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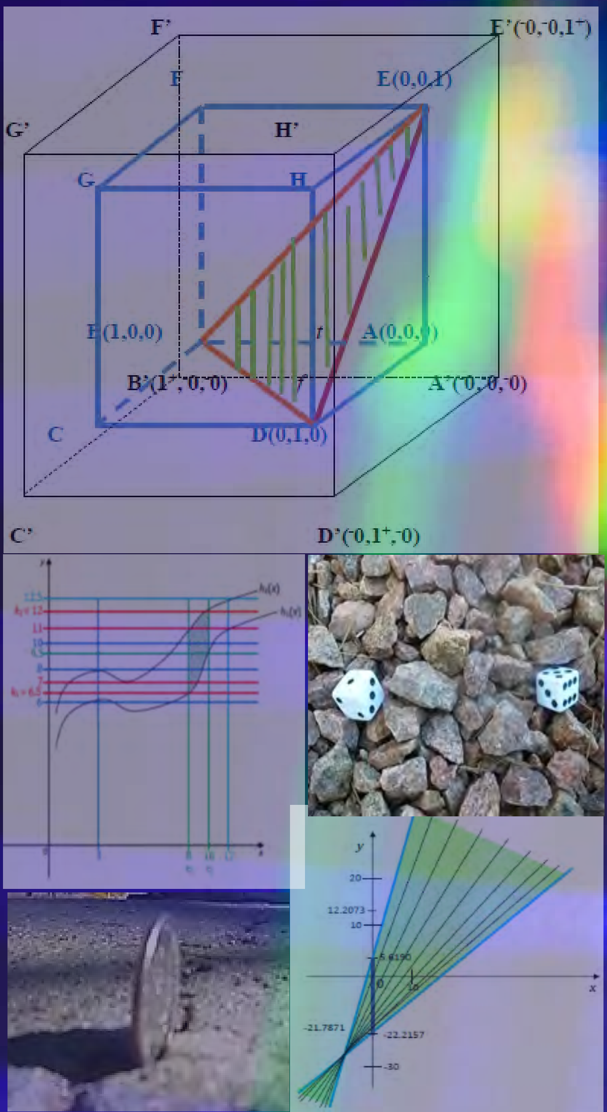

<A> <neutA> <antiA>

Florentin Smarandache . Mohamed Abdel-Basset
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"Neutrosophic Sets and Systems" has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Google Dictionaries have translated the neologisms "**neutrosophy**" (1) and"**neutrosophic**" (2), coined in 1995 for the first time, into about 100 languages.
FOLDOC Dictionary of Computing (**1, 2**), Webster Dictionary (**1, 2**), Wordnik (**1**),Dictionary.com, The Free Dictionary (**1**), Wiktionary (**2**), YourDictionary (**1, 2**),OneLook Dictionary (**1, 2**), Dictionary / Thesaurus (**1**), Online Medical Dictionary (**1,2**), Encyclopedia (**1, 2**), Chinese Fanyi Baidu Dictionary (**2**), Chinese Youdao Dictionary (**2**) etc. have included these scientific neologisms.

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March 20, 2019

Prof. Florentin Smarandache
Univ New Mexico, Gallup Campus

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Contents

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A Hybrid Approach of Neutrosophic with MULTIMOORA in Application of Personnel Selection

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Abstract: Personnel selection is an important key for the success of human resource management in organizations. The main challenge faces organization is to determine the most proper candidates. To match organization requirements, the decision-makers do their best to achieve the most appropriate solutions. The process of choosing between candidates is a very complex and confused task. The environment of decision making is a multi-criteria decision making (MCDM) of various and conflicting criteria and alternatives in addition to the environmental conditions of uncertainty and incomplete information. Hence, this paper contributes to support the personnel selection process with non-classical methods by the integration of neutrosophic theory with MULTIMOORA. .A case study is applied on Telecommunication Company in smart village Cairo Egypt. The case study applies the hybrid approach to attain to most appropriate solutions in the problem of personnel selection.

Keywords: Personnel selection, Multi-criteria decision making (MCDM), Neutrosophic Sets, MULTIMOORA.

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1. Introduction

The competitiveness of organizations can be achieved by the ability of efficient employment [1]. For organization, the most effective part of Human Resource Management is the personnel selection process [2]. The classical methods are used in organizations to select candidates were not sufficient enough and need to be enhanced, to continue proceeding with globalization and rivalry [3]. The numerous and conflict personal criteria make the decision maker confused [4]. The fuzzy set theory appears as an important tool to provide a decision framework that incorporates imprecise judgments inherent in the personnel selection process [5, 6] The Analytical Hierarchy Process (AHP) is used to format the complex problems into a hierarchical form **of** criterions, alternatives, and goals to support decision makers in the selection process [7]. Classical AHP method has been stretched to numerous fuzzy versions, because of partial information and ambiguity. Although the theories of fuzzy have been developed and generalized but cannot deal with all kinds of uncertainties in real problems. Indeed, sure kinds of uncertainties, such as indeterminate and inconsistent information, cannot be managed. Therefore, some new theories are required to present the truth membership, indeterminacy membership and falsity membership simultaneously this called neutrosophic sets. Unlike fuzzy, the neutrosophic sets deal with uncertain, inconsistent, and incomplete information in many researches [32-40]. The personnel selection is a multi-criteria decision-making (MCDM) problem that contains multiple criterions, alternatives, and decision makers to obtain the best candidate to be hire in organization [8]. The use of neutrosophic in personnel selection aids decision makers in the case of uncertainty and inconsistent information to achieve organizations objectives [9]. Sometimes neither of candidates satisfies the vision and objectives of organizations. Therefore, in this study we extend the neutrosophic personnel selection with MULTIMOORA method to encompass the measurement value the method reference level.

The Multi-Objective Optimization by Ratio Analysis (MOORA) method has been introduced by [10]. The MOORA is composed of ratio system, reference point [11-13]. The method MOORA enhanced to MULTIMOORA by adding full Multiplicative Form and employing Dominance Theory to obtain a final rank [2]. The ordinary MULTIMOORA method has been proposed for usage with crisp numbers. MUTIMOORA can solve larger numbers of complex decision-making problems by adding several extensions to solve wide range of problems. The hybrid approach handles the current obstacles and challenges by recommending the most appropriate candidates in the environment of uncertainty and incomplete information.

The structure of this paper ordered as follows: section 2 illustrates some related studies of personnel selection. Section 3 represents the hybrid methodology of neutrosophic with MUTIMOORA method to aid decision makers to choose most appropriate candidate to achieve the goal of organization. Section 4 represents an empirical case study for the proposed hybrid approach. Section 5 summarizes the research key pints and the future trends.

2. Related Studies

The processes of personnel selection in organizations can be affected by many conditions e.g. change the nature of work, governmental regulations, client's behavior, development of new technology, and others [14-16]. The traditional methods are not appropriate enough to keep on globalization. Hence organizations needs to make enhancement on personnel selection problem especially in the field of the judgments of decision makers by integrating advanced tools to decision support system [17,18]. In [19-22] describe the method of AHP with a fuzzy multi-criteria decision making algorithms for solving the personnel selection problems. In [23-25] describe the fuzzy MCDM with TOPSIS method to solve personnel selection problem using linguistic and numerical scales with different data sources to permit decision makers to evaluate candidate's information. In [19] illustrate the AHP method combined with fuzzy to solve personnel selection problem for information systems.

The MULTIMOORA method is extended by researchers to handle several MCDM problems [26, 27]. In [2,] the use of MULTMOORA with a fuzzy MCDM were not the most appropriate methodology. Due to the situations of uncertainty and incomplete information, researches recommend to integrate neutrosophic sets in personnel selection problem [28, 29]. We propose to be the first to applying the neutrosophic sets with MULTIMOORA method to aid decision makers to achieve to the most appropriate candidates.

3. Methodology

A hybrid MULTIMOORA method with neutrosophic is applied in personnel selection problem to select the best candidate to hire in organization. The MULTIMOORA method is used to solve personnel selection problem. In Fig. 1 represents conceptual flow of personnel selection to obtain ideal solution. In Fig. 2 represents the structure of methodology phase to apply MULTIMOORA method with neutrosophic. The phases for the hybrid approach are mentioned as follows:

Figure 1. conceptual flow of personnel selection problem.

***Phase1**:* Acquire expert information in neutrosophic environment.

- Determine the study goal, criteria, and alternative.
- Use neutrosophic scale mentioned in Table 1 [30].
- Create pairwise matrix of decision making judgments using the following form:

$$C^M = \begin{bmatrix} B_{11}^M & \cdots & B_{1z}^M \\ \vdots & \ddots & \vdots \\ B_{y1}^M & \cdots & B_{yz}^M \end{bmatrix} \tag{1}$$

- Aggregate pairwise matrix by:

$$B_{uv} = \frac{\sum_{M=1}^{M} <(l_{uv}^M, m_{uv}^M, u_{uv}^M); T_{uv}^M, I_{uv}^M, F_{uv}^M>}{M} \tag{2}$$

Where, M represents number of decision makers, $l_{uv}^M, m_{uv}^M, u_{uv}^M$ are lower, middle and upper bound of neutrosophic number, $T_{uv}^M, I_{uv}^M, F_{uv}^M$ are truth, indeterminacy and falsity.

- Construct the initial pairwise comparison matrix as mentioned:

$$C = \begin{bmatrix} B_{11} & \cdots & B_{1z} \\ \vdots & \ddots & \vdots \\ B_{y1} & \cdots & B_{yz} \end{bmatrix} \tag{3}$$

- Convert neutrosophic scales to crisp values by using score function of B_{uv} [31]:

$$s(B_{uv}) = \left| (l_{uv} * m_{uv} * u_{uv})^{\frac{T_{uv}+I_{uv}+F_{uv}}{9}} \right| \tag{4}$$

where l, m, u represents lower, middle and upper of the scale neutrosophic numbers.

Phase2: Calculate weights of criteria.

- Compute the average of row

$$w_u = \frac{\sum_{v=1}^{z}(B_{uv})}{z}; u = 1,2,3, \dots \dots . y; v = 1,2,3, \dots \dots . z; \tag{5}$$

- The normalization of crisp value is calculated using the following equation

$$w_u^y = \frac{w_u}{\sum_{u=1}^{y} w_u}; u = 1,2,3, \dots \dots . y \qquad (6)$$

Phase3: *Evaluate expert judgement using consistency rate*

Check the conistency of matrix using table 2 and for detailed information in [31]

- Compute weighted columns by multiplying the weight of priority by each value in the pairwise comparison matrix [31].
- The weighted sum values are divided with the corresponding priority.
- Compute the mean of the previous step denoted as λ_{max}.
- Compute consistency index $CI = \frac{\lambda_{max} - \text{n}}{n-1}$,where n the number of criteria.
- Calculate consistency ratio by the use for the mentioned equation

$$CR = \frac{CI}{RI} \qquad (7)$$

Where, CR is the consistency rate, CI is consistency Index. RI is the random index for consistency matrix as mentioned in Table 3.

Phase4: *MULTIMOORA Method*

The decision judgments between criterions and alternatives will be collected and obtained by the use of form (1). Then, apply Equation (2) to make a general vision of aggregation of experts. Finally, apply Equation (4) to change neutrosophic scale values to crisp values. The MULTIMOORA method consists of: ratio system, reference point and full multiplicative form.

Phase4.1: *Ratio System*

- The first step of ratio system is to calculate the normalize of the decision matrix as mentioned:

$$B_{uv}^{*} = \frac{B_{uv}}{\sqrt[2]{\sum_{u=1}^{y} B_{uv}^{2}}} \quad u = 1,2,3, \dots \dots , y \text{ and } v = 1,2,3 \dots \dots , z. \qquad (8)$$

- Compute the beneficial criteria (Y^{+}) is the summation of beneficial criteria of weight normalized elements of matrix. Then non-beneficial criteria denoted as (Y^{-}). Finally subtract sum of beneficial criteria from sum of non-beneficial criteria. (NB. In this study all criterions are beneficial)

$$Y^{+} = \sum_{v=1}^{g} w_v B_{uv}^{*} \qquad (9)$$

$$Y^{-} = \sum_{v=1}^{z} w_v B_{UV}^{*} \qquad (10)$$

- The next formula represents number of criteria to be maximized and (z-g) represents number of criteria to be minimized.

$$Y^{*} = \sum_{v=1}^{g} w_v B_{uv}^{*} - \sum_{v=g+1}^{z} w_v B_{uv}^{*} \qquad (11)$$

,where w_v is the weight of criteria

- Finally, Rank the alternatives

Phase4.2: *Reference point*

The second step of neutrosophic MULTIMOORA is reference point

- Compute reference point to be maximized

$$r_v = \max_u (w_v (B_z^*)_{uv}). \quad (12)$$

- Compute reference point to be minimized

$$r_v = \min_u (w_v (B_z^*)_{uv}). \quad (13)$$

- Compute deviation of reference point

$$\min_v \left(\max_u |(r_u - w_v (x_z^*)_{uv})| \right). \quad (14)$$

Phase4.3: *Full multiplicative form*

The third step of neutrosophic MULTIMOORA is full multiplicative form

- Compute utility of the alternative

$$U_u = \frac{E_u}{F_u} \quad (15)$$

$$E_u = \prod_{v=1}^{g} w_v (B_Z^*)_{uv} \quad (16)$$

$$F_u = \prod_{v=g+1}^{g} w_v (B_Z^*)_{uv} \quad (17)$$

The first component E_u represents the product of criteria of u th alternative to be maximized. The second component F_u represents the product criteria of uth alternative to be minimized.

- Finally apply the dominance theory to obtain final rank

Table1. Neutrosophic triangular scale (linguistic terms)

Saaty scale	Caption	Neutrosophic triangular scale
1	Evenly significant	$\tilde{1}$ = < <1 ,1, 1>;0.50, 0.50, 0.50>
3	A little significant	$\tilde{3}$ = < <2 ,3, 4>;0.30, 0.75, 0.70>
5	Powerfully significant	$\tilde{5}$ = < <4 ,5, 6>;0.80, 0.15, 0.20>
7	Completely Powerfully significant	$\tilde{7}$ = < <6 ,7 ,8>;0.90, 0.10, 0.10>
9	Absolutely significant	$\tilde{9}$ = < <9 ,9 ,0>;1.00, 0.00, 0.00>
2	Sporadic values between two close scales	$\tilde{2}$ = < <1 ,2, 3>;0.40, 0.60, 0.65>
4		$\tilde{4}$ = < <3 ,4, 5>;0.35, 0.60, 0.40>
6		$\tilde{6}$ = < <5 ,6, 7>;0.70, 0.25, 0.30>
8		$\tilde{8}$ = < <7 ,8 ,9>;0.85, 0.10, 0.15>

Table 2. The consistency rate for pair-wise comparison matrix

N	4 × 4	5 × 5	N > 4
$CR \leq$	0.58	0.90	1.12

Table 3. Random Consistency index for various criterions

Size of matrix	1	2	3	4	5	6	7	8	9	10
Random Consistency	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

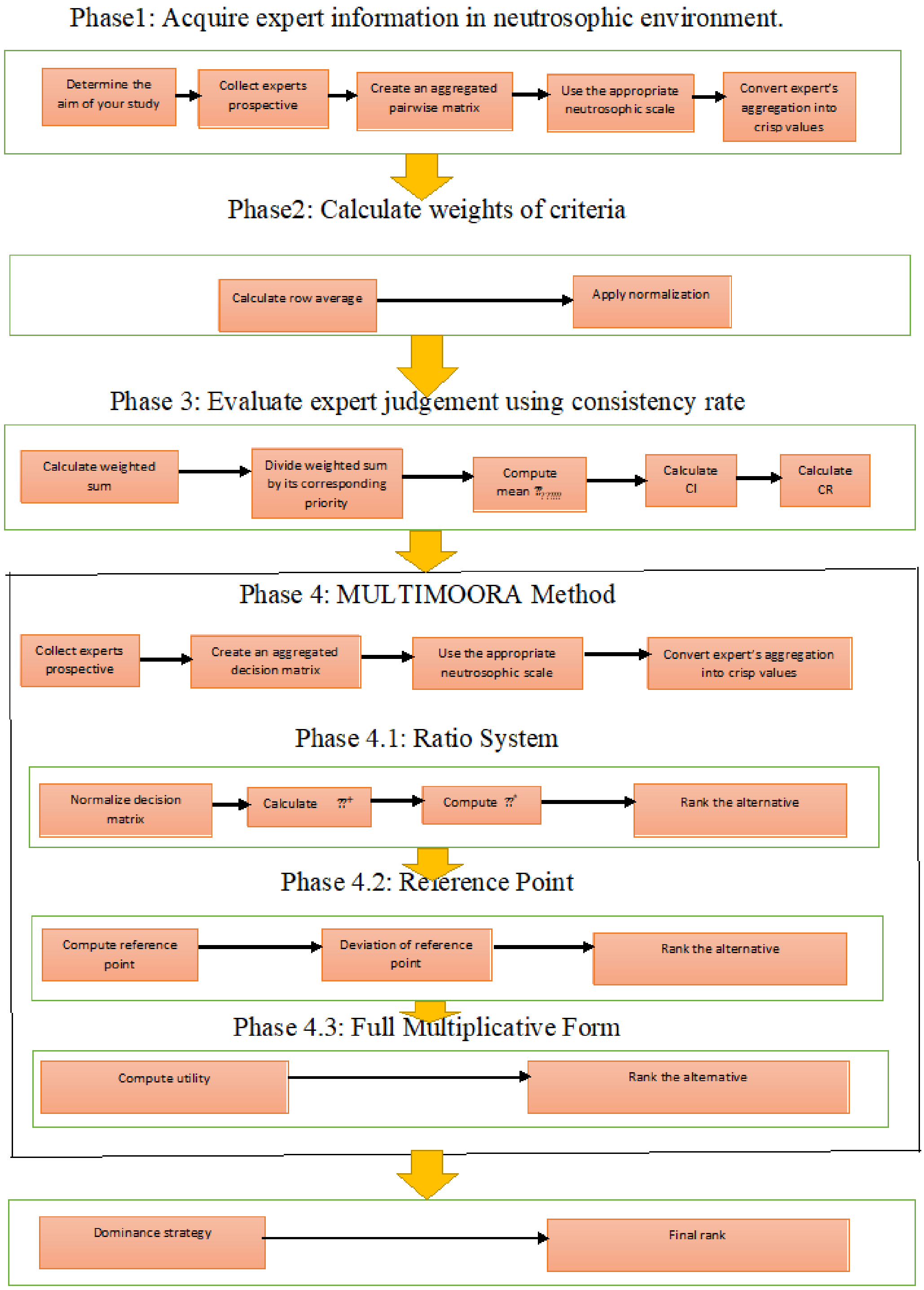

Figure 2. Personnel selection and MULTIMOORA method

4. An Empirical Case Study

In this section, the case study is about personnel selection in a telecommunication company in smart village in Egypt. The case study applies the hybrid methodology of neutrosophic with MULTIMOORA method. In order to make a general image for the telecommunication company, we adopt eight criterions, seven alternatives, and four decision makers. Figure 3 shows the relations between criterions and alternatives. The telecommunication goal is to hire best candidate to achieve competitive organization goals.

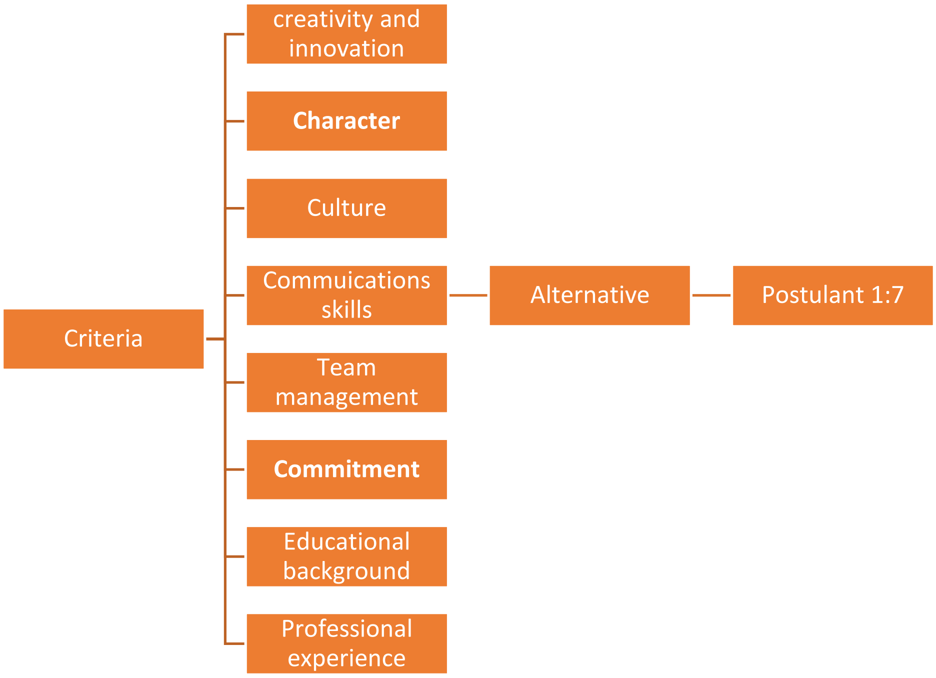

Figure 3. The AHP Structure for criteria and alternatives

Phase 1: *Represent expert judgments in neutrosophic environment*

- Create neutrosophic triangular scale (linguistic term) in Table 1.
- Create the general vision pairwise comparison matrix of criteria in Table 4 in form (1).
- Aggregate pairwise comparison matrix of criteria using Equations (2) and form in (3).
- Convert aggregate pairwise comparison matrix of criteria to crisp value in Table 5 using Equation (4).

Table 4. The pairwise comparison matrix of criteria of decision maker judgments

		C1	C2	C3	C4	C5	C6	C7	C8
	C1	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>
	C2	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>
	C3	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <1, 1, 1 >;0.50,	< <1, 1, 1 >;0.50,	< <5 ,6, 7>;0.70,	< <4 ,5, 6>;0.80,	< <1 ,2, 3>;0.40,	< <4 ,5, 6>;0.80,	< <7 ,8 ,9>;0.85,

			0.50, 0.50>	0.50, 0.50>	0.25, 0.30>	0.15, 0.20>	0.60, 0.65>	0.15, 0.20>	0.10, 0.15>
	C4	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	1/<<4 ,5, 6>;0.80, 0.15, 0.20>	1/<<5 ,6, 7>;0.70, 0.25, 0.30>	<<1, 1, 1 >;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	<<1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>
DM1	C5	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/<<4 ,5, 6>;0.80, 0.15, 0.20>	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	<<1, 1, 1 >;0.50, 0.50, 0.50>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <4 ,5, 6>;0.80, 0.15, 0.20>	<<1, 1, 1 >;0.50, 0.50, 0.50>
	C6	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/<<4 ,5, 6>;0.80, 0.15, 0.20>	1/<<1 ,2, 3>;0.40, 0.60, 0.65>	1/<<4 ,5, 6>;0.80, 0.15, 0.20>	1/<<3 ,4, 5>;0.35, 0.60, 0.40>	<<1, 1, 1 >;0.50, 0.50, 0.50>	< <3 ,4, 5>;0.35, 0.60, 0.40>	<<1, 1, 1 >;0.50, 0.50, 0.50>
	C7	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	1/<<4 ,5, 6>;0.80, 0.15, 0.20>	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	1/<<4 ,5, 6>;0.80, 0.15, 0.20>	1/<<4 ,5, 6>;0.80, 0.15, 0.20>	<<1, 1, 1 >;0.50, 0.50, 0.50>	<<1, 1, 1 >;0.50, 0.50, 0.50>
	C8	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/<<4 ,5, 6>;0.80, 0.15, 0.20>	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	<<1, 1, 1 >;0.50, 0.50, 0.50>
	C1	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	<<1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <3 ,4, 5>;0.35, 0.60, 0.40>	<<1, 1, 1 >;0.50, 0.50, 0.50>
	C2	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	<<1, 1, 1 >;0.50, 0.50, 0.50>	<<1, 1, 1 >;0.50, 0.50, 0.50>	<<1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <3 ,4, 5>;0.35, 0.60, 0.40>
	C3	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	<<1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	<<1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>
DM2	C4	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	1/<<1, 1, 1 >;0.50, 0.50, 0.50>	1/<<5 ,6, 7>;0.70, 0.25, 0.30>	<<1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <3 ,4, 5>;0.35, 0.60, 0.40>

	C5	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	C6	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <3 ,4, 5>;0.35, 0.60, 0.40>
	C7	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	C8	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1, 1, 1 >;0.50, 0.50, 0.50>
	C1	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <3 ,4, 5>;0.35, 0.60, 0.40>
	C2	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>
DM3	C3	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1, 1, 1 >;0.50, 0.50, 0.50>
	C4	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>
	C5	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	C6	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <3 ,4, 5>;0.35, 0.60, 0.40>

	C7	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	C8	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1, 1, 1 >;0.50, 0.50, 0.50>
DM4	C1	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1, 1, 1 >;0.50, 0.50, 0.50>
	C2	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1, 1, 1 >;0.50, 0.50, 0.50>
	C3	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <3 ,4, 5>;0.35, 0.60, 0.40>
	C4	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <3 ,4, 5>;0.35, 0.60, 0.40>
	C5	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>
	C6	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	C7	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	< <1, 1, 1 >;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>
	C8	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <1, 1, 1 >;0.50, 0.50, 0.50>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <3 ,4, 5>;0.35, 0.60, 0.40>	1/< <7 ,8 ,9>;0.85, 0.10, 0.15>	1/< <5 ,6, 7>;0.70, 0.25, 0.30>	1/< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1, 1, 1 >;0.50, 0.50, 0.50>

Table 5. Crisp value of aggregated pairwise comparison matrix of criteria.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8
C1	1	1.88288	1.88288	1.85098	2.01946	2.04291	2.03948	1.76092
C2	0.53110	1	1.77829	1.82446	1.94923	1.93354	1.53537	1.66246
C3	0.53110	0.56233	1	2.05393	1.79510	2.02662	1.89927	1.95726
C4	0.54025	0.54810	0.48687	1	2.01743	1.85375	1.82446	1.97178
C5	0.48949	0.51302	0.55707	0.49568	1	1.88588	1.58172	2.01743
C6	0.48949	0.51718	0.49343	0.53944	0.53025	1	1.71033	1.81143
C7	0.49032	0.65130	0.52651	0.54810	0.63222	0.58468	1	1.89927
C8	0.56788	0.60151	0.51091	0.50715	0.45991	0.55205	0.52651	1

Phase 2: *Calculate weight of criteria* as mentioned in Fig. (4).

- Compute the average of row.

$$w_1 = 14.47951 \text{ w2} = 12.21445 \text{ w3} = 11.82561 \text{ w4} = 10.24264 \text{ w5} = 8.54029 \text{ w6} = 7.09155 \text{ w7} = 6.3324 \text{ w8} = 4.72592$$

- The normalization of crisp value is calculated.

$$w_1 = 0.1919026\ w_2 = 0.1618829\ w_3 = 0.1567294\ w_4 = 0.1357497\ w_5 = 0.1131878\ w_6 = 0.0939871\ w_7 = 0.0839257\ w_8 = 0.0626344$$

$$\sum w_i = 1\,.$$

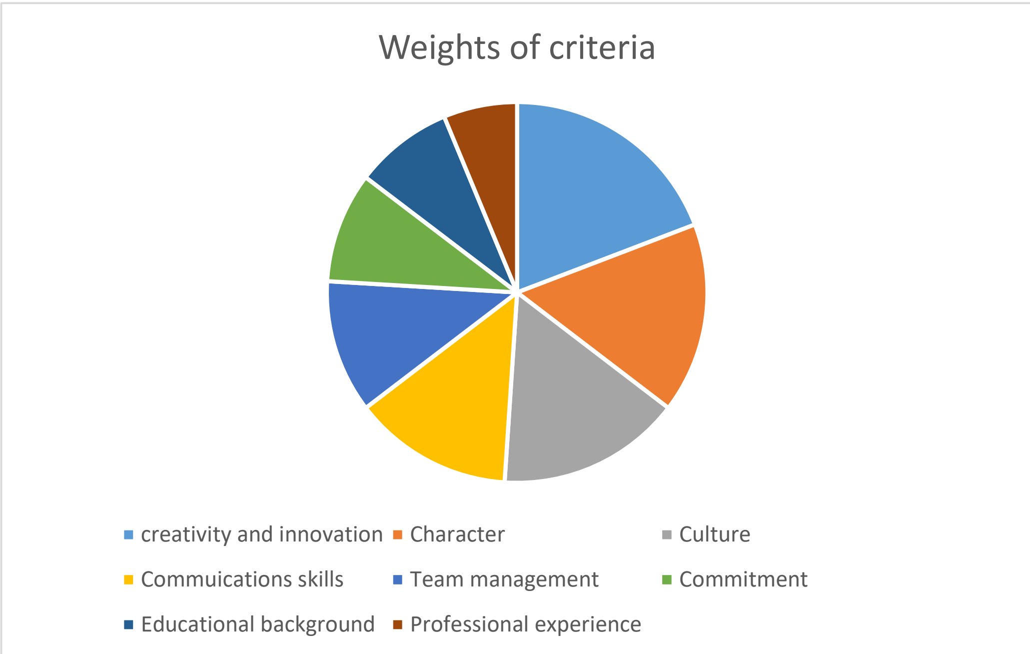

Figure 4. Pie chart weights of criteria

Phase 3: *Check consistency rate*

- Compute weighted sum

$$w_1 = 1.74501\ w_2 = 1.4254\ w_3 = 1.30403\ w_4 = 1.08356\ w_5 = 0.88104\ w_6 = 0.73916\ w_7 = 0.68578\ w_8 = 0.56598$$

- Divide weighted sum by weight of criteria

$$w_1 = 9.09320\ w_2 = 8.80513\ w_3 = 8.32026\ w_4 = 7.98204\ w_5 = 7.78387\ w_6 = 7.86448\ w_7 = 8.17127\ w_8 = 9.03624$$

- Divide summation of Weighted sum by the number of criteria 8
- Compute $\lambda_{max} = 8.38206$
- Compute $CI = \frac{\lambda_{max} - \mathrm{n}}{n-1} = \frac{8.38206 - 8}{8-1} = 0.05458$
- Compute $CR = \frac{\mathrm{CI}}{RI} = \frac{0.05458}{1.41} = 0.03870.$

Hence, the pair-wise comparison matrix is consistent and fellow the next phase of MULTIMOORA Method

Phase 4: *MULTIMOORA Method Calculations*

- A session is performed with four decision makers and the collected judgments presented in table 6.
- Aggregate judgments of decision matrix of four decision makers using Equation (2).
- Compute crisp value of aggregated decision matrix using Equation (4) and mentioned in Table 7.

Table 6. The judgments for multiple decision makers

	Criteria/ Alternatives	C1	C2	C3	C4	C5	C6	C7	C8
	A1	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,2, 3>;0.40, 0.60, 0.65>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>
DM1	A2	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>
	A3	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>
	A4	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,2, 3>;0.40, 0.60, 0.65>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1 ,2, 3>;0.40, 0.60, 0.65>	< <1 ,2, 3>;0.40, 0.60, 0.65>	< <1 ,1, 1>;0.50, 0.50, 0.50
	A5	< <7 ,8 ,9>;0.85,	< <1 ,1, 1>;0.50,	< <1 ,1, 1>;0.50,	< <4 ,5, 6>;0.80,	< <4 ,5, 6>;0.80,	< <7 ,8 ,9>;0.85,	< <7 ,8 ,9>;0.85,	< <4 ,5, 6>;0.80,

		0.10, 0.15>	0.50, 0.50>	0.50, 0.50>	0.15, 0.20>	0.15, 0.20>	0.10, 0.15>	0.10, 0.15>	0.15, 0.20>
	A6	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>
	A7	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>
DM2	A1	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	A2	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	A3	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>
	A4	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,2, 3>;0.40, 0.60, 0.65>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1 ,2, 3>;0.40, 0.60, 0.65>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>
	A5	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>
	A6	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>
	A7	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1 ,1, 1>;0.50, 0.50, 0.50>

DM3	A1	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	A2	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>
	A3	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,2, 3>;0.40, 0.60, 0.65>
	A4	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,2, 3>;0.40, 0.60, 0.65>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>
	A5	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>
	A6	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>
	A7	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1 ,1, 1>;0.50, 0.50, 0.50>
DM4	A1	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	A2	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <5 ,6, 7>;0.70, 0.25, 0.30>
	A3	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <5 ,6, 7>;0.70, 0.25, 0.30>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>

	A4	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,2, 3>;0.40, 0.60, 0.65>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,2, 3>;0.40, 0.60, 0.65>	< <1 ,2, 3>;0.40, 0.60, 0.65>
	A5	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <3 ,4, 5>;0.35, 0.60, 0.40>	< <1 ,1, 1>;0.50, 0.50, 0.50>
	A6	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>
	A7	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <1 ,1, 1>;0.50, 0.50, 0.50>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <7 ,8 ,9>;0.85, 0.10, 0.15>	< <4 ,5, 6>;0.80, 0.15, 0.20>	< <4 ,5, 6>;0.80, 0.15, 0.20>

Table 7. The aggregated pairwise matrix for multiple decision maker's judgments

Criteria/ Alternatives	C1	C2	C3	C4	C5	C6	C7	C8
A1	1.88288	1.96309	2.01160	1.93540	1.88606	1.99504	1.99504	2.03414
A2	1.38248	2.00514	1.97958	2.073329	1.98669	2.25679	2.073329	2.12321
A3	1.88288	2.06542	1.985350	1.95726	1.99504	2.03414	1.382488	2.063838
A4	1.98669	1.96418	1.77208	1.55075	1.99504	1.73960	1.21198	1.11336
A5	1.77829	1.75314	1.382488	1.77829	1.617809	1.915488	2.042910	1.88288
A6	1.61780	1.98669	1.88288	1.38248	1.38248	1.93354	1.986697	1.996661
A7	1.88288	1.88288	1.93354	1	1.762838	1.93354	1.97178	1.617809

Phase 4.1: The ratio system

- Calculate normalization of decision matrix in using Equation (8), and mentioned in Table 8.
- Calculate Y^+ (weight normalized) using Equation (9) in Table 9.
- $Y^- = 0$ because all criteria are beneficial.
- The ranks of ratio system ranking are mentioned in Table 10.

Table 8. The normalization matrix

Criteria/ Alternatives	C1	C2	C3	C4	C5	C6	C7	C8
A1	0.39896	0.38088	0.40856	0.42899	0.39241	0.38124	0.41009	0.41269
A2	0.29293	0.38904	0.40205	0.45956	0.41335	0.43126	0.42618	0.43076
A3	0.39896	0.40074	0.40322	0.43383	0.41508	0.38872	0.24817	0.41872
A4	0.42095	0.38109	0.35991	0.34373	0.41508	0.33243	0.24912	0.22588
A5	0.37680	0.34015	0.28078	0.39416	0.33659	0.36604	0.41993	0.38200
A6	0.34279	0.38546	0.38241	0.30643	0.28763	0.36949	0.40837	0.40509
A7	0.39896	0.36532	0.39270	0.22165	0.36677	0.36949	0.40530	0.32822

Table 9. The Y^+ (Weighted normalized)

Criteria/ Alternatives	C1	C2	C3	C4	C5	C6	C7	C8
A1	0.076561	0.061657	0.064033	0.058235	0.044416	0.035831	0.034417	0.025848
A2	0.056214	0.062978	0.063013	0.062385	0.046786	0.040532	0.035767	0.026980
A3	0.076561	0.064872	0.063196	0.058892	0.046981	0.036534	0.020827	0.026226
A4	0.080781	0.061691	0.056408	0.046661	0.046981	0.031244	0.020907	0.014147
A5	0.072308	0.055064	0.044006	0.053507	0.038097	0.034403	0.035242	0.023926
A6	0.065782	0.062399	0.059934	0.041597	0.032556	0.034727	0.034272	0.025372
A7	0.076561461	0.059139061	0.061547635	0.030088921	0.041513889	0.034727294	0.034015086	0.020557863

Table 10. The ranks of Ratio system

Alternatives	Y^*	Ranking
A1	0.401001	1
A2	0.394658	2
A3	0.394094	3
A4	0.358825	4
A5	0.356557	7
A6	0.356643	6
A7	0.358151	5

Phase 4.2: *The reference point*

- Calculate Reference point r_v using Eq. (12) in table 11
- Calculate deviations from reference point using Eq. (14) in table 12
- The Reference point ranking mentioned in table 13.

Table 11. Reference point

Criteria	C1	C2	C3	C4	C5	C6	C7	C8
R_j	0.080781399	0.064872953	0.064033364	0.062385132	0.046981992	0.040532877	0.035767455	0.026980394

Table 13. Deviations from reference point.

Criteria/Alternative	C1	C2	C3	C4	C5	C6	C7	C8
A1	0.004219938	0.003214994	0.00000000	0.004149868	0.002565967	0.004701235	0.001350365	0.001131803
A2	0.02456737	0.00189403	0.001020309	0.00000000	0.000195815	0.00000000	0.00000000	0.00000000
A3	0.004219938	0.00000000	0.000836935	0.00349284	0.00000000	0.003998211	0.014939614	0.000754118
A4	0.00000000	0.003180999	0.007624886	0.015723888	0.00000000	0.009288745	0.014859885	0.012832536
A5	0.008472499	0.009808485	0.020026883	0.00887803	0.00888411	0.006129839	0.000524536	0.003054053
A6	0.014999107	0.00247357	0.004098474	0.020787351	0.014425785	0.005805583	0.001494717	0.001607825
A7	0.004219938	0.005733892	0.002485729	0.032296211	0.005468103	0.005805583	0.001752369	0.006422531

Table13. Rank reference point

Alternative	Max value (Deviations from reference point)	Rank reference point
A1	0.004701235	7
A2	0.02456737	2
A3	0.014939614	6
A4	0.015723888	5
A5	0.020026883	4
A6	0.020787351	3
A7	0.032296211	1

Phase 4.3: *Full multiplicative form*

- Compute utility of the alternative using Equation (15), (16) and (17) in Table 14.
- The full Multiplicative form ranking in Table 15.

According to Table 16 and Fig. 5, the final rank recommends alternative one as the best alternative, while alternative four as the worst alternative.

Table 14. Utility and Rank of full multiplicative form.

Alternatives	Utility (U_u)	Rank Multiplicative form
A1	2.49235E-11	2
A2	2.54691E-11	1
A3	1.73317E-11	3
A4	5.69554E-12	7
A5	1.03618E-11	4
A6	1.00614E-11	5
A7	8.45311E-12	6

Table15. The final rank according to the proposed hybrid methodology

Alternatives	Ratio system	Reference point	Full multiplicative	(Final Rank)
A1	1	7	2	1
A2	2	2	1	2
A3	3	6	3	3
A4	4	5	7	7
A5	7	4	4	4
A6	6	3	5	6
A7	5	1	6	5

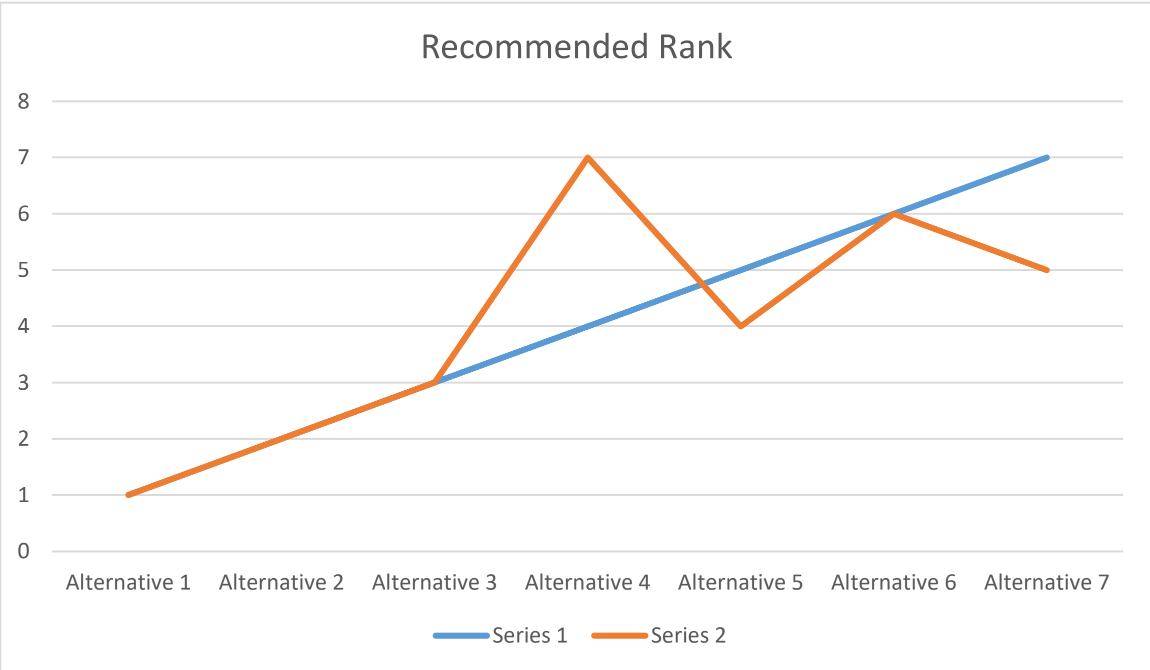

Figure 5. The final rank recommendation

5. Conclusions

Personnel selection is an important issue that effect on the competitive advantages for organizations. Decision makers take decisions for complex problems with various criterions and alternatives with surrounded environment of uncertain and incomplete information. The traditional methods cannot achieve to the proper solutions. In addition fuzzy cannot handle the conditions of uncertainty and inconsistency. The study proposes to use neutrosophic sets to handle the environmental conditions of uncertainty and inconsistent information, in addition extend study with MULTIMOORA method to choose the most appropriate candidate. A case study is applied on smart village Cairo, Egypt, on Telecommunication Company shows the effectiveness for the proposed method and provides final decision to hire the most appropriate candidate for attaining success of enterprises. The future work includes evolutionary algorithms for selecting the most effective criterions. In addition, applies other methodologies e.g. DEMTAL to improve the selection process.

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Conflicts of Interest

The authors declare no conflict of interest.

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A New Approach to Operations on Bipolar Neutrosophic Soft Sets and Bipolar Neutrosophic Soft Topological Spaces

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Abstract: In this study, we re-define some operations on bipolar neutrosophic soft sets differently from the studies [2]. On this operations are given interesting examples and them basic properties. In the direction of these newly defined operations, we construct the bipolar neutrosophic soft topological spaces. Finally, we introduce basic definitions and theorems on bipolar neutrosophic soft topological spaces

Keywords: Bipolar neutrosophic soft set; bipolar neutrosophic soft operations; bipolar neutrosophic soft topological space; bipolar neutrosophic soft interior; bipolar neutrosophic soft closure.

1. Introduction

Set theory which is inducted by Cantor is one of the main topic in mathematics and is frequently used while solving the problems with the mathematical methods. However the real life problems which we meet in several areas as medicine, economics, engineering and etc. include vagueness and this leads to break the precise of data and makes the mathematical solutions unusable. To overpass this lack alternative theories are developed as theory of fuzzy sets [25], theory of intuitionistic fuzzy sets[4], theory of soft sets [15] and etc. But all these approaches have their implicit crisis in solving the problems involving indeterminate and inconsistent data due to inadequacy of parameterization tools. Smarandache [20] studied the idea of neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy and persistent data. Smarandache introduced the neutrosophic set theory as a generalization of many theories such as fuzzy set, intuitionistic fuzzy set etc. Neutrosophic set theory is still popular today. Researchers are working intensively on this set theory [1, 3, 14, 19].

Molodtsov [15] claimed that the theory of soft sets is free from the difficulties seen in the fuzzy set theory. Recently this new theory is used extensively both in mathmetics and in different areas. [6, 10, 21, 23, 24]. As it is known, in Boolean logic a property is either present or absent, i.e. it takes values in the set $\{0,1\}$ and also the theories developed for vagueness focus only on the existence of a property and so in these approaches coexistence of a property is ignored. Hence, it is impossible to model the coexistence of a property with these approaches. Coexistence is associated with bipolarity of an information. For this reason, bipolarity is also an important characteristic of the data which should be considered. In 2013, Shabir and Naz [22] defined bipolar soft sets and basic operations of union, intersection and complementation for bipolar soft sets. They gave examples of bipolar soft sets and an application of bipolar soft sets in a decision making problem. Many different studies have been conducted on bipolar soft set theory [11, 17]. The bipolar neutrosophic soft set theory was first presented by M. Ali at al.[2]. In their study, the structure of theory and the operations on this set structure are defined. However, when the study is examined carefully, one can see that some definitions need to be corrected and re-defined.

In our study, bipolar neutrosophic soft subset, empty bipolar neutrosophic soft set, absolute bipolar neutrosophic soft set, bipolar neutrosophic soft union and bipolar neutrosophic soft intersection are re-defined different from the paper written by M.Ali et al. [2] and also new algebraic operations are presented. Then the topology on the bipolar neutrosophic soft set is built. Closure and interior concepts of the obtained topological spaces are defined and basic theorems are presented. All of these presented notions are constructed with supporting examples.

2. Preliminary

In this section, we will give some preliminary information for the present study.

Definition 2.1 *[20] A neutrosophic set A on the universe of discourse X is defined as:*

$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$, where$T,\ I,\ F: X \to]^{-}0, 1^{+}[$ and $^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

Definition 2.2 *[15] Let X be an initial universe, E be a set of all parameters and $P(X)$ denotes the power set of X. A pair (F, E) is called a soft set over X, where F is a mapping given by $F: E \to P(X)$.*

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F, E), or as the set of $e-$approximate elements of the soft set, i.e.,

$$(F, E) = \{(e, F(e)) : e \in E, F: E \to P(X)\}.$$

Firstly, neutrosophic soft set defined by Maji [12] and later this concept has been modified by Deli and Bromi [9] as given below:

Definition 2.3 *Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set $(\tilde{F}, E)$ over X is a set defined by a set valued function $\tilde{F}$ representing a mapping $\tilde{F}: E \to P(X)$ where $\tilde{F}$ is called approximate function of the neutrosophic soft set $(\tilde{F}, E)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs,*

$$(\tilde{F}, E) = \{(e, \langle x, T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x)\rangle : x \in X) : e \in E\}$$

where $T_{\tilde{F}(e)}(x)$, $I_{\tilde{F}(e)}(x)$, $F_{\tilde{F}(e)}(x) \in 0,1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\tilde{F}(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{\tilde{F}(e)}(x) + I_{\tilde{F}(e)}(x) + F_{\tilde{F}(e)}(x) \leq 3$ is obvious.

Definition 2.4 *[16] Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X and $\overset{NSS}{\tau} \subset NSS(X, E)$. Then $\overset{NSS}{\tau}$ is said to be a neutrosophic soft topology on X if*

1. $0_{(X,E)}$ and $1_{(X,E)}$ belongs to $\overset{NSS}{\tau}$

2. The union of any number of neutrosophic soft sets in $\overset{NSS}{\tau}$ belongs to $\overset{NSS}{\tau}$

3. The intersection of finite number of neutrosophic soft sets in $\overset{NSS}{\tau}$ belongs to $\overset{NSS}{\tau}$.

Then $\left(X, \overset{NSS}{\tau}, E\right)$ is said to be a neutrosophic soft topological space over X.

Definition 2.5 *[2] Let X be a universe and E be a set of parameters that are describing the elements of X. A bipolar neutrosophic soft set $(\tilde{B}, E)$ in X is defined as;*

$$(\tilde{B}, E) = \{(e, \langle x, (T^+_{B(e)}(x), I^+_{B(e)}(x), F^+_{B(e)}(x), T^-_{B(e)}(x), I^-_{B(e)}(x), F^-_{B(e)}(x))\rangle : x \in X) : e \in E\}$$

where T^+_B, I^+_B, $F^+_B \to 0,1]$ and T^-_B, I^-_B, $F^-_B \to -1,0]$. The positive membership degree $T^+_{B(e)}(x)$, $I^+_{B(e)}(x)$, $F^+_{B(e)}(x)$ denotes the truth membership, indeterminate membership and false membership of an element corresponding to a bipolar neutrosophic soft set $(\tilde{B}, E)$ and the negative membership degree $T^-_{B(e)}(x)$, $I^-_{B(e)}(x)$, $F^-_{B(e)}(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic soft set $(\tilde{B}, E)$.

Definition 2.6 *[2] Let $(\tilde{B}, E)$ be a bipolar neutrosophic soft set over X. Then, the complement of a bipolar neutrosophic soft set $(\tilde{B}, E)$, is denoted by $(\tilde{B}, E)^c$, is defined as;*

$$(\tilde{B}, E)^c = \left\{\left(e, \left\langle x, \begin{pmatrix} F^+_{B(e)}(x), 1 - I^+_{B(e)}(x), T^+_{B(e)}(x), \\ F^-_{B(e)}(x), -1 - I^-_{B(e)}(x), T^-_{B(e)}(x) \end{pmatrix} \right\rangle : x \in X\right) : e \in E\right\}.$$

3. A New Approach to Operations on Bipolar Neutrosophic Soft Sets

In this section, we re-defined some concepts as absolute bipolar neutrosophic soft set, empty bipolar neutrosophic soft set, bipolar neutrosophic soft subset, bipolar neutrosophic soft union and intersection . In addition, basic properties of these operations was presented.

Definition 3.1 *An empty bipolar neutrosophic soft set $(\tilde{B}^{\emptyset}, E)$ over X is defined by;*

$$(\tilde{B}^{\emptyset}, E) = \{(e, \langle x, (0,0,1,-1,-1,0)\rangle : x \in X) : e \in E\}.$$

An absolute bipolar neutrosophic soft set $(\tilde{B}^X, E)$ over X is defined by;

$$(\tilde{B}^X, E) = \{(e, \langle x, (1,1,0,0,0,-1)\rangle : x \in X) : e \in E\}.$$

Clearly, $(\tilde{B}^{\emptyset}, E)^c = (\tilde{B}^X, E)$ and $(\tilde{B}^X, E)^c = (\tilde{B}^{\emptyset}, E)$.

Definition 3.2 *Let $(\tilde{B}_1, E)$ and $(\tilde{B}_2, E)$ be two bipolar neutrosophic soft sets over X. $(\tilde{B}_1, E)$ is said to be bipolar neutrosophic soft subset of $(\tilde{B}_2, E)$ if $T^+_{\tilde{B}_1(e)}(x) \le T^+_{\tilde{B}_2(e)}(x)$, $I^+_{\tilde{B}_1(e)}(x) \le I^+_{\tilde{B}_2(e)}(x)$, $F^+_{\tilde{B}_1(e)}(x) \ge F^+_{\tilde{B}_2(e)}(x)$, $T^-_{\tilde{B}_1(e)}(x) \le T^-_{\tilde{B}_2(e)}(x)$, $I^-_{\tilde{B}_1(e)}(x) \le I^-_{\tilde{B}_2(e)}(x)$ and $F^-_{\tilde{B}_1(e)}(x) \ge F^-_{\tilde{B}_2(e)}(x)$ for all $(e, x) \in E \times X$. It is denoted by $(\tilde{B}_1, E) \sqsubseteq (\tilde{B}_2, E)$.*

$(\tilde{B}_1, E)$ is said to be bipolar neutrosophic soft equal to $(\tilde{B}_2, E)$ if $(\tilde{B}_1, E)$ is bipolar neutrosophic soft subset of $(\tilde{B}_2, E)$ and $(\tilde{B}_2, E)$ is bipolar neutrosophic soft subset of $(\tilde{B}_1, E)$. It is denoted by $(\tilde{B}_1, E) = (\tilde{B}_2, E)$.

Example 3.3 *Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. If*

$$(\tilde{B}_1, E) = \begin{Bmatrix} (e_1, \langle x_1, (0.6,0.5,0.3,-0.4,-0.8,-0.4)\rangle, \langle x_2, (0.5,0.4,0.6,-0.4,-0.6,-0.3)\rangle), \\ (e_2, \langle x_1, (0.5,0.7,0.4,-0.3,-0.6,-0.5)\rangle, \langle x_2, (0.3,0.5,0.8,-0.3,-0.4,-0.2)\rangle) \end{Bmatrix}$$

and

$$(\tilde{B}_2, E) = \begin{Bmatrix} (e_1, \langle x_1, (0.7,0.8,0.1,-0.2,-0.5,-0.6)\rangle, \langle x_2, (0.6,0.6,0.3,-0.3,-0.5,-0.7)\rangle), \\ (e_2, \langle x_1, (0.6,0.9,0.2,-0.1,-0.4,-0.7)\rangle, \langle x_2, (0.4,0.7,0.6,-0.2,-0.3,-0.6)\rangle) \end{Bmatrix}$$

then, $(\tilde{B}_1, E) \sqsubseteq (\tilde{B}_2, E)$.

Definition 3.4 *Let* $(\tilde{B}_i, E) = \left\{\left(e, \left\langle x, \left(T^+_{B_i(e)}(x), I^+_{B_i(e)}(x), F^+_{B_i(e)}(x), T^-_{B_i(e)}(x), I^-_{B_i(e)}(x), F^-_{B_i(e)}(x)\right)\right\rangle : x \in X\right) : e \in E\right\}$ *for* $i = 1,2$ *be two bipolar neutrosophic soft sets over* X*. Then their union is denoted by* $(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)$ *and is defined as;*

$$\underset{i=1}{\overset{2}{\sqcup}} (\tilde{B}_i, E) = \left\{\left(e, \left\langle x, \begin{pmatrix} \max\{T^+_{B_i(e)}(x)\}, \max\{I^+_{B_i(e)}(x)\}, \min\{F^+_{B_i(e)}(x)\}, \\ \max\{T^-_{B_i(e)}(x)\}, \max\{I^-_{B_i(e)}(x)\}, \min\{F^-_{B_i(e)}(x)\} \end{pmatrix} \right\rangle : x \in X\right) : e \in E\right\}.$$

Definition 3.5 *Let* $(\tilde{B}_i, E) = \left\{\left(e, \left\langle x, \left(T^+_{B_i(e)}(x), I^+_{B_i(e)}(x), F^+_{B_i(e)}(x), T^-_{B_i(e)}(x), I^-_{B_i(e)}(x), F^-_{B_i(e)}(x)\right)\right\rangle : x \in X\right) : e \in E\right\}$ *for* $i = 1,2$ *be two bipolar neutrosophic soft sets over* X*. Then their intersection is denoted by* $(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)$ *and is defined as;*

$$\underset{i=1}{\overset{2}{\sqcap}} (\tilde{B}_i, E) = \left\{\left(e, \left\langle x, \begin{pmatrix} \min\{T^+_{B_i(e)}(x)\}, \min\{I^+_{B_i(e)}(x)\}, \max\{F^+_{B_i(e)}(x)\}, \\ \min\{T^-_{B_i(e)}(x)\}, \min\{I^-_{B_i(e)}(x)\}, \max\{F^-_{B_i(e)}(x)\} \end{pmatrix} \right\rangle : x \in X\right) : e \in E\right\}.$$

Definition 3.6 *Let* $(\tilde{B}_i, E) = \left\{\left(e, \left\langle x, \left(T^+_{B_i(e)}(x), I^+_{B_i(e)}(x), F^+_{B_i(e)}(x), T^-_{B_i(e)}(x), I^-_{B_i(e)}(x), F^-_{B_i(e)}(x)\right)\right\rangle : x \in X\right) : e \in E\right\}$ *for* $i \in I$ *be a family of bipolar neutrosophic soft sets over* X*. Then,*

$$\underset{i\in I}{\sqcup} (\tilde{B}_i, E) = \left\{\left(e, \left\langle x, \begin{pmatrix} \sup\{T^+_{B_i(e)}(x)\}, \sup\{I^+_{B_i(e)}(x)\}, \inf\{F^+_{B_i(e)}(x)\}, \\ \sup\{T^-_{B_i(e)}(x)\}, \sup\{I^-_{B_i(e)}(x)\}, \inf\{F^-_{B_i(e)}(x)\} \end{pmatrix} \right\rangle : x \in X\right) : e \in E\right\},$$

$$\underset{i\in I}{\sqcap} (\tilde{B}_i, E) = \left\{\left(e, \left\langle x, \begin{pmatrix} \inf\{T^+_{B_i(e)}(x)\}, \inf\{I^+_{B_i(e)}(x)\}, \sup\{F^+_{B_i(e)}(x)\}, \\ \inf\{T^-_{B_i(e)}(x)\}, \inf\{I^-_{B_i(e)}(x)\}, \sup\{F^-_{B_i(e)}(x)\} \end{pmatrix} \right\rangle : x \in X\right) : e \in E\right\}.$$

Proposition 3.7 *Let* $(\tilde{B}^{\emptyset}, E)$ *and* $(\tilde{B}^{X}, E)$ *be the empty bipolar neutrosophic soft set and absolute bipolar neutrosophic soft set over* X*, respectively. Then,*

1. $(\tilde{B}^{\emptyset}, E) \sqsubseteq (\tilde{B}^{X}, E)$,
2. $(\tilde{B}^{\emptyset}, E) \sqcup (\tilde{B}^{X}, E) = (\tilde{B}^{X}, E)$,
3. $(\tilde{B}^{\emptyset}, E) \sqcap (\tilde{B}^{X}, E) = (\tilde{B}^{\emptyset}, E)$.

Proof. Straightforward.

Remark 3.8 *When we consider the definitions of absolute bipolar neutrosophic soft set, empty bipolar neutrosophic soft set, bipolar neutrosophic soft subset, bipolar neutrosophic soft union and intersection presented by M.Ali et al. in [1] then Proposition 3.7 does not hold.*

Definition 3.9 *Let* $(\tilde{B}_1, E)$ *and* $(\tilde{B}_2, E)$ *be two bipolar neutrosophic soft sets over* X*. Then "*$(\tilde{B}_1, E)$ *difference* $(\tilde{B}_2, E)$*" operation on them is denoted by* $(\tilde{B}_1, E) \backslash (\tilde{B}_2, E) = (\tilde{B}_3, E)$ *and is defined by* $(\tilde{B}_3, E) = (\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)^c$ *as follows:*

$$(\tilde{B}_3, E) = \left\{\left(e, \left\langle x, \begin{pmatrix} T^+_{B_3(e)}(x), I^+_{B_3(e)}(x), F^+_{B_3(e)}(x), \\ T^-_{B_3(e)}(x), I^-_{B_3(e)}(x), F^-_{B_3(e)}(x) \end{pmatrix} \right\rangle : x \in X\right) : e \in E\right\}$$

where

$$T^+_{B_3(e)}(x) = \min\{T^+_{B_1(e)}(x), F^+_{B_2(e)}(x)\}, T^-_{B_3(e)}(x) = \min\{T^-_{B_1(e)}(x), F^-_{B_2(e)}(x)\},$$
$$I^+_{B_3(e)}(x) = \min\{I^+_{B_1(e)}(x), 1 - I^+_{B_2(e)}(x)\}, I^-_{B_3(e)}(x) = \min\{I^-_{B_1(e)}(x), -1 - I^-_{B_2(e)}(x)\},$$
$$F^+_{B_3(e)}(x) = \max\{F^+_{B_1(e)}(x), T^+_{B_2(e)}(x)\}, F^-_{B_3(e)}(x) = \max\{F^-_{B_1(e)}(x), T^-_{B_2(e)}(x)\}.$$

Definition 3.10 *Let* $(\tilde{B}_1, E)$ *and* $(\tilde{B}_2, E)$ *be two bipolar neutrosophic soft sets over* X*. Then "AND" operation on them is denoted by* $(\tilde{B}_1, E) \wedge (\tilde{B}_2, E) = (\tilde{B}_3, E \times E)$ *and is defined by:*

$$(\tilde{B}_3, E \times E) = \left\{\left((e_1, e_2), \left\langle x, \begin{pmatrix} T^+_{B_3(e_1,e_2)}(x), I^+_{B_3(e_1,e_2)}(x), F^+_{B_3(e_1,e_2)}(x), \\ T^-_{B_3(e_1,e_2)}(x), I^-_{B_3(e_1,e_2)}(x), F^-_{B_3(e_1,e_2)}(x) \end{pmatrix} \right\rangle : x \in X\right) : (e_1, e_2) \in E \times E\right\}$$

where

$$T^{+}_{B_3(e_1,e_2)}(x) = \min\{T^{+}_{B_1(e_1)}(x), T^{+}_{B_2(e_2)}(x)\}, T^{-}_{B_3(e_1,e_2)}(x) = \min\{T^{-}_{B_1(e_1)}(x), T^{-}_{B_2(e_2)}(x)\},$$
$$I^{+}_{B_3(e_1,e_2)}(x) = \min\{I^{+}_{B_1(e_1)}(x), I^{+}_{B_2(e_2)}(x)\}, I^{-}_{B_3(e_1,e_2)}(x) = \min\{I^{-}_{B_1(e_1)}(x), I^{-}_{B_2(e_2)}(x)\},$$
$$F^{+}_{B_3(e_1,e_2)}(x) = \max\{F^{+}_{B_1(e_1)}(x), F^{+}_{B_2(e_2)}(x)\}, F^{-}_{B_3(e_1,e_2)}(x) = \max\{F^{-}_{B_1(e_1)}(x), F^{-}_{B_2(e_2)}(x)\}.$$

Definition 3.11 *Let $(\tilde{B}_1, E)$ and $(\tilde{B}_2, E)$ be two bipolar neutrosophic soft sets over X. Then "OR" operation on them is denoted by $(\tilde{B}_1, E) \vee (\tilde{B}_2, E) = (\tilde{B}_3, E \times E)$ and is defined by:*

$$(\tilde{B}_3, E \times E) = \left\{\left((e_1, e_2), \left\langle x, \begin{pmatrix} T^{+}_{B_3(e_1,e_2)}(x), I^{+}_{B_3(e_1,e_2)}(x), F^{+}_{B_3(e_1,e_2)}(x), \\ T^{-}_{B_3(e_1,e_2)}(x), I^{-}_{B_3(e_1,e_2)}(x), F^{-}_{B_3(e_1,e_2)}(x) \end{pmatrix} \right\rangle : x \in X \right) : (e_1, e_2) \in E \times E \right\}$$

where

$$T^{+}_{B_3(e_1,e_2)}(x) = \max\{T^{+}_{B_1(e_1)}(x), T^{+}_{B_2(e_2)}(x)\}, T^{-}_{B_3(e_1,e_2)}(x) = \max\{T^{-}_{B_1(e_1)}(x), T^{-}_{B_2(e_2)}(x)\},$$
$$I^{+}_{B_3(e_1,e_2)}(x) = \max\{I^{+}_{B_1(e_1)}(x), I^{+}_{B_2(e_2)}(x)\}, I^{-}_{B_3(e_1,e_2)}(x) = \max\{I^{-}_{B_1(e_1)}(x), I^{-}_{B_2(e_2)}(x)\},$$
$$F^{+}_{B_3(e_1,e_2)}(x) = \min\{F^{+}_{B_1(e_1)}(x), F^{+}_{B_2(e_2)}(x)\}, F^{-}_{B_3(e_1,e_2)}(x) = \min\{F^{-}_{B_1(e_1)}(x), F^{-}_{B_2(e_2)}(x)\}.$$

Example 3.12 *Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. If*

$$(\tilde{B}_1, E) = \begin{Bmatrix} (e_1, \langle x_1, (0.3, 0.5, 0.7, -0.6, -0.5, -0.7)\rangle, \langle x_2, (0.3, 0.5, 0.4, -0.2, -0.5, -0.8)\rangle), \\ (e_2, \langle x_1, (0.4, 0.4, 0.3, -0.7, -0.4, -0.3)\rangle, \langle x_2, (0.5, 0.8, 0.9, -0.1, -0.9, -0.7)\rangle) \end{Bmatrix}$$

and

$$(\tilde{B}_2, E) = \begin{Bmatrix} (e_1, \langle x_1, (0.4, 0.6, 0.8, -0.5, -0.3, -0.9)\rangle, \langle x_2, (0.4, 0.6, 0.2, -0.3, -0.2, -0.3)\rangle), \\ (e_2, \langle x_1, (0.3, 0.3, 0.5, -0.3, -0.6, -0.8)\rangle, \langle x_2, (0.4, 0.5, 0.3, -0.6, -0.1, -0.3)\rangle) \end{Bmatrix}$$

then

$$(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E) = \begin{Bmatrix} (e_1, \langle x_1, (0.4, 0.6, 0.7, -0.5, -0.3, -0.9)\rangle, \langle x_2, (0.4, 0.6, 0.2, -0.2, -0.2, -0.8)\rangle), \\ (e_2, \langle x_1, (0.4, 0.4, 0.3, -0.3, -0.4, -0.8)\rangle, \langle x_2, (0.5, 0.8, 0.3, -0.1, -0.1, -0.7)\rangle) \end{Bmatrix},$$

$$(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E) = \begin{Bmatrix} (e_1, \langle x_1, (0.3, 0.5, 0.8, -0.6, -0.5, -0.7)\rangle, \langle x_2, (0.3, 0.5, 0.4, -0.3, -0.5, -0.3)\rangle), \\ (e_2, \langle x_1, (0.3, 0.3, 0.5, -0.7, -0.6, -0.3)\rangle, \langle x_2, (0.4, 0.5, 0.9, -0.6, -0.9, -0.3)\rangle) \end{Bmatrix},$$

$$(\tilde{B}_1, E) \backslash (\tilde{B}_2, E) = \begin{Bmatrix} (e_1, \langle x_1, (0.3, 0.4, 0.7, -0.9, -0.7, -0.5)\rangle, \langle x_2, (0.2, 0.4, 0.4, -0.3, -0.8, -0.3)\rangle), \\ (e_2, \langle x_1, (0.4, 0.4, 0.3, -0.8, -0.4, -0.3)\rangle, \langle x_2, (0.3, 0.5, 0.9, -0.3, -0.9, -0.6)\rangle) \end{Bmatrix},$$

$$(\tilde{B}_1, E) \wedge (\tilde{B}_2, E) = \begin{Bmatrix} ((e_1, e_1), \langle x_1, (0.3, 0.5, 0.8, -0.6, -0.5, -0.7)\rangle, \langle x_2, (0.3, 0.5, 0.4, -0.3, -0.5, -0.3)\rangle), \\ ((e_1, e_2), \langle x_1, (0.3, 0.3, 0.7, -0.6, -0.6, -0.7)\rangle, \langle x_2, (0.3, 0.5, 0.4, -0.6, -0.5, -0.3)\rangle), \\ ((e_2, e_1), \langle x_1, (0.4, 0.4, 0.8, -0.7, -0.4, -0.3)\rangle, \langle x_2, (0.4, 0.6, 0.9, -0.3, -0.9, -0.3)\rangle), \\ ((e_2, e_2), \langle x_1, (0.3, 0.3, 0.5, -0.7, -0.6, -0.3)\rangle, \langle x_2, (0.4, 0.5, 0.9, -0.6, -0.9, -0.3)\rangle) \end{Bmatrix},$$

$$(\tilde{B}_1, E) \vee (\tilde{B}_2, E) = \begin{Bmatrix} ((e_1, e_1), \langle x_1, (0.4, 0.6, 0.7, -0.5, -0.3, -0.9)\rangle, \langle x_2, (0.4, 0.6, 0.2, -0.2, -0.2, -0.8)\rangle), \\ ((e_1, e_2), \langle x_1, (0.3, 0.5, 0.5, -0.3, -0.5, -0.8)\rangle, \langle x_2, (0.4, 0.5, 0.3, -0.2, -0.1, -0.8)\rangle), \\ ((e_2, e_1), \langle x_1, (0.4, 0.6, 0.3, -0.5, -0.3, -0.9)\rangle, \langle x_2, (0.5, 0.8, 0.2, -0.1, -0.2, -0.7)\rangle), \\ ((e_2, e_2), \langle x_1, (0.4, 0.4, 0.3, -0.3, -0.4, -0.8)\rangle, \langle x_2, (0.5, 0.8, 0.3, -0.1, -0.1, -0.7)\rangle) \end{Bmatrix}.$$

Proposition 3.13 *Let $(\tilde{B}_1, E)$, $(\tilde{B}_2, E)$ and $(\tilde{B}_3, E)$ be bipolar neutrosophic soft sets over X. Then,*

1. $(\tilde{B}_1, E) \sqcup [(\tilde{B}_2, E) \sqcup (\tilde{B}_3, E)] = [(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)] \sqcup (\tilde{B}_3, E)$ and
   $(\tilde{B}_1, E) \sqcap [(\tilde{B}_2, E) \sqcap (\tilde{B}_3, E)] = [(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)] \sqcap (\tilde{B}_3, E)$;

2. $(\tilde{B}_1, E) \sqcup [(\tilde{B}_2, E) \sqcap (\tilde{B}_3, E)] = [(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)] \sqcap [(\tilde{B}_1, E) \sqcup (\tilde{B}_3, E)]$ and
   $(\tilde{B}_1, E) \sqcap [(\tilde{B}_2, E) \sqcup (\tilde{B}_3, E)] = [(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)] \sqcup [(\tilde{B}_1, E) \sqcap (\tilde{B}_3, E)]$;

3. $(\tilde{B}_1, E) \sqcup (\tilde{B}^{\emptyset}, E) = (\tilde{B}_1, E)$ and $(\tilde{B}_1, E) \sqcap (\tilde{B}^{\emptyset}, E) = (\tilde{B}^{\emptyset}, E)$;

4. $(\tilde{B}_1, E) \sqcup (\tilde{B}^{X}, E) = (\tilde{B}^{X}, E)$ and $(\tilde{B}_1, E) \sqcap (\tilde{B}^{X}, E) = (\tilde{B}_1, E)$;

5. $(\tilde{B}^{\emptyset}, E) \backslash (\tilde{B}^{X}, E) = (\tilde{B}^{\emptyset}, E)$ and $(\tilde{B}^{X}, E) \backslash (\tilde{B}^{\emptyset}, E) = (\tilde{B}^{X}, E)$

Proof. Straightforward.

Proposition 3.14 *Let* $(\tilde{B}_1, E)$ *and* $(\tilde{B}_2, E)$ *be two bipolar neutrosophic soft sets over* X. *Then,*

1. $[(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]^c = (\tilde{B}_1, E)^c \sqcap (\tilde{B}_2, E)^c$;
2. $[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]^c = (\tilde{B}_1, E)^c \sqcup (\tilde{B}_2, E)^c$.

