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New Open Sets in N-Neutrosophic Supra Topological Spaces

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Abstract: The neutrosophic set is an imprecise set to deal the concepts of uncertainty, vagueness and irregularity, which consists of three independent functions called truth-membership, indeterminacy-membership and falsity-membership. This set is a generalization of Atanassov's intuitionistic fuzzy sets. The neutrosophic supra topological space is a set together with neutrosophic supra topology. The intension of this paper is to develop the concept of N-neutrosophic supra topological spaces. We further investigate the closure and interior operators in N-neutrosophic supra topological spaces. Moreover, some weak form of N-neutrosophic supra topological open sets are defined and establish their relations with suitable examples.

Keywords: N-neutrosophic supra topology; N-neutrosophic supra α-open set; N-neutrosophic supra semi- open set; N-neutrosophic supra pre-open set; N-neutrosophic supra β-open set.

1. Introduction

A. Lottif Zadeh[1] developed a new set to analyze imprecise, vagueness and ambiguity information, namely fuzzy set, it discuss each element along with the membership value. Fuzzy set theory [2, 3, 4, 5] was applied in various fields such control systems, artificial intelligence, biology, medical diagnosis, economics and probability. C. L. Chang [6] introduced the concept of fuzzy topological space. R. Lowen [7] further studied about the fuzzy topological compactness. AbdMonsef and Ramadan [9] introduced fuzzy supra topological spaces and its continuous mappings. In 1986, K. Atanassov [10] introduced intuitionistic fuzzy set as a generalization of the fuzzy set, by taking into account both the degrees of membership and of non-membership of an element subject to the condition that their sum does not exceed 1. Some researchers [11, 12, 13, 14, 15, 16, 17] used the intuitionistic fuzzy sets in pattern recognition, medical diagnosis, data mining process. Dogan Coker [18] generalized the fuzzy topological spaces into intuitionistic fuzzy topological spaces and further Reza Saadati and Jin Han Park [19] studied the properties of intuitionistic fuzzy topological spaces. The concept of intuitionistic fuzzy supra topological space was initiated by N. Turnal [20]. Neutrosophic set is the generalization of Atanassov's intuitionistic fuzzy set, developed by Florentin Samarandache [21, 22, 23] which is a set considering the degree of membership, the degree of indeterminacy-membership and the degree of falsity-membership whose values are real standard or non-standard subset of unit interval] 0^- ; 1^+[. Recently many researchers [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37] introduced neutrosophic numbers, several similarity measures and single-valued neutrosophic sets, which are applied in attribute decision making, information system quality, medical diagnosis, control systems, artificial intelligence, etc. Salama et al. [38, 39] defined the neutrosophic crisp set and neutrosophic topological space. In 1963, Norman Levine [40] initiated the concept of semi open sets and discussed the continuous functions in classical spaces. O.Njastad [41] showed that the family of all α-open sets forms a topology. Mashhour et al. [42] investigated the properties of pre open sets. Andrijevic [43] discussed the behavior of β-open sets in classical topology. By relaxing one of the topological axioms, Mashhour et al. [44] further developed the concept of supra topological space with the properties. Devi et al. [45] introduced the properties of α-open sets and α-continuous functions in supra topological spaces. Supra topological pre-open sets and its continuous functions are defined by O.R.Sayed [46]. Saeid Jafari et al. [47] investigated the properties of supra β-open sets and its continuity. In 2016, Lellis Thivagar et al. [48] developed a new theory called N-topological spaces and its own open sets. Apart from this, M. Lellis Thivagar and M.Arockia Dasan [49] derived some new N-topologies by the help of weak open sets and mappings in N-topological spaces. Recently, G.Jayaparthasarathy et al. [50] defined the concept of neutrosophic supra topological spaces and proposed a new method to solve medical diagnosis problems by using single valued neutrosophic score function.

The present paper is organized as follows: The second section gives some basic properties of fuzzy, intuitionistic, neutrosophic sets and neutrosophic supra topological spaces. The third section extends the concept of neutrosophic supra topological spaces into N-neutrosophic supra topological spaces with the properties of closure and interior operators. In the next section, we introduce some weak open sets in N-neutrosophic supra topological spaces, namely N-neutrosophic supra α-open sets, N-neutrosophic supra semi-open sets, N-neutrosophic supra pre-open sets and N-neutrosophic supra β-open sets. The fifth section discusses the relationship between N-neutrosophic supra topological closed sets. In the next section, we compare the neutrosophic supra topological spaces and N-neutrosophic supra topological spaces with their limitations. The seventh section states the conclusion and future work of this paper. Finally all the necessary references of this paper are given.

2. Preliminaries

In this section, we discuss some basic definitions and properties of fuzzy, intuitionistic, neutrosophic sets and neutrosophic supra topological spaces which are useful in sequel.

Definition 2.1 [1] Let X be a non empty set and a fuzzy set A on X is of the form $A = \{(x, \mu_A(x)) : x \in X\}$, where $0 \leq \mu_A(x) \leq 1$ represents the degree of membership function of each $x \in X$ to the set A. For X, I^X denotes the collection of all fuzzy sets of X.

Definition 2.2 [10] Let X be a non empty set. An intuitionistic set A is of the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A(x)$ and $\gamma_A(x)$ represent the degree of membership and non membership function respectively of each $x \in X$ to the set A and

$0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$. The set of all intuitionistic sets of X is denoted by I(X).

Definition 2.3 [21] Let X be a non empty set. A neutrosophic set A having the form $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x) \in\ ]^-0,1^+[$ represent the degree of membership (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) respectively of each $x \in X$ to the set A such that $-_0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^+$ for all $x \in X$. For X, $N(X)$ denotes the collection of all neutrosophic sets of X.

Definition 2.4. [22] The following statements are true for neutrosophic sets A and B on X:

$\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$ if and only if $A \subseteq B$.

$A \subseteq B$ and $B \subseteq A$ if and only if $A = B$.

$A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \min\{\sigma_A(x), \sigma_B(x)\}, \max\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.

$A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \max\{\sigma_A(x), \sigma_B(x)\}, \min\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.

More generally, the intersection and the union of a collection of neutrosophic sets $\{A_i\}_{i \in \Lambda}$, are defined by $\bigcap_{i \in \Lambda} A_i = \{(x, inf_{i \in \Lambda}\{\mu_{A_i}(x)\}, inf_{i \in \Lambda}\{\sigma_{A_i}(x)\}, sup_{i \in \Lambda}\{\gamma_{A_i}(x)\}) : x \in X\}$ and $\bigcup_{i \in \Lambda} A_i = \{(x, sup_{i \in \Lambda}\{\mu_{A_i}(x)\}, sup_{i \in \Lambda} \sigma_{A_i}(x)\}, inf_{i \in \Lambda}\{\gamma_{A_i}(x)\}) : x \in X\}$.

