, n \)

\[
\iff \left| s_i \circ T_X(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k) \right| + \left| s_i \circ I_X(e_j)(z_k) - s_i \circ I_Y(e_j)(z_k) \right| + \left| s_i \circ F_X(e_j)(z_k) - s_i \circ F_Y(e_j)(z_k) \right| = 0
\]

for all \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, q \) and \( k = 1, 2, \ldots, n \)

\[
\iff \left| s_i \circ T_X(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k) \right| = 0
\]

for all \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, q \) and \( k = 1, 2, \ldots, n \)
\[ s_i \circ T_X(e_j)(z_k) = s_i \circ T_Y(e_j)(z_k), \]
\[ s_i \circ I_X(e_j)(z_k) = s_i \circ I_Y(e_j)(z_k), \]
\[ s_i \circ F_X(e_j)(z_k) = s_i \circ F_Y(e_j)(z_k) \]
for all \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, q \) and \( k = 1, 2, \ldots, n \)

\[ \iff \quad \Omega_X = \Psi_Y \]

(3) \( d_H(\Omega_X, \Psi_Y) = d_H(\Psi_Y, \Omega_X) \)

(4) For any three mNS sets \( \Omega_X, \Psi_Y \) and \( \Lambda_W \)

\[ |s_i \circ T_X(e_j)(z_k) - s_i \circ T_W(e_j)(z_k)| \]
\[ + |s_i \circ I_X(e_j)(z_k) - s_i \circ I_W(e_j)(z_k)| \]
\[ + |s_i \circ F_X(e_j)(z_k) - s_i \circ F_W(e_j)(z_k)| \]
for all \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, q \) and \( k = 1, 2, \ldots, n \)

\[ \leq |s_i \circ T_X(e_j)(z_k) - s_i \circ T_W(e_j)(z_k)| + |s_i \circ T_W(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k)| \]
\[ + |s_i \circ I_X(e_j)(z_k) - s_i \circ I_W(e_j)(z_k)| + |s_i \circ I_Y(e_j)(z_k)| \]
\[ + |s_i \circ F_X(e_j)(z_k) - s_i \circ F_W(e_j)(z_k)| + |s_i \circ F_Y(e_j)(z_k)| \]
for all \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, q \) and \( k = 1, 2, \ldots, n \)

\[ = \{ |s_i \circ T_X(e_j)(z_k) - s_i \circ T_W(e_j)(z_k)| + |s_i \circ I_X(e_j)(z_k) - s_i \circ I_W(e_j)(z_k)| \}
\[ + |s_i \circ F_X(e_j)(z_k) - s_i \circ F_W(e_j)(z_k)| \} + \{|s_i \circ T_W(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k)| \}
\[ + |s_i \circ I_W(e_j)(z_k) - s_i \circ I_Y(e_j)(z_k)| + |s_i \circ F_W(e_j)(z_k) - s_i \circ F_Y(e_j)(z_k)| \}
for all \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, q \) and \( k = 1, 2, \ldots, n \)

Thus,

\[ d_H(\Omega_X, \Psi_Y) \leq d_H(\Omega_X, \Lambda_W) + d_H(\Lambda_W, \Psi_Y) \]

5.2. Similarity Measure

**Definition 5.2** [16] The SM of \( \Omega_X \) and \( \Psi_Y \) is defined as

\[ S(\Omega_X, \Psi_Y) = \frac{1}{1 + d(\Omega_X, \Psi_Y)} \quad (5) \]

where \( d(\Omega_X, \Psi_Y) \) is any of the above distance.