Proof. 1. For all $e \in E$ and $x \in X$,

$$\overset{2}{\underset{i=1}{\sqcup}} (\tilde{B}_i, E) = \left\{ e, \left\langle x, \begin{pmatrix} \max\{T^+_{B_1(e)}(x), T^+_{B_2(e)}(x)\}, \max\{I^+_{B_1(e)}(x), I^+_{B_2(e)}(x)\}, \min\{F^+_{B_1(e)}(x), F^+_{B_2(e)}(x)\}, \\ \max\{T^-_{B_1(e)}(x), T^-_{B_2(e)}(x)\}, \max\{I^-_{B_1(e)}(x), I^-_{B_2(e)}(x)\}, \min\{F^-_{B_1(e)}(x), F^-_{B_2(e)}(x)\} \end{pmatrix} \right\rangle \right\}$$

$$\left[\overset{2}{\underset{i=1}{\sqcup}} (\tilde{B}_i, E)\right]^c = \left\{ e, \left\langle x, \begin{pmatrix} \min\{F^+_{B_1(e)}(x), F^+_{B_2(e)}(x)\}, 1 - \max\{I^+_{B_1(e)}(x), I^+_{B_2(e)}(x)\}, \max\{T^+_{B_1(e)}(x), T^+_{B_2(e)}(x)\}, \\ \min\{F^-_{B_1(e)}(x), F^-_{B_2(e)}(x)\}, -1 - \max\{I^-_{B_1(e)}(x), I^-_{B_2(e)}(x)\}, \max\{T^-_{B_1(e)}(x), T^-_{B_2(e)}(x)\} \end{pmatrix} \right\rangle \right\}.$$

Now,

$$(\tilde{B}_1, E)^c = \{e, \langle x, (F^+_{B_1(e)}(x), 1 - I^+_{B_1(e)}(x), T^+_{B_1(e)}(x), F^-_{B_1(e)}(x), -1 - I^-_{B_1(e)}(x), T^-_{B_1(e)}(x))\rangle\},$$
$$(\tilde{B}_2, E)^c = \{e, \langle x, (F^+_{B_2(e)}(x), 1 - I^+_{B_2(e)}(x), T^+_{B_2(e)}(x), F^-_{B_2(e)}(x), -1 - I^-_{B_2(e)}(x), T^-_{B_2(e)}(x))\rangle\}.$$

Then,

$$\overset{2}{\underset{i=1}{\sqcap}} (\tilde{B}_i, E)^c = \left\{ e, \left\langle x, \begin{pmatrix} \min\{F^+_{B_1(e)}(x), F^+_{B_2(e)}(x)\}, \min\{(1 - I^+_{B_1(e)}(x)), (1 - I^+_{B_2(e)}(x))\}, \max\{T^+_{B_1(e)}(x), T^+_{B_2(e)}(x)\} \\ \min\{F^-_{B_1(e)}(x), F^-_{B_2(e)}(x)\}, \min\{(-1 - I^-_{B_1(e)}(x)), (-1 - I^-_{B_2(e)}(x))\}, \max\{T^-_{B_1(e)}(x), T^-_{B_2(e)}(x)\} \end{pmatrix} \right\rangle \right\}$$
$$= \left\{ e, \left\langle x, \begin{pmatrix} \min\{F^+_{B_1(e)}(x), F^+_{B_2(e)}(x)\}, 1 - \max\{I^+_{B_1(e)}(x), I^+_{B_2(e)}(x)\}, \max\{T^+_{B_1(e)}(x), T^+_{B_2(e)}(x)\}, \\ \min\{F^-_{B_1(e)}(x), F^-_{B_2(e)}(x)\}, -1 - \max\{I^-_{B_1(e)}(x), I^-_{B_2(e)}(x)\}, \max\{T^-_{B_1(e)}(x), T^-_{B_2(e)}(x)\} \end{pmatrix} \right\rangle \right\}.$$

Thus, $[(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]^c = (\tilde{B}_1, E)^c \sqcap (\tilde{B}_2, E)^c$.

2. It is obtained in a similar way.

Proposition 3.15 *Let* $(\tilde{B}_1, E)$ *and* $(\tilde{B}_2, E)$ *be two bipolar neutrosophic soft sets over* X. *Then,*

1. $[(\tilde{B}_1, E) \vee (\tilde{B}_2, E)]^c = (\tilde{B}_1, E)^c \wedge (\tilde{B}_2, E)^c$;
2. $[(\tilde{B}_1, E) \wedge (\tilde{B}_2, E)]^c = (\tilde{B}_1, E)^c \vee (\tilde{B}_2, E)^c$.

Proof. 1. For all $(e_1, e_2) \in E \times E$ and $x \in X$,

$$\overset{2}{\underset{i=1}{\vee}} (\tilde{B}_i, E) = \left\{ (e_1, e_2), \left\langle x, \begin{pmatrix} \max\{T^+_{B_1(e_1)}(x), T^+_{B_2(e_2)}(x)\}, \max\{I^+_{B_1(e_1)}(x), I^+_{B_2(e_2)}(x)\}, \min\{F^+_{B_1(e_1)}(x), F^+_{B_2(e_2)}(x)\}, \\ \max\{T^-_{B_1(e_1)}(x), T^-_{B_2(e_2)}(x)\}, \max\{I^-_{B_1(e_1)}(x), I^-_{B_2(e_2)}(x)\}, \min\{F^-_{B_1(e_1)}(x), F^-_{B_2(e_2)}(x)\} \end{pmatrix} \right\rangle \right\},$$

$$\left[\overset{2}{\underset{i=1}{\vee}} (\tilde{B}_i, E)\right]^c = \left\{ (e_1, e_2), \left\langle x, \begin{matrix} \min\{F^+_{B_1(e_1)}(x), F^+_{B_2(e_2)}(x)\}, 1 - \max\{I^+_{B_1(e_1)}(x), I^+_{B_2(e_2)}(x)\}, \max\{T^+_{B_1(e_1)}(x), T^+_{B_2(e_2)}(x)\}, \\ \min\{F^-_{B_1(e_1)}(x), F^-_{B_2(e_2)}(x)\}, -1 - \max\{I^-_{B_1(e_1)}(x), I^-_{B_2(e_2)}(x)\}, \max\{T^-_{B_1(e_1)}(x), T^-_{B_2(e_2)}(x)\} \end{matrix} \right\rangle \right\}.$$

On the other hand,

$$(\tilde{B}_1, E)^c = \{e_1, \langle x, F^+_{B_1(e_1)}(x), 1 - I^+_{B_1(e_1)}(x), T^+_{B_1(e_1)}(x), F^-_{B_1(e_1)}(x), -1 - I^-_{B_1(e_1)}(x), T^-_{B_1(e_1)}(x)\rangle : e_1 \in E\},$$
$$(\tilde{B}_2, E)^c = \{e_2, \langle x, F^+_{B_2(e_2)}(x), 1 - I^+_{B_2(e_2)}(x), T^+_{B_2(e_2)}(x), F^-_{B_2(e_2)}(x), -1 - I^-_{B_2(e_2)}(x), T^-_{B_2(e_2)}(x)\rangle : e_2 \in E\}.$$

Then,

$$\overset{2}{\underset{i=1}{\wedge}} (\tilde{B}_i, E)^c = \left\{ (e_1, e_2), \left\langle \begin{matrix} x, \min\{F^+_{B_1(e_1)}(x), F^+_{B_2(e_2)}(x)\}, \min\{(1 - I^+_{B_1(e_1)}(x)), (1 - I^+_{B_2(e_2)}(x))\}, \max\{T^+_{B_1(e_1)}(x), T^+_{B_2(e_2)}(x)\} \\ \min\{F^-_{B_1(e_1)}(x), F^-_{B_2(e_2)}(x)\}, \min\{(-1 - I^-_{B_1(e_1)}(x)), (-1 - I^-_{B_2(e_2)}(x))\}, \max\{T^-_{B_1(e_1)}(x), T^-_{B_2(e_2)}(x)\} \end{matrix} \right\rangle \right\}$$
$$= \left\{ (e_1, e_2), \left\langle x, \begin{matrix} \min\{F^+_{B_1(e_1)}(x), F^+_{B_2(e_2)}(x)\}, 1 - \max\{I^+_{B_1(e_1)}(x), I^+_{B_2(e_2)}(x)\}, \max\{T^+_{B_1(e_1)}(x), T^+_{B_2(e_2)}(x)\}, \\ \min\{F^-_{B_1(e_1)}(x), F^-_{B_2(e_2)}(x)\}, -1 - \max\{I^-_{B_1(e_1)}(x), I^-_{B_2(e_2)}(x)\}, \max\{T^-_{B_1(e_1)}(x), T^-_{B_2(e_2)}(x)\} \end{matrix} \right\rangle \right\}.$$

Hence, $[(\tilde{B}_1, E) \vee (\tilde{B}_2, E)]^c = (\tilde{B}_1, E)^c \wedge (\tilde{B}_2, E)^c$.

2. It is obtained in a similar way.

4. Bipolar Neutrosophic Soft Topological Spaces

In this section we defined neutrosophic soft topology by the revised form of neutrosophic soft sets and also we gave the basic structures of the bipolar neutrosophic soft topological spaces.

Definition 4.1 *Let $BNSS(X,E)$ be the family of all bipolar neutrosophic soft sets over X and $\tau^{BN} \subset BNSS(X,E)$. Then τ^{BN} is said to be a bipolar neutrosophic soft topology on X if*

1. $(\tilde{B}^{\emptyset}, E)$ and $(\tilde{B}^{X}, E)$ belongs to τ^{BN}
2. the union of any number of bipolar neutrosophic soft sets in τ^{BN} belongs to τ^{BN}
3. the intersection of finite number of bipolar neutrosophic soft sets in τ^{BN} belongs to τ^{BN}.

Then (X, τ^{BN}, E) is said to be a bipolar neutrosophic soft topological space over X. Each members of τ^{BN} is said to be bipolar neutrosophic soft open set.

Definition 4.2 *Let (X, τ^{BN}, E) be a bipolar neutrosophic soft topological space over X and $(\tilde{B}, E)$ be a bipolar neutrosophic soft set over X. Then $(\tilde{B}, E)$ is said to be bipolar neutrosophic soft closed set iff its complement is a bipolar neutrosophic soft open set.*

Proposition 4.3 *Let (X, τ^{BN}, E) be a bipolar neutrosophic soft topological space over X. Then*

1. $(\tilde{B}^{\emptyset}, E)$ and $(\tilde{B}^{X}, E)$ are bipolar neutrosophic soft closed sets over X
2. the intersection of any number of bipolar neutrosophic soft closed sets is a bipolar neutrosophic soft closed set over X
3. the union of finite number of bipolar neutrosophic soft closed sets is a bipolar neutrosophic soft closed set over X.

Proof. It is easily obtained from the definition bipolar neutrosophic soft topological space and Proposition 2.

Definition 4.4 *Let $BNSS(X,E)$ be the family of all bipolar neutrosophic soft sets over the universe set X.*

1. If $\tau^{BN} = \{(\tilde{B}^{\emptyset}, E), (\tilde{B}^{X}, E)\}$, then τ^{BN} is said to be the bipolar neutrosophic soft indiscrete topology and (X, τ^{BN}, E) is said to be a bipolar neutrosophic soft indiscrete topological space over X.
2. If $\tau^{BN} = BNSS(X,E)$, then τ^{BN} is said to be the bipolar neutrosophic soft discrete topology and (X, τ^{BN}, E) is said to be a bipolar neutrosophic soft discrete topological space over X.

Proposition 4.5 *Let (X, τ_1^{BN}, E) and (X, τ_2^{BN}, E) be two bipolar neutrosophic soft topological spaces over the same universe set X. Then $(X, \tau_1^{BN} \cap \tau_2^{BN}, E)$ is bipolar neutrosophic soft topological space over X.*

Proof. 1. Since $(\tilde{B}^{\emptyset}, E)$, $(\tilde{B}^{X}, E) \in \tau_1^{BN}$ and $(\tilde{B}^{\emptyset}, E)$, $(\tilde{B}^{X}, E) \in \tau_2^{BN}$, then $(\tilde{B}^{\emptyset}, E)$, $(\tilde{B}^{X}, E) \in \tau_1^{BN} \cap \tau_2^{BN}$.
2. Suppose that $\{(\tilde{B}_i, E) | i \in I\}$ be a family of bipolar neutrosophic soft sets in $\tau_1^{BN} \cap \tau_2^{BN}$. Then $(\tilde{B}_i, E) \in \tau_1^{BN}$ and $(\tilde{B}_i, E) \in \tau_2^{BN}$ for all $i \in I$, so $\sqcup_{i \in I} (\tilde{B}_i, E) \in \tau_1^{BN}$ and $\sqcup_{i \in I} (\tilde{B}_i, E) \in \tau_2^{BN}$. Thus $\sqcup_{i \in I} (\tilde{B}_i, E) \in \tau_1^{BN} \cap \tau_2^{BN}$.
3. Let $\{(\tilde{B}_i, E) | i = \overline{1,n}\}$ be a family of the finite number of bipolar neutrosophic soft sets in $\tau_1^{BN} \cap \tau_2^{BN}$. Then $(\tilde{B}_i, E) \in \tau_1^{BN}$ and $(\tilde{B}_i, E) \in \tau_2^{BN}$ for $i = \overline{1,n}$, so $\sqcap_{i=1}^{n} (\tilde{B}_i, E) \in \tau_1^{BN}$ and $\sqcap_{i=1}^{n} (\tilde{B}_i, E) \in \tau_2^{BN}$. Thus $\sqcap_{i=1}^{n} (\tilde{B}_i, E) \in \tau_1^{BN} \cap \tau_2^{BN}$.

Remark 4.6 *The union of two bipolar neutrosophic soft topologies over X may not be a bipolar neutrosophic soft topology on X.*

Example 4.7 *Let $X=\{x_1,x_2\}$ be an initial universe set, $E=\{e_1,e_2\}$ be a set of parameters and $\tau_1^{BN}=\{(\tilde{B}^{\emptyset},E),(\tilde{B}^{U},E),(\tilde{B}_1,E),(\tilde{B}_2,E),(\tilde{B}_3,E)\}$ and $\tau_2^{BN}=\{(\tilde{B}^{\emptyset},E),(\tilde{B}^{U},E),(\tilde{B}_2,E),(\tilde{B}_4,E)\}$ be two bipolar neutrosophic soft topologies over X. Here, the bipolar neutrosophic soft sets $(\tilde{B}_1,E)$, $(\tilde{B}_2,E)$, $(\tilde{B}_3,E)$ and $(\tilde{B}_4,E)$ over X are defined as following:*

$$(\tilde{B}_1,E)=\begin{cases}e_1,\langle x_1,(0.9,0.4,0.3,-0.2,-0.3,-0.7)\rangle,\langle x_2,(0.5,0.6,0.5,-0.1,-0.2,-0.8)\rangle\\ e_2,\langle x_1,(0.7,0.3,0.4,-0.4,-0.5,-0,4)\rangle,\langle x_2,(0.6,0.6,0.2,-0.6,-0.7,-0.5)\rangle\end{cases},$$
$$(\tilde{B}_2,E)=\begin{cases}e_1,\langle x_1,(0.7,0.4,0.5,-0.3,-0.4,-0.6)\rangle,\langle x_2,(0.4,0.5,0.5,-0.2,-0.3,-0.7)\rangle\\ e_2,\langle x_1,(0.6,0.2,0.4,-0.5,-0.6,-0.3)\rangle,\langle x_2,(0.5,0.4,0.3,-0.7,-0.8,-0.4)\rangle\end{cases},$$
$$(\tilde{B}_3,E)=\begin{cases}e_1,\langle x_1,(0.5,0.3,0.6,-0.4,-0.5,-0.5)\rangle,\langle x_2,(0.3,0.4,0.7,-0.3,-0.4,-0.6)\rangle\\ e_2,\langle x_1,(0.4,0.1,0.5,-0.6,-0.7,-0.2)\rangle,\langle x_2,(0.4,0.3,0.4,-0.8,-0.9,-0.3)\rangle\end{cases},$$
$$(\tilde{B}_4,E)=\begin{cases}e_1,\langle x_1,(0.8,0.5,0.4,-0.1,-0.2,-0.8)\rangle,\langle x_2,(0.5,0.6,0.3,-0.1,-0.1,-0.9)\rangle\\ e_2,\langle x_1,(0.7,0.3,0.3,-0.3,-0.4,-0.5)\rangle,\langle x_2,(0.6,0.5,0.1,-0.5,-0.6,-0.6)\rangle\end{cases}.$$

Since $(\tilde{B}_1,E)\cup(\tilde{B}_4,E)\notin\tau_1^{BN}\sqcup\tau_2^{BN}$, then $\tau_1^{BN}\sqcup\tau_2^{BN}$ is not a bipolar neutrosophic soft topology over X.

Proposition 4.8 *Let (X,τ^{BN},E) be a bipolar neutrosophic soft topological space over X and $\tau^{BN}=\{(\tilde{B}_i,E):(\tilde{B}_i,E)\in BNSS(X,E)\}$ where*
$(\tilde{B}_i,E)=\{(e,\langle x,(T^+_{B_i(e)}(x),I^+_{B_i(e)}(x),F^+_{B_i(e)}(x),T^-_{B_i(e)}(x),I^-_{B_i(e)}(x),F^-_{B_i(e)}(x))\rangle:x\in X):e\in E\}$ for $i\in I$.
Then
$\overset{NSS}{\tau}=\{(\tilde{B}_i^+,E)=\{(e,\langle x,(T^+_{B_i(e)}(x),I^+_{B_i(e)}(x),F^+_{B_i(e)}(x))\rangle:x\in X):e\in E\}:(\tilde{B}_i^+,E)\in NSS(X,E)\}$
define neutrosophic soft topology on X.

Proof. Straightforward.

Definition 4.9 *Let (X,τ^{BN},E) be a bipolar neutrosophic soft topological space over X and $(\tilde{B},E)\in BNSS(X,E)$ be a bipolar neutrosophic soft set. Then, bipolar neutrosophic soft interior of $(\tilde{B},E)$, denoted $(\tilde{B},E)^{\circ}$, is defined as the bipolar neutrosophic soft union of all bipolar neutrosophic soft open subsets of $(\tilde{B},E)$.* Clearly, $(\tilde{B},E)^{\circ}$ is the biggest bipolar neutrosophic soft open set contained by $(\tilde{B},E)$.

Example 4.10 *Let us consider the bipolar neutrosophic soft topology τ_1^{BN} given in Example 4.7. Suppose that an any $(\tilde{B},E)\in BNSS(X,E)$ is defined as following:*

$$(\tilde{B},E)=\begin{cases}e_1,\langle x_1,(0.8,0.4,0.2,-0.1,-0.2,-0.6)\rangle,\langle x_2,(0.4,0.7,0.3,-0.2,-0.1,-0.9)\rangle\\ e_2,\langle x_1,(0.9,0.2,0.3,-0.3,-0.6,-0.5)\rangle,\langle x_2,(0.7,0.5,0.1,-0.4,-0.6,-0.6)\rangle\end{cases}.$$

Then $(\tilde{B}^{\emptyset},E)$, $(\tilde{B}_2,E)$, $(\tilde{B}_3,E)\sqsubseteq(\tilde{B},E)$. Therefore, $(\tilde{B},E)^{\circ}=(\tilde{B}^{\emptyset},E)\sqcup(\tilde{B}_2,E)\sqcup(\tilde{B}_3,E)=(\tilde{B}_2,E)$.

Theorem 4.11 *Let (X,τ^{BN},E) be a bipolar neutrosophic soft topological space over X and $(\tilde{B},E)\in BNSS(X,E)$. $(\tilde{B},E)$ is a bipolar neutrosophic soft open set iff $(\tilde{B},E)=(\tilde{B},E)^{\circ}$.*

Proof. Let $(\tilde{B},E)$ be a bipolar neutrosophic soft open set. Then the biggest bipolar neutrosophic soft open set that is contained by $(\tilde{B},E)$ is equal to $(\tilde{B},E)$. Hence, $(\tilde{B},E)=(\tilde{B},E)^{\circ}$.
Conversely, it is known that $(\tilde{B},E)^{\circ}$ is a bipolar neutrosophic soft open set and if $(\tilde{B},E)=(\tilde{B},E)^{\circ}$, then $(\tilde{B},E)$ is a bipolar neutrosophic soft open set.

Theorem 4.12 *Let* (X, τ^{BN}, E) *be a bipolar neutrosophic soft topological space over* X *and* $(\tilde{B}_1, E), (\tilde{B}_2, E) \in BNSS(X, E)$. *Then,*

1. $[(\tilde{B}_1, E)^{\circ}]^{\circ} = (\tilde{B}_1, E)^{\circ}$,
2. $(\tilde{B}^{\emptyset}, E)^{\circ} = (\tilde{B}^{\emptyset}, E)$ and $(\tilde{B}^{X}, E)^{\circ} = (\tilde{B}^{X}, E)$,
3. $(\tilde{B}_1, E) \sqsubseteq (\tilde{B}_2, E) \Rightarrow (\tilde{B}_1, E)^{\circ} \sqsubseteq (\tilde{B}_2, E)^{\circ}$,
4. $[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]^{\circ} = (\tilde{B}_1, E)^{\circ} \sqcap (\tilde{B}_2, E)^{\circ}$,
5. $(\tilde{B}_1, E)^{\circ} \sqcup (\tilde{B}_2, E)^{\circ} \sqsubseteq [(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]^{\circ}$.

Proof. 1. Let $(\tilde{B}_1, E)^{\circ} = (\tilde{B}_2, E)$. Then $(\tilde{B}_2, E) \in \tau^{BN}$ iff $(\tilde{B}_2, E) = (\tilde{B}_2, E)^{\circ}$. So, $[(\tilde{B}_1, E)^{\circ}]^{\circ} = (\tilde{B}_1, E)^{\circ}$.
2. Straighforward.
3. It is known that $(\tilde{B}_1, E)^{\circ} \sqsubseteq (\tilde{B}_1, E) \sqsubseteq (\tilde{B}_2, E)$ and $(\tilde{B}_2, E)^{\circ} \sqsubseteq (\tilde{B}_2, E)$. Since $(\tilde{B}_2, E)^{\circ}$ is the biggest bipolar neutrosophic soft open set contained in $(\tilde{B}_2, E)$ and so, $(\tilde{B}_1, E)^{\circ} \sqsubseteq (\tilde{B}_2, E)^{\circ}$.
4. Since $(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E) \sqsubseteq (\tilde{B}_1, E)$ and $(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E) \sqsubseteq (\tilde{B}_2, E)$, then $[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]^{\circ} \sqsubseteq (\tilde{B}_1, E)^{\circ}$ and $[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]^{\circ} \sqsubseteq (\tilde{B}_2, E)^{\circ}$ and so, $[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]^{\circ} \sqsubseteq (\tilde{B}_1, E)^{\circ} \sqcap (\tilde{B}_2, E)^{\circ}$.
On the other hand, since $(\tilde{B}_1, E)^{\circ} \sqsubseteq (\tilde{B}_1, E)$ and $(\tilde{B}_2, E)^{\circ} \sqsubseteq (\tilde{B}_2, E)$, then $(\tilde{B}_1, E)^{\circ} \sqcap (\tilde{B}_2, E)^{\circ} \sqsubseteq (\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)$. Besides, $[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]^{\circ} \sqsubseteq (\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)$ and it is the biggest bipolar neutrosophic soft open set. Therefore, $(\tilde{B}_1, E)^{\circ} \sqcap (\tilde{B}_2, E)^{\circ} \sqsubseteq [(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]^{\circ}$.
Thus, $[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]^{\circ} = (\tilde{B}_1, E)^{\circ} \sqcap (\tilde{B}_2, E)^{\circ}$.
5. Since $(\tilde{B}_1, E) \sqsubseteq (\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)$ and $(\tilde{B}_2, E) \sqsubseteq (\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)$, then $(\tilde{B}_1, E)^{\circ} \sqsubseteq [(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]^{\circ}$ and $(\tilde{B}_2, E)^{\circ} \sqsubseteq [(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]^{\circ}$. Therefore, $(\tilde{B}_1, E)^{\circ} \sqcup (\tilde{B}_2, E)^{\circ} \sqsubseteq [(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]^{\circ}$.

Definition 4.13 *Let* (X, τ^{BN}, E) *be a bipolar neutrosophic soft topological space over* X *and* $(\tilde{B}, E) \in BNSS(X, E)$ *be a bipolar neutrosophic soft set. Then, bipolar neutrosophic soft closure of* $(\tilde{B}, E)$, *denoted* $\overline{(\tilde{B}, E)}$, *is defined as the bipolar neutrosophic soft intersection of all bipolar neutrosophic soft closed supersets of* $(\tilde{B}, E)$.
Clearly, $\overline{(\tilde{B}, E)}$ is the smallest bipolar neutrosophic soft closed set that containing $(\tilde{B}, E)$.

Example 4.14 *Let us consider the bipolar neutrosophic soft topology* τ_1^{BN} *given in Example 4.7. Suppose that an any* $(\tilde{B}, E) \in BNSS(X, E)$ *is defined as following:*

$$(\tilde{B}, E) = \begin{Bmatrix} e_1, \langle x_1, (0.1, 0.4, 0.9, -0.8, -0.9, -0.1) \rangle, \langle x_2, (0.4, 0.2, 0.7, -0.9, -0.8, -0.1) \rangle \\ e_2, \langle x_1, (0.2, 0.3, 0.8, -0.6, -0.7, -0{,}2) \rangle, \langle x_2, (0.1, 0.2, 0.8, -0.6, -0.7, -0.4) \rangle \end{Bmatrix}.$$

Obviously, $(\tilde{B}^{\emptyset}, E)$, $(\tilde{B}^{U}, E)$, $(\tilde{B}_1, E)^c$, $(\tilde{B}_2, E)^c$ and $(\tilde{B}_3, E)^c$ are all bipolar neutrosophic soft closed sets over (X, τ_1^{BN}, E). They are given as following:

$$(\tilde{B}^{\emptyset}, E)^c = (\tilde{B}^{U}, E), (\tilde{B}^{U}, E)^c = (\tilde{B}^{\emptyset}, E)$$
$$(\tilde{B}_1, E)^c = \begin{Bmatrix} e_1, \langle x_1, (0.3, 0.6, 0.9, -0.7, -0.7, -0.2) \rangle, \langle x_2, (0.5, 0.4, 0.5, -0.8, -0.8, -0.1) \rangle \\ e_2, \langle x_1, (0.4, 0.7, 0.7, -0.4, -0.5, -0{,}4) \rangle, \langle x_2, (0.2, 0.4, 0.6, -0.5, -0.3, -0.6) \rangle \end{Bmatrix},$$
$$(\tilde{B}_2, E)^c = \begin{Bmatrix} e_1, \langle x_1, (0.5, 0.6, 0.7, -0.6, -0.6, -0.3) \rangle, \langle x_2, (0.5, 0.5, 0.4, -0.7, -0.7, -0.2) \rangle \\ e_2, \langle x_1, (0.4, 0.8, 0.6, -0.3, -0.4, -0{,}5) \rangle, \langle x_2, (0.3, 0.6, 0.5, -0.4, -0.2, -0.7) \rangle \end{Bmatrix},$$
$$(\tilde{B}_3, E)^c = \begin{Bmatrix} e_1, \langle x_1, (0.6, 0.7, 0.5, -0.5, -0.5, -0.4) \rangle, \langle x_2, (0.7, 0.6, 0.3, -0.6, -0.6, -0.3) \rangle \\ e_2, \langle x_1, (0.5, 0.9, 0.4, -0.2, -0.3, -0{,}6) \rangle, \langle x_2, (0.4, 0.7, 0.4, -0.3, -0.1, -0.8) \rangle \end{Bmatrix}.$$

Then $(\tilde{B}^{U}, E)^c$, $(\tilde{B}_1, E)^c$, $(\tilde{B}_2, E)^c, (\tilde{B}_3, E)^c \sqsupseteq (\tilde{B}, E)$. Therefore, $\overline{(\tilde{B}, E)} = (\tilde{B}^{U}, E)^c \sqcap (\tilde{B}_1, E)^c \sqcap (\tilde{B}_2, E)^c \sqcap (\tilde{B}_3, E)^c = (\tilde{B}_1, E)^c$.

Theorem 4.15 *Let* (X, τ^{BN}, E) *be a bipolar neutrosophic soft topological space over* X *and* $(\tilde{B}, E) \in BNSS(X, E)$. $(\tilde{B}, E)$ *is bipolar neutrosophic soft closed set iff* $(\tilde{B}, E) = \overline{(\tilde{B}, E)}$.

Proof. Straightforward.

Theorem 4.16 *Let* (X, τ^{BN}, E) *be a bipolar neutrosophic soft topological space over* X *and* $(\tilde{B}_1, E), (\tilde{B}_2, E) \in BNSS(X, E)$. *Then,*

1. $\overline{\left[\overline{(\tilde{B}_1, E)}\right]} = \overline{(\tilde{B}_1, E)}$,
2. $\overline{(\tilde{B}^{\emptyset}, E)} = (\tilde{B}^{\emptyset}, E)$ and $\overline{(\tilde{B}^{X}, E)} = (\tilde{B}^{X}, E)$
3. $(\tilde{B}_1, E) \sqsubseteq (\tilde{B}_2, E) \Rightarrow \overline{(\tilde{B}_1, E)} \sqsubseteq \overline{(\tilde{B}_2, E)}$,
4. $\overline{[(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]} = \overline{(\tilde{B}_1, E)} \sqcup \overline{(\tilde{B}_2, E)}$,
5. $\overline{[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]} \sqsubseteq \overline{(\tilde{B}_1, E)} \sqcap \overline{(\tilde{B}_2, E)}$.

Proof. 1. Let $\overline{(\tilde{B}_1, E)} = (\tilde{B}_2, E)$. Then, $(\tilde{B}_2, E)$ is a bipolar neutrosophic soft closed set. Hence, $(\tilde{B}_2, E)$ and $\overline{(\tilde{B}_2, E)}$ are equal. Therefore, $\overline{\left[\overline{(\tilde{B}_1, E)}\right]} = \overline{(\tilde{B}_1, E)}$.

2. Straightforward.

3. It is known that $(\tilde{B}_1, E) \sqsubseteq \overline{(\tilde{B}_1, E)}$ and $(\tilde{B}_2, E) \sqsubseteq \overline{(\tilde{B}_2, E)}$ and so, $(\tilde{B}_1, E) \sqsubseteq (\tilde{B}_2, E) \sqsubseteq \overline{(\tilde{B}_2, E)}$. Since $\overline{(\tilde{B}_1, E)}$ is the smallest bipolar neutrosophic soft closed set containing $(\tilde{B}_1, E)$, then $\overline{(\tilde{B}_1, E)} \sqsubseteq \overline{(\tilde{B}_2, E)}$.

4. Since $(\tilde{B}_1, E) \sqsubseteq (\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)$ and $(\tilde{B}_2, E) \sqsubseteq (\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)$, then $\overline{(\tilde{B}_1, E)} \sqsubseteq \overline{[(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]}$ and $\overline{(\tilde{B}_2, E)} \sqsubseteq \overline{[(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]}$ and so, $\overline{(\tilde{B}_1, E)} \sqcup \overline{(\tilde{B}_2, E)} \sqsubseteq \overline{[(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]}$.
Conversely, since $(\tilde{B}_1, E) \sqsubseteq \overline{(\tilde{B}_1, E)}$ and $(\tilde{B}_2, E) \sqsubseteq \overline{(\tilde{B}_2, E)}$, then $(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E) \sqsubseteq \overline{(\tilde{B}_1, E)} \sqcup \overline{(\tilde{B}_2, E)}$. Besides, $\overline{[(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]}$ is the smallest bipolar neutrosophic soft closed set that containing $(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)$. Therefore, $\overline{[(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]} \sqsubseteq \overline{(\tilde{B}_1, E)} \sqcup \overline{(\tilde{B}_2, E)}$. Thus, $\overline{[(\tilde{B}_1, E) \sqcup (\tilde{B}_2, E)]} = \overline{(\tilde{B}_1, E)} \sqcup \overline{(\tilde{B}_2, E)}$.

5. Since $(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E) \sqsubseteq \overline{(\tilde{B}_1, E)} \sqcap \overline{(\tilde{B}_2, E)}$ and $\overline{[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]}$ is the smallest bipolar neutrosophic soft closed set that containing $(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)$, then $\overline{[(\tilde{B}_1, E) \sqcap (\tilde{B}_2, E)]} \sqsubseteq \overline{(\tilde{B}_1, E)} \sqcap \overline{(\tilde{B}_2, E)}$.

Theorem 4.17 *Let* (X, τ^{BN}, E) *be a bipolar neutrosophic soft topological space over* X *and* $(\tilde{B}, E) \in BNSS(X, E)$. *Then,*

1. $\left[\overline{(\tilde{B}, E)}\right]^c = \left[(\tilde{B}, E)^c\right]^{\circ}$,
2. $\left[(\tilde{B}, E)^{\circ}\right]^c = \overline{\left[(\tilde{B}, E)^c\right]}$.

Proof. 1. $\overline{(\tilde{B}, E)} = \sqcap_{i \in I} \{(\tilde{B}_i, E) \in (\tau^{BN})^c : (\tilde{B}_i, E) \sqsupseteq (\tilde{B}, E)\}$

$$\Rightarrow \left[\overline{(\tilde{B}, E)}\right]^c = \left[\sqcap_{i \in I} \{(\tilde{B}_i, E) \in (\tau^{BN})^c : (\tilde{B}_i, E) \sqsupseteq (\tilde{B}, E), \forall i \in I\}\right]^c$$

$$= \sqcup_{i \in I} \left\{(\tilde{B}_i, E)^c \in \overset{NSS}{\tau} : (\tilde{B}_i, E)^c \sqsubseteq (\tilde{B}, E)^c\right\} = \left[(\tilde{B}, E)^c\right]^{\circ}.$$

2. $(\tilde{B}, E)^{\circ} = \sqcup_{i \in I} \{(\tilde{B}_i, E) \in \tau^{BN} : (\tilde{B}_i, E) \sqsubseteq (\tilde{B}, E)\}$

$$\Rightarrow \left[(\tilde{B}, E)^{\circ}\right]^c = \left[\sqcup_{i \in I} \left\{(\tilde{B}_i, E) \in \overset{NSS}{\tau} : (\tilde{B}_i, E) \sqsubseteq (\tilde{F}, E)\right\}\right]^c$$

$$= \sqcap_{i \in I} \{(\tilde{B}_i, E)^c \in (\tau^{BN})^c : (\tilde{B}_i, E)^c \sqsupseteq (\tilde{B}, E)^c\} = \overline{\left[(\tilde{B}, E)^c\right]}.$$

5. Conclusions

Re-defined operations in this study are placed on a suitable system to present topological structure on bipolar neutrosophic soft sets. Later, bipolar neutrosophic soft topological spaces are defined and their structural properties are presented. Since this study is the basic characteristic of bipolar neutrosophic soft set theory, it will be able to lead the study of bipolar neutrosophic soft set structure in all sub-branches of mathematics. It can be also considered as a preliminary study of the theory mentioned in topology.

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Conflicts of Interest

The authors declare no conflict of interest.

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Analysis of Technological Innovation Contribution to Gross Domestic Product Based on Neutrosophic Cognitive Maps and Neutrosophic Numbers

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Abstract: Sustained growth and progress towards more equitable societies with better opportunities for all depends on how competitive a country could be, which in turn depends on the productivity of its economic sectors. The study aims to analyze the influence of technological innovation to Ecuador's gross domestic product, using a neutrosophic cognitive map that defines the factors that directly affect technological innovation. The PESTEL framework is used to identify the political, economic, social, technological, ecological, and legal factors that contribute to technological innovation in Ecuador's gross domestic product. For this purpose, a quantitative analysis was carried out based on the static analysis and neutrosophic numbers, which facilitated the applicability of the proposal. The main contribution present work is the analysis of interrelations and uncertainty/indeterminacy for analysis of technological innovation. The results show the importance of political and legal factors related to technological innovation projects to gross domestic products growth in Ecuador. The work ends with the conclusion and recommendations for future work.

Keywords: Technological innovation; PESTEL; neutrosophic numbers, neutrosophic cognitive maps; static analysis

1. Introduction

Latin America has made significant progress in stabilizing macroeconomic policies that have kept its economies growing, even in an adverse international context. However, sustained growth and progress towards more equitable societies with better opportunities for all depend on how competitive the region can be, which in turn depends on the productivity of its economic sectors. It is a fact that Latin America has significant lags in productivity and competitiveness compared to other developing regions [1].

Ecuador is not an exception, macroeconomic stability has improved, and gross domestic product (GDP) grew more than 5% according to [2]. However, behind this past growth, there is a little diversified economy that concentrates on products and exports that are not very intensive in specialized knowledge and added value. This entails a risk for the country's growth in the long term, which is as imminent as it is worthy.

The issue of innovation must be analyzed with a systemic approach, which addresses not only the individual performance of the parties but also their interactions. Investment in innovation, acquisition, absorption, modification, and creation of technological and non-technological knowledge are indispensable activities for the development of any economy [3]. When dealing with activities that demand sophisticated inputs, which involve risks and face market failures, their success depends on the systemic and systematic interaction of the public sector, the private sector and the entities capable of generating knowledge.

These coordination needs require a national strategy with short, medium and long term objectives. It is also for this reason that the theme of innovation must be analyzed with a systemic approach, which addresses not only the individual performance of the parties but also their interactions.

The National Innovation System of Ecuador is characterized by unprecedented public investment in innovation activities and the creation of a highly qualified human talent base. This analysis benefits from unprecedented quantitative information on the subject of entrepreneurship and highlights the presence of a critical mass of entrepreneurs who innovate and generate growth opportunities for the country, especially in the services sector.

It should be noted that Ecuador has shown a relatively good economic performance in recent years, but its low starting point means that it still has a way to go before reaching the average level of per capita income in the region. Even high levels of poverty and inequality pose the imperative of growth.

One of the weakest points for Ecuador's growth is the low level of total factor productivity (TFP), which explains more than 70% of the income gap with the United States are is where the role of innovation as an engine of economic growth and productivity takes relevance.

The existence of a causal link between innovation (especially I+D) and growth is reflected in the positive social returns of innovation activities. In the case of Ecuador, the social return rate of investment in I+D would be around 47% and that of investment in physical capital around 12%. This would imply that investing in I+D is almost four times more profitable than an investment in capital, which shows the vast space that exists to invest in I+D and generate value.

Despite the above, innovation does not occur at optimum levels automatically, since there is a set of problems or failures that make the investment in innovation by agents less than the social optimum. These problems can be grouped into four categories:

1. Insufficient appropriation of benefits
2. Information asymmetries
3. High uncertainty
4. Coordination problems

From the analysis of existing indicators and the processing of quantitative information, it is observed that Ecuador has a long way to go. Concerning the regulatory framework and the business climate, in Ecuador, people need a lot of days, procedures and money to open a company.

As for the protection of intellectual property, it is inferior to that of all the reference countries in the region. The levels of use of standards remain low compared to the rest of the region

Tax schemes and benefits need higher specificity: they are incentives that favor the retention of profits, which affects the investment in working capital, but they do not point to invest in innovation in a particular way. On the positive side, levels of broadband penetration have increased steadily in recent years and are expected to continue to do so; even Ecuador has been the country in Latin America where the use of the Internet has grown fastest in recent years.

Respectively, different inputs for innovation are analyzed, both empirically and conceptually for the Ecuadorian case, where countries of the region and developed economies are used as a point of comparison. Specifically, investment in I+D and its composition, human talent, and access to credit through the financial market are studied.

The indicator traditionally used to measure the intensity of innovation activities in an economy is the expenditure made in I+D. Human talent is another indicator that is used to measure innovation concerning the Gross Domestic Product, in this sense, Ecuador has achieved significant improvements in the enrolment of students in educational institutions and adequate access to higher education of the students lower quintiles.

Concerning the quality of children's education, Ecuador has participated in some international comparative learning tests, in which it has been documented that the quality of a year in school for the average child in this country is well below international standards and, in the Latin American context, it is among the lowest. On the other hand, both the quality and relevance of the education of higher education also present deficiencies.

It should be noted that Ecuador is one of the Latin American countries with the lowest number of professionals trained in the fields of engineering and sciences. However, in recent years, the public sector has committed a significant amount of resources to reverse this situation. Along with the efforts aimed at raising the coverage and quality of education that is taught in the country, those aimed at promoting the advanced training of professionals, particularly abroad, stand out.

Economic growth, productivity, and innovation have unique importance concerning access to financing; specific data are not available for innovation activities for Ecuador. However, there is a history of access to credit by companies in general that have a direct impact on the Gross Domestic Product.

The main variables that allow us to estimate how successful the results of the inputs are in the contribution of technological innovation to the gross domestic product in Ecuador are those related to patents, publications, and the export of technology. With regard to the evolution of the number of applications entered and the registration of intellectual property in the Ecuadorian Institute of Intellectual Property (IEPI in Spanish), the country has not experienced a substantial change, but only minimal variations are recorded.

Regarding high technology exports, Ecuador has a very low share compared to the rest of the region. These pieces of evidence allow us to see in a general way the current panorama of the National System of Innovation (SNI in Spanish) of Ecuador, an economy that has made great efforts to strengthen its innovation activities, but with significant challenges still to be solved.

Consequently, the level of investment in innovation of an economy is determined by a series of factors, both on the side of inputs and environmental conditions, as well as the results that these inputs and the characteristics that the economy generate. On the side of the environmental factors that facilitate innovation, it is worth mentioning:

The regulatory framework

Protection of intellectual property

Quality control, standardization, and metrology

Tax incentives

Information and communication technologies (TIC)

Productivity is essential for economic growth and the competitiveness of an economy since it reflects the efficiency level of that economy in the generation of its product. Productivity is not everything, but in the long term, it is almost everything. A country's ability to improve its standard of living over time depends almost exclusively on its ability to increase its output per worker [4].

Total factor productivity represents economic growth that is not explained by productive factors, capital, and labor. The technology produces improvements in efficiency, as well as positive externalities that contribute to an increase in production. Therefore, if the productive factors were increased, production would grow more than proportionally, since technological improvement affects the final result.

Current approaches lack analysis of interrelations and uncertainty/indeterminacy for analysis of technological innovation contribution to gross domestic. The use of neutrosophy in cognitive maps is useful because it contributes to the treatment of indetermination and inconsistent information [5].

Neutrosophic cognitive maps (NCM) are an extension of fuzzy cognitive maps, including indetermination in causal relations [6, 7]. Fuzzy cognitive maps do not include an indeterminate relationship [8], making it less suitable for real-world applications.

In the present study, an analysis of the proposal is made where the possibility of dealing with the interdependencies, the feedback, and indetermination of the technological innovation, and its contribution to the Gross Domestic Product through the use of neutrosophic cognitive maps are presented.

Fuzzy cognitive maps (FCM) are a tool for modeling causal relations interrelations [9]. Connections in FCMs are just numeric, and the relationship between two events should be linear [10]. On the other hand, neutrosophy operates with indeterminate and inconsistent information, while fuzzy sets and intuitionistic fuzzy do not [5]. Neutrosophic cognitive maps (NCM) are an extension of FCM where was included the concept indeterminacy [6, 7], whereas of fuzzy cognitive maps fails to deal with this kind of relation [8]. Neutrosophics decision support is an area of active research with new development in areas of application [11, 12, 13] and group decision making for example [14,15].

In this paper, a model for the analysis of Technological Innovation projects contribution to Gross Domestic Product based on neutrosophic cognitive maps and PESTEL analysis is presented, providing methodological support and making possible dealing with real-world facts like interdependence, indeterminacy and feedback, indeterminacy. This paper continues as follows: Section 2 reviews some essential concepts about NCM. In Section 3, a framework for the show a static analysis based on NCM. Section 4, displays a case study of the proposed model. The paper finishes with conclusions and additional work recommendations.

2. Neutrosophic cognitive maps

Neutrosophic Logic (NL) is a generalization of the fuzzy logic that was introduced in 1995 [16]. According to this theory, a logical proposition P is characterized by three neutrosophic components:

$$NL(P) = (T, I, F) \quad (1)$$

Where the neutrosophic component the degree of true is T, the degree of falsehood is F, and I is the degree of indeterminacy [9]. Neutrosophic set (NS) was introduced by F. Smarandache, who introduced the degree of indeterminacy (i) as an independent component [11].

Additionally, a neutrosophic matrix is a matrix where the elements are $a = (a_{ij})$ have been replaced by elements in $\langle R \cup I \rangle$. A neutrosophic graph is a graph with at least one neutrosophic edge [7]. If a cognitive map includes indetermination, it is called the neutrosophic cognitive map (NCM) [9]. NCM is based on neutrosophic logic to represent uncertainty and indeterminacy in cognitive maps to deal with real-world problems [17]. An NCM is a directed graph in which at least one edge is an indeterminate border and is indicated by dashed lines [7] (Figure 2).

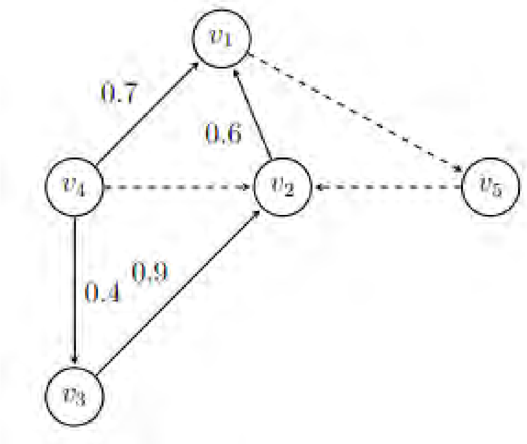

Figure 1. Neutrosophic Cognitive Maps example.

In [9] a static analysis of an NCM is presented. The result of the static analysis is in the form of neutrosophic numbers (a+bI, where I = indeterminacy). A neutrosophic number is a number as follows [14] :

$$N = d + I \quad (2)$$

Where d is the determinacy part, and i is the indeterminate part. For example s: a=5 +I si $i \in [5, 5.4]$ is equivalent to $a \in [5, 5.4]$.

Let $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ be two neutrosophic numbers then the following operational relation of neutrosophic numbers are defined as follows [17]:

$N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I$;

$N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I$

A de-neutrosophication process as proposed by Salmeron and Smarandache could be applied giving final ranking values [13]. In the de-neutrosophication process, a neutrosophic value is converted in an interval with two values, the maximum and the minimum value for I. The neutrosophic centrality measure will be an area where the upper limit has I =1 and the lower limit has I = 0.

3. Proposed Framework

The aim was to develop and further detail a framework based on PESTEL and NCM [15] to analyze the contribution of technology to Gross national product (GNP). The model was made in five steps (graphically, figure 3).

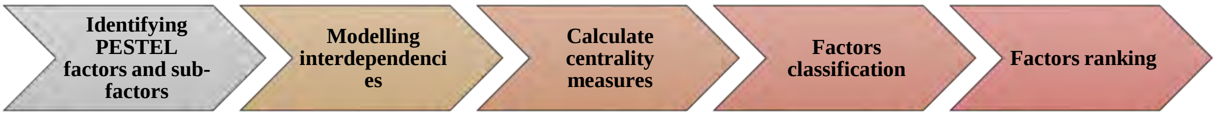

Figure. 2. The proposed framework for PESTEL analysis [15]

.3.1 Factors and sub-factors identification in the PESTEL method

In this step, the significant PESTEL factors and sub-factors were recognized. Identify factors and subfactors to form a hierarchical structure of the PESTEL model. Sub-factors are categorized according to the literature [18].

3.2 Modelling interdependencies

In this step, causal interdependencies between PESTEL sub-factors are modeled, consists of the construction of NCM of subfactors following the point views of an expert or a group of experts.

If a group of experts (k) participates, the adjacency matrix of the collective NCM is calculated as follows:

$$\mathrm{E} = \mu(\mathrm{E}_1, \mathrm{E}_2, \dots, \mathrm{E}_k) \quad (3)$$

The μ operator is usually the arithmetic mean [20].

3.3 Calculate centrality measures

Centrality measures are calculated [21] with absolute values of the adjacency matrix from the NCM [19]:

- Outdegree od (v_i) is the summation of the row of absolute values of a variable in the neutrosophic adjacency matrix and shows the aggregated strengths of connections (c_{ij}) leaving the node.

$$od(v_i) = \sum_{i=1}^{N} c_{ij} \quad (4)$$

- Indegree $id(v_i)$ is the summation of the column of absolute values of a variable, and it shows the total strength of variables entering into the node.
  $id(v_i) = \sum_{i=1}^{N} c_{ji}$ (5)
- The centrality degree (total degree $td(v_i)$), of a variable is the total sum of its indegree and outdegree
  $td(v_i) = od(v_i) + id(v_i)$ (6)

3.4 Factors classification and ranking

The factors were categorized according to the next rules:

- The variables are a Transmitter (T) when having a positive or indeterminacy outdegree, $od(v_i)$ and zero indegree, $id(v_i)$.
- The variables give a Receiver (R) when having a positive indegree or indeterminacy, $id(v_i)$., and zero outdegree, $od(v_i)$.
- Variables receive the Ordinary (O) name when they have a non-zero degree, and these Ordinary variables can be considered more or less as receiving variables or transmitting variables, depending on the relation of their indegrees and outdegrees.

The de-neutrosophication process provides a range of numbers for centrality using as a ground the maximum & minimum values of I. A neutrosophic value is changed to a value an interval from I=0 to I=1.

The importance of a variable in an NCM can be known by calculating its degree of centrality, which shows how the node is connected to other nodes and what is the total force of these connections. The median of the extreme values as proposed by Merigo [23] is used to give a real number as a centrality value :

$$\lambda([a_1, a_2]) = \frac{a_1 + a_2}{2} \quad (7)$$

Then

$$A > B \Leftrightarrow \frac{a_1 + a_2}{2} > \frac{b_1 + b_2}{2} \quad (8)$$

Finally, a ranking of variables is given.

3.3 Factor prioritization

The numerical value obtained in the previous step is used for sub-factor ranking and/or reduction [21,22]. Threshold values may be set for subfactor reduction. Additionally, sub-factor could be grouped to extend the analysis to ecological, economic, legal, political, social and technological general factors.

4. Case Study

Figure 4 shows the factors from the PESTEL model that are obtained for the analysis of the factors that have the greatest impact on technological innovation and that have an impact on Ecuador's gross domestic product.

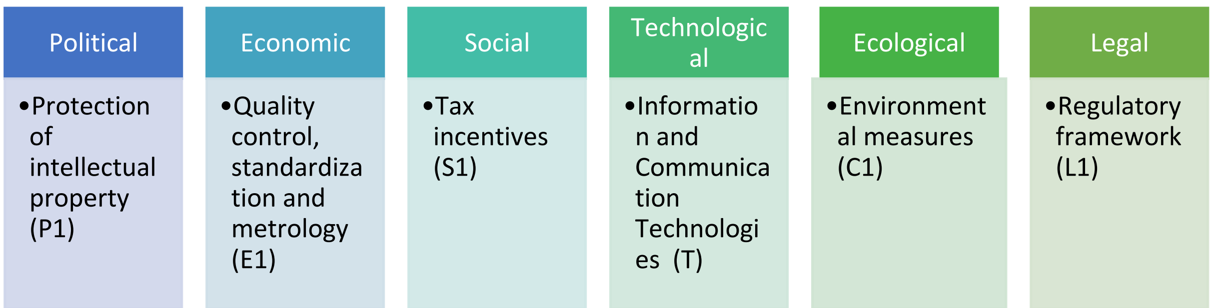

Figure 4. Factors identified through the PESTEL technique.

Obtained the characteristics corresponding to the factors of the PESTEL model, later are analyzed taking into account that the PESTEL model is a strategic analysis technique to define the context of a determined area through the analysis of a series of external factors [18, 19]. The PESTEL analysis incorporates in PEST analysis the ecological and legal factors into the so that in the present investigation, a PEST analysis was previously carried out and extended to include those factors.

In the present study, neutrosophic cognitive maps, for better interpretability is used as a tool for modeling the characteristics that are related the factors that affect technological innovation and that have an impact on Ecuador's gross domestic product.

For the evaluation of the PESTEL factors are modeling with a neutrosophic cognitive map. The factors found with the PESTEL technique and the causal connection to each factor that was represented in figure 4 are taken into account. NCM is used as a tool for modeling the characteristics that are related to the factors that affect technological innovation and that have an impact on Ecuador's gross domestic product. The neutrosophic cognitive map in the present study is developed through experts' knowledge. The neutrosophic adjacency matrix obtained is shown in Table 1.

Table 1. Neutrosophic adjacency matrix.

	P1	**E1**	**S1**	**T1**	**C1**	**L1**
P1	0	0	0	0	0	0
E1	0	0	0	0	0	0
S1	0.4	0	0	0	0	0
T1	0	0	0	0	0	0
C1	0	0	0	0	0.25	0
L1	0	0	0	0	0.25	0

Based on the neutrosophic adjacency matric centralities measures are calculates (Table 2)

Table 2. Measures of centrality, outdegree, indegree

Node	**Id**	**Od**
P1	0.4	0
E1	0	0
S1	0	0.4
T1	0	0
C1	0.25	0.25
L1	0	0.25

When the centrality measures are calculated, the nodes of the neutrosophic cognitive map are classified according to rules presented in section 3.4.

Table 3. Classification of the nodes.

	Transmitter node	**Receiving node**	**Ordinary**
P1		x	
E1			
S1	x		
T1			
C1			x
L1	x		

The total centrality (total degree $td\ (vi)$), is calculated through equation 6. Finally, we work with the mean of the extreme values, which is calculated through equation 7, which is useful to obtain a real number value [24]. A value that contributes to the identification of the characteristics to be prioritized according to the factors obtained with the PESTEL framework. The results are the same as those shown in Table 4.

Table 4. Total centrality.

	td
P1	0.4
E1	0
S1	0.4
T1	0
C1	0.50
L1	0.25

From these numerical values, the following ranking is obtained:

$$\mathbf{C_1 \succ P_1 \approx S_1 \succ L_1 \succ E_1 \approx T_1}$$

Factors to address in terms of technological innovation, which have an impact on Ecuador's gross domestic product, are mainly ecological, political, social and legal. The measures of the central position of the factors obtained through the PESTEL technique and analyzed according to the use of the static analysis in NCMS are shown in Figure 5. Each sub-factor were grouped to obtain the results.

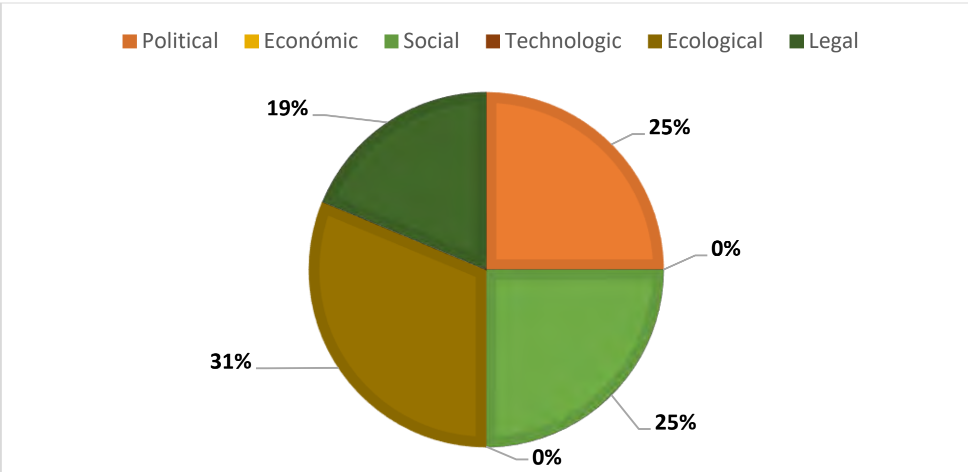

Figure 5. Central position values grouped by factors.

The results show the importance of political and legal factors related to technological innovation projects to gross domestic products growth in Ecuador. Furthermore, economical and technology factor have least importance but further work need to be developed. Handling the problem as a multiobjetive / multicriteria one [28,29], the use of SVN numbers and another neutrosophic tool for better interpretability are among future improvements in the method proposed in this paper [30, 31].

5. Conclusions

In the present study, a characterization of the factors to be attended in terms of technological innovation is carried out, according to its impact on Ecuador's gross domestic product. The PESTEL technique was used, which contributed to the analysis of the environment, identifying the fundamental factors that have a significant impact on technological innovation factors impacting Ecuador's gross domestic product. The characteristics were modeled using neutrosophic cognitive maps, taking into account the indeterminacy and interdependencies between the characteristics and the factors identified with the PESTEL technique. A quantitative analysis based on the static analysis provided by the use was made of neutrosophic cognitive maps and centrality measures. It is shown that technological innovation, which has an impact on Ecuador's gross domestic product, must be addressed in terms of ecological, political, social and legal factors mainly. The case study shows the importance of political and legal factors related to technological innovation projects to gross domestic products growth in Ecuador

Future work will concentrate on extending the model to express importance and interrelation using Fuzzy/Neutrosophic Decisions Maps. Another are of future work is development of a software tool to support the process.

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Conflicts of Interest

The authors declare no conflict of interest.

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Neutrosophic Extended Triplet Group Action and Burnside's Lemma

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Abstract: The aim of this article is mainly to discuss the neutrosophic extended triplet (NET) group actions and Burnside's lemma of NET group. We introduce NET orbits, stabilizers, conjugates and NET group action. Then, we give and proof the Orbit stabilizer formula for NET group by utilizing the notion of NET set theory. Moreover, some results related to NET group action, and Burnside's lemma are obtained.

Keywords: NET group action; NET orbit; NET stabilizer; NET conjugate; Burnside's lemma; NET fixed points; The fundamental theorem about NET group action.

1. Introduction

Galois is well known as the first researcher associating group theory and field theory, along the theory particularly called Galois theory. The concept of groupoid gives a more flexible and powerful approach to the concept of symmetry (see [1]). Symmetry groups come out in the review of combinatorics outline and algebraic number theory, along with physics and chemistry. For instance, Burnside's lemma can be utilized to compute combinatorial objects related along symmetry groups. A group action is a precise method of solving the technique wither the elements of a group meet transformations of any space in a method such protects the structure of a certain space. Just as there is a natural similarity among the set of a group elements and the set of space transformations, a group can be explained as acting on the space in a canonical way. A familiar method of defining no-canonical groups is to express a homomorphism f from a group G to the group of symmetries (an object is invariant to some of different transformations; including reflection, rotation) of a set X. The action of an element $g \in G$ on a point $x \in X$ is supposed to be similar to the action of its image $f(g) \in Sym(X)$ on the point x. The stabilizers of the action are the vertex groups, and the orbits of the action are the elements, of the action groupoid. Some other facts about group theory can be revealed in [2-5].

Neutrosophy is a new branch of philosophy, presented by Florentic Smarandache [6] in 1980, which studies the interactions with different ideational spectra in our everyday life. A NET is an object of the structure $(x, e^{neut(x)}, e^{anti(x)})$, for $\in x \quad N$, was firstly presented by Florentin Smarandache [7-9] in 2016. In this theory, the extended neutral and the extended opposites can similar or non-identical from the classical unitary element and inverse element respectively. The NETs are depend on real triads: (friend, neutral, enemy), (pro, neutral, against), (accept, pending, reject), and in general $(x, neut(x), anti(x))$ as in neutrosophy is a conclusion of Hegel's dialectics that is depend on x and $anti \ x(\ )$. This theory acknowledges every concept or idea x together along its opposite and along their spectrum of neutralities $neut(x)$ among them. Neutrosophy is the foundation of neutrosophic logic, neutrosophic set, neutrosophic probability, and neutrosophic statistics that are utilized or applied in engineering (like software and information fusion), medicine, military, airspace, cybernetics, and physics. Kandasamy and Smarandache [10] introduced many new neutrosophic notions in graphs and applied it to the case of neutrosophic cognitive and relational maps. The same researchers [11] were introduced the concept of neutrosophic algebraic structures for groups, loops, semi groups and groupoids and also their $N-$ algebraic structures in 2006. Smarandache and Mumtaz Ali [12] proposed neutrosophic triplets and by utilizing these they defined NTG and the application areas of NTGs. They also define NT field [13] and NT in physics [14]. Smarandache investigated physical structures of hybrid NT ring [15]. Zhang et al [16] examined the Notion of cancellable NTG and group coincide in 2017. Şahın and Kargın [17], [18] firstly introduced new structures called NT normed space and NT inner product respectively. Smarandache et al [19] studied new algebraic structure called NT G-module which is constructed on NTGs and NT vector spaces. The above set theories have been applied to many different areas including real decision making problems [20-44]. Furthermore, Abdel Basset et al applied this theory to decision making approach for selecting supply chain sustainability metrics [48], an approach of TOPSIS technique [49, 51], iot-based enterprises [50, 52], calculation of the green supply chain management [53] and neutrosophic ANP and VIKOR method for achieving sustainable supplier selection [54].

The paper deals with action of a NET set on NETGs and Burnside's lemma. We provide basic definitions, notations, facts, and examples about NETs which play a significant role to define and build new algebraic structures. Then, the concept of NET orbits, stabilizers, fixed points and conjugates are given and their difference between the classical structures are briefly discussed. Finally, some results related to NET group actions and Burnside's lemma are obtained.

2. Preliminaries

Since some properties of NETs are used in this work, it is important to have a keen knowledge of NETs. We will point out some few NETs and concepts of NET group, NT normal subgroup, and NT cosets according to what needed in this work.

Definition 2.1 [12, 14] A NT has a form $(a,\ neut(a),\ anti(a))$, for $(a, neut(a), anti(a)) \in N$, accordingly $neut(a)$ and $anti(a) \in N$ are neutral and opposite of a, that is different from the unitary element, thus: $a * neut(a) = neut(a) * a = a$ and $a * anti(a) = anti(a) * a = neut(a)$ respectively. In general, a may have one or more than one neut's and one or more than one anti's.

Definition 2.2 [8, 14] A NET is a NT, defined as definition 1, but where the neutral of a (symbolized by $e^{neut(a)}$ and called "extended neutral") is equal to the classical unitary element. As a consequence, the "extended opposite" of a, symbolized by $e^{anti(a)}$ is also same to the classical inverse element. Thus, a NET has a form $(a, e^{neut(a)}, e^{anti(a)})$, for $a \in N$, where $e^{neut(a)}$ and $e^{anti(a)}$ in N are the extended neutral and negation of a respectively, thus :

$$a * e^{neut(a)} = e^{neut(a)} * a = a,$$

which can be the same or non-identical from the classical unitary element if any and

$$a * e^{anti(a)} = e^{anti(a)} * a = e^{neut(a)}.$$

Generally, for each a ∈ N there are one or more $e^{neut(a)}$'s and $e^{anti(a)}$'s.

Definition 2.3 [12, 14] Suppose $(N, *)$ is a NT set. Subsequently $(N, *)$ is called a NTG, if the axioms given below are holds.

(1) $(N, *)$ is well-defined, i.e. for and $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in N,$ one has $(a, neut(a), anti(a)) * (b, neut(b), anti(b) \in N.$

(2) $(N, *)$ is associative, i.e. for any

one has $(a, neut(a), anti(a)) * (b, neut(b), anti(b) * (c, neut(c), anti(c)) \in N.$

Theorem 2.4 [46] Let $(N,*)$ be a commutative NET relating to $*$ and $(a, neut(a), anti(a)), (b, neut(b), anti(b)) \in N$;

(i) $neut(a) * neut(b) = neut(a * b)$;

(ii) $anti(a) * anti(b) = anti(a * b)$;

Definition 2.5 [8, 14] Assume $(N,*)$ is a NET strong set. Subsequently $(N,*)$ is called a NETG, if the axioms given below are holds.

(1) $(N,*)$ is well-defined, i.e. for any $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in N$, one has $(a, neut(a), anti(a)) * (b, neut(b), anti(b) \in N$.

(2) $(N,*)$ is associative, i.e. for any $(a, neut(a), anti(a)), (b, neut(b), anti(b)), (c, neut(c), anti(c)) \in N$, one has

$$(a, neut(a), anti(a)) * \big((b, neut(b), anti(b)) * (c, neut(c), anti(c))\big)$$
$$= \big((a, neut(a), anti(a)) * (b, neut(b), anti(b))\big) * (c, neut(c), anti(c)).$$

Definition 2.6 [47] Assume that $(N_1,*)$ and $(N_2,\circ)$ are two NETG's. A mapping $f : N_1 \to N_2$ is called a neutro-homomorphism if:

(1) For any $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in N_1$, we have

$$f\big((a, neut(a), anti(a)) * (b, neut(b), anti(b))\big)$$
$$= f\big((a, neut(a), anti(a))\big) * f\big((b, neut(b), anti(b))\big)$$

(2) If $(a, neut(a), anti(a))$ is a NET from N_1, Then $f(neut(a)) = neut(f(a))$ and $f(anti(a)) = anti(f(a))$.

Definition 2.7 [45] Assume that $(N_1,*)$ is a NETG and H is a subset of N_1. H is called a NET subgroup of N if itself forms a NETG under $*$. On other hand it means :

(1) $e^{neut(a)}$ lies in H.

(2) For any $(a, neut(a), anti(a)), (b, neut(b), anti(b) \in H$, $(a, neut(a), anti(a)) * (b, neut(b), anti(b) \in H$.

(3) If $(a, neut(a), anti(a)) \in H$, then $e^{anti(a)} \in H$.

Definition 2.8 [45] A NET subgroup H of a NETG N is called a NT normal subgroup of N if $(a, neut(a), anti(a))H = H(a, neut(a), anti(a)), \forall (a, neut(a), anti(a)) \in N$ and we represent it as $H (\quad N$.

3. NET Group Action

A NETG action is a representation of the elements of a NETG as a symmetries of a NET set. It is a precise method of solving the technique in which the elements of a NETG meet transformations of any space in a method that maintains the structure of that space. Just as a group action plays an important role in the classical group theory, NETG action enacts identical role in the theory of NETG theory.

Definition 3.1 An action of N on X (left NETG action) is a map $N \times X \to X$ denoted

$$\big((n, neut(n), anti(n)), (x, neut(x), anti(x))\big) \to (n, neut(n), anti(n))(x, neut(x), anti(x))$$

as shown $\quad 1(x, neut(x), anti(x)) = (x, neut(x), anti(x))$

and
$$(n, neut(n), anti(n))\big((h, neut(h), anti(h))(x, neut(x), anti(x))\big)$$
$$= \big((n, neut(n), anti(n))(h, neut(h), anti(h))\big)(x, neut(x), anti(x))$$

for all in X and $(n, neut(n), anti(n)), (h, neut(h), anti(h))$ in N. Given a NET action of N on X, we call X a $N-$set. A $N-$map between $N-$sets X and Y is a map $f : X \to Y$ of NET sets that respects the $N-$action, meaning that,

$f\left((n,neut(n),anti(n))(x,neut(x),anti(x))\right)=(n,neut(n),anti(n))f\left((x,neut(x),anti(x))\right)$ for all in X and $(n,neut(n),anti(n))$ in N. To give a NET action of N on X is equivalent to giving a NETG neutro-homomorphism from N to the NETG of bijections of X. Note that a NETG action is not the same thing as a binary structure, we combine two elements of X to get a third element of X (we combine two apples and get an apple). In a NETG action, we combine an element of N with an element of X to get an element of X (we combine an apple and an orange and get another orange).

It is critical to note that $(n,neut(n),anti(n))\cdot\left((h,neut(h),anti(h))\cdot(x,neut(x),anti(x))\right)$ has two actions of N on elements of X. under other conditions

$$\left((n,neut(n),anti(n))(h,neut(h),anti(h))\right)\cdot(x,neut(x),anti(x))$$

has one multiplication in the NETG $\left((n,neut(n),anti(n))(h,neut(h),anti(h))\right)$ and then one action of an element of N on X.

Example 3.2 For a NET subgroup $H\subset N$, consider the left NT coset space $N/H=\{(a,neut(a),anti(a))H:(a,neut(a),anti(a))\in N\}$. (We do not care wether or not $H\lhd N$, as we are just thinking about N/H as a set.) Let N act on N/H by left multiplication. That is for $(n,neut(n),anti(n))\in N$ and a left NT coset $(a,neut(a),anti(a))H$ ($(a,neut(a),anti(a))\in N$), set

$$(n,neut(n),anti(n))\cdot(a,neut(a),anti(a))H=(n,neut(n),anti(n))(a,neut(a),anti(a))H$$

$$=\begin{Bmatrix}(n,neut(n),anti(n))(y,neut(y),anti(y)): \\ (y,neut(y),anti(y))\in(a,neut(a),anti(a))H\end{Bmatrix}.$$

This is an action of N on N/H, since $1_N(a,neut(a),anti(a))H=(a,neut(a),anti(a))H$ and

$$(n_1,neut(n_1),anti(n_1))\cdot\left((n_2,neut(n_2),anti(n_2))\cdot(a,neut(a),anti(a))H\right)$$
$$=(n_1,neut(n_1),anti(n_1))\cdot\left((n_2,neut(n_2),anti(n_2))(a,neut(a),anti(a))H\right)$$

$$=(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))(a,neut(a),anti(a))H$$
$$=\left((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\right)(a,neut(a),anti(a))H.$$

Note: NET Groups Acting Independently by Multiplication

All NETG acts independently like so, NET set $N=N$ and $X=N$. Then for $(n,neut(n),anti(n))\in N$ and $(n,neut(n),anti(n))\in X=N$, we define
$(n,neut(n),anti(n))\cdot\left((n,neut(n),anti(n))\right)$
$=(n,neut(n),anti(n))\left((n,neut(n),anti(n))\right)\in X=N$.

Example 3.3 Each NETG N acts independently $\left(X=N\right)$ by left multiplication functions. In other words, we set $\pi_{(n,neut(n),anti(n))}:N\to N$ by

$$\pi_{(n,neut(n),anti(n))}\left((h,neut(h),anti(h))\right)=(n,neut(n),anti(n))(h,neut(h),anti(h))$$

for all $(n,neut(n),anti(n))\in N$ and $(h,neut(h),anti(h))\in H$. Subsequently, the axioms for being a NETG action are $1_N(h,neut(h),anti(h))=(h,neut(h),anti(h))$ for all $(h,neut(h),anti(h))\in N$ and

$$(n_1,neut(n_1),anti(n_1))\left((n_2,neut(n_2),anti(n_2))(h,neut(h),anti(h)\right)$$
$$=\left((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\right)(h,neut(h),anti(h))$$

for all $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2)),(h,neut(h),anti(h))\in N$, which are both true whereby 1_N is a neutrality and multiplication in N is associative.

The notation for the NET effect of N is $\pi(n,neut(n),anti(n))$ or

$$\pi(n,neut(n),anti(n))\big((x,neut(x),anti(x))\big)$$

simply as $(n,neut(n),anti(n))\cdot(x,neut(x),anti(x))$ or

$$(n,neut(n),anti(n))(x,neut(x),anti(x)).$$

In this explanation, the conditions for the left NETG action take the succeeding shape:

i. for all $(x,neut(x),anti(x))\in X, 1_N(x,neut(x),anti(x))=(x,neut(x),anti(x))$.

ii. for every $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2))\in N$ an $(x,neut(x),anti(x))\in X$,

$$\begin{aligned}&(n_1,neut(n_1),anti(n_1))\cdot\big((n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x))\big)\\&=\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)\cdot(x,neut(x),anti(x)).\end{aligned}$$

Theorem 3.4 Let a NETG action N act on the NET set X. If $(x,neut(x),anti(x))\in X,(n,neut(n),anti(n))\in N$, and

$$(y,neut(y),anti(y))=(n,neut(n),anti(n))(x,neut(x),anti(x)),$$

then $(x,neut(x),anti(x))=(n,neut(n),anti(n))^{-1}\cdot(y,neut(y),anti(y))$.

If $(x,neut(x),anti(x))\neq(x',neut(x'),anti(x'))$ then

$$(n,neut(n),anti(n))\cdot(x,neut(x),anti(x))\neq(n,neut(n),anti(n))\cdot(x',neut(x'),anti(x')).$$

Proof : From $(y,neut(y),anti(y))=(n,neut(n),anti(n))\cdot(x,neut(x),anti(x))$ we get

$$\begin{aligned}&(n,neut(n),anti(n))^{-1}\cdot(y,neut(y),anti(y))\\&=(n,neut(n),anti(n))^{-1}\big((n,neut(n),anti(n))(x,neut(x),anti(x))\big)\\&=\big((n,neut(n),anti(n))^{-1}(n,neut(n),anti(n))\big)(x,neut(x),anti(x))\\&=1_N(x,neut(x),anti(x))=(x,neut(x),anti(x)).\end{aligned}$$

To show $(x,neut(x),anti(x))\neq(x',neut(x'),anti(x'))\Rightarrow$

$$(n,neut(n),anti(n))(x,neut(x),anti(x))\neq(n,neut(n),anti(n))(x',neut(x'),anti(x')),$$

we show the contrapositive : if

$$(n,neut(n),anti(n))(x,neut(x),anti(x))=(n,neut(n),anti(n))(x',neut(x'),anti(x'))$$

then applying $(n,neut(n),anti(n))^{-1}$ to both sides gives

$$\begin{aligned}&(n,neut(n),anti(n))^{-1}\cdot\big((n,neut(n),anti(n))\cdot(x,neut(x),anti(x))\big)\\&=(n,neut(n),anti(n))^{-1}\cdot\big((n,neut(n),anti(n))\cdot(x',neut(x'),anti(x'))\big)\end{aligned}$$

so

$$\begin{aligned}&\big((n,neut(n),anti(n))^{-1}(n,neut(n),anti(n))\big)\cdot(x,neut(x),anti(x))\\&=\big((n,neut(n),anti(n))^{-1}(n,neut(n),anti(n))\big)\cdot(x',neut(x'),anti(x'))\end{aligned}$$

so

$$(x, neut(x), anti(x)) = (x', neut(x'), anti(x')).$$

On the other hand to imagine action of a NETG on a NET set is such it's a definite neutro-homomorphism. On hand are the facts.