Corollary 2.5. [23] The following statements are true for the neutrosophic sets A, B, C and D on X :

$A \cap C \subseteq B \cap D$ and $A \cup C \subseteq B \cup D$, if $A \subseteq B$ and $C \subseteq D$.

$A \subseteq B \cap C$, if $A \subseteq B$ and $A \subseteq C$. $A \cup B \subseteq C$, if $A \subseteq C$ and $B \subseteq C$.

$A \subseteq C$, if $A \subseteq B$ and $B \subseteq C$.

Definition 2.6. [50] Let A, B be two neutrosophic sets of X, then the difference of A and B is a neutrosophic set on X, defined as $A \setminus B = \{(x, |\mu_A(x) - \mu_B(x)|, |\sigma_A(x) - \sigma_B(x)|, 1 - |\gamma_A(x) - \gamma_B(x)|) : x \in X\}$. Clearly $X^c = X \setminus X = (x, 0,0,1) = \emptyset$ and $\emptyset^c = X \setminus \emptyset = (x, 1,1,0) = X$.

Notation 2.7. Let X be a non empty set. We consider the neutrosophic empty set as $\emptyset = \{(x, 0, 0, 1) : x \in X\}$ and the neutrosophic whole set as $X = \{(x, 1, 1, 0) : x \in X\}$.

Corollary 2.8. [50] The following statements are true for the neutrosophic sets $\{A\}_{i=1}^{\infty}$, A, B on X:

$(i) \cap_{i\in\Lambda} (A_i)^c = \cup_{i\in\Lambda} Ai^c, (\cup_{i\in\Lambda} Ai)^c = \cap_{i\in\Lambda} Ai^c$.

(ii) $(A^c)^c = A$.

iii) $B^c \subseteq A^c$, if $A \subseteq B$.

Definition 2.9. [39] Let X be a non empty set. A subfamily τ_n of $N(X)$ is said to be a neutrosophic topology on X if the neutrosophic sets $X$and $\emptyset$ belong to τ_n, $\tau_n$is closed under arbitrary union and $\tau_n$is closed under finite intersection. Then (X, τ_n) is called neutrosophic topological space (shortly nts), members of $\tau_n$are known as neutrosophic open sets and their complements are neutrosophic closed sets. For a neutrosophic set $A$of X, the interior and closure of A are respectively defined as: $int_n(A) = \cup\{G : G \subseteq A, G \in \tau_n\}$ and $cl_n(A) = \cap\{F : A \subseteq F, F^c \in \tau_n\}$.

Definition 2.10. [50] Let Xbe a non empty set. A sub collection$\tau_n{}^* \subseteq N(X)$ is said to be a neutrosophic supra topology on X if the sets $\emptyset$, X$\in \tau_n{}^*$and $\tau_n{}^*$is closed under arbitrary union. Then the ordered pair $(X, \tau_n{}^*)$ is called neutrosophic supra topological space on X(for short nsts). The elements of $\tau_n{}^*$are known as neutrosophic supra open sets and its complement is called neutrosophic supra closed. Let (X, τ_n) be a neutrosophic topological space, then a neutrosophic supra topology $\tau_n{}^*$on $X$is said to be an associated neutrosophic supra topology with τ_n if $\tau_n \subseteq \tau_n{}^*$. Every neutrosophic topology on X is neutrosophic supra topology on X.

Definition 2.11. [50] Let A be a neutrosophic set on nsts$(X, \tau_n{}^*)$, then the $int_{\tau_n{}^*}(A)$ and $cl\tau_n{}^*(A)$ are respectively defined as: $int_{\tau_n{}^*}(A) = \cup\{G : G \subseteq A$ and $G \in \tau_n{}^*\}$ and $cl_{\tau_n{}^*}(A) = \cap\{F : A \subseteq F$ and$F^c \in \tau_n{}^*\}$.

3. *N*-Neutrosophic Supra Topological Spaces

In this section, we introduce N-neutrosophic supra topological spaces and investigate the properties of closure, interior operators in *N*-neutrosophic supra topological spaces.

Definition 3.1. Let X be a non empty set, $\tau_{n_1}{}^*, \tau_{n_2}{}^*, \ldots, \tau_{n_N}{}^*$ be *N*-arbitrary neutrosophic supra topologies defined on X. Then the collection $N\tau_n{}^* = \{S \subseteq X : S = \cup_{i=1}^{N} Ai, Ai \in \tau_{n_i}{}^*\}$ is said to be a *N*-neutrosophic supra topology if it satisfies the following axioms:

$X, \emptyset \in N\tau_n{}^*$.

$\cup_{i=1}^{\infty} Si \in N\tau_n{}^* for\ all\ S_i \in N\tau_n{}^*$.

Then the *N*-neutrosophic supra topological space is the non empty set X together with the collection $N\tau_n{}^*$,denoted by $(X, N\tau_n{}^*)$ and its elements are known as $N\tau_n{}^*$-open sets on X. A neutrosophic subset A of X is said to be $N\tau_n{}^*$-closed on X if $X \setminus A$ is $N\tau_n{}^*$-open on X. The set of all $N\tau_n{}^*$-open sets on X and the set of all $N\tau_n{}^*$-closed sets on $X$are respectively denoted by $N\tau_n{}^*O(X)$ and $N\tau_n{}^*C(X)$.

Remark 3.2. For instance, if $N = 1$, then $(X, 1\tau_n{}^* = \tau_n{}^*)$ is called the classical neutrosophic supra topological space [50]. If $N = 2$, then $(X, 2\tau_n{}^*)$ is called the bi neutrosophic supra topological space. If $N = 3$, then$(X, 3\tau_n{}^*)$ is called the tri neutrosophic supra topological space defined on $X$and so on.

Example 3.3. Let $X = \{a, b, c\}, N = 4$, assume the neutrosophic supra topologies $\tau_{n_1}{}^* = \{\emptyset, X, ((0.5, 0 .5, 0.5), (1, 1, 0), (0, 0, 1))\}, \tau_{n_2}{}^* = \{\emptyset, X, ((0.25, 0.25, 0.75),$

$(0, 0, 1), (1, 1, 0))\}$, $\tau_{n_3}{}^* = \{\emptyset, X, ((0.5, 0.5, 0.5), (1, 1, 0), (1, 1, 0))\}$ and

$\tau_{n_4}{}^* = \{\emptyset, X, ((0.5, 0.5, 0.5), (1, 1, 0), (0, 0, 1)), ((0.5, 0.5, 1), (1, 1, 0), (1, 1, 0))\}$.