Saeed et al., Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making
5.3. Similarity of two \( mN \) Soft Set

**Definition 5.3** [16] The two \( mN \) soft sets \( \Omega_X \) and \( \Psi_Y \) are \( \gamma \) similar if and only if

\[
S(\Omega_X, \Psi_Y) \geq \gamma, \text{ i.e.,}
\]

\[
\Omega_X \approx^\gamma \Psi_Y \iff S(\Omega_X, \Psi_Y) \geq \gamma, \gamma \in (0, 1)
\]

(6)

\( \Omega_X \) and \( \Psi_Y \) are significantly similar if \( S(\Omega_X, \Psi_Y) \geq 0.5 \)

**Theorem 5.3**

The \( SM \) of \( \Omega_X \) and \( \Psi_Y \) over \( Z \) satisfies the following.

1. \( 0 \leq S(\Omega_X, \Psi_Y) \leq 1 \)
2. \( S(\Omega_X, \Psi_Y) = S(\Psi_Y, \Omega_X) \)
3. \( S(\Omega_X, \Psi_Y) = 1 \iff \Omega_X = \Psi_Y \)

6. Application of \( SM \) for \( mN \) Soft Set

In this section, we utilize similarity measure for \( mNS \) set in two different real-life applications like as in medical diagnosis and decision-making for selection of a lecturer for university.

6.1. Case Study I

We use the notion of Similarity Measure to analyze whether the patient has dengue fever or not. An algorithm is given as follows

6.1.1. Algorithm

Step 1: Construct set of parameters \( E = \{e_1, e_2, \cdots, e_q\} \) as all symptoms of a disease.

Step 2: Construct an \( mN \) soft set \( \Omega_X \) of disease by a medical expert.

Step 3: Construct an \( mN \) soft set \( \Psi_Y \) by the medical report of the patient.

Step 4: Compute the distance between \( \Omega_X \) and \( \Psi_Y \) by using the distance formula

\[
d_H(\Omega_X, \Psi_Y) = \frac{1}{3m} \sum_{i=1}^{m} \sum_{j=1}^{q} \sum_{k=1}^{n} \left( |s_i \circ T_X(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k)|
+
|s_i \circ I_X(e_j)(z_k) - s_i \circ I_Y(e_j)(z_k)|
+
|s_i \circ F_X(e_j)(z_k) - s_i \circ F_Y(e_j)(z_k)| \right)
\]

Step 5: Calculate similarity measure between \( \Omega_X \) and \( \Psi_Y \) by using formula

\[
S(\Omega_X, \Psi_Y) = \frac{1}{1 + d(\Omega_X, \Psi_Y)}
\]

Step 6: Analyze the result using similarity.
6.1.2. Problem Formulation and Assumptions

The proposed algorithm can be utilized in medical diagnosis problems, here we are giving one numerical example of solution for such medical diagnosis problem in the light of mathematics. This proposed algorithm can be applied for any medical disease diagnosis problems. We consider dengue fever disease as an medical diagnosis problem, whether a considered patient has dengue fever or not, since many of the symptoms of dengue fever are matched with other diseases such as malaria. For specification of disease we applied similarity measure on \( mNS \) structure to get insured and accurate results. The \( m \)-polar structure gives us data of \( m \) medical experts evaluation for particular disease.

6.1.3. Application of Algorithm

Now we consider a universal set \( Z = \{ z_1 = \text{dengue fever}, z_2 = \text{not dengue fever} \} \)
We consider set of parameters \( E = \{ e_1 = \text{High Fever}, e_2 = \text{Bleeding}, e_3 = \text{Severe Pain} \} \) as some of the symptoms of dengue fever disease, where these parameters can be described as,
The patient may have "High Fever" may also suffering from irritability and headache
"Bleeding" from gums or under the skin or from nose
"Severe Pain" in joints or in muscles
Let \( X, Y \subseteq E \). Then we construct an 3–NS set \( \Omega_X \) with the help of 3 medical expert (doctor) as follows:

\[
\begin{array}{c|ccc|ccc}
\Omega_X & z_1 & & z_2 & & \\
 & (0.69,0.52,0.61),(0.37,0.44,0.23),(0.46,0.37,0.29) & & (0.54,0.63,0.55),(0.48,0.44,0.26),(0.63,0.47,0.59) \\
e_1 & (0.63,0.57,0.54),(0.47,0.46,0.32),(0.62,0.75,0.67) & & (0.45,0.71,0.50),(0.50,0.43,0.26),(0.61,0.50,0.47) \\
e_2 & (0.43,0.66,0.62),(0.48,0.45,0.53),(0.47,0.52,0.36) & & (0.17,0.23,0.29),(0.37,0.41,0.47),(0.53,0.59,0.61) \\
e_3 & (0.34,0.47,0.27),(0.46,0.48,0.37),(0.75,0.58,0.69) & & (0.58,0.53,0.55),(0.37,0.35,0.32),(0.65,0.63,0.59) \\
\end{array}
\]

Table 1: 3–NS set by 3 medical expert (doctor)

Then we construct a 3–NS set \( \Psi_Y \) by a medical report of the patient as follows:

\[
\begin{array}{c|ccc|ccc}
\Psi_Y & z_1 & & z_2 & & \\
 & (0.63,0.57,0.54),(0.47,0.46,0.32),(0.62,0.75,0.67) & & (0.45,0.71,0.50),(0.50,0.43,0.26),(0.61,0.50,0.47) \\
e_1 & (0.47,0.59,0.69),(0.53,0.50,0.60),(0.43,0.58,0.32) & & (0.15,0.25,0.25),(0.32,0.40,0.43),(0.53,0.60,0.60) \\
e_2 & (0.27,0.38,0.24),(0.58,0.37,0.47),(0.65,0.69,0.70) & & (0.47,0.46,0.64),(0.44,0.40,0.30),(0.61,0.60,0.68) \\
\end{array}
\]

Table 2: 3–NS set by a medical report of a patient
Computing distances between \( \Omega_X \) and \( \Psi_Y \) and the results are
\[
\begin{align*}
d_H(\Omega_X, \Psi_Y) &= 0.1381 \\
d_{NH}(\Omega_X, \Psi_Y) &= 0.0690 \\
d_E(\Omega_X, \Psi_Y) &= 0.0195 \\
d_{NE}(\Omega_X, \Psi_Y) &= 0.0097
\end{align*}
\]
Using Eculidean distance to calculate similarity measure of \( \Omega_X \) and \( \Psi_Y \) and result is as follows
\[
S(\Omega_X, \Psi_Y) = 0.98 \geq 0.5
\]
Since \( S(\Omega_X, \Psi_Y) \) is greater than 0.5, i.e. the similarity measure of two 3-NS sets is significantly similar, this implies a patient is suffering from dengue fever.

6.2. Case Study II

Here, we generate an example of selecting lecturer for university after seeing candidate’s interview reports. An algorithm is given as follows;

6.2.1. Algorithm

Step 1: Construct a set of attribute of selection purpose as \( E = \{e_1, e_2, \cdots, e_q\} \)

Step 2: Construct an \( m \)NS set \( \Omega_X \) as the requirements of a firm concluded by decision-making team.

Step 3: Construct \( t m \)NS sets \( \Psi_Y^h \) by the help of evaluation of different alternatives given by decision-making team, where \( h = 1, 2, \cdots, t \)

Step 4: Compute the distance between \( \Omega_X \) and \( \Psi_Y^h \) by using the distance formula

\[
d_E(\Omega_X, \Psi_Y) = \frac{1}{3mq} \sum_{i=1}^{m} \sum_{j=1}^{q} \sum_{k=1}^{n} ((s_i \circ T_X(e_j)(z_k) - s_i \circ T_Y(e_j)(z_k))^2 \\
+ (s_i \circ I_X(e_j)(z_k) - s_i \circ I_Y(e_j)(z_k))^2 \\
+ (s_i \circ F_X(e_j)(z_k) - s_i \circ F_Y(e_j)(z_k))^2) \frac{1}{2}
\]

Step 5: Calculate the similarity measure between \( \Omega_X \) and \( \Psi_Y^h \) using formula

\[
S(\Omega_X, \Psi_Y^h) = \frac{1}{1 + d(\Omega_X, \Psi_Y^h)}
\]

Step 6: Analyze the result using similarity that which alternative is more suitable to be select as a lecturer.