Theorem 3.5 Actions of the NETG N on the NET set X are identical NETG neutro-homeomorphisms from $N \to Sym(X)$, the NETG of permutations of X.

Proof: Assume we've an action of N on the NET set X. We observe $(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$ as a function of (with $(n, neut(n), anti(n))$ fixed). That is, for each $(n, neut(n), anti(n)) \in N$ we have a function $\pi_{(n,neut(n),anti(n))}: X \to X$ by

$$\pi_{(n,neut(n),anti(n))}\big((x, neut(x), anti(x))\big) = (n, neut(n), anti(n)) \cdot (x, neut(x), anti(x)).$$

The axiom $1_N \cdot (x, neut(x), anti(x)) = (x, neut(x), anti(x))$ says π_1 is the neutrality function on X.

The axiom

$$(n_1, neut(n_1), anti(n_1))\big((n_2, neut(n_2), anti(n_2)) \cdot (x, neut(x), anti(x)\big)$$

$$=\big((n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2))\big) \cdot (x, neut(x), anti(x))$$

says

$$\pi_{(n_1,neut(n_1),anti(n_1))} \circ \pi_{(n_2,neut(n_2),anti(n_2))}$$
$$= \pi_{(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))},$$

so structure of functions on X match multiplication in N. Additionally, $\pi_{(n,neut(n),anti(n))}$ is an invertible function whereby $\pi_{(n_1,neut(n_1),anti(n_1))^{-1}}$ is an anti-neutral: the composite of $\pi_{(n_1,neut(n_1),anti(n_1))}$ and $\pi_{(n_1,neut(n_1),anti(n_1))^{-1}}$ is π_1, which is the neutral function on X. Therefore, $\pi_{(n_1,neut(n_1),anti(n_1))} \in Sym(X)$ and $(n, neut(n), anti(n)) \to \pi_{(n_1,neut(n_1),anti(n_1))}$ is a neutro-homomorphism $N \to Sym(X)$.

Contrariwise, assume we've a homomorphism $f : N \to Sym(X)$. For every $(n, neut(n), anti(n))$, we have a permutation $f\big((n, neut(n), anti(n))\big)$ on X, and

$$f\big((n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2))\big)$$

$$= f\big((n_1, neut(n_1), anti(n_1))\big) \circ f\big((n_2, neut(n_2), anti(n_2))\big).$$

Setting $(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$
$$= f\big((n, neut(n), anti(n))\big)\big((x, neut(x), anti(x))\big)$$

introduces a NETG action of N on X, whereby the neutro-homomorphism properties of f submits the defining properties of a NETG action. From this view point, the NET set of $(n, neut(n), anti(n)) \in N$ that act trivially

$$\left((n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))\right) = (x, neut(x), anti(x))$$

for all $(x, neut(x), anti(x)) \in X$ is straightforwardly the neutrosophic kernel of the neutro-homomorphism $N \to Sym(X)$ related to the action. Consequently the above mentioned $(n, neut(n), anti(n))$ such act trivially on X are assumed to lie in the neutrosophic kernel of the action.

Example 3.6 To build N act independently by conjugation, take $X = N$ and let

$$\begin{aligned}&(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))\\ &=(n,neut(n),anti(n))(x,neut(x),anti(x))(n,neut(n),anti(n))^{-1}.\end{aligned}$$

Here, $(n, neut(n), anti(n)) \in N$ and $(x, neut(x), anti(x)) \in N$. Since

$$1_N \cdot (x, neut(x), anti(x)) = 1_N(x, neut(x), anti(x))1_N{}^{-1} = (x, neut(x), anti(x))$$

and

$$\begin{aligned}&(n_1, neut(n_1), anti(n_1)) \cdot \left((n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x))\right)\\ &= (n_1, neut(n_1), anti(n_1)) \cdot\\ &\left((n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x))(n_2,neut(n_2),anti(n_2))^{-1}\right)\\ &=(n_1,neut(n_1),anti(n_1))\\ &\left((n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x))(n_2,neut(n_2),anti(n_2))^{-1}\right)\\ &(n_1,neut(n_1),anti(n_1))^{-1}\\ &=\left((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\right)(x,neut(x),anti(x))\\ &\left((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\right)^{-1}\\ &= \left((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\right) \cdot (x, neut(x), anti(x)),\end{aligned}$$

neutrosophic conjugation is a NET action.

Definition 3.7 Assume such N is a NETG and X is a NET set. A right NETG action of N on X is a rule for merging elements $(n, neut(n), anti(n)) \in N$ and elements $(x, neut(x), anti(x)) \in X$, symbolized by $(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$, $(n, neut(n), anti(n)) \cdot \left((x, neut(x), anti(x))\right) \in X$ for all $(x, neut(x), anti(x)) \in X$ and $(n, neut(n), anti(n)) \in N$. We also need the succeeding conditions.

I. $(x,neut(x),anti(x))1_N=(x,neut(x),anti(x))$ for all $(x, neut(x), anti(x)) \in X$.

II. $$\big((x,neut(x),anti(x))\cdot(n_2,neut(n_2),anti(n_2))\big)\cdot(n_1,neut(n_1),anti(n_1))$$
$$=(x,neut(x),anti(x))\big((n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\big)$$

for all $(x,neut(x),anti(x))\in X$ and $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2))\in N.$

Remark 3.8 Left NETG actions are not very distinct from right NETG actions. The only distinction exists in condition (ii).

- For left NETG actions, implementing $(n_2,neut(n_2),anti(n_2))$ to an element and then applying $(n_1,neut(n_1),anti(n_1))$ to the result is the same as applying

$$(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\in N.$$

- For right NETG actions applying $(n_2,neut(n_2),anti(n_2))$ and then $(n_1,neut(n_1),anti(n_1))$ is the same as applying $(n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\in N.$

Let us see the example of a right NETG action (beyond the Rubik's cube example, which as we wrote things is a right NETG action). Also it is easy to do matrices multiplying vectors from the right.

Example 3.9 (A NETG acting on a NET set of NT cosets). Assume such N is a NETG and H is a NET subgroup. Examine the NET set $X=\{Ha/(a,neut(a),anti(a))\in N\}$ of right NT cosets of H. subsequently N acts on X by right multiplication, That is, we describe

$$\big(H(a,neut(a),anti(a))\big)\cdot(n,neut(n),anti(n))$$
$$=H\big((a,neut(a),anti(a))(n,neut(n),anti(n))\big)$$

for $(n,neut(n),anti(n))\in N$ and $H(a,neut(a),anti(a))\in X$. First let's chect that this is well defined, hence assume such $H(a,neut(a),anti(a))=H(a',neut(a'),anti(a'))$, then $(a',neut(a'),anti(a'))(a,neut(a),anti(a))^{-1}\in H$. Now, we have to prove that

for any $(n,neut(n),anti(n))\in N$. But $(a',neut(a'),anti(a'))(a,neut(a),anti(a))^{-1}\in H$ so that

$$(a',neut(a'),anti(a'))(n,neut(n),anti(n))$$
$$=\big((a',neut(a'),anti(a'))(a,neut(a),anti(a))^{-1}\big)\begin{pmatrix}(a,neut(a),\\ anti(a))(n,neut(n),anti(n))\end{pmatrix}$$
$$\in H\big((a,neut(a),anti(a))(n,neut(n),anti(n))\big)$$

$$H\big((a,neut(a),anti(a))(n,neut(n),anti(n))\big)=H\big((a',neut(a'),anti(a'))(n,neut(n),anti(n))\big)$$

so that

$$(a',neut(a'),anti(a'))(n,neut(n),anti(n))\in H\begin{pmatrix}(a,neut(a),anti(a))(n,neut(n),\\ anti(n))\end{pmatrix}.$$

But certainly $H\big((a',neut(a'),anti(a'))(n,neut(n),anti(n))\big)$ also contains

$$1_N\left((a',neut(a'),anti(a'))(n,neut(n),anti(n))\right) = (a',neut(a'),anti(a'))(n,neut(n),anti(n)).$$

Thus the two cosets $H\left((a,neut(a),anti(a))(n,neut(n),anti(n))\right)$ and

$H\left((a',neut(a'),anti(a'))(n,neut(n),anti(n))\right)$ have the elements

$(a',neut(a'),anti(a'))(n,neut(n),anti(n))$ in common. This proves that

$H\left((a,neut(a),anti(a))(n,neut(n),anti(n))\right) = H\left((a',neut(a'),anti(a'))(n,neut(n),anti(n))\right)$

since NT cosets are either same or separate.

Now we've proved that this is well defined, we have to show it is also an action. Definitely axiom (i) is holds since

$$\left(H(a,neut(a),anti(a))\right)\cdot 1_N = H\left((a,neut(a),anti(a))1_N\right) = H(a,neut(a),anti(a)).$$

Lastly, we have to show axiom (ii). Assume such

$(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2))\in N.$ Then

$$\begin{aligned}
&\left(\left(H(a,neut(a),anti(a))\right)\cdot(n_2,neut(n_2),anti(n_2))\right)\cdot(n_1,neut(n_1),anti(n_1))\\
&=\left(H\left((a,neut(a),anti(a))(n_2,neut(n_2),anti(n_2))\right)\right)\cdot(n_1,neut(n_1),anti(n_1))\\
&=H\left(\left((a,neut(a),anti(a))(n_2,neut(n_2),anti(n_2))\right)\right)(n_1,neut(n_1),anti(n_1))\\
&=H\left((a,neut(a),anti(a))\left((n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\right)\right)\\
&=\left(H(a,neut(a),anti(a))\right)\cdot\left((n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\right)
\end{aligned}$$

which proves (ii) and ends the proof. Of course, N also acts on the set of left NT cosets of H by multiplication on the left.

Definition 3.10 A NETG action of N on X is called NET faithful if distinct elements of N act on X in dis-similar methods: when $(n_1,neut(n_1),anti(n_1))\neq(n_2,neut(n_2),anti(n_2))$ in N, there is an $(x,neut(x),anti(x))\in X$ such that

$$(n_1,neut(n_1),anti(n_1))\cdot(x,neut(x),anti(x))\neq(n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x)).$$

Note that when we say $(n_1,neut(n_1),anti(n_1))$ and $(n_2,neut(n_2),anti(n_2))$ act distinctly, we signify they act distinctly somewhere, not all place. This is consistent with what it signifies to say two functions are disjoint. They take distinct values somewhere, not all place.

Example 3.11 The action of N independently by left multiplication is faithful: distinct elements send 1_N to distinct places.

Example 3.12 When H is a NET subgroup of N and N acts on N/H left multiplication $(n_1, neut(n_1), anti(n_1))$ and $(n_2, neut(n_2), anti(n_2))$ in N act in the similar method on N/H exactly when

$$(n_1, neut(n_1), anti(n_1))(n, neut(n), anti(n))H = (n_2, neut(n_2), anti(n_2))(n, neut(n), anti(n))H$$

for all $(n, neut(n), anti(n)) \in N$, which means

$$(n_2, neut(n_2), anti(n_2))^{-1}(n_1, neut(n_1), anti(n_1)) \in \bigcap_{(n, neut(n), anti(n))}$$
$$\in N(n, neut(n), anti(n))H(n, neut(n), anti(n))^{-1}.$$

So the left multiplication action of N on N/H is NET faithful in the case that the NET subgroups $(n, neut(n), anti(n))H(n, neut(n), anti(n))^{-1}$ (as $(n, neut(n), anti(n))$ varies) have trivial intersection.

Viewing NETG actions as neutro-homeomorphisms, a NET faithful action of N on X is an injective neutro-homomorphism $N \to Sym(X)$. Non faithful actions are not injective as NETG neutro-homeomorphisms, and many important homeomorphisms are not injective.

Remark 3.13 What we've been calling a NETG action could be a left and right NETG action. The difference among left and right actions is how a product $(n, neut(n), anti(n))(n', neut(n'), anti(n'))$ acts: in a left action $(n', neut(n'), anti(n'))$ acts first and $(n, neut(n), anti(n))$ acts second, while in a right action $(n, neut(n), anti(n))$ acts first and $(n', neut(n'), anti(n'))$ acts second.

We can introduce the NET conjugate of $(h, neut(h), anti(h))$ by $(n, neut(n), anti(n))$ as

$$(n, neut(n), anti(n))(h, neut(h), anti(h))(n, neut(n), anti(n))$$

Instead $\quad (n, neut(n), anti(n))(h, neut(h), anti(h))(n, neut(n), anti(n))^{-1},$

and this convention fits well with the right NET conjugation action but not left action : setting

$$(h, neut(h), anti(h))^{(n, neut(n), anti(n))} = (n, neut(n), anti(n))^{-1}(h, neut(h), anti(h))(n, neut(n), anti(n))$$

we have $(h, neut(h), anti(h))^{1_N} = (h, neut(h), anti(h))$ and

$$\left((h, neut(h), anti(h))^{(n_1, neut(n_1), anti(n_1))}\right)^{(n_2, neut(n_2), anti(n_2))}$$
$$= (h, neut(h), anti(h))^{(n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2))}.$$

The distinction among left and right actions of a NETG is mostly unreal, whereby subsetituting $(n, neut(n), anti(n))$ with $(n, neut(n), anti(n))^{-1}$ in the NETG changes left actions into right actions and contrarily since inversion backwards the order of multiplication in N. So for us "NETG action" means "left NETG action".

Definition 3.14 Let a NETG N act on NET set X. For each $(x, neut(x), anti(x)) \in X$, its orbit is

$$Orb(x,neut(x),anti(x)) = \{(n,neut(n),anti(n))(x,neut(x),anti(x)):(n,neut(n),anti(n))\in N\} \subset X$$

and its stabilizer is

$$Stab(x,neut(x),anti(x)) = \{(n,neut(n),anti(n))\in N:(n,neut(n),anti(n))(x,neut(x),anti(x))\} \subset N.$$

(The stabilizer of NET is symbolized by $N(x,neut(x),anti(x))$, where N is NETG.) We call a NET fixed point for the action when

$$(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x)) = (x, neut(x), anti(x))$$

for every $(n, neut(n), anti(n)) \in N$, that is, when

$$Orb(x,neut(x),anti(x)) = \{(x,neut(x),anti(x))\}$$

(or equivalently, when $Stab(x,neut(x),anti(x)) = N$). The orbit of NETs of a point is a geometric notion: it is the NET set of places where the points can be moved by the NETG action. Under other conditions, the stabilizer of a NET of a point is an algebraic notion: it is the NET set of NETG elements that fix the point. Mostly we'll denote the elements of X as points and we'll denote the size of a NET orbit as its length.

Definition 3.15 Let N be a NETG, $(n, neut(n), anti(n)) \in N$, and let H be a NET subgroup of N.

$$\begin{aligned}&(a, neut(a), anti(a))H(a, neut(a), anti(a))^{-1}\\ &= \begin{Bmatrix}(a, neut(a), anti(a))(h, neut(h), anti(h))(a, neut(a), anti(a))^{-1}: \\ (h, neut(h), anti(h)) \in H\end{Bmatrix}\end{aligned}$$

is called a NET conjugate of H and the NET center of N is

$$Z_N = \begin{Bmatrix}(a,neut(a),anti(a))\in N:(a,neut(a),anti(a))(n,neut(n),anti(n)) \\ =(n,neut(n),anti(n))(a,neut(a),anti(a)):\forall(n,neut(n),anti(n))\in N\end{Bmatrix}.$$

Remark 3.16 When we imagine about a NET set as a geometric object, it is useful to describe to its elements as points. For instance, when we imagine about N/H as a NET set on which N acts, it is helpful to imagine about the NT cosets of H, which are the elements N/H, as the points in N/H. simultaneously, though, a NT coset is a NET subset of N.

All of our applications of NETG actions to group theory will flow from the similarities among NET orbits, stabilizers, and fixed points, which we now build explicit in our the following fundamental examples of NETG actions.

Example 3.17 When a NETG N acts independently by conjugation,

a) the NET orbit of $(a, neut(a), anti(a))$ is

$$Orb(a,neut(a),anti(a)) = \left\{ \begin{matrix} (n,neut(n),anti(n))(a,neut(a),anti(a)) \\ (n,neut(n),anti(n))^{-1} : (n,neut(n),anti(n)) \in N \end{matrix} \right\},$$

which is the conjugacy class of $(a, neut(a), anti(a))$,

b) $$Stab(a,neut(a),anti(a)) = \left\{ \begin{matrix} (n,neut(n),anti(n)) : (n,neut(n),anti(n)) \\ (a,neut(a),anti(a))(n,neut(n),anti(n))^{-1} \\ =(a,neut(a),anti(a)) \end{matrix} \right\}$$

c) $$Z(a,neut(a),anti(a)) = \left\{ \begin{matrix} (n,neut(n),anti(n)) \\ :(n,neut(n),anti(n))(a,neut(a),anti(a)) \end{matrix} \right.$$
$$=(a,neut(a),anti(a))(n,neut(n),anti(n)) \}$$

is the NET centralizer of $(a, neut(a), anti(a))$.

d) $(a, neut(a), anti(a))$ is a NET fixed point when it commutes with all elements of N, and thus the NET fixed points of conjugation form the NET center of N, and thus the NET fixed points of NET conjugation form the center of N.

Example 3.18 When H acts on N by conjugation,

i. the orbit of $(a, neut(a), anti(a))$ is

$$Orb(a,neut(a),anti(a)) = \left\{ \begin{matrix} (h,neut(h),anti(h))(a,neut(a),anti(a)) \\ (h,neut(h),anti(h))^{-1} : (h,neut(h),anti(h)) \in H \end{matrix} \right\},$$

which has no special name (elements of N that are $H-$conjugate to $(a, neut(a), anti(a))$),

ii. $$\begin{aligned} Stab(a,neut(a),anti(a)) &= \{(h,neut(h),anti(h)) : \\ &(h, neut(h), anti(h))(a, neut(a), anti(a))(h, neut(h), anti(h))^{-1} \\ &= (h, neut(h), anti(h)) \} \\ &= \{(h, neut(h), anti(h)) : (h, neut(h), anti(h))(a, neut(a), anti(a)) \\ &= (a, neut(a), anti(a))(h, neut(h), anti(h)) \} \end{aligned}$$

is the elements of H commuting with $(a, neut(a), anti(a))$ (this is $H \cap Z((a,neut(a),anti(a)))$ is the NET centralizer of $(a, neut(a), anti(a))$ in N).

iii. $(a, neut(a), anti(a))$ is a NET fixed point when it commutes with all elements of H, so the NET fixed points of $H-$conjugation on N shape the NET centralizer of H in $N.$

Theorem 3.19 the Fundamental Theorem about NETG Action

Let a NETG N act on a NET set $X.$

a. Different NET orbits of the action are disjoint and form a portion of $X.$

b. For each $(x,neut(x),anti(x)) \in X, Stab(x,neut(x),anti(x))$ is a NET subgroup of N and

$$Stab(n,neut(n),anti(n))(x,neut(x),anti(x)) = (n, neut(n), anti(n))$$
$$Stab(x,neut(x),anti(x)) Stab(n,neut(n),anti(n))(n, neut(n), anti(n))^{-1}$$

for all $(n, neut(n), anti(n)) \in N.$

c. For each $(x, neut(x), anti(x)) \in X$, there is a bijections

$$Orb(x,neut(x),anti(x)) \rightarrow N / Stab(x,neut(x),anti(x)) \text{ by}$$

$$(n, neut(n), anti(n))(x, neut(x), anti(x)) \rightarrow (n, neut(n), anti(n)) Stab(x,neut(x),anti(x)).$$

More concretely, $(n, neut(n), anti(n))(x, neut(x), anti(x)) = (n', neut(n'), anti(n'))(x, neut(x), anti(x))$

in the case that $(n, neut(n), anti(n))$ and $(n', neut(n'), anti(n'))$ lie in the similar NET coset of $Stab(x,neut(x),anti(x))$, and different NT left cosets of $Stab(x,neut(x),anti(x))$ correspond to different points in $Orb(x,neut(x),anti(x))$. In particular, if and $(y, neut(y), anti(y))$ are in the same NET orbit then

$$\left\{ \begin{array}{l} (n, neut(n), anti(n)) \in N : (n, neut(n), anti(n))(x, neut(x), anti(x)) \\ = (y, neut(y), anti(y)) \end{array} \right\}$$

is a NT left coset of $Stab(x,neut(x),anti(x))$, and

$$\left|Orb(x,neut(x),anti(x))\right|=\left[N:Stab(x,neut(x),anti(x))\right].$$

Parts b and c Show the role of conjugate NET subgroups and neutrosophic triplet cosets of a NET subgroup when working with NETG actions. The formula in part c that relates the length of a NET orbit to the index in N of a NET stabilizer for a point in the NET orbit, is named the NET orbit-stabilizer formula.

Proof:

a) We show distinct NET orbits in a NETG action are not equal by showing that two NET orbits that overlap must coexist. Assume $Orb(x,neut(x),anti(x))$ and $Orb(y,neut(y),anti(y))$ have a common element $(z,neut(z),anti(z))$.

$$(z,neut(z),anti(z))=(n_1,neut(n_1),anti(n_1))(x,neut(x),anti(x))$$
$$(z,neut(z),anti(z))=(n_2,neut(n_2),anti(n_2))(y,neut(y),anti(y)).$$

We want to show $Orb(x,neut(x),anti(x))$ and $Orb(y,neut(y),anti(y))$. It suffices to show $Orb(x,neut(x),anti(x))\subset Orb(y,neut(y),anti(y))$, since then we can switch the roles of and $(y,neut(y),anti(y))$ to obtain the converse insertion. For each point $(u,neut(u),anti(u))\in Orb(x,neut(x),anti(x))$, write

$$(u,neut(u),anti(u))=(n,neut(n),anti(n))(x,neut(x),anti(x))$$

for some $(n,neut(n),anti(n))\in N$. Since

$$(x,neut(x),anti(x))$$
$$=(n_1,neut(n_1),anti(n_1))^{-1}(z,neut(z),anti(z)),(u,neut(u),anti(u))$$
$$=(u,neut(u),anti(u))\left((n_1,neut(n_1),anti(n_1))^{-1}(z,neut(z),anti(z))\right)$$
$$=\left((n,neut(n),anti(n))(n_1,neut(n_1),anti(n_1))^{-1}\right)(z,neut(z),anti(z))$$
$$=\left((n,neut(n),anti(n))(n_1,neut(n_1),anti(n_1))^{-1}\right)\begin{pmatrix}(n_2,neut(n_2),anti(n_2))\\(y,neut(y),anti(y))\end{pmatrix}$$
$$=\left((n,neut(n),anti(n))(n_1,neut(n_1),anti(n_1))^{-1}(n_2,neut(n_2),anti(n_2))\right)$$
$$(y,neut(y),anti(y)),$$

which shows us that $(u,neut(u),anti(u))\in Orb(y,neut(y),anti(y))$. Therefore $Orb(x,neut(x),anti(x))\subset Orb(y,neut(y),anti(y))$. Every element of X is in some NET orbit (its own NET orbits), so the NET orbits partition X into disjoint NET subsets.

b) To see that $Stab(x,neut(x),anti(x))$ is a NET subgroup of N, we've $1_N\in Stab(x,neut(x),anti(x))$ since $1_N(x,neut(x),anti(x))=(x,neut(x),anti(x))$, and if $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2))\in Stab(x,neut(x),anti(x))$, then

$$\begin{aligned}
&\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)(x,neut(x),anti(x))\\
&=(n_1,neut(n_1),anti(n_1))\big((n_2,neut(n_2),anti(n_2))(x,neut(x),anti(x))\big)\\
&=(n_1,neut(n_1),anti(n_1))(x,neut(x),anti(x))\\
&=(x,neut(x),anti(x)),
\end{aligned}$$

so $(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\in Stab(x,neut(x),anti(x))$. Thus $Stab(x,neut(x),anti(x))$ is closed under multiplication. Lastly,

$$(n_1,neut(n_1),anti(n_1))(x,neut(x),anti(x))=(x,neut(x),anti(x))$$

$$\begin{aligned}
&\Rightarrow (n,neut(n),anti(n))^{-1}\big((n,neut(n),anti(n))(x,neut(x),anti(x))\big)\\
&=(n,neut(n),anti(n))^{-1}(x,neut(x),anti(x))\\
&\Rightarrow (x,neut(x),anti(x))=(n,neut(n),anti(n))^{-1}(x,neut(x),anti(x)),
\end{aligned}$$

so $Stab(x,neut(x),anti(x))$ is closed under inversion. To prove

$$\begin{aligned}
&Stab(n,neut(n),anti(n))(x,neut(x),anti(x))\\
&=(n,neut(n),anti(n))Stab(x,neut(x),anti(x))(n,neut(n),anti(n))^{-1},
\end{aligned}$$

for all $(x,neut(x),anti(x))\in X$ and $(n,neut(n),anti(n))\in N,$ observe that

$$\begin{aligned}
&(h,neut(h),anti(h))\in Stab(n,neut(n),anti(n))(x,neut(x),anti(x))\\
&\Leftrightarrow (h,neut(h),anti(h))\cdot\big((n,neut(n),anti(n))(x,neut(x),anti(x))\big)\\
&=(n,neut(n),anti(n))(x,neut(x),anti(x))
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow \big((h,neut(h),anti(h))(n,neut(n),anti(n))\big)(x,neut(x),anti(x))\\
&=(n,neut(n),anti(n))(x,neut(x),anti(x))
\end{aligned}$$

$$\Leftrightarrow (n, neut(n), anti(n))^{-1}\begin{pmatrix}((h, neut(h), anti(h))(n, neut(n), anti(n)))\\(x, neut(x), anti(x))\end{pmatrix}$$
$$= (n, neut(n), anti(n))^{-1}\left((n, neut(n), anti(n))(x, neut(x), anti(x))\right)$$
$$\Leftrightarrow \left((n, neut(n), anti(n))^{-1}(h, neut(h), anti(h))(n, neut(n), anti(n))\right)$$
$$(x, neut(x), anti(x)) = (x, neut(x), anti(x))$$
$$\Leftrightarrow (n,neut(n),anti(n))^{-1}(h,neut(h),anti(h))(n,neut(n),anti(n))$$
$$\in Stab_{(x,neut(x),anti(x))}$$
$$\Leftrightarrow (h,neut(h),anti(h)) \in (n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}$$
$$(n,neut(n),anti(n))^{-1},$$

so

$$Stab_{(x,neut(x),anti(x))}(x, neut(x), anti(x))$$
$$= (n, neut(n), anti(n))Stab_{(x,neut(x),anti(x))}(n, neut(n), anti(n))^{-1}.$$

C) The condition

$$(n, neut(n), anti(n))(x, neut(x), anti(x)) = (n', neut(n'), anti(n'))(x, neut(x), anti(x))$$

is equivalent to

$$(x, neut(x), anti(x)) = \left((n, neut(n), anti(n))^{-1}(n', neut(n'), anti(n'))\right)(x, neut(x), anti(x)),$$

which means $(n,neut(n),anti(n))^{-1}(n',neut(n'),anti(n')) \in Stab_{(x,neut(x),anti(x))}$, or

$$(n',neut(n'),anti(n')) \in (n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}.$$

Therefore $(n, neut(n), anti(n))$ and $(n', neut(n'), anti(n'))$ have the same effect on in the case that $(n, neut(n), anti(n))$ and $(n', neut(n'), anti(n'))$ lie in the similar NT coset of $Stab_{(x,neut(x),anti(x))}$. (Recall that for all NET subgroups H and

$$N, (n', neut(n'), anti(n')) \in (n, neut(n), anti(n))H$$
$$(n', neut(n'), anti(n'))H = (n, neut(n), anti(n))H.$$

Whereby $Orb(x,neut(x),anti(x))$ consists of the points $(n, neut(n), anti(n))(x, neut(x), anti(x))$ for varying $(n, neut(n), anti(n))$, and we showed elements of N have the similar effect on if and only if they lie in the similar

NT left coset of $Stab(x,neut(x),anti(x))$, we get a bijections between the points in the NET orbit of and the NT left cosets of $Stab(x,neut(x),anti(x))$ by

$$(n,neut(n),anti(n))(x,neut(x),anti(x)) \to (n,neut(n),anti(n))Stab(x,neut(x),anti(x)).$$

Therefore the cardinality of the NET orbit of $(x,neut(x),anti(x))$, which is $\left|Orb(x,neut(x),anti(x))\right|$ equals the cardinality of the NT left cosets of $Stab(x,neut(x),anti(x))$ in N.

Remark 3.20 that the NET orbits of a NETG action are a partition results in a NETG theory: conjugacy classes are a partitioning of a NETG and the NT left cosets of a NET subgroup partition the NETG. The first result utilizes the action of a NETG independently by NET conjugation, having NET conjugacy classes as its NET orbits. The second result utilizes the right inverse multiplication action of the NET subgroup on the NETG.

Corollary 3.21 Let a finite NETG act on a NET set.

a) The length of every NET orbit divides the size of N.
b) Points in a common NET orbit have conjugate stabilizers, and in particular the size of the NET stabilizer is the similar for all points in a NET orbit.

Proof: **a)** The length of NET orbit is an index of a NET subgroup, so it divides $|N|$.

b) If and $(y,neut(y),anti(y))$ are in the same NET orbit, write

$$(y,neut(y),anti(y)) = (n,neut(n),anti(n))(x,neut(x),anti(x)).$$

Then,

$$Stab(y,neut(y),anti(y)) = Stab(n,neut(n),anti(n))(x,neut(x),anti(x))$$
$$=(n,neut(n),anti(n))Stab(x,neut(x),anti(x))(n,neut(n),anti(n))^{-1},$$

so the NET stabilizers of and $(y,neut(y),anti(y))$ are conjugate NET subgroups.

A converse of part b is not generally true: points with NET conjugate stabilizers need not be in the same NET orbit. Even points with the same NET stabilizer need nor be in the same NET orbit. For example, if N acts on itself trivially then all points have NET stabilizer N and all orbits have size 1.

Corollary 3.22 Let a NETG N acts on a NET set X, where X is finite. Let the distinct NET orbits of X be symbolized by $(x_1,neut(x_1),anti(x_1)),\ldots,(x_t,neut(x_t),anti(x_t))$. Then

$$|X| = \sum_{i=1}^{t}\left|Orb(x_i,neut(x_i),anti(x_i))\right| = \sum_{i=1}^{t}\left[N : Stab(x_i,neut(x_i),anti(x_i))\right].$$

Proof: The NET set X can be written as the union of its NET orbits, which are mutually disjoint. The NET orbit-stabilizer formula tells us how large each NET orbit is.

Example 3.23 As an application of the NET orbit-stabilizer formula we describe why

$|HK| = {|H||K|}\Big/{|H \cap K|}$ for NET subgroups H and K of a finite NETG N. At this point

$$HK = \begin{Bmatrix} (h,neut(h),anti(h)),(k,neut(k),anti(k)) : (h,neut(h),anti(h)) \in H, \\ (K,neut(K),anti(K)) \in K \end{Bmatrix}$$

is the NET set of products, which usually is just a subset of N. To count the size of HK, let the direct product of NETG $H \times K$ act on the NET set HK like this :

$$\begin{aligned} &\big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big)\cdot(x,neut(x),anti(x)) \\ &= (h,neut(h),anti(h))(x,neut(x),anti(x))(h,neut(h),anti(h))^{-1}, \end{aligned}$$

which gives us a NETG action (the NETG is $H \times K$ and the NET set is HK). There is only 1 NET orbit where by $1_N = 1_N 1_N \in HK$ and

$$(h,neut(h),anti(h)),(k,neut(k),anti(k)) = \Big((h,neut(h),anti(h)),(k,neut(k),anti(k))^{-1}\Big)\cdot 1_N.$$

So that the NET orbit-stabilizer formula shows us

$$|HK| = \frac{|H \times K|}{|Stab 1_N|}$$

$$= \frac{|H||K|}{\left|\begin{Bmatrix} \big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big) : (h,neut(h),anti(h)),(k,neut(k),anti(k))\cdot 1_N \\ = 1_N \end{Bmatrix}\right|}.$$

The condition $\big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big)\cdot 1_N = 1_N$ means

$$(h,neut(h),anti(h))(k,neut(k),anti(k))^{-1} = 1_N, \text{ so}$$

$$Stab 1_N = \Big\{\big((h,neut(h),anti(h))(h,neut(h),anti(h))\big) : (h,neut(h),anti(h)) \in H \cap K\Big\}.$$

So that $|Stab 1_N| = |H \cap K|$ and $|HK| = {|H||K|}\Big/{|H \cap K|}$.

Theorem 3.24 Burnside's Lemma

Let a finite NETG N act on a finite NET set X in relation to r NET orbits. Subsequently r is the average number of NET fixed points of the elements of the NETG.

$$r = \frac{1}{|N|} \sum_{(n,neut(n),anti(n)) \in N} \left| Fix_{(n,neut(n),anti(n))}(X) \right|,$$

where

$$Fix_{(n,neut(n),anti(n))}\left(X\right)=\begin{Bmatrix}(x,neut(x),anti(x))\in X:(n,neut(n),anti(n))\\(x,neut(x),anti(x))=(x,neut(x),anti(x))\end{Bmatrix}$$

is the NET set of elements of X fixed by $(n, neut(n), anti(n))$.

Don't confuse the NET set $Fix_{(n,neut(n),anti(n))}\left(X\right)$ in relation to the NET fixed points of the action: $Fix_{(n,neut(n),anti(n))}\left(X\right)$ is only the points fixed by the elements $(n, neut(n), anti(n))$. The NET set of NET fixed points for the action of N is the intersection of the NET sets $Fix_{(n,neut(n),anti(n))}\left(X\right)$ as $(n, neut(n), anti(n))$ runs over the NETG.

Proof: we will count

$$\begin{Bmatrix}\left((n,neut(n),anti(n)),(x,neut(x),anti(x))\right)\in N\times X:\\(n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x))\end{Bmatrix}$$

in two ways. By counting over $(n, neut(n), anti(n))$'s first we have to add up the number of $(x, neut(x), anti(x))'s$ with

$$(n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x)), \text{ so}$$

$$\left|\begin{Bmatrix}\left((n,neut(n),anti(n)),(x,neut(x),anti(x))\right)\in N\times X:\\(n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x))\end{Bmatrix}\right|$$

$$=\sum_{(n,neut(n),anti(n))\in N}\left|Fix_{(n,neut(n),anti(n))}(X)\right|.$$

Next we count over the 's and have to add up the number of $(n, neut(n), anti(n))$'s with $(n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x))$, i.e., with $(n,neut(n),anti(n))\in Stab_{(x,neut(x),anti(x))}$:

$$\left|\begin{Bmatrix}\left((n,neut(n),anti(n)),(x,neut(x),anti(x))\right)\in N\times Y:\\(n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x))\end{Bmatrix}\right|$$

$$=\sum_{(X,neut(X),anti(X))\in X}\left|Stab_{(x,neut(x),anti(x))}\right|.$$

Equating these two counts gives

$$= \sum_{(n,neut(n),anti(n))\in N} \left|Fix_{(n,neut(n),anti(n))}(X)\right|$$
$$= \sum_{(X,neut(X),anti(X))\in X} \left|Stab_{(x,neut(x),anti(x))}\right|.$$

By the NET orbit-stabilizer formula, $|N| \big/ \left|Stab_{(x,neut(x),anti(x))}\right| = \left|Orb_{(x,neut(x),anti(x))}\right|$, so

$$\sum_{(n,neut(n),anti(n))\in N} \left|Fix_{(n,neut(n),anti(n))}(X)\right|$$
$$= \sum_{(X,neut(X),anti(X))\in X} \frac{|N|}{\left|Orb_{(x,neut(x),anti(x))}\right|}.$$

Divide by $|N|$:

$$\frac{1}{|N|}\sum_{(n,neut(n),anti(n))\in N} \left|Fix_{(n,neut(n),anti(n))}(X)\right|$$
$$= \sum_{(x,neut(x),anti(x))\in X} \frac{1}{\left|Orb_{(x,neut(x),anti(x))}\right|}.$$

Let's examine the benefaction to the right side from points in a single NET orbit. If a NET orbit has n points in it, subsequently the sum over the points in that NET orbit is a sum of $\frac{1}{n}$ for n terms, and in other words equal to 1. Consequently the part of the sum over points in a NET orbit is 1, which makes the sum on the right side equal to the number of NET orbits, which is r.

Definition 3.25 Two actions of NETG N on a NET sets X and Y are called NET equivalent if there is a bijection $f: X \to Y$ as shown

$$f\left((n,neut(n),anti(n))(x,neut(x),anti(x))\right) = (n,neut(n),anti(n))f\left((x,neut(x),anti(x))\right)$$

for all $(n,neut(n),anti(n)) \in N$ and $(x,neut(x),anti(x)) \in X$.

Actions of N on two NET sets are equivalent when N permutes elements in the similar method on the two NET sets following matching up the NET sets properly. When $f: X \to Y$ is a NET equivalence of NETG actions on X and Y,

$$(n,neut(n),anti(n))(x,neut(x),anti(x)) = (x,neut(x),anti(x))$$

if and only if

$$(n,neut(n),anti(n))\left(f\left((x,neut(x),anti(x))\right)\right) = f\left((x,neut(x),anti(x))\right),$$

so the NET stabilizer subgroups of $(x, neut(x), anti(x)) \in X$ and $f(x, neut(x), anti(x)) \in Y$ are the same.

Example 3.26 Let H and K be NET subgroup of N. The NETG N acts by left multiplication on N/H and N/K. If H and K are NET conjugate subgroups then these actions are equivalent: fix a representation $K=(n_0,neut(n_0),anti(n_0))H(n_0,neut(n_0),anti(n_0))^{-1}$ for some $(n_0,neut(n_0),anti(n_0))\in N$ and let $f : N/H \to N/K$ by

$$f\left((n, neut(n), anti(n))H\right) = (n, neut(n), anti(n))(n_0, neut(n_0), anti(n_0))^{-1}K.$$

This is well-defined (independent of the NT coset representatives for $(n, neut(n), anti(n))H$) since, for $(h, neut(h), anti(h)) \in H$,

$$\begin{aligned}
&f\left((n,neut(n),anti(n))h,neut(h),anti(h))H\right)\\
&=(n,neut(n),anti(n))(h,neut(h),anti(h))(n_0,neut(n_0),anti(n_0))^{-1}K\\
&=(n,neut(n),anti(n))(h,neut(h),anti(h))(n_0,neut(n_0),anti(n_0))^{-1}H(n_0,neut(n_0),anti(n_0))^{-1}\\
&=(n,neut(n),anti(n))H(n_0,neut(n_0),anti(n_0))^{-1}=(n,neut(n),anti(n))(n_0,neut(n_0),anti(n_0))^{-1}K.
\end{aligned}$$

There can be multiple equivalences between two equivalent NETG actions, just as there can be multiple neutro-isomorphisms between two isomorphic NETGs. If H and K are not NET conjugate then the actions have the same NET stabilizer subgroup, but the NET stabilizer subgroups of left NT cosets in N/H are NET conjugate to K, and none of the former and the latter are equal.

Theorem 3.27 An action of N that has one NET orbit is equivalent to the left multiplication action of N on some left NT coset space of N.

Proof : Assume that N acts on the NET set X in relation to one NET orbit. $Fix_{(x_0,neut(x_0),anti(x_0))} \in X$ and let $H = Stab_{(x_0,neut(x_0),anti(x_0))}$. We will Show the action of N on X is equivalent to the left multiplication action of N on N/H. Every $(x, neut(x), anti(x)) \in X$ has the form $(n,neut(n),anti(n))(x_0,neut(x_0),anti(x_0))$ for some $(n, neut(n), anti(n)) \in N$, and all elements in a left NT coset $(n, neut(n), anti(n))H$ have the same effect on $(x_0, neut(x_0), anti(x_0))$: for all $(h, neut(h), anti(h)) \in H$,

$$\begin{aligned}&\big((n,neut(n),anti(n))(h,neut(h),anti(h))\big)\big((x_0,neut(x_0),anti(x_0))\big)\\&=(n,neut(n),anti(n))\big((h,neut(h),anti(h))(x_0,neut(x_0),anti(x_0))\big).\end{aligned}$$

Let $f : N/H \to X$ by $f\big((n,neut(n),anti(n))H\big)=(n,neut(n),anti(n))(x_0,neut(x_0),anti(x_0))$.

This is well defined, as we just saw. Moreover,

$$\big((n, neut(n), anti(n)) \cdot (n', neut(n'), anti(n'))H\big) = (n, neut(n), anti(n)) f\big((n', neut(n'), anti(n'))H\big)$$

since both sides equal

$$(n,neut(n),anti(n))(n',neut(n'),anti(n'))\big((n,neut(n),anti(n))\cdot(x_0,neut(x_0),anti(x_0))\big).$$

We will show f is a bijection. Since X has one NET orbit,

$$\begin{aligned}X &= \{(n,neut(n),anti(n))(x_0,neut(x_0),anti(x_0)):(n,neut(n),anti(n))\in N\}\\&=\{f\big((n, neut(n), anti(n))H\big) : (n, neut(n), anti(n)) \in N\},\end{aligned}$$

so f is onto. If $f\big((n_1,neut(n_1),anti(n_1))H\big)=f\big((n_2,neut(n_2),anti(n_2))H\big)$ then

$$(n_1, neut(n_1), anti(n_1))(x_0, neut(x_0), anti(x_0)) = (n_2, neut(n_2), anti(n_2))(x_0, neut(x_0), anti(x_0)),$$

so

$$(n_2, neut(n_2), anti(n_2))^{-1}(n_1, neut(n_1), anti(n_1))(x_0, neut(x_0), anti(x_0)) = (x_0, neut(x_0), anti(x_0)).$$

Since $(x_0,neut(x_0),anti(x_0))$ has NET stabilizer H,

$$(n_2,neut(n_2),anti(n_2))^{-1}(n_1,neut(n_1),anti(n_1))\in H, \text{ so}$$

$$(n_1,neut(n_1),anti(n_1))H=(n_2,neut(n_2),anti(n_2))H.$$

Consequently f is one – to –one.
A special condition of this theorem tells that an action of N is equivalent to the left multiplication action of N independently in the case that the action has one NET orbit and the NET stabilizer subgroup are trivial.

5. Conclusion

The most important point of this research is first to define the NETs and subsequently use these NETs in order to describe the NETG action, NET orbits, stabilizers, and fixed point. We further introduced the Burnside's Lemma. Finally, we allow rise to a new field called NET Structures (namely, the neutrosophic extended triplet group action and Burnside's Lemma. Another researchers can work on the application of NETG action to NT vector spaces (representation of the NETG), number theory, analysis, geometry, and topological spaces.

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Conflicts of Interest

The authors declare no conflict of interest.

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Design and Application of A Questionnaire for the Development of the Knowledge Management Audit Using Neutrosophic Iadov Technique

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Abstract: This paper aims to design a new kind of questionnaire to be applied in the Knowledge Management audit. For illustration purpose, we analyse the knowledge management audit in a grain storage and conservation company. This proposal is based on 18 well-known questionnaires to audit knowledge management. We recommend using neutrosophic Iadov to process the obtained answers. Neutrosophy is combined with Iadov technique to model uncertainty and indeterminacy which characterize the possible answers given by the interviewed persons, as well as to evaluate according to a linguistic scale. Our contribution is that we propose a more generic questionnaire on knowledge management audit which can process indeterminate information and knowledge, and additionally we confirm it with one case study.

Keywords: knowledge management audit, questionnaire, processes, neutrosophic Iadov technique.

1. Introduction

The progress of humanity and its organizations has been associated with the development of knowledge, and has made it possible to obtain the means to survive [1]. That is why, organizations give more and more attention to the solution of problems that arise associated with knowledge management (KM) and its use in processes [2]. The KM contributes to raise the knowledge of the organization through the increase of the capabilities of the employees and the learning that is obtained in the solution of the problems associated with the fulfillment of its strategic objectives [3]. In this sense, authors such as GONZÁLEZ GUITIÁN and PONJUÁN DANTE [4] propose to carry out knowledge audit processes in organizations, given that the information and knowledge resources in the different departments may be duplicated or in deficit and there is not always an awareness about its value [5]. The importance of the knowledge management audit (KMA) is attested by the numerous methodologies that exist in the literature [6] and corroborated by GONZÁLEZ GUITIÁN *et al.* [7] when it relates to applications in the areas of information science, social sciences, business, computing, and finance. Likewise, the absence of a single procedure is recognized as an international reference and a useful tool for the development of KM strategies that identify and describe the organizational knowledge, its use, and also the gaps and duplicities within the organization. Among the most common methods used to capture data in the KM is the questionnaire. This technique, which obeys different needs and the research problem that originates it, has been used in a large part of the studies on KMA, and this is confirmed by the results obtained in MEDINA NOGUEIRA, YULY ESTHER *et al.* [8], where its use is seen in 43% of the proposals, both in the diagnosis [9] and in the different stages that make up the methodologies analysed [10; 11]. Likewise, it can be affirmed that the questionnaires constitute the main tool for the data collection [12] as a key factor for the development of the KMA [13].

Additionally, from the study of 18 questionnaires for the KMA, MEDINA NOGUEIRA, YULY ESTHER *et al.* [14] identifies little flexibility in the designs analysed, since they are focused on specific purposes in the organization. On the other hand, it denotes some limitations in how the processes are evaluated of the KM (acquire, organize, distribute, use and measure), and that are an indispensable basis for the creation of the knowledge value chain. In this sense, the present research aims to propose and apply a questionnaire for the development of the KMA, based on previous research, which guarantees its use in any organization, and that allows to evaluate the development of the KM processes from of the significant variables for the development of the KMA.

2. Development of the questionnaire

The organization selected as a case study is a national company whose mission is the storage, refrigeration and conservation of grains for animal and human consumption.

Step 1. Sample design

The sample selected was made up of 19 management workers who represent 100% of the members of the board of directors and the leaders of the processes. They are classified into nine (9) Directors: Chief Executive Officer (CEO), Deputy Manager (DM), Chief Technical Officer (CTO), Chief Industrial Officer (CIO), Chief Operating Officer (COO), Control and Analysis Manager (CAM), Chief Financial Officer (CFO), Chief Human Resources Officer (CHRO), Chief of Logistics and Transportation Business Unit (CLT); eleven (11) Process Leaders and two (2) employees who participate in the board of directors and are considered experts within the company. The sampling method to be applied is non-probabilistic. It is based on the researcher's judgment for the selection of an element of the population to be part of the sample. Subsequently, the error of the sample committed is calculated and it is verified that it is in the corresponding limits.

Step 2. Design of the questionnaire

From the previous studies carried out on 47 definitions of KMA and 28 methodologies, the questionnaire developed by LONDOÑO GALEANO and GARCÍA OSPINA [15] based on the following elements is selected as a basis for its subsequent modification: it is relatively short; the questions are closed type, formulated in a clear, simple and understandable way; the terms used on KM are simple and concise, which facilitates their interpretation and, finally, evaluates the processes of the KM from the components established by Probst (1998). The questionnaire has totally closed questions and 47 items: eight items (8) associated to the process of use, eight (8) to culture, eight (8) to identification, eight (8) to retention, seven (7) to transfer and eight (8) to sources. The questions are formulated on a 4-level Likert scale, with the following assessment:

1 = Never, 2 = Sometimes, 3 = Often, 4 = Always

The modifications that were made were aimed at: simplifying the number of elements of the questionnaire and the magnitude of some questions; achieve its applicability in any organization; evaluate the processes of the KM defined by MEDINA NOGUEIRA, DAYLIN *et al.* [16], as well as the significant variables for the development of the KMA.

The preliminary instrument was submitted to the evaluation of eight researchers on the subject of the KM and according to their suggestions, some questions were eliminated and others added or modified. Likewise, aspects related to the ability to diagnose KM processes based on the criteria of MEDINA NOGUEIRA, DAYLIN *et al.* [16] were specified, hence, the proposed version consists of 38 items: seven items (7) associated to the process of acquiring, eight (8) to organizing, eight (8) to distributing, five (5) to use, nine (9) to measuring and one question that integrates all the processes.

According to the type of response, the questionnaire can be classified as mixed; according to the moment of coding: pre-coded and, according to the form of administration: self-administered. Next, in Table 1, the version of the questionnaire used is shown. Next, we proceed to check the presence of the variables evaluated in the questionnaire and check its relevance.

Table 1. Questionnaire used for the Knowledge Management Audit.

<table>
<tr><th colspan="2">Questions</th><th>Never</th><th>Hardly ever</th><th>Sometimes</th><th>Usually</th><th>Always</th></tr>
<tr><td rowspan="5">1. Do you consider that the company has sufficient human, material, technological and infrastructure resources for activities related to:</td><td>The acquisition of new knowledge</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>The organization of new knowledge</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Knowledge distribution</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Knowledge use</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Knowledge measurement</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="3">2. The company, for the improvement of its processes, is an organization that learns from:</td><td>The interaction with the environment (customers, suppliers, regulations and regulations)</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Other organizations</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Their own procedure and experience</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="7">3. Mark the ways in which you acquire the necessary knowledge for the performance of your job:
__Postgraduate courses __Search engines on the Internet __ Specialized web publications __Exchange of experiences (live) __Exchange of information (e-mail) __Work meetings __ Use of phone
__ Participation in scientific events __ Other. Which?</td></tr>
<tr><td colspan="2">4. Does the company verify the effectiveness of the training received by its workers?</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">5. Did the training received at the company allow me</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

to improve my job performance?					
6. Does the company have established mechanisms to detect the training needs of workers?					
7. Does the company have the knowledge that is required to adequately perform my job?					
8. Does the company have identified the difference between the knowledge I have and the knowledge I should have in order to perform my work optimally?					
9. Mark the routes through which you have identified the knowledge required to adequately perform my job: __ Regulations and manuals __ Tutorial videos __ Knowledge maps __ Web portal __ Data base __ None __ Other what?					
10. Does the company evaluate the future knowledge needs of workers?					
11. Does the company develop plans to meet the future knowledge needs of workers?					
12. All that I know how to do is transferred to other workers within the company?					
13. The company uses the knowledge of workers to: Design Training programs for other workers					
13. The company uses the knowledge of workers to: The development of new projects					
13. The company uses the knowledge of workers to: The improvement in the processes					
14. Is the information of my process accessible to all interested parties?					
15. Is the knowledge generated in the different processes of the company made available to the entire company?					
16. Mark the ways in which the knowledge generated in the different processes of the company is made available to the entire company: __Scientific sessions in the center __ Specialized web publications __Exchange of experiences (live) __Exchange of information (e-mail) __ Work meetings __Thesis applied in the company __Use of the landline phone __In scientific events developed by the center __Other. Which?					
17. Does my process learn from other processes within the organization?					
18. Is the existing knowledge in the company inventoried?					
19. Are the experts in the various subjects clearly identified in the company to consult them when necessary?					

20. If I have questions to perform the activities in my process I ask to: (Name / Responsibility) (1) ____________________ (2) ____________________ (3) ____________________						
21. Does the company have identified external persons or entities that can contribute to the development of knowledge of it?						
22. Does the company use specialized software to share information? Which software?						
23. The evaluation of workers takes into account:	Their contributions to the development of organizational knowledge					
	Training programs					
	Participation in scientific events					
	Scientific publications					
24. Does my immediate boss attend to my training needs?						
25. Does the company motivate the process of sharing knowledge?						
26. Does the management formally recognize the achievements of its workers for making improvements in their process?						
27. Do you consider that the company manages the necessary knowledge for the development and improvement of the activities related to its process?						

Table 2 verifies the correspondence between the questions and the processes that evaluates the KM; as well as, the presence of the variables of the KMA.

Table 2. List of questionnaire questions, KM processes and variables present in the definitions of KMA.

Questions		**KM process**	**KMA variables**
1. Do you consider that the company has sufficient human, material, technological and infrastructure resources for activities related to:	The acquisition of new knowledge	To acquire	-Firm strategy
	The organization of new knowledge	To organize	-Firm strategy
	Knowledge distribution	To distribute	-Firm strategy
	Knowledge use	To use	-Firm strategy -Use of knowledge
	Knowledge measurement	To measure	- Firm strategy
2. The company, for the improvement of its	The interaction with the environment (customers,	To acquire	-Process approach -Organizational culture

processes, is an organization that learns from:	suppliers, regulations and regulations)		-Sources of knowledge
	Other organizations	To acquire	-Process approach -Organizational culture -Sources of knowledge
	Their own procedure and experience	To acquire	-Process approach -Organizational culture -Sources of knowledge
3. Mark the ways in which you acquire the necessary knowledge for the performance of your job: __ Postgraduate courses __ Search engines on the Internet __ Specialized web publications __ Exchange of experiences (live) __ Exchange of information (e-mail) __ Work meetings __ Use of landline phone __ Participation in scientific events __ Other. Which?		To acquire	-Identification of information -Process approach
4. Does the company verify the effectiveness of the training received by its workers?		To measure	-Firm strategy -KM strategy -Existing knowledge
5. Did the training received at the company allow me to improve my job performance?		To use	-Existing knowledge -Use of knowledge
6. Does the company have established mechanisms to detect the training needs of workers?		To measure	-Knowledge required -Analysis of gaps
7. Does the company have the knowledge that is required to adequately perform my job?		To organize	-Knowledge required
8. Does the company have identified the difference between the knowledge I have and the knowledge I should have in order to perform my work optimally?		To measure	- Analysis of gaps
9. Mark the routes through which you have identified the knowledge required to adequately perform my job: __ Regulations and manuals __ Tutorial videos __ Knowledge maps __ Web portal __ Data base __ None __ Other what?		To organize	-Identification of information -Sources of knowledge -Techniques used in the KMA
10. Does the company evaluate the future knowledge needs of workers?		To measure	- Analysis of gaps -Continuous auditing
11. Does the company develop plans to meet the future knowledge needs of workers?		To organize	-Firm strategy - Analysis of gaps
12. All that I know how to do is transferred to other workers within the company?		To distribute	-Social networks
13. The company uses the knowledge of	Design Training programs for other workers	To use	-Use of knowledge -KM strategy

workers to:	The development of new projects	To use	- KM strategy - Use of knowledge
	The improvement in the processes	To use	-KM strategy -Process approach -Use of knowledge
14. Is the information of my process accessible to all interested parties?		To distribute	-Identification of information
15. Is the knowledge generated in the different processes of the company made available to the entire company?		To distribute	-Process approach -KM strategy -Social networks
16. Mark the ways in which the knowledge generated in the different processes of the company is made available to the entire company: __Scientific sessions in the center __ Specialized web publications __Exchange of experiences (live) __Exchange of information (e-mail) __ Work meetings __Thesis applied in the company __Use of the landline phone __In scientific events developed by the center __Other. Which?		To distribute	-Identification of information
17. Does my process learn from other processes within the organization?		To acquire	-Process approach -Organizational culture -Sources of knowledge
18. Is the existing knowledge in the company inventoried?		To organize	-Existing knowledge -Techniques used in the KMA
19. Are the experts in the various subjects clearly identified in the company to consult them when necessary?		To organize	-Firm strategy -Sources of knowledge -Decision making
20. If I have questions to perform the activities in my process I ask (Name / Responsibility): (1) ____________________ (2) ____________________ (3) ____________________		To acquire	-Sources of knowledge
21. Does the company have identified external persons or entities that can contribute to the development of knowledge of it?		To organize	-Firm strategy -Sources of knowledge
22. Does the company use specialized software to share information? Which software?		To distribute	-Identification of information
23. The evaluation of workers takes into account:	Their contributions to the development of organizational knowledge	To measure	-Firm strategy -Existing knowledge
	Training courses	To measure	-Firm strategy -Existing knowledge

	Participation in scientific events	To measure	-Firm strategy -Existing knowledge
	Scientific publications	To measure	-Firm strategy -Existing knowledge
24. Does my immediate boss attend to my training needs?		To organize	-Organizational culture - Analysis of gaps
25. Does the company motivate the process of sharing knowledge?		To distribute	-Firm strategy -KM strategy -Social networks
26. Does the management formally recognize the achievements of its workers for making improvements in their process?		To distribute	-Firm strategy -Organizational culture
27. Does the management formally recognize the achievements of its workers for making improvements in their process?		Includes the value chain of the KM	-Firm strategy -KM strategy

Step 3. Fieldwork development

The survey, applied in May 2018, was accompanied by an introductory conference on the work to be carried out and all the pertinent information was provided about the instrument to be applied and the guarantee of the confidentiality of the answers. Throughout the process, a member of the audit team was present to directly address the doubts and concerns of the workers involved. The participation was 100% and, at the time of delivery of the questionnaire, it was checked that all the questions were answered; however, some participants left questions unanswered.

Step 4. Database creation and information analysis

Of the 38 questions, 34 are closed and are formulated on a five-level Likert scale (1 = Never, 2 = Almost never, 3 = Sometimes, 4 = Almost always and 5 = Always). The remaining four are: three semi-closed and one open, and were designed to obtain the means by which knowledge is acquired, organized and distributed in the organization; as well as, the people that can be considered as assets of knowledge within it.

Once the 19 surveys were applied, the information was reviewed and entered into the electronic sheet and codified for the creation of the database that was analysed statistically through the SPSS® software.

For the analysis of reliability and validity of the survey, the Cronbach's Alpha test is used, with a value of α= 0.928 that indicates consistency, homogeneity and reliability of the results and the Correlation Coefficient (R^2) with a value of 1 indicates a high correlation between the variables, which confirms the validity of the instrument used.

Step 5. Validation of the survey by the Iadov Neutrosophic Technique

Neutrosophy is a new branch that studies the origin, nature and scope of neutralities [17]. Etymologically neutrosophy [French neutre <Latin neuter, neutral, and Greek Sophia, knowledge]

means knowledge of neutral thoughts [18]. The basic definitions of Neutrosophy, which are those of neutrosophic sets and single-valued neutrosophic sets are formally defined in the following:

Definition 1. Let X be a universe of discourse, a space of points (objects) and x denotes a generic element of X. A *neutrosophic set A* in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Where, $T_A(x)$, $I_A(x)$, $F_A(x) \subseteq]^-0, 1^+[$, i.e., they are real standard or nonstandard subsets of the interval $]^-0, 1^+[$. These functions do not satisfy any restriction, that is to say, the following inequalities hold:

$^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2. Let X be a universe of discourse, a space of points (objects) and x denotes a generic element of X. A *Single Valued Neutrosophic Set* (SVNS) A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function falseness membership function $F_A(x)$. Where, $T_A(x)$, $I_A(x)$, $F_A(x)$: $X \rightarrow [0, 1]$ such that: $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. A *single valued neutrosophic number* (SVNN) is symbolized by <T,I,F> for convenience, where T, I, F$\in$ [0, 1] and $0 \leq$ T+ I+ F≤ 3.

Therefore, $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$ or more straightforwardl$A = \langle\, T_A(x), I_A(x), F_A(x)\rangle$, for every $x \in X$.

Given A and B two SVNSs, they satisfy the following relationships:

1. $A \subseteq B$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$ and $F_A(x) \geq F_B(x)$. Particularly, $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
2. $A \cup B = \langle \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$, for every $x \in X$.
3. $A \cap B = \langle \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$, for every $x \in X$.

Definition 3. The *Neutrosophic Logic* (NL) is the generalization of the fuzzy logic, where a logical proposition P is characterized by three components:

$$\mathrm{NL(P)} = (\mathrm{T,I,F}) \quad (1)$$

Where the neutrosophic component T is the degree of truthfulness, F is the degree of falsehood, and I is the degree of indeterminacy.

Definition 4. Let (T_1, I_1, F_1) and (T_2, I_2, F_2) be elements of NL where the sum of the elements of the triplet is 1. The logical connectives of { ¬, ∧, ∨ } can be defined in the following way:

1. $\neg(T_1,I_1,F_1) = (F_1,I_1,T_1)$,
2. $(T_1,I_1,F_1) \wedge (T_2,I_2,F_2) = (T = \min\{T_1,T_2\}, I = 1 - (T+F), F= \max\{F_1,F_2\})$,
3. $(T_1,I_1,F_1) \vee (T_2,I_2,F_2) = (T = \max \{T_1,T_2\}, I =1- (T + F), F = \min \{F_1,F_2\})$.

This Neutrosophic Logic is denoted by NL_1.

To analyse the result, a *scoring function* is established to order alternatives:

$$\mathrm{S(V) = T - F - I} \quad (2)$$

Where V is the valuation of proposition P in the NL_1.

The use of questionnaires as a tool for validation or obtaining information always has the characteristic that the information obtained is permeated or affected by the mental models and internal representations of the external reality of each participating individual. It means this, before the same external reality, each individual could have varied internal representations. These representations are modelled preferably by means of causal representations in the presence of uncertainty [17], make it easy to understand them and explain why a conclusion is reached? [19].

The Iadov Neutrosophic Technique, as it raises the original technique, the related criteria of answers to intercalated questions whose relation the subject does not know, at the same time the unrelated or complementary questions serve as introduction and sustenance of objectivity to the respondent who uses them to locate and contrast the answers [20]. The inclusion of the Neutrosophy allows to deal with the non-determination in the answers [19].

The introduction of Neutrosophic estimation seeks to solve the problems of indeterminacy that appear universally in the evaluations of surveys and other instruments, taking advantage of not only the opposing and opposing positions, but also the neutral or ambiguous ones. Part of that every idea <A> tends to be neutralized, diminished, balanced by the ideas, in clear rupture with the binary doctrines in the explanation and understanding of the phenomena [17]. To measure satisfaction and assess satisfaction with the instrument created, a questionnaire is used that includes open and closed questions. The closed ones are related by the Iadov procedure. The scale used is represented by the form, where a valuation as programming techniques to structure propositional formulas to, and consider each proposition P. The usual fuzzy operators utilized to solve Group Decision problems are the aggregation operators. This notion can be extended to the neutrosophic framework. Neutrosophic Aggregation Operators are formally defined in Definition 5.

Definition 5. Let *X* be a universe of discourse, a space of points (objects) and *x* denotes a generic element of *X*. ***A*** is a *Single Valued Neutrosophic Aggregation Operator* (SVNAO) if it is a mapping ***A***: $\cup_{n\in\mathbb{N}}\ ([0,1]^3)^n \rightarrow [0,1]^3$. One example of SVNAO is the *Weighted Average* operator (WA), which is shown in Equation 3.

$$WA(a_1, a_2, \cdots, a_n) = \sum_{i=1}^{n} w_i a_i \tag{3}$$

Where, $a_i = (T_i, I_i, F_i)$ are SVNNs and $w_i \in [0, 1]$ for every i = 1, 2, …, n; which satisfy the condition $\sum_{i=1}^{n} w_i = 1$. The a_is are the values obtained for the i^th alternative assessment, and w_i denote the weight which represents the importance given to the alternative a_i.

Where w_i represents the importance / relevance of the data source a_i. In order to achieve the verification of the necessary elements in decision-making, the single-valued neutrosophic numbers were presented; to increase the quantitative analysis in the comprehension models of suggestions to clearly assess the indeterminacy (Table 3). In the case of the undefined result, the de-neutrosophication process is used, as it was proposed by SALMERON and SMARANDACHE [21]. In this case, I ϵ [-1,1], is replaced by its maximum and minimum values. Finally, we work with the average of the extreme values to obtain a single value, see Equation (4).

Table 3. Iadov Scale

Semantic indicator	SVN Number	Score
Satisfied	(1 , 0, 0)	1
More satisfied that dissatisfied	(1, 0.25, 0.25)	0.5
Neutral	I	0
More dissatisfied that satisfied	(0.25, 0,25, 1)	-0.5
Total satisfied	(0,0,1)	-1
Opposites	(1,0,1)	0

Source: SALMERON and SMARANDACHE [21].

$$\lambda([a_1, a_2]) = \frac{a_1+a_2}{2} \qquad (4)$$

We can rank the variables by the using Equation 5.

$$\text{Then } A \succ B \iff \frac{a_1+a_2}{2} > \frac{b_1+b_2}{2} \qquad (5)$$

The application of the questionnaire is done to the 19 people to whom the instrument was applied and three academics with research experience in the subject are added for a total of 22. The survey was developed with seven (7) questions, three closed questions interspersed in four open questions; of which one (1) fulfilled the introductory function and three functioned as reaffirmation and support of objectivity to the respondent. Table 4 shows the logical process of Iadov.

Table 4. Iadov Logical Process.

<table>
<tr><td rowspan="4">5- Does the design of the designed questionnaire meet your expectations and do you consider that it responds to the processes of knowledge management?</td><td colspan="9">6- Would it be feasible to dispense with the development of knowledge management in the organization as a way to achieve strategic objectives?</td></tr>
<tr><td colspan="3">Not (N)</td><td colspan="3">I don't know (IDK)</td><td colspan="3">Yes (Y)</td></tr>
<tr><td colspan="9">7- Do you consider that the development of knowledge management audit processes and the use of surveys in them would favor the determination of existing knowledge, the necessary knowledge and, therefore, the gaps to be overcome?</td></tr>
<tr><td>Y</td><td>IDK</td><td>N</td><td>Y</td><td>IDK</td><td>N</td><td>Y</td><td>IDK</td><td>N</td></tr>
<tr><td>Very satisfied</td><td>1(14)</td><td>2(3)</td><td>6</td><td>2</td><td>2</td><td>6</td><td>6</td><td>6</td><td>6</td></tr>
<tr><td>Partially satisfied</td><td>2 (12)</td><td>2(2)</td><td>3</td><td>2 (1)</td><td>3</td><td>3</td><td>6</td><td>3</td><td>6</td></tr>
<tr><td>Does not matter to me.</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr>
<tr><td>More in</td><td>3</td><td>3</td><td>6</td><td>3</td><td>4</td><td>4</td><td>3</td><td>4</td><td>4</td></tr>
</table>

satisfied than satisfied									
Not satisfied at all.	6	6	6	6	4	4	6	4	5
I do not know what to say.	2	3	6	3	3	3	6	3	4

In this case, the following results are obtained (Table 5).

Table 5. Results using the Iadov scale.

Semantic Indicator	Total	Percentage
Satisfied	14	64
Very satisfied that dissatisfied	8	36
Neutral	0	0
Very dissatisfied that satisfied	0	0
Total satisfied	0	0
Opposites	0	0

Source: (Mesa Mariscal and Ordoñez Lago, 2010).

The calculation of the score is made and the calculation of Iadov is determined in this case each one is assigned a value in the weight vector equal to: $w_1 = w_2 = \cdots = w_{22} = 0.055$. The final result that shows a high level of satisfaction yields the value of: ISG =0.818 (Figure 1).

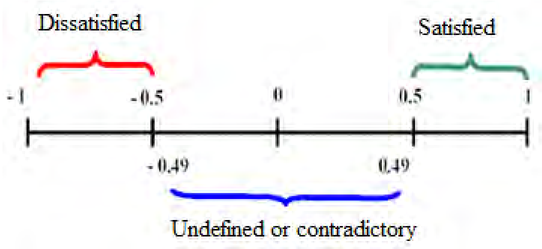

Figure 1. Iadov Scale.

Step 6. Interpretation of the results and final report

The average total result by items is recommended to be determined by the sum of the scores obtained in it and its division by the total of respondents. To obtain the average total result by category (KM processes), the sum of the average scores obtained in the items that comprise it and its division among the total of questions by category is performed. The scale of valuation of the instrument is established in the 1 in approximation to the processing carried out by LONDOÑO GALEANO and GARCÍA OSPINA [15] (Table 6).

Table 6. Scale of the values considered low, acceptable and good.

Assessment		Low		Acceptable		Good
Scale	1	1,8	2,6	3,4	4,2	5

To obtain the valuation scale, the major and minor values of the scale (5) and (1) are subtracted and the result (4) is divided by the number of divisions in which the scale is to be fragmented. In this case, it is divided by 5 to obtain higher valuation ranges, for a result of 0.8. This value is added to the lowest value of the scale (1) until reaching the highest value of the scale (5). As a result, a rating scale of Low (from 1 to 2.6), Acceptable (from 2.6 to 4.2) and Good (from 4.2 to 5) is obtained. As a result of the application of the questionnaire, table 3 shows the value obtained and the scale in which each process of the KM is located, as well as the percentage of questions in each of the scales. Figure 1 summarizes these results and compares them with good standards and reflects values of: 4.31 and 4.35 with evaluation of good to acquire and use; 4.07, 4.17 and 4.01 evaluation of acceptable to organize, disclose and measure respectively. In turn, the company's knowledge management has an average of 4.18; so its assessment is acceptable. Question 27 that evaluates all the processes of the KM has an average of 4.21; when compared with the general average obtained (4.18), it can be seen that they do not differ, so the veracity of the answers obtained is evident. Next, an analysis is shown in each of the processes by the respective questions that evaluate it.

Figure 2 shows the evaluation obtained in the process of acquiring according to the behavior of the measured variables of the KMA. (Green: Minimal value for a good evaluation of each KM process).

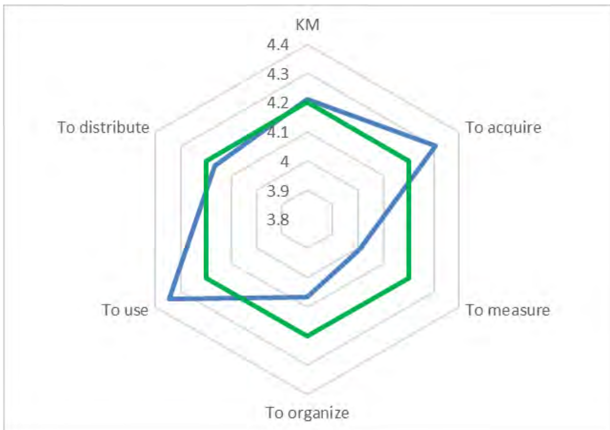

Figure 2. Summary of the results of the questionnaire for each KM process.

Figure 3. Scales obtained in the five KM processes.

Table 7. Improvement actions for each knowledge management process.

KM processes	**Improvement actions**
To Acquire	Recognize the sources of knowledge external to the organization and allow the improvement of processes. Apply knowledge management tools in at least one of the productive organizations for later generalization to the rest of the country. Among the tools to apply are: questionnaire, social network analysis, knowledge maps.
To organize	Make individual improvement plans to meet the needs detected. Formalize (document and standardize) the knowledge inventory in the organization. This inventory is the basis for the field work to be performed. It allows to establish the knowledge-competence relationship and its insertion in the manual of functions through the occupational description method (DACUM).

To distribute	To expose all the investigations carried out in the company, both in the national office and in the UEB, silos and mills of the country and through a repository or digital library.
To use	Take actions so that process leaders rely on the sources of knowledge detected to implement the organization's strategies.
To measure	Evaluate in the company future knowledge needs to eliminate the gaps between existing and required knowledge. Develop continuous auditing to acquire, organize, disseminate, use and measure (through AGC techniques) the required and existing knowledge for continuous improvement in the company's processes.