$4\tau_n{}^* = \{\emptyset, X, ((0.5, 0.5, 0.5), (1, 1, 0), (0, 0, 1)), ((0.25, 0.25, 0.75), (0, 0, 1), (1, 1, 0)),$

$((0.5, 0.5, 0.5), (1, 1, 0), (1,1,0))\}$ and $(4\tau_n{}^*)^c = \{X, \emptyset, ((0.5, 0.5, 0.5), (0, 0, 1), (1, 1, 0)),$

$((0.75, 0.75, 0.25), (1, 1, 0), (0, 0, 1)), ((0.5, 0.5, 0.5), (0, 0, 1), (0, 0, 1))\}$. Therefore $(X, 4\tau_n{}^*)$ is a quad neutrosophic supra topological space on X.

Remark 3.4. (i) If $N = 1$, then $N\tau_n{}^* = \tau_n{}^*$.

(ii) Union of two N-neutrosophic supra topologies is again an N-neutrosophic supra topology.

(iii) Intersection of two N-neutrosophic supra topologies is again an N-neutrosophic supra topology.

Proof. (i): The proof is trivial.

(ii): Let $(N\tau_n{}^*)_1$ and $(N\tau_n{}^*)_2$ be two N-neutrosophic supra topologies on X. Clearly, X and $\emptyset$ are the elements of$(N\tau_n{}^*)_1 \cup (N\tau_n{}^*)_2$. Let $\{A_i\}_{i\in\Lambda} \in (N\tau_n{}^*)_1 \cup (N\tau_n{}^*)_2$, then by definition of N-neutrosophic supra topology, $\cup_{i\in\Lambda} A_i \in (N\tau_n{}^*)_1 \cup (N\tau_n{}^*)_2$. Thus the union of two N-neutrosophic supra topologies is a N-neutrosophic supra topology.

(iii): Let $(N\tau_n{}^*)_1$ and $(N\tau_n{}^*)_2$be two N-neutrosophic supra topologies on X. Clearly, $X$and $\emptyset$ are the elements of$(N\tau_n{}^*)_1 \cap (N\tau_n{}^*)_2$. Let $\{A_i\}_{i\in\Lambda} \in (N\tau_n{}^*)_1 \cap (N\tau_n{}^*)_2$, then,$\cup_{i\in\Lambda} A_i \in (N\tau_n{}^*)_1$, $\cup_{i\in\Lambda} A_i \in (N\tau_n{}^*)_2$ and so $\cup_{i\in\Lambda} A \in (N\tau_n{}^*)_1 \cap (N\tau_n{}^*)_2$. Thus the intersection of two N-neutrosophic supra topologies is a N-neutrosophic supra topology.

Remark 3.5. In classical N-topological spaces, the union of two N-topologies need not be a N-topology. But this statement is not true in N-neutrosophic supra topological spaces as proved above. Thus the union of two N-neutrosophic supra topologies is a N-neutrosophic supra topology.

Definition 3.6. Let $(X, N\tau_n{}^*)$ be a N-neutrosophic supra topological space and A be a neutrosophic set of X. Then

$N\tau_n{}^*$ -interior of A is defined by $int_{N\tau_n{}^*}(A) = \cup\{G : G \subseteq S$ and G is $N\tau_n{}^*$-open$\}$.

$N\tau_n{}^*$-closure of A is defined by $cl_{N\tau_n{}^*}(A) = \cap\{F : A \subseteq F$ and F is $N\tau_n{}^*$-closed$\}$.

Theorem 3.7. The following are true for neutrosophic sets A and B of N-neutrosophic supra topological space $(X, N\tau_n{}^*)$:

$A = cl_{N\tau_n{}^*}(A)$ if and only if A is N-neutrosophic supra closed.

$A = int_{N\tau_n{}^*}(A)$ if and only if A is N-neutrosophic supra open.

$cl_{N\tau_n{}^*}(A) \subseteq cl_{N\tau_n{}^*}(B)$, if $A \subseteq B$.

$int_{N\tau_n{}^*}(A) \subseteq int_{N\tau_n{}^*}(A)$, if $A \subseteq B$.

$cl_{N\tau_n{}^*}(A) \cup cl_{N\tau_n{}^*}(B) \subseteq cl_{N\tau_n{}^*}(A \cup B)$.

$int_{N\tau_n{}^*}(A) \cup int_{N\tau_n{}^*}(B) \subseteq int_{N\tau_n{}^*}(A \cup B)$.

$cl_{N\tau_n{}^*}(A) \cap cl_{N\tau_n{}^*}(B) \supseteq cl_{N\tau_n{}^*}(A \cap B)$.

$int_{N\tau_n{}^*}(A) \cap int_{N\tau_n{}^*}(B) \supseteq int_{N\tau_n{}^*}(A \cap B)$.

$int_{N\tau_n{}^*}(A^c) = (cl_{N\tau_n{}^*}(A))^c$.

$(int_{N\tau_n{}^*}(A))^c = cl_{N\tau_n{}^*}(A^c)$.

Proof. (i): Since $A = cl_{N\tau_n{}^*}(A)$ and by definition $cl_{N\tau_n{}^*}(A)$ is N-neutrosophic supra closed, then A is N-neutrosophic supra closed. Conversely, if B is any N-neutrosophic supra closed containing A, and since $cl_{N\tau_n{}^*}(A)$ is the intersection of all N-neutrosophic supra closed sets containing A, then $cl_{N\tau_n{}^*}(A) \subseteq B$ and $cl_{N\tau_n{}^*}(A)$ is the smallest N-neutrosophic supra closed set containing A. Since A is N-neutrosophic supra closed, then the smallest N-neutrosophic supra closed set containing A is A itself. Therefore, $A = cl_{N\tau_n{}^*}(A)$.

(ii): Since $A = int_{N\tau_n^*}(A)$ and by definition $int_{N\tau_n^*}(A)$ is N-neutrosophic supra open, then A is N-neutrosophic supra open. Conversely, if B is any N-neutrosophic supra open contained in A, and since $int_{N\tau_n^*}(A)$ is the union of all N-neutrosophic supra open sets contained in A, then $int_{N\tau_n^*}(A) \supseteq B$ and $int_{N\tau_n^*}(A)$ is the largest N-neutrosophic supra open set contained in A. Since A is N-neutrosophic supra open, then the largest N-neutrosophic supra open set contained in A is A itself. Therefore, $A = int_{N\tau_n^*}(A)$.