6.2.2. Problem Formulation and Assumption

A Similarity measure of \( m \)NS sets can help in decision-making problem of selection of best alternative corresponds to attribute of selection purpose. It can be applicable in problems of group decision making where a group of people gives their own evaluation to all alternatives corresponds to attribute and wants an alternative to be selected which fulfills or near to the evaluation that was given individually by them. We consider one example of such type of problems that is department of mathematics wants to hire a new lecturer for university. The new lecturer must well aware of both Pure and Applied Mathematics. They made a decision-making team of three-person for the selection of a lecturer. All member of team first evaluate the requirements of a department for selecting the purpose of a new lecturer individually, then they took interviews and demo classes of four applicants who are willing to be a lecturer in that university and made reports of every applicant for decision-making process.
6.2.3. Application of Algorithm

Consider a universal set \( Z = \{ z_1 = \text{Pure Math}, z_2 = \text{Applied Math} \} \) and set of attributes for the selection purpose as \( E = \{ e_1 = \text{Teaching Techniques}, e_2 = \text{Research Work}, e_3 = \text{Expertise} \} \). Let \( X = Y \subseteq E \), then we construct a 3–NS set \( \Omega_X \) as requirements of a department.

Now we will construct four 3–NS sets \( \Psi^1_Y \), \( \Psi^2_Y \), \( \Psi^3_Y \), and \( \Psi^4_Y \) of different applicants \( A_1 \), \( A_2 \), \( A_3 \) and \( A_4 \) respectively with the help of reports made by decision-making team.

The Euclidean distance between \( \Omega_X \) and \( \Psi^h_Y \) is calculated as

\[
d_E(\Omega_X, \Psi^h_Y) = 0.3798 \\
d_E(\Omega_X, \Psi^2_Y) = 0.3610
\]
\[ d_E(\Omega_X, \Psi^1_Y) = 0.0076 \]
\[ d_E(\Omega_X, \Psi^4_Y) = 0.1087 \]

The Similarity Measure of \( \Omega_X \) and \( \Psi^1_Y \) is calculated as
\[ S(\Omega_X, \Psi^1_Y) = 0.72 \]
\[ S(\Omega_X, \Psi^2_Y) = 0.73 \]
\[ S(\Omega_X, \Psi^3_Y) = 0.99 \]
\[ S(\Omega_X, \Psi^4_Y) = 0.90 \]

Since, similarity measure of \( S(\Omega_X, \Psi^3_Y) = 0.99 \) is greater than 0.5 and greater than all others, the two 3–NS sets are significantly similar, which shows applicant 3 \( (A_3) \) is more suitable and he also fulfills requirements of the university.

7. Conclusion

The existing multi-polar information is not completely defined by using the existing methods. now multi-polar neutrosophic models explain the things in a better way to solve the undetermined data having multi-polar information and having the vast applications in different fields. similarity measures based on distance play an important role to solve the problems that have indeterminacy. In this paper, we defined some basic operations and their properties on \( mN \) soft sets. Moreover, we have defined the distance-based similarity measure of multipolar neutrosophic soft sets. We have used the concept of distance-based similarity measures in medical diagnosis and decision-making of the selection of lecturers for university with along algorithms. Moreover, we defined some basic operations and their properties on \( mN \) soft sets. In the future, Group MCDM problems can be solved using different methods of MCDM (TOPSIS, VIKOR, etc).

References

1. L. A. Zadeh, Fuzzy sets, Inf. and Cont. 8, No. 3 (1965) 338-353
2. K. T. Atanassov Intuitionistic fuzzy sets, Fuzzy sets and Syst. 20, No. 1 (1986) 87-96

Saeed et al., Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making

Saeed et al., Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making

Received: 9, February, 2020 / Accepted: 13, April, 2020