The improvement actions to be carried out are outlined below: (1) to carry out knowledge inventories in a systematic way, to determine the existing knowledge, the required knowledge and the gaps between them; (2) perfect the bank of problems detected by the company and propose solutions based on investigations carried out through consultancies or continue the link with the university. In addition, Table 3 shows other actions to be taken that are more specific and directed to each process of knowledge management. Likewise, improvement actions for each of the KM processes are established and an analysis of the values obtained for each variable of the KMA is made. Table 4 shows the 16 variables evaluated and the percentage of questions in each of the scales: nine variables presented good, six acceptable and the variable identification of the information presented a low value.

3. Considerations about KMA results

The firm needs to apply knowledge identification tools to locate the existing and requiring knowledge for the development of their processes. Developing the KMA process continuously for each of the KM processes: acquire, organize, distribute, use and measure and the continuous improvement of the processes of the company.

The main forms in which knowledge is acquired were determined: postgraduate courses, meetings and exchange of experiences live and via e-mail. The means by which the knowledge generated by the processes is distributed to all workers are mainly: the exchange of experiences, work meetings, the exchange of information using e-mail and the investigations (thesis) applied in the company.

The knowledge acquisition is achieved in work meetings (mainly), live exchange and the use of the telephone. However, it is recognized what the regulations, manuals and databases provide, which is where the knowledge required to adequately perform the work is identified. The people who are most consulted in the company and can be considered valuable assets of knowledge are: the CEO, the CTO and the CFO.

Table 4. Variables evaluated and the percentage of questions in each of the scales.

KMA Variables	Value	Scale		
Firm strategy	4.26	GOOD		
KM key factors	4.18		ACCEPTABLE	
KM strategy	4.37	GOOD		
KM value chain	4.18		ACCEPTABLE	
Process approach	4.36	GOOD		
Organizational culture	4.50	GOOD		
Knowledge required	4.08		ACCEPTABLE	
Existing knowledge	4.02		ACCEPTABLE	
Use of knowledge	4.39	GOOD		
Identification of information	2.46			LOW
Sources of knowledge	4.37	GOOD		
Social networks	4.35	GOOD		
Analysis of gaps	4.42	GOOD		
Techniques used in the KMA	3.21		ACCEPTABLE	
Decision making	4.74	GOOD		
Continuous auditing	3.63		ACCEPTABLE	

4. Conclusions

The KMA is a useful tool for the development of KM strategies and identifies and describes organizational knowledge, its use, gaps and duplication within the organization. The existing methodologies for the KMA are characterized by the use of questionnaires as a common method of acquiring data in the KM. In this paper we designed a questionnaire and applied it to assess the knowledge management audit in a grain storage and conservation company. Usually, the possible answers to the questionnaire can contain uncertainty and indeterminacy, thus, we applied the neutrosophic Iadov technique for processing the survey, where the undefined or contradictory information are also included. Moreover, neutrosophic Iadov contains linguistic terms for evaluating, which facilitates to answering the questions. The proposed questionnaire is composed of 38 items and the correspondence between the proposed questions is achieved with all the processes and the significant variables of knowledge management. It was successfully applied to 100% of people to be surveyed, its reliability and validity are demonstrated; where it is concluded that: the company presents an acceptable KM performance with a value of 4.18; the use and purchase categories obtained better scores and are considered to be in good condition; while the categories to show, organize and measure obtained results considered acceptable.

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Conflicts of Interest

The authors declare no conflict of interest.

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Neutrosophic Bitopological Spaces

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Abstract: In this study, bitopological structure which is a more general structure than topological spaces is built on neutrosophic sets. The necessary arguments which are pairwise neutrosophic open set, pairwise neutrosophic closed set, pairwise neutrosophic closure, pairwise neutrosophic interior are defined and their basic properties are presented. The relations of these concepts with their counterparts in neutrosophic topological spaces are given and many examples are presented.

Keywords: Neutrosophic set; neutrosophic bitopological space; pairwise neutrosophic open (closed) set; pairwise neutrosophic interior; pairwise neutrosophic closure; pairwise neutrosophic neighbourhood.

1. Introduction

In recent years, the major factor in the progress of natural sciences and its sub-branches is the construction of new set structures in mathematics. It is the fuzzy set theory defined by Zadeh [19] that leads to these set structures. This set structure is followed by intuitionistic set theory [7], intuitionistic fuzzy set theory [1] and soft set theory [15]. Later, as a generalization of fuzzy set and intuitionistic fuzzy set, Samarandache [17] introduced neutrosophic set. Neutrosophic set N consist of three independent object called truth-membership $T_N(x)$, interminancy-membership $I_N(x)$ and falsity-memebership $F_N(x)$ whose values are real standard or non-standard subset of unit interval $]^-0,1^+[$. Scientists continue to intensively study in different fields with this set structure [3, 4, 8, 14, 15, 17, 18, 19, 20, 21, 22]. These set structures have been studied by some authors in topology [2, 5, 6, 16, 18].

The concept of bitopological spaces was introduced by Kelly [13] as an extension of topological spaces in 1963. This concept has been studied with interest in other set structures [10, 12]. Therefore, we find it necessary and important to construct a bitopological spaces on the neutrosophic set structure.

In this study, we presented bitopological spaces on neutrosophic set structure and some basic notions of this spaces, open (closed) set, closure, interior, neighbourhood systems are defined. In addition, the theorems required for this structure are proved and their relations with neutrosophic topological spaces are investigated.

2. Preliminary

In this section, we will give some preliminary information for the present study.

Definition 2.1 [23] Let X be a non empty set, then $N=\{\langle x,T_N(x),I_N(x),F_N(x)\rangle : x\in X\}$ is called a neutrosophic set on X, where $^-0\le T_N(x)+I_N(x)+F_N(x)\le 3^+$ for all $x\in X$, $T_N(x)$, $I_N(x)$ and $F_N(x)\in]^-0,1^+[$ are the degree of membership (namely $T_N(x)$), the degree of indeterminacy (namely

$I_N(x)$) and the degree of non membership (namely $F_N(x)$) of each $x \in X$ to the set N respectively. For X, $\aleph(X)$ denotes the collection of all neutrosophic sets of X.

Definition 2.2 [23] The following statements are true for neutrosophic sets N and M on X:
i) $T_N(x) \le T_M(x)$, $I_N(x) \le I_M(x)$ and $F_N(x) \ge F_M(x)$ for all $x \in X$ iff $N \subseteq M$.
ii) $N \subseteq M$ and $M \subseteq N$ iff $N = M$.
iii) $N \cap M = \{\langle x, \min\{T_N(x), T_M(x)\}, \min\{I_N(x), I_M(x)\}, \max\{F_N(x), F_M(x)\}\rangle : x \in X\}$.
iv) $N \cup M = \{\langle x, \max\{T_N(x), T_M(x)\}, \max\{I_N(x), I_M(x)\}, \min\{F_N(x), F_M(x)\}\rangle : x \in X\}$.
More generally, the intersection and the union of a collection of neutrosophic sets $\{N_i\}_{i\in I}$, are defined by:

$$\bigcap_{i\in I} N_i = \left\{\langle x, \inf\{T_{N_i}(x)\}, \inf\{I_{N_i}(x)\}, \sup\{F_{N_i}(x)\}\rangle : x \in X\right\},$$
$$\bigcup_{i\in I} N_i = \left\{\langle x, \sup\{T_{N_i}(x)\}, \sup\{I_{N_i}(x)\}, \inf\{F_{N_i}(x)\}\rangle : x \in X\right\}.$$

v) N is called neutrosophic universal set, denoted by 1_X, if $T_N(x) = 1$, $I_N(x) = 1$ and $F_N(x) = 0$ for all $x \in X$.

vi) N is called neutrosophic empty set, denoted by 0_X, if $T_N(x) = 0$, $I_N(x) = 0$ and $F_N(x) = 1$ for all $x \in X$.

vii) $N\backslash M = \{\langle x, |T_N(x) - T_M(x)|, |I_N(x) - I_M(x)|, 1 - |F_N(x) - F_M(x)|\rangle : x \in X\}$. Clearly, the neutrosophic complements of 1_X and 0_X are defined:

$$(1_X)^c = 1_X\backslash 1_X = \langle x, 0,0,1\rangle = 0_X,$$
$$(0_X)^c = 1_X\backslash 0_X = \langle x, 1,1,0\rangle = 1_X.$$

Proposition 2.1 [23] Let N_1, N_2, N_3 and $N_4 \in \aleph(X)$. Then followings hold:
i) $N_1 \cap N_3 \subseteq N_2 \cap N_4$ and $N_1 \cup N_3 \subseteq N_2 \cup N_4$, if $N_1 \subseteq N_2$ and $N_3 \subseteq N_4$,
ii) $(N_1^c)^c = N_1$ and $N_1 \subseteq N_2$, if $N_2^c \subseteq N_1^c$,
iii) $(N_1 \cap N_2)^c = N_1^c \cup N_2^c$ and $(N_1 \cup N_2)^c = N_1^c \cap N_2^c$.

Definition 2.3 [22] Let X be a non empty set. A neutrosophic topology on X is a subfamily τ^N of $\aleph(X)$ such that 1_X and 0_X belong to τ^n, τ^n is closed under arbitrary union and τ^n is closed finite intersection. Then (X, τ^n) is called neutrosophic topological space, members of τ^n are known as neutrosophic open sets and their complements are neutrosophic closed sets. For a neutrosophic set N over X, the neutrosophic interior and the neutrosophic closure of N are defined as: $int^n(N) = \cup\{G : G \subseteq N, G \in \tau^n\}$ and $cl^n(N) = \cap\{F : N \subseteq F, F^c \in \tau^n\}$.

Definition 2.4 [9] Let X be a non empty set. If α, β, γ be real standard or non standard subsets of $]^-0, 1^+[$, then the neutrosophic set $x_{\alpha,\beta,\gamma}$ is called a neutrosophic point in given by

$$x_{\alpha,\beta,\gamma}(y) = \begin{cases} (\alpha, \beta, \gamma), & \text{if } x = y \\ (0,0,1), & \text{if } x \ne y \end{cases}$$

for $y \in X$ is called the support of $x_{\alpha,\beta,\gamma}$.
It is clear that every neutrosophic set is the union of its neutrosophic points.

Definition 2.5 [9] Let $N \in \aleph(X)$. We say that $x_{\alpha,\beta,\gamma} \in N$ read as belonging to the neutrosophic set N whenever $\alpha \le T_N(x)$, $\beta \le I_N(x)$ and $\gamma \ge F_N(x)$.

Definition 2.6 [11] A subcollection τ_n^* of neutrosophic sets on a non empty set X is said to be a neutrosophic supra topology on X if the sets 1_X, $0_X \in \tau_n^*$ and $\bigcup_{i=1}^{\infty} N_i \in \tau_n^*$ for $\{N_i\}_{i=1}^{\infty} \in \tau_n^*$. Then (X, τ_n^*) is called neutrosophic supra topological space on X.

3. Neutrosophic Bitopological Spaces

Definition 3.1 Let (X,τ_1^n) and (X,τ_2^n) be the two different neutrosophic topologies on X. Then (X,τ_1^n,τ_2^n) is called a neutrosophic bitopological space.

Definition 3.2 Let (X,τ_1^n,τ_2^n) be a neutrosophic bitopological space. A neutrosophic set $N=\{\langle x,T_N(x),I_N(x),F_N(x)\rangle: x\in X\}$ over X is said to be a pairwise neutrosophic open set in (X,τ_1^n,τ_2^n) if there exist a neutrosophic open set $N_1=\{\langle x,T_{N_1}(x),I_{N_1}(x),F_{N_1}(x)\rangle: x\in X\}$ in τ_1^n and a neutrosophic open set $N_2=\{\langle x,T_{N_2}(x),I_{N_2}(x),F_{N_2}(x)\rangle: x\in X\}$ in τ_2^n such that $N=N_1\cup N_2=\{\langle x,\max\{T_{N_1}(x),T_{N_2}(x)\},\max\{I_{N_1}(x),I_{N_2}(x)\},\min\{F_{N_1}(x),F_{N_2}(x)\}\rangle: x\in X\}$.

Definition 3.3 Let (X,τ_1^n,τ_2^n) be a neutrosophic bitopological space. A neutrosophic set N over X is said to be a pairwise neutrosophic closed set in (X,τ_1^n,τ_2^n) if its neutrosophic complement is a pairwise neutrosophic open set in (X,τ_1^n,τ_2^n). Obviously, a neutrosophic set $C=\{\langle x,T_C(x),I_C(x),F_C(x)\rangle: x\in X\}$ over X is a pairwise neutrosophic closed set in (X,τ_1^n,τ_2^n) if there exist a neutrosophic closed set $C_1=\{\langle x,T_{C_1}(x),I_{C_1}(x),F_{C_1}(x)\rangle: x\in X\}$ in $(\tau_1^n)^c$ and a neutrosophic closed set $C_2=\{\langle x,T_{C_2}(x),I_{C_2}(x),F_{C_2}(x)\rangle: x\in X\}$ in $(\tau_2^n)^c$ such that $C=C_1\cap C_2=\{\langle x,\min\{T_{C_1}(x),T_{C_2}(x)\},\min\{I_{C_1}(x),I_{C_2}(x)\},\max\{F_{C_1}(x),F_{C_2}(x)\}\rangle: x\in X\}$, where

$$(\tau_i^n)^c=\{N^c\in \aleph(X): N\in\tau_i^n\}, i=1,2.$$

The family of all pairwise neutrosophic open (closed) sets in (X,τ_1^n,τ_2^n) is denoted by $PNO(X,\tau_1^n,\tau_2^n)$ $[PNC(X,\tau_1^n,\tau_2^n)]$, respectively.

Example 3.1 Let $X=\{a,b,c\}$. We think that following neutrosophic set over X.

$$N_1=\{\langle a,0.3,0.2,0.5\rangle,\langle b,0.6,0.5,0.3\rangle,\langle c,0.7,0.1,0.9\rangle\},$$
$$N_2=\{\langle a,0.4,0.1,0.3\rangle,\langle b,0.2,0.6,0.7\rangle,\langle c,0.1,0.3,0.4\rangle\},$$
$$N_3=\{\langle a,0.3,0.1,0.5\rangle,\langle b,0.2,0.5,0.7\rangle,\langle c,0.1,0.1,0.9\rangle\},$$
$$N_4=\{\langle a,0.4,0.2,0.3\rangle,\langle b,0.6,0.6,0.3\rangle,\langle c,0.7,0.3,0.4\rangle\}$$

and

$$M_1=\{\langle a,0.1,0.2,0.3\rangle,\langle b,0.2,0.1,0.4\rangle,\langle c,0.5,0.2,0.4\rangle\},$$
$$M_2=\{\langle a,0.7,0.3,0.1\rangle,\langle b,0.7,0.8,0.2\rangle,\langle c,0.9,0.8,0.3\rangle\}.$$

Then (X,τ_1^n,τ_2^n) is a neutrosophic bitopological space, where

$$\tau_1^n=\{0_X,1_X,N_1,N_2,N_3,N_4\},$$
$$\tau_2^n=\{0_X,1_X,M_1,M_2\}.$$

Obviously,

$$\tau_{12}^n=\tau_1^n\cup\tau_2^n\cup\{N_1\cup M_1,N_2\cup M_1,N_3\cup M_1\}$$

because the neutrosophic sets $N_1\cup M_1$, $N_2\cup M_1$ and $N_3\cup M_1$ not belong to either τ_1^n nor τ_2^n.

Theorem 3.1 Let (X,τ_1^n,τ_2^n) be a neutrosophic bitopological space. Then,

1. 0_X and 1_X are pairwise neutrosophic open sets and pairwise neutrosophic closed sets.

2. An arbitrary neutrosophic union of pairwise neutrosophic open sets is a pairwise neutrosophic open set.

3. An arbitrary neutrosophic intersection of pairwise neutrosophic closed sets is a pairwise neutrosophic closed set.

Proof. 1. Since $0_X\in\tau_1^n$, τ_2^n and $0_X\cup 0_X=0_X$, then 0_X is a pairwise neutrosophic open set. Similarly, 1_X is a pairwise neutrosophic open set.

2. Let $\{(N_i): i\in I\}\subseteq PNO(X,\tau_1^n,\tau_2^n)$. Then N_i is a pairwise neutrosophic open set for all $i\in I$, therefore there exist $N_i^1\in\tau_1^n$ and $N_i^2\in\tau_2^n$ such that $N_i=N_i^1\cup N_i^2$ for all $i\in I$ which implies that

$$\bigcup_{i\in I} N_i=\bigcup_{i\in I}[N_i^1\cup N_i^2]=\left[\bigcup_{i\in I} N_i^1\right]\cup\left[\bigcup_{i\in I} N_i^2\right].$$

Now, since τ_1^n and τ_2^n are neutrosophic topologies, then $\left[\bigcup_{i\in I} N_i^1\right]\in\tau_1^n$ and $\left[\bigcup_{i\in I} N_i^2\right]\in\tau_2^n$. Therefore, $\bigcup_{i\in I} N_i$ is a pairwise neutrosophic open set.

3. It is immediate from the Definition 9, Proposition 1.

Corollary 3.1 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space. Then, the family of all pairwise neutrosophic open sets is a supra neutrosophic topology on X. This supra neutrosophic topology we denoted by τ_{12}^n.

Remark 3.1 The Example 1 show that:
1. τ_{12}^n is not neutrosophic topology in general.
2. The finite neutrosophic intersection of pairwise neutrosophic open sets need not be a pairwise neutrosophic open set.
3. The arbitrary neutrosophic union of pairwise neutrosophic closed sets need not be a pairwise neutrosophic closed set.

Theorem 3.2 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space. Then,

1. Every τ_i^n −open neutrosophic set is a pairwise neutrosophic open set $i = 1,2$, i.e., $\tau_1^n \cup \tau_2^n \subseteq \tau_{12}^n$.
2. Every τ_i^n −closed neutrosophic set is a pairwise neutrosophic closed set $i = 1,2$, i.e., $(\tau_1^n)^c \cup (\tau_2^n)^c \subseteq (\tau_{12}^n)^c$.
3. If $\tau_1^n \subseteq \tau_2^n$, then $\tau_{12}^n = \tau_2^n$ and $(\tau_{12}^n)^c = (\tau_2^n)^c$.

Proof. Straightforward.

Definition 3.4 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space and $N \in \aleph(X)$. The pairwise neutrosophic closure of N, denoted by $cl_p^n(N)$, is the neutrosophic intersection of all pairwise neutrosophic closed super sets of N, i.e.,

$$cl_p^n(N) = \cap \{C \in (\tau_{12}^n)^c : N \subseteq C\}.$$

It is clear that $cl_p^n(N)$ is the smallest pairwise neutrosophic closed set containing N.

Example 3.2 Let (X, τ_1^n, τ_2^n) be the same as in Example 1 and
$G = \{\langle a, 0.7,0.8,0.7\rangle, \langle b, 0.5,0.4,0.6\rangle, \langle c, 0.8,0.7,0.5\rangle\}$ be a neutrosophic set over X.
Now, we need to determine pairwise neutrosophic closed sets in (X, τ_1^n, τ_2^n) to find $cl_p^n(G)$. Then,

$$N_1^c = \{\langle a, 0.7,0.8,0.5\rangle, \langle b, 0.4,0.5,0.7\rangle, \langle c, 0.3,0.9,0.1\rangle\},$$
$$N_2^c = \{\langle a, 0.6,0.9,0.7\rangle, \langle b, 0.8,0.4,0.3\rangle, \langle c, 0.9,0.7,0.6\rangle\},$$
$$N_3^c = \{\langle a, 0.7,0.9,0.5\rangle, \langle b, 0.8,0.5,0.3\rangle, \langle c, 0.9,0.9,0.1\rangle\},$$
$$N_4^c = \{\langle a, 0.6,0.8,0.7\rangle, \langle b, 0.4,0.4,0.7\rangle, \langle c, 0.3,0.7,0.6\rangle\},$$

$$M_1^c = \{\langle a, 0.9,0.8,0.7\rangle, \langle b, 0.8,0.9,0.6\rangle, \langle c, 0.5,0.8,0.6\rangle\},$$
$$M_2^c = \{\langle a, 0.3,0.7,0.9\rangle, \langle b, 0.3,0.2,0.8\rangle, \langle c, 0.1,0.2,0.7\rangle\}.$$

and

$$(N_1 \cup M_1)^c = \{\langle a, 0.7,0.8,0.7\rangle, \langle b, 0.4,0.5,0.7\rangle, \langle c, 0.3,0.8,0.6\rangle\}$$
$$(N_2 \cup M_1)^c = \{\langle a, 0.6,0.8,0.7\rangle, \langle b, 0.8,0.4,0.6\rangle, \langle c, 0.5,0.7,0.6\rangle\}$$
$$(N_3 \cup M_1)^c = \{\langle a, 0.7,0.8,0.7\rangle, \langle b, 0.8,0.5,0.6\rangle, \langle c, 0.5,0.8,0.6\rangle\}$$

In here, the pairwise neutrosophic closed sets which contains G are N_3^c and 1_X it follows that $cl_p^n(G) = N_3^c \cap 1_X$. Therefore, $cl_p^n(G) = N_3^c$.

Theorem 3.3 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space and $N, M \in \aleph(X)$. Then,
1. $cl_p^n(0_X) = 0_X$ and $cl_p^n(1_X) = 1_X$.
2. $N \subseteq cl_p^n(N)$.
3. N is a pairwise neutrosophic closed set iff $cl_p^n(N) = N$.
4. $N \subseteq M \Rightarrow cl_p^n(N) \subseteq cl_p^n(M)$.
5. $cl_p^n(N) \cup cl_p^n(M) \subseteq cl_p^n(N \cup M)$.
6. $cl_p^n[cl_p^n(N)] = cl_p^n(N)$, i.e., $cl_p^n(N)$ is a pairwise neutrosophic closed set.

Proof. Straightforward.

Theorem 3.4 Let (X,τ_1^n,τ_2^n) be a neutrosophic bitopological space and $N \in \aleph(X)$. Then,

$$x_{\alpha,\beta,\gamma} \in cl_p^n(N) \Leftrightarrow U_{x_{\alpha,\beta,\gamma}} \cap N \neq 0_X, \forall U_{x_{\alpha,\beta,\gamma}} \in \tau_{12}^n(x_{\alpha,\beta,\gamma}),$$

where $U_{x_{\alpha,\beta,\gamma}}$ is any pairwise neutrosophic open set contains $x_{\alpha,\beta,\gamma}$ and $\tau_{12}^n(x_{\alpha,\beta,\gamma})$ is the family of all pairwise neutrosophic open sets contains $x_{\alpha,\beta,\gamma}$.

Proof. Let $x_{\alpha,\beta,\gamma} \in cl_p^n(N)$ and suppose that there exists $U_{x_{\alpha,\beta,\gamma}} \in \tau_{12}^n(x_{\alpha,\beta,\gamma})$ such that $U_{x_{\alpha,\beta,\gamma}} \cap N = 0_X$. Then $N \subseteq \left(U_{x_{\alpha,\beta,\gamma}}\right)^c$, thus $cl_p^n(N) \subseteq cl_p^n\left(U_{x_{\alpha,\beta,\gamma}}\right)^c = \left(U_{x_{\alpha,\beta,\gamma}}\right)^c$ which implies $cl_p^n(N) \cap U_{x_{\alpha,\beta,\gamma}} = 0_X$, a contradiction.
Conversely, assume that $x_{\alpha,\beta,\gamma} \notin cl_p^n(N)$, then $x_{\alpha,\beta,\gamma} \in [cl_p^n(N)]^c$. Thus, $[cl_p^n(N)]^c \in \tau_{12}^n(x_{\alpha,\beta,\gamma})$, so, by hypothesis, $[cl_p^n(N)]^c \cap N \neq 0_X$, a contradiction.

Theorem 3.5 Let (X,τ_1^n,τ_2^n) be a neutrosophic bitopological space. A neutrosophic set N over X is a pairwise neutrosophic closed set iff $N = cl_{\tau_1}^n(N) \cap cl_{\tau_2}^n(N)$.

Proof. Suppose that N is a pairwise neutrosophic closed set and $x_{\alpha,\beta,\gamma} \notin N$. Then, $x_{\alpha,\beta,\gamma} \notin cl_p^n(N)$. Thus, [by Theorem 4], there exists $U_{x_{\alpha,\beta,\gamma}} \in \tau_{12}^n(x_{\alpha,\beta,\gamma})$ such that $U_{x_{\alpha,\beta,\gamma}} \cap N = 0_X$. Since $U_{x_{\alpha,\beta,\gamma}} \in \tau_{12}^n(x_{\alpha,\beta,\gamma})$, then there exists $M_1 \in \tau_1^n$ and $M_2 \in \tau_2^n$ such that $U_{x_{\alpha,\beta,\gamma}} = M_1 \cup M_2$. Hence, $(M_1 \cup M_2) \cap N = 0_X$ it follows that $M_1 \cap N = 0_X$ and $M_2 \cap N = 0_X$. Since $x_{\alpha,\beta,\gamma} \in U_{x_{\alpha,\beta,\gamma}}$, then $x_{\alpha,\beta,\gamma} \in M_1$ or $x_{\alpha,\beta,\gamma} \in M_2$ implies, $x_{\alpha,\beta,\gamma} \notin cl_{\tau_1}^n(N)$ or $x_{\alpha,\beta,\gamma} \notin cl_{\tau_2}^n(N)$. Therefore, $x_{\alpha,\beta,\gamma} \notin cl_{\tau_1}^n(N) \cap cl_{\tau_2}^n(N)$. Thus, $cl_{\tau_1}^n(N) \cap cl_{\tau_2}^n(N) \subseteq N$. On the other hand, we have $N \subseteq cl_{\tau_1}^n(N) \cap cl_{\tau_2}^n(N)$. Hence, $N = cl_{\tau_1}^n(N) \cap cl_{\tau_2}^n(N)$.
Conversely, suppose that $N = cl_{\tau_1}^n(N) \cap cl_{\tau_2}^n(N)$. Since, $cl_{\tau_1}^n(N)$ is a neutrosophic closed set in (X,τ_1^n) and $cl_{\tau_2}^n(N)$ is a neutrosophic closed set in (X,τ_2^n), then, [by Definition 9], $cl_{\tau_1}^n(N) \cap cl_{\tau_2}^n(N)$ is a pairwise neutrosophic closed set in (X,τ_1^n,τ_2^n), so N is a pairwise neutrosophic closed set.

Corollary 3.2 Let (X,τ_1^n,τ_2^n) be a neutrosophic bitopological space. Then,

$$cl_p^n(N) = cl_{\tau_1}^n(N) \cap cl_{\tau_2}^n(N), \forall N \in \aleph(X).$$

Definition 3.5 An operator $\Psi: \aleph(X) \to \aleph(X)$ is called a neutrosophic supra closure operator if it satisfies the following conditions for all N, $M \in \aleph(X)$.

1. $\Psi(0_X) = 0_X$,
2. $N \subseteq \Psi(N)$,
3. $\Psi(N) \cup \Psi(M) \subseteq \Psi(N \cup M)$
4. $\Psi(\Psi(N)) = \Psi(N)$.

Theorem 3.6 Let (X,τ_1^n,τ_2^n) be a neutrosophic bitopological space. Then, the operator $cl_p^n: \aleph(X) \to \aleph(X)$ which defined by

$$cl_p^n(N) = cl_{\tau_1}^n(N) \cap cl_{\tau_2}^n(N)$$

is neutrosophic supra closure operator and it is induced, a unique neutrosophic supra topology given by $\{N \in \aleph(X): cl_p^n(N^c) = N^c\}$ which is precisely τ_{12}^n.

Proof. Straightforward.

Definition 3.6 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space and $N \in \aleph(X)$. The pairwise neutrosophic interior of N, denoted by $int_p^n(N)$, is the neutrosophic union of all pairwise neutrosophic open subsets of N, i.e.,

$$int_p^n(N) = \cup \{M \in \tau_{12}^n : M \subseteq N\}.$$

Obviously, $int_p^n(N)$ is the biggest pairwise neutrosophic open set contained in N.

Example 3.3 Let (X, τ_1^n, τ_2^n) be the same as in Example 1 and $M = \{\langle a, 0.3, 0.4, 0.2\rangle, \langle b, 0.5, 0.7, 0.1\rangle, \langle c, 0.8, 0.7, 0.3\rangle\}$ be a neutrosophic set over X. Then the pairwise neutrosophic open sets which containing in M are N_3, M_1, $N_3 \cup M_1$ and 0_X. Therefore,

$$\begin{aligned} int_p^n(M) &= N_3 \cup M_1 \cup (N_3 \cup M_1) \cup 0_X \\ &= N_3 \cup M_1 \\ &= \{\langle a, 0.3, 0.2, 0.3\rangle, \langle b, 0.2, 0.5, 0.4\rangle, \langle c, 0.5, 0.2, 0.4\rangle\}. \end{aligned}$$

Theorem 3.7 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space and $N, M \in \aleph(X)$. Then,

1. $int_p^n(0_X) = 0_X$ and $int_p^n(1_X) = 1_X$,
2. $int_p^n(N) \subseteq N$,
3. N is a pairwise neutrosophic open set iff $int_p^n(N) = N$,
4. $N \subseteq M \Rightarrow int_p^n(N) \subseteq int_p^n(M)$,
5. $int_p^n(N \cap M) \subseteq int_p^n(N) \cap int_p^n(M)$,
6. $int_p^n[int_p^n(N)] = int_p^n(N)$.

Proof. Starightforward.

Theorem 3.8 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space and $N \in \aleph(X)$. Then, $x_{\alpha,\beta,\gamma} \in int_p^n(N) \Leftrightarrow \exists U_{x_{\alpha,\beta,\gamma}} \in \tau_{12}^n(x_{\alpha,\beta,\gamma})$ such that $U_{x_{\alpha,\beta,\gamma}} \subseteq N$.

Proof. Starightforward.

Theorem 3.9 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space. A neutrosophic set N over X is a pairwise neutrosophic open set iff $N = int_{\tau_1}^n(N) \cup int_{\tau_2}^n(N)$.

Proof. Let N be a pairwise neutrosophic open set. Since, $int_{\tau_i}^n(N) \subseteq N$, $i = 1,2$, then $int_{\tau_1}^n(N) \cup int_{\tau_2}^n(N) \subseteq N$. Now, let $x_{\alpha,\beta,\gamma} \in N$. Then, there exists $U_{x_{\alpha,\beta,\gamma}}^1 \in \tau_1^n$ such that $U_{x_{\alpha,\beta,\gamma}}^1 \subseteq N$ or there exists $U_{x_{\alpha,\beta,\gamma}}^2 \in \tau_2^n$ such that $U_{x_{\alpha,\beta,\gamma}}^2 \subseteq N$, thus $x_{\alpha,\beta,\gamma} \in int_{\tau_1}^n(N)$ or $x_{\alpha,\beta,\gamma} \in int_{\tau_2}^n(N)$. Hence, $x_{\alpha,\beta,\gamma} \in int_{\tau_1}^n(N) \cup int_{\tau_2}^n(N)$. Therefore, $N = int_{\tau_1}^n(N) \cup int_{\tau_2}^n(N)$.
Coversely, since $int_{\tau_1}^n(N)$ is a neutrosophic open set in (X, τ_1^n) and $int_{\tau_2}^n(N)$ is a neutrosophic open set in (X, τ_2^n), then, [by Definition 8], $int_{\tau_1}^n(N) \cup int_{\tau_2}^n(N)$ is a pairwise neutrosophic open set in (X, τ_1^n, τ_2^n). Thus, N is a pairwise neutrosophic open set.

Corollary 3.3 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space. Then,

$$int_p^n(N) = int_{\tau_1}^n(N) \cup int_{\tau_2}^n(N).$$

Definition 3.7 An operator $I: \aleph(X) \to \aleph(X)$ is called a neutrosophic supra interior operator if it satisfies the following conditions for all N, $M \in \aleph(X)$.

1. $I(0_X) = 0_X$,
2. $I(N) \subseteq N$,
3. $I(N \cap M) \subseteq I(N) \cap I(M)$
4. $I(I(N)) = I(N)$.

Theorem 3.10 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space. Then, the operator $\mathrm{int}_p^n: \aleph(X) \to \aleph(X)$ which defined by

$$\mathrm{int}_p^n(N) = \mathrm{int}_{\tau_1}^n(N) \cup \mathrm{int}_{\tau_2}^n(N)$$

is neutrosophic supra interior operator and it is induced, a unique neutrosophic supra topology given by $\{N \in \aleph(X): \mathrm{int}_p^n(N) = N\}$ which is precisely τ_{12}^n.

Proof. Straightforward.

Theorem 3.11 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space and $N \in \aleph(X)$. Then,

1. $\mathrm{int}_p^n(N) = \left(\mathrm{cl}_p^n(N^c)\right)^c$.
2. $\mathrm{cl}_p^n(N) = \left(\mathrm{int}_p^n(N^c)\right)^c$.

Proof. Starightforward.

Definition 3.8 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space, $N \in \aleph(X)$ and $x_{\alpha,\beta,\gamma} \in \aleph(X)$. Then N is said to be a pairwise neutrosophic neighborhood of $x_{\alpha,\beta,\gamma}$, if there exists a pairwise neutrosophic open set U such that $x_{\alpha,\beta,\gamma} \in U \subseteq N$. The family of pairwise neutrosophic neighborhood of neutrosophic point $x_{\alpha,\beta,\gamma}$ denoted by $N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$.

Theorem 3.12 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space and $N \in \aleph(X)$. Then N is pairwise neutrosophic open set iff N is a pairwise neutrosophic neighborhood of its neutrosophic points.

Proof. Let N be a pairwise neutrosophic open set and $x_{\alpha,\beta,\gamma} \in N$. Then $x_{\alpha,\beta,\gamma} \in N \subseteq N$. Therefore N is a pairwise neutrosophic neighborhood of $x_{\alpha,\beta,\gamma}$ for each $x_{\alpha,\beta,\gamma} \in N$.
Conversely, suppose that N is a pairwise neutrosophic neighborhood of its neutrosophic points and $x_{\alpha,\beta,\gamma} \in N$. Then there exists a pairwise neutrosophic open set U such that $x_{\alpha,\beta,\gamma} \in U \subseteq N$. Since

$$N = \bigcup_{x_{\alpha,\beta,\gamma} \in N} \{x_{\alpha,\beta,\gamma}\} \subseteq \bigcup_{x_{\alpha,\beta,\gamma} \in N} U \bigcup_{x_{\alpha,\beta,\gamma} \in N} N = N$$

it follows that N is an union of pairwise neutrosophic open sets. Hence, N is a pairwise neutrosophic open set.

Proposition 3.2 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space and $\{N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma}): x_{\alpha,\beta,\gamma} \in \aleph(X)\}$ be a system of pairwise neutrosophic neighborhoods. Then,

1. For every $N \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$, $x_{\alpha,\beta,\gamma} \in N$;
2. $N \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$ and $N \subseteq M \Rightarrow M \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$;
3. $N \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma}) \Rightarrow \exists M \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$ such that $M \subseteq N$ and $M \in N_{\tau_{12}^n}\left(y_{\alpha',\beta',\gamma'}\right)$, for every $y_{\alpha',\beta',\gamma'} \in M$.

Proof. Proofs of 1 and 2 are straightforward.
3. Let N be a pairwise neutrosophic neighborhood of $x_{\alpha,\beta,\gamma}$, then there exists a pairwise neutrosophic open set $M \in \tau_{12}^n$ such that $x_{\alpha,\beta,\gamma} \in M \subseteq N$. Since $x_{\alpha,\beta,\gamma} \in M \subseteq M$ can be written, then $M \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$. From the Theorem 12, if M is pairwise neutrosophic open set then N is a pairwise neutrosophic neighborhood of its neutrosophic points, i.e., $M \in N_{\tau_{12}^n}\left(y_{\alpha',\beta',\gamma'}\right)$, for every $y_{\alpha',\beta',\gamma'} \in M$.

Remark 3.2 Generally, $N, M \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma}) \Rightarrow N \cap M \notin N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$. Actually, if $N, M \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$, there exist $U_1, U_2 \in \tau_{12}^n$ such that $x_{\alpha,\beta,\gamma} \in U_1 \subseteq N$ and $x_{\alpha,\beta,\gamma} \in U_2 \subseteq M$. But $U_1 \cap U_2$ need not be a pairwise neutrosophic open set. Therefore, $N \cap M$ need not be a pairwise neutrosophic neighborhood of $x_{\alpha,\beta,\gamma}$.

Theorem 3.13 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space. Then

$$N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma}) = N_{\tau_1^n}(x_{\alpha,\beta,\gamma}) \cup N_{\tau_2^n}(x_{\alpha,\beta,\gamma})$$

for each $x_{\alpha,\beta,\gamma} \in \aleph(X)$.

Proof. Let $x_{\alpha,\beta,\gamma} \in \aleph(X)$ be any neutrosophic point and $N \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$. Then there exists a pairwise neutrosophic open set $M \in \tau_{12}^n$ such that $x_{\alpha,\beta,\gamma} \in M \subseteq N$. If $M \in \tau_{12}^n$, there exist $M_1 \in \tau_1^n$ and $M_2 \in \tau_2^n$ such that $M = M_1 \cup M_2$. Since $x_{\alpha,\beta,\gamma} \in M = M_1 \cup M_2$, then $x_{\alpha,\beta,\gamma} \in M_1$ or $x_{\alpha,\beta,\gamma} \in M_2$. So, $x_{\alpha,\beta,\gamma} \in M_1 \subseteq M \subseteq N$ or $x_{\alpha,\beta,\gamma} \in M_2 \subseteq M \subseteq N$. In this case, $N \in N_{\tau_1^n}(x_{\alpha,\beta,\gamma})$ or $N \in N_{\tau_2^n}(x_{\alpha,\beta,\gamma})$, i.e., $N \in N_{\tau_1^n}(x_{\alpha,\beta,\gamma}) \cup N_{\tau_2^n}(x_{\alpha,\beta,\gamma})$.
Conversely, suppose that $N \in N_{\tau_1^n}(x_{\alpha,\beta,\gamma}) \cup N_{\tau_2^n}(x_{\alpha,\beta,\gamma})$. Then $N \in N_{\tau_1^n}(x_{\alpha,\beta,\gamma})$ or $N \in N_{\tau_2^n}(x_{\alpha,\beta,\gamma})$. Hence, there exists $x_{\alpha,\beta,\gamma} \in M_1 \in \tau_1^n$ or $x_{\alpha,\beta,\gamma} \in M_2 \in \tau_2^n$ such that $x_{\alpha,\beta,\gamma} \in M_1 \subseteq N$ and $x_{\alpha,\beta,\gamma} \in M_2 \subseteq N$. As a result, $x_{\alpha,\beta,\gamma} \in M_1 \cup M_2 = M \subseteq N$ such that $M \in \tau_{12}^n$ i.e., $N \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})$.

Definition 3.9 An operator $\nu: \aleph(X) \to \aleph(X)$ is called a neutrosophic supra neighborhood operator if it satisfies the following conditions for all N, $M \in \aleph(X)$.

1. $\forall N \in \nu(x_{\alpha,\beta,\gamma})$, $x_{\alpha,\beta,\gamma} \in N$;
2. $N \in \nu(x_{\alpha,\beta,\gamma})$ and $N \subseteq M \Rightarrow M \in \nu(x_{\alpha,\beta,\gamma})$;
3. $N \in \nu(x_{\alpha,\beta,\gamma}) \Rightarrow \exists M \in \nu(x_{\alpha,\beta,\gamma})$ such that $N \subseteq M$ and $M \in \nu\left(y_{\alpha',\beta',\gamma'}\right)$, $y_{\alpha',\beta',\gamma'} \in M$.

Theorem 3.14 Let (X, τ_1^n, τ_2^n) be a neutrosophic bitopological space. Then, the operator $N_{\tau_{12}^n}: \aleph(X) \to \aleph(X)$ which defined by

$$N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma}) = N_{\tau_1^n}(x_{\alpha,\beta,\gamma}) \cup N_{\tau_2^n}(x_{\alpha,\beta,\gamma})$$

is neutrosophic supra neighboorhod operator and it is induced, a unique neutrosophic supra topology given by $\{N \in \aleph(X): \forall x_{\alpha,\beta,\gamma} \in N for N \in N_{\tau_{12}^n}(x_{\alpha,\beta,\gamma})\}$ which is precisely τ_{12}^n.

4. Conclusions

In this paper, neutrosophic bitopological spaces are presented. By defining open (closed) sets, interior, closure and neighbourhood systems, fundamentals theorems for neutrosophic bitopological spaces are proved and some examples on the subject are given. This paper is just a beginning of a new structure and we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application. In the future, using these notions, various classes of mappings on neutrosophic bitopological space, separation axioms on the neutrosophic bitopological spaces and many researchers can be studied

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Conflicts of Interest

The authors declare no conflict of interest.

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Neutrosophic Goal Programming Approach to A Green Supplier Selection Model with Quantity Discount

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Abstract: In this study, we have proposed a supplier selection problem with the goals of minimizing the net cost, minimizing the net rejections, minimizing the net late deliveries, and minimizing the net green house gas emission subject to realistic constraints like suppliers' capacity, buyer's demand etc. Due to uncertainty, the buyer's demand is fuzzy in nature and can be represented as a triangular neutrosophic number. We have also considered that quantity discounts are provided by the suppliers. The weights for different criteria are calculated using neutrosophic analytical hierarchy process. The neutrosophic goal programming approach has been applied in this article for solving the proposed supplier selection problem. An illustration has been given with comparison between fuzzy goal programming approach to demonstrate the effectiveness of the proposed model.

Keywords: Supplier selection; Quantity discounts; Green house gas; Neutrosophic goal programming; Triangular neutrosophic number; Neutrosophic analytical hierarchy process

1. Introduction

The supplier selection problem (SSP) is the problem of determining the right suppliers and their quota allocations. In designing a supply chain, a decision maker needs to consider decisions regarding the selection of the right suppliers and their quota allocation (Kumar, Vrat, & Shankar, 2004). Dickson(Dickson, 1966) was the first to identify 23 different criteria for various supplier selection problems. According to him quality was the most important criterion while delivery, price, geographical location and capacity were also very important factors in the supplier selection process. Weber and Current(Weber & Current, 1993) took a multi-objective approach to solve a supplier selection problem where net price, net late deliveries, net rejected unit delivered were minimized subject to a constant demand and capacity constraint. Kumar et al.(Kumar et al., 2004) applied fuzzy goal programming to solve a similar problem as Weber and Current(Weber & Current, 1993) with some additional constrains such as budget restriction for each retailer, supplier's quota flexibility etc. Wang
and Yang(Wang & Yang, 2009) considered quantity discount in supplier selection problem and applied fuzzy goal programming to find out a compromise solution. They also used analytical hierarchy process (AHP) to find out weights of different goals. Shaw et al.(Shaw, Shankar, Yadav, & Thakur, 2012) developed a supplier selection model with the amount of carbon emission by the suppliers as an objective function. They used fuzzy AHP to figure out weights for different objective functions. They also considered the aggregate demand as a fuzzy triangular number. To solve the problem, they also used fuzzy goal programming approach. Abdel-Basset et al.(Abdel-Basset,

Manogaran, Gamal, & Smarandache, 2018) used neutrosophic set for decision making and evaluation method to analyze and determine the factors influencing the selection of supply chain management suppliers. Gamal et al.(Gamal, Ismail, & Smarandache, 2018) used Multi-Objective Optimization on the basis of Ratio Analysis with the help of neutrosophic trapezoidal number to a supplier selection problem.

Zadeh(Zadeh, 1965) was the first to introduce the concept of fuzzy set. Bellman and Zadeh(Bellman & Zadeh, 1970) demonstrated decision making in fuzzy systems. Zimmermann(Zimmermann, 1978) applied the fuzzy set theory concept with some suitable membership functions to solve linear programming problem with several objective functions. Atanassov(Atanassov, 1986) developed the idea of intuitionistic fuzzy set, which is characterized by the membership degree as well as non-membership degree such that the sum of these two values is less than equal to one. Angelov(Angelov, 1997) gave the idea of optimization in intuitionistic fuzzy environment. In this article, he maximized the degree of acceptance of intuitionistic fuzzy objective(s) and minimized the degree of rejection of intuitionistic fuzzy objectives subject to the constraints of the problem.

Intuitionistic fuzzy sets cannot handle when indeterminate information is present in the concerned problem. In decision making theory, sometimes decision makers find it hard to decide due to presence of indeterminate information in the problem. So generalization of the concept of intuitionistic fuzzy sets was needed. So, Smarandache(Smarandache, 1999) incorporated the concept of indeterminacy by adding another independent membership function called as indeterminacy membership along with truth and falsity membership functions. Hezam et al.(Hezam, Abdel-Baset, & Smarandache, 2015) used neutrosophic theory in multi-objective linear programming problem. M. Hezam et al.(M. Hezam, Smarandache, & Abdel-Baset, 2016) introduced goal programming to neutrosophic fuzzy environment. In that paper, they established two models to solve an optimization problem. Here, they maximized truth and indeterminacy membership function and minimized the falsity membership function. Pramanik(Pramanik, 2016) also presented a neutrosophic linear goal programming problem. But instead of maximizing the indeterminacy membership function, he minimized it along with maximizing truth membership function and minimizing the falsity membership function. He also pointed out that minimizing the indeterminacy membership function is decision maker's best option. Islam and Kundu(Islam & Kundu, 2018) developed the geometric goal programming in neutrosophic environment and applied it to a Bridge Network Reliability Model. Islam and Ray(Islam & Ray, 2018) applied neutrosophic goal programming in multi-objective portfolio selection model. Rizk-Allah et al.(Rizk-Allah, Hassanien, & Elhoseny, 2018) used neutrosophic goal programming in a multi-objective transportation problem. (Abdel-Basset, Saleh, Gamal, & Smarandache, 2019) used type 2 neutrosophic number in supplier selection model. Plithogenic decision-making approach has been applied in selecting supply chain sustainability metrics in (Abdel-Basset, Mohamed, Zaied, & Smarandache, 2019).

Neutrosophic theory has been applied to internet of things (IoT) in (Abdel-Basset, Nabeeh, El-Ghareeb, & Aboelfetouh, 2019; Nabeeh, Abdel-Basset, El-Ghareeb, & Aboelfetouh, 2019). In (Abdel-Basset, El-hoseny, Gamal, & Smarandache, 2019; Abdel-Basset, Manogaran, Gamal, & Chang, 2019) neutrosophic theory has been applied in medical sciences.

As much as we know, neutrosophic goal programming has never been used before in a supplier selection problem. Also, there have not been many studies, in which quantity discounts offered by the suppliers. Our objective in this study is to give a computational algorithm for solving multi-objective supplier selection problem with quantity discount with the help of neutrosophic goal programming and neutrosophic analytical hierarchy process. The rest of the article is organized as follows: Section 2 presents some assumptions, notations and model description. Section 3 discusses some preliminaries and the neutrosophic analytical hierarchy process. Section 4 presents the fuzzified version of our model. Section 5 presents the computational algorithm. Section 6 provides a numerical example with comparison between neutrosophic goal programming approach and fuzzy goal programming approach. Finally, Section 7 gives some conclusions regarding the effectiveness of our proposed model.

2. Supplier Selection Model

A Supplier Selection Problem (SSP) is a very important problem for most of the manufacturing firms. The main goal of an SSP is to identify the supplier who has the most potential to meet the firm's demands with minimizing different costs for the firm in the process. An SSP is typically a multi-objective problem. Also, mostly it has conflicting goals. The assumptions and notations for our model are as follow:

2.1. Assumptions

- Single type of item is considered.
- Quantity discounts are offered by the suppliers.
- No shortage of the item is permitted for any supplier.

2.2. Notations

2.2.1. Index

- i: index for suppliers, $\forall$ i = 1,2, ..., n
- m(i): number of quantity ranges in supplier-i's price level
- j: index for price level for the suppliers, $\forall$ 1,2,...,m(i)
- k: index for objective functions,

2.2.2. Decision Variables

- x_{ij}:ordered quantity for the supplier-i at the price level j
- $y_{ij}: \begin{cases} 1 & \{\text{if supplier} - \text{i is selected at price level j}\} \\ 0 & \text{otherwise} \end{cases}$

2.2.3. Parameters

D: aggregate demand of the item over a fixed planning period

a_{ij}: j^{th} price level for supplier-i

p_{ij}: the unit price of the supplier-i at price level j

η_i: percentage of units delivered late by the supplier-i

ϑ_i : percentage of rejected units delivered by supplier-i

g_i: green house gas emission (GHGE) for product supplied by supplier i.

n: number of suppliers

C_i: maximum capacity of supplier-i

B_i: budget allocated to supplier-i

2.3. Model Description and Formulation:

In this article, we study the case in which a single firm buys raw materials or semi-products from n-suppliers. Suppliers sell the products at different prices and emit different amount of greenhouse gases. The suppliers may deliver some rejected items and also they may fail to deliver in time as agreed before by the both parties. The firm requires to minimize the above mentioned costs and shortcomings. Hence a multi-objective linear programming problem has been formed to find out the optimal purchasing quantity from each supplier for the firm.

A multi-objective linear programming problem(MOLP) is of the form,

Maximize $Z_k(x_i) = [Z_1(x_i), Z_2(x_i), \ldots\ldots, Z_K(x_i)]$, k=1,2,3,...,K

Minimize $Y_l(x_i) = [Y_1(x_i), Y_2(x_i), \ldots\ldots, Y_L(x_i)]$, l=1,2,...,L

subject to,

$f_m(x_i) \leq a_m$, m=1,2,...,M

$g_t(x_i) = b_t$, t=1,2,...,T

$h_o(x_i) \geq c_o$, o=1,2,...,O

$x_i \in X$, X is the solution space. Now, the multi-objective linear programming problem for this supplier selection problem (MOLP-SSP) is,

Minimize $Z_1(x_{ij}) =$
$\Sigma_{i=1}^{n}\Sigma_{j=1}^{m(i)} p_{ij}.x_{ij}$ mizeZ_1(x_ij)=?Σ_i=1^n?Σ_j=1^m(i)p_ij.x_ij (2.1)

Minimize $Z_2(x_{ij}) =$
$\Sigma_{i=1}^{n}\eta_i.\Sigma_{j=1}^{m(i)} x_{ij}$ mizeZ_2(x_ij)=?Σ_i=1^nη_i.?Σ_j=1^m(i)x_ij (2.2)

Minimize $Z_3(x_{ij}) =$
$\Sigma_{i=1}^{n}\vartheta_i.\Sigma_{j=1}^{m(i)} x_{ij}$ mizeZ_3(x_ij)=?Σ_i=1^nϑ_i.?Σ_j=1^m(i)x_ij (2.3)

Minimize $Z_4(x_{ij}) =$
$\Sigma_{i=1}^{n}g_i.\Sigma_{j=1}^{m(i)} x_{ij}$ mizeZ_4(x_ij)=?Σ_i=1^ng_i.?Σ_j=1^m(i)x_ij (2.4)

$$\Sigma_{i=1}^{n}\Sigma_{j=1}^{m(i)} x_{ij} = D, \tag{2.5}$$

$$\Sigma_{j=1}^{m(i)} x_{ij} \leq C_i, \quad \text{for } i = 1,2,\ldots,n, \tag{2.6}$$

$$y_{ij} = \begin{pmatrix} 1 \; if \; x_{ij} > 0 \\ 0 \; if \; x_{ij} = 0 \end{pmatrix}, \quad \text{for } i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i), \tag{2.7}$$

$$a_{ij-1}y_{ij-1} \leq x_{ij} < a_{ij}y_{ij}, \quad \text{for } i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i), \tag{2.8}$$

$$\Sigma_{j=1}^{m(i)} y_{ij} \leq 1, \quad \text{fori} = 1,2,\ldots,n, \tag{2.9}$$

$$\Sigma_{j=1}^{m(i)} p_{ij}.x_{ij} \leq B_i, \quad \text{fori} = 1,2,\ldots,n, \tag{2.10}$$

$$x_{ij} \geq 0, \quad i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i). \tag{2.11}$$

- Objective function (2.1) minimizes the total cost for the purchased items.
- Objective function (2.2) minimizes the net number of late delivered items from the suppliers.
- Objective function (2.3) minimizes the total number of rejected items from the suppliers.
- Objective function (2.4) minimizes the total amount of green house gas emission by the suppliers.
- The constraint (2.5) ensures that the overall demand is met for the firm.
- The constraint (2.6) puts restrictions on the capacities of the suppliers.
- The constraint (2.7) ensures the binary nature of the supplier selection decision.
- The constraint (2.8) is a quantity range constraint to meet the number of quantity ranges in a supplier's price level.
- The constraint (2.9) guarantees that at most one price level per supplier can be chosen.
- The constraint (2.10) prevents negative orders.
- The constraint (2.11) puts restrictions on the budget amount allocated to the suppliers.

In a real life problem of supplier selection, there are many elements, which can not be known properly and they create vagueness in the decision environment. This vagueness cannot be translated perfectly by a deterministic model. Therefore, the deterministic models are not suited for real life problems ((Kumar et al., 2004; Shaw et al., 2012)). For example, the predicted aggregate demand may not be accurate. So, the aggregate demand can be taken as a triangular neutrosophic number. Also, the objective functions for the firm are conflicting in nature because e.g. one supplier may charge less for the items but it may also deliver a lot of rejected/unusable items. So, the firm will want to find a compromise solution. Hence neutrosophic goal programming has been used in this study to find out the optimal trade-off for the firm.

3. Preliminaries

3.1. *Some Definitions*

Definition 3.1.1 (Fuzzy sets): *As in* (Zadeh, 1965) *, a fuzzy set* $\tilde{A}$ *in a universe of discourse X is defined as the ordered pairs* $\tilde{A} = \{(x, M_{\tilde{A}}(x)) : x \in X\}$ *where* $M_{\tilde{A}}: X \to [0,1]$ *is a function known as the membership function of the set* $\tilde{A}$*.* $M_{\tilde{A}}(x)$ *is the degree of membership of* $x \in X$ *in the fuzzy set* $\tilde{A}$*. Higher value of* $M_{\tilde{A}}(x)$ *indicates a higher degree of membership in* $\tilde{A}$*.*

Definition 3.1.2. (Neutrosophic sets): *As in* (Smarandache, 1999)*, let X be a universe of discourse and let* $x \in X$*. A neutrosophic set A in X is characterized by a truth-membership function* $T_A(x)$*, an indeterminacy-membership function* $I_A(x)$ *, and a falsity- membership function* $F_A(x)$ *, where* $T_A(x), I_A(x), F_A(x) \in (0,1), \forall x \in X$ *and* $0^+ \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^-$*.*

Definition 3.1.3. (Single valued neutrosophic sets): *According to* (Haibin, Smarandache, Zhang, & Sunderraman, 2010)*, if X is a universe of discourse and if* $x \in X$*, a single valued neutrosophic set A is characterized by a truth-membership function* $T_A(x)$*, an indeterminacy-membership function* $I_A(x)$*, and a falsity- membership function* $F_A(x)$ *, where* $T_A(x), I_A(x), F_A(x) \in [0,1], \forall x \in X$ *and* $0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3$pT_A(x)+supI_A(x)+supF_A(x)≤3.

Definition 3.1.4. (Intersection of two Single valued neutrosophic number): *As in* (Salama & Alblowi, 2012) *, the intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as* $C = A \cap B$B *its truth, indeterminacy and falsity membership functions are given by,*

$$T_C(x) = min(T_A(x), T_B(x)), \tag{3.1}$$

$$I_C(x) = max(I_A(x), I_B(x)), \tag{3.2}$$

$$F_C(x) = max(F_A(x), F_B(x)) \tag{3.3}$$

for all x in X.

Definition 3.1.5. (Triangular neutrosophic numbers) *As in* (Abdel-Basset, Mohamed, Zhou, & M. Hezam, 2017)*, a triangular neutrosophic number is a special kind of neutrosophic set on the real number set* $\mathbb{R}$ *denoted as* $\tilde{a} =< (a_1, b_1, c_1); \widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} >$*,where* $\widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} \in [0,1]$*. The truth-membership, indeterminacy-membership and falsity-membership functions are defined as follows:*

$$T_{\tilde{a}}(x) = \begin{cases} \frac{(x-a_1)\widetilde{\delta_a}}{b_1-a_1}, & if \;\; a_1 \leq x \leq b_1 \\ \widetilde{\delta_a}, & if \;\; x = b_1 \\ \frac{(c_1-x)\widetilde{\delta_a}}{(c_1-b_1)}, & if \;\; b_1 < x \leq c_1 \\ 0 \;\; , otherwise & \end{cases} \tag{3.4}$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{b_1 - x + \widetilde{\theta_a}(x - a_1)}{b_1 - a_1} \ , & if \ \ a_1 \leq x \leq b_1 \\ \widetilde{\theta_a} \ , & if \ \ x = b_1 \\ \frac{x - b_1 + \widetilde{\theta}_a(c_1 - x)}{c_1 - b_1} \ , & if \ \ b_1 < x \leq c_1 \\ 1 \ , otherwise & \end{cases} \tag{3.5}$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{b_1 - x + \widetilde{\lambda_a}(x - a_1)}{b_1 - a_1} \ , & if \ \ a_1 \leq x \leq b_1 \\ \widetilde{\lambda_a} \ , & if \ \ x = b_1 \\ \frac{x - b_1 + \widetilde{\lambda}_a(c_1 - x)}{c_1 - b_1} \ , & if \ \ b_1 < x \leq c_1 \\ 1 \ , otherwise & \end{cases} \tag{3.6}$$

where $\delta_a, \theta_a, \lambda_a$ are the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree respectively.

3.2. *Neutrosophic Goal Programming Technique*

A minimizing type multi-objective linear programming is of the form,

$$\begin{aligned} &min\ [Z_1(x), Z_2(x), \ldots, Z_K(x)] \\ &g_t(x) \leq b_t, \text{t} = 1,2, \ldots, \text{T} \end{aligned} \tag{3.7}$$

Let, the fuzzy goal for each objective function be denoted as G_k for all k=1,2,...,K and the fuzzy constraints be denoted as C_t for all t=1,2,...,T. Then, the neutrosophic decision set D^N, which is a conjunction of neutrosophic objectives and constraints, is defined by,

$$D^N = (\cap_1^K G_K)(\cap_1^T C_T) = (x, T_{D^n}, I_{D^n}, F_{D^n}) \tag{3.8}$$

$$T_{D^n} = min(T_{G_1}(x), T_{G_2}(x), \ldots, T_{C_k}(x); T_{C_1}(x), T_{C_2}(x), \ldots, T_{C_k}(x)), \forall x \in X \tag{3.9}$$

$$I_{D^n} = max(I_{G_1}(x), I_{G_2}(x), \ldots, I_{C_k}(x); I_{C_1}(x), I_{C_2}(x), \ldots, I_{C_k}(x)), \forall x \in X \tag{3.10}$$

$$F_{D^n} = max(F_{G_1}(x), F_{G_2}(x), \ldots, F_{C_k}(x); F_{C_1}(x), F_{C_2}(x), \ldots, F_{C_k}(x)), \forall x \in X \tag{3.11}$$

, where $T_{D^n}, I_{D^n}, F_{D^n}$ are truth, indeterminacy and falsity membership function of the neutrosophic decision set D^N respectively. Now the transformed linear programming problem of the problem in eq. (3.7) can be written as the following crisp programming problem,

$$\begin{aligned} &\text{min } \ (1 - \alpha) + \ \ \gamma + \beta \\ &\text{subject to,} \\ &\quad T_{D^n}(X) \quad \geq \alpha \\ &\quad I_{D^n}(x) \quad \leq \gamma \\ &\quad F_{D^n}(X) \quad \leq \beta \\ &\quad 0 \leq \alpha + \quad \beta + \gamma \leq 3 \\ &\quad \alpha \quad \geq \beta \\ &\quad \alpha \quad \geq \gamma \\ &\quad \alpha, \beta, \gamma \quad \in [0,1] \end{aligned} \tag{3.12}$$

3.3. *Neutrosophic Analytical Hierarchy Process*

The analytical hierarchy process was first introduced by Saaty(Saaty, 1980). The process has been applied to a wide variety of decision making problems. It also gives a structured method for determining the weights of criteria. The Neutrosophic Analytical Hierarchy Process(NAHP) was introduced by Abdel-Basset et al.(Abdel-Basset et al., 2017) The process of calculating weight criteria by means of NAHP is described below briefly:

- A pairwise comparison matrix based on relative importance of each criterion is formed. If A=($\widetilde{a_{ij}}$) represents the matrix then, $\tilde{a}ij$ is a neutrosophic triangular number.

• We take $\widetilde{a_{ij}} = \tilde{1}$ if i and j are equally important, $\widetilde{a_{ij}} = \tilde{3}$ if i is moderately important than j, $\widetilde{a_{ij}} = \tilde{5}$ if i is strongly important than j, $\widetilde{a_{ij}} = \tilde{7}$ if i is very strongly important than j, $\widetilde{a_{ij}} = \tilde{9}$ if i is extremely important than j. We may also take $\tilde{a} = \tilde{2}, \tilde{4}, \tilde{6}\ or\ \tilde{8}$ for different importance.

• Next, the neutrosophic pair-wise comparison matrix is transformed into a deterministic pair-wise comparison matrix, using the following equations: if $\tilde{a} =< (a_1, b_1, c_1); \widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} >$ be a single valued triangular neutrosophic number then

$$\begin{aligned} s_{ij} &= \frac{(a_1+b_1+c_1)(2+\widetilde{\delta_a}-\widetilde{\theta_a}-\widetilde{\lambda_a})}{16} \\ \widetilde{a_{ij}} &= s_{ij} \\ \widetilde{a_{ji}} &= \frac{1}{s_{ij}} \end{aligned} \tag{3.13}$$

• After forming the deterministic matrix, each column entries are normalized by dividing each entry by column sum.

• Then, we average each row to get the required weights(w_l).

• Finally, we check the consistency of the comparison matrix with the help of consistency index (CI) and consistency ratio (CR) ((Abdel-Basset et al., 2017; Saaty, 1980)):

$$\begin{aligned} CI &= \frac{\lambda_{max}-n}{n-1} \\ CR &= \frac{CI}{RI} \end{aligned} \tag{3.14}$$

where n is the number of items being compared, and RI is the consistency index of a randomly generated pair-wise comparison matrix of similar size *(Saaty, 1980)*. If CR<0.1, the comparison matrix is consistent.

4. Fuzzy Supplier Selection Model

In this model, the decision maker/ firm tries to achieve a certain goal for each objective function. The goals are a fuzzy in nature. As well as, we assumed in this study demand cannot be known precisely. So, the aggregate demand is also fuzzy in nature. After fuzzification, the eqs. (2.1) to (2.11) can be represented as follows:

Find x_{ij} to satisfy,

$$\begin{aligned} Z_k(x_{ij}) &\cong \widetilde{Z_k} && \text{for } k = 1,2,3,4 \\ \Sigma_{i=1}^{n}\Sigma_{j=1}^{m(i)} x_{ij} &\cong \widetilde{D}, \\ \Sigma_{j=1}^{m(i)} x_{ij} &\leq C_i, && \text{for } i = 1,2,\ldots,n, \\ y_{ij} &= \begin{cases} 1 & if\ x_{ij} > 0 \\ 0 & if\ x_{ij} = 0 \end{cases}, && \text{for } i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i), \\ a_{ij-1}y_{ij-1} &\leq x_{ij} < a_{ij}y_{ij}, && \text{for } i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i), \\ \Sigma_{j=1}^{m(i)} y_{ij} &\leq 1, && \text{fori} = 1,2,\ldots,n, \\ \Sigma_{j=1}^{m(i)} p_{ij}.x_{ij} &\leq B_i. \\ x_{ij} &\geq 0, && i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i). \end{aligned} \tag{4.1}$$

where $\widetilde{Z_k}$ is the aspiration level for each objective and $\widetilde{D}$ is the fuzzified demand. Hence, the aggregate demand can be taken as fuzzy triangular number or triangular neutrosophic number.

5. Computational Algorithm

In this study, NAHP and neutrosophic goal programming approach has been used to solve the problem. The solution steps to solve this model are as follows:

Step 1: Firstly, identification of supplier selection criteria with multi-supplier quantity discounts is done.

Step 2: A panel of experts in the fields of supply chain and operations is formed. To get the weights(w_l) for different criteria they are asked to fill a nine-point-scale questionnaire to form the pairwise comparison matrix using eq. (3.13). Then, consistency property of each expert's comparison results must be checked using eq. (3.14). If it is not consistent they are ask to fill the questionnaire again. They are also asked to approximate the market demand and how much it may fluctuate.

Step 3: Objective functions for the Supplier selection model are formed. These objective functions are purchasing cost, total amount of rejected items, total amount of late deliveries and the total amount of green- house gas emitted by the suppliers.

Step 4: Each objective is solved dismissing the other objective functions subject to the constrains and using the approximate demand as predicted by the experts in step 2. Using the values of all objective function at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{pmatrix} Z_1(x_{ij}^1) & Z_2(x_{ij}^1) & Z_3(x_{ij}^1) & Z_4(x_{ij}^1) \\ Z_1(x_{ij}^2) & Z_2(x_{ij}^2) & Z_3(x_{ij}^2) & Z_4(x_{ij}^2) \\ Z_1(x_{ij}^3) & Z_2(x_{ij}^3) & Z_3(x_{ij}^3) & Z_4(x_{ij}^3) \\ Z_1(x_{ij}^4) & Z_2(x_{ij}^4) & Z_3(x_{ij}^4) & Z_4(x_{ij}^4) \end{pmatrix}, \text{where } x_{ij}^k \text{ for } k = 1,2,3,4 \text{ is the ideal solution for } Z_k$$

Step 5: For each objective function Z_k the lower bound L_k, which is the aspiration level ($\widetilde{Z_k}$) and the upper bound U_k are formed as: $L_k = \widetilde{Z_k} = min_k(Z_k(x_{ij}^k))$ and $U_k = max_k(Z_k(x_{ij}^k))$ for k=1,2,3,4.

Step 6: The bounds for the neutrosophic environment can be calculated as follows:

$U_k^T = U_k, L_k^T = L_k$, for truth membership function (5.1)

$U_k^I = U_k, L_k^I = L_k + s_k(U_k - L_k)$, for indeterminacy membership function (5.2)

$U_k^F = U_k, L_k^F = L_k + t_k(U_k - L_k)$, for falsity membership function (5.3)

, where $s_k, t_k \in (0,1)$.

Step 7: For the objective functions the truth, indeterminacy and falsity membership functions are formed as follow:

$$T_k(Z_k(x_{ij})) = \begin{cases} 1 & \text{, if } Z_k(x_{ij}) \le L_k^T \\ \frac{U_k^T - Z_k(x_{ij})}{U_k^T - L_k^T} & \text{, if } L_k^T \le Z_k(x_{ij}) \le U_k^T \\ 0 & \text{, if } Z_k(x_{ij}) \ge U_k^T \end{cases} \tag{5.4}$$

$$I_k(Z_k(x_{ij})) = \begin{cases} 0 & \text{, if } Z_k(x_{ij}) \le L_k^I \\ \frac{Z_k(x_{ij}) - L_k^I}{U_k^I - L_k^I} & \text{, if } L_k^I \le Z_k(x_{ij}) \le U_k^I \\ 1 & \text{, if } Z_k(x_{ij}) \ge U_k^I \end{cases} \tag{5.5}$$

$$F_k(Z_k(x_{ij})) = \begin{cases} 0 & \text{, if } Z_k(x_{ij}) \le L_k^F \\ \frac{Z_k(x_{ij}) - L_k^F}{U_k^F - L_k^F} & \text{, if } L_k^F \le Z_k(x_{ij}) \le U_k^F \\ 1 & \text{, if } Z_k(x_{ij}) \ge U_k^F \end{cases} \tag{5.6}$$

Step 8: Using the information in Step 2, a neutrosophic triangular number is formed for the aggregate demand as: $\tilde{D} =< (D_1, D_2, D_3); \widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} >, where\ \widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} \in [0,1]$ and the values of D_1, D_2, D_3 are given by the experts. The truth, indeterminacy and falsity membership functions are denoted by $T_{\tilde{D}}(D), I_{\tilde{D}}(D)$ and $F_{\tilde{D}}(D)$ respectively and can be calculated using equations (3.4)-(3.6).

Step 9: Now modifying the neutrosophic goal programming technique which was described in section 3.2, the problem in eq. (4.1) can be written as the following crisp programming problem,

$$min\ \Sigma_{l=1}^{5} w_l((1 - \alpha_l) + (\gamma_l) + \beta_l)$$?Σ_l=1^5w_l((1-α_l)+(γ_l)+β_l)

subject to,

$$
\begin{array}{llllll}
T_k(Z_k(x_{ij})) & \geq \alpha_k, & \Sigma_{j=1}^{m(i)} x_{ij} & \leq C_i, & & \\
I_k(Z_k(x_{ij})) & \leq \gamma_k, & y_{ij} & = \begin{pmatrix} 1 & if & x_{ij} > 0 \\ 0 & if & x_{ij} = 0 \end{pmatrix}, & & \\
F_k(Z_k(x_{ij})) & \leq \beta_k, & a_{ij-1} y_{ij-1} & \leq x_{ij} < a_{ij} y_{ij}, & & \\
T_{\tilde{D}}(D) & \geq \alpha_5, & \Sigma_{j=1}^{m(i)} y_{ij} & \leq 1, & & (5.7) \\
I_{\tilde{D}}(D) & \leq \gamma_5, & x_{ij} & \geq 0, & & \\
F_{\tilde{D}}(D) & \leq \beta_5, & \Sigma_{j=1}^{m(i)} p_{ij} . x_{ij} & \leq B_i, & & \\
0 \leq \alpha_l + & \beta_l + \gamma_l \leq 3, & \alpha_l & \geq \gamma_l, & & \\
\alpha_l & \geq \beta_l, & \alpha_l, \beta_l, \gamma_l & \in [0,1] & &
\end{array}
$$

,for all i=1,2,...,n, j=1,2,...,m(i), k=1,2,3,4,l=1,2,3,4,5.

Step 10: Finally, use LINGO software to get the results.

6. Numerical Example

The following example shows the usefulness of the proposed model. Here, considering the same weights for the objectives, we have done a comparative study between Fuzzy Goal Programming(FGP) approach and Neutrosophic Goal Programming (NGP) approach for our model. The weights have been calculated by using NAHP. Here Six suppliers have been considered in the evaluation process. Most of the data used in this example have been derived from the articles (Wang & Yang, 2009; Weber & Desai, 1996). A panel of experts (as in Step 2 of section5) will predict the aggregate demand and how much it will fluctuate as oppose to in those above studies where the aggregate demand has been taken as a fixed number. The data which is given by those experts will be used to calculate the triangular neutrosophic number and fuzzy triangular number for the aggregate demand. Moreover, there is no consideration of greenhouse gas emission for the suppliers in those studies. We assumed the amount of greenhouse gas emission for the suppliers for the example.

Table 1: supplier quantity discounts.