(iii):
$cl_{N\tau_n^*}(B) = \cap\{G : G^c \in N\tau_n^*, B \subseteq G\} \supseteq \cap\{G : G^c \in N\tau_n^*, A \subseteq G\} = cl_{N\tau_n^*}(A)$.

Thus, $cl_{N\tau_n^*}(A) \subseteq cl_{N\tau_n^*}(B)$.

(iv):
$int_{N\tau_n^*}(B) = \cup\{G : G \in N\tau_n^*, B \supseteq G\} \supseteq \cup\{G : G \in N\tau_n^*, A \supseteq G\} = int_{N\tau_n^*}(A)$.
Thus, $int_{N\tau_n^*}(A) \subseteq int_{N\tau_n^*}(B)$.

(v): Since $A \cup B \supseteq A, B$, then by part (iii) $cl_{N\tau_n^*}(A) \cup cl_{N\tau_n^*}(B) \subseteq cl_{N\tau_n^*}(A \cup B)$.

(vi): Since $A \cup B \supseteq A, B$, then by part (iv) $int_{N\tau_n^*}(A) \cup int_{N\tau_n^*}(B) \subseteq int_{N\tau_n^*}(A \cup B)$.

(vii): Since $A \cap B \subseteq A, B$, then by part (iii) $int_{N\tau_n^*}(A) \cap int_{N\tau_n^*}(B) \supseteq int_{N\tau_n^*}(A \cap B)$.

(viii): Since $A \cap B \subseteq A, B$, then by part (iv) $int_{N\tau_n^*}(A) \cap int_{N\tau_n^*}(B) \subseteq int_{N\tau_n^*}(A \cap B)$.

(ix): $cl_{N\tau_n^*}(A) = \cap\{G : G^c \in N\tau_n^*, G \supseteq A\}$, $(cl_{N\tau_n^*}(A))^c = \cup\{G^c : G^c$ is a N-neutrosophic supra open in X and $G^c \subseteq A^c\} = int_{N\tau_n^*}(A^c)$. Thus, $(cl_{N\tau_n^*}(A))^c = int_{N\tau_n^*}(A^c)$

(x): $int_{N\tau_n^*}(A) = \cup\{G : G \in N\tau_n^*, G \subseteq A\}, (int_{N\tau_n^*}(A))^c = \cap\{G^c : G^c$ is a N-neutrosophic supra closed in X and $G^c \supseteq A^c\} = cl_{N\tau_n^*}(A^c)$. Thus, $(int(A))^c = int_{N\tau_n^*}(A^c)$.

Remark 3.8. If we take complement of either side of (ix) and (x) of previous theorem, we get

(i) $cl_{N\tau_n^*}(A) = (int_{N\tau_n^*}(A^c))^c$.

(ii) $int_{N\tau_n^*}(A) = (cl_{N\tau_n^*}(A^c))^c$.

Theorem 3.9. Let $(X, N\tau_n^*)$ be a N-neutrosophic supra topological space and A be a neutrosophic set of X. Then

(i) $int_{N\tau_n^*}(A) \supseteq int_{\tau_{n_1}^*}(A) \cup int_{\tau_{n_2}^*}(A) \cup \ldots \cup int_{\tau_{n_N}^*}(A)$.

(ii) $cl_{N\tau_n^*}(A) \subseteq cl_{\tau_{n_1}^*}(A) \cap cl_{\tau_{n_2}^*}(A) \cap \ldots \cap cl_{\tau_{n_N}^*}(A)$.

Proof. (i): By definition of N-neutrosophic supra topological space, we have $N\tau_n^* = \{S \subseteq X : S = \cup_{i=1}^{N} A_i,\ A_i \in \tau_{n_i}^*\} \supseteq \tau_{n_1}^* \cup \tau_{n_2}^* \cup \ldots \cup \tau_{n_N}^*$.

Therefore, $int_{N\tau_n^*}(A) \supseteq int_{\tau_{n_1}^*}(A) \cup int_{\tau_{n_2}^*}(A) \cup \ldots \cup int_{\tau_{n_N}^*}(A)$.

(ii): Since $int_{N\tau_n^*}(A^c) \supseteq int_{\tau_{n_1}^*}(A^c) \cup int_{\tau_{n_2}^*}(A^c) \cup \ldots \cup int_{\tau_{n_N}^*}(A^c)$, then $(cl_{N\tau_n^*}(A))^c \supseteq (cl_{\tau_{n_1}^*}(A))^c \cup (cl_{\tau_{n_2}^*}(A))^c \cup \ldots \cup (cl_{\tau_{n_N}^*}(A))^c$ which implies $cl_{N\tau_n^*}(A) \subseteq cl_{\tau_{n_1}^*}(A) \cap cl_{\tau_{n_2}^*}(A) \cap \ldots \cap cl_{\tau_{n_N}^*}(A)$.

4. N-Neutrosophic Supra Topological Weak Open Sets

In this section, we introduce some new classes of N-neutrosophic supra topological open sets and discuss the relationship between them.

Definition 4.1. A neutrosophic set A of a N-neutrosophic supra topological space $(X, N\tau_n^*)$ is called

N-neutrosophic supra α-open set if $A \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(A)))$.

N-neutrosophic supra semi-open set if $A \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(A))$.

N-neutrosophic supra pre-open set if $A \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(A))$.

N-neutrosophic supra β-open set if $A \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(cl_{N\tau_n^*}(A)))$.

The set of all N-neutrosophic supra α-open (resp. N-neutrosophic supra semi-open, N-neutrosophic supra pre-open and N-neutrosophic supra β-open) sets of $(X, N\tau_n^*)$ is denoted by $N\tau_n^*O(X)$ (resp. $N\tau_n^*SO(X), N\tau_n^*PO(X)$ and $N\tau_n^*\beta O(X)$.

Theorem 4.2. Let *A* be a subset of N-neutrosophic supra topological space $(X, N\tau_n^*)$. Then

every N-neutrosophic supra open set is N-neutrosophic supra α-open.

every N-neutrosophic supra α-open set is N-neutrosophic supra semi-open.

every N-neutrosophic supra α-open set is N-neutrosophic supra pre-open every N-neutrosophic supra semi-open set is N-neutrosophic supra β-open.

every N-neutrosophic supra pre-open set is N-neutrosophic supra β-open.