Supplier-i	a_{i0}	p_{i1}	a_{i1}(K)	p_{i2}	a_{i2}(K)	p_{i3}	a_{i3}(M)	p_{i4}
1	0	0.2020	50	0.1990	100	0.1980	1	0.1958
2	0	0.1900	10	0.1890	200	0.1881	-	-
3	0	0.2350	10	0.2300	100	0.2250	1	0.2204
4	0	0.2200	20	0.2150	500	0.2100	2	0.2081
5	0	0.2250	50	0.2200	500	0.2150	1	0.2118
6	0	0.2200	10	0.2170	500	0.2140	1	0.2096

Table 2: supplier source data.

	suppliers					
	1	2	3	4	5	6
Rejection rate(%)	1.2	0.8	0.0	2.1	2.3	1.2
Late delivery rate(%)	5.0	7.0	0.0	0.0	3.0	4.0
GHGE(kg)	0.1	0.2	0.25	0.15	0.3	0.1
Capacity(C_i)	2.4 M	360 K	2.783 M	3.0 M	2.966 M	2.5 M
Budget constraint(B_i)($)	600000	100000	650000	500000	500000	300000

Table 3: Comparison matrix

	Cost	**Lead time**	**Quality**	**GHGE**	**Demand**
Cost	$\tilde{1}$	$\tilde{2}$	$\tilde{3}^{-1}$	$\tilde{6}^{-1}$	$\tilde{5}^{-1}$
Lead time	$\tilde{2}^{-1}$	$\tilde{1}$	$\tilde{5}^{-1}$	$\tilde{8}^{-1}$	$\tilde{1}$
Quality	$\tilde{3}$	$\tilde{5}$	$\tilde{1}$	$\tilde{3}^{-1}$	$\tilde{2}^{-1}$
GHGE	$\tilde{6}$	$\tilde{8}$	$\tilde{3}$	$\tilde{1}$	$\tilde{3}^{-1}$
Demand	$\tilde{5}$	$\tilde{1}$	$\tilde{2}$	$\tilde{3}$	$\tilde{1}$

The suppliers provide quantity discounts with the anticipation that the firm will increase order quantity in each order, thereby reducing the supplier's order processing cost. The data for quantity discounts are given in table 1. The data for other parameters are given in table 2. The comparison matrix for the criteria given in table 3.

The objective functions are,

$$\begin{aligned}
Z_1 = \;& 0.202x_{11} + 0.199x_{12} + 0.198x_{13} + 0.1958x_{14} + 0.19x_{21} + 0.189x_{22} + 0.1881x_{23} + 0.235x_{31} + \\
& 0.23x_{32} + 0.225x_{33} + 0.2204x_{34} + 0.22x_{41} + 0.215x_{42} + 0.21x_{43} + 0.2081x_{44} + 0.225x_{51} + \\
& 0.22x_{52} + 0.215x_{53} + 0.2118x_{54} + 0.22x_{61} + 0.217x_{62} + 0.214x_{63} + 0.2096x_{64} \\
Z_2 = \;& 0.05(x_{11} + x_{12} + x_{13} + x_{14}) + 0.07(x_{21} + x_{22} + x_{23}) + \\
& 0.03(x_{51} + x_{52} + x_{53} + x_{54}) + 0.04(x_{61} + x_{62} + x_{63} + x_{64}) \qquad (6.1) \\
Z_3 = \;& 0.012(x_{11} + x_{12} + x_{13} + x_{14}) + 0.008(x_{21} + x_{22} + x_{23}) + 0.021(x_{41} + x_{42} + x_{43} + x_{44}) + \\
& 0.023(x_{51} + x_{52} + x_{53} + x_{54}) + 0.012(x_{61} + x_{62} + x_{63} + x_{64}) \\
Z_4 = \;& 0.1(x_{11} + x_{12} + x_{13} + x_{14}) + 0.2(x_{21} + x_{22} + x_{23}) + 0.25(x_{31} + x_{32} + x_{33} + x_{34}) + \\
& 0.15(x_{41} + x_{42} + x_{43} + x_{44}) + 0.3(x_{51} + x_{52} + x_{53} + x_{54}) + 0.1(x_{61} + x_{62} + x_{63} + x_{64})
\end{aligned}$$

Subject to the constraints,

$$\begin{aligned}
& x11 + x12 + x13 + x14 \le 2400K, \quad x21 + x22 + x23 \le 360K \quad x31 + x32 + x33 + x34 \le 2783K \\
& x41 + x42 + x43 + x44 \le 3000K, \quad x51 + x52 + x53 + x54 \le 2966K, \quad x61 + x62 + x63 + x64 \le 2500K
\end{aligned}$$

(6.2)

$$y_{ij} = \begin{cases} 1 \ if \ x_{ij} > 0 \\ 0 \ if \ x_{ij} = 0 \end{cases}, \quad \Sigma_{j=1}^{m(i)} y_{ij} \le 1, \quad 0 \le x_{11} < 50000y_{11},$$

$$\begin{array}{llllll} 50000y_{11} \le x_{12} & < 100000y_{12} & 100000y_{12} \le x_{13} & < 1000000y_{13}, & x_{14} \ge & 1000000y_{14}, \\ 0 \le x_{21} & < 10000y_{21}, & 10000y_{21} \le x_{22} & < 200000y_{22}, & x_{23} \ge & 200000y_{23}, \\ 0 \le x_{31} & < 10000y_{31}, & 10000y_{31} \le x_{32} & < 100000y_{32}, & 100000y_{32} \le x_{33} < & 1000000y_{33}, \\ x_{34} & \ge 1000000y_{34}, & 0 \le x_{41} & < 20000y_{41}, & 20000y_{41} \le x_{42} < & 500000y_{42}, \\ 500000y_{42} \le x_{43} & < 2000000y_{43}, & x_{44} & \ge 2000000y_{44}, & 0 \le x_{51} < & 50000y_{51}, \\ 50000y_{51} \le x_{52} & < 500000y_{52}, & 500000y_{52} \le x_{53} & < 1000000y_{53}, & x_{54} \ge & 1000000y_{54}, \\ 0 \le x_{61} & < 10000y_{61}, & 10000y_{61} \le x_{62} & < 500000y_{62}, & 500000y_{62} \le x_{63} < & 1000000y_{63}, \\ x_{64} & \ge 1000000y_{64}, & x_{ij} \ge & 0. & & \end{array} \quad (6.3)$$

$$\begin{array}{rl} 0.202x_{11} + 0.199x_{12} + 0.198x_{13} + 0.1958x_{14} \le & 600000 \\ 0.19x_{21} + 0.189x_{22} + 0.1881x_{23} \le & 100000 \\ 0.235x_{31} + 0.23x_{32} + 0.225x_{33} + 0.2204x_{34} \le & 650000 \\ 0.22x_{41} + 0.215x_{42} + 0.21x_{43} + 0.2081x_{44} \le & 500000 \\ 0.225x_{51} + 0.22x_{52} + 0.215x_{53} + 0.2118x_{54} \le & 500000 \\ 0.22x_{61} + 0.217x_{62} + 0.214x_{63} + 0.2096x_{64} \le & 300000 \end{array} \quad (6.4)$$

$$\begin{array}{rl} D = & x_{11} + x_{12} + x_{13} + x_{14} + x_{21} + x_{22} + x_{23} + x_{31} + x_{32} + x_{33} + x_{34} + \\ & x_{41} + x_{42} + x_{43} + x_{44} + x_{51} + x_{52} + x_{53} + x_{54} + x_{61} + x_{62} + x_{63} + x_{64}. \end{array} \quad (6.5)$$

To find the weights for different objective functions we have taken $\tilde{1}$=<(0.6,1,5);(0.9,0.2,0.3)>, $\tilde{2}$=<(1,2,6);(0.8,0.4,0.2)>, $\tilde{3}$=<(0,3,9)(0.6,0.3,0.2)>, $\tilde{5}$=<(2,5,10);(0.6,0.3,0.2)>,$\tilde{6}$=<(2,6,9);(0.7,0.5,0.1)>, $\tilde{8}$=<(3,8,11);(0.7,0.5,0.1)>. From the discussions in section 3.3, we have the following weights: $w_1 = 0.126469, w_2 = 0.131538, w_3 = 0.207651, w_4 = 0.272911, w_5 = 0.26143$. For these set of weights we get CI=0.0540024. RI equal to 1.12 for five criteria, which is derived from (Saaty, Vargas, & others, 2006). So, we have CR=.0482164<0.1 and hence the consistency property holds. We calculate the aspiration levels for each objective function, dismissing other objective functions. From eqs. (5.1) to (5.3) for $s_k = .3, t_k = .2, \forall k = 1,2,3,4,$ we can calculate the bounds for truth, indeterminacy and falsity membership functions. The results are given in table 4. Here, the aggregate demand is taken as fuzzy triangular number for the FGP approach and triangular neutrosophic number for the NGP approach. We are Using LINGO to get the results which are given in table 5 and table 6.

Table 4: Bounds of each objective function, dismissing other objectives.

	$\mathbf{Z_1}$	$\mathbf{Z_2}$	$\mathbf{Z_3}$	$\mathbf{Z_4}$
$\mathbf{L_k = L_k^T}$	2221790	170620	119367	1644500
$\mathbf{U_k = U_k^T}$	2293665.6	321100	182870	2239650
$\mathbf{L_k^I}$	2243352.68	215764	138417.9	1823045
$\mathbf{U_k^I}$	2293665.6	321100	182870	2239650
$\mathbf{L_k^F}$	2236165.12	200716	132067.6	1763530
$\mathbf{U_k^F}$	2293665.6	321100	182870	2239650

For the FGP approach the demand is predicted to be 10900000 and assumed to vary between 10500000 and 12000000. The FGP approach can be written as (Similarly as (Shaw et al., 2012; Wang & Yang, 2009)),

$$\max \ \Sigma_{l=1}^{5} w_l \lambda_l$$
subject to,
$$\begin{aligned}
&\frac{2293665.6-Z_1}{2293665.6-2221790} \geq \lambda_1,\\
&\frac{321100-Z_2}{321100-170620} \geq \lambda_2,\\
&\frac{182870-Z_3}{182870-119367} \geq \lambda_3,\\
&\frac{2239650-Z_4}{2239650-1644500} \geq \lambda_4,\\
&\frac{12000000-D}{1100000} \geq \lambda_5,\\
&\frac{D-10500000}{400000} \geq \lambda_5,
\end{aligned} \tag{6.6}$$

where Z_1, Z_2, Z_3, Z_4, D are given in eqs. (6.1) and (6.5), along with the constraints in eqs. (6.2) to (6.4).

For the NGP approach, we take $D_1 = 10500000, D_2 = 10900000, D_3 = 12000000, \delta_D = .99, \theta_D = .3, \lambda_D = .01$. One can calculate easily the truth, indeterminacy, falsity membership functions for $\widetilde{D}$ and the objective functions using eqs. (3.4), (3.5), (3.6) and (5.1), (5.2), (5.3) and table 4 respectively. The NGP approach is given as follow (5.7):

$$\min \ \Sigma_{l=1}^{5} w_l((1-\alpha_l)+(\gamma_l)+\beta_l)$$
subject to the constrains,
$$\begin{array}{llllllll}
\frac{2293665.6-Z_1}{71875.6} & \geq \alpha_1 & \frac{Z_1-2243352.68}{50312.9} & \leq \gamma_1 & \frac{Z_1-2236165.12}{57500.5} & \leq \beta_1 \\
\frac{321100-Z_2}{150480} & \geq \alpha_2 & \frac{Z_2-215764}{105336.} & \leq \gamma_2 & \frac{Z_2-200716}{120384} & \leq \beta_2 \\
\frac{182870-Z_3}{63503} & \geq \alpha_3 & \frac{Z_3-138417.9}{44452.1} & \leq \gamma_3 & \frac{Z_3-132067.6}{50802.4} & \leq \beta_3 \\
\frac{2239650-Z4}{595150} & \geq \alpha_4 & \frac{Z_4-1823045}{416605} & \leq \gamma_4 & \frac{Z_4-1763530}{476120} & \leq \beta_4 \\
\frac{(D-10500000).99}{400000} & \geq \alpha_5 & \frac{(12000000-D).99}{1100000} & \geq \alpha_5 & \frac{7750000-0.7D}{400000} & \leq \gamma_5 \\
\frac{0.7D-7300000}{1100000} & \leq \gamma_5 & \frac{9850000-0.9D}{400000} & \leq \beta_5 & \frac{0.9D-9700000}{1100000} & \leq \beta_5
\end{array} \tag{6.7}$$

where Z_1, Z_2, Z_3, Z_4, D are given in eqs. (6.1) and (6.5), along with the constraints in eqs. (6.2) to (6.4).

Table 5:

	Z_1	Z_2	Z_3	Z_4
FGP approach (6.6)	2273582.988	248142.2467	134341.3432	1968186.806
NGP approach(with weights(6.7))	2243352.680	243860.3333	131058.5429	1925367.672
NGP approach(without weights (3.12)	2258260.159	245971.8743	132677.3910	1946483.082

Table 6:

	x_1	x_2	x_3	x_4	x_5	x_6
FGP approach (6.6)	2400000	360000	2783000	2402691	**1523011**	1431297
NGP approach(with weights(6.7))	2400000	360000	2783000	2402691	**1380280**	1431297
NGP approach(without weights (3.12)	2400000	360000	2783000	2402691	**1450665**	1431297

Table 7:

Weights	$\mathbf{Z_1}$	$\mathbf{Z_2}$	$\mathbf{Z_3}$	$\mathbf{Z_4}$
$w_1 = 0.1, w_2 = 0.3, w_3 = 0.2, w_4 = 0.2, w_5 = 0.2$	2236165.120	227233.7668	134751.5086	1939102.007
$w_1 = 0.15, w_2 = 0.25, w_3 = 0.1, w_4 = 0.2, w_5 = 0.3$	2243352.680	243860.3333	131058.5429	1925367.672
$w_1 = 0.1, w_2 = 0.1, w_3 = 0.1, w_4 = 0.3, w_5 = 0.4$	2273582.988	248142.2467	134341.3432	1968186.806

As it can be seen in table 5, the NGP approach (with weights) yields the best result among other methods for each objective function for the chosen weights. Finally, we provide the results of the proposed NGP approach for different weights. The results are given in table 7.

7. Conclusion

On its own, a supplier selection problem in a quantity discount environment is a very complicated task. Also, there may exist vagueness and imprecision in the goals of the decision maker and market demand. To approximate the imprecise aggregate demand, we have used the triangular neutrosophic numbers and to deal with the vagueness we have used neutrosophic goal programming. The proposed generalized models can deal with imprecise market demand as well as the vagueness present in the goals of the decision maker. As oppose to the studies that already exist, our study also includes the case where the decision maker cannot decide about the goals with certainty, by including indeterminacy membership function. As shown in the numerical example, neutrosophic goal programming method yield better value for the objective functions than the fuzzy goal programming method for the given weights.

This study has been done assuming that no shortages are allowed. We also assumed that a single type of item is being supplied.

The proposed model can be expanded if we assume shortages are allowed as well as multi-item are consided . The proposed model can be solved using particle swarm optimization.

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Neutrosophic Intelligent Energy Efficient Routing for Wireless Ad-hoc Network Based on Multi-criteria Decision Making

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Abstract: A wireless ad-hoc network is a decentralized ad-hoc network which has no access point earlier time. In this network, data from every node is transferred to another node dynamically based on network connectivity and existing routing algorithm. Many authors introduced various routing techniques to handle the issues in wireless ad-hoc networks. The main concept of this paper is to develop a new network design to improve the service of wireless ad-hoc network by equipping the routes energy efficient using neutrosophic technique. Multi-criteria decision making method under neutrosophic environment is used for making the routes of the network efficiently here. Since neutrosophic set is the generalization of fuzzy and intuitionistic fuzzy sets, the parameters involved in this method like hop-count, data packets, distance and energy are taken from neutrosophic sets. Mathematical analysis for the proposed network design is carried out and results are also discussed here.

Keywords: Neutrosophic set; WANET; Multi-criteria; Neutrosophic energy function; Neutrosophic distance function.

1. Introduction

Ad-hoc is a communication setting that allows computers to communicate with each other directly without a route. Ad-hoc networks play an important role in emergency situations like military conflicts, natural disasters etc., because of its minimal configuration and quick deployment. Ad-hoc networks are analyzed by various features like uncertain connectivity changes; erratic wireless medium etc., According to these features, ad-hoc networks creates numerous types of failures including failure of nodes and links, data transmission errors, congestions and route breakages.

WANET is a self-configured network which can be shared to various devices like sensors, laptops, personal communication systems for weather conditions, airlines schedules etc.[20]WANET has no established infrastructure in advance. Nodes in wanet are dynamic and easily movable. Since wanet is a decentralized one, it helps to improve the network system more efficient than wireless controlled networks [5, 7, 8, 9].Due to lack of energy and physical damages, some nodes of this network will not be able to use and the total system will be affected. In such situations, the lifetime of wanet is reduced. So many authors in [10, 12] established different types of protocols for improving the lifetime of wanet by considering data packets, hop count, energy and distance parameters. The present network design focused on introducing neutrosophic logic for analyzing intelligent energy efficient routing for wanet based on multicriteria decision making and the analysis of the proposed method is compared with one of the existing methods to validate the results.

Neutrosophic set was introduced by Florentin Smarandache [22] which is the generalization of fuzzy set, intuitionistic set fuzzy set, classical set and paraconsistent set etc., In intuitionistic fuzzy sets, the uncertainty is dependent on the degree of belongingness and degree of non-belongingness. In case of neutrosophy theory, the indeterminacy factor is independent of truth and falsity membership-values. Also neutrosophic sets are more general than IFS, because there are no conditions between the degree of truth, degree of indeterminacy and degree of falsity. Multi-criteria decision making in neutrosophic sets are developed in the book [23] edited by Florentin Smarandache and Surapati Pramanik in 2016 and Faruk Karaaslan introduced Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making in[11]. Also many authors discussed about multi-criteria decision making in neutrosophic sets and its applications in [14,15,16,17,18,19,24].Decision analysis and expert system was developed in[5,13] and various types of shortest route algorithms in neutrosophic environment are established in [1,2,3,4].

The main concept of this paper is to develop a new network design to improve the lifetime of wireless ad-hoc network by equipping the routes energy efficient using neutrosophic technique. Multicriteria decision making method under neutrosophic environment is used for making the routes of the network efficiently here. The parameters involved in this method like hop-count, data packets, distance and energy are taken from neutrosophic sets. Using this method, we can reduce the energy consumption and route breakages due to high level data packet transmission and maximum hop count. The neutrosophic technique is implemented here will give better energy efficient routes for WANET. The rest of the paper is organized as follows: Section 2 provides preliminaries about each of the set theories. Section 3 describes proposed network design with neutrosophic rule matrix and section 4 gives conclusions and future research.

2. Preliminaries

This section includes some basic definitions that are very useful to the proposed network model.

Definition 2.1[22]:

Let E be a universe. Then a fuzzy set X over E is a function defined as follows: $X = (\mu_x(x)/x): x \in E$, where $\mu_x: E \to [0.1]$. Here, μ_x is called membership function of X, and the value $\mu_x(x)$ is called the grade of membership om $x \in E$. The value represents the degree of x belonging to the fuzzy set X.

Several authors [1, 2, 9-12] used fuzzy set theory in ad-hoc network and wireless sensor network to solve routing problems. The logic in fuzzy set theory is vastly used in all fields of mathematics like networks, graphs, topological space etc.

Definition 2.2[20]:

Intuitionistic Fuzzy Sets are the extension of usual fuzzy sets. All outcomes which are applicable for fuzzy sets can be derived here also. Almost all the research works for fuzzy sets can be used to draw information of IFSs. Further, there have been defined over IFSs not only operations similar to those of ordinary fuzzy sets, but also operators that cannot be defined in the case of ordinary fuzzy sets.

Definition 2.3[20]:

Adroit system [3,4] is a computer program that efforts to act like a human effect in a particular subject area to give the solution to the particular unpredictable problem. Sometimes, adroit systems are used instead of human minds. Its main parts are knowledge based system and inference engine. In that the software is the knowledge based system which can be solved by artificial intelligence technique to find efficient route. The second part is inference engine which processes data by using rule based knowledge.

Definition 2.4[20]:

Let E be a universe. A neutrosophic sets A in E is characterized by a truth-membership function $\mathrm{T_A}$, a indeterminacy-membership function $\mathrm{I_A}$ and a falsity-membership function $\mathrm{F_A}$. $\mathrm{T_A(x)}$; $\mathrm{I_A(x)}$ and $\mathrm{F_A(x)}$ are real standard elements of [0,1]. It can be written as

$$\mathrm{A} = \{< x, (\mathrm{T_A(x), I_A(x), F_A(x)}) >: x \in E, \mathrm{T_A(x), I_A(x), F_A(x)} \in]^{-}0, 1^{+}[\}$$

There is no restriction on the sum of $\mathrm{T_A(x)}$, $\mathrm{I_A(x)}$ and $\mathrm{F_A(x)}$, so $0^{-} \leq \mathrm{T_A(x) + I_A(x) + F_A(x)} \leq 3^{+}$.

Definition 2.5[20]:

Let E be a universe. A single valued neutrosophic sets A, which can be used in real scientific and engineering applications, in E is characterized by a truth-membership function $\mathrm{T_A}$, a indeterminacy-membership function $\mathrm{I_A}$ and a falsity-membership function $\mathrm{F_A}$. $\mathrm{T_A(x)}$; $\mathrm{I_A(x)}$ and $\mathrm{F_A(x)}$ are real standard elements of [0,1]. It can be written as

$$\mathrm{A} = \{< x, (\mathrm{T_A(x), I_A(x), F_A(x)}) >: x \in E, \mathrm{T_A(x), I_A(x), F_A(x)} \in [^{-}0, 1^{+}]\}$$

There is no restriction on the sum of $\mathrm{T_A(x)}$, $\mathrm{I_A(x)}$ and $\mathrm{F_A(x)}$, so $0 \leq \mathrm{T_A(x) + I_A(x) + F_A(x)} \leq 3$.

Definition 2.6[20]:

Let $\tilde{\mathrm{a}} =< (\mathrm{a_1, b_1, c_1}); \widetilde{\mathrm{w_a}}, \widetilde{\mathrm{u_a}}, \widetilde{\mathrm{y_a}} >$, and $\tilde{\mathrm{b}} =< (\mathrm{a_2, b_2, c_2}); \widetilde{\mathrm{w_b}}, \widetilde{\mathrm{u_b}}, \widetilde{\mathrm{y_b}} >$ be two single valued triangular neutrosophic numbers and $\gamma \neq 0$ be any real number. Then,

1. $\tilde{\mathrm{a}} + \tilde{\mathrm{b}} =< (\mathrm{a_1 + a_2, b_1 + b_2, c_1 + c_2}); \widetilde{\mathrm{w_a}}$â^§$\widetilde{\mathrm{w_b}}, \widetilde{\mathrm{u_a}}$â^¨$\widetilde{\mathrm{u_b}}, \widetilde{\mathrm{y_a}}$â^¨$\widetilde{\mathrm{y_b}} >$
2. $\tilde{\mathrm{a}} - \tilde{\mathrm{b}} =< (\mathrm{a_1 - c_2, b_1 - b_2, c_1 - a_2}); \widetilde{\mathrm{w_a}}$â^§$\widetilde{\mathrm{w_b}}, \widetilde{\mathrm{u_a}}$â^¨$\widetilde{\mathrm{u_b}}, \widetilde{\mathrm{y_a}}$â^¨$\widetilde{\mathrm{y_b}} >$

Definition 2.7[20]:

Let $\widetilde{\mathrm{A_1}} =< \mathrm{T_1},\ \mathrm{I_1},\ \mathrm{F_1} >$ be a single valued neutrosophic number. Then, the score function $\mathrm{s}(\widetilde{\mathrm{A_1}})$, accuracy function$\mathrm{a}(\widetilde{\mathrm{A_1}})$, and certainty function $\mathrm{c}(\widetilde{\mathrm{A_1}})$ of an single valued neutrosophic numbers are defind

1. $\mathrm{s}(\widetilde{\mathrm{A_1}}) = (\mathrm{T_1} + 1 - \mathrm{I_1} + 1 - \mathrm{F_1})/3$
2. $\mathrm{a}(\widetilde{\mathrm{A_1}}) = \mathrm{T_1} - \mathrm{F_1}$
3. $c(\widetilde{A_1}) = T_1$

3. Proposed Network Protocol

The proposed system is neutrosophic intelligent energy efficient routing for WANET based on multicriteria decision making, which divides the entire system into three stages. These three stages are assessed by intelligent system through multicriteria rule based system. The above three stages are as follows:

(i). Neutrosophic multicriteria intelligent

(ii). Construction of neutrosophic intelligent route (iii). Selection of neutrosophic energy efficient route

Stage (i) describes the neutrosophic membership functions of hop counts, data packets, distance and energy for the proposed system briefly.

In stage (ii), rating of each and every neutrosophic route is established with the help of skilled system using rating formula.

Stage (iii) handles the selection process of neutrosophic energy efficient route using rule matrix after rating of neutrosophic routes.

3.1. Stage(i): Neutrosophic multicriteria intelligence

In this stage, neutrosophic membership functions of hop count, data packets, distance and energy are given as the input variables and the rating scale of neutrosophic routes as output variable. These input and output variables are categorized as the linguistic variables(low, medium and high). In this network model, the input variables hop count, data packet, distance and energy are considered as 30 (Nos.), 600(Mbps), 260(Meters) and 80(Joules).The membership functions of input variables are given in Table1, Table 2, Table 3, and Table 4 and output variable inTable 5.

Table:1 Neutrosophic membership function of hop count(Nos.)

Linguistic Values	Notation	Neutrosophic Range	Neutro. Base value
Low	H_L^N	$[H_{L1}^N, H_{L2}^N]$	(0,0,15)(0,0,30)(0,0,45)
Medium	H_M^N	$[H_{M1}^N, H_{M2}^N]$	(0,15,30)(0,15,45)(0,15,60)
High	H_H^N	$[H_{H1}^N, H_{H2}^N]$	(15,30,30)(10,30,45)(9,30,60)

Table:2 Neutrosophic membership function of Data packet(Mbps)

Linguistic Values	Notation	Neutrosophic Range	Neutro. Base value
Low	DP_L^N	$[DP_{L1}^N, DP_{L2}^N]$	(0,0,300)(0,0,600)(0,0,900)
Medium	DP_L^N	$[DP_{M1}^N, DP_{M2}^N]$	(0,300,600)(150,300,750)(270,300,900)
High	DP_L^N	$[DP_{H1}^N, DP_{H2}^N]$	(300,600,600)(500,600,800)(700,600,850)

Table:3 Neutrosophic membership function of Distance(Meters)

Linguistic Values	Notation	Neutrosophic Range	Neutro. Base value
Low	D_L^N	$[D_{L1}^N, D_{L2}^N]$	(0,0,100)(0,0,200)(0,0,250)
Medium	D_L^N	$[D_{M1}^N, D_{M2}^N]$	(40,100,220)(70,100,250)(90,100,270)
High	D_L^N	$[D_{H1}^N, D_{H2}^N]$	(140,260,260)(170,260,290)(190,260,300)

Table4: Neutrosophic membership function of Energy(Joules)

Linguistic Values	Notation	Neutrosophic Range	Neutro. Base value
Low	E_L^N	$[E_{L1}^N, E_{L2}^N]$	(0,0,32)(0,0,64)(0,0,96)
Medium	E_M^N	$[E_{M1}^N, E_{M2}^N]$	(8,40,72)(16,40,82)(24,40,92)
High	E_H^N	$[E_{H1}^N, E_{H2}^N]$	(48,80,80)(68,80,90)(78,80,100)

The rating scale of different neutrosophic routes are classified in the following table.

Table5: Neutrosophic membership function of Energy(Joules)

Linguistic Variable	Very Bad	Bad	Satisfactory	Medium	Less Good	Good	Very Good	Excellent	Very Excellent
Notation	$R^N{}_{VB}$	$R^N{}_B$	$R^N{}_S$	$R^N{}_M$	$R^N{}_{LG}$	$R^N{}_G$	$R^N{}_{VG}$	$R^N{}_E$	$R^N{}_{VE}$

3.2. Stage(ii): Construction of neutrosophic intelligent

In stage(ii), the rules and formulas for construction of neutrosophic intelligent routes are established. Usually, in ad-hoc networks while sending and receiving data packets energy consumption is occurred.Also the total network system is affected and lifetime of network is reduced at the time of power failure. The amount of input variables should be reduced in order to give the energy efficient routes for improving lifetime and performance of network system in such situations. Since energy plays an important role in network performance, the other input variables(hop count, data packet, distance) are combined with energy and the rules are framed for construction of intelligent route as follows:

Table 6: Rules for construction of neutrosophic route)

Rule	Energy and Hop Count level	Rating of Neutrosophic Route
R1	Low energy and high hop count	Very Bad
R2	Low energy and medium hop count	Bad
R3	Low energy and low hop count	Satisfactory
R4	Medium energy and high hop count	Medium
R5	Medium energy and medium hop count	Less Good
R6	Medium energy and low hop count	Good
R7	High energy and high hop count	Very Good
R8	High energy and medium hop count	Excellent
R9	High energy and low hop count	Very Excellent
	Energy and Data Packet level	
R10	Low energy and high data packet	Very Bad
R11	R11 Low energy and medium data packet	Bad
R12	Low energy and low data packet	Satisfactory
R13	Medium energy and high data packet	Medium
R14	R14 Medium energy and medium data packet	Less Good
R15	Medium energy and low data packet	Good
R16	High energy and high data packet	Very Good
R17	High energy and medium data packet	Excellent
R18	High energy and low data packet	Very Excellent
	Energy and Distance level	
R19	Low energy and high distance	Very Bad
R20	Low energy and medium distance	Bad
R21	Low energy and low distance	Satisfactory
R22	Medium energy and high distance	Medium
R23	Medium energy and medium distance	Less Good
R24	Medium energy and low distance	Good
R25	High energy and high distance	Very Good
R26	High energy and medium distance	Excellent
R27	High energy and low distance	Very Excellent

In Table 7, different types of neutrosophic states are established by using the formula

NR_{pq} = mean value of neutrosophic energy / mean value of other parameters

Rating of neutrosophic routes(Table.8) is calculated by using neutrosophic states in Table 7 and by using Table.8, the ascending order of rating of neutrosophic routes and linguistic nature of different neutrosophic rating of routes are calculated and given in Table.9 and Table.10.

Table 7: Different types of neutrosophic states

Neutro. Energy and Hop count		**Neutro. Energy and Data packet**		**Neutro. Energy and Distance**	
Neutro.State	Neutro.Value	Neutro. State	Neutro.Value	Neutro. State	Neutro.Value
NS11	2.133	NS21	0.10665	NS31	0.349
NS12	1.0665	NS22	0.0537	NS32	0.1548
NS13	0.7412	NS23	0.03458	NS33	0.09013
NS14	5.4	NS24	0.27	NS34	0.8836
NS15	2.7	NS25	0.1361	NS35	0.39192
NS16	1.8765	NS26	0.0875	NS36	0.2281
NS17	7.822	NS27	0.3911	NS37	1.2799
NS18	3.911	NS28	0.19719	NS38	0.5677
NS19	2.7182	NS29	0.1268	NS39	0.3305

Table 8: Different types of neutrosophic rating of routes

Neutro. Energy and Hop count		**Neutro. Energy and Data packet**		**Neutro. Energy and Distance**	
Neutro.Route	Neutro. Rating	Neutro.Route	Neutro. Rating	Neutro. Route	Neutro.Rating
NS11	3.911	NS21	0.19555	NS31	0.63995
NS12	1.955	NS22	0.097775	NS32	0.25598
NS13	1.3036	NS23	0.06518	NS33	0.159987
NS14	0.9777	NS24	0.04888	NS34	1.59987
NS15	0.48885	NS25	0.02444	NS35	0.6399
NS16	0.3259	NS26	0.01629	NS36	3.99968
NS17	0.6518	NS27	0.03258	NS37	2.5598
NS18	0.16295	NS28	0.00814	NS38	1.02392
NS19	0.1086	NS29	0.00543	NS39	0.63995

Table 9: Ascending order of rating of neutrosophic routes

Based on hop count rating
NR11 > NR12 > NR13 > NR14 > NR17 > NR15 > NR16 > NR18 > NR19
Based on data packets rating
NR21 > NR22 > NR23 > NR24 > NR27 > NR25 > NR26 > NR28 > NR29
Based on distance rating
NR36 > NR37 > NR34 > NR38 > NR35 > NR31;NR39 > NR32 > NR33

Table 10: Linguistic nature of di_erent neutrosophic rating of routes

S.No.	Linguistic nature	Neutrosophic Rating
1	NRV E	NR11, NR21, NR36
2	NRE	NR12, NR22, NR37
3	NRV G	NR13, NR23, NR34
4	NRG	NR14, NR24, NR38
5	NRLG	NR17, NR27, NR35
6	NRM	NR15, NR25, NR31, NR39
7	NRS	NR16, NR26, NR32
8	NRB	NR18, NR28, NR33
9	NRV B	NR19, NR29

3.3. Stage(iii): Selection of neutrosophic energy efficient route

Neutrosophic energy efficient route is evaluated using neutrosophic rule matrix in Table.11, Table.12 and Table.13. These three matirices are framed by combining energy with other parameters hop count, data packet and distance. Each route selected by these matrices have a particular value in the proposed ad-hoc network. After evaluated the routes using rule matrices, it is analysed that if the source node is in the positions NR19 or NR29 having lowest neutrosophic energy with high neutrosophic hop count or high neutrosophic data packets or long distance from destination, then it will receice the lowest neutrosophic rating value $\mathrm{NR_{VB}}$ and if the source node is in the positions NR11, NR21 or NR36 having high neutrosophic energy with low neutrosophic hop count or low neutrosophic data packets or shortest distance from the destination, then it will receive highest neutrosophic rating value $\mathrm{NR_{VE}}$.

Table 11: Neutrosophic rule matrix based on energy and hop count

Neutro. energy / Hop count	H_L^N	H_L^N	H_L^N
E_L^N	NR_S	NR_B	NR_{VB}
E_M^N	NR_G	NR_{LG}	NR_M
E_H^N	NR_{VE}	NR_E	NR_{VG}

Table 12: Neutrosophic rule matrix based on data packet and energy

Neutro. energy / Hop count	DP_L^N	DP_L^N	Dp_L^N
E_L^N	NR_S	NR_B	NR_{VB}
E_M^N	NR_G	NR_{LG}	NR_M
E_H^N	NR_{VE}	NR_E	NR_{VG}

Table 13: Neutrosophic rule matrix based on distance and energy

Neutro. energy / Hop count	D_L^N	D_L^N	D_L^N
E_L^N	NR_S	NR_B	NR_{VB}
E_M^N	NR_G	NR_{LG}	NR_M
E_H^N	NR_{VE}	NR_E	NR_{VG}

Finally, by analysing the the different types of neurtrosophic energy efficient rating of routes as given in figure.1, the process of wanet is improved in this stage by identifying the neutrosophic intelligent energy efficient route.

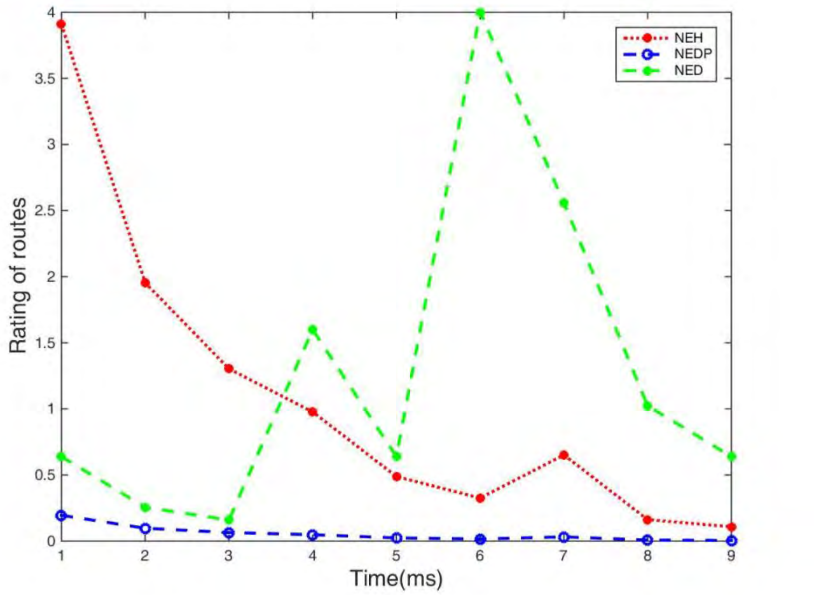

Figure 1: Analysis of neutrosophic intelligent energy efficient rating of routes.

4. Conclusions

In this paper, a new network design is developed to improve the service of wireless ad-hoc network by equipping the routes energy efficient using neutrosophic technique. Multi-criteria decision making method under neutrosophic environment is used for making the routes of the network efficiently here. From the mathematical analysis of the proposed network design, we conclude that the neutrosophic route is very efficient when source node is in the position NR11, NR21 or NR36, since the node with low energy, high hopcout, high transmitted data packets and long distance from the destination causes breakage of route and data packet retransmission. This neutrosophic energy efficient routing for wanet under multi-criteria decision making is better than other existing methods in uncertain environment. Various protocols for the efficiency of ad-hoc network system using neutrosophic sets will be established in future.

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Neutrosophic Triplet Group Based on Set Valued Neutrosophic Quadruple Numbers

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Abstract: Smarandache introduced neutrosophic quadruple sets and neutrosophic quadruple numbers [45] in 2015. These sets and numbers are real or complex number valued. In this study, we firstly introduce set valued neutrosophic quadruple sets and numbers. We give some known and special operations for set valued neutrosophic quadruple numbers. Furthermore, Smarandache and Ali obtained neutrosophic triplet groups [30] in 2016. In this study, we firstly give neutrosophic triplet groups based on set valued neutrosophic quadruple number thanks to operations for set valued neutrosophic quadruple numbers. In this way, we define new structures using the together set valued neutrosophic quadruple number and neutrosophic triplet group. Thus, we obtain new results for set valued neutrosophic quadruple numbers and neutrosophic triplet groups based on set valued neutrosophic quadruple number.

Keywords: Neutrosophic triplet set, neutrosophic triplet group, neutrosophic triplet quadruple set, neutrosophic triplet quadruple number, set valued neutrosophic triplet quadruple set, set valued neutrosophic triplet quadruple number

1 Introduction

Smarandache defined neutrosophic logic and neutrosophic set [1] in 1998. In neutrosophic logic and neutrosophic sets, there is T degree of membership, I degree of indeterminacy and F degree of non-membership. These degrees are defined independently of each other. It has a neutrosophic value (T, I, F) form. In other words, a condition is handled according to both its accuracy and its inaccuracy and its uncertainty. Therefore, neutrosophic logic and neutrosophic set help us to explain many uncertainties in our lives. In addition, many researchers have made studies on this theory [2 - 27] and [52-57].
In fact, fuzzy logic and fuzzy set [28] were obtained by Zadeh in 1965. In the concept of fuzzy logic and fuzzy sets, there is only a degree of membership. In addition, intuitionistic fuzzy logic and intuitionistic fuzzy set [29] were obtained by Atanassov in 1986. The concept of intuitionistic fuzzy logic and intuitionistic fuzzy set includes membership degree, degree of indeterminacy and degree of non-membership. But these degrees are defined dependently of each other. Therefore, neutrosophic set is a generalized state of fuzzy and intuitionistic fuzzy set.
Furthermore, Smarandache and Ali obtained neutrosophic triplet set (NTS) and neutrosophic triplet groups (NTG) [30]. For every element "x" in NTS A, there exist a neutral of "x" and an opposite of "x". Also, neutral of "x" must different from the classical neutral element. Therefore, the NTS is different from the classical set. Furthermore, a neutrosophic triplet (NT) "x" is showed by <x, neut(x), anti(x)>. Also, many researchers have introduced NT structures [31-44]
Also, Smarandache introduced neutrosophic quadruple sets (NQS) and neutrosophic quadruple number (NQN) [45]. The NQSs are generalized state of neutrosophic set. A NQS is shown by {(x, yT, zI, tF): x, y, z, t ∈ ℝ or ℂ}. Where, x is called the known part and (yT, zI, tF) is called the unknown part

and T, I, F have their usual neutrosophic logic means. Recently, researchers studied NQS and NQN. Akinleye, Smarandache, Agboola studied NQ algebraic structures [46]; Jun, Song, Smarandache obtained NQ BCK/BCI-algebras [47]; Muhiuddin, Al-Kenani, Roh, Jun introduced implicative NQ BCK-algebras and ideals [48]; Li, Ma, Zhang, Zhang studied neutrosophic extended triplet group based on NQNs [49]; Ma, Zhang, and Smarandache studied neutrosophic quadruple rings [50]; Kandasamy, Kandasamy and Smarandache obtained neutrosophic quadruple vector spaces and their properties [51].

In this study, we firstly introduce set valued neutrosophic quadruple set (SVNQS) and set valued neutrosophic quadruple number (SVNQN). In the neutrosophic quadruples, real or complex numbers were taken as variables, while in this study we took sets as variables. So, we will expand the applications of neutrosophic quadruples. Because things or variables in any application will be more useful than real numbers or complex numbers. Also we give NT group (NTG) based on SVNQN. In Section 2, we give definitions and properties for NQS, NQN [45] and NTS, NTG [30]. In Section 3, we define SVNQS and SVNQN. Also, we give operations for these structures. In Section 4, we obtain some NTG based on SVNQN thanks to operations for SVNQN. In this way, we define new structures using the together SVNQN and NTG.

2 Preliminaries

Definition 2.1: [45] A NQN is a number of the form (x, yT, zI, tF), where T, I, F have their usual neutrosophic logic means and x, y, z, t ∈ ℝ or ℂ. The NQS defined by NQ = {(x, yT, zI, tF): x, y, z, t ∈ ℝ or ℂ}.

For a NQN (x, yT, zI, tF), representing any entity which may be a number, an idea, an object, etc., x is called the known part and (yT, zI, tF) is called the unknown part.

Definition 2.2: [45] Let a = (a_1, a_2T, a_3I, a_4F) and b = (b_1, b_2T, b_3I, b_4F) ∈ NQ be NQNs. We define the following:

a + b = (a_1 +b_1, (a_2+b_2)T, (a_3+b_3)I, (a_4+b_4)F)

a - b = (a_1 - b_1, (a_2 - b_2)T, (a_3 - b_3)I, (a_4 - b_4)F)

Definition 2.3: [45] Consider the set {T, I, F}. Suppose in an optimistic way we consider the prevalence order T>I>F. Then we have:

TI = IT = max{T, I} = T,

TF = FT = max{T, F} = T,

FI = IF = max{F, I} = I,

TT = T^2 = T,

II = I^2 = I,

FF = F^2 = F.

Analogously, suppose in a pessimistic way we consider the prevalence order T < I < F. Then we have:

TI = IT = max{T, I} = I,

TF = FT = max{T, F} = F,

FI = IF = max{F, I} = F,

TT = T^2 = T,

II = I^2 = I,

FF = F^2 = F.

Definition 2.4: [45] Let

a = (a_1, a_2T, a_3I, a_4F),

b = (b_1, b_2T, b_3I, b_4F) ∈ NQ;

T < I < F.

Then a*b = (a_1 , a_2 T, a_3 I, a_4 F)* (b_1 , b_2 T, b_3 I, b_4 F) = (a_1b_1 , (a_1b_2 + a_2b_1 + a_2b_2)T, (a_1b_3 + a_2b_3 + a_3b_1 + a_3b_2 + a_3b_3)I, (a_1b_4 + a_2b_4 + a_3b_4 + a_4b_1 + a_4b_2 + a_4b_3 + a_4b_4)F)

Definition 2.5: [45] Let

a = (a_1, a_2T, a_3I, a_4F),

b = (b_1, b_2T, b_3I, b_4F) ∈ NQ,

T > I > F

Then a#b = (a_1, a_2T, a_3I, a_4F) # (b_1, b_2T, b_3I, b_4F) = (a_1b_1, (a_1b_2 + a_2b_1 + a_2b_2 + a_3b_2 + a_4b_2 + a_2b_3 + a_2b_4)T, (a_1b_3 + a_3b_3 + a_3b_4 + a_4b_3)I, (a_1b_4 + a_4b_1 + a_4b_4)F)

Definition 2.6: [30]: Let # be a binary operation. A NTS (X, #) is a set such that for x ∈ X,

i) There exists neutral of "x" such that x#neut(x) = neut(x)#x = x,

ii) There exists anti of "x" such that x#anti(x) = anti(x)#x = neut(x).

Also, a neutrosophic triplet "x" is showed with (x, neut(x), anti(x)).

Definition 2.7: [30] Let (X, #) be a NT set. Then, X is called a NTG such that

a) for all a, b ∈ X, a*b ∈ X.

b) for all a, b, c ∈ X, (a*b)*c = a*(b*c)

3 Set Valued Neutrosophic Quadruple Numbers

Definition 3.1: Let N be a non – empty set and P(N) be power set of N. A SVNQN shown by the form (A_1, A_2T, A_3I, A_4F). Where, T, I and F are degree of membership, degree of undeterminacy, degree of non-membership in neutrosophic theory, respectively. Also, A_1, A_2, A_3, A_4 ∈ P(N). Then, a SVNQS shown by N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}.

Where, similar to NQS, A_1 is called the known part and (A_1, A_2T, A_3I, A_4F) is called the unknown part.

Definition 3.2: Let A = (A_1, A_2T, A_3I, A_4F) and B = (B_1, B_2T, B_3I, B_4F) be SVNQNs. We define the following operations, well known operators in set theory, such that

A ∪ B = ($A_1 \cup B_1$, ($A_2 \cup B_2$)T, ($A_3 \cup B_3$)I, ($A_4 \cup B_4$)F)

A ∩ B = ($A_1 \cap B_1$, ($A_2 \cap B_2$)T, ($A_3 \cap B_3$)I, ($A_4 \cap B_4$)F)

A \ B = (A_1 \ B_1, (A_2 \ B_2)T, (A_3 \ B_3)I, (A_4 \ B_4)F)

A' = (A'_1, A'_2T, A'_3I, A'_4F)

Now, we define specific operations for SVNQN.

Definition 3.3: Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) be SVNQNs and T < I < F. We define the following operations

A*₁B = (A_1, A_2T, A_3I, A_4F) *₁ (B_1, B_2T, B_3I, B_4F) = ($A_1 \cap B_1$, $((A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2))$T, $((A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3))$I, $((A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4))$F) and

A*₂B = (A_1, A_2T, A_3I, A_4F) *₂ (B_1, B_2T, B_3I, B_4F) = ($A_1 \cup B_1$, $((A_1 \cup B_2) \cap (A_2 \cup B_1) \cap (A_2 \cup B_2))$T, $((A_1 \cup B_3) \cap (A_2 \cup B_3) \cap (A_3 \cup B_1) \cap (A_3 \cup B_2) \cap (A_3 \cup B_3))$I, $((A_1 \cup B_4) \cap (A_2 \cup B_4) \cap (A_3 \cup B_4) \cap (A_4 \cup B_1) \cap (A_4 \cup B_2) \cap (A_4 \cup B_3) \cap (A_4 \cup B_4))$F).

Definition 3.4: Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) be SVNQNs and T > I > F. We define the following operations

A #₁B = (A_1, A_2T, A_3I, A_4F) #₁ (B_1, B_2T, B_3I, B_4F) = ($A_1 \cap B_1$, $((A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2) \cup (A_3 \cap B_2) \cup (A_4 \cap B_2) \cup (A_2 \cap B_3) \cup (A_2 \cap B_4))$T, $((A_1 \cap B_3) \cup (A_3 \cap B_3) \cup (A_3 \cap B_4) \cup (A_4 \cap B_3))$I, $((A_1 \cap B_4) \cup (A_4 \cap B_2) \cup (A_4 \cap B_4))$F) and

A#₂B = (A_1, A_2T, A_3I, A_4F) #₂ (B_1, B_2T, B_3I, B_4F) = ($A_1 \cup B_1$, $((A_1 \cup B_2) \cap (A_2 \cup B_1) \cap (A_2 \cup B_2) \cap (A_3 \cup B_2) \cap (A_4 \cup B_2) \cap (A_2 \cup B_3) \cap (A_2 \cup B_4))$T, $((A_1 \cup B_3) \cap (A_3 \cup B_3) \cap (A_3 \cup B_4) \cap (A_4 \cup B_3))$I, $((A_1 \cup B_4) \cap (A_4 \cup B_2) \cap (A_4 \cup B_4))$F).

Definition 3.5: Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) be SVNQNs. If $A_1 \subset B_1$, $A_2 \subset B_2$, $A_3 \subset B_3$, $A_4 \subset B_4$, then it is called that A is subset of B. It is shown by A⊂ B.

Definition 3.6: Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) be SVNQNs If A⊂ B and $B \subset A$., then it is called that A is equal to B. It is shown by A = B.

Example 3.7: Let X = {x, y, z} be a set. Thus, we have P(X) ={∅ , {x}, {y}, {z}, {y, z}, {x, z}, {x, y} ,{x, y, z}}. Also, X_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(X)} is a SVNQS. For example,

A_1 = ({y, z}, {x, y, z}T, {x, y}I, {z}F) and A_2 = ({ z}, {x, z}T, {x, y}I, ∅F) are two SVNQNs in X_q. Furthermore,

$A_1 \cup A_2$ = ({y, z}, {x, y, z}T, {x, y}I, {z}F) = A_1.

$A_1 \cap A_2$ = ({ z}, {x, z}T, {x, y}I, ∅F) = A_2.

Thus, we have $A_2 \subset A_1$. Also,

$A_1{}'$ = ({x}, ∅T, {z}I, {x, y}F)

$A_1 \setminus A_2$ = ({y}, { y}T, ∅I, {z}F)

4 Neutrosophic Triplet Group Based on Set Valued Neutrosophic Quadruple Numbers

Theorem 4.1: Let N be a non – empty set and N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}be a SVNQS. Then,
a) (N_q, ∪) is a NTS.
b) (N_q, ∩) is a NTS.
Proof:
a) Let A = (A_1, A_2T, A_3I, A_4F) be a SVNQN in N_q. From Definition 3.2, it is clear that
A ∪ A = (A_1, A_2T, A_3I, A_4F) ∪ (A_1, A_2T, A_3I, A_4F) = ($A_1 \cup A_1$, ($A_2 \cup A_2$)T, ($A_3 \cup A$)I, ($A_4 \cup A_4$)F) = (A_1, A_2T, A_3I, A_4F) = A.
Hence, we can take neut(A) = A. Also, if neut(A) = A, then we have anti(A) = A. Thus, (N_q, ∪) is a neutrosophic triplet set with neut(A) = A and anti(A) = A.
b) a) Let A = (A_1, A_2T, A_3I, A_4F) be a SVNQN in N_q. From Definition 3.2, it is clear that
A ∩ A = (A_1, A_2T, A_3I, A_4F) ∩ (A_1, A_2T, A_3I, A_4F) = ($A_1 \cap A_1$, ($A_2 \cap A_2$)T, ($A_3 \cap A$)I, ($A_4 \cap A_4$)F) = (A_1, A_2T, A_3I, A_4F) = A.
Hence, we can take neut(A) = A. Also, if neut(A) = A, then we have anti(A) = A. Thus, (N_q, ∩) is a neutrosophic triplet set with neut(A) = A and anti(A) = A.

Theorem 4.2: Let N be a non – empty set and N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}be a SVNQS. Then,
a) (N_q, ∪) is a NTG.
b) (N_q, ∩) is a NTG.
Proof:
a) From Theorem 4.1, (N_q, ∪) is a NTS with neut(A) = A and anti(A) = A. Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) and C = (C_1, C_2T, C_3I, C_4F) ∈ N_q.
i) We have that A ∪ B ∈ N_q since P(N) is power set of N and A, B ∈ P(N). Because, if A, B ∈ P(X), then A ∪ B ∈ P(N).
ii) (A ∪ B) ∪ C = [($A_1 \cup B_1$, ($A_2 \cup B_2$)T, ($A_3 \cup B_3$)I, ($A_4 \cup B_4$)F)] ∪ (C_1, C_2T, C_3I, C_4F) =
[($A_1 \cup B_1$) ∪ C_1, (($A_2 \cup B_2$) ∪ C_2)T, (($A_3 \cup B_3$) ∪ C_3)I, (($A_4 \cup B_4$) ∪ C_4))F)] =
[$A_1 \cup (B_1 \cup C_1)$, ($A_2 \cup (B_2 \cup C_2)$)T, ($A_3 \cup (B_3 \cup C_3)$)I, ($A_4 \cup (B_4 \cup C_4)$)F)] = A ∪ (B ∪ C).
Thus, (N_q, ∪) is a NTG.
b) From Theorem 4.1, (N_q, ∩) is a NTS with neut(A) = A and anti(A) = A. Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) and C = (C_1, C_2T, C_3I, C_4F) ∈ N_q.
i) We have that A ∩ B ∈ N_q since P(N) is power set of N and A, B ∈ P(N). Because, if A, B ∈ P(N), then A ∩ B ∈ P(N).
iii) (A ∩ B) ∩ C = [($A_1 \cap B_1$, ($A_2 \cap B_2$)T, ($A_3 \cap B_3$)I, ($A_4 \cap B_4$)F)] ∩ (C_1, C_2T, C_3I, C_4F) =[($A_1 \cap B_1$) ∩ C_1, (($A_2 \cap B_2$) ∩ C_2)T, (($A_3 \cap B_3$) ∩ C_3)I, (($A_4 \cap B_4$) ∩ C_4))F)] = [$A_1 \cap (B_1 \cap C_1)$, ($A_2 \cap (B_2 \cap C_2)$)T, ($A_3 \cap (B_3 \cap C_3)$)I, ($A_4 \cap (B_4 \cap C_4)$)F)] = A ∩ (B ∩ C).
Thus, (N_q, ∩) is a NTG.

Theorem 4.3: Let N be a non – empty set and N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}be a SVNQS. Then,
a) (N_q, $*_1$) is a NTS with binary operation $*_1$ in Definition 3.3.
b) (N_q, $*_2$) is a NTS with binary operation $*_2$ in Definition 3.3.
Proof:
a) Let A = (A_1, A_2T, A_3I, A_4F) be a SVNQN in N_q. From Definition 3.3, we obtain
A $*_1$ A = (A_1, A_2T, A_3I, A_4F) $*_1$ (A_1, A_2T, A_3I, A_4F) =

$(A_1 \cap A_1,\ ((A_1 \cap A_2) \cup (A_2 \cap A_1) \cup (A_2 \cap A_2))T,\ ((A_1 \cap A_3) \cup (A_2 \cap A_3) \cup (A_3 \cap A_1) \cup (A_3 \cap A_2) \cup (A_3 \cap A_3))I,\ ((A_1 \cap A_4) \cup (A_2 \cap A_4) \cup (A_3 \cap A_4) \cup (A_4 \cap A_1) \cup (A_4 \cap A_2) \cup (A_4 \cap A_3) \cup (A_4 \cap A_4))F) = (A_1, A_2T, A_3I, A_4F) = A$

since

$A_2 \cap A_2 = A_2$ and $(A_1 \cap A_2), (A_2 \cap A_2) \subset A_2$;

$A_3 \cap A_3 = A_3$ and $(A_1 \cap A_3), (A_2 \cap A_3), (A_3 \cap A_3) \subset A_3$;

$A_4 \cap A_4 = A_4$ and $(A_1 \cap A_4), (A_2 \cap A_4), (A_3 \cap A_4), (A_4 \cap A_4) \subset A_4$.

Hence, we can take neut(A) = A. Also, if neut(A) = A, then we have anti(A) = A. Thus, $(N_q, *_1)$ is a NTS with neut(A) = A and anti(A) = A.

b) Let A = (A_1, A_2T, A_3I, A_4F) be a SVNQN in N_q. From Definition 3.3, we obtain

A $*_2$ A = $(A_1, A_2T, A_3I, A_4F) *_2 (A_1, A_2T, A_3I, A_4F) = (A_1 \cup A_1,\ ((A_1 \cup A_2) \cap (A_2 \cup A_1) \cap (A_2 \cup A_2))T,\ ((A_1 \cup A_3) \cap (A_2 \cup A_3) \cap (A_3 \cup A_1) \cap (A_3 \cup A_2) \cap (A_3 \cup A_3))I,\ ((A_1 \cup A_4) \cap (A_2 \cup A_4) \cap (A_3 \cup A_4) \cap (A_4 \cup A_1) \cap (A_4 \cup A_2) \cap (A_4 \cup A_3) \cap (A_4 \cup A_4))F) = (A_1, A_2T, A_3I, A_4F) = A$

since

$A_2 \cup A_2 = A_2$ and $(A_1 \cup A_2), (A_2 \cup A_2) \supset A_2$;

$A_3 \cup A_3 = A_3$ and $(A_1 \cup A_3), (A_2 \cup A_3), (A_3 \cup A_3) \supset A_3$;

$A_4 \cup A_4 = A_4$ and $(A_1 \cup A_4), (A_2 \cup A_4), (A_3 \cup A_4), (A_4 \cup A_4) \supset A_4$.

Hence, we can take neut(A) = A. Also, if neut(A) = A, then we have anti(A) = A. Thus, $(N_q, *_2)$ is a NTS with neut(A) = A and anti(A) = A.

Theorem 4.4: Let N be a non – empty set and N_q= $\{(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 \in P(N)\}$be a SVNQS. Then,

a) $(N_q, *_1)$ is a NTG with binary operation $*_1$ in Definition 3.3.

b) $(N_q, *_2)$ is a NTG with binary operation $*_2$ in Definition 3.3.

Proof:

a) From Theorem 4.3, $(N_q, *_1)$ is a neutrosophic triplet set. Let

A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) and C = $(C_1, C_2T, C_3I, C_4F) \in N_q$,

i) We obtain A $*_1$ B $\in N_q$since P(N) is power set of N and A, B $\in$ P(N).

ii)

(A $*_1$ B) $*_1$ C =

$(A_1 \cap B_1,\ ((A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2))T,\ ((A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3))I,\ ((A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4))F) *_1 (C_1, C_2T, C_3I, C_4F)$ =

$([A_1 \cap B_1] \cap C_1,$

$(([A_1 \cap B_1] \cap C_2) \cup ([(A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2)] \cap C_1) \cup ([(A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2)] \cap C_2))T,$

$([A_1 \cap B_1] \cap C_3) \cup ([(A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2)] \cap C_3) \cup ([A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3)] \cap C_1) \cup ([A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3)] \cap C_2) \cup ([A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3)] \cap C_3))I,$

$(([A_1 \cap B_1] \cap C_4) \cup ([(A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2)] \cap C_4) \cup ([A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3)] \cap C_4) \cup ([(A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4)] \cap C_1) \cup ([(A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4)] \cap C_2) \cup ([(A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4)] \cap C_3) \cup ([(A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4)] \cap C_4))F)$ =

$(A_1 \cap [B_1 \cap C_1],$

$((A_1 \cap [(B_1 \cap C_2) \cup (B_2 \cap C_1) \cup (B_2 \cap C_2)]) \cup (A_2 \cap [B_1 \cap C_1]) \cup (A_2 \cap [(B_1 \cap C_2) \cup (B_2 \cap C_1) \cup (B_2 \cap C_2)]))T,$

$((A_1 \cap [(B_1 \cap C_3) \cup (B_2 \cap C_3) \cup (B_3 \cap C_1) \cup (B_3 \cap C_2) \cup (B_3 \cap C_3)]) \cup (A_2 \cap [(B_1 \cap C_3) \cup (B_2 \cap C_3) \cup (B_3 \cap C_1) \cup (B_3 \cap C_2) \cup (B_3 \cap C_3)]) \cup (A_3 \cap [B_1 \cap C_1]) \cup (A_3 \cap [(B_1 \cap C_2) \cup (B_2 \cap C_1) \cup (B_2 \cap C_2)]) \cup (A_3 \cap [(B_1 \cap C_3) \cup (B_2 \cap C_3) \cup (B_3 \cap C_1) \cup (B_3 \cap C_2) \cup (B_3 \cap C_3)]))I,$

$((A_1 \cap [(B_1 \cap C_4) \cup (B_2 \cap C_4) \cup (B_3 \cap C_4) \cup (B_4 \cap C_1) \cup (B_4 \cap C_2) \cup (B_4 \cap C_3) \cup (B_4 \cap C_4)] \cup (A_2 \cap [(B_1 \cap C_4) \cup (B_2 \cap C_4) \cup (B_3 \cap C_4) \cup (B_4 \cap C_1) \cup (B_4 \cap C_2) \cup (B_4 \cap C_3) \cup (B_4 \cap C_4)]) \cup (A_3 \cap$

$[(B_1 \cap C_4) \cup (B_2 \cap C_4) \cup (B_3 \cap C_4) \cup (B_4 \cap C_1) \cup (B_4 \cap C_2) \cup (B_4 \cap C_3) \cup (B_4 \cap C_4)]) \cup (A_4 \cap [B_1 \cap C_1]) \cup (A_4 \cap [(B_1 \cap C_2) \cup (B_2 \cap C_1) \cup (B_2 \cap C_2)]) \cup (A_4 \cap [(B_1 \cap C_3) \cup (B_2 \cap C_3) \cup (B_3 \cap C_1) \cup (B_3 \cap C_2) \cup (B_3 \cap C_3)]) \cup (A_4 \cap [(B_1 \cap C_4) \cup (B_2 \cap C_4) \cup (B_3 \cap C_4) \cup (B_4 \cap C_1) \cup (B_4 \cap C_2) \cup (B_4 \cap C_3) \cup (B_4 \cap C_4)]\ ))F)$
$= A *_1 (B *_1 C)$.
Thus, $(N_q, *_1)$ is a NTG with binary operation $*_1$ in Definition 3.3.
b) This proof can be made similar to a.

Theorem 4.5: Let N be a non – empty set and N_q= $\{(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 \in P(N)\}$be a SVNQS. Then,
a) $(N_q, *_1)$ is a NTS with binary operation $\#_1$ in Definition 3.4.
b) $(N_q, *_2)$ is a NTS with binary operation $\#_2$ in Definition 3.4.
Proof: These proofs can be made similar to Theorem 4.3.

Theorem 4.6: Let N be a non – empty set and N_q= $\{(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 \in P(N)\}$be a SVNQS. Then,
a) $(N_q, *_1)$ is a NTG with binary operation $\#_1$ in Definition 3.4.
b) $(N_q, *_2)$ is a NTG with binary operation $\#_2$ in Definition 3.4.
Proof: These proofs can be made similar to Theorem 4.4.

Conclusion
In this study, we firstly obtain set valued neutrosophic quadruple sets and numbers. Also, we introduce some known and special operations for set valued neutrosophic quadruple numbers. In the neutrosophic quadruples, real or complex numbers were taken as variables, while in this study we took sets as variables. So, we will expand the applications of neutrosophic quadruples. Because things or variables in any application will be more useful than real numbers or complex numbers. Furthermore, we give some neutrosophic triplet groups based on set valued neutrosophic quadruple number thanks to operations for set valued neutrosophic quadruple numbers. Thus, we have added a new structure to neutrosophic triplet structures and neutrosophic quadruple structures. Thanks to set valued neutrosophic quadruple sets and numbers other neutrosophic triplet structures can be defined similar to this study. For example, neutrosophic triplet metric space based on set valued neutrosophic quadruple numbers; neutrosophic triplet vector space based on set valued neutrosophic quadruple numbers; neutrosophic triplet normed space based on set valued neutrosophic quadruple numbers. Also, set valued neutrosophic quadruple sets can be used decision making applications due to the its set valued structure. For example, in a medical application in which more than one drug is used, this structure may be used.

Abbreviations
NT: Neutrosophic triplet
NTS: Neutrosophic triplet set
NTG: Neutrosophic triplet group
NQ: Neutrosophic quadruple
NQS: Neutrosophic quadruple set
NQN: Neutrosophic quadruple number
SVNQS: Set valued neutrosophic quadruple set
SVNQN: Set valued neutrosophic quadruple number

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Conflicts of Interest

The authors declare no conflict of interest.

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Neutrosophic Semi-Baire Spaces

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Abstract: In this paper, we introduce the concept of Neutrosophic Semi Baire spaces in Neutrosophic Topological Spaces. Also we define Neutrosophic Semi-nowhere dense, Neutrosophic Semi-first category and Neutrosophic Semi-second category sets. Some of its characterizations of Neutrosophic Semi-Baire spaces are also studied. Several examples are given to illustrate the concepts

Keywords: Neutrosophic semi-open set, Neutrosophic semi-nowhere dense set, Neutrosophic semi-first category, Neutrosophic semi-second category and Neutrosophic semi-Baire spaces

1. Introduction and Preliminaries

The fuzzy idea has invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [29]. The important concept of fuzzy topological space was offered by C. L. Chang [9] and from that point forward different ideas in topology have been reached out to fuzzy topological space. The concept of "intuitionistic fuzzy set" was first presented by Atanassov [5]. He and his associates studied this useful concept [6 - 8]. Afterward, this idea was generalized to "intuitionistic L – fuzzy sets" by Atanassov and Stoeva [6]. The idea of somewhat fuzzy continuous functions and somewhat fuzzy open hereditarily irresolvable were introduced and investigated by by G. Thangaraj and G. Balasubramanian in [25]. The idea of intuitionistic fuzzy nowhere dense set in intuitionistic fuzzy topological space presented and studied by Dhavaseelan and et al. in [16]. The concepts of neutrosophy and Neutrosophic set were introduced by F. Smarandache [[22], [23]]. Afterwards, the works of Smarandache inspired A. A. Salama and S. A. Alblowi[21] to introduce and study the concepts of Neutrosophic crisp set and Neutrosophic crisp topological spaces. The Basic definitions and Proposition related to Neutrosophic topological spaces was introduced and discussed by Dhavaseelan et al. [17]. The concepts of Neutrosophic Baire spaces are introduced by R. Dhavaseelan, S. Jafari ,R. Narmada Devi, Md. Hanif Page [16]

Definition 1.1. [22, 23] Let T,I,F be real standard or non standard subsets of $]0^-, 1^+[$, with

$sup_T = t_{sup}$ T ; $inf_T = t_{inf}$

$Sup_I = i_{sup}$; $inf_I = i_{inf}$

$Sup_F = f_{sup}$; $inf_F = f_{inf}$

n - $sup = t_{sup} + i_{sup} + f_{sup}$

n-$inf = t_{inf}+i_{inf}+f_{inf}$. T, I, F are Neutrosophic components.

Definition 1.2. [22, 23] Let X is a nonempty fixed set. A Neutrosophic set [briefly Ne.S] K is an object having the form $K = \{\langle x, \mu_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$ where $\mu_K(x), \sigma_K(x) and\, \gamma_K(x)$ which represents the degree of membership function (namely $\mu_K(x)$), the degree of indeterminacy (namely $\sigma_k(x)$) and the degree of non-membership (namely $\gamma_K(x)$) respectively of each element $x \in X$ to the set K.

Remark 1.2. [22, 23]

(1) A Ne.S $K = \{\langle x, \mu_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_K, \sigma_K, \gamma_K\rangle$ in $]0^-, 1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol K = $\langle \mu_K, \sigma_K, \gamma_K\rangle$ for the Ne.S $K = \{\langle x, \mu_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$

Definition 1.3. [22, 23] Let X be a nonempty set and the Ne.Sets K and L in the form $K = \{\langle x, \mu_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$, L= $\{\langle x, \mu_L(x), \sigma_L(x), \gamma_L(x)\rangle : x \in X\}$. Then

(a) $K \subseteq L$ iff $\mu_K(x) \le \mu_L(x)$, $\sigma_K(x) \le \sigma_L(x)$, $\gamma_K(x) \ge \gamma_L(x)$ for all $x \in X$;

(b) $K = L$ iff $K \subseteq L$ and $L \subseteq K$;

(c) $\overline{K} = \{\langle x, \gamma_L(x), \sigma_K(x), \mu_L(x)\rangle : x \in X\}$; [Complement of K]

(d) K $\cap$ L= $\left\{\langle x, \mu_K(x) \wedge \mu_L(x) , \sigma_K(x) \wedge \sigma_L(x) , \gamma_K(x) \vee \gamma_L(x)\rangle : x \in X\right\}$;

(e) K $\cup$ L= $\left\{\langle x, \mu_K(x) \vee \mu_L(x), \sigma_K(x) \vee \sigma_L(x) , \gamma_K(x) \wedge \gamma_L(x)\rangle : x \in X\right\}$;

(f) [ ]K = $\{\langle x, \mu_K(x), \sigma_K(x), 1 - \mu_K(x)\rangle : x \in X\}$;

(g) $\langle\,\rangle K = \{\langle x, 1 - \gamma_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$

Definition 1.4. [22, 23] Let $\{K_i : i \in J\}$ be an arbitrary family of Ne.Sets in X. Then

(a) $\cap K_i = \{\langle x, \wedge \mu_{Ki}(x), \wedge \sigma_{Ki}(x), \vee \gamma_{Ki}(x)\rangle : x \in X\}$,

(b) $\cup K_i = \{\langle x, \vee \mu_{Ki}(x), \vee \sigma_{Ki}(x), \wedge \gamma_{Ki}(x)\rangle : x \in X\}$,

Since our main purpose is to construct the tools for developing Ne.T.Spaces, we introduce the Ne.Sets 0_N and 1_N in X as follows:

Definition 1.5. [22, 23]

$$0_N = \{\langle x, 0,0,1\rangle : x \in X\}\, and \quad 1_N = \{\langle x, 1,1,0\rangle : x \in X\}$$

Definition 1.6. [21]

A Neutrosophic topology (Ne.T) on a nonempty set X is a family N_T of Ne.Sets in X satisfying the following axioms:

(i) $0_N , 1_N \in N_T$,

(ii) $G_1 \cap G_2 \in N_T$ for any $G_1, G_2 \in N_T$.

(iii)$\cup G_i$ for arbitrary family $\{G_i | \, i \in \wedge \}$.

In this case the ordered pair (X, N_T) or simply X is called a Neutrosophic Topological Space (briefly Ne.T.S) and each Ne.S in N_T is called a Neutrosophic open set (briefly Ne.O.S). The complement K of a Ne.O.S K in X is called a Neutrosophic closed set (briefly Ne.C.S) in X.

Definition 1.7. [9]

Let K be a Ne.S in a Ne.T.S X. Then

Ne.int(K) = $\cup \{G \mid G\ is\ Neutrosophic\ open\ set\ in\ X\ and\ G \subseteq K\}$

is called the Neutrosophic interior of K;

Ne.cl(K) = $\cap \{G \mid G\ is\ Neutrosophic\ closed\ set\ in\ X\ and\ G \supseteq K\}$

is called the Neutrosophic closure of K.

Definition 1.8: [13] A Ne.S K in a Ne.T.S X is said to a Neutrosophic Semi Open set (Ne.S.O.S) if $K \subseteq Ne.cl(Ne.\text{int}(K))$ and Neutrosophic Semi Closed set (Ne.S.C.S) if $Ne.\text{int}(Ne.cl(K)) \subseteq K$.

Definition 1.9:[13] Let K be a Ne.S in a Ne.T.S X. Then

Ne.S.int(K) = $\cup \{G \mid G\ is\ Neutrosophic\ semi\ open\ set\ in\ X\ and\ G \subseteq K\}$

is called the Neutrosophic semi interior of K;

Ne.S.cl(K) = $\cap \{G \mid G\ is\ Neutrosophic\ semi\ closed\ set\ in\ X\ and\ G \supseteq K\}$

is called the Neutrosophic semi closure of K;

Result: 1.9 Let K be a Ne.S in a Ne.T.S X. Then

Ne.S.cl(K) = $K \cup Ne.\text{int}(Ne.cl(K))$

Ne.S.int(K) = $K \cap Ne.cl(Ne.\text{int}(K))$

2. Neutrosophic Semi-nowhere dense sets

Definition 2.1 A Ne.S K in Ne.T.S (X, N_T) is called Neutrosophic semi nowhere dense (briefly Ne.S.N.D) if there exists no non-zero Ne.S.O.S L in (X; N_T) such that $L \subset Ne.S.cl(K)$. That is $Ne.S.\text{int}(Ne.S.cl(K)) = 0_N$

Example 2.1 Let *X = {k, l}*. Define the Ne.S K, L and M on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.6}\right), \left(\frac{k}{0.5}, \frac{l}{0.2}\right), \left(\frac{k}{0.4}, \frac{l}{0.5}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.2}, \frac{l}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.1}\right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L, K \cup L, K \cap L\}$ is Ne.T on X. Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, \overline{K \cup L}$ are Ne.S.N.D set

Proposition 2.1. If K is a Ne.S.N.D set in (X; N_T), then $\overline{K}$ is a Ne.S.D set in (X, T)

Proposition 2.2. Let K be a set. If K is a Ne.S.C.S in (X, N_T) with Ne.S.int(K) = 0_N, then K is a Ne.S.N.D set in (X; N_T).

Definition 2.2. Let K be a Neutrosophic semi first category set (Ne.S.F.C.) in (X, N_T). Then $\overline{K}$ is called a Neutrosophic residual set in (X; N_T).

Proposition 2.3. The complement of a Ne.S.N.D. set in a Ne.T.S (X, N_T) need not be Ne.S.N.D. set.

Proof: For, in example 2.1, $\overline{K}$ is a Ne.S.N.D. set in (X, N_T) whereas K is not a Ne.S.N.D. set in (X, N_T).

Proposition 2.4. If K & L are Ne.S.N.D. sets in a Ne.T.S (X, N_T), then $K \cup L$ need not be Ne.S.N.D. set in

(X, N_T).

Proof: For, in example 2.1, $\overline{K} \& \overline{L}$ is Ne.S.N.D. sets in (X, N_T). But $\overline{K} \cup \overline{L}$ implies that Ne.S.int(Ne.S.cl($\overline{K} \cup \overline{L}$) ≠ 0_N. Therefore union of Ne.S.N.D. sets need not be Ne.S.N.D. set in (X, N_T).

Proposition 2.5: If the Ne.Sets K and L are Ne.S.N.D. sets in a Ne.T.S (X, N_T) then K$\cap L$ is a Ne.S.N.D. set in (X, N_T).

Proof: Let the fuzzy sets K and L be Ne.S.N.D. sets in (X, N_T). Now Ne.S.int (Ne.S.cl ($K \cap L$)) $\subseteq$ Ne.S.int (Ne.S.cl (K)) $\cap$ Ne.S.int (Ne.S.cl (L)) = $0_N \cap 0_N$ (since Ne.S.int (Ne.S.cl (K)) = 0_N and Ne.S.int(Ne.S.cl(B)) = 0_N). That is, Ne.S.int(Ne.S.cl (K$\cap L$) = 0_N. Hence (K$\cap L$) is a Ne.S.N.D. set in (X, N_T).

Proposition 2.6: If K is a Ne.S.N.D. set in a Ne.T.S (X, N_T) then Ne. S.int (K) = 0_N.

Proof: Let K be a Ne.S.N.D. set in (X, N_T). Then, we have Ne.S.int (Ne.S.cl (K)) = 0_N. Now K $\subseteq$ Ne.S.cl (K) we have Ne.S.int (K) $\subseteq$ Ne.S.int (Ne.S.cl (K))= 0_N. Hence Ne.S.int (K) = 0_N

Proposition 2.7:

If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T) then Ne.int (Ne.S.cl (K)) = 0.

Proof: Let K be a Ne.S.N.D. sets in (X, N_T). Then, we have Ne.int(Ne.cl (K)) = 0_N and Ne.int (K) = 0_N. Now Ne.S.cl (K) = K, since K is fuzzy semi-closed set in (X, N_T) implies that Ne.int (Ne.S.cl(K)) =Ne.int (K) = 0_N. Hence Ne.int (Ne.S.cl (K)) = 0_N.

Proposition 2.8: If K is a Ne.S.N.D. set and L is any Ne.Set in a Ne.T.S. (X, N_T), then (K$\cap L$) is a Ne.S.N.D. set in (X, N_T).

Proof: Let K be a Ne.S.N.D. set in (X, N_T). Then, Ne.S.int (Ne.S.cl (K)) = 0. Now Ne.S.int (Ne.S.cl (K$\cap L$)) $\subseteq$ Ne.S.int (Ne.S.cl (K)) $\cap$ Ne.S.int (Ne.S.cl (L)) $\subseteq$ $0_N \cap$ Ne.S.int (Ne.S.cl (L)) = 0_N. That is, Ne.S.int (Ne.S.cl (K$\cap L$) = 0_N. Hence (K$\cap L$) is a Ne.S.N.D. set in (X, N_T).

Definition 2.3 A Ne.S. K in Ne.T.S. (X; N_T) is called Neutrosophic semi dense(Ne.S.D.) if there exists no Ne.S.C.set L in (X; N_T) such that $K \subset L \subset 1_N$ .That is $Ne.S.cl(K) = 1_N$

Proposition2.9 If K is a Ne.S.D. and Ne.S.O. set in a Ne.T.S. (X, N_T) and if L $\subseteq$ 1 - K then L is a Ne.S.N.D. set in (X, N_T).

Proof: Let K be a Ne.S.D. set in (X, N_T). Then we have Ne.S.cl (K) = 1_N and Ne.S.int (K) = K. Now L $\subseteq$1-K implies that Ne.S.cl (L) $\subseteq$ Ne.S.cl (1 - K). Then Ne.S.cl (L) $\subseteq$ 1- Ne.S.int (K) = 1 - K. Hence Ne.S.cl (L) $\subseteq$ (1 - K), which implies that Ne.S.int (Ne.S.cl (L)) $\subseteq$ Ne.S.int(1- K) = 1-Ne.S.cl (K) = 1 – 1 = 0_N. That is, Ne.S.int(Ne.S.cl (L))= 0_N. Hence L is a Ne.S.N.D. set in (X, N_T).

Proposition 2.10: If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T), then 1 - K is a Ne.S.D. set in (X, N_T).

Proof: Let K b e a Ne.S.N.D. set in (X, N_T). Then, Ne.S.int (Ne.S.cl(K) = 0_N. Now K $\subseteq$ Ne.S.cl (K) implies that Ne.S.int(K) $\subseteq$ Ne.S.int (Ne.S.cl(K) = 0_N. Then Ne.S.int (K) = 0_N and Ne.S.cl(1 - K) = 1 – Ne.S.int(K) = 1 – 0_N = 1_N and hence 1 - K is a fuzzy semi-dense set in (X, N_T).

Proposition 2.11: If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T), then Ne.S.cl (K) is also a Ne.S.N.D. set in (X, N_T).

Proof: Let K be a Ne.S.N.D. set in (X, N_T). Then, Ne.S.int (Ne.S.cl (K) = 0_N. Now Ne.S.cl (Ne.S.cl (K)) = Ne.S.cl (K). Hence Ne.S.int(Ne.S.cl (Ne.S.cl (K))) = Ne.S.int (Ne.S.cl (K)) = 0_N. Therefore Ne.S.cl (K) is also a Ne.S.N.D. set in (X, N_T).

Proposition 2.12: If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T), then 1 – Ne.S.cl (K) is a Ne.S.D. set in (X, N_T).

Proof: Let K be a Ne.S.N.D. set in (X, N_T). Then, by proposition 2.11, Ne.S.cl (K) is a Ne.S.N.D. set in (X, T). Also by proposition 2.10, 1 – Ne.S.cl (K) is a Ne.S.D. set in (X, N_T).

Proposition 2.13: Let K be a Ne.S.D. set in a Ne.T.S. (X, N_T). If L is any Ne. set in (X, N_T), then L is a Ne.S.N.D. set in (X, N_T) if and only if K∩ L is a Ne.S.N.D. set in (X, N_T).

Proof: Let L be a Ne.S.N.D. set in (X, N_T). Then, Ne.S.int (Ne.S.cl (L) = 0_N. Now Ne.S.int (Ne.S.cl (K∩ L)) $\subseteq$ Ne.S.int (Ne.S.cl (K) ∩ Ne.S.int (Ne.S.cl (L)) $\subseteq$ Ne.S.int (Ne.S.cl (K)) ∩ 0_N = 0_N. That is, Ne.S.int(Ne.S.cl (K∩ L)) = 0_N. Hence (K∩ L) is a Ne.S.N.D. set in (X, N_T). Conversely, let (K∩ L) be a Ne.S.N.D. set in (X, N_T). Then Ne.S.int Ne.S.cl (K∩ L) = 0_N. Then, Ne.S.int (Ne.S.cl (K)) ∩ Ne.S.int(Ne.S.cl(L)) = 0_N. Since K is a Ne.S.D. set in (X, N_T), Ne.S.cl (K) = 1_N. Then, Ne.S.int (1_N) ∩ Ne.S.int (Ne.S.cl (L))= 0_N. That is, (1_N) ∩ Ne.S.int (Ne.S.cl (L)) = 0_N. Hence Ne.S.int (Ne.S.cl (L)) = 0_N, which means that L is a Ne.S.N.D. set in (X, N_T).

3. Neutrosophic Semi Baire Spaces

Definition 3.1. Let (X, N_T) be a Ne.T.S. A Ne. Set K in (X, N_T) is called Neutrosophic semi first category(Ne.S.F.C.) if A = $\bigcup_{i=1}^{\infty} A_i$ where A_i's are Ne.S.N.D. sets in (X, N_T). Any other Ne. set in (X, N_T) is said to be of Neutrosophic semi second category(Ne.S.S.C.).

Example 3.1: Let $X = \{k, l\}$. Define the Ne. set K, L ,M and N on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.6}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right) \right\rangle$$

$$M = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.5}\right) \right\rangle$$

$$N = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.7}\right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L\}$ is Ne.T. on X. Thus (X, N_T) is a Ne.T.S.. Now the sets $\overline{K}, \overline{L}, M, N$ are Ne.S.N.D. set and $\left[\overline{K} \cup \overline{L} \cup M \cup N\right] = \overline{L}$ is Ne.S.F.C. set in (X, N_T)

Definition 3.2: Let K be a Ne.S.F.C. set in a Ne..S. (X, N_T). Then 1 - K is called a Neutrosophic semi-residual (Ne.S.R.) set in (X, N_T).

Proposition 3.1: If K is a Ne.S.F.C. set in a Ne.T.S. (X, N_T), then 1–K = $\bigcap_{i=1}^{\infty} K_i$, where Ne.S.cl(L_i) = 1_N.

Proof: Let K be a Ne.S.F.C. set in (X, N_T). Then we have K = $\bigcup_{i=1}^{\infty} K_i$), where K_i 's are Ne.S.N.D. in (X, N_T). Now 1– K = $\bigcap_{i=1}^{\infty} K_i$. Let L_i = 1 – K_i. Then 1-K = $\bigcap_{i=1}^{\infty} L_i$. Since K_i's are Ne.S.N.D. sets in (X, N_T), by proposition 2.10, we have 1-K 's are Ne.S.D. sets in (X, N_T). Hence Ne.S.cl (L_i) = Ne.S.cl (1- K_i) =1_N. Therefore we have 1-K = $\bigcap_{i=1}^{\infty} L_i$ where Ne.S.cl (L_i) = 1_N.

Definition 3.3: A Ne.T.S. (X, N_T) is called a Ne.S.F.C. space if the Ne. set 1_N is a Ne.S.F.C. set in (X, N_T). That is, 1_N = $\bigcup_{i=1}^{\infty} K_i$ where K_i's are Ne.S.N.D. sets in (X, N_T). Otherwise (X, N_T) will be called a Ne.S.S.C. space.

Proposition 3.2: If K is a Ne.S.C. set in a Ne.T.S. (X, N_T) and if Ne.S.int (K) = 0_N, then K is a NeS.N.D. set in (X, N_T).

Proof: Let K be a Ne.S.C. set in (X, N_T). Then we have Ne.S.cl (K) = K. Now Ne.S.int (Ne.S.cl (K) = Ne.S.int (K) and Ne.S.int(K) = 0_N, implies that Ne.S.int(Ne.S.cl(K))= 0_N. Hence K is a Ne.S.N.D. set in (X, N_T).

Definition 3.4: Let (X, N_T) be a Ne.T.S.. Then (X, N_T) is called a Neutrosophic semi-Baire space(Ne.S.B.) if Ne.S.int [$\bigcup_{i=1}^{\infty} K_i$] = 0_N, where K_i's are Ne.S.N.D. sets in (X, N_T).

Example 3.2: Let *X = {k, l}*. Define the Ne. set k, L ,M and N on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right)\right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.6}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right)\right\rangle$$

$$M = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.5}\right)\right\rangle$$

$$N = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.7}\right)\right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L\}$ is Ne.T. on X. Thus (X, N_T) is a Ne.T.S.. Now the sets $\overline{K}, \overline{L}, M, N$ are Ne.S.N.D. set and $\left[\overline{K} \cup \overline{L} \cup M \cup N\right]$=Ne.S.int ($\overline{L}$) = 0_N is Ne.S.B. space.

Example 3.3: Let *X = {k, l}*. Define the Ne.Sets K, L and M on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.6}\right), \left(\frac{k}{0.5}, \frac{l}{0.2}\right), \left(\frac{k}{0.4}, \frac{l}{0.5}\right)\right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.2}, \frac{l}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.1}\right)\right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L, K \cup L, K \cap L\}$ is Ne.T on X. Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, \overline{K \cup L}$ are Ne.S.N.D set and Ne.S.int $(\overline{K} \cup \overline{L} \cup (\overline{K \cup L})) = Ne.S.\text{int}(\overline{K \cap L}) \neq 0_N$. Hence the Ne.T.S. (X, N_T) is not Ne.S.B. space.

Proposition 3.3:` If Ne.S.int $(\bigcup_{i=1}^{\infty} K_i)$, = 0_N, where Ne.S.int (K_i) = 0_N and K_i 's are Ne.S.C. sets in a Ne.T.S. (X, N_T), then (X, N_T) is a Ne.S.B. space.

Proof: Let K_i 's be Ne.S.C. sets in (X, N_T). Since Ne.S.int (K_i) = 0_N, by proposition 3.2, the K_i 's are Ne.S.N.D. sets in (X, N_T). Therefore we have Ne.S.int $(\bigcup_{i=1}^{\infty}(K_i))$ = 0_N, where K_i 's are fuzzy semi-nowhere dense sets in (X, N_T). Hence (X, N_T) is a Ne.S.B. space.

Proposition 3.4:

If Ne.S.cl$(\bigcap_{i=1}^{\infty}(K_i))$ = 1_N, where K_i's are Ne.S.D. and Ne.S.O. sets in a Ne.T.S. (X, N_T), then (X, N_T) is a Ne.S.B. space.

Proof:

Now Ne.S.cl $(\bigcap_{i=1}^{\infty}(K_i))$ = 1_N implies that 1-Ne.S.cl $(\bigcap_{i=1}^{\infty}(K_i))$ = 0_N. Then we have

Ne.S.int (1-$\bigcap_{i=1}^{\infty} K_i$) = 0_N,which implies that Ne.S.int $(\bigcup_{i=1}^{\infty} 1-K_i)$= 0_N. Since K_i's are Ne.S.D. sets in (X, N_T), Ne.S.cl (K_i) = 1_N and Ne.S.int(1- K_i) = 1-Ne.S.cl (K_i) = 1-1_N = 0_N. Hence we have Ne.S.int $(\bigcup_{i=1}^{\infty}(1-K_i))$ = 0_N, where Ne.S.int (1- K_i) = 0 and (1- K_i)'s are Ne.S.C. sets in (X, N_T). Then, by proposition 3.3, (X, N_T) is a Ne.S.B. space.

Proposition 3.5: Let (X, N_T) be a Ne.T.S. The$\bigcup_{i=1}^{\infty} K_i$ n the following are equivalent:

(1). (X, N_T) is a Ne.S.B. space.
(2). Ne.S.int (K) = 0_N for everyone.S.F.C. set K in (X, N_T).
(3). Ne.S.cl (L) = 1_N for every Ne.S.R. set in (X, N_T).

Proof: (1) → (2). Let K be a Ne.S.F.C. set in (X, N_T). Then K =$\bigcup_{i=1}^{\infty} K_i$, where K_i's are Ne.S.N.D. sets in (X, N_T). Now Ne.S.int (K) = Ne.S.int $(\bigcup_{i=1}^{\infty} K_i)$ = 0_N (since (X, N_T) is a Ne.S.B. space). Therefore Ne.S.int (K) = 0_N.

(2) → (3). Let L be a Ne.S.R. set in (X, N_T). Then 1-L is a Ne.S.F.C set in (X, N_T). By hypothesis, Ne.S.int (1-L) = 0_N which implies that 1- Ne.S.cl (L) = 0_N.
Hence we have Ne.S.cl (L) = 1_N.

(3)→ (1). Let K be a Ne.S.F.C.set in (X, N_T). Then K = $\bigcup_{i=1}^{\infty} K_i$ where K_i's are Ne.S.N.D.sets in (X, N_T). 1-K is a Ne.S.R. set in (X, N_T). Since K is a Ne.S.F.C. set in (X, N_T), By hypothesis, we have Ne.S.cl (1-

K) = 1_N. Then 1-Ne.S.int (K) = 1_N, which implies that Ne.S.int (K) = 0_N. Hence Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = 0_N where K_i's are Ne.S.N.D. sets in (X, N_T). Hence (X, N_T) is a Ne.S.B. space.

Proposition 3.6: If a fuzzy topological space (X, N_T) is a Ne.S.B. space, then (X, N_T) is a Ne.S.S.C.space.

Proof: Let (X, N_T) be a Ne.S.B. space. Then Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = 0_N where K_i's are Ne.S.N.D. sets in (X, N_T). Then $\bigcup_{i=1}^{\infty} K_i \neq 1_N$. (Suppose, $\bigcup_{i=1}^{\infty} K_i = 1_N$ implies that Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = Ne.S.int($1_N$) which implies that $0_N = 1_N$, a contradiction). Hence (X, N_T) is a Ne.S.S.C. space.

Remarks 3.6: The converse of the above proposition need not be true. A Ne.S.S.C. space need not be Ne.S.B. space.

Example 3.4: Let $X = \{k, l\}$. Define the Ne.Sets K and L on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.6}\right), \left(\frac{k}{0.5}, \frac{l}{0.2}\right), \left(\frac{k}{0.4}, \frac{l}{0.5}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.2}, \frac{l}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.1}\right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L, K \cup L, K \cap L\}$ is Ne.T on X. Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, \overline{K \cup L}$ are Ne.S.N.D set and $(\overline{K} \cup \overline{L} \cup \overline{(K \cup L)}) = (\overline{K \cap L}) \neq 1_N$ & $Ne.S.\text{int}(\overline{K \cap L}) \neq 0_N$.

Hence the Ne.S.S.C. space need not be Ne.S.B.space.

Proposition 3.7: If a Ne.T.S. (X, N_T) is a Ne.S.B. space, then no non-zero Ne.S.O. set in (X, N_T) is a fuzzy semi-first category set in (X, N_T).

Proof: Suppose that K is a non-zero Ne.S.O. set in (X, N_T) such that K = $\bigcup_{i=1}^{\infty} K_i$, where K_i 's are Ne.S.N.D. sets in (X, N_T). Then we have Ne.S.int (K) = Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$). Since K is a non-zero Ne.S.O. set in (X, N_T) Ne.S.int(K) = K. Then Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = K ≠ 0. But this is a contradiction to (X, N_T) being a Ne.S.B. space, in which Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = 0, where K_i 's are Ne.S.N.D. sets in (X, N_T). Hence we must have A ≠ ($\bigcup_{i=1}^{\infty} K_i$).

Therefore no non-zero Ne.S.O. set in (X, N_T) is a Ne.S.F.C. set in (X, N_T).

Proposition 3.8: A Ne.S.B. space is a Ne.B. space. For consider the following example:

Example 3.5: Let $X = \{k, l\}$. Define the Ne. set K, L ,M and N on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.6}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right) \right\rangle$$

$$M = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.5}\right) \right\rangle$$

$$N = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.7}\right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L\}$ is Ne.T. on X. Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, M, N$ are Ne.S.N.D. set and Ne.S.int$\left[\overline{K} \cup \overline{L} \cup M \cup N\right]$= Ne.S.int($\overline{L}$)= 0_N Hence the Ne.T.S. (X, N_T) is Ne.S.B. space.

Here the sets $\overline{K}, \overline{L}, M, N$ are Ne.N.D. set and Ne.int$\left[\overline{K} \cup \overline{L} \cup M \cup N\right]$= Ne.int($\overline{L}$)= 0_N .Hence Ne.S.B. space is a Ne.B. space

Conclusions

Many different forms of closed sets have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, : In this paper, we introduced the concept of Neutrosophic Semi Baire spaces in Neutrosophic Topological Spaces. Also we define Neutrosophic Semi-nowhere dense, Neutrosophic Semi-first category and Neutrosophic Semi-second category sets. Some of its characterizations of Neutrosophic Semi-Baire spaces are also studied. This shall be extended in the future Research with some applications

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Conflicts of Interest

The authors declare no conflict of interest.

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Decomposition of Matrix under Neutrosophic Environment

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Abstract: Matrices help for the effective representation of systems of linear equations and analyzing any sort of data. The decomposition of any matrix allows for the efficient implementation of matrix-based algorithms. Spectral decomposition is one of the approaches commonly used for square symmetric matrices in order to spell out variation for each of the involved components. The Neutrosophic environment is based on square symmetric matrices and likely to call Spectral decomposition. Neutrosophic is the branch of philosophy that deals with nature, the scope of neutralities and their associations with changed ideational spectra. It is the generalization of the classical set, classical fuzzy set, and intuitionistic fuzzy set. These set theories often limited to handle the problem of uncertainty. Neutrosophic basically based on three possibilities; like Degree of Truth (T), Degree of Falsehood (F) and Degree of Indeterminacy (I).In real-life uncertainties commonly happened and so neutrosophic plays an important role to measure those uncertainties such as inexplicit statements, specious or inadequate information. In order to measure the indeterminacy, a neutrosophic matrix approach is purposed and matrix named "Square-Symmetric Neutrosophic (SSN) matrix". The SSN matrix is computed using the spectral decomposition of matrices; which do factorization of a matrix into canonical form. The increasing level of indeterminacy restrains from reaching to exact decision. If indeterminacy in (any two) SSN matrices increases, then this leads to reduce variation in data. The process is checked through the Eigenvectors which suggests that through spectral decomposition the variation of the indeterminacy in SSN matrices can be minimized.

Keywords: Neutrosophic set, Square Neutrosophic matrices, and Spectral decomposition.

1. Introduction

Neutrosophic philosophy was presented by Florentin Smarandache (Smarandache, 1999) which based on three components namely Degree of Truth(T), Degree of Falsehood(F) and Degree of Indeterminacy(I) defined on the sample space X, where these three components are fully independent. This theory has many applications in different fields such as (Ansari, Biswas, & Aggarwal, 2011; Broumi & Smarandache, 2013; Cheng & Guo, 2008; Kharal, 2014) where inconsistent, and indeterminate problems occurred. Two types of measure for bipolar and interval-valued bipolar neutrosophic sets proposed by (Abdel-Basset, Mohamed, Elhoseny, Chiclana, & Zaied, 2019). A robust ranking method with the neutrosophic set theory proposed by (Abdel-Baset, Chang, & Gamal, 2019) study the environmental performance of green supply chain management. The uncertainty mostly handle with the support of set theories but neutrosophic theory generalize these set theories (Azizzadeh, Zadeh, Zahed, & Zadeh, 1965). In decision-making problems the neutrosophic approach is used that deal and overcome the ambiguity (Abdel-Basset, Atef, & Smarandache, 2019). A neutrosophic method for assessment of Hospital medical care systems which based on plithogenic data sets presented by(Abdel-Basset, El-hoseny, Gamal, & Smarandache, 2019). For Supply Chain Sustainability a neutrosophic method is presented by (Abdel-Basset, Mohamed, Zaied, & Smarandache, 2019). Matrices play a big role in science and technology. When uncertainty involved in classical matrix different fuzzy matrices are developed using the fuzzy relation system. For this purpose different square neutrosophic matrices were proposed by (Dhar, Broumi, & Smarandache, 2014). The descriptive neutrosophic statistics using the neutrosophic logic Proposed by (Smarandache, 2014) and Neutrosophic Probability, Set, and Logic also proposed by (Smarandache, 1998). Later on, (Aslam, 2018), (Aslam, Bantan, & Khan) and (Aslam, 2019) introduced the inferential neutrosophic statistics and neutrosophic statistical quality control. (Alhabib, Ranna, Farah, & Salama, 2018) presented Some continuous Neutrosophic Probability models including the Poisson model, Exponential model and Uniform model that are applicable when uncertainty involved in data. The neutrosophic matrix operations first time introduced by (Ye, 2017) and solution methods including addition method, substitution method and inverse method also developed. (Basu & Mondal, 2015) proposed different types of Neutrosophic Soft matrix along with various mathematical operations. In medical science this application is applicable.(Uma, Murugadas, & Sriram) developed the methods of determinant and adjoint of Fuzzy Neutrosophic Matrices. (Varol & Aygün, 2019) proposed a neutrosophic matrix, whose elements are based on single-valued neutrosophic sets. In this paper, they proposed various theorems on neutrosophic matrix with basic operations. (Sumathi & Arockiarani, 2014) discussed some operations on fuzzy neutrosophic matrix and developed a decision method scheme that deal uncertainty. (Kavitha, Murugadas, & Sriram, 2018) studied the powers of a fuzzy neutrosophic soft square matrix under the function of max and min. Our aim in this paper to propose a neutrosophic matrix called "Square-Symmetric Neutrosophic (SSN) matrix, whose entries based on indeterminate part. The SSN matrix is computed using the spectral decomposition of matrices.

1.1 Fundamental and basic concepts

Definition 1.1.1 (Broumi, Bakali, Talea, Smarandache, & Selvachandran, 2017)(Neutrosophic Set)

Suppose Y be a sample space and let y ε Y. A neutrosophic set $\overline{U}$ in Y based on three components

such as truth part $T_{\overline{u}}$, an in determinant part $I_{\overline{u}}$ and falsehood part that is $F_{\overline{u}}$. All these three components are independent to each other and based on standard or on standard subsets such as $]0^-,1^+[$. In real-life applications such as engineering and scientific problems, it is recommended to use the interval [0, 1] instead of $]0^-,1^+[$ as it reduces the complicity of system. The Neutrosophic set can be defined as

$$\overline{U}=\{((y, T_{\overline{u}}(y)), (I_{\overline{u}}(y)), (F_{\overline{u}}(y)): y\varepsilon Y\} \tag{1}$$

Where the sum of these three neutrosophic components are

$$0^- \leq T_{\bar{u}}(y) + I_{\bar{u}}(y) + F_{\bar{u}}(y) \leq 3^+ \tag{2}$$

Definition 1.1.2 (Dhar et al., 2014) (Square Neutrosophic Matrix)

Let $A_{m\times m}$ *and* $B_{n\times n}$ be two square Neutrosophic matrices where indeterminacy involved in the matrices

$$A_{m\times m}=\begin{bmatrix} a_{11}I & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } B_{n\times n}=\begin{bmatrix} b_{11}I & b_{12}I & b_{13} \\ b_{21}I & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

2. Methodology

Spectral Decomposition

The spectral theorem states that any symmetric mx m or nx n matrix which has real entries have exactly m or n real but possibly not different Eigenvalues and analogous to those Eigenvalues there are mutually independent Eigenvectors. Where Eigenvector based on a linear transformation whose direction does not change when a scalar is multiplied and Eigenvalue is a scalar that is used to transform an Eigenvector. Both are used to reduce variation in data. They can also help to improve the model efficiency (LI, 2016).

Consider two square neutrosophic matrices of the same dimension and let λ be an Eigenvalue of these two matrices

If x any y be two nonzero vectors ($x \neq 0$) and ($y \neq 0$) such that $Ax = \lambda x$ and $By = \lambda y$ (3)

then x is said to be an Eigenvector of the matrix A linked with Eigenvalue λ and y is said to be an Eigenvector of matrix B linked with the Eigen value λ. An equivalent condition for λ to be a solution of the Eigenvalue- Eigenvector equation is $|A - \lambda I| = 0$ and $|B - \lambda I| = 0$.

Let $A_{m\times m}$ and $B_{n\times n}$ be two symmetric matrices. Then these two matrices can be expressed in terms of its m and n Eigen value-Eigen vector pairs (λ_i, e_i) as

$$A_{m\times m}=\sum_{i=1}^{m} \lambda_i e_i e_i' \text{ and } B_{n\times n}=\sum_{i=1}^{n} \lambda_i e_i e_i' \tag{4}$$

3. Results

The results using the proposed methodology for various values of K and I are given in Table 1.

4 Comparison

In this section, we compare the performance of the proposed method with the method under classical statistics. It is important to note that the proposed methodology of neutrosophic statistics reduces under classical statistics when K=1 and I=0. From Table 1, we note that in matrix A_k where indeterminacy involved in the first variable, so as I is increased, the variation is reduced in the first variable checked through the Eigenvectors. The same two indeterminate variables situation is presented in the matrix B_k where variation in the first two variables also reduces checked through the Eigenvectors as I increase. Therefore, it is concluded that through spectral decomposition the indeterminacy in SSN matrices can be minimized. By this comparison, it is concluded that the proposed methodology under neutrosophic statistics is useful to reduce the variation as compared to classical statistics.

Table 1: Neutrosophic matrices based on different indeterminacy (I) values.

	Consider a Neutrosophic square and symmertic matrix			
	$A_k = \begin{bmatrix} 2.2I & 0.4 \\ 0.4 & 2.8 \end{bmatrix}$		$B_k = \begin{bmatrix} 2.2I & 0.4I & 0.2 \\ 0.4I & 2.8 & 1.5 \\ 0.2 & 1.5 & 1.5 \end{bmatrix}$	
	Eigen values	**Eigen vectors**	**Eigen values**	**Eigen vectors**
K=1 and I=0	λ_1=2.856 λ_2=-0.056	$e_1{}'$=[0.139,0.99] $e_2{}'$=[-0.99,0.139]	λ_1=3.79 λ_2=0.564 λ_3=-0.052	$e_1{}'$=[-0.03,-0.83,-0.55] $e_2{}'$=[-0.28,0.53,-0.79] $e_3{}'$=[-0.96,0.132,-0.25
K=2 and I=1	λ_1=3 λ_2=2	$e_1{}'$=[0.45,0.89] $e_2{}'$=[-0.8,0.44]	λ_1=3.9 $\lambda_2 = 2.1$ λ_3=0.5	$e_1{}'$=[-0.25,-0.81,-0.53] $e_2{}'$=[0.97,-0.19,-0.17] $e_3{}'$=[0.03,-0.55,0.83]
K=3 and I=2	λ_1=4.5 λ_2=2.7	$e_1{}'$=[-.097,-0.23] $e_2{}'$=[0.23,-0.97]	λ_1=4.9 $\lambda_2 = 3.3$ λ_3=0.5	$e_1{}'$=[0.83,0.49,0.26] $e_2{}'$=[0.55,-0.66,-0.50] $e_3{}'$=[0.07,-0.56,0.82
K=4 and I=3	λ_1=6.6 λ_2=2.8	$e_1{}'$=[-099,-0.10] $e_2{}'$=[0.10,-0.99]	λ_1=7.02 $\lambda_2 = 3.41$ λ_3=0.47	$e_1{}'$=[0.94,0.31,0.12] $e_2{}'$=[0.32,-0.76,-0.56] $e_3{}'$=[0.09,-0.57,0.82]
K=5 and I=5	λ_1=11.02 λ_2=2.78	$e_1{}'$=[-0.99,-0.05] $e_2{}'$=[0.05,-0.99]	λ_1=11.49 λ_2 =3.38 λ_3=0.43	$e_1{}'$=[0.971,0.232,0.054] $e_2{}'$=[0.219,-0.775,-0.593] $e_3{}'$=[0.096,-0.588,0.830]
K=6 and I=10	λ_1=22.01 λ_2=2.79	$e_1{}'$=[-0.99,-0.021] $e_2{}'$=[0.021,-0.99]	λ_1=22.8 λ_2 =3.19 λ_3=0.29	$e_1{}'$=[0.980,0.198,0.023] $e_2{}'$=[0.166,-0.748,-0.642] $e_3{}'$=[0.109,-0.633,0.767]
K=7 and I=20	λ_1=44 λ_2=2.79	$e_1{}'$=[-0.999,-0.009] $e_2{}'$=[0.009,-0.999)]	λ_1=45.5 λ_2 =2.83 λ_3=-0.04	$e_1{}'$=[0.98,0.18,0.01] $e_2{}'$=[0.134,-0.669,-0.730] $e_3{}'$=[0.128,-0.719,0.683]
K=8 and I=50	λ_1=110 λ_2=2.79	$e_1{}'$=[-0.999,-0.004] $e_2{}'$=[0.004,-0.99]	λ_1=113.6 λ_2 =2.2 λ_3=-1.5	$e_1{}'$=[0.984,0.178,0.004] $e_2{}'$=[-0.081,0.425,0.901] $e_3{}'$=[0.158,-0.887,0.433]
K=9 and I=100	λ_1=220 λ_2=2.79	$e_1{}'$=[-0.999,-0.002] $e_2{}'$=[0.002,-0.999]	λ_1=227.13 λ_2 =1.84 λ_3=-4.67	$e_1{}'$=[0.984,0.176,0.002] $e_2{}'$=[-0.042,0.224,0.974] $e_3{}'$=[0.17,-0.96,0.23]
K=10 and I=200	λ_1=440 λ_2=2.79	$e_1{}'$=[-0.99,-0.0009] $e_2{}'$=[0.0009,-0.99]	λ_1=454.18 λ_2 =1.66 λ_3=-11.54	$e_1{}'$=[0.984,0.175,0.001] $e_2{}'$=[-0.020,0.108,0.994] $e_3{}'$=[0.173,-0.979,0.109]

5 Conclusions

Sometime the simple matrix theory often limited to handle the problem of uncertainty. The neutrosophic matrix deals the uncertainty, which based on three components including truth component, an indeterminate component and falsehood component. This paper focused on SSN matrices where indeterminacy involved in its variables. So the spectral decomposition analysis is performed that requires a square and symmetric matrix. The proposed method is quite effective to be applied in indeterminacy. The increasing level of indeterminacy restrains from reaching to exact decision. If indeterminacy in two SSN matrices increases, then this leads to reduce variation in data. The process is checked through the Eigenvectors, which suggests that through spectral decomposition the variation of the indeterminacy in SSN matrices can be minimized.

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Conflicts of Interest

The authors declare no conflict of interest.

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Multi-Valued Interval Neutrosophic Soft Set: Formulation and Theory

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Abstract: Neutrosophic set is a powerful general formal framework. A lot of studies on neutrosophic had been proposed and recently, in multi-valued interval values. However, sometimes there is problem involving elements of ambiguity and uncertainties in which the function of membership is difficult to be set in a particular case. Clearly, these problems can be solved by soft set since it is able to solve the lack of parameterization tool of theory. Thus, this paper introduces a concept of multi-valued interval neutrosophic soft set which amalgamates multi-valued interval neutrosophic set and soft set. The proposed set extends the notions of fuzzy set, intuitionistic fuzzy set, neutrosophic set, interval-valued neutrosophic set, multi-valued neutrosophic set, soft set and neutrosophic soft set. Further, we study some basic operations such as complement, equality, inclusion, union, intersection, "AND" and "OR" for multi-valued interval neutrosophic soft elements and discuss its associated properties. Moreover, the derivation of its properties, related examples and some proofs on the propositions are included.

Keywords: multi-valued interval neutrosophic set; multi-valued interval neutrosophic soft set; neutrosophic set, soft set

1. Introduction

Fuzzy set (FS) was firstly initiated by Zadeh [1] in order to solve the decision-making problems with fuzzy information. However, FS only considers single membership function to represent vague data. Moreover, the membership degree alone is unable to describe the information in some cases of decision-making problems. Thus, Atanassov [2] introduced intuitionistic fuzzy set (IFS) in order to measure both membership degree and non-membership degree of elements in universal set. Then, the IFSs have been extended by many researchers and have been applied in some real applications. However, the membership and non-membership degrees values in IFSs are independent with the sum of degrees of membership and non-membership is less than unity. Moreover, it is unable to cope with the indefinite and inconsistent information which exist in belief system. Both FSs and IFSs may not deal with indeterminacy in real decision-making problem. Indeterminacy is an important part in decision-making process. For example, in a survey form, there are three choices 'YES / NO/ N. A.', while for gender, Male/ Female/ Others. So, different types of uncertainty and ambiguity with indeterminacy cannot be explained by the fuzzy concept or intuitionistic fuzzy concept. Thus, Smarandache [3] proposed the theory of neutrosophic set (NS) in 1995. The concept of NS which introduced by Smarandache [4] is a mathematical tool that handles the problems with inconsistent and imprecise data. It also has been proved that the NS is a continuation of the intuitionistic fuzzy sets [5]. An NS is represented by the truth-membership function, indeterminacy-membership function, and falsity-membership function respectively, where $]^{-}0,1^{+}[$ is the non-standard interval. Basically, it is the generalization to the standard interval in the intuitionistic fuzzy sets [2] which is $[0,1]$. The uncertainty that represented by the indeterminacy factor is independent of truth and falsity values, while the integrated ambiguity is dependent of the degree of belongingness and the degree of non-belongingness in IFS. Nowadays, the studies on the NS theory have been developed actively [6]–[13]. However, since operators necessary to be specified, there is difficulty to apply NS in some real situations. Thus, Wang et al. [14] proposed single-valued neutrosophic set (SVNS) and since then, there are many researches related to SVNS have been conducted [9–18].

Despite its success, the truth-membership, indeterminacy-membership and falsity-membership in SVNS may not be written in one specific number for some cases. Thus, interval-valued neutrosophic set (IVNS) was introduced by Wang et al. [25], so that the values of truth-membership, indeterminacy-membership and falsity-membership are determined in intervals rather than real numbers. Also, IVNS may represent the indefinite, inaccurate, inadequate and inconsistent information which is always exist in real world. Numerous real world applications of IVNS have been studied by number of researchers [20–25]. In another perspective, the value of neutrosophic elements also not always be a single real number. Thus, Wang and Li [32] generalized SVNS into multi-valued neutrosophic set (MVNS), where the values of truth-membership, indeterminacy-membership and falsity-membership are represented in several real numbers rather than one single real number [27–30]. Nevertheless, in some complicated decision problems, several decision makers can refuse to give any evaluation values if they are unfamiliar with the characteristics of decision-making. Consequently, Broumi et al. [37] proposed multi-valued interval neutrosophic set (MVINS) in order to cope with complex decision problems which involving multiple decision makers and the evaluation values of decision makers are given in form of multi-valued interval neutrosophic values. Then, it has been discussed by other scholars such as Fan and Ye [38], Yang and Pang [39] and Samuel and Narmadhagnanam [40].

Apart from NS based sets, the soft set is just another set that can be used to deal with uncertain and vague information. Molodtsov [41] who is a Russian mathematician, had solved the difficult problem involving uncertainty by proposing a new mathematical tool called "soft set theory". This theory is free from the difficulties on how to set the function of membership in a particular case and inadequacy of parameterization tool of theory. After Molodtsov's work, the soft set (SS) theory has been studied widely in numerous applications, like lattices [36–38], topology [39–41], algebraic structures [42–46], game theory [47,48], medical diagnosis [55], perron integration [56], data analysis and operations research [51–54], optimization [61] and decision-making under uncertainty [56–59]. In recent years, SS theory has been extended by embedding the ideas of other sets. For example, Maji et al. [66] firstly integrated the beneficial properties of SS and FS. Theory of fuzzy soft set (FSS) has been studied by many scholars. For instance, Cagman et al. [67] defined the theory of fuzzy soft set (FSS) and studied the related properties. Roy and Maji [68] discussed some results on the implementation of FSS in solving the problem of object recognition. Kong et al. [69] gave a comment on Roy and Maji's paper [68], by providing a counter-example to show the problem. Then, Maji [70] studied the theory of NS which proposed by Smarandache [4] and combined it with soft set to become a novel mathematical model, which is called neutrosophic soft set (NSS). After the introduction of the NSS, Karaaslan [71] redefined the NSS notion and its operations to make it become more useful. The NSS has been applied to solve decision-making problem. Mukherjee and Sarkar [72] also discussed about NSSs. They solved a medical diagnosis decision-making problem based on the NSS. Şahin and Küçük [73] introduced a novel style of NSS notion and studied some algebraic properties. Sumathi and Arockiarani [74] also studied the NSSs. Cuong et al. [75] reanalyzed the notion of NSS and discussed the basic properties of NSS, neutrosophic soft relations and neutrosophic soft compositions. Hussain and Shabir [76] investigated the algebraic operations of NSS and the properties related to the operations. Mukherjee and Sarkar [77] defined new similarity measure and weighted similarity measure between two NSSs. Maji [78] verified some operations of weighted NSSs. Chatterjee et al. [79] studied the single-valued NSSs and some uncertainty based measures. Marei [80] proposed single valued neutrosophic soft approach to rough sets based on neutrosophic right minimal structure. Then, some scholars generalized the NSS into interval form by combining the IVNS with SS. This combination is known as interval-valued neutrosophic soft set (IVNSS) and it can deal with the problem in interval form with uncertainty. Deli [81] firstly introduced the definitions and operations of IVNSS and developed decision-making approach based on level soft sets of IVNSS. Mukherjee and Sarkar [82] defined Hamming and Euclidean distance for two IVNSSs. They also studied the similarity measure based on set theoretic approach. Broumi et al. [83] introduced the relations on IVNSS and presented the several properties such as symmetry, reflexivity and transitivity of the proposed relations. Another extension of NSS set has been done by some researchers to solve the problem in several real numbers with uncertainty. The multi-valued neutrosophic soft set (MVNSS) was proposed by Alkhazaleh [84]. A theoretical study on MVNSS properties and operations have been made and an MCDM approach based on the proposed set has been provided. Alkhazaleh and Hazaymeh [85] also discussed about the MVNSS and introduced an MCDM approach based on the set. It can be seen that there are a lot of researches that integrate the NS theory with SS theory. However, the NSS need to be specified from a point of view and since very little information of MVINS combines with NS is available in literatures, thus, we fill this gap by presenting a new set which integrate two existing concepts of MVINS introduced by Broumi et al. [37] and SS introduced by Molodtsov [41]. To accompaniment the concept of MVINSS, some basic operations for MVINSS which namely complement, union, intersection, equality, inclusion, "AND" and "OR" operations the proposed. The structure of this paper is listed as follows. In section 2, the related definitions and concepts for developing MVINSS are presented. Some proving on the propositions are included. Section 3 proposes the MVINSS and its associated properties together with example. Finally, we conclude the paper in section 4.

2. Preliminaries

In this section, we present some definitions and properties which are related to neutrosophic set, single-valued neutrosophic set, interval-valued neutrosophic set, multi-valued neutrosophic set, soft set and neutrosophic soft set.

2.1. Neutrosophic Set

Definition 2.1 [3] Let U be a universe of discourse, then NS A can be defined as
$A=\{<\tau_A(y),\,\delta_A(y),\,\lambda_A(y)>/\,y, y\in U\}$
where $\tau,\delta,\lambda:U\to\,]^-0,1^+[$ define the degree of truth-membership $\tau_A(y)$, degree of indeterminacy $\delta_A(y)$ and degree of falsity $\lambda_A(y)$ respectively and there is no restriction on the sum of $\tau_A(y),\delta_A(y)$ and $\lambda_A(y)$, so $^-0\le\tau_A(y)+\delta_A(y)+\lambda_A(y)\le 3^+.$
From philosophical point of view, the NS takes the value from real standard or non-standard subsets of $]^-0,1^+[$. Thus for technical applications, we need to take the interval $[0,1]$ instead of $]^-0,1^+[$ because it is hard to apply in the real applications such as problems in scientific and engineering.

2.2. Single-Valued Neutrosophic Set

Definition 2.2 [14] Let U be a universal set, with generic element of U denoted by $y.$ An SVNS A over U is defined as $A=\{<\tau_A(y),\delta_A(y),\lambda_A(y)>/y,\,y\in U\}$ It is characterized by a truth-membership function $\tau_A(y),$ indeterminacy-membership function $\delta_A(y)$ and falsity-membership function $\lambda_A(y),$ with for each $y\in U,\tau_A(y),\delta_A(y),\lambda_A(y)\in[0,1]$ and $0\le\tau_A(y)+\delta_A(y)+\lambda_A(y)\le 3.$

2.3. Interval-Valued Neutrosophic Set

Definition 2.3 [25] Let U be a space of points with generic elements in U denoted by $y.$ An IVNS $\hat{A}$ over U is characterized by truth-membership interval $\hat{\tau}_{\hat{A}}(y),$ indeterminacy-membership interval $\hat{\delta}_{\hat{A}}(y)$ and falsity-membership interval $\hat{\lambda}_{\hat{A}}(y).$ It can be defined as

$$\hat{A}=\{<\hat{\tau}_{\hat{A}}(y),\hat{\delta}_{\hat{A}}(y),\hat{\lambda}_{\hat{A}}(y)>/y,\,y\in U\}$$

$\hat{\tau}_{\hat{A}}(y)=[\hat{\tau}^-_{\hat{A}}(y),\,\hat{\tau}^+_{\hat{A}}(y)],\,\hat{\delta}_{\hat{A}}(y)=[\hat{\delta}^-_{\hat{A}}(y),\,\hat{\delta}^+_{\hat{A}}(y)],\,\hat{\lambda}_{\hat{A}}(y)=[\hat{\lambda}^-_{\hat{A}}(y),\,\hat{\lambda}^+_{\hat{A}}(y)]\subseteq[0,1]$ and $0\le[\hat{\tau}^+_{\hat{A}}(y)+\hat{\delta}^+_{\hat{A}}(y)+\hat{\lambda}^+_{\hat{A}}(y)]\le 3,\,y\in U.$ It only considers the subunitary interval of $[0,1].$

2.4. Multi-Valued Neutrosophic Set

Definition 2.4 [32] Let U be a space of points (objects), with a generic element in U denoted by $y.$

An MVNS $\tilde{A}$ over U is characterized by $\tilde{A}=\{<\tilde{\tau}^l_{\tilde{A}}(y),\tilde{\delta}^m_{\tilde{A}}(y),\tilde{\lambda}^n_{\tilde{A}}(y)>/y,\,y\in U\}$

where $\tilde{\tau}^l_{\tilde{A}}(y)=\tilde{\tau}^1_{\tilde{A}}(y),\,\tilde{\tau}^2_{\tilde{A}}(y),\ldots,\,\tilde{\tau}^q_{\tilde{A}}(y),$ $\tilde{\delta}^m_{\tilde{A}}(y)=\tilde{\delta}^1_{\tilde{A}}(y),\,\tilde{\delta}^2_{\tilde{A}}(y),\ldots,\,\tilde{\delta}^r_{\tilde{A}}(y)$ and $\tilde{\lambda}^n_{\tilde{A}}(y)=\tilde{\lambda}^1_{\tilde{A}}(y),\,\tilde{\lambda}^2_{\tilde{A}}(y),\ldots,\,\tilde{\lambda}^s_{\tilde{A}}(y)$ are three sets in the form of subset of $[0,1],$ denoting the truth-membership sequence $\tilde{\tau}^l_{\tilde{A}}(y),$ indeterminacy-membership sequence $\tilde{\delta}^m_{\tilde{A}}(y)$ and falsity-membership sequence $\tilde{\lambda}^n_{\tilde{A}}(y)$ respectively, satisfying $0\le\tilde{\tau}^l_{\tilde{A}}(y),\,\tilde{\delta}^m_{\tilde{A}}(y),\,\tilde{\lambda}^n_{\tilde{A}}(y)\le 1$ and $0\le\tilde{\tau}^l_{\tilde{A}}(y),\,\tilde{\delta}^m_{\tilde{A}}(y),\,\tilde{\lambda}^n_{\tilde{A}}(y)\le 3$ for $l=1,2,\ldots,q,\;m=1,2,\ldots,r,\;n=1,2,\ldots,s$ for all $y\in U.$ Also, l,m,n are called as the dimension of MVNS.
If U has only one element, then $\tilde{A}$ is called a multi-valued neutrosophic number (MVNN), denoted by $\tilde{A}=\left\langle\tilde{\tau}^l_{\tilde{A}}(y),\tilde{\delta}^m_{\tilde{A}}(y),\tilde{\lambda}^n_{\tilde{A}}(y)\right\rangle.$ For convenience, an MVNN can be denoted by $\tilde{A}=\left\langle\tilde{\tau}^l_{\tilde{A}},\tilde{\delta}^m_{\tilde{A}},\tilde{\lambda}^n_{\tilde{A}}\right\rangle.$
The set of all MVNNs is represented as MVNS.

2.5. Multi-Valued Interval Neutrosophic Set

Definition 2.5 [37] Let U be a space of points (objects), with a generic element in U denoted by $y.$ An MVINS $\dddot{A}$ over U can be defined as

$$\dddot{A} = \{< \dddot{\tau}_{\dddot{A}}^{l} \leq (y), \dddot{\delta}_{\dddot{A}}^{m}(y), \dddot{\lambda}_{\dddot{A}}^{n}(y) > / y,\ y \in U\}$$

where

$\dddot{\tau}_{\dddot{A}}^{l}(y)=[\dddot{\tau}_{\dddot{A}}^{1\,-}(y), \dddot{\tau}_{\dddot{A}}^{1\,+}(y)], [\dddot{\tau}_{\dddot{A}}^{2\,-}(y), \dddot{\tau}_{\dddot{A}}^{2\,+}(y)], \ldots, [\dddot{\tau}_{\dddot{A}}^{q\,-}(y), \dddot{\tau}_{\dddot{A}}^{q\,+}(y)]$, $\dddot{\delta}_{\dddot{A}}^{m}(y)=[\dddot{\delta}_{\dddot{A}}^{1\,-}(y), \dddot{\delta}_{\dddot{A}}^{1\,+}(y)], [\dddot{\delta}_{\dddot{A}}^{2\,-}(y), \dddot{\delta}_{\dddot{A}}^{2\,+}(y)], \ldots, [\dddot{\delta}_{\dddot{A}}^{r\,-}(y), \dddot{\delta}_{\dddot{A}}^{r\,+}(y)]$, $\dddot{\lambda}_{\dddot{A}}^{n}(y)=[\dddot{\lambda}_{\dddot{A}}^{1\,-}(y), \dddot{\lambda}_{\dddot{A}}^{1\,+}(y)], [\dddot{\lambda}_{\dddot{A}}^{2\,-}(y), \dddot{\lambda}_{\dddot{A}}^{2\,+}(y)], \ldots, [\dddot{\lambda}_{\dddot{A}}^{s\,-}(y), \dddot{\lambda}_{\dddot{A}}^{s\,+}(y)] \in U\}$ such that $0 \leq \tilde{\tau}_{\tilde{A}}^{l+}(y), \tilde{\delta}_{\tilde{A}}^{m+}(y), \tilde{\lambda}_{\tilde{A}}^{n+}(y) \leq 3,$ for all $l=1, 2, \ldots, q,\ m=1, 2, \ldots, r,\ n=1, 2, \ldots, s.$

In this research, dimension of the interval truth-membership sequence $\dddot{\tau}_{\dddot{A}}^{l}(y),$ interval indeterminacy-membership sequence $\dddot{\delta}_{\dddot{A}}^{m}(y)$ and interval falsity-membership sequence $\dddot{\lambda}_{\dddot{A}}^{n}(y)$ of the element y are considered as equal that is $q=r=s,$ respectively. Also, l, m, n are called the dimension of MVINS $A.$ Obviously, when the values of upper and lower of $\dddot{\tau}_{\dddot{A}}^{l}(y), \dddot{\delta}_{\dddot{A}}^{m}(y), \dddot{\lambda}_{\dddot{A}}^{n}(y)$ are equal, then the MVINS is reduced to MVNS.

2.6. Soft Set

Definition 2.6 [41] Let U be an initial universe set and E be a set of parameters. Consider $A \subset E.$ Let $P(U)$ denotes the power SS of U. A pair (L, A) is called a SS over U and the function L is a mapping defined by $L: A \to P(U)$ such that $L(\varepsilon)(y)=\phi$ if $y \notin U.$

Here, $L(\varepsilon)$ is called approximate function of the soft set $(L, A),$ and the value $L(\varepsilon)(y)$ is a set called x-element of the soft set for all $y \in U.$ The sets may be arbitrary, empty, or have non-empty intersection.

2.7. Neutrosophic Soft Set

Definition 2.7 [70]

Let U be an initial universe set and E be a set of parameters. Consider $A \subset E.$ Let $P(U)$ denotes the set of all NSS of U. The collection (L, A) is called an NSS over U and the function $L(\varepsilon)$ is a mapping defined by $L: A \to P(U)$ such that $L(\varepsilon)(y)=\phi$ if $y \notin U.$

(L, A) is characterized by $\tau_{L(\varepsilon)}(y), \delta_{L(\varepsilon)}(y)$ and $\lambda_{(\varepsilon)}(y).$ in the form of subset of $[0,1]$ and here, $L(\varepsilon)$ is called approximate function of the NSS $(L, A),$ such that

$(L,\ A) = \{< \tau_{L(\varepsilon)}(y),\ \delta_{L(\varepsilon)}(y),\ \lambda_{L(\varepsilon)}(y) > / y;\ \forall \varepsilon \in A,\ y \in U\}$

where $\tau_{L(\varepsilon)}(y), \delta_{L(\varepsilon)}(y)$ and $\lambda_{L(\varepsilon)}(y)$ are the truth-membership, indeterminacy-membership and falsity-membership values of object y respectively that object y holds on parameter $\varepsilon.$

2.8. Interval-Valued Neutrosophic Soft Set

Definition 2.8 [81]

Let U be an initial universe set and E be a set of parameters. Consider $A \subset E.$ Let $P(U)$ denotes the set of all IVNSS of U. The collection $(\hat{L}, A)$ is called an IVNSS over U and the function $\hat{L}(\varepsilon)$ is a mapping defined by $\hat{L}: A \to P(U)$ such that $\hat{L}(\varepsilon)(y)=\phi$ if $y \notin U.$

$(\hat{L}, A)$ is characterized by $\hat{\tau}_{\hat{L}(\varepsilon)}(y), \hat{\delta}_{\hat{L}(\varepsilon)}(y)$ and $\hat{\lambda}_{\hat{L}(\varepsilon)}(y)$ in the interval form of subset of $[0,1]$ and here, $\hat{L}(\varepsilon)$ is called approximate function of the IVNSS $(\hat{L}, A),$ such that

$$(\hat{L},\ A) = \{< \hat{\tau}_{\hat{L}(\varepsilon)}(y),\ \hat{\delta}_{\hat{L}(\varepsilon)}(y),\ \hat{\lambda}_{\hat{L}(\varepsilon)}(y) > / y;\ \forall \varepsilon \in A,\ y \in U\}$$

where $\hat{\tau}_{\hat{L}(\varepsilon)}(y)=[\hat{\tau}_{\hat{L}(\varepsilon)}^{-}(y),\hat{\tau}_{\hat{L}(\varepsilon)}^{+}(y)]$, $\hat{\delta}_{\hat{L}(\varepsilon)}(y)=[\hat{\delta}_{\hat{L}(\varepsilon)}^{-}(y),\hat{\delta}_{\hat{L}(\varepsilon)}^{+}(y)]$ and $\hat{\lambda}_{\hat{L}(\varepsilon)}(y)=[\hat{\lambda}_{\hat{L}(\varepsilon)}^{-}(y),\hat{\lambda}_{\hat{L}(\varepsilon)}^{+}(y)]$ are the interval truth-membership, interval indeterminacy-membership and interval falsity-membership respectively that object y holds on parameter ε. s

2.9. Multi-Valued Neutrosophic Soft Sets

Definition 2.9 [86] Let U be an initial universe set and E be a set of parameters. Consider $A\subset E$. Let $P(U)$ denotes the set of all MVNSS of U. The collection $(\tilde{L},A)$ is called an MVNSS over U and the function $\tilde{L}(\varepsilon)$ is a mapping defined by $\tilde{L}:A\to P(U)$ such that $\tilde{L}(\varepsilon)(y)=\phi$ if $y\notin U$.
$(\tilde{L},A)$ is characterized by $\tilde{\tau}_{\tilde{L}(\varepsilon)}(y),\tilde{\delta}_{\tilde{L}(\varepsilon)}(y)$ and $\tilde{\lambda}_{\tilde{L}(\varepsilon)}(y)$ in the form of subset of $[0,1]$ and here, $\tilde{L}(\varepsilon)$ is called approximate function of the MVNSS $(\tilde{L},A)$, such that

$$(\tilde{L},A)=\{<\tilde{\tau}^{l}_{\tilde{L}(\varepsilon)}(y),\tilde{\delta}^{m}_{\tilde{L}(\varepsilon)}(y),\tilde{\lambda}^{n}_{\tilde{L}(\varepsilon)}(y)>/y;\forall\varepsilon\in A,\ y\in U\}$$

where $\tilde{\tau}^{l}_{\tilde{L}(\varepsilon)}(y)=\tilde{\tau}^{1}_{\tilde{L}(\varepsilon)}(y),\tilde{\tau}^{2}_{\tilde{L}(\varepsilon)}(y),\ldots,\tilde{\tau}^{q}_{\tilde{L}(\varepsilon)}(y)$, $\tilde{\delta}^{m}_{\tilde{L}(\varepsilon)}(y)=\tilde{\delta}^{1}_{\tilde{L}(\varepsilon)}(y),\tilde{\delta}^{2}_{\tilde{L}(\varepsilon)}(y),\ldots,\tilde{\delta}^{r}_{\tilde{L}(\varepsilon)}(y)$ and $\tilde{\lambda}^{n}_{\tilde{L}(\varepsilon)}(y)=\tilde{\lambda}^{1}_{\tilde{L}(\varepsilon)}(y),\tilde{\lambda}^{2}_{\tilde{L}(\varepsilon)}(y),\ldots,\tilde{\lambda}^{s}_{\tilde{L}(\varepsilon)}(y)$ are the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence respectively that object y holds on parameter ε.

3. Proposed Multi-Valued Interval Neutrosophic Soft Set

In this section, we propose the definition of a multi-valued interval neutrosophic soft set (MVINSS) and its basic operations such as complement, inclusion, equality, union, intersection, "AND" and "OR" are defined as follows.

Definition 3.1
The pair $(\ddot{L},A)$ is called an MVINSS over $\ddot{P}(U)$, where $\ddot{L}$ is a mapping given by $\ddot{L}:A\to\ddot{P}(U)$. $\ddot{P}(U)$ denotes the set of all MVINSS of U with parameters from A and the function $\ddot{L}(\varepsilon)$ is a mapping defined by

$\ddot{L}:A\to\ddot{P}(U)$ such that $\ddot{L}(\varepsilon)(y)=\phi$ if $y\notin U$.

$(\ddot{L},A)$ is characterized by $\ddot{\tau}_{\ddot{L}(\varepsilon)}(y),\ddot{\delta}_{\ddot{L}(\varepsilon)}(y)$ and $\ddot{\lambda}_{\ddot{L}(\varepsilon)}(y)$ in the form of subset of $[0,1]$ and can be defined as follows:

$$(\ddot{L},A)=\left\{<\ddot{\tau}^{l}_{\ddot{L}(\varepsilon)}(y),\ \ddot{\delta}^{m}_{\ddot{L}(\varepsilon)}(y),\ \ddot{\lambda}^{n}_{\ddot{L}(\varepsilon)}(y)>/y;\ \forall\varepsilon\in A,\ y\in U\right\}$$

where

$\ddot{\tau}^{l}_{\ddot{L}(\varepsilon)}(y)=[\ddot{\tau}^{1}_{\ddot{L}(\varepsilon)}{}^{-}(y),\ddot{\tau}^{1}_{\ddot{L}(\varepsilon)}{}^{+}(y)],[\ddot{\tau}^{2}_{\ddot{L}(\varepsilon)}{}^{-}(y),\ddot{\tau}^{2}_{\ddot{L}(\varepsilon)}{}^{+}(y)],\ldots,[\ddot{\tau}^{q}_{\ddot{L}(\varepsilon)}{}^{-}(y),\ddot{\tau}^{q}_{\ddot{L}(\varepsilon)}{}^{+}(y)]$, $\ddot{\delta}^{m}_{\ddot{L}(\varepsilon)}(y)=[\ddot{\delta}^{1}_{\ddot{L}(\varepsilon)}{}^{-}(y),\ddot{\delta}^{1}_{\ddot{L}(\varepsilon)}{}^{+}(y)],[\ddot{\delta}^{2}_{\ddot{L}(\varepsilon)}{}^{-}(y),\ddot{\delta}^{2}_{\ddot{L}(\varepsilon)}{}^{+}(y)],$ $\ldots,[\ddot{\delta}^{r}_{\ddot{L}(\varepsilon)}{}^{-}(y),\ddot{\delta}^{r}_{\ddot{L}(\varepsilon)}{}^{+}(y)]$ and $\ddot{\lambda}^{n}_{\ddot{L}(\varepsilon)}(y)=[\ddot{\lambda}^{1}_{\ddot{L}(\varepsilon)}{}^{-}(y),\ddot{\lambda}^{1}_{\ddot{L}(\varepsilon)}{}^{+}(y)],[\ddot{\lambda}^{2}_{\ddot{L}(\varepsilon)}{}^{-}(y),\ddot{\lambda}^{2}_{\ddot{L}(\varepsilon)}{}^{+}(y)],\ldots,[\ddot{\lambda}^{s}_{\ddot{L}(\varepsilon)}{}^{-}(y),\ddot{\lambda}^{s}_{\ddot{L}(\varepsilon)}{}^{+}(y)]$ are the interval truth-membership sequence, interval indeterminacy-membership sequence and interval falsity-membership sequence respectively that object y holds on parameter ε.

An example of an MVINSS is given as follows.

Example 3.1 Let $U=\{y_1,y_2,y_3\}$ be the set of laptops under consideration and A is a set of parameters which describes the attractiveness of the laptop. Consider $A=\{\varepsilon_1=thin,\varepsilon_2=light,\varepsilon_3=cheap,\varepsilon_4=large\}$. Define a mapping $\ddot{L}:A\to\ddot{P}(U)$ as

$$\ddot{L}(\varepsilon_1)=\left\{\frac{\langle([0.2,0.6],[0.1,0.3]),([0.3,0.5],[0.1,0.4]),([0.2,0.6],[0.4,0.8])\rangle}{y_1},\right.$$
$$\frac{\langle([0.1,0.3],[0.2,0.4]),([0.3,0.6],[0.4,0.8]),([0.3,0.5],[0.2,0.7])\rangle}{y_2},$$
$$\left.\frac{\langle([0.1,0.6],[0.2,0.7]),([0.2,0.5],[0.3,0.5]),([0.5,0.8],[0.3,0.8])\rangle}{y_3}\right\},$$

$$\ddot{L}(\varepsilon_2)=\left\{\frac{\langle([0.4,0.6],[0.2,0.5]),([0.2,0.6],[0.4,0.7]),([0.6,0.9],[0.5,0.8])\rangle}{y_1},\right.$$
$$\frac{\langle([0.3,0.6],[0.3,0.5]),([0.5,0.8],[0.5,0.7]),([0.4,0.8],[0.6,0.9])\rangle}{y_2},$$
$$\left.\frac{\langle([0.6,0.9],[0.3,0.6]),([0.1,0.4],[0.4,0.8]),([0.2,0.5],[0.7,0.9])\rangle}{y_3}\right\},$$

$$\ddot{L}(\varepsilon_3)=\left\{\frac{\langle([0.5,0.9],[0.1,0.4]),([0.2,0.4],[0.6,0.7]),([0.3,0.7],[0.2,0.5])\rangle}{y_1},\right.$$
$$\frac{\langle([0.6,0.9],[0.1,0.5]),([0.3,0.8],[0.5,0.8]),([0.2,0.6],[0.1,0.5])\rangle}{y_2},$$
$$\left.\frac{\langle([0.1,0.4],[0.1,0.5]),([0.6,0.8],[0.2,0.5]),([0.6,0.9],[0.6,0.8])\rangle}{y_3}\right\},$$

$$\ddot{L}(\varepsilon_4)=\left\{\frac{\langle([0.1,0.5],[0.2,0.5]),([0.2,0.5],[0.7,0.9]),([0.3,0.5],[0.1,0.5])\rangle}{y_1},\right.$$
$$\frac{\langle([0.2,0.6],[0.3,0.7]),([0.7,0.8],[0.2,0.5]),([0.1,0.6],[0.4,0.7])\rangle}{y_2},$$
$$\left.\frac{\langle([0.6,0.8],[0.6,0.7]),([0.3,0.6],[0.4,0.5]),([0.6,0.9],[0.2,0.4])\rangle}{y_3}\right\}.$$

Then, the multi-valued interval neutrosophic soft set $(\ddot{L},A)$ can be written as the following collection of approximations:

$$(\ddot{L},A)=\left\{\left(\varepsilon_1,\left\{\frac{\langle([0.2,0.6],[0.1,0.3]),([0.3,0.5],[0.1,0.4]),([0.2,0.6],[0.4,0.8])\rangle}{y_1},\frac{\langle([0.1,0.3],[0.2,0.4]),([0.3,0.6],[0.4,0.8]),([0.3,0.5],[0.2,0.7])\rangle}{y_2},\frac{\langle([0.1,0.6],[0.2,0.7]),([0.2,0.5],[0.3,0.5]),([0.5,0.8],[0.3,0.8])\rangle}{y_3}\right\}\right),\right.$$

$$\left(\varepsilon_2,\left\{\frac{\langle([0.4,0.6],[0.2,0.5]),([0.2,0.6],[0.4,0.7]),([0.6,0.9],[0.5,0.8])\rangle}{y_1},\frac{\langle([0.3,0.6],[0.3,0.5]),([0.5,0.8],[0.5,0.7]),([0.4,0.8],[0.6,0.9])\rangle}{y_2},\frac{\langle([0.6,0.9],[0.3,0.6]),([0.1,0.4],[0.4,0.8]),([0.2,0.5],[0.7,0.9])\rangle}{y_3}\right\}\right),$$

$$\left(\varepsilon_3,\left\{\frac{\langle([0.5,0.9],[0.1,0.4]),([0.2,0.4],[0.6,0.7]),([0.3,0.7],[0.2,0.5])\rangle}{y_1},\frac{\langle([0.6,0.9],[0.1,0.5]),([0.3,0.8],[0.5,0.8]),([0.2,0.6],[0.1,0.5])\rangle}{y_2},\frac{\langle([0.1,0.4],[0.1,0.5]),([0.6,0.8],[0.2,0.5]),([0.6,0.9],[0.6,0.8])\rangle}{y_3}\right\}\right),$$

$$\left.\left(\varepsilon_4,\left\{\frac{\langle([0.1,0.5],[0.2,0.5]),([0.2,0.5],[0.7,0.9]),([0.3,0.5],[0.1,0.5])\rangle}{y_1},\frac{\langle([0.2,0.6],[0.3,0.7]),([0.7,0.8],[0.2,0.5]),([0.1,0.6],[0.4,0.7])\rangle}{y_2},\frac{\langle([0.6,0.8],[0.6,0.7]),([0.3,0.6],[0.4,0.5]),([0.6,0.9],[0.2,0.4])\rangle}{y_3}\right\}\right)\right\}.$$

The MVINSS can be represented in tabular form. The entries are c_{ij} corresponding to the laptop y_i and the parameter ε_j where c_{ij} refers to interval truth-membership sequence of y_i interval The MVINSS can be represented in tabular form. The entries are indeterminacy-membership sequence of y_i, and interval falsity-membership sequence of y_i, in $\ddot{L}(\varepsilon_j)$.
The tabular representation of multi-valued interval neutrosophic soft set $(\ddot{L},A)$ is as follow:

Table 1. The tabular representation of $(\ddot{L},A)$

U	$\varepsilon_1 = thin$	$\varepsilon_2 = light$
y_1	$\langle([0.2,0.6],[0.1,0.3]),([0.3,0.5],[0.1,0.4]),([0.2,0.6],[0.4,0.8])\rangle$	$\langle([0.4,0.6],[0.2,0.5]),([0.2,0.6],[0.4,0.7]),([0.6,0.9],[0.5,0.8])\rangle$
y_2	$\langle([0.1,0.3],[0.2,0.4]),([0.3,0.6],[0.4,0.8]),([0.3,0.5],[0.2,0.7])\rangle$	$\langle([0.3,0.6],[0.3,0.5]),([0.5,0.8],[0.5,0.7]),([0.4,0.8],[0.6,0.9])\rangle$
y_3	$\langle([0.1,0.6],[0.2,0.7]),([0.2,0.5],[0.3,0.5]),([0.5,0.8],[0.3,0.8])\rangle$	$\langle([0.6,0.9],[0.3,0.6]),([0.1,0.4],[0.4,0.8]),([0.2,0.5],[0.7,0.9])\rangle$

U	$\varepsilon_3 = cheap$	$\varepsilon_4 = large$
y_1	$\langle([0.5,0.9],[0.1,0.4]),([0.2,0.4],[0.6,0.7]),([0.3,0.7],[0.2,0.5])\rangle$	$\langle([0.6,0.9],[0.1,0.5]),([0.3,0.8],[0.5,0.8]),([0.2,0.6],[0.1,0.5])\rangle$
y_2	$\langle([0.1,0.5],[0.2,0.5]),([0.2,0.5],[0.7,0.9]),([0.3,0.5],[0.1,0.5])\rangle$	$\langle([0.2,0.6],[0.3,0.7]),([0.7,0.8],[0.2,0.5]),([0.1,0.6],[0.4,0.7])\rangle$
y_3	$\langle([0.1,0.4],[0.1,0.5]),([0.6,0.8],[0.2,0.5]),([0.6,0.9],[0.6,0.8])\rangle$	$\langle([0.6,0.8],[0.6,0.7]),([0.3,0.6],[0.4,0.5]),([0.6,0.9],[0.2,0.4])\rangle$

Suppose $(\dddot{L}, A)$ is a multi-valued interval neutrosophic soft set in $MVINSS(U)$ where $U = \{y_1, y_2, y_3\}$. The basic operations on MVINSS are given as follows:

We also define the complement operation for MVINSS and give an illustrative example.

Definition 3.2 The complement of a multi-valued interval neutrosophic soft set $(\dddot{L}, A)$ is denoted by $(\dddot{L}, A)^C$ and is defined as $(\dddot{L}, A)^C = (\dddot{L}^C, A)$ where $\dddot{L}^C : A \to MVINSS(U)$ is a mapping given by $\dddot{L}^C(\varepsilon) = c(\dddot{L}(\varepsilon))$, so that $(\dddot{L}, A)^C = \{< \dddot{\lambda}_{\dddot{L}(\varepsilon)}(y), 1 - \dddot{\delta}_{\dddot{L}(\varepsilon)}(y), \dddot{\delta}\,(y) > / y;\ \forall \varepsilon \in A;\ y \in U\}$.