Proof.(i): Assume A is N-neutrosophic supra open, $int_{N\tau_n^*}(A) = A$.

Since $A \subseteq cl_{N\tau_n^*}(A)$, $int_{N\tau_n^*}(A) \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(A))$.

Then $A \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(A)))$. Therefore, A is N-neutrosophic supra semi-open.

(ii): Assume A is N-neutrosophic supra α-open and since $int_{N\tau_n^*}(A) \subseteq A$, then $A \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(A))) \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(A))$. Therefore, A is N-neutrosophic supra semi-open.

(iii): Assume A is N-neutrosophic supra α-open and since $int_{N\tau_n^*}(A) \subseteq A$, then

$cl_{N\tau_n^*}(int_{N\tau_n^*}(A)) \subseteq cl_{N\tau_n^*}(A)$. Then $A \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(A))) \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(A))$. Therefore, A is N-neutrosophic supra pre-open.

(iv): Assume A is N-neutrosophic supra semi-open and since $A \subseteq cl_{N\tau_n^*}(A)$, then $int_{N\tau_n^*}(A) \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(A)$). Then $A \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(A)) \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}\left(cl_{N\tau_n^*}(A)\right))$. Therefore, A is N-neutrosophic supra β-open.

(v): Assume A is N-neutrosophic supra pre-open and since $A \subseteq cl_{N\tau_n^*}(A)$, then $A \subseteq cl_{N\tau_n^*}(A) \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(cl_{N\tau_n^*}(A)))$. Therefore, A is N-neutrosophic supra β-open.

The converse of the above theorem need not be true as shown in the following examples.

Example4.3. Let $X = \{a, b\}$ and $N = 2$, assume $\tau_{n_1}{}^* = \{\emptyset, X, ((0.3,0.4),(0.3,0.4),(0.4,0.5))\}$ $\tau_{n_2}{}^* = \{\emptyset, X, \left((0.4,0.2),(0.4,0.2),(0.5,04)\right)\}$. Then $2\tau_n{}^* = \{\emptyset, X, ((0.3,0.4),(0.3,0.4),(0.4,0.5)),((0.4,0.2),(0.4,0.2),(0.5,0.4)),$

$((0.4,0.4),(0.4,0.4),(0.4,0.4))\}$ is a bi neutrosophic supra topology on X. Then the neutrosophic set $A = ((0.4,0.6),(0.4,0.6),(0.3,0.4))$ is 2-neutrosophic supra α-open but not 2-neutrosophic supra open.

Example4.4. Let $X = \{a, b\}$ and $N = 2$, assume $\tau_{n_1}{}^* = \{\emptyset, X, ((0.3, 0.5), (0.3, 0.5), (0.4, 0.5))\}$, $\tau_{n_2}{}^* = \{\emptyset, X, ((0.4, 0.3), (0.4, 0.3), (0.5, 0.2))\}$. Then

$2\tau_n{}^* = \{\emptyset, X, ((0.3, 0.5), (0.3, 0.5), (0.4, 0.5)), ((0.4, 0.3), (0.4, 0.3), (0.5, 0.2)),$

$((0.4, 0.5), (0.4, 0.5), (0.4, 0.2))\}$ is a bi neutrosophic supra topology on X. Then the neutrosophic set $A = ((0.4, 0.5), (0.4, 0.5), (0.5, 0.4))$ is 2-neutrosophic supra pre-open, 2-neutrosophic supra β-open, but not 2-neutrosophic supra α-open and not 2-neutrosophic supra semi-open.

Example4.5. Let $X = \{a, b\}$ and $N = 3$, assume

$\tau_{n_1}{}^* = \{\emptyset, X, ((0.3, 0.5), (0.3, 0.5), (0.4, 0.5))\}$, $\tau_{n_2}{}^* =$
$\{\emptyset, X, ((0.4, 0.3), (0.4, 0.3), (0.5, 0.6))\}$

and $\tau_{n_3}{}^* = \{\emptyset, X, ((0.4, 0.5), (0.4, 0.5), (0.4, 0.5))\}$. Then

$3\tau_n{}^* = \{\emptyset, X, ((0.3, 0.5), (0.3, 0.5), (0.4, 0.5)),$

$((0.4, 0.3), (0.4, 0.3), (0.5, 0.6)), ((0.4, 0.5), (0.4, 0.5), (0.4, 0.5))\}$ is a tri neutrosophic supra topology on X. Then $A = ((0.4, 0.5), (0.4, 0.5), (0.4, 0.5))$ is 3-neutrosophic supra semi-open and 3-neutrosophic supra β-open, but not 3-neutrosophic supra α-open and not 3-neutrosophic supra pre-open.

Theorem 4.6. A neutrosophic set A in a N-neutrosophic supra topological space $(X, N\tau_n{}^*)$ is N-neutrosophic supra α-open set if and only if A is both N-neutrosophic supra semi-open and N-neutrosophic supra pre-open.

Proof. Assume that A is N-neutrosophic supra α-open set, then $A \subseteq int_{N\tau_n{}^*}(cl_{N\tau_n{}^*}(int_{N\tau_n{}^*}(A))) \subseteq cl_{N\tau_n{}^*}(int_{N\tau_n{}^*}(A))$. Since $int_{N\tau_n{}^*}(A) \subseteq A$, then

$A \subseteq int_{N\tau_n{}^*}(cl_{N\tau_n{}^*}(int_{N\tau_n{}^*}(A))) \subseteq int_{N\tau_n{}^*}(cl_{N\tau_n{}^*}(A))$. Therefore, A is both N-neutrosophic supra semi-open and N-neutrosophic supra pre-open. On the other hand, assume that A is both N-neutrosophic supra semi-open and N-neutrosophic supra pre-open. Then $A \subseteq int_{N\tau_n{}^*}(cl_{N\tau_n{}^*}(A)) \subseteq int_{N\tau_n{}^*}(cl_{N\tau_n{}^*}(int_{N\tau_n{}^*}(A)))$. Therefore, A is N-neutrosophic supra α-open.

Lemma 4.7. The arbitrary union of N-neutrosophic supra α-open (resp. N-neutrosophic supra semi-open, N-neutrosophic supra pre-open, N-neutrosophic supra β-open) sets is

N-neutrosophic supra α-open (resp. N-neutrosophic supra semi-open, N-neutrosophic supra pre-open, N-neutrosophic supra β-open).