Example 3.2 Consider Example 3.1, then $(\dddot{L}, A)^C$ is given by

$$
\begin{aligned}
(\dddot{\mathrm{L}}, A)^C = \Bigg\{ \Bigg(\varepsilon_1, \Bigg\{ & \frac{\langle([0.2,0.6],[0.4,0.8]),([0.5,0.7],[0.6,0.9]),([0.2,0.6],[0.1,0.3])\rangle}{y_1}, \\
& \frac{\langle([0.3,0.5],[0.2,0.7]),([0.4,0.7],[0.2,0.6]),([0.1,0.3],[0.2,0.4])\rangle}{y_2}, \\
& \frac{\langle([0.5,0.8],[0.3,0.8]),([0.5,0.8],[0.5,0.7]),([0.1,0.6],[0.2,0.7])\rangle}{y_3} \Bigg\} \Bigg), \\
\Bigg(\varepsilon_2, \Bigg\{ & \frac{\langle([0.6,0.9],[0.5,0.8]),([0.4,0.8],[0.3,0.6]),([0.4,0.6],[0.2,0.5])\rangle}{y_1}, \\
& \frac{\langle([0.4,0.8],[0.6,0.9]),([0.2,0.5],[0.3,0.5]),([0.3,0.6],[0.3,0.5])\rangle}{y_2}, \\
& \frac{\langle([0.2,0.5],[0.7,0.9]),([0.6,0.9],[0.2,0.6]),([0.6,0.9],[0.3,0.6])\rangle}{y_3} \Bigg\} \Bigg), \\
\Bigg(\varepsilon_3, \Bigg\{ & \frac{\langle([0.3,0.7],[0.2,0.5]),([0.6,0.8],[0.3,0.4]),([0.5,0.9],[0.1,0.4])\rangle}{y_1}, \\
& \frac{\langle([0.2,0.6],[0.1,0.5]),([0.2,0.7],[0.2,0.5]),([0.6,0.9],[0.1,0.5])\rangle}{y_2}, \\
& \frac{\langle([0.6,0.9],[0.6,0.8]),([0.2,0.4],[0.5,0.8]),([0.1,0.4],[0.1,0.5])\rangle}{y_3} \Bigg\} \Bigg), \\
\Bigg(\varepsilon_4, \Bigg\{ & \frac{\langle([0.3,0.5],[0.1,0.5]),([0.5,0.8],[0.1,0.3]),([0.1,0.5],[0.2,0.5])\rangle}{y_1}, \\
& \frac{\langle([0.1,0.6],[0.4,0.7]),([0.2,0.3],[0.5,0.8]),([0.2,0.6],[0.3,0.7])\rangle}{y_2}, \\
& \frac{\langle([0.6,0.9],[0.2,0.4]),([0.4,0.7],[0.5,0.6]),([0.6,0.8],[0.6,0.7])\rangle}{y_3} \Bigg\} \Bigg) \Bigg\}.
\end{aligned}
$$

We will next define the subset hood of two MVINSS and give an illustrative example.

Definition 3.3 Let $(\dddot{L}, A)$ and $(\dddot{M}, B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. $(\dddot{L}, A)$ is a multi-valued interval neutrosophic soft subset of $(\dddot{M}, B)$ denoted by $(\dddot{L}, A) \subseteq (\dddot{M}, B)$ if and only if $A \subseteq B$ and $\forall \varepsilon \in A$, $\dddot{L}(\varepsilon)$ is a multi-valued interval neutrosophic soft subset of $\dddot{M}(\varepsilon)$.

Example 3.3 Consider Table 1 and $(\dddot{M}, B)$ is another MVINSS over the common universe U. Let B be a set of parameters which describes the size of the laptops. Consider $B = \{\varepsilon_4 = large, \varepsilon_5 = small\}$ and given $(\dddot{M}, B)$ is represented in tabular form as follows.

Table 2. The tabular representation of $(\dddot{M}, B)$

U	$\varepsilon_4 = large$	$\varepsilon_5 = small$
y_1	$\langle([0.3, 0.6], [0.3, 0.5]), ([0.5, 0.8], [0.5, 0.7]), ([0.4, 0.8], [0.6, 0.9])\rangle$	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.8], [0.5, 0.8]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.3, 0.5], [0.1, 0.4]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.7, 0.8], [0.2, 0.5]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.5, 0.9], [0.1, 0.4]), ([0.2, 0.4], [0.6, 0.7]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.2, 0.5], [0.7, 0.9]), ([0.3, 0.5], [0.1, 0.5])\rangle$

It is clear that $(\dddot{M}, B) \subseteq (\dddot{L}, A)$.

Definition 3.4 Let $(\dddot{L}, A)$ and $(\dddot{M}, B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. $(\dddot{L}, A)$ is equal to $(\dddot{M}, B)$ denoted by $(\dddot{L}, A) = (\dddot{M}, B)$ if and only if $(\dddot{L}, A) \subseteq (\dddot{M}, B)$ and $(\dddot{M}, B) \subseteq (\dddot{L}, A)$.

In the following, we define the union of two NVSSs and give an illustrative example.

Definition 3.5 Let $(\dddot{L}, A)$ and $(\dddot{M}, B)$ be two multi-valued neutrosophic soft sets over the common universe U. Then the union of $(\dddot{L}, A)$ and $(\dddot{M}, B)$ is denoted by '$(\dddot{L}, A) \cup (\dddot{M}, B)$' and is defined by $(\dddot{L}, A) \cup (\dddot{M}, B) = (\ddot{N}, C)$ where $C = A \cup B$ and $(\ddot{N}, C) = \{< \dddot{\tau}^{l}_{\ddot{N}(\varepsilon)}(y), \dddot{\delta}^{m}_{\ddot{N}(\varepsilon)}(y), \dddot{\lambda}^{n}_{\ddot{N}(\varepsilon)}(y) > / y;\ y \in U\}$ such that

$$
\begin{array}{lll}
\dddot{\tau}^{l}_{\ddot{N}(\varepsilon)}(y) & = [\dddot{\tau}^{1\,-}_{\dddot{L}(\varepsilon)}(y), \dddot{\tau}^{1\,+}_{\dddot{L}(\varepsilon)}(y)], [\dddot{\tau}^{2\,-}_{\dddot{L}(\varepsilon)}(y), \dddot{\tau}^{2\,+}_{\dddot{L}(\varepsilon)}(y)], \ldots, [\dddot{\tau}^{q\,-}_{\dddot{L}(\varepsilon)}(y), \dddot{\tau}^{q\,+}_{\dddot{L}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A - B; \\
 & = [\dddot{\tau}^{1\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{1\,+}_{\dddot{M}(\varepsilon)}(y)], [\dddot{\tau}^{2\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{2\,+}_{\dddot{M}(\varepsilon)}(y)], \ldots, [\dddot{\tau}^{q\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{q\,+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in B - A; \\
 & = [\dddot{\tau}^{1\,-}_{\dddot{L}(\varepsilon)}(y) \vee \dddot{\tau}^{1\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{1\,+}_{\dddot{L}(\varepsilon)}(y) \vee \dddot{\tau}^{1\,+}_{\dddot{M}(\varepsilon)}(y)],, \ldots, [\dddot{\tau}^{q\,-}_{\dddot{L}(\varepsilon)}(y) \vee \dddot{\tau}^{q\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{q\,+}_{\dddot{L}(\varepsilon)}(y) \vee \dddot{\tau}^{q\,+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A \cap B; \\
\dddot{\delta}^{m}_{\ddot{N}(\varepsilon)}(y) & = [\dddot{\delta}^{1\,-}_{\dddot{L}(\varepsilon)}(y), \dddot{\delta}^{1\,+}_{\dddot{L}(\varepsilon)}(y)], [\dddot{\delta}^{2\,-}_{\dddot{L}(\varepsilon)}(y), \dddot{\delta}^{2\,+}_{\dddot{L}(\varepsilon)}(y)], \ldots, [\dddot{\delta}^{r\,-}_{\dddot{L}(\varepsilon)}(y), \dddot{\delta}^{r\,+}_{\dddot{L}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A - B; \\
 & = [\dddot{\delta}^{1\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\delta}^{1\,+}_{\dddot{M}(\varepsilon)}(y)], [\dddot{\delta}^{2\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\delta}^{2\,+}_{\dddot{M}(\varepsilon)}(y)], \ldots, [\dddot{\delta}^{r\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\delta}^{r\,+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in B - A; \\
 & = \dfrac{[\dddot{\delta}^{1\,-}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}^{1\,-}_{\dddot{M}(\varepsilon)}(y),}{2}, \dfrac{\dddot{\delta}^{1\,+}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}^{1\,+}_{\dddot{M}(\varepsilon)}(y)]}{2}, \ldots, \dfrac{[\dddot{\delta}^{r\,-}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}^{r\,-}_{\dddot{M}(\varepsilon)}(y),}{2}, \dfrac{\dddot{\delta}^{r\,+}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}^{r\,+}_{\dddot{M}(\varepsilon)}(y)]}{2}; & \text{if } \varepsilon \in A \cap B; \\
\dddot{\lambda}^{n}_{\ddot{N}(\varepsilon)}(y) & = [\dddot{\lambda}^{1\,-}_{\dddot{L}(\varepsilon)}(y), \dddot{\lambda}^{1\,+}_{\dddot{L}(\varepsilon)}(y)], [\dddot{\lambda}^{2\,-}_{\dddot{L}(\varepsilon)}(y), \dddot{\lambda}^{2\,+}_{\dddot{L}(\varepsilon)}(y)], \ldots, [\dddot{\lambda}^{s\,-}_{\dddot{L}(\varepsilon)}(y), \dddot{\lambda}^{s\,+}_{\dddot{L}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A - B; \\
 & = [\dddot{\lambda}^{1\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{1\,+}_{\dddot{M}(\varepsilon)}(y)], [\dddot{\lambda}^{2\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{2\,+}_{\dddot{M}(\varepsilon)}(y)], \ldots, [\dddot{\lambda}^{s\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{s\,+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in B - A; \\
 & = [\dddot{\lambda}^{1\,-}_{\dddot{L}(\varepsilon)}(y) \wedge \dddot{\lambda}^{1\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{1\,+}_{\dddot{L}(\varepsilon)}(y) \wedge \dddot{\lambda}^{1\,+}_{\dddot{M}(\varepsilon)}(y)],, \ldots, [\dddot{\lambda}^{s\,-}_{\dddot{L}(\varepsilon)}(y) \wedge \dddot{\lambda}^{s\,-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{s\,+}_{\dddot{L}(\varepsilon)}(y) \wedge \dddot{\lambda}^{s\,+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A \cap B;
\end{array}
$$

It can be simplified as:

$$
(\ddot{N}, C)(\varepsilon) = \begin{cases} \dddot{L}(\varepsilon) & \text{if } \varepsilon \in A - B; \\ \dddot{M}(\varepsilon) & \text{if } \varepsilon \in B - A; \\ \max(\dddot{\tau}_{\dddot{L}(\varepsilon)}(\mathrm{y}), \dddot{\tau}_{\dddot{M}(\varepsilon)}),\ \dfrac{\dddot{\delta}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}_{\dddot{M}(\varepsilon)}(y)}{2},\ \min(\dddot{\lambda}_{\dddot{L}(\varepsilon)}(\mathrm{y}), \dddot{\lambda}_{\dddot{M}(\varepsilon)}) & \text{if } \varepsilon \in A \cap B. \end{cases}
$$

Refer to Example 3.3, the union of $(\dddot{L}, A)$ and $(\dddot{M}, B)$ can be represented as follows.

Table 3. The union of $(\dddot{L}, A)$ and $(\dddot{M}, B)$

U	$\varepsilon_1 = thin$	$\varepsilon_2 = light$
y_1	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.3, 0.5], [0.1, 0.4]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.4, 0.6], [0.2, 0.5]), ([0.2, 0.6], [0.4, 0.7]), ([0.6, 0.9], [0.5, 0.8])\rangle$
y_2	$\langle([0.1, 0.3], [0.2, 0.4]), ([0.3, 0.6], [0.4, 0.8]), ([0.3, 0.5], [0.2, 0.7])\rangle$	$\langle([0.3, 0.6], [0.3, 0.5]), ([0.5, 0.8], [0.5, 0.7]), ([0.4, 0.8], [0.6, 0.9])\rangle$
y_3	$\langle([0.1, 0.6], [0.2, 0.7]), ([0.2, 0.5], [0.3, 0.5]), ([0.5, 0.8], [0.3, 0.8])\rangle$	$\langle([0.6, 0.9], [0.3, 0.6]), ([0.1, 0.4], [0.4, 0.8]), ([0.2, 0.5], [0.7, 0.9])\rangle$

U	$\varepsilon_3 = cheap$	$\varepsilon_4 = large$
y_1	$\langle([0.5, 0.9], [0.1, 0.4]), ([0.2, 0.4], [0.6, 0.7]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.6, 0.9], [0.3, 0.5]), ([0.4, 0.8], [0.5, 0.75]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.2, 0.5], [0.7, 0.9]), ([0.3, 0.5], [0.1, 0.5])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.5, 0.65], [0.15, 0.45]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.1, 0.4], [0.1, 0.5]), ([0.6, 0.8], [0.2, 0.5]), ([0.6, 0.9], [0.6, 0.8])\rangle$	$\langle([0.6, 0.9], [0.6, 0.7]), ([0.25, 0.5], [0.5, 0.6]), ([0.3, 0.7], [0.2, 0.4])\rangle$

U	$\varepsilon_5 = small$
y_1	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.8], [0.5, 0.8]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.7, 0.8], [0.2, 0.5]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.2, 0.5], [0.7, 0.9]), ([0.3, 0.5], [0.1, 0.5])\rangle$

Then, we present the definition of intersection operation and give an illustrative example.

Let $(\dddot{L}, A)$ and $(\dddot{M}, B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. Then the intersection of $(\dddot{L}, A)$ and $(\dddot{M}, B)$ is denoted by $'(\dddot{L}, A) \cap (\dddot{M}, B)'$ and is defined by $(\dddot{L}, A) \cap (\dddot{M}, B) = (\dddot{N}, C)$ where $C = A \cap B$ and $(\dddot{N}, C) = \{< \dddot{\tau}^{l}_{\dddot{N}(\varepsilon)}(y), \dddot{\delta}^{m}_{\dddot{N}(\varepsilon)}(y), \dddot{\lambda}^{n}_{\dddot{N}(\varepsilon)}(y) > / y;\ y \in U\}$ such that for every $\varepsilon \in C$,

$$\dddot{\tau}^{l}_{\dddot{N}(\varepsilon)}(y) = [\dddot{\tau}^{1}_{\dddot{L}(\varepsilon)}{}^{-}(y) \wedge \dddot{\tau}^{1}_{\dddot{M}(\varepsilon)}{}^{-}(y),\ \dddot{\tau}^{1}_{\dddot{L}(\varepsilon)}{}^{+}(y) \wedge \dddot{\tau}^{1}_{\dddot{M}(\varepsilon)}{}^{+}(y)], \ldots, [\dddot{\tau}^{q}_{\dddot{L}(\varepsilon)}{}^{-}(y) \wedge \dddot{\tau}^{q}_{\dddot{M}(\varepsilon)}{}^{-}(y),\ \dddot{\tau}^{q}_{\dddot{L}(\varepsilon)}{}^{+}(y) \wedge \dddot{\tau}^{q}_{\dddot{M}(\varepsilon)}{}^{+}(y)];$$

$$\dddot{\delta}^{m}_{\dddot{N}(\varepsilon)}(y) = \frac{[\dddot{\delta}^{1}_{\dddot{L}(\varepsilon)}{}^{-}(y) + \dddot{\delta}^{1}_{\dddot{M}(\varepsilon)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{1}_{\dddot{L}(\varepsilon)}{}^{+}(y) + \dddot{\delta}^{1}_{\dddot{M}(\varepsilon)}{}^{+}(y)]}{2}, \ldots, \frac{[\dddot{\delta}^{r}_{\dddot{L}(\varepsilon)}{}^{-}(y) + \dddot{\delta}^{r}_{\dddot{M}(\varepsilon)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{r}_{\dddot{L}(\varepsilon)}{}^{+}(y) + \dddot{\delta}^{r}_{\dddot{M}(\varepsilon)}{}^{+}(y)]}{2};$$

$$\dddot{\lambda}^{l}_{\dddot{N}(\varepsilon)}(y) = [\dddot{\lambda}^{1}_{\dddot{L}(\varepsilon)}{}^{-}(y) \vee \dddot{\lambda}^{1}_{\dddot{M}(\varepsilon)}{}^{-}(y),\ \dddot{\lambda}^{1}_{\dddot{L}(\varepsilon)}{}^{+}(y) \vee \dddot{\lambda}^{1}_{\dddot{M}(\varepsilon)}{}^{+}(y)], \ldots, [\dddot{\lambda}^{s}_{\dddot{L}(\varepsilon)}{}^{-}(y) \vee \dddot{\lambda}^{s}_{\dddot{M}(\varepsilon)}{}^{-}(y),\ \dddot{\lambda}^{s}_{\dddot{L}(\varepsilon)}{}^{+}(y) \vee \dddot{\lambda}^{s}_{\dddot{M}(\varepsilon)}{}^{+}(y)];$$

Refer to Example 3.3, the intersection of $(\dddot{L}, A)$ and $(\dddot{M}, B)$ can be represented as follows.

Table 4. The intersection of $(\dddot{L}, A)$ and $(\dddot{M}, B)$

U	$\varepsilon_4 = large$
y_1	$\langle([0.3, 0.6], [0.1, 0.5]), ([0.4, 0.8], [0.5, 0.75]), ([0.4, 0.8], [0.6, 0.9])\rangle$
y_2	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.5, 0.65], [0.15, 0.45]), ([0.2, 0.6], [0.4, 0.8])\rangle$
y_3	$\langle([0.5, 0.8], [0.1, 0.4]), ([0.25, 0.5], [0.5, 0.6]), ([0.6, 0.9], [0.2, 0.5])\rangle$

Some properties of union and intersection are derived as follows.

Proposition 3.1

Idempotency Laws:

(1) $(\dddot{L}, A) \cup (\dddot{L}, A) = (\dddot{L}, A)$

(2) $(\tilde{F}, A) \cap (\tilde{F}, A) = (\tilde{F}, A)$.

Commutative Laws:

(3) $(\dddot{L}, A) \cup (\dddot{M}, B) = (\dddot{M}, B) \cup (\dddot{L}, A)$

(4) $(\dddot{L}, A) \cap (\dddot{M}, B) = (\dddot{M}, B) \cap (\dddot{L}, A)$

Proof 1

Let ε be an arbitrary element of $(\dddot{L}, A) \cup (\dddot{L}, A)$. Then, $\varepsilon \in (\dddot{L}, A)$ or $\varepsilon \in (\dddot{L}, A)$. Hence $\varepsilon \in (\dddot{L}, A)$. Thus, $(\dddot{L}, A) \cup (\dddot{L}, A) \subseteq (\dddot{L}, A)$. Conversely, if ε is an arbitrary element of $(\dddot{L}, A)$, then $\varepsilon \in (\dddot{L}, A) \cup (\dddot{L}, A)$ since it is in $(\dddot{L}, A)$. Therefore $(\dddot{L}, A) \subseteq (\dddot{L}, A) \cup (\dddot{L}, A)$.

$\therefore (\dddot{L}, A) \cup (\dddot{L}, A) = (\dddot{L}, A)$ □

Proof 2

Let ε be an arbitrary element of $(\ddot{L},A)\cap(\ddot{L},A)$. Then, $\varepsilon\in(\ddot{L},A)$ and $\varepsilon\in(\ddot{L},A)$. Hence $\varepsilon\in(\ddot{L},A)$. Thus, $(\ddot{L},A)\cap(\ddot{L},A)\subseteq(\ddot{L},A)$. Conversely, if $\varepsilon\in(\ddot{L},A)$ is arbitrary, then $\varepsilon\in(\ddot{L},A)$ and $\varepsilon\in(\ddot{L},A)$. Therefore $(\ddot{L},A)\subseteq(\ddot{L},A)\cap(\ddot{L},A)$..

$\therefore(\ddot{L},A)\cap(\ddot{L},A)=(\ddot{L},A)$ □

Proof 3

Let ε is any element in $(\ddot{L},A)\cup(\ddot{M},B)$. Then, by definition of union, $\varepsilon\in(\ddot{L},A)$ or $\varepsilon\in(\ddot{M},B)$. But, if ε is in $(\ddot{L},A)$ or $(\ddot{M},B)$, then it is in $(\ddot{M},B)$, or $(\ddot{L},A)$ and by definition of union, this means $\varepsilon\in(\ddot{L},A)\cup(\ddot{M},B)$. Therefore, $(\ddot{L},A)\cup(\ddot{M},B)\subseteq(\ddot{M},B)\cup(\ddot{L},A)$.

The other inclusion is identical. If ε is any element of $(\ddot{M},B)\cup(\ddot{L},A)$. Then, $\varepsilon\in(\ddot{M},B)$ or $\varepsilon\in(\ddot{L},A)$. But, $\varepsilon\in(\ddot{M},B)$ or $\varepsilon\in(\ddot{L},A)$. implies that ε is in $(\ddot{L},A)$ or $(\ddot{M},B)$. Hence, $\varepsilon\in(\ddot{M},\mathrm{B})\cup(\ddot{L},B)$. Therefore $(\ddot{M},B)\cup(\ddot{L},A)\subseteq(\ddot{L},A)\cup(\ddot{M},B)$.

$\therefore(\ddot{L},A)\cup(\ddot{M},B)=(\ddot{M},B)\cup(\ddot{L},A)$ □

Proof 4

Let ε is any element in $(\ddot{L},A)\cap(\ddot{M},B)$. Then, by definition of intersection, $\varepsilon\in(\ddot{L},A)$ and $\varepsilon\in(\ddot{M},B)$. Hence, $\varepsilon\in(\ddot{M},B)$. and $\varepsilon\in(\ddot{L},A)$. So, $\varepsilon\in(\ddot{M},B)\cap(\ddot{L},A)$. Therefore, $(\ddot{L},A)\cap(\ddot{M},B)\subseteq(\ddot{M},B)\cap(\ddot{L},A)$.
The reverse inclusion is again identical. If ε is any element of $(\ddot{M},B)\cap(\ddot{L},A)$. Then, $\varepsilon\in(\ddot{M},B)$. and $\varepsilon\in(\ddot{L},A)$. Hence, $\varepsilon\in(\ddot{L},A)$. and $\varepsilon\in(\ddot{M},B)$. This implies $\varepsilon\in(\ddot{L},A)\cap(\ddot{M},B)$. Therefore $(\ddot{M},B)\cap(\ddot{L},A)\subseteq(\ddot{L},A)\cap(\ddot{M},B)$.

$\therefore(\ddot{L},A)\cap(\ddot{M},B)=(\ddot{M},B)\cap(\ddot{L},A)$ □

For three multi-valued neutrosophic soft sets $(\ddot{L},A),(\ddot{M},B)$ and $(\ddot{N},C)$ over the common universe U, we have the following propositions:

Proposition 3.2

Associative Laws:

1. $(\ddot{L},A)\cup[(\ddot{M},B)\cup(\ddot{N},C)]=[(\ddot{L},A)\cup(\ddot{M},B)]\cup(\ddot{N},C)$.
2. $(\ddot{L},A)\cap[(\ddot{M},B)\cap(\ddot{N},C)]=[(\ddot{L},A)\cap(\ddot{M},B)]\cap(\ddot{N},C)$.

Distributive Laws:

3. $(\ddot{L},A)\cup[(\ddot{M},B)\cap(\ddot{N},C)]=[(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$.
4. $(\ddot{L},A)\cap[(\ddot{M},B)\cup(\ddot{N},C)]=[(\ddot{L},A)\cap(\ddot{M},B)]\cup[(\ddot{L},A)\cap(\ddot{N},C)]$.

Proof 1

Let $\varepsilon\in(\ddot{L},A)\cup[(\ddot{M},B)\cup(\ddot{N},C)]$. If $\varepsilon\in(\ddot{L},A)\cup[(\ddot{M},B)\cup(\ddot{N},C)]$,
then ε is either in $(\ddot{L},A)$ or in $[(\ddot{M},B)$ or $(\ddot{N},C)]$.

$\Rightarrow\varepsilon\in(\ddot{L},A)$ or $\varepsilon\in[(\ddot{M},B)$ or $(\ddot{N},C)]$

$\Rightarrow\varepsilon\in(\ddot{L},A)$ or $\{\varepsilon\in(\ddot{M},B)$ or $\varepsilon\in(\ddot{N},C)\}$

$\Rightarrow\{\varepsilon\in(\ddot{L},A)$ or $\varepsilon\in(\ddot{M},B)\}$ or $\{\varepsilon\in(\ddot{L},A)$ or $\varepsilon\in(\ddot{N},C)\}$

$\Rightarrow\varepsilon\in[(\ddot{L},A)$ or $(\ddot{M},B)]$ or $\varepsilon\in[(\ddot{L},A)$ or $(\ddot{N},C)]$

$\Rightarrow\varepsilon\in[(\ddot{L},A)\cup(\ddot{M},B)]\cup\varepsilon\in[(\ddot{L},A)\cup(\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$

Since $\exists \varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$ such that $\varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$, therefore $(\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)] \subseteq [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$.

Let $\varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$. If $\varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$, then ε is in $[(\ddot{L},A)$ or $(\ddot{M},B)]$ or ε is in $[(\ddot{L},A)$ or $(\ddot{N},C)]$.

$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $(\ddot{M},B)]$ or $\varepsilon \in (\ddot{L},A)$ or $(\ddot{N},C)]$

$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{M},B)\}$ or $\Rightarrow \{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $\{\varepsilon \in (\ddot{M},B)$ or $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $\{\varepsilon \in [(\ddot{M},B)$ or $(\ddot{N},C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cup \{\varepsilon \in [(\ddot{M},B) \cup (\ddot{N},C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$

Since $\exists \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$ such that $\varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$, therefore $[(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)] \subseteq (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$.

$\therefore (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)] = [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$ □

Proof 2

Let $\varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$. If $\varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$, then ε is either in $(\ddot{L},A)$ and in $[(\ddot{M},B)$ and $(\ddot{N},C)]$.

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\varepsilon \in [(\ddot{M},B)$ and $(\ddot{N},C)]$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\{\varepsilon \in (\ddot{M},B)$ and $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{M},B)\}$ and $\{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in [(\ddot{L},A)$ and $(\ddot{M},B)]$ and $\varepsilon \in [(\ddot{L},A)$ and $(\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap \varepsilon \in [(\ddot{L},A) \cap (\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$

Since $\exists \varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$ such that $\varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$, therefore $(\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)] \subseteq [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$.

Let $\varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$. If $\varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$, then ε is in $[(\ddot{L},A)$ and $(\ddot{M},B)]$ and ε is in $[(\ddot{L},A)$ and $(\ddot{N},C)]$.

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $(\ddot{M},B)]$ and $\varepsilon \in (\ddot{L},A)$ and $(\ddot{N},C)]$

$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{M},B)\}$ and $\{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\{\varepsilon \in (\ddot{M},B)$ and $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\{\varepsilon \in [(\ddot{M},B)$ and $(\ddot{N},C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cap \{\varepsilon \in [(\ddot{M},B) \cap (\ddot{N},C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$

Since $\exists \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$ such that $\varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$, therefore $[(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)] \subseteq (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$.

$\therefore (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)] = [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$ □

Proof 3

Let $\varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cap (\ddot{N},C)]$. If $\varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cap (\ddot{N},C)]$,
then ε is either in $(\ddot{L},A)$ or in $[(\ddot{M},B)$ and $(\ddot{N},C)]$.
$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $\varepsilon \in [(\ddot{M},B)$ and $(\ddot{N},C)]$
$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $\{\varepsilon \in (\ddot{M},B)$ and $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{M},B)\}$ and $\{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ or $(\ddot{M},B)]$ and $\varepsilon \in [(\ddot{L},A)$ or $(\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cap \varepsilon \in [(\ddot{L},A) \cup (\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cap [(\ddot{L},A) \cup (\ddot{N},C)]$
$\Rightarrow \varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cap (\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cap [(\ddot{L},A) \cup (\ddot{N},C)]$
Since $\exists \varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cap (\ddot{N},C)]$ such that $\varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cap [(\ddot{L},A) \cup (\ddot{N},C)]$,
therefore $(\ddot{L},A) \cup [(\ddot{M},B) \cap (\ddot{N},C)] \subseteq [(\ddot{L},A) \cup (\ddot{M},B)] \cap [(\ddot{L},A) \cup (\ddot{N},C)]$.

Let $\varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cap [(\ddot{L},A) \cup (\ddot{N},C)]$. If $\varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cap [(\ddot{L},A) \cup (\ddot{N},C)]$,
then ε is in $[(\ddot{L},A)$ or $(\ddot{M},B)]$ and ε is in $[(\ddot{L},A)$ or $(\ddot{N},C)]$.
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ or $(\ddot{M},B)]$ and $\varepsilon \in [(\ddot{L},A)$ or $(\ddot{N},C)]$
$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{M},B)\}$ and $\{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ or $\{\varepsilon \in (\ddot{M},B)$ and $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ or $\{\varepsilon \in [(\ddot{M},B)$ and $(\ddot{N},C)]\}$
$\Rightarrow \varepsilon \in (\ddot{L},A) \cup \{\varepsilon \in [(\ddot{M},B) \cap (\ddot{N},C)]\}$
$\Rightarrow \varepsilon \in (\ddot{L},A) \cup (\ddot{M},B) \cap (\ddot{N},C)]$
Since $\exists \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cap [(\ddot{L},A) \cup (\ddot{N},C)]$ such that $\varepsilon \in (\ddot{L},A) \cup (\ddot{M},B) \cap (\ddot{N},C)]$,
therefore $[(\ddot{L},A) \cup (\ddot{M},B)] \cap [(\ddot{L},A) \cup (\ddot{N},C)] \subseteq (\ddot{L},A) \cup (\ddot{M},B) \cap (\ddot{N},C)]$.

$\therefore (\ddot{L},A) \cup [(\ddot{M},B) \cap (\ddot{N},C)] = [(\ddot{L},A) \cup (\ddot{M},B)] \cap [(\ddot{L},A) \cup (\ddot{N},C)]$. □

Proof 4

Let $\varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cup (\ddot{N},C)]$. If $\varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cup (\ddot{N},C)]$,
then ε is in $(\ddot{L},A)$ and $[(\ddot{M},B)$ or $(\ddot{N},C)]$.
$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\varepsilon \in [(\ddot{M},B)$ or $(\ddot{N},C)]$
$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\{\varepsilon \in (\ddot{M},B)$ or $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{M},B)\}$ or $\{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ and $(\ddot{M},B)]$ or $\varepsilon \in [(\ddot{L},A)$ and $(\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cup \varepsilon \in [(\ddot{L},A) \cap (\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cup [(\ddot{L},A) \cap (\ddot{N},C)]$
$\Rightarrow \varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cup (\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cup [(\ddot{L},A) \cap (\ddot{N},C)]$
Since $\exists \varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cup (\ddot{N},C)]$ such that $\varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cup [(\ddot{L},A) \cap (\ddot{N},C)]$,
therefore $(\ddot{L},A) \cap [(\ddot{M},B) \cup (\ddot{N},C)] \subseteq [(\ddot{L},A) \cap (\ddot{M},B)] \cup [(\ddot{L},A) \cap (\ddot{N},C)]$.

Let $\varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cup [(\ddot{L},A) \cap (\ddot{N},C)]$. If $\varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cup [(\ddot{L},A) \cap (\ddot{N},C)]$,
then ε is in $[(\ddot{L},A)$ and $(\ddot{M},B)]$ or ε is in $[(\ddot{L},A)$ and $(\ddot{N},C)]$.
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ and $(\ddot{M},B)]$ or $\varepsilon \in [(\ddot{L},A)$ and $(\ddot{N},C)]$

$\Rightarrow \{\varepsilon \in (\dddot{L}, A)$ and $\varepsilon \in (\dddot{M}, B)\}$ or $\{\varepsilon \in (\dddot{L}, A)$ and $\varepsilon \in (\dddot{N}, C)\}$

$\Rightarrow \varepsilon \in [(\dddot{L}, A)$ and $\{\varepsilon \in (\dddot{M}, B)$ or $\varepsilon \in (\dddot{N}, C)\}$

$\Rightarrow \varepsilon \in [(\dddot{L}, A)$ and $\{\varepsilon \in [(\dddot{M}, B)$ or $(\dddot{N}, C)]\}$

$\Rightarrow \varepsilon \in (\dddot{L}, A) \cap \{\varepsilon \in [(\dddot{M}, B) \cup (\dddot{N}, C)]\}$

$\Rightarrow \varepsilon \in (\dddot{L}, A) \cap (\dddot{M}, B) \cup (\dddot{N}, C)]$

Since $\exists \varepsilon \in [(\dddot{L}, A) \cap (\dddot{M}, B)] \cup [(\dddot{L}, A) \cap (\dddot{N}, C)]$ such that $\varepsilon \in (\dddot{L}, A) \cap (\dddot{M}, B) \cup (\dddot{N}, C)]$,
therefore $[(\dddot{L}, A) \cap (\dddot{M}, B)] \cup [(\dddot{L}, A) \cap (\dddot{N}, C)] \subseteq (\dddot{L}, A) \cap (\dddot{M}, B) \cup (\dddot{N}, C)]$.

$\therefore (\dddot{L}, A) \cap [(\dddot{M}, B) \cup (\dddot{N}, C)] = [(\dddot{L}, A) \cap (\dddot{M}, B)] \cup [(\dddot{L}, A) \cap (\dddot{N}, C)]$ □

Then, we introduce the definition of 'AND' and 'OR' operations and give the illustrative example.

Definition 3.6

Let $(\dddot{L}, A)$ and $(\dddot{M}, B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. Then the 'AND' operation between $(\dddot{L}, A)$ and $(\dddot{M}, B)$ is denoted by $'(\dddot{L}, A) \wedge (\dddot{M}, B)'$ and is defined by $'(\dddot{L}, A) \wedge (\dddot{M}, B)' = (\dddot{N}, A \times B)$ where $(\dddot{N}, A \times B) = \{< \dddot{\tau}^{l}_{\dddot{N}(\alpha,\beta)}(y), \dddot{\delta}^{m}_{\dddot{N}(\alpha,\beta)}(y), \dddot{\lambda}^{n}_{\dddot{N}(\alpha,\beta)}(y) > / y; y \in U\}$ such that for every $\alpha \in A, \beta \in B, y \in U$.

$$\dddot{\tau}^{l}_{\dddot{N}(\alpha,\beta)}(y) = [\dddot{\tau}^{1}_{\dddot{L}(\alpha)}{}^{-}(y) \wedge \dddot{\tau}^{1}_{\dddot{M}(\beta)}{}^{-}(y), \dddot{\tau}^{1}_{\dddot{L}(\alpha)}{}^{+}(y) \wedge \dddot{\tau}^{1}_{\dddot{M}(\beta)}{}^{+}(y)], \ldots, [\dddot{\tau}^{q}_{\dddot{L}(\alpha)}{}^{-}(y) \wedge \dddot{\tau}^{q}_{\dddot{M}(\beta)}{}^{-}(y), \dddot{\tau}^{q}_{\dddot{L}(\alpha)}{}^{+}(y) \wedge \dddot{\tau}^{q}_{\dddot{M}(\beta)}{}^{+}(y)];$$

$$\dddot{\delta}^{m}_{\dddot{N}(\alpha,\beta)}(y) = \frac{[\dddot{\delta}^{1}_{\dddot{L}(\alpha)}{}^{-}(y) + \dddot{\delta}^{1}_{\dddot{M}(\beta)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{1}_{\dddot{L}(\alpha)}{}^{+}(y) + \dddot{\delta}^{1}_{\dddot{M}(\beta)}{}^{+}(y)]}{2}, \ldots, \frac{[\dddot{\delta}^{r}_{\dddot{L}(\alpha)}{}^{-}(y) + \dddot{\delta}^{r}_{\dddot{M}(\beta)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{r}_{\dddot{L}(\alpha)}{}^{+}(y) + \dddot{\delta}^{r}_{\dddot{M}(\beta)}{}^{+}(y)]}{2};$$

$$\dddot{\lambda}^{n}_{\dddot{N}(\alpha,\beta)}(y) = [\dddot{\lambda}^{1}_{\dddot{L}(\alpha)}{}^{-}(y) \vee \dddot{\lambda}^{1}_{\dddot{M}(\beta)}{}^{-}(y), \dddot{\lambda}^{1}_{\dddot{L}(\alpha)}{}^{+}(y) \vee \dddot{\lambda}^{1}_{\dddot{M}(\beta)}{}^{+}(y)], \ldots, [\dddot{\lambda}^{s}_{\dddot{L}(\alpha)}{}^{-}(y) \vee \dddot{\lambda}^{s}_{\dddot{M}(\beta)}{}^{-}(y), \dddot{\lambda}^{s}_{\dddot{L}(\alpha)}{}^{+}(y) \vee \dddot{\lambda}^{s}_{\dddot{M}(\beta)}{}^{+}(y)];$$

Refer to Example 3.3, the 'AND' operation of $(\dddot{L}, A)$ and $(\dddot{M}, B)$ can be represented as follows.

Table 5. The 'AND' operation of $(\dddot{L}, A)$ and $(\dddot{M}, B)$

U	$(thin, large)$	$(thin, small)$
y_1	$\langle([0.3, 0.6], [0.1, 0.5]), ([0.4, 0.73], [0.4, 0.65]), ([0.4, 0.8], [0.6, 0.9])\rangle$	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.7], [0.4, 0.7]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.4, 0.6], [0.2, 0.53]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.6, 0.75], [0.25, 0.58]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.1, 0.6], [0.1, 0.4]), ([0.2, 0.45], [0.55, 0.7]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.2, 0.5], [0.6, 0.8]), ([0.3, 0.5], [0.1, 0.5])\rangle$

U	$(light, large)$	$(light, small)$
y_1	$\langle([0.3, 0.6], [0.2, 0.5]), ([0.35, 0.7], [0.45, 0.7]), ([0.6, 0.9], [0.6, 0.9])\rangle$	$\langle([0.4, 0.6], [0.1, 0.5]), ([0.25, 0.7], [0.45, 0.75]), ([0.6, 0.9], [0.5, 0.8])\rangle$
y_2	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.4, 0.65], [0.3, 0.55]), ([0.4, 0.8], [0.6, 0.9])\rangle$	$\langle([0.2, 0.6], [0.3, 0.5]), ([0.6, 0.8], [0.35, 0.6]), ([0.4, 0.8], [0.6, 0.9])\rangle$
y_3	$\langle([0.5, 0.9], [0.1, 0.4]), ([0.15, 0.4], [0.5, 0.75]), ([0.3, 0.7], [0.7, 0.9])\rangle$	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.15, 0.45], [0.55, 0.85]), ([0.3, 0.5], [0.7, 0.9])\rangle$

U	$(cheap, large)$	$(cheap, small)$
y_1	$\langle([0.3, 0.6], [0.1, 0.4]), ([0.35, 0.6], [0.55, 0.7]), ([0.4, 0.8], [0.6, 0.9])\rangle$	$\langle([0.5, 0.9], [0.1, 0.4]), ([0.25, 0.6], [0.55, 0.75]), ([0.3, 0.7], [0.2, 0.5])\rangle$
y_2	$\langle([0.1, 0.5], [0.1, 0.3]), ([0.25, 0.5], [0.4, 0.65]), ([0.3, 0.6], [0.4, 0.8])\rangle$	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.45, 0.65], [0.45, 0.7]), ([0.3, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.1, 0.4], [0.1, 0.4]), ([0.4, 0.6], [0.4, 0.6]), ([0.6, 0.9], [0.6, 0.8])\rangle$	$\langle([0.1, 0.4], [0.1, 0.5]), ([0.4, 0.65], [0.45, 0.7]), ([0.6, 0.9], [0.6, 0.8])\rangle$

U	$(large, large)$	$(large, small)$
y_1	$\langle([0.3, 0.6], [0.1, 0.5]), ([0.4, 0.8], [0.5, 0.75]), ([0.4, 0.8], [0.6, 0.9])\rangle$	$\langle([0.6, 0.9], [0.6, 0.5]), ([0.3, 0.8], [0.5, 0.8]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.5, 0.65], [0.15, 0.45]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.7, 0.8], [0.2, 0.5]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.5, 0.8], [0.1, 0.4]), ([0.25, 0.5], [0.5, 0.6]), ([0.6, 0.9], [0.2, 0.5])\rangle$	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.25, 0.55], [0.55, 0.7]), ([0.6, 0.9], [0.2, 0.5])\rangle$

Definition 3.7 Let $(\ddot{L}, A)$ and $(\ddot{M}, B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. Then, the 'OR' operation between $(\ddot{L}, A)$ and $(\ddot{M}, B)$ is denoted by $'(\ddot{L}, A) \vee (\ddot{M}, B)'$ and is defined by $(\ddot{L}, A) \vee (\ddot{M}, B) = (\ddot{N}, A \times B)$ where $(\ddot{N}, A \times B) = \{< \dddot{\tau}^{l}_{\ddot{N}(\alpha,\beta)}(y), \dddot{\delta}^{m}_{\ddot{N}(\alpha,\beta)}(y), \dddot{\lambda}^{n}_{\ddot{N}(\alpha,\beta)}(y) > / y; y \in U\}$ such that for every $\alpha \in A, \beta \in B, y \in Y$,

$$\dddot{\tau}^{l}_{\ddot{N}(\alpha,\beta)}(y) = [\dddot{\tau}^{1}_{\ddot{L}(\alpha)}{}^{-}(y) \vee \dddot{\tau}^{1}_{\ddot{M}(\beta)}{}^{-}(y), \dddot{\tau}^{1}_{\ddot{L}(\alpha)}{}^{+}(y) \vee \dddot{\tau}^{1}_{\ddot{M}(\beta)}{}^{+}(y)], \ldots, [\dddot{\tau}^{q}_{\ddot{L}(\alpha)}{}^{-}(y) \vee \dddot{\tau}^{q}_{\ddot{M}(\beta)}{}^{-}(y), \dddot{\tau}^{q}_{\ddot{L}(\alpha)}{}^{+}(y) \vee \dddot{\tau}^{q}_{\ddot{M}(\beta)}{}^{+}(y)];$$

$$\dddot{\delta}^{m}_{\ddot{N}(\alpha,\beta)}(y) = \frac{[\dddot{\delta}^{1}_{\ddot{L}(\alpha)}{}^{-}(y) + \dddot{\delta}^{1}_{\ddot{M}(\beta)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{1}_{\ddot{L}(\alpha)}{}^{+}(y) + \dddot{\delta}^{1}_{\ddot{M}(\beta)}{}^{+}(y)]}{2}, \ldots, \frac{[\dddot{\delta}^{r}_{\ddot{L}(\alpha)}{}^{-}(y) + \dddot{\delta}^{r}_{\ddot{M}(\beta)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{r}_{\ddot{L}(\alpha)}{}^{+}(y) + \dddot{\delta}^{r}_{\ddot{M}(\beta)}{}^{+}(y)]}{2};$$

$$\dddot{\lambda}^{n}_{\ddot{N}(\alpha,\beta)}(y) = [\dddot{\lambda}^{1}_{\ddot{L}(\alpha)}{}^{-}(y) \wedge \dddot{\lambda}^{1}_{\ddot{M}(\beta)}{}^{-}(y), \dddot{\lambda}^{1}_{\ddot{L}(\alpha)}{}^{+}(y) \wedge \dddot{\lambda}^{1}_{\ddot{M}(\beta)}{}^{+}(y)], \ldots, [\dddot{\lambda}^{s}_{\ddot{L}(\alpha)}{}^{-}(y) \wedge \dddot{\lambda}^{s}_{\ddot{M}(\beta)}{}^{-}(y), \dddot{\lambda}^{s}_{\ddot{L}(\alpha)}{}^{+}(y) \wedge \dddot{\lambda}^{s}_{\ddot{M}(\beta)}{}^{+}(y)];$$

Refer to Example 3.3, the 'OR' operation of $(\ddot{L}, A)$ and $(\ddot{M}, B)$ can be represented as follows.

Table 6. The 'OR' operation of $(\ddot{L}, A)$ and $(\ddot{M}, B)$

U	$(thin, large)$	$(thin, small)$
y_1	$\langle([0.3, 0.6], [0.3, 0.5]), ([0.4, 0.73], [0.4, 0.65]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.7], [0.4, 0.7]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.2, 0.4]), ([0.4, 0.6], [0.2, 0.53]), ([0.2, 0.5], [0.2, 0.7])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.6, 0.75], [0.25, 0.58]), ([0.1, 0.5], [0.2, 0.7])\rangle$
y_3	$\langle([0.5, 0.9], [0.2, 0.7]), ([0.2, 0.45], [0.55, 0.7]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.1, 0.6], [0.2, 0.7]), ([0.2, 0.5], [0.6, 0.8]), ([0.3, 0.5], [0.1, 0.5])\rangle$

U	$(light, large)$	$(light, small)$
y_1	$\langle([0.4, 0.6], [0.3, 0.5]), ([0.35, 0.7], [0.45, 0.7]), ([0.4, 0.8], [0.5, 0.8])\rangle$	$\langle([0.6, 0.9], [0.2, 0.5]), ([0.25, 0.7], [0.45, 0.75]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.3, 0.6], [0.3, 0.5]), ([0.4, 0.65], [0.3, 0.55]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.3, 0.6], [0.3, 0.7]), ([0.6, 0.8], [0.35, 0.6]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.6, 0.9], [0.3, 0.6]), ([0.15, 0.4], [0.5, 0.75]), ([0.2, 0.5], [0.2, 0.5])\rangle$	$\langle([0.6, 0.9], [0.3, 0.6]), ([0.15, 0.45], [0.55, 0.85]), ([0.2, 0.5], [0.1, 0.5])\rangle$

U	$(cheap, large)$	$(cheap, small)$
y_1	$\langle([0.5, 0.9], [0.3, 0.5]), ([0.35, 0.6], [0.55, 0.7]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.25, 0.6], [0.55, 0.75]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.2, 0.5]), ([0.25, 0.5], [0.4, 0.65]), ([0.2, 0.5], [0.1, 0.5])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.45, 0.65], [0.45, 0.7]), ([0.1, 0.5], [0.1, 0.5])\rangle$
y_3	$\langle([0.5, 0.9], [0.1, 0.5]), ([0.4, 0.6], [0.4, 0.6]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.4, 0.65], [0.45, 0.7]), ([0.3, 0.5], [0.1, 0.5])\rangle$

U	$(large, large)$	$(large, small)$
y_1	$\langle([0.6, 0.9], [0.3, 0.5]), ([0.4, 0.8], [0.5, 0.75]), ([0.2, 0.6], [0.1, 0.5])\rangle$	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.8], [0.5, 0.8]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.5, 0.65], [0.15, 0.45]), ([0.1, 0.6], [0.4, 0.7])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.7, 0.8], [0.2, 0.5]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.6, 0.9], [0.6, 0.7]), ([0.25, 0.5], [0.5, 0.6]), ([0.3, 0.7], [0.2, 0.4])\rangle$	$\langle([0.6, 0.8], [0.6, 0.7]), ([0.25, 0.55], [0.55, 0.7]), ([0.3, 0.5], [0.1, 0.4])\rangle$

For three multi-valued interval neutrosophic soft sets $(\ddot{L}, A)$, $(\ddot{M}, B)$ and $(\ddot{N}, C)$ over the common universe, then De Morgan's Law are given as follows.

Preposition 3

(1) $(\ddot{L}, A)^C \vee (\ddot{M}, B)^C = [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$

(2) $(\ddot{L}, A)^C \wedge (\ddot{M}, B)^C = [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$

(3) $(\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C = [(\ddot{L}, A) \wedge (\ddot{M}, B) \wedge (\ddot{N}, C)]^C$

(4) $(\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C = [(\ddot{L}, A) \vee (\ddot{M}, B) \vee (\ddot{N}, C)]^C$

Proof 1

Let $\varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ or $\varepsilon \in (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ or $\varepsilon \notin (\ddot{M}, B)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A) \wedge (\ddot{M}, B)$

$\Rightarrow \varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$

Since $\exists \varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$ such that $\varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$,

Therefore $(\ddot{L}, A)^C \vee (\ddot{M}, B)^C \subseteq [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$.

Then consider $\varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A) \wedge (\ddot{M}, B)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ or $\varepsilon \notin (\ddot{M}, B)$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ or $\varepsilon \in (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$

Since $\exists \varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$ such that $\varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$,

Therefore $[(\ddot{L}, A) \wedge (\ddot{M}, B)]^C \subseteq (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$.

$\therefore (\ddot{L}, A)^C \vee (\ddot{M}, B)^C = [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$ □

Proof 2

Let $\varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ and $\varepsilon \in (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ and $\varepsilon \notin (\ddot{M}, B)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A) \vee (\ddot{M}, B)$

$\Rightarrow \varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$

Since $\exists \varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$ such that $\varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$,

Therefore $(\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \subseteq [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$.

Then consider $\varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A) \vee (\ddot{M}, B)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ and $\varepsilon \notin (\ddot{M}, B)$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ and $\varepsilon \in (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$

Since $\exists \varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$ such that $\varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$,

Therefore $[(\ddot{L}, A) \vee (\ddot{M}, B)]^C \subseteq (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$.

$\therefore (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C = [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$ □

Proof 3

Let $\varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ or $\varepsilon \in (\ddot{M}, B)^C$ or $\varepsilon \in (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ or $\varepsilon \notin (\ddot{M}, B)$ or $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \wedge (\ddot{M}, A)]$ or $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]$

$\Rightarrow \varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]^C$

Since $\exists \varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$ such that $\varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]^C$,

Therefore $(\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C \subseteq [(\ddot{L}, A) \wedge (\ddot{M}, B) \wedge (\ddot{N}, C)]^C$.

Then consider $\varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]^C$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \wedge (\ddot{M}, A)]$ or $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ or $\varepsilon \notin (\ddot{M}, B)$ or $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ or $\varepsilon \in (\ddot{M}, B)^C$ or $\varepsilon \in (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$

Since $\exists \varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]^C$ such that $\varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$,

Therefore $[(\ddot{L}, A) \wedge (\ddot{M}, B) \wedge (\ddot{N}, C)]^C \subseteq (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$.

$\therefore (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C = [(\ddot{L}, A) \wedge (\ddot{M}, B) \wedge (\ddot{N}, C)]^C$ □

Proof 4

Let $\varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ and $\varepsilon \in (\ddot{M}, B)^C$ and $\varepsilon \in (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ and $\varepsilon \notin (\ddot{M}, B)$ and $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \vee (\ddot{M}, A)]$ and $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]$

$\Rightarrow \varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]^C$

Since $\exists \varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$ such that $\varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]^C$,

Therefore $(\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C \subseteq [(\ddot{L}, A) \vee (\ddot{M}, B) \vee (\ddot{N}, C)]^C$.

Then consider $\varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]^C$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \vee (\ddot{M}, A)]$ and $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ and $\varepsilon \notin (\ddot{M}, B)$ and $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ and $\varepsilon \in (\ddot{M}, B)^C$ and $\varepsilon \in (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$

Since $\exists \varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]^C$ such that $\varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$,

Therefore $[(\ddot{L}, A) \vee (\ddot{M}, B) \vee (\ddot{N}, C)]^C \subseteq (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$.

$\therefore (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C = [(\ddot{L}, A) \vee (\ddot{M}, B) \vee (\ddot{N}, C)]^C$ □

The definition of MVINSS, its arithmetic operations and properties would provide a good insight in mining a new knowledge of NS.

4. Conclusions

In this paper, the concept of multi-valued interval neutrosophic soft set (MVINSS) has been successfully proposed by integrating the multi-valued interval neutrosophic set and soft set. It is already known that neutrosophic soft set considers the indeterminate and inconsistent information. But the proposed set was introduced to improve the result in decision-making problem with multi-valued interval neutrosophic soft elements. The proposed set has several significant features. Firstly, it emphasized the hesitant, indeterminate and uncertainty and can be used more practical to solve decision-making problem. Secondly, some basic properties of MVINSS such as complement, equality, inclusion, union, intersection, "AND" and "OR" were well defined. The propositions related to the proposed properties were mathematically proven and some examples were provided. For future work, this novel proposed set can be applied and utilized in solving supply chain, time series forecasting and decision-making problem such as partner selection, wastewater treatment selection and renewable energy selection.

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Neutrosophic Generalized Pre Regular Closed Sets

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Abstract: As a generalization of fuzzy sets and intuitionistic fuzzy sets, Neutrosophic sets have been developed by Smarandache to represent imprecise, incomplete and inconsistent information existing in the real world. A neutrosophic set is characterized by a truth value, an indeterminacy value and a falsity value. In this paper, we introduce and study a new class of Neutrosophic generalized closed set, namely Neutrosophic generalized pre regular closed sets and Neutrosophic generalized pre regular open sets in Neutrosophic topological spaces. Also we study the separation axioms of Neutrosophic generalized pre regular closed sets, namely Neutrosophic pre regular $T_{1/2}$ space and Neutrosophic pre regular $T^*_{1/2}$ space and their properties are discussed.

Keywords: Neutrosophic generalized pre regular closed sets, Neutrosophic generalized pre regular open sets, $NprT_{1/2}$ space and $NprT^*_{1/2}$ space.

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1. Introduction

In 1970, Levine [12] introduced the concept of g-closed sets in general topology. Generalized closed sets play a very important role in general topology and they are now the research topics of many researchers worldwide. In 1965, Zadeh [19] introduced the notion of fuzzy sets [FS]. Later, fuzzy topological space was introduced by Chang [6] in 1968 using fuzzy sets. In 1986, Atanassov [5] introduced the notion of intuitionistic fuzzy sets [IFS], where the degree of membership and degree of non-membership of an element in a set X are discussed. In 1997, Intuitionistic fuzzy topological spaces were introduced by Coker [7] using intuitionistic fuzzy sets.

Neutrality the degree of indeterminacy as an independent concept was introduced by Florentin Smarandache [8]. He also defined the Neutrosophic set on three components, namely Truth (membership), Indeterminacy, Falsehood (non-membership) from the fuzzy sets and intuitionistic fuzzy sets. Smarandache's Neutrosophic concepts have wide range of real time applications for the fields of [1, 2, 3&4] Information systems, Computer science, Artificial Intelligence, Applied Mathematics and Decision making.

In 2012, Salama A. A and Alblowi [14] introduced the concept of Neutrosophic topological spaces by using Neutrosophic sets. Salama A. A. [15] introduced Neutrosophic closed set and Neutrosophic continuous functions in Neutrosophic topological spaces. Further the basic sets like Neutrosophic regular-open sets, Neutrosophic semi-open sets, Neutrosophic pre-open sets, Neutrosophic α-open sets and Neutrosophic generalized closed sets are introduced in Neutrosophic topological space and their properties are studied by various authors [10], [15], [17], [13]. In this direction, we introduce and analyze a new class of Neutrosophic generalized closed set called Neutrosophic generalized pre regular closed sets and Neutrosophic generalized pre regular open sets in Neutrosophic topological spaces. Also we study the separation axioms of Neutrosophic generalized pre regular closed sets, namely Neutrosophic pre regular $T_{1/2}$ space and Neutrosophic pre regular $T^*_{1/2}$ space in Neutrosophic topological spaces. Many examples are given to justify the results.

2. Preliminaries

We recall some basic definitions that are used in the sequel.

Definition 2.1: [14] Let X be a non-empty fixed set. A Neutrosophic set (NS for short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ where the functions $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

Remark 2.2: [14] A Neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ can be identified to an ordered triple $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ in non-standard unit interval $]^-0, 1^+[$on X.

Remark 2.3: [14] For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \nu_A \rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$.

Example 2.4: [14] Every IFS A is a non-empty set in X is obviously on NS having the form $A = \{\langle x, \mu_A(x), 1 - (\mu_A(x) + \nu_A(x)), \nu_A(x)\rangle : x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the NS 0_N and 1_N in X as follows:

0_N may be defined as:

(0_1) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$

(0_2) $0_N = \{\langle x, 0, 1, 1\rangle : x \in X\}$

(0_3) $0_N = \{\langle x, 0, 1, 0\rangle : x \in X\}$

(0_4) $0_N = \{\langle x, 0, 0, 0\rangle : x \in X\}$

1_N may be defined as:

(1_1) $1_N = \{\langle x, 1, 0, 0\rangle : x \in X\}$

(1_2) $1_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$

(1_3) $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$

(1_4) $1_N = \{\langle x, 1, 1, 1\rangle : x \in X\}$

Definition 2.5: [14] Let $A = \langle \mu_A, \sigma_A, \nu_A \rangle$ be a NS on X, then the complement of the set A [C(A) for short] may be defined as three kind of complements:

(C_1) $C(A) = \{\langle x, 1-\mu_A(x), 1-\sigma_A(x), 1-\nu_A(x)\rangle : x \in X \}$

(C_2) $C(A) = \{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

(C_3) $C(A) = \{\langle x, \nu_A(x), 1-\sigma_A(x), \mu_A(x)\rangle : x \in X\}$

Definition 2.6: [14] Let X be a non-empty set and Neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$. Then we may consider two possible definitions for subsets ($A \subseteq B$).

(1) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x)$ and $\mu_A(x) \ge \mu_B(x)$ $\forall$ $x \in X$

(2) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x)$ and $\mu_A(x) \ge \mu_B(x)$ $\forall$ $x \in X$

Proposition 2.7: [14] For any Neutrosophic set A, the following conditions hold:

$0_N \subseteq A$, $0_N \subseteq 0_N$

$A \subseteq 1_N$, $1_N \subseteq 1_N$

Definition2.8: [14] Let X be a non-empty set and $A = \{<x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$, $B = \{<x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$ are NSs. Then $A \cap B$ may be defined as:

(I_1) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)\rangle$

(I_2) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)\rangle$

A∪B may be defined as:
(U1) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x) \rangle$
(U2) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x) \rangle$

We can easily generalize the operations of intersection and union in Definition 2.8., to arbitrary family of NSs as follows:

Definition 2.9: [14] Let $\{A_j: j \in J\}$ be an arbitrary family of NSs in X, then
$\cap A_j$ may be defined as:
(i) $\cap A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \wedge_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \nu_{A_j}(x) \rangle$
(ii) $\cap A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \vee_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \nu_{A_j}(x) \rangle$

$\cup A_j$ may be defined as:
(i) $\cup A_j = \langle x, \vee_{j \in J} \mu_{A_j}(x), \vee_{j \in J} \sigma_{A_j}(x), \wedge_{j \in J} \nu_{A_j}(x) \rangle$
(ii) $\cup A_j = \langle x, \vee_{j \in J} \mu_{A_j}(x), \wedge_{j \in J} \sigma_{A_j}(x), \wedge_{j \in J} \nu_{A_j}(x) \rangle$

Proposition 2.10: [14] For all A and B are two Neutrosophic sets then the following conditions are true:

$C(A \cap B) = C(A) \cup C(B)$; $C(A \cup B) = C(A) \cap C(B)$.

Definition 2.11: [14] A Neutrosophic topology [NT for short] is a non-empty set X is a family τ of Neutrosophic subsets in X satisfying the following axioms:
(NT1) $0_N, 1_N \in \tau$,
(NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(NT3) $\cup G_i \in \tau$ for every $\{G_i : i \in J\} \subseteq \tau$.

Throughout this paper, the pair (X, τ) is called a Neutrosophic topological space (NTS for short). The elements of τ are called Neutrosophic open sets [NOS for short]. A complement C(A) of a NOS A in NTS (X, τ) is called a Neutrosophic closed set [NCS for short] in X.

Example 2.12: [14] Any fuzzy topological space (X, τ) in the sense of Chang is obviously a NTS in the form $\tau = \{A: \mu_A \in \tau\}$ wherever we identify a fuzzy set in X whose membership function is μ_A with its counterpart.

The following is an example of Neutrosophic topological space.

Example 2.13: [14] Let X = {x} and A = {⟨x, 0.5, 0.5, 0.4⟩: x ∈ X}, B = {⟨x, 0.4, 0.6, 0.8⟩: x ∈ X}, C = {⟨x, 0.5, 0.6, 0.4⟩: x ∈ X}, D = {⟨x, 0.4, 0.5, 0.8⟩: x ∈ X}. Then the family $\tau = \{0_N, A, B, C, D, 1_N\}$ of NSs in X is Neutrosophic topological space on X.

Now, we define the Neutrosophic closure and Neutrosophic interior operations in Neutrosophic topological spaces:
Definition 2.14: [14] Let (X, τ) be NTS and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle: x \in X\}$ be a NS in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by
$NCl(A) = \cap\{K : K$ is a NCS in X and $A \subseteq K\}$
$NInt(A) = \cup\{G : G$ is a NOS in X and $G \subseteq A\}$
It can be also shown that NCl(A) is NCS and NInt(A) is a NOS in X.
a) A is NOS if and only if A = NInt(A),
b) A is NCS if and only if A = NCl(A).

Proposition 2.15: [14] For any Neutrosophic set A is (X, τ) we have
a) NCl(C(A)) = C(NInt(A)),
b) NInt(C(A)) = C(NCl(A)).

Proposition 2.16: [14] Let (X, τ) be NTS and A, B be two Neutrosophic sets in X. Then the following properties are holds:

a) NInt(A) ⊆ A,
b) A ⊆ NCl(A),
c) A ⊆ B ⇒ NInt(A) ⊆ NInt(B),
d) A ⊆ B ⇒ NCl(A) ⊆ NCl(B),
e) NInt(NInt(A)) = NInt(A),
f) NCl(NCl(A)) = NCl(A),
g) NInt(A ∩ B) = NInt(A) ∩ NInt(B),
h) NCl(A ∪ B) = NCl(A) ∪ NCl(B),
i) NInt(0_N) = 0_N,
j) NInt(1_N) = 1_N,
k) NCl(0_N) = 0_N,
l) NCl(1_N) = 1_N,
m) A ⊆ B ⇒ C(A) ⊆ C(B),
n) NCl(A ∩ B) ⊆ NCl(A) ∩ NCl(B),
o) NInt(A ∪ B) ⊇ NInt(A) ∪ NInt(B).

Definition 2.17: [9] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be

(i) Neutrosophic regular closed set (NRCS for short) if A = NCl(NInt(A)),
(ii) Neutrosophic regular open set (NROS for short) if A = NInt(NCl(A)),
(iii) Neutrosophic semi closed set (NSCS for short) if NInt(NCl(A)) ⊆ A,
(iv) Neutrosophic semi open set (NSOS for short) if A ⊆ NCl(NInt(A)),
(v) Neutrosophic pre closed set (NPCS for short) if NCl(NInt(A)) ⊆ A,
(vi) Neutrosophic pre open set (NPOS for short) if A ⊆ NInt(NCl(A)),
(vii) Neutrosophic α- closed set (NSCS for short) if NCl(NInt(NCl(A))) ⊆ A,
(viii) Neutrosophic α- open set (NSOS for short) if A ⊆ NInt(NCl(NInt(A))).

Definition 2.18: [18] Let (X, τ) be NTS and A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} be a NS in X. Then the Neutrosophic pre closure and Neutrosophic pre interior of A are defined by

NPCl(A) = ∩{K : K is a NPCS in X and A ⊆ K},
NPInt(A) = ∪{G : G is a NPOS in X and G ⊆ A}.

Definition 2.18: [13] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized closed set (NGCS for short) if NCl(A) ⊆ U whenever A ⊆ U and U is a NOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic generalized open set (NGOS for short) if C(A) is a NGCS in (X, τ).

Definition 2.20: [11] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic α- generalized closed set (NαGCS for short) if NαCl(A) ⊆ U whenever A ⊆ U and U is a NOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic α- generalized open set (NαGOS for short) if C(A) is a NαGCS in (X, τ).

Definition 2.21: [16] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic ω closed set (NωCS for short) if NCl(A) ⊆ U whenever A ⊆ U and U is a NSOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic ω open set (NωOS for short) if C(A) is a NωCS in (X, τ).

Definition 2.22: [9] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic regular generalized closed set (NRGCS for short) if NCl(A) ⊆ U whenever A ⊆ U and U is a NROS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic regular generalized open set (NRGOS for short) if C(A) is a NRGCS in (X, τ).

Definition 2.23: [18] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized pre closed set (NGPCS for short) if NPCl(A) ⊆ U whenever A ⊆ U and U is a NOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic generalized pre open set (NGPOS for short) if C(A) is a NGPCS in (X, τ).

Definition 2.24: [9] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic regular α generalized closed set (NRαGCS for short) if NαCl(A) ⊆ U whenever A ⊆ U and U is a NROS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic regular α generalized open set (NRαGOS for short) if C(A) is a NRGCS in (X, τ).

3. Neutrosophic Generalized Pre Regular Closed Sets

In this section we introduce Neutrosophic generalized pre regular closed sets in the Neutrosophic topological space and study some of their properties.

Definition 3.1: A NS A in a NTS (X, τ) is said to be a Neutrosophic generalized pre regular closed set (NGPRCS for short) if NPCl(A) ⊆ U whenever A ⊆ U and U is a NROS in (X, τ). The family of all NGPRCSs of a NTS(X, τ) is denoted by NGPRC(X).

Example 3.2: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.2, 0.1, 0.7), (0.4, 0.4, 0.7)⟩ is a NGPRCS in (X, τ). Since A ⊆ U and U is a NROS, we have NPCl(A) = A ⊆ U.

Theorem 3.3: Every NCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NCS in (X, τ), we have NCl (A) = A. Therefore NPCl(A) ⊆ NCl (A) = A ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.4: In Example 3.2., the NS A= A= ⟨(0.2, 0.1, 0.7), (0.4, 0.4, 0.7)⟩ is a NGPRCS but not NCS in (X, τ).

Theorem 3.5: Every NαCS in (X, τ) is an NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NαCS in (X, τ), we have NCl(NInt(NCl(A))) ⊆ A, now A ⊆ NCl(A), NCl(NInt(A)) ⊆ NCl(NInt(NCl(A))) ⊆ A. Therefore NPCl(A) = A∪ NCl(NInt(A)) ⊆ A∪A = A ⊆ U. Hence A is a NGPRCS in (X, τ).

Example 3.6: In Example 3.2., the NS A= A= ⟨(0.2, 0.1, 0.7), (0.4, 0.4, 0.7)⟩ is a NGPRCS but not NαCS in (X, τ).

Theorem 3.7: Every NωCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NωCS in (X, τ), we have NCl (A) ⊆ U because every NROS is NSOS in (X, τ). Therefore NPCl(A) ⊆ NCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.8: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.6, 0.5, 0.2), (0.7, 0.5, 0.1)⟩ and V = ⟨(0.5, 0.4, 0.7), (0.4, 0.5, 0.6)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.4, 0.3, 0.7), (0.3, 0.2, 0.6)⟩ is a NGPRCS in (X, τ). Since A ⊆ V and V is a NROS, we have NPCl(A) = A ⊆ V. But A is not NωCS in (X, τ). Since A ⊆ V and V is a NSOS, we have NCl(A) = C(V) ⊈ V.

Theorem 3.9: Every NPCS in (X, τ) is an NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NPCS in (X, τ), we have NCl(NInt(A)) ⊆ A. Therefore NPCl(A) = A∪ NCl(NInt(A)) ⊆ A∪A = A ⊆ U. Hence A is a NGPRCS in (X, τ).

Example 3.10: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.3, 0.2, 0.6), (0.1, 0.2, 0.7)⟩ and V = ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩ is a NGPRCS in (X, τ). Since A ⊆ 1_N, we have NPCl(A) = 1_N ⊆ 1_N. But A is not NPCS in (X, τ). Since NCl(NInt(A)) = 1_N ⊈ A.

Theorem 3.11: Every NGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NPCl(A) ⊆ NCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.12: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.3, 0.5, 0.7), (0.4, 0.5, 0.6)⟩ and V = ⟨(0.8, 0.5, 0.2), (0.7, 0.5, 0.3)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.3, 0.5, 0.7), (0.3, 0.5, 0.7)⟩ is a NGPRCS in (X, τ). Since A ⊆ U and U is a NROS, we have NPCl(A) = A ⊆ U. But A is not NGCS in (X, τ). Since A ⊆ U and U is a NOS, we have NCl(A) = C(U) ⊈ U.

Theorem 3.13: Every NαGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NαGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NPCl(A) ⊆ NαCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.14: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.4, 0.3, 0.6), (0.3, 0.4, 0.7)⟩ is a NGPRCS in (X, τ). Since A ⊆ U and U is a NROS, we have NPCl(A) = A ⊆ U. But A is not NαGCS in (X, τ). Since A ⊆ U and U is a NOS, we have NαCl(A) = C(U) ⊈ U.

Theorem 3.15: Every NRαGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NRαGCS in (X, τ). Therefore NPCl(A) ⊆ NαCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.16: In Example 3.14., the NS A= ⟨(0.4, 0.3, 0.6), (0.3, 0.4, 0.7)⟩ is a NGPRCS but not NRαGCS in (X, τ).

Theorem 3.17: Every NGPCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NGPCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NPCl(A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.18: In Example 3.10., the NS A= ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩ is a NGPRCS in (X, τ). Since A ⊆ 1_N, we have NPCl(A) = 1_N ⊆ 1_N. But A is not NGPCS in (X, τ). Since A ⊆ V and V is a NOS, we have NPCl(A) = 1_N ⊈ V.

Theorem 3.19: Every NRGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NRGCS in (X, τ). Therefore NPCl(A) ⊆ NCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.20: In Example 3.8., the NS A= ⟨(0.4, 0.3, 0.7), (0.3, 0.2, 0.6)⟩ is a NGPRCS but not NRGCS in (X, τ).

Theorem 3.21: Every NαGCS in (X, τ) is a NRαGCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A $\subseteq$ U. Since A is NαGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NαCl (A) $\subseteq$ U, by hypothesis. Hence A is a NRαGCS in (X, τ).

Example 3.22: In Example 3.10., the NS A= ⟨(0.7, 0.2, 0.3), (0.8, 0.2, 0.2)⟩ is a NRαGCS but not NαGCS in (X, τ).

Theorem 3.23: Every NGCS in (X, τ) is a NαGCS in (X, τ) but not conversely.

Proof: Let U be a NOS in (X, τ) such that A $\subseteq$ U. Since A is NGCS in (X, τ). Therefore NαCl(A) $\subseteq$ NCl (A) $\subseteq$ U, by hypothesis. Hence A is a NαGCS in (X, τ).

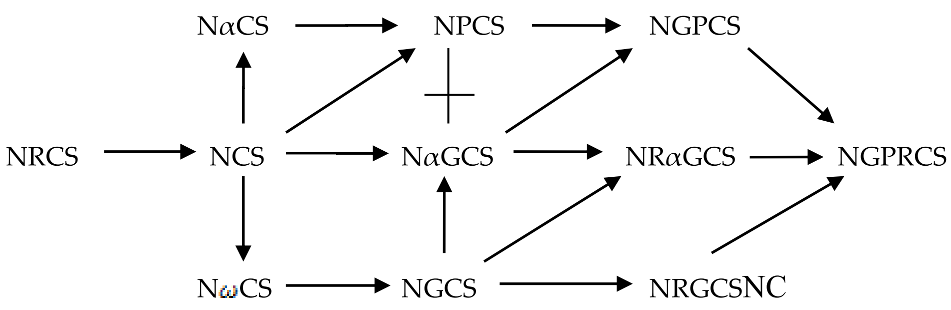
Example 3.24: Let X= {a} and $\tau = \{0_N, U, V, 1_N\}$ where U= ⟨0.5, 0.4, 0.7⟩ and V = ⟨0.8, 0.5, 0.2)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨0.2, 0.2, 0.8⟩ is a NαGCS in (X, τ). Since A $\subseteq$ U and U is a NOS, we have NαCl(A) = A $\subseteq$ U. But A is not NGCS in (X, τ). Since A $\subseteq$ U, we have NCl(A) = C(V) $\nsubseteq$ U.

Theorem 3.25: Every NGCS in (X, τ) is a NRGCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A $\subseteq$ U. Since A is NGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NCl (A) $\subseteq$ U, by hypothesis. Hence A is a NRGCS in (X, τ).

Example 3.26: Let X= {a, b, c} and $\tau = \{0_N, U, 1_N\}$ where U= ⟨(0.6, 0.4, 0.3), (0.8, 0.5, 0.2), (0.7, 0.4, 0.8)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.5, 0.6, 0.6), (0.3, 0.5, 0.3), (0.5, 0.4, 0.3)⟩ is a NRGCS in (X, τ). Since A $\subseteq$ 1_N, we have NCl(A) = $1_N \subseteq 1_N$. but A is not NGCS in (X, τ). Since A $\subseteq$ U and U is a NOS, we have NCl(A) = 1_N $\nsubseteq$ U.

The following diagram, we have provided the relation between NGPRCS and the other existed NSs.



In this diagram by A ⟶ B means A implies B but not conversely and A ⟷̸ B means A & B are independent.

Remark 3.27: The union of any two NGPRCSs in (X, τ) is not an NGPRCS in (X, τ) in general as seen from the following example.

Example 3.28: Let X = {a, b} and $\tau = \{0_N, U, V, 1_N\}$ where U = ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then the NSs A = ⟨(0.2, 0.1, 0.7), (0.4, 0.4, 0.7)⟩ and B=⟨(0.5, 0.3, 0.6), (0.2, 0.2, 0.8)⟩ are NGPRCSs in (X, τ) but A⋃B=⟨(0.5,0.3,0.6), (0.4,0.4,0.7)⟩ is not a NGPRCS in (X, τ). Since A⋃B ⊆ U but NPCl(A⋃B) = C(U) ⊈ U.

Remark 3.29: The intersection of any two NGPRCSs in (X, τ) is not an NGPRCS in (X, τ) in general as seen from the following example.

Example 3.30: Let X = {a, b} and τ = {0_N, U, V, 1_N} where U = ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then the NSs A = ⟨(0.5, 0.5, 0.4), (0.7, 0.6, 0.7)⟩ and B = ⟨(0.6, 0.3, 0.6), (0.4, 0.4, 0.3)⟩ are NGPRCSs in (X, τ) but A∩B = ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ is not a NGPRCS in (X, τ). Since A∩B ⊆ U but NPCl(A∩B) = C(U) ⊈ U.

Theorem 3.31: Let (X, τ) be a NTS. Then for every A ∈ NGPRC(X) and for every NS B ∈ NS(X), A ⊆ B ⊆ NPCl(A) implies B ∈ NGPRC(X).

Proof: Let B ⊆ U and U is a NROS in (X, τ). Since A ⊆ B, then A ⊆ U. Given A is a NGPRCS, it follows that NPCl(A) ⊆ U. Now B ⊆ NPCl(A) implies NPCl(B) ⊆ NPCl(NPCl(A)) = NPCl(A). Thus, NPCl(B) ⊆ U. This proves that B ∈ NGPRC(X).

Theorem 3.32: If A is a NROS and a NGPRCS in (X, τ), then A is a NPCS in (X, τ).

Proof: Since A ⊆ A and A is a NROS in (X, τ), by hypothesis, NPCl(A) ⊆ A. But since A ⊆ NPCl(A). Therefore NPCl(A)= A. Hence A is a NPCS in (X, τ).

Theorem 3.33: Let (X, τ) be a NTS and NPC(X) (resp. NRO(X)) be the family of all NPCSs (resp. NROSs) of X. If NPC(X) = IRO(X) then every Neutrosophic subset of X is NGPRCS in (X, τ).

Proof: If NPC(X) = IRO(X) and A is any Neutrosophic subset of X such that A ⊆ U where U is NROS in X. Then by hypothesis, U is NPCS in X which implies that NPCl(U) = U. Then NPCl(U) ⊆ NPCl(U) = U. Therefore A is NGPRCS in (X, τ).

Definition 3.34: Let (X, τ) be a NTS and A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} be the subset of X. Then
NGPRCl(A) = ∩{K : K is a NGPRCS in X and A ⊆ K} and
NGPRInt(A) = ∪{G : G is a NGPROS in X and G ⊆ A}.

Lemma 3.35: Let A and B be subsets of (X, τ). Then the following results are obvious.

a) NGPRCl(0_N) = 0_N.
b) NGPRCl(1_N) = 1_N.
c) A ⊆ NGPRCl(A).
d) A ⊆ B ⇒ *NGPRCl(A)* ⊆ *NGPRCl(B).*

4. Neutrosophic Generalized Pre Regular Open Sets

In this section we introduce Neutrosophic generalized pre regular open sets in Neutrosophic topological space.

Definition 4.1: A NS A in a NTS (X, τ) is said to be a Neutrosophic generalized pre regular open set (NGPROS for short) if NPInt(A) ⊇ U whenever A ⊇ U and U is a NRCS in (X, τ). Alternatively, A NS A is said to be a Neutrosophic generalized pre regular open set (NGPROS for short) if the complement of C(A) is a NGPRCS in (X, τ).
The family of all NGPROSs of a NTS(X, τ) is denoted by NGPRO(X).

Example 4.2*:* Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.8, 0.9, 0.2), (0.9, 0.6, 0.1)⟩ is a NGPROS in (X, τ). Since A $\supseteq$ C(U) and C(U) is a NRCS, we have NPInt(A) = A $\supseteq$ C(U).

Theorem 4.3: Every NOS is a NGPROS in (X, τ) but the converses may not be true in general.

Proof: Let U be a NRCS in (X, τ) such that A $\supseteq$ U. Since A is NOS, NInt(A) = A. By hypothesis, NPInt(A) = A ∩ NInt(NCl(A)) = A ∩ NCl(A) $\supseteq$ A ∩ A = A $\supseteq$ U. Therefore A is a NGPROS in (X, τ).

Example 4.4: In Example 4.2., the NS A= ⟨(0.8, 0.9, 0.2), (0.9, 0.6, 0.1)⟩ is an NGPROS in (X, τ) but not a NOS in (X, τ).

Theorem 4.5*:* Every NαOS, NWOS, NPOS, NGOS, NαGOS, NGPOS, NRGOS, NRαGOS is a NGPROS in (X, τ) but the converses are not true in general.

Example 4.6: Let X= {a, b} and τ = {0_N, U, 1_N} where U = ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A = ⟨(0.2, 0.8, 0.6), (0.6, 0.4, 0.9)⟩ is a NGPROS in (X, τ). Since A $\supseteq$ 0_N, we have NPInt(A) = 0_N $\supseteq$ 0_N. but A is not a NαOS, NWOS, NPOS in (X, τ).

Example 4.7: Let X= {a, b} and τ = {0_N, U, 1_N} where U = ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A = ⟨(0.3, 0.8, 0.4), (0.7, 0.4, 0.8)⟩ is a NGPROS in (X, τ). Since A $\supseteq$ 0_N, we have NPInt(A) = 0_N $\supseteq$ 0_N. but A is not a NGOS, NαGOS, NGPOS in (X, τ).

Example 4.8*:* Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.6, 0.5, 0.2), (0.7, 0.5, 0.1)⟩ and V = ⟨(0.5, 0.4, 0.7), (0.4, 0.5, 0.6)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.8, 0.8, 0.2), (0.7, 0.9, 0.3)⟩ is a NGPROS in (X, τ). Since A $\supseteq$ C(V) and C(V) is a NRCS, we have NPInt(A) = A $\supseteq$ C(V). but A is not NRGOS, NRαGOS in (X, τ).

Theorem 4.9: Let (X, τ) be a NTS. Then for every A ∈ NGPRO(X) and for every B ∈ NP(X), NPInt(A) $\subseteq$ B $\subseteq$ A implies B ∈ NGPRO(X).

Proof: Let A be any NGPROS of (X, τ) and B be any NS of X. By hypothesis NPInt(A) $\subseteq$ B $\subseteq$ A. Then C(A) is an NGPRCS in (X, τ) and C(A) $\subseteq$ C(B) $\subseteq$ NPCl(C(A)). By Theorem 3.31., C(B) is an NGPRCS in (X, τ). Therefore B is an NGPROS in (X, τ). Hence B ∈ NGPRO(X).

Theorem 4.10: A NS A of a NTS (X, τ) is a NGPROS in (X, τ) if and only if F $\subseteq$ Npint(A) whenever F is a NRCS in (X, τ) and F $\subseteq$ A.

Proof: Necessity*:* Suppose A is a NGPROS in (X, τ). Let F be a NRCS in (X, τ) such that F $\subseteq$ A. Then C(F) is a NROS and C(A) $\subseteq$ C(F). By hypothesis C(A) is a NGPRCS in (X, τ), we have NPCl(C(A)) $\subseteq$ C(F). Therefore F $\subseteq$ Npint(A).

Sufficiency: Let U be a NROS in (X, τ) such that C(A) $\subseteq$ U. By hypothesis, C(U) $\subseteq$ Npint(A). Therefore NPCl(C(A)) $\subseteq$ U and C(A) is a NGPRCS in (X, τ). Hence A is a NGPROS in (X, τ).

Theorem 4.11*:* Let (X, τ) be a NTS and NPO(X) (resp. NGPRO(X)) be the family of all NPOSs (resp. NGPROSs) of X. Then NPO(X) ⊆ NGPRO(X).

Proof*:* Let A ∈ NPO(X). Then C(A) is NPCS and so NGPRCS in (X, τ). This implies that A is NGPROS in (X, τ). Hence A ∈ NGPRO(X). Therefore NPO(X) ⊆ NGPRO(X).

5. Separation Axioms of Neutrosophic Generalized Pre Regular Closed Sets

In this section we have provide some applications of Neutrosophic generalized pre regular closed sets in Neutrosophic topological spaces.

Definition 5.1: If every NGPRCS in (X, τ) is a NPCS in (X, τ), then the space (X, τ) can be called a Neutrosophic pre regular $T_{1/2}$ ($NPRT_{1/2}$ for short) space.

Theorem 5.2: An NTS (X, τ) is a $NPRT_{1/2}$ space if and only if NPOS(X) = NGPRO(X).

Proof: Necessity: Let (X, τ) be a $NPRT_{1/2}$ space. Let A be a NGPROS in (X, τ). By hypothesis, C(A) is a NGPRCS in (X, τ) and therefore A is a NPOS in (X, τ). Hence NPO(X) = NGPRO(X).
Sufficiency: Let NPO(X, τ) = NGPRO(X, τ). Let A be a NGPRCS in (X, τ). Then C(A) is a NGPROS in (X, τ). By hypothesis, C(A) is a NPOS in (X, τ) and therefore A is a NPCS in (X, τ). Hence (X, τ) is a $NPRT_{1/2}$ space.

Definition 5.3: A NTS (X, τ) is said to be a Neutrosophic pre regular $T^*_{1/2}$ space ($NPRT^*_{1/2}$ space for short) if every NGPRCS is a NCS in (X, τ).

Remark 5.4: Every $NPRT^*_{1/2}$ space is a $NPRT_{1/2}$ space but not conversely.

Proof: Assume be a $NPRT^*_{1/2}$ space. Let A be a NGPRCS in (X, τ). By hypothesis, A is an NCS. Since every NCS is a NPCS, A is a NPCS in (X, τ). Hence (X, τ) is a $NPRT_{1/2}$ space.

Example 5.8: Let X= {a, b} and let τ = {0N, U, 1N} where U= ⟨(0.5, 0.4, 0.7), (0.4, 0.5, 0.6)⟩. Then (X, τ) is a NPRT1/2 space, but it is not $NPRT^*_{1/2}$ space. Here the NS A= ⟨(0.2, 0.3, 0.8), (0.3, 0.4, 0.8)⟩ is a NGPRCS but not a NCS in (X, τ).

Theorem 5.9: Let (X, τ) be a $NPRT^*_{1/2}$ space then,

(i) the union of NGPRCSs is NGPRCS in (X, τ)
(ii) the intersection of NGPROSs is NGPROS in (X, τ)

Proof: (i) Let $\{Ai\}_{i\in J}$ be a collection of NGPRCSs in a $NPRT^*_{1/2}$ space (X, τ). Thus, every NGPRCSs is a NCS. However, the union of NCSs is a NCS in (X, τ). Therefore the union of NGPRCSs is NGPRCS in (X, τ). (ii) Proved by taking the complement in (i).

6. Conclusion

In this paper, we have defined new class of Neutrosophic generalized closed sets called Neutrosophic generalized pre regular closed sets; Neutrosophic generalized pre regular open sets and studied some of their properties in Neutrosophic topological spaces. Furthermore, the work was extended as the separation axioms of Neutrosophic generalized pre regular closed sets, namely Neutrosophic pre regular $T_{1/2}$ space and Neutrosophic pre regular $T^*_{1/2}$ space and discussed their properties. Further, the relation between Neutrosophic generalized pre regular closed set and existing Neutrosophic closed sets in Neutrosophic topological spaces were established. Many examples are given to justify the results.

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An Approach to Similarity Measure between Neutrosophic Soft Sets

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Abstract: In this paper, we have defined different types of similarity measures between Neutrosophic Soft (NS) sets and studied some of their properties. Finally we have solve a real life problem by using similarity measure of neutrosophic soft sets.

Keywords: Neutrosophic set, Soft Set, Neutrosophic Soft set, Similarity Measure, Neutrosophic Soft Similarity Measure.

1. Introduction

Theory of probability, fuzzy sets, rough sets, vague sets etc. are the some established theories in the world to solve the problems related to uncertainty. Molodtstov introduced the Soft Set theory [32] as a parametric tool to deal the uncertain data of many mathematical problems. Later Maji, Roy and Biswas [24, 25] have further studied the theory of soft sets. Gradually research in soft set theory (SST) are grown up in many areas like algebra, entropy calculation, solving decision making problems etc. [27 - 30], for example). Prof. Florentin Smarandache [34] introduced the neutrosophic logic and sets. In this logic, every statement consists a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F) and all of these degrees lie between, the non-standard unit intervals. Works on soft sets and neutrosophic sets are progressing very rapidly [10, 11, 19, 21, 28, 29, 30, 31, 32, 33]. In 2013, P.K. Maji introduced the theory of Neutrosophic Soft (NS) sets [26]. Similarity measure technique is a well-known process to compare two sets. Similarity measure on Fuzzy sets, Soft sets, Neutrosophic sets etc. are done by several authors in their papers [14, 15, 16, 17, 18, 19, 22]. In this paper we have tried to build up the theory of similarity measures between two NS sets. We organized the paper in the following manner. In Section 2, we have given some preliminary definitions and results. We have given a similarity measure of NS in Section 3. In Section 4 and Section 5 are devoted on weighted similarity measure of NS sets and measuring distances of NS sets respectively. We have discussed Distanced Based Similarity Measure of NS sets in Section 6. A real life application of similarity measure of two NS sets are shown in Section 8. Section 9 is the conclusion of our paper.

2. Preliminaries

Neutrosophic sets has several applications in different areas of physical systems, biological systems etc. and even in daily life problems. Most of the preliminary ideas can be easily found in any standard reference say [1—11, 31, 34, 35] .However we will discuss some definitions and terminologies regarding neutrosophic sets which will be used in the rest of the paper.

Definition 1 *[34] Let X be a universal set. A neutrosophic set A on X is characterized by a truth membership function t_A, an indeterminacy function i_A and a falsity function f_A, where $t_A, i_A, f_A: \to [0,1]$, are functions and $\forall x \in X, x = x(t_A(x), i_A(x), f_A(x)) \in A$ is a single valued neutrosophic element of A.*

Definition 2 *[25] Suppose U be an initial universal set and let E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U if and only if F is a mapping given by $F: A \to P(U)$.*

Example 3 *As an illustration, consider the following example. Suppose a soft set (F, E) describes choice of places which the authors are going to visit with his family. Consider U = the set of places under consideration $= \{x_1, x_2, x_3, x_4, x_5\}$. E = {desert, forest, mountain, sea beach} = $\{e_1, e_2, e_3, e_4\}$. Let $F(e_1) = \{x_1, x_2\}$, $F(e_2) = \{x_1, x_2, x_3\}$, $F(e_3) = \{x_4\}$, $F(e_4) = \{x_2, x_5\}$. So, the soft set (F, E) is a family $\{F(e_i); i = 1, \ldots, 4\}$ of U*.In 2012, P.K. Maji gives the idea of Neutrosophic Soft Set in his paper [26] as follows:

Definition 4 *[26] Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $N(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by $F: A \to N(U)$.*

Example 5 Let X and E be the set of buses and condition of buses i.e. the set of parameters respectively. Each parameter is either a neutrosophic word or sentence involving neutrosophic words. Consider E = {beautiful, eco-friendly, costly, good seating arrangement}. Now, to define a NS set means to sort out beautiful buses, eco-friendly buses etc. Suppose, there are four buses in the universe X given by $U = \{h_i;\ i = 1,2,3,4\}$ and the set of parameters $E = \{e_i;\ i = 1,2,3,4\}$, where e_1 stands for the parameter beautiful, e_2 stands for the parameter eco-friendly, e_3 stands for the parameter costly and the parameter e_4 stands for good seating arrangement. Let

$$
\begin{aligned}
\text{F(beautiful)} &= \{(h_1, 0.4, 0.7, 0.3), (h_2, 0.3, 0.6, 0.2), (h_3, 0.4, 0.4, 0.2), (h_4, 0.6, 0.5, 0.4)\},\\
\text{F(eco-friendly)} &= \{(h_1, 0.6, 0.7, 0.8), (h_2, 0.5, 0.5, 0.1), (h_3, 0.2, 0.3, 0.6)\},\\
\text{F(costly)} &= \{(h_2, 0.3, 0.3, 0.4), (h_3, 0.5, 0.4, 0.8), (h_4, 0.8, 0.7, 0.8)\},\\
\text{F(good-seating arrangement)} &= \{(h_1, 0.4, 0.1, 0.4), (h_2, 0.3, 0.7, 0.4), (h_4, 0.9, 0.6, 0.8)\}.
\end{aligned}
$$

Then (F, E) is a neutrosophic soft set (NSS) over X.

The most of the terminologies regarding Neutrosophic soft set can be found in [26]. Thus it is our request to follow the paper [26] thoroughly for terminologies, operations etc of NS set. Several authors have defined Similarity measure between two fuzzy sets. Prof. Chen have given the following definition of Similarity measure based on a matching function S.

Definition 6 *[12] Suppose A and B are two fuzzy sets with membership functions μ_A and μ_B respectively. Then the similarity measure between A and B is denoted by $S(A, B)$ and*

$$S(A,B)=\frac{\vec{A}.\,\vec{B}}{\overrightarrow{A^2}\ \vee\overrightarrow{B^2}}$$

where $\vec{A}\ =\ (\mu_A(x_1),\mu_A(x_2),,\ldots,\mu_A(x_n)\,)$ *and* $\vec{B}=(\mu_B(x_1),\,\mu_B(x_2),..,\,\mu_B(x_n)\,)$.

Prof P. Majumdar have defined similarity measure for two soft sets in his paper [27]. For details on similarity measures on two Soft sets, one can follow [27].

3. Similarity measure of two NS sets

Consider the NS set (F,E) over the set. Now we will express the NS set (F,E) as a NS soft matrix M as follows:

$$M=\begin{bmatrix} * & F(e_1) & F(e_2) & F(e_3) & F(e_4)\\ h_1 & (0.4,0.7,0.3) & (0.6,0.7,0.8) & (0,0,0) & (0.4,0.1,0.4)\\ h_2 & (0.2,0.3,0.6) & (0.5,0.5,0.1) & (0.3,0.3,0.4) & (0.3,0.7,0.4)\\ h_3 & (0.4,0.4,0.2) & (0.2,0.3,0.6) & (0.5,0.4,0.8) & (0,0,0)\\ h_4 & (0.6,0.5,0.4) & (0,0,0) & (0.8,0.7,0.8) & (0.9,0.6,0.8)\end{bmatrix}$$

Then with the above interpretation the NS set (F,E) is represented by the matrix M and we write

(F,E) = M. Clearly, the complement of (F,E), i.e. $(F,E)^{C}$ will be represented by another matrix M^{C}

where

$$M^{C}=\begin{bmatrix} * & F(e_1) & F(e_2) & F(e_3) & F(e_4)\\ h_1 & (0.3,0.7,0.4) & (0.8,0.7,0.6) & (0,0,0) & (0.4,0.1,0.4)\\ h_2 & (0.6,0.3,0.2) & (0.1,0.5,0.5) & (0.4,0.3,0.3) & (0.4,0.7,0.3)\\ h_3 & (0.2,0.4,0.4) & (0.6,0.3,0.2) & (0.8,0.4,0.5) & (0,0,0)\\ h_4 & (0.4,0.5,0.6) & (0,0,0) & (0.8,0.7,0.8) & (0.8,0.6,0.9)\end{bmatrix}$$

Hence for any given matrix representation *M*, we can retrieve the NS set (*F*, *E*) and also vice versa in an obvious way. Henceforth, we will denote each column of membership matrix by the vector $\overrightarrow{F(e_\iota)}$ or simply by $\overrightarrow{F(e_i)}$

i.e. here $\overrightarrow{F(e_1)}$ = {(0.3, 0.7, 0.4), (0.6, 0.3, 0.2), (0.2, 0.4, 0.4), (0.4, 0.5, 0.6)} in M. Now we will define a similarity measure between two NS sets (F_1,E_1) and (F_2,E_2) over *U*. We try to formulate with the help of a matching function *S*.

Definition 7 The similarity between NS sets (F_1,E_1) and (F_2,E_2) is defined by

$$S(F_1,F_2)=\frac{\sum_i\overrightarrow{F_1(e_\iota)}.\ \overrightarrow{F_2(e_\iota)}}{\sum_i[\overrightarrow{F_1(e_\iota)^{\,2}}\ \vee\overrightarrow{F_2(e_\iota)^2}]}$$

provided,

(i) $E_1=E_2$

(ii) $\sum_i\overrightarrow{F_1(e_\iota)}.\ \overrightarrow{F_2(e_\iota)}=\sum_i(t_{F_1(e_i)}.t_{F_2(e_i)}+i_{F_1(e_i)}.i_{F_2(e_i)}+f_{F_1(e_i)}.f_{F_2(e_i)}\,)$

(iii) $\sum_i[\overrightarrow{F_1(e_\iota)^{\,2}}\ \vee\overrightarrow{F_2(e_\iota)^2}]\ =\ \sum_i(\boldsymbol{t}_{F_1(e_i)^{\,2}}\vee t_{F_2(e_i)}+i_{F_1(e_i)^{\,2}}\vee i_{F_2(e_i)^{\,2}}+f_{F_1(e_i)^{\,2}}\vee f_{F_2(e_i)^{\,2}}\,)$

If $E_1\neq E_2,\ E=E_1\cap E_2\neq\emptyset$, then we will consider $\overrightarrow{F_1(e_1)}=(0,0,0)$ for $e_1\in E_1\backslash E$ and $\overrightarrow{F_2(e_2)}=(0,$

$0, 0)$ for $e_2 \in E_2 \setminus E$. Then the similarity measure $S(F_1, F_2)$ is obtained from Definition 7.

Remark 8 *If $E_1 \cap E_2 = \emptyset$, then we have* $S(F_1, F_2) = 0$.

The following lemmas are quite obvious:

Lemma 9 *Suppose (F_1, E_1) and (F_2, E_2) be two NS sets over the same finite universe. Then we have the following:*

(i) $S(F_1, F_2) = S(F_2, F_1)$ *(ii)* $0 \le S(F_1, F_2) \le 1$ *(iii)* $S(F_1, F_1) = 1$

Lemma 10 *Suppose $(F_1, E), (F_2, E), (F_3, E)$ be three NS sets such that $(F_1, E) \subseteq (F_2, E) \subseteq (F_3, E)$ then, $S(F_1, F_3) \le S(F_2, F_3)$.*

Example 11 *Consider another NS set (G, E) over the same universe U, where $E = \{e_1, e_2, e_3, e_4\}$ whose NS matrix representation N is as following:*

$$N = \begin{bmatrix} * & F(e_1) & F(e_2) & F(e_3) & F(e_4) \\ h_1 & (0.3, 0.7, 0.3) & (0.6, 0.1, 0.8) & (0.5, 0.1, 0.5) & (0.4, 0.5, 0.4) \\ h_2 & (0.4, 0.4, 0.9) & (0, 0, 0) & (0.3, 0.3, 0.4) & (0.3, 0.7, 0.4) \\ h_3 & (0.2, 0.6, 0.2) & (0.2, 0.6, 0.6) & (0, 0, 0) & (0.4, 0.2, 0.8) \\ h_4 & (0.6, 0.5, 0.4) & (0.3, 0.9, 0.5) & (0.8, 0.7, 0.8) & (0.3, 0.7, 0.4) \end{bmatrix}$$

Then we have $S(F, G) = 0.22147$.

4. Weighted Similarity measure between two NS sets

Definition 12 *Suppose $U = \{u_1, u_2, \ldots, u_n\}$ be the universe and w_i be the weight of u_i and $w_i \in [0, 1]$, but not all zero, $1 \le i \le n$. Suppose (F_1, E) and (F_2, E)be two NS sets over U. We define their weighted similarity as follows*

$$W(F_1, F_2) = \frac{\sum_i w_i \ \overrightarrow{F_1(e_\iota)} . \overrightarrow{F_2(e_\iota)}}{\sum_i w_i \ \overrightarrow{[F_1(e_\iota)}^2 \vee \overrightarrow{F_2(e_\iota)^2}}$$

provided,

(i) $E_1 = E_2$

(ii) $\sum_i \overrightarrow{F_1(e_\iota)} . \overrightarrow{F_2(e_\iota)} = \sum_i (t_{F_1(e_i)} . t_{F_2(e_i)} + i_{F_1(e_i)} . i_{F_2(e_i)} + f_{F_1(e_i)} . f_{F_2(e_i)})$

(iii) $\sum_i \overrightarrow{[F_1(e_\iota)}^2 \vee \overrightarrow{F_2(e_\iota)^2}] = \sum_i (\boldsymbol{t}_{F_1(e_i)^2} \vee t_{F_2(e_i)} + i_{F_1(e_i)^2} \vee i_{F_2(e_i)^2} + f_{F_1(e_i)^2} \vee f_{F_2(e_i)^2})$

Example 13 *Consider the two NS sets (F, E) and (G, E) in Example 11. We assign weights to the elements $\{u_i, i = 1, \ldots, 4\}$ of X i.e.*

$w(u_1) = 0.3, w(u_2) = 0.1, w(u_3) = 0.4, w(u_4) = 0.7$.

Then we have $W(F, G) = 0.13864$.

Definition 14 *Consider the set of all NS sets $N_1(U)$ over the set U. Suppose $(F_1, E), (F_2, E) \in N_1(U)$. If $S(F_1, F_2) \ge \alpha, \alpha \in (0, 1)$, then the two NS sets (F_1, E) and (F_2, E) are said to be α-similar and we denote the similarity relation between two aforesaid sets as $(F_1, E) \cong_\propto (F_2, E)$.*

It can be easily seen that similarity is an equivalence relation.

Lemma 15 $\underset{C}{\cong}\propto$ *is a reflexive as well as symmetric relation but not an equivalence relation.*

From Lemma 9, we can easily see that $\underset{C}{\cong}\propto$ is a reflexive as well as symmetric relation. To see that $\underset{C}{\cong}\propto$ is not a transitive relation, we consider the following example:

$$N=\begin{bmatrix} * & F(e_1) & F(e_2) & F(e_3) & F(e_4) \\ h_1 & (0.3,0.7,0.4) & (0.8,0.7,0.8) & (0.1,0.1,0.2) & (0.6,0.2,0.8) \\ h_2 & (0,0,0) & (0,0,0) & (0.5,0.6,0.1) & (0,0,0) \\ h_3 & (0.4,0.5,0.2) & (0.4,0.1,0.2) & (0,0,0) & (0.4,0.2,0.8) \\ h_4 & (0.8,0.4,0.8) & (0.6,0.3,0.1) & (0.5,0.6,0.5) & (0.1,0.8,0.8) \end{bmatrix}$$

Example 16 *Consider a NS set* (H,E) *over the same universe, where* $E = \{e_1, e_2, e_3, e_4\}$ *who's NS matrix representation N is as above. Then* $S(G,F) = 0.22147, S(F,H) = 0.88609, S(G,H) = 0.54576.$

Definition 17 *Suppose* (F_1,E_1) *and* (F_2,E_2) *be two NS sets over the set . Then the two NS sets* (F_1,E_1) *and* (F_2,E_2) *are said to be significantly similar if*

$$S(F_1,F_2) > 1/2$$

Example 18 $S(F,H)$ *is significantly similar whereas* $S(F,G)$ *is not similar.*

5. Two sets and their measuring distances.

Throughout this section, we will consider $\boldsymbol{U}$ to be finite, namely $\boldsymbol{U} = \{\boldsymbol{h}_1, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_n\}$ and universal parameter set $\boldsymbol{E} = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_m\}$. Now for any NS set $(\boldsymbol{F},\boldsymbol{A}) \in \boldsymbol{N}(\boldsymbol{U})$, $\boldsymbol{A}$ is a subset of $\boldsymbol{E}$. Consider an extension of the NS set $(\boldsymbol{F},\boldsymbol{A})$ to the NS set $(\widehat{\boldsymbol{F}},\boldsymbol{E})$ where $\widehat{\boldsymbol{F}}(e_i)\{\boldsymbol{h}_j\} = \varphi$ where $\boldsymbol{e}_i \notin \boldsymbol{A}$. Now onwards we will take the parameter subset of any NS set over $\boldsymbol{N}(\boldsymbol{U})$ to be the same as the parameter set $\boldsymbol{E}$ without loss of generality.

Definition 19: *For two NS sets* $(\hat{F},E)$ *and* $(\widehat{G},E)$,

(i) The mean Hamming distance $D^S(F,G)$ between two NS sets is defined as follows

$$D^S(F,G) = \frac{1}{m}\{\sum_{i=1}^m \sum_{j=1}^n |F(e_i)(x_j) - G(e_i)(x_j)|\}$$

$$= \frac{1}{m}\left\{\sum_{i=1}^m \sum_{j=1}^n \left|t_{F(e_i)(x_j)} - t_{G(e_i)(x_j)}\right| + \left|i_{F(e_i)(x_j)} - i_{G(e_i)(x_j)}\right| + \left|f_{F(e_i)(x_j)} - f_{G(e_i)(x_j)}\right|\right\}$$

(ii) The normalized Hamming distance $L^S(F, G)$ is defined as follows:

$$L^S(F,G) = \frac{1}{mn}\{\sum_{i=1}^m \sum_{j=1}^n |F(e_i)(x_j) - G(e_i)(x_j)|\}$$

$$= \frac{1}{mn}\left\{\sum_{i=1}^m \sum_{j=1}^n \left|t_{F(e_i)(x_j)} - t_{G(e_i)(x_j)}\right| + \left|i_{F(e_i)(x_j)} - i_{G(e_i)(x_j)}\right| + \left|f_{F(e_i)(x_j)} - f_{G(e_i)(x_j)}\right|\right\}$$

(iii) The Euclidean distance $E^S(F, G)$ is defined as follows:

$$E^S(F,G) = \sqrt{\frac{1}{m}\{\sum_{i=1}^m \sum_{j=1}^n |F(e_i)(x_j) - G(e_i)(x_j)|^2\}}$$

$$= \sqrt{\frac{1}{m}\left\{\sum_{i=1}^m \sum_{j=1}^n \left|t_{F(e_i)(x_j)} - t_{G(e_i)(x_j)}\right|^2 + \left|i_{F(e_i)(x_j)} - i_{G(e_i)(x_j)}\right|^2 + \left|f_{F(e_i)(x_j)} - f_{G(e_i)(x_j)}\right|^2\right\}}.$$

(iv) The normalized Euclidean distance Q^S (F, G) is defined as follows:

$$Q^S(F,G) = \sqrt{\frac{1}{mn}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}|F(e_i)(x_j) - G(e_i)(x_j)|^2\right\}}$$

$$= \sqrt{\frac{1}{mn}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\left|t_{F(e_i)(x_j)} - t_{G(e_i)(x_j)}\right|^2 + \left|i_{F(e_i)(x_j)} - i_{G(e_i)(x_j)}\right|^2 + \left|f_{F(e_i)(x_j)} - f_{G(e_i)(x_j)}\right|^2\right\}}$$

Example 20 *Consider the two NS sets (F, E) and (G, E) in Example 11. Then we have the following:*

(i) $D^S(G, H) = 2.8$.

(ii) $L^S(F, G) = 1.67$.

(iii) $E^S(F, G) = 1.09$.

(iii) $Q^S(F, G) = 0.544$.

The following result is quite obvious.

Lemma 21 *For any two NS sets (F, E) and (G, E) of N (U), the following inequalities hold.*

(i) $D^S(F, G) \leq n$.

(ii) $L^S(F, G) \leq 1$.

(iii) $E^S(F, G) \leq \sqrt{n}$.

(iv) $Q^S(F, G) \leq 1$.

The following theorem can also be easily proved.

Theorem 22 *The functions* $D^S, L^S, E^S, Q^S\colon N(U) \longrightarrow R^+$ given by Definition 19 respectively are metrics, where R^+ is the set of all nonnegative numbers.

6. Distance based similarity measure of NS sets

We have defined several types of distances between a pair of NS sets (*F, E*) and (*G, E*) over the set *N* (*U*) in the previous section. Now using these distances we can also define similarity measures for NS sets. In the following, we now define a similarity measure based on Hamming Distance.

$$S'(F,G) = \frac{1}{1 + D^S(F,G)}$$

Also we can define another similarity measure as: $S'(F,G) = e^{-\alpha D^S(F,G)}$, where α is a positive real number (parameter) called the steepness measure. Similarly using Euclidian distance, similarity measure can be defined as follows:

$$S''(F,G) = \frac{1}{1 + E^S(F,G)}$$

Also we can define another similarity measure as: $S''(F,G) = e^{-\alpha E^S(F,G)}$, where α is a positive real number (parameter) called the steepness measure.

Lemma 23 *For a pair of NS sets* (F, E) *and* (G, E) *over the set* $N(U)$, *the following holds:*

$$(i)\ \ 0 \le S'(F,G) \le 1.$$

(ii) $S'(F,G) = S'(G,F)$.

(iii) $S'(F,G) = 1 \Longleftrightarrow (F,G) = (G,F)$.

The proof of the above lemma easily follows from definition.

7. Comparison between $S(F,G)$ and $S'(F,G)$:

Suppose $S_{M,N}$ denote the similarity measure between two NS sets (F,E) and (G,E) whose membership matrices are M and N. Now we compare the properties of the two measures of similarity of NS sets discussed here. Although most of the properties are common between them but some of these are different. Here we have the following:

(i) $Common\ Properties$: $S_{M,N} = S_{N,M}, 0 \le S_{M,N} \le 1, S_{M,N} = 1\ if\ M = N$.

(ii) $Distinct\ Property$: $S_{M,N} = 1 \Longrightarrow M = N$.

8. A real life application

The process of measuring similarity between two Neutrosophic soft sets can be applied to solve real life situations. A particular disease occurs to a patient or not can be easily determined by us using similarity measure. To see, consider the following problem: India is a polio-effected country in the last century. After taking several measurement by Govt of India, WHO declares India as a Polio-Free Nation from 2015. It is seen in the past that several situations like high population, literacy factor, socio-economic background, Govt initiative etc. are quite responsible for polio disease. Suppose U be the set of only three elements h_1, h_2, h_3 where h_1, h_2, h_3 denotes symptoms of the high growth of polio disease, average growth of polio disease, and low growth of polio disease.

We have tried to formulate the problem in terms of NS sets. . Here we list the set of parameters E is the factors which are responsible for polio disease. Suppose $E = \{e_1, e_2, e_3, e_4\}$ where e_1, e_2, e_3, e_4 denotes high population, literacy factor, socio-economic background, Govt initiative of a Murshidabad District, West Bengal, India. Now consider a NS matrix P of a neutrosophic set (F, E) of a polio effected patient X_1 based on the data available from a Govt. report [33] as follows:

$$P = \begin{bmatrix} * & F(e_1) & F(e_2) & F(e_3) & F(e_4) \\ h_1 & (0.7, 0.2, 0.3) & (0.6, 0.1, 0.3) & (0.8, 0.3, 0.5) & (0.7, 0.2, 0.4) \\ h_2 & (0.6, 0.3, 0.2) & (0.1, 0.5, 0.5) & (0.4, 0.3, 0.3) & (0.4, 0.7, 0.3) \\ h_3 & (0.2, 0.6, 0.7) & (0.2, 0.4, 0.4) & (0,1,0) & (0.3, 0.2, 0.7) \end{bmatrix}$$

Here the entry $F(e_1)(h_1)$ in the matrix P denotes the positive impact, the uncertainties impact, and negative impact of high population to positive growth of polio symptoms respectively. Consider two persons Rajibul and Rupam, both live in Bhagabangola village of Murshidabad District but belongs to different category. Both of them have polio disease symptoms with some positive, average, low growth rate. Let we denote both Rajibul and Rupam's health condition with two NS set (G, E) and (H, E) over U whose NS matrices Q, S respectively are given below:

$$Q=\begin{bmatrix} * & F(e_1) & F(e_2) & F(e_3) & F(e_4) \\ h_1 & (0.8,0.3,0.5) & (0.7,0.4,0.3) & (0.8,0.6,0.7) & (1,0,0) \\ h_2 & (0.2,0.5,0.6) & (0.1,0.1,0.8) & (0.4,0.1,0.5) & (0.3,0.3,0.4) \\ h_3 & (0,0,0) & (0.1,0.3,0.3) & (1,1,0) & (0,0,0) \end{bmatrix}$$

$$S=\begin{bmatrix} * & F(e_1) & F(e_2) & F(e_3) & F(e_4) \\ h_1 & (0.8,0.4,0.8) & (0.6,0.3,0.1) & (0.5,0.6,0.5) & (0.7,0.2,0.4) \\ h_2 & (0,0,0) & (0,1,1) & (0.3,0.1,0.1) & (0.2,0.5,0.4) \\ h_3 & (0.2,0.6,0.2) & (0.2,0.6,0.6) & (0,0,0) & (0.4,0.2,0.8) \end{bmatrix}$$

After calculating similarity measure, we have $S(F,G) = 0.64, S(F,H) = 0.69$. From this result we can conclude that Rajibul and Rupam both have the chances to be effected by polio disease. Both of their symptoms are significantly similar to a natural polio effected person. Beside this, Rupam's condition is more significantly similar than Rajibul condition since$S(F,G) = 0.64 < S(F,H) = 0.69$.

9. Conclusion

To deal with uncertain real life situations, Molodtstov gave the concept of soft set theory in his paper [32]. Later on Prof P.K. Maji introduced NSS theory and have shown the properties and application of NSS ([26]). In this paper we have defined similarity measure properties of two NS sets and studied some of its important properties and applied it in a decision making problem. In future, we will study some another applications of similarity measures of two NS sets and will try to solve the uncertainty using NS similarity measure technique. One may try to solve many realistic health diagnosis problem using the similarity measure technique between NS sets.

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Conflicts of Interest

The authors declare no conflict of interest.

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A Study on Neutrosophic Zero Rings

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Abstract: Let $N(R, I)$ be a Neutrosopic ring corresponding to the classical ring R and indeterminate I. In this paper, we introduced the Neutrosophic zero rings $N(R, I)^0$ and $N(R^0, I)$ corresponding to the ring R and the zero ring R^0 respectively, and also studied structural properties of these Neutrosophic zero rings. Among many properties, it is shown that $N(R, I) \neq N(R, I)^0$ and $|N(R, I)| = |N(R, I)^0|$. Particularly, we prove that $N(R, I)^0$ is not a Boolean ring and the characteristics of $N(R, I)$ and $N(R, I)^0$ are equal. For every classical ring R, the Neutrosophic zero ring $N(R, I)^0$ is isomorphic to Neutrosophic zero ring $M_2(R, I)^0$ of all 2×2 matrices of the form $\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(a+bI) \end{pmatrix}$ with entries from $N(R, I)$. We also find a necessary and sufficient condition for the classical zero rings R^0 and Neutrosophic zero ring $N(R^0, I)$ to be isomorphic under the following actions $r \leftrightarrow \begin{pmatrix} r & -r \\ r & -r \end{pmatrix}$ and $r+sI \leftrightarrow \begin{pmatrix} r+sI & -(r+sI) \\ r+sI & -(r+sI) \end{pmatrix}$.

Keywords: Neutrosophic rings; Neutrosophic zero rings; Neutrosophic square zero matrices; Neutrosophic Boolean rings

1. Introduction

Abstract algebra is largely concerned with the study of abstract sets endowed with one, or, more binary operations along with few axioms. In this paper, we consider one of the basic algebraic structures known as a ring, called a classical ring. A ring $R = (R, +, \cdot)$ is a non-empty set with two binary operations, namely addition (+) and multiplication ($\cdot$) defined on R satisfying some natural axioms, see [1]. A ring $R = (0)$ is called a trivial ring, otherwise R is called nontrivial. A ring R is called commutative if $ab = ba$ for all a and b in R. An element u in R is called a unit if there exists v in R such that $uv = 1 = vu$, where u and v are both multiplicative inverses in R. The set of units of R is denoted by $U(R)$. However, the set $R-U(R)$ is denoted by $Z(R)$ and called zero-divisors of R. For any commutative ring R with unity, we have every non zero elements of R is either unit or, zero divisors. Clearly, $R = U(R) \cup Z(R)$. The Characteristic of R denoted $Char(R)$ is the smallest nonnegative n such that $n \cdot 1 = 0$. If no such n exists then we define the $Char(R) = 0$. Next, a ring R is called cyclic ring if $(R, +)$ is a cyclic group. Every cyclic ring is commutative and these rings have been investigated in [2]. The theory of finite rings occupies a central position in modern mathematics and engineering science. Recently, finite rings play a central role in many research areas such as digital image processing, algebraic coding theory, encryption systems, QUAM signals and linear coding theory; see [4-7].

The notion of zero rings was considered by Buck [2] in 2004. A zero ring R^0 is a triplet $(R^0, +, \cdot)$ where $(R^0, +)$ is an abelian group and $a \cdot b = 0$ for all $a, b \in R^0$. Every zero is a commutative cyclic ring but a cyclic ring need not be a zero ring. For instance, $(Z_6, \oplus, \odot)$ is a cyclic ring but not a zero ring under addition and multiplication modulo 6.

Neutrosophy is a part of philosophical reasoning, introduced by Smarandache in 1980, which concentrates the origin, nature and extent of neutralities, comparable to their cooperation with particular ideational spectra. Neutrosophy is the premise of Neutrosophic Logic, Neutrosophic likelihood, Neutrosophic set and Neutrosophic realities in [8]. Handling of indeterminacy present in real-world data is introduced in [9, 10] as Neutrosophy. Neutralities and indeterminacies spoken to Neutrosophic Logic have been utilized in the analysis of genuine world and engineering problems. In 2004, the creators Vasantha Kanda Swami and Smarandache presented the ideas of Neutrosophic arithmetical hypothesis and they were utilized in Neutrosophic mathematical structures and build up numerous structures such as Neutrosophic semigroups, groups, rings, fields which are different from classical algebraic structures and are presented and analyzed their application to fuzzy and Neutrosophic models are developed in [11].

Now we begin our attention to the Neutrosophic ring $N(R, I)$, we are considering in this paper. The basic study on Neutrosophic rings was given by Vasantha Kandasamy and Smarandache [11], and there are many interesting properties of Neutrosophic rings available in the literature, see [12-16]. Let I be the indeterminate of the real-world problem with two fundamental properties such as $I^2 = I$ and I^{-1} does not exists. Then generally we define the Neutrosophic set $N(R, I) = \{a + bI : a, b \in R, I^2 = I\}$ which is a nonempty set of Neutrosophic elements $a + bI$ and it is generated by a ring R and indeterminate I under the following Neutrosophic operations.

(1) $(a+bI) + (c+dI) = (a+c) + (b+d)I$ and

(2) $(a+bI)\,(c+dI) = ac + (ad+bc+bd)I$

for all $a + bI$, $c + dI$ in $N(R, I)$. More specifically, the indeterminate I satisfies the following algebraic properties. (1) $I^2 = I$, (2) $0I = 0$ and $1I = I$ but $I \neq 0, 1$, (3) I^{-1} does not exist with respect to Neutrosophic multiplication but $-I = (-1)\mathrm{I}$ exists with respect to Neutrosophic addition such that $\mathrm{I} + (-\mathrm{I}) = 0$ and $-I \neq I$, and (4) $I + I = 2I$ and $I + I \neq I$. Recently, Agboola, Akinola and Oyebola studied further properties of Neutrosophic rings in [13, 14]. In [15-17], Chalapathi and Kiran established relations between units and Neutrosophic units of rings, fields, Neutrosophic rings and Neutrosophic fields. However, we have $|N(R, I)| \geq 4$ for any finite ring R with $|R| > 1$. This clears that $4 \leq |N(R, I)| \leq |R|^2$.

In numerous certifiable circumstances, it is regularly seen that the level of indeterminacy assumes a significant job alongside the fulfillment and disappointment levels of the decision-makers in any decision making process and Internet clients. Because of some uncertainty or dithering, it might important for chiefs to take suppositions from specialists which lead towards a lot of clashing qualities with respect to fulfillment, indeterminacy and dis-fulfillment level of choice makers. So as to feature the previously mentioned understanding, the authors Abdel-Basset et al. [18-20] built up a successful structure which mirrors the truth engaged with any basic decision-making process. In this investigation, a multi-objective nonlinear programming issue has been planned in the assembling framework. Another calculation, Neutrosophic reluctant fluffy programming approach, dependent on single esteemed Neutrosophic reluctant fuzzy decision set has been proposed which contains the idea of indeterminacy reluctant degree alongside truth and lie reluctant degrees of various objectives.

Web of Things associates billions of items and gadgets to outfit a genuine viable open door for the enterprises. Fourth industrial and mechanical upset must guarantee proficient correspondence and work by thinking about the components of expenses and execution. Transition to the fourth industrial and mechanical transformation creates and generates challenges for enterprises. In [21, 22], the authors recognize the fundamental difficulties influencing the change procedure utilizing non-conventional techniques and proposed a hybrid combination between the systematic various leveled process as a Neutrosophic criteria decision-making approach for IoT-based ventures and furthermore Neutrosophic hypothesis to effectively distinguish and deal with the uncertainty and irregularity challenges.

2. Neutrosophic zero rings of rings

In this section, we studied Neutrosophic zero rings of various classical rings and presented their basic properties with many suitable illustrations and examples. First, the language of Neutrosophic element makes it possible to work with indeterminate I and it relationships much as we work with equalities and powers only. Prior to the consideration of Neutrosophic element $a+bI$, the notation $(a+bI)^{-1}$ used for reciprocity relationships but it is not applicable for every element a and b in the classical ring R. So the introduction of a convenient Neutrosophic multiplication notation helped accelerate the development of Neutrosophic theory. For this reason, the Neutrosophic mathematical concepts establish solutions to many problems with indeterminacy.

In working with Neutrosophic multiplications, we will sometimes need to translate them into further Neutrosophic algebraic structures. The following definition is one.

Definition 2.1. Let R be a ring. Then $N(R, I)$ is called a **Neutrosophic zero ring** if the product of any two Neutrosophic elements of $N(R, I)$ is 0, where $0=0+0I$ is the Neutrosophic additive identity.

For any ring R, there is a Neutrosophic zero ring and is denoted by $N(R, I)^0$. This statement connects the relation $N(R, I) \neq N(R, I)^0$ for every $R \neq (0)$. In particular, if $R = (0)$ then $N(R, I) = (0)$ and $N(R, I)^0 = (0)$. For any ring $R \neq (0)$, the actual construction of Neutrosophic zero rings $N(R, I)^0$ appear below. If R is not a zero ring, then $N(R, I)$ is never a Neutrosophic zero ring. This means that, the only Neutrosophic ring $N(R, I)$ that cannot be described as a Neutrosophic zero ring when R is either finite or infinite. For this reason, the construction of Neutrosophic zero rings depends on the collection Neutrosophic matrices and which are up to Neutrosophic isomorphism. The next definition deals with these constructions.

Definition 2.2. Let $\mathrm{M}_2(R, I)^0$ be the non-empty subset of 2×2 Neutrosophic matrices

$$\mathrm{M}_2(R, I)=\left\{\begin{pmatrix} a+bI & c+dI \\ e+fI & g+hI \end{pmatrix} : a+bI, c+dI, e+fI, g+hI \in N(R, I)\right\}.$$

Then we define $\mathrm{M}_2(R, I)^0$ as follows

$$\mathrm{M}_2(R,\ I)^0 = \left\{ \begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(a+bI) \end{pmatrix} : a+bI \in N(R,\ I) \right\}$$

and this collection is called **Neutrosophic square zero matrices**.

Example 2.3. For the ring $Z_2 = \{0,\ 1\}$ under addition and multiplication modulo 2, the Neutrosophic ring and corresponding Neutrosophic square matrices are

$N(Z_{2,}\ I) = \{0,\ 1,\ I,\ 1+I\}$ and $\mathrm{M}_2(\mathrm{Z}_2,\ I)^0 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} I & -I \\ I & -I \end{pmatrix}, \begin{pmatrix} 1+I & -(1+I) \\ 1+I & -(1+I) \end{pmatrix} \right\}$,respectively.

To determine the structure of Neutrosophic zero ring $N(R,\ I)^0$, we must derive a result for determining when an element of $N(R,\ I)^0$ is a Neutrosophic unit, or, Neutrosophic zero divisor. Recall that in a commutative Neutrosophic ring $N(R,\ I)$ a non zero Neutrosophic element $a+bI$ is called a Neutrosophic zero divisor provided there is a non zero Neutrosophic element $c+dI$ in $N(R,\ I)$ such that $(a+bI)(c+dI)=0$. No Neutrosophic element of $N(R,\ I)$ can be both a Neutrosophic unit and Neutrosophic zero divisor, but there are Neutrosophic rings such as $N(\mathrm{Z},\ I)$, $N(\mathrm{Q},\ I)$, $N(\mathrm{R},\ I)$, $N(\mathrm{C},\ I)$ and $N(\mathrm{Z[i]},\ I)$,, with non zero Neutrosophic elements that are neither Neutrosophic units nor Neutrosophic zero divisors, , where Z , Q , R , C and $Z[i]$ are ring of integers, rationals, real numbers, complex numbers, and Gaussian integers, respectively. However, when $N(R,\ I)$ is finite, every non zero Neutrosophic elements of $N(R,\ I)$ is either Neutrosophic unit, or, Neutrosophic zero divisor. In particular, this result is true for $N(\mathrm{Z}_n,\ I)$, $N(\mathrm{Z}_n \times \mathrm{Z}_n,\ I)$, $N(\mathrm{Z}_n[\mathrm{x}]/(\mathrm{x}^n),\ I)$, and $N(\mathrm{Z}_n[\mathrm{i}],\ I)$, where Z_n , $\mathrm{Z}_n \times \mathrm{Z}_n$, $\mathrm{Z}_n[x]/(x^n)$ and $\mathrm{Z}_n[i]$ are finite commutative rings with usual notions under modulo n . We develop this fact in Theorem [2.4]. Since $N(R,\ I)^0 \not\subset N(R,\ I)$ and $N(R,\ I) \not\subset N(R,\ I)^0$, it is not surprising that there is a connection between the Neutrosophic units in the Neutrosophic zero rings.

Theorem 2.4. For any ring R with unity, we have $U(N(R,\ I)^0)$ is empty.

Proof. Assume that $U(N(R,\ I)^0)$ is nonempty. Suppose that $a+bI \in U(N(R,\ I)^0)$. Then there exists some $u+vI$ in $U(N(R,\ I)^0)$ such that $(u+vI)(a+bI)=1$. This implies that $(u+vI)^2(a+bI)^2 = 1^2$, or, it is equivalent to $0=1$ because $(u+vI)^2 = 0$ and $(a+bI)^2 = 0$, a contradiction. So our assumption is not true, and hence $U(N(R,\ I)^0) = \phi$.

In general, it is not easy to classify Neutrosophic rings and their corresponding Neutrosophic zero rings by determining their orders. For this reason, we must follow a better approach which is shown below.

Theorem 2.5. For any Neutrosophic ring $N(R,\ I)$, we have

$$N(R,\ I)^0 \cong M_2(R,\ I)^0 .$$

Proof. Let R be any ring. Then there exists $N(R,\ I)$ and $N(R,\ I)^0$. Now we want to show that $N(R,\ I)^0 \cong M_2(R,\ I)^0$. For this, we define a map $f : N(R,\ I)^0 \to M_2(R,\ I)^0$ by the following relation

$$f(a+bI) = \begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}$$

for every $a+bI \in N(R, I)^0$. If $a+bI \in N(R, I)^0$, then $(a+bI)^2 = (a+bI)\ (a+bI) = 0$. That is, there exists a Neutrosophic matrix $\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}$ in $M_2(R, I)^0$ such that

$$\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(a+bI) \end{pmatrix} = \begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

implying that f makes sense. Therefore f is well defined. Because $f(0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ and $f(I) = \begin{pmatrix} I & -I \\ I & -I \end{pmatrix}$, one can easily verify that f is a Neutrosophic ring homomorphism.

Now, we show that f is one-one and onto. For every two Neutrosophic elements $a+bI$ and $c+dI$ in $N(R, I)^0$, we have

$$f(a+bI) = f(c+dI) \Rightarrow \begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix} = \begin{pmatrix} c+dI & -(\mathrm{c}+dI) \\ c+dI & -(\mathrm{c}+dI) \end{pmatrix} \Rightarrow a+bI = c+dI .$$

Consequently, f is one-one, and also the unique part shows f is surjective. Therefore, f is a Neutrosophic isomorphism from $N(R, I)^0$ onto $M_2(R, I)^0$. Hence, $N(R, I)^0 \cong M_2(R, I)^0$.

Recall that $N(R, I)$ is not equal to $N(R, I)^0$ but the following theorem shows that $N(R, I)$ is equivalent to $N(R, I)^0$, that is we shall show that there is a one-one correspondence between $N(R, I)$ and $N(R, I)^0$.

Theorem 2.6. For any ring R, we have $|N(R, I)| = |N(R, I)^0|$.

Proof. By the Theorem [2.5], we know that $N(R, I)^0 \cong M_2(R, I)^0$. We shall show that $|N(R, I)| = |N(R, I)^0|$. For this, we must show that $|M_2(R, I)^0| = |N(R, I)|$. Define a map $\psi : M_2(R, I)^0 \to N(R, I)$ by the connection

$$\psi\left(\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}\right) = a+bI$$

for every element $\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}$ in $M_2(R, I)^0$. Every element $a+bI$ in $\mathrm{N}(R, I)$ has the following form $a+bI = \psi\left(\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}\right)$ for some $\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}$ in $M_2(R, I)^0$. Then the map ψ is clearly onto; it is one-one because for every

$A^0 = \begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}$, $B^0 = \begin{pmatrix} c+dI & -(c+dI) \\ c+dI & -(c+dI) \end{pmatrix}$ in $M_2(R, I)^0$, we have

$$\psi\left(A^0\right) = \psi\left(B^0\right) \Rightarrow \psi\left(\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}\right) = \psi\left(\begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix}\right)$$

$$\Rightarrow a+bI = c+dI$$

$$\Rightarrow \begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(\mathrm{a}+bI) \end{pmatrix} = \begin{pmatrix} c+dI & -(c+dI) \\ c+dI & -(c+dI) \end{pmatrix}$$

$$\Rightarrow A^0 = B^0.$$

Therefore, the correspondence $a+bI \leftrightarrow \begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(a+bI) \end{pmatrix}$ pairs every element in each of two sets $\mathrm{N}(R, I)$ and $M_2(R, I)^0$ with exactly one element of the other set. Hence, $\mathrm{N}(R, I)$ and $M_2(R, I)^0$ contains the same number of elements, and we write this as $|\mathrm{N}(R, I)| = |M_2(R, I)^0|$.

Now because of the Theorem [2.5], we conclude that $|\mathrm{N}(R, I)^0| = |\mathrm{N}(R, I)|$.

This is all somewhat vague; of course, let us look at a concrete example.

Example 2.7. For the ring $Z_2 = \{0, 1\}$, the correspondence ψ from $\mathrm{N}(\mathrm{Z}_2, I)$ onto $M_2(\mathrm{Z}_2, I)^0$ with actions given by the following arrow diagrams:

$$0 \leftrightarrow \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},\ 1 \leftrightarrow \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix},\ I \leftrightarrow \begin{pmatrix} I & -I \\ I & -I \end{pmatrix} \text{and}\ 1+I \leftrightarrow \begin{pmatrix} 1+I & -(1+I) \\ 1+I & -(1+I) \end{pmatrix}.$$

These actions illustrate that $|\mathrm{N}(\mathrm{Z}_2, I)| = 4$, $|M_2(\mathrm{Z}_2, I)^0| = 4$, and hence $|\mathrm{N}(\mathrm{Z}_2, I)^0| = 4$. This shows that $|\mathrm{N}(\mathrm{Z}_2, I)| = |\mathrm{N}(\mathrm{Z}_2, I)^0|$ but $\mathrm{N}(\mathrm{Z}_2, I) \neq \mathrm{N}(\mathrm{Z}_2, I)^0$.

We now change focus somewhat take up the study of Neutrosophic isomorphism between $\mathrm{N}(R, I)$ and $\mathrm{N}(R, I)^0$. Particularly we observe that nothing is known of Neutrosophic isomorphism between $\mathrm{N}(R, I)$ and $\mathrm{N}(R, I)^0$. For instance, the Neutrosophic ring $\mathrm{N}(\mathrm{Z}_2, I)$ and Neutrosophic zero ring $\mathrm{N}(\mathrm{Z}_2, I)^0$ are not isomorphic with respect to Neutrosophic isomorphism because $I^2 = I$ in $\mathrm{N}(\mathrm{Z}_2, I)$ but $\begin{pmatrix} I & -I \\ I & -I \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ in $\mathrm{N}(\mathrm{Z}_2, I)^0$. This observation takes place according to Theorem [2.8].

Theorem 2.8. Let R be any non-trivial ring. Then, $\mathrm{N}(R, I)$ is not isomorphic to $\mathrm{N}(R, I)^0$.

Proof. Assume that the element $A^0 = \begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(a+bI) \end{pmatrix} \neq 0$ in $M_2(R, I)^0$ satisfies the condition $(A^0)^2 = 0$, where $a+bI \neq 0$. Suppose that the Neutrosophic mapping $g : M_2(R, I)^0 \to N(R, I)$ is a Neutrosophic isomorphism. If $a+bI = g(\mathrm{A}^0)$, then

$$(a+bI)^2 = g(A^0)^2 \Rightarrow (a+bI)^2 = g((A^0)^2)$$

$$\Rightarrow (a+bI)^2 = g(0), \text{ since } (A^0)^2 = 0$$

$$\Rightarrow (a+bI)^2 = 0,\ g(0) = 0.$$

But $(a+bI)^2 = 0$ in $N(R, I)$ implies that $a+bI = 0$, giving $A^0 = \begin{pmatrix} a+bI & -(a+bI) \\ a+bI & -(a+bI) \end{pmatrix} = 0$ because g is one-one. This is a contradiction to the fact that $A^0 \neq 0$, so no such isomorphism g can exist between $M_2(R, I)^0$ and $N(R, I)$. But $N(R, I)^0 \cong M_2(R, I)^0$, and hence $\mathrm{N}(R, I)$ is not isomorphic to $\mathrm{N}(R, I)^0$.

Theorem 2.9. Let R be a finite ring with unity. Then, $Char(N(R, I)^0) = Char(R)$.

Proof. Suppose $|R|$ is finite and $1 \in R$. Then, by the definition of the characteristic of a ring,

$$\begin{aligned} Char(R) = n &\Leftrightarrow o(1) = n \text{ in the additive group } (R, +) \\ &\Leftrightarrow n \cdot 1 = 0 \text{ in the additive group } (R, +) \\ &\Leftrightarrow n \cdot 1 = 0,\ n \cdot (-1) = 0 \text{ in the additive group } (R, +) \\ &\Leftrightarrow n \cdot \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} n \cdot 1 & n \cdot (-1) \\ n \cdot 1 & n \cdot (-1) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ &\Leftrightarrow Char(\mathrm{M}_2(R, I)^0) = n \\ &\Leftrightarrow Char(\mathrm{N}(R, I)^0) = n. \end{aligned}$$

A ring R is called Boolean ring if $a^2 = a$ for all a in R. Every finite Boolean ring with unity is isomorphic to the ring $Z_2{}^n$, where $Z_2{}^n$ is the Cartesian product of n copies of the ring $Z_2 = \{0, 1\}$ with respect to addition and multiplication modulo 2. Therefore, $N(Z_2{}^n, I)$ is a Neutrosophic Boolean ring with the property that $|N(Z_2{}^n, I)| = 2^{4n}$. Now we move on to verify that the structure of $N(Z_2{}^n, I)^0$ is Neutrosophic Boolean ring, or, not.

Theorem 2.10. Every Neutrosophic zero ring of a Boolean ring is not a Neutrosophic Boolean ring.
Proof. Suppose $n > 1$ is a positive integer. By the Theorem [2.5], we know that $N(Z_2{}^n, I)^0$ is isomorphic to the Neutrosophic zero ring $\mathrm{M}_2(Z_2{}^n, I)^0$. In anticipation of a contradiction, let us assume that $\mathrm{M}_2(Z_2{}^n, I)^0$ is a Neutrosophic Boolean ring, then for any $\alpha = a + bI \neq 0$ in $N(Z_2{}^n, I)^0$ such that $\begin{pmatrix} \alpha & -\alpha \\ \alpha & -\alpha \end{pmatrix}$ is in $\mathrm{M}_2(Z_2{}^n, I)^0$. Under the condition of Neutrosophic Boolean ring, we have

$$\begin{aligned} \begin{pmatrix} \alpha & -\alpha \\ \alpha & -\alpha \end{pmatrix}^2 = \begin{pmatrix} \alpha & -\alpha \\ \alpha & -\alpha \end{pmatrix} &\Rightarrow \begin{pmatrix} \alpha & -\alpha \\ \alpha & -\alpha \end{pmatrix}\begin{pmatrix} \alpha & -\alpha \\ \alpha & -\alpha \end{pmatrix} = \begin{pmatrix} \alpha & -\alpha \\ \alpha & -\alpha \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} \alpha & -\alpha \\ \alpha & -\alpha \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ &\Rightarrow \alpha = a + bI = 0. \end{aligned}$$

This is not true. Hence, we conclude that every Neutrosophic zero ring of a Boolean ring is not a Neutrosophic Boolean ring.

3. Neutrosophic zero rings of zero rings

This section introduces Neutrosophic Zero rings associated with zero rings. First, we recall that R^0 is a zero ring if the product any two elements in R^0 is zero. If $R^0 \neq (0)$ then clearly $|R^0| \geq 2$ and R^0 is never a field structure. By the Buck's [2] research in 2004, for any ring R with $R \neq R^0$, the zero rings R^0 isomorphic to the zero rings of all 2×2 matrices of the form

$$M_2(R)^0 = \left\{ \begin{pmatrix} r & -r \\ r & -r \end{pmatrix} : r \in R \right\}$$

with the same cardinality of R, that is, $|\mathrm{M}_2(R)^0| = |R|$. For example, the zero ring

$$M_2(Z_3)^0 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 2 & -2 \\ 2 & -2 \end{pmatrix} \right\}$$

with an order 3 under usual matrix addition and multiplication of modulo 3. This observation concludes that, if R is not a zero ring then $N(R, I)$ is never a zero ring. However, the following definition gives a concise way of referring to the definition of Neutrosophic zero rings associated with zero rings.

Definition 3.1. If R^0 is a zero ring, then $N(R^0, I) = \{a + bI : a, b \in R^0\}$ is called **Neutrosophic zero ring** corresponding to the zero ring R^0.

Example 3.2. Suppose that $R^0 = \{0, 3, 6\}$ is a zero ring under addition and multiplication modulo 9. Then

$N(R^0, I) = \{0, 3, 6, 3I, 6I, 3+3I, 3+6I, 6+3I, 6+6I\}$ and

$$N(R^0, I)^0 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 3 & -3 \\ 3 & -3 \end{pmatrix}, \begin{pmatrix} 6 & -6 \\ 6 & -6 \end{pmatrix}, \begin{pmatrix} 3I & -3I \\ 3I & -3I \end{pmatrix}, \begin{pmatrix} 6I & -6I \\ 6I & -6I \end{pmatrix}, \begin{pmatrix} 3+3I & -(3+3I) \\ 3+3I & -(3+3I) \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 3+6I & -(3+6I) \\ 3+3I & -(3+6I) \end{pmatrix}, \begin{pmatrix} 6+3I & -(6+3I) \\ 6+3I & -(6+3I) \end{pmatrix}, \begin{pmatrix} 6+6I & -(6+6I) \\ 6+6I & -(6+6I) \end{pmatrix} \right\}$$

Properties of $N(R^0, I)$.

(1) $N(R^0, I)$ is generated by R^0 and I.

(2) $N(R^0, I)$ is a Neutrosophic square zero ring.

(3) $|N(R^0, I)| = |R^0|^2$.

(4) $N(R^0, I) \neq N(R, I)^0$.

(5) $|N(R^0, I)| = |N(R^0, I)^0|$.

Theorem 3.3. For any finite zero ring R^0, the following equality holds good

$$|N(R^0, I)| = |R^0|^2.$$

Proof. The Cartesian product of R^0 is defined by $R^0 \times R^0 = \{(a, b) : a, b \in R^0\}$. Now define the map $\tau : R^0 \times R^0 \to N(R^0, I)$ by the relation $\tau((a, b)) = a + bI$ for every $(a, b) \in R^0 \times R^0$.

For any two elements (a, b) and (c, d) in the zero ring $R^0 \times R^0$, we have

$$\tau((a, b)) = \tau((c, d)) \Leftrightarrow a + bI = c + dI$$
$$\Leftrightarrow a = b, c = d \text{, since } I \neq 0.$$
$$\Leftrightarrow (a, b) = (c, d).$$

Thus the mapping τ is a well-defined one-one function. Also τ is onto function, because for any $\alpha \in \tau(R^0 \times R^0)$, there exists $\beta \in R^0 \times R^0$ such that $\alpha = \tau(\beta)$. Therefore, the map $\tau : R^0 \times R^0 \to N(R^0, I)$ is one-one correspondence from $R^0 \times R^0$ onto $N(R^0, I)$, and clear that

$$|N(R^0, I)| = |R^0 \times R^0| = |R^0|^2.$$

Recall that $U(R)$ and $U(N(R, I))$ denotes the set of all units and Neutrosophic units of R and $N(R, I)$, respectively, see [17]. Note that, if at least one of $U(R)$ and $U(N(R, I))$ is non-empty, then there is nothing to the existence of Neutrosophic zero ring. The next hurdle that stands in our way is to establish that a relation between $U(N(R, I))$ and its corresponding Neutrosophic zero ring.

Theorem 3.4. If the set $U(N(R, I)) = \phi$, then there is a Neutrosophic zero ring with at least four elements.

Proof. There is no harm in assuming that $|R| > 1$, and automatically $|\mathrm{N}(R, I)| \geq 4$ is true. Suppose $U(N(R, I)) \neq \phi$. Then there are at least two elements in $U(N(R, I))$. If $u + vI$ and $u' + v'I$ are the two distinct elements in $U(N(R, I))$, then, bearing in mind that u, u', v, v' are elements in $U(R)$. As a result, the Neutrosophic product $(u + vI)\ (u' + v'I)$ is given by

$$(u + vI)\ (u' + v'I) = uu' + (uv' + vu' + vv')I .$$

It is never zero because $uu' \in U(R)$. This contraposition proves our result.

Theorem [3.4] indicates that every commutative Neutrosophic zero ring is without unity. For this fact, the following theorem is essential in our paper.

Theorem 3.5. The Neutrosophic ring $N(R, I)$ is a Neutrosophic zero ring if and only if R is isomorphic to zero ring. In particular, $R \cong R^0 \Leftrightarrow N(R, I) \cong N(R^0, I)$.

Proof. Suppose R is isomorphic to a zero ring R^0. Then there exists a Neutrosophic ring $N(R^0, I)$ which is also Neutrosophic zero ring because

$$R^0 \cong M_2(R)^0 \Leftrightarrow N(R^0, I) \cong N(M_2(R)^0, I)$$

under the following actions

$$r \leftrightarrow \begin{pmatrix} r & -r \\ r & -r \end{pmatrix} \Leftrightarrow r + sI \leftrightarrow \begin{pmatrix} r + sI & -(r + sI) \\ r + sI & -(r + sI) \end{pmatrix}$$

4. Conclusions

In this work, another Neutrosophic Algebraic structure, for the Neutrosophic speculation, in view of the traditional Ring Theory was proposed. This study understands the new structure basis in Neutrosophic hypothesis which builds up another idea for the comparison of two ring structures dependent on the use of the indeterminacy idea and the structural information. The Neutrosophic zero ring structure was characterized utilizing the identical classes of traditional zero rings, to be equipped for choosing any Neutrosophic element of the class. Additionally, we built up a connection between the various zero rings and matrix zero rings R^0, $M_2(R)^0$,, $M_2(R, I)^0$, $N(R^0, I)$, $N(R, I)^0$ and $N(M_2(R)^0, I)$ such as $N(R, I)^0 \cong M_2(R, I)^0$ and $R^0 \cong M_2(R)^0 \Leftrightarrow N(R^0, I) \cong N(M_2(R)^0, I)$. The future work will recommend a Neutrosophic square zero elements and Neutrosophic square zero matrices to speak to all Neutrosophic mathematical frameworks, and apply the properties of these frameworks for identifying the total number of Neutrosophic zero subrings and Neutrosophic zero ideals.

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Correlation Measure for Pythagorean Neutrosophic Sets with T and F as Dependent Neutrosophic Components

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Abstract: In this paper, we study the new concept of Pythagorean neutrosophic set with T and F as dependent neutrosophic components [PNS]. Pythagorean neutrosophic set with T and F as dependent neutrosophic components [PNS] is introduced as a generalization of neutrosophic set (In neutrosophic sets, there are three special cases, here we take one of the special cases. That is, membership and non-membership degrees are dependent components and indeterminacy is independent) and Pythagorean fuzzy set. In PNS sets, membership, non-membership and indeterminacy degrees are gratifying the condition $0 \leq \left(u_A(x)\right)^2 + \left(\zeta_A(x)\right)^2 + \left(v_A(x)\right)^2 \leq 2$ instead of $u_A(x) + \zeta_A(x) + v_A(x) > 2$ as in neutrosophic sets. We investigate the basic operations of PNS sets. Also, the correlation measure of PNS set is proposed and proves some of their basic properties. The concept of this correlation measures of PNS set is the extension of correlation measures of Pythagorean fuzzy set and neutrosophic set. Then, using correlation of PNS set measure, the application of medical diagnosis is given.

Keywords: Pythagorean fuzzy set, Pythagorean Neutrosophic set with T and F as dependent neutrosophic components [PNS], Correlation measure and Medical diagnosis.

Introduction

Fuzzy sets were firstly initiated by L.A.Zadeh [36] in 1965. Zadeh's idea of fuzzy set evolved as a new tool having the ability to deal with uncertainties in real-life problems and discussed only membership function. After the extensions of fuzzy set theory Atanassov [7] generalized this concept and introduced a new set called intuitionistic fuzzy set (IFS) in 1986, which can be describe the non-membership grade of an imprecise event along with its membership grade under a restriction that the sum of both membership and non-membership grades does not exceed 1. IFS has its greatest use in practical multiple attribute decision making problems.In some practical problems.In some practical problems, the sum of membership and non-membership degree to which an alternative satisfying attribute provided by decision maker(DM) may be bigger than 1.

Yager [30] was decided to introduce the new concept known as Pythagorean fuzzy sets. Pythagorean fuzzy sets has limitation that their square sum is less than or equal to 1. IFS was failed to deal with indeterminate and inconsistent information which exist in beliefs system, therefore, Smarandache [22] in 1995 introduced new concept known as neutrosophic set(NS) which generalizes fuzzy sets and intuitionistic fuzzy sets and so on. A neutrosophic set includes truth membership, falsity membership and indeterminacy membership.