Proof. Here we only prove for N-neutrosophic supra α-open sets and similarly we can prove for N-neutrosophic supra semi-open, N-neutrosophic supra pre-open, N-neutrosophic supra β-open sets. Assume that $\{A_i\}_{i\in\Lambda} \in N\,\tau_n{}^*\alpha O(X)$, then $A_i \subseteq int_{N\tau_n{}^*}(cl_{N\tau_n{}^*}(int_{N\tau_n{}^*}(A_i)))$. Since $\cup_{i\in\Lambda} int_{N\tau_n{}^*}(A_i) \subseteq int_{N\tau_n{}^*}(\cup_{i\in\Lambda} A_i)$,
$\cup_{i\in\Lambda} cl_{N\tau_n{}^*}(int_{N\tau_n{}^*}(A_i)) \subseteq cl_{N\tau_n{}^*}(int_{N\tau_n{}^*}(\cup_{i\in\Lambda} A_i))$.
Then $\cup_{i\in\Lambda} A_i \subseteq \cup_{i\in\Lambda} int_{N\tau_n{}^*}(cl_{N\tau_n{}^*}(int_{N\tau_n{}^*}(A_i))) \subseteq int_{N\tau_n{}^*}(cl_{N\tau_n{}^*}(int_{N\tau_n{}^*}(\cup_{i\in\Lambda} A_i)))$.
Therefore, $\cup_{i\in\Lambda} A_i$ is a N-neutrosophic supra α-open set.

Remark 4.8. Intersection of any two N-neutrosophic supra α-open (resp. N-neutrosophic supra semi-open, N-neutrosophic supra pre-open, N-neutrosophic supra β-open) sets need not be a N-neutrosophic supra α -open (resp. N-neutrosophic supra semi-open, N-neutrosophic supra pre-open, N-neutrosophic supra β-open) set.

Example 4.9. Let $X = \{a, b\}$ and $N = 3$, assume $\tau_{n_1}{}^* = \{\emptyset, X, ((0.3,0.5),(0.3,0.5),(0.4,0.5))\}$, $\tau_{n_2}{}^* = \{\emptyset, X, ((0.4,0.3),(0.4,0.3),(0.5,0.4))\}$ and $\tau_{n_3}{}^* = \{\emptyset, X, ((0.4,0.5),(0.4,0.5),(0.4,0.4))\}$. Then $3\,\tau_n{}^* = \{\emptyset, X,$

$((0.3,0.5),(0.3,0.5),(0.4,0.5)),((0.4,0.3),(0.4,0.3),(0.5,0.4)),$
$((0.4,0.5),(0.4,0.5),(0.4,\ 0.4))\}$
is a tri neutrosophic supra topology on X and $(X, 3\tau_n{}^*)$ is a tri neutrosophic supra topological space on X. Here $A = ((0.3,\ 0.5),(0.3,0.5),(0.4,$ 0.5)) and $B = ((0.4,0.3),(0.4,\ 0.3),(0.5,0.4))$ are both 3-neutrosophic supra α -open and 3-neutrosophic supra semi open, but $A \cap B$ is not 3-neutrosophic supra α-open and not 3-neutrosophic supra semi-open.

Example4.10. Let $X = \{a, b, c\}, N = 3$, assume the neutrosophic supra topologies $\tau_{n_1}{}^* = \{\emptyset, X\}, \tau_{n_2}{}^* = \{\emptyset, X, ((0.6,0,0),(0.4,0.1,0),(0,0,1))\}$, $\tau_{n_3}{}^* = \{\emptyset, X, ((0.3,0.7,1),$

$(0.7,0.6,1),(1,1,0))\}$. Then $3\,\tau_n{}^* = \{\emptyset, X, ((0.6,0,0),(0.4,0.1,0),(0,0,1)),$

$((0.3,0.7,1),(0.7,0.6,1),(1,1,0)),((0.6,0.7,1),(0.7,0.6,1),(0,0,0))\}$ is a tri neutrosophic supra topology on X and $(X, 3\,\tau_n{}^*)$ is a tri neutrosophic supra topological space on X. Here the neutrosophic sets $A = ((0.6,0,0),(0.4,0.1,0),(0,0,1))$ and $B = ((0.3,0.7,1),(0.7,0.6,1),(1,1,0))$ are 3-neutrosophic supra pre-open and

3-neutrosophic supra β-open, but $A \cap B$ is not 3-neutrosophic supra pre-open and 3-neutrosophic supra β-open.

Remark 4.11. In classical topological spaces, O. Njastad [41] proved that the collection of all α-open sets form a topology which is finer than the collection of all open sets. This statement need not be true in neutrosophic topological spaces as shown in the following example, that is, the collection of all neutrosophic α-open sets need not be a neutrosophic topology, but this collection forms a neutrosophic supra topology.

Example4.12. Let $X = \{a, b\}$, assume the neutrosophic topology $\tau_n = \{\emptyset, X, ((0.3, 0.6), (0.5, 0.2), (0.4, 0.5)), ((0.2, 0.5), (0.6, 0.3), (0.7, 0.1)), ((0.3, 0.6), (0.6, 0.3), (0.4, 0.1)), ((0.2, 0.5), (0.5, 0.2), (0.7, 0.5))\}$ and (X, τ_n) is a neutrosophic topological space on X. Here $A = ((0.4, 0.8), (0.6, 0.3), (0.4, 0.4))$ and $B = ((1, 0.5), (0.9, 0.7), (0.2, 0))$ are neutrosophic α-open, but $A \cap B$ is not neutrosophic α-open.

Lemma 4.13. Let $A, B \in N(X)$ and A be a N-neutrosophic supra open set such that $int_{N\tau_n^*}(A) \subseteq B \subseteq A$, then B is N-neutrosophic supra open.

Proof. Assume that A is a N-neutrosophic supra open set such that $int_{N\tau_n^*}(A) \subseteq B \subseteq A$. Then $B \subseteq A = int_{N\tau_n^*}(A) = int_{N\tau_n^*}(B) \subseteq B$. Therefore, B is N-neutrosophic supra open.

Lemma 4.14. Let $A, B \in N(X)$ and A be a N-neutrosophic supra α-open set such that $int_{N\tau_n^*}(A) \subseteq B \subseteq A$, then B is N-neutrosophic supra α-open.

Proof. Assume that A is a N-neutrosophic supra α-open set such that $int_{N\tau_n^*}(A) \subseteq B \subseteq A$. Then $B \subseteq A \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(A))) = int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(B)))$. Therefore, B is N-neutrosophic supra α-open.

Lemma 4.15. Let $A, B \in N(X)$ and A be a N-neutrosophic supra semi-open set such that $int_{N\tau_n^*}(A) \subseteq B \subseteq A$, then B is N-neutrosophic supra semi-open.

Proof. Assume that A is a N-neutrosophic supra semi-open set such that $int_{N\tau_n^*}(A) \subseteq B \subseteq A$. Then $B \subseteq A \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(A)) = cl_{N\tau_n^*}(int_{N\tau_n^*}(B))$. Therefore, B is N-neutrosophic supra semi-open.

Lemma 4.16. Let $A, B \in N(X)$ and A be a N-neutrosophic supra pre-open set such that $cl_{N\tau_n^*}(A) \subseteq B \subseteq A$, then B is N-neutrosophic supra pre-open.

Proof. Assume that A is a N-neutrosophic supra pre-open set such that $cl_{N\tau_n^*}(A) \subseteq B \subseteq A$. Then $B \subseteq A \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(A)) \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(B))$. Therefore, B is N-neutrosophic supra pre-open.

Lemma 4.17. Let $A, B \in N(X)$ and A be a N-neutrosophic supra β-open set such that $cl_{N\tau_n^*}(A) \subseteq B \subseteq A$, then B is N-neutrosophic supra β-open.

Proof. Assume that A is a N-neutrosophic supra β-open set such that $cl_{N\tau_n^*}(A) \subseteq B \subseteq A$. Then $B \subseteq A \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(cl_{N\tau_n^*}(A))) \subseteq cl_{N\tau_n^*}(A)(int_{N\tau_n^*}(int_{N\tau_n^*}(B)))$. Therefore, B is N-neutrosophic supra β-open.

5. N-Neutrosophic Supra Topological Weak Open Sets

In this section, we introduce some weak closed sets in N-neutrosophic supra topological spaces and investigate the relationship between them.

Definition 5.1. A neutrosophic set A of a N-neutrosophic supra topological space $(X, N\tau_n^*)$ is called N-neutrosophic supra α-closed (resp. N-neutrosophic supra semi-closed, Nneutrosophic supra pre-closed and N-neutrosophic supra β-closed) if the complement of A is N-neutrosophic supra α-open (resp. N-neutrosophic supra semi-open, N-neutrosophic supra pre-open and N-neutrosophic supra β-open). The set of all N-neutrosophic supra α-closed (resp. N-neutrosophic supra semi-closed, N-neutrosophic supra pre-closed and N-neutrosophic supra β-closed) sets of $(X, N\tau_n^*)$ is denoted by $N\tau_n^*\alpha C(X)$ (resp. $N\tau_n^*SC(X), N\tau_n^*PC(X)$ and $N\tau_n^*\beta C(X)$.

Theorem 5.2. A neutrosophic set A of a N-neutrosophic supra topological space$(X, N\tau_n^*)$ is

N-neutrosophic supra α-closed if $cl_{N\tau_n^*}(int_{N\tau_n^*}(cl_{N\tau_n^*}(A))) \subseteq A$.

N-neutrosophic supra semi-closed if $int_{N\tau_n^*}(cl_{N\tau_n^*}(A)) \subseteq A$.

N-neutrosophic supra pre-closed if $cl_{N\tau_n^*}(int_{N\tau_n^*}(A)) \subseteq A$.

N-neutrosophic supra β-closed if $int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(A))) \subseteq A$.

Proof. : Here we shall prove parts (i) only and the remaining parts similarly follows. Assume A is N-neutrosophic supra α-closed, then A^c is N-neutrosophic supra α-open and $A^c \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(A^c)))$. Then $A \supseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(cl_{N\tau_n^*}(A)))$.

Theorem 5.3. Let A be a subset of N-neutrosophic supra topological space $(X, N\tau_n^*)$. Then

every N-neutrosophic supra closed set is N-neutrosophic supra α-closed.

every N-neutrosophic supra α-closed set is N-neutrosophic supra semi-closed.

every N-neutrosophic supra α-closed set is N-neutrosophic supra pre-closed.

every N-neutrosophic supra semi-closed set is N-neutrosophic supra β-closed.

every N-neutrosophic supra pre-closed set is N-neutrosophic supra β-closed.

Proof. The proof follows from theorem 4.2 and definition 5.1.

The converse of the above theorem need not be true as shown in the following examples.

Example 5.4. Consider example 4.3, the neutrosophic set $A = ((0.6, 0.4), (0.6, 0.4), (0.7, 0.6))$ is 2-neutrosophic supra α-closed but not 2-neutrosophic supra closed. Consider example 4.4, the neutrosophic set $B = ((0.6, 0.5), (0.6, 0.5), (0.5, 0.6))$ is 2-neutrosophic supra pre-closed, 2-neutrosophic supra β-closed, but not 2-neutrosophic supra α-closed and not 2-neutrosophic supra semi-closed. Consider example 4.5, the neutrosophic set $C = ((0.6, 0.6), (0.6, 0.6), (0.5, 0.6))$ is 3-neutrosophic supra semi-closed and 3-neutrosophic supra β-closed, but not 3-neutrosophic supra α-closed and not 3-neutrosophic supra pre-closed.

Theorem 5.5. A neutrosophic set A in a N-neutrosophic supra topological space$(X, N\,\tau_n{}^*)$ is N-neutrosophic supra α-closed set if and only if A is both N-neutrosophic supra semi-closed and N-neutrosophic supra pre-closed.

Proof. The proof follows directly from theorem 4.6 and definition 5.1.

Lemma 5.6. The arbitrary intersection of N-neutrosophic supra α-closed (resp. N-neutrosophic supra semi-closed, N-neutrosophic supra pre-closed, N-neutrosophic supra β-closed) sets is N-neutrosophic supra α-closed (resp. N-neutrosophic supra semi-closed, N-neutrosophic supra pre-closed, N-neutrosophic supra β-closed).

Proof. The proof follows directly from lemma 4.7 and definition 5.1.

Remark 5.7. Union of any two N-neutrosophic supra α-closed (resp. N-neutrosophic supra semi-closed, N-neutrosophic supra pre-closed, N-neutrosophic supra β-closed) sets need not be a N-neutrosophic supra α-closed (resp. N-neutrosophic supra semi-closed, N-neutrosophic supra pre-closed, N-neutrosophic supra β-closed) set.

Example5.8. Consider example 4.9, the neutrosophic sets $A = ((0.7,\ 0.5), (0.7, 0.5), (0.6, 0.5))$ and $B = ((0.6, 0.7), (0.6, 0.7), (0.5, 0.6))$ are both 3-neutrosophic supra α-closed and 3-neutrosophic supra semi-closed, but $A \cup B$ is not 3-neutrosophic supra α-closed and not 3-neutrosophic supra semi-closed. Consider example 4.10, the neutrosophic sets $A = ((0.4, 1, 1), (0.6, 0.9, 1), (1, 1, 0))$ and $B = ((0.7, 0.3, 0), (0.3, 0.4, 0), (0, 0, 1))$ are 3-neutrosophic supra pre-closed and 3-neutrosophic supra β-closed, but $A \cup B$ is not 3-neutrosophic supra pre-closed and 3-neutrosophic supra β-closed.

Lemma5.9. Let $A, B \in N(X)$ and A be a N-neutrosophic supra α-closed set such that $A \subseteq B \subseteq cl_{N\tau_n^*}(A)$, then B is N-neutrosophic supra α-closed.

Proof. Assume that A is a N-neutrosophic supra α-closed set such that $A \subseteq B \subseteq cl_{N\tau_n^*}(A)$. Then $cl_{N\tau_n^*}\left(int_{N\tau_n^*}\left(cl_{N\tau_n^*}(B)\right)\right) \subseteq cl_{N\tau_n^*}\left(int_{N\tau_n^*}\left(cl_{N\tau_n^*}(A)\right)\right) \subseteq A \subseteq B$. Therefore, B is N-neutrosophic supra α-closed.

Lemma 5.10. Let $A, B \in N(X)$ and A be a N-neutrosophic supra semi-closed set such that $A \subseteq B \subseteq cl_{N\tau_n^*}(A)$, then B is N-neutrosophic supra semi-closed.

Proof. Assume that A is a N-neutrosophic supra semi-closed set such that $A \subseteq B \subseteq cl_{N\tau_n^*}(A)$. Then $cl_{N\tau_n^*}(B) \subseteq cl_{N\tau_n^*}(A)$ and $int_{N\tau_n^*}(cl_{N\tau_n^*}(B)) \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(A)) \subseteq A \subseteq B$. Therefore, B is N-neutrosophic supra semi-closed.

Lemma 5.11. Let $A, B \in N(X)$ and A be a N-neutrosophic supra pre-closed set such that $int_{N\tau_n^*}(A) \supseteq B \supseteq A$, then B is N-neutrosophic supra pre-closed.

Proof. Assume that A is a N-neutrosophic supra pre-closed set such that $int_{N\tau_n^*}(A) \supseteq B \supseteq A$. Then $B \supseteq A \supseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(A)) \supseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(B))$. Therefore, B is N-neutrosophic supra pre-closed.

Lemma 5.12. Let $A, B \in N(X)$ and A be a N-neutrosophic supra β-closed set such that $int_{N\tau_n^*}(A) \supseteq B \supseteq A$, then B is N-neutrosophic supra β-closed.

Proof. Assume that A is a N-neutrosophic supra β-closed set such that $int_{N\tau_n^*}(A) \supseteq B \supseteq A$. Then $B \supseteq A \supseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(A))) \supseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(B)))$. Therefore, B is N-neutrosophic supra β-closed.

6.Comparison and Limitations

S.No	Neutrosophic supra topological spaces	N-Neutrosophic supra topological spaces
1	A sub collection τ_n^* of neutrosophic sets on a non empty set X is said to be a neutrosophic supra topology on X if the sets $\emptyset, X \in \tau_n^*$ and $\bigcup_{i=1}^{\infty} A_i \in \tau_n^*$, for $\{A_i\}_{i=1}^{\infty} \in \tau_n^*$. A non empty set X together with the collection τ_n^* is called neutrosophic supra topological	Let X be a non empty set, $\tau_{n_1}{}^*$, $\tau_{n_2}{}^*, \ldots, \tau_{n_N}{}^*$ be N-arbitrary neutrosophic supra topologies defined on X. Then the collection

	space on X (for short nsts) denoted by the ordered pair(X, τ_n^*). The members of τ_n^* are known as neutrosophic supra open sets.	$N\tau_n{}^* = \{S \subseteq X : S = \bigcup_{i=1}^{N} A_i, A_i \in \tau_{n_i}{}^*\}$ is said to be a N-neutrosophic supra topology if it satisfies the following axioms: (i) $X, \emptyset \in N\tau_n{}^*$. (ii) $\bigcup_{i=1}^{\infty} S_i \in N\tau_n{}^* for\ all\ S_i \in N\tau_n{}^*$ The *N*-neutrosophic supra topological space is the non empty set X together with the collection $N\tau_n{}^*$, denoted by $(X, N\tau_n{}^*)$. The elements of $N\tau_n{}^*$are known as $N\tau_n{}^*$-open sets on X.
2	It is a generalization of intuitionistic supra topological spaces.	It is an extension of neutrosophic supra topological spaces.
3	Every neutrosophic topology is neutrosophic supra topology.	Every *N*-neutrosophic topology is *N*-neutrosophic supra topology.
4	It is a particular case of *N*-neutrosophic supra topology, that is if *N*=1, then we have neutrosophic supra topology.	It is a general form of neutrosophic supra topology.
5	Union of two neutrosophic supra topologies is again a neutrosophic supra topology. Intersection of two neutrosophic supra topologies is again a neutrosophic supra topology. These two properties may not true in neutrosophic topology.	Union of two *N*-neutrosophic supra topologies is again an *N*-neutrosophic supra topology. Intersection of two *N*-neutrosophic supra topologies is again an *N*-neutrosophic supra topology. These two properties may not true in *N*-neutrosophic topology.
6	The collection of neutrosophic supra α -open sets need not form a neutrosophic topology, but it is a neutrosophic supra topology.	The collection of *N*-neutrosophic supra α-open sets need not form an *N*-neutrosophic topology, but this collection is an *N*-neutrosophic supra topology.

7. Conclusions and Future Work

Neutrosophic topological space is a generalization intuitionistic fuzzy topological space to deal the concept of vagueness. This paper has developed N -neutrosophic supra topological spaces and its closure operator. Moreover, we have defined some weak form of open sets in N-neutrosophic supra topological spaces and established their relations. Apart from this, we have observed that the collection of weak open sets in N-neutrosophic supra topological spaces need not form an N-neutrosophic topology, but this forms an N-neutrosophic supra topology. We can be developed and implement these N-neutrosophic supra topological open sets to other research areas of topology such as Nano topology, Rough topology, Digital topology and so on.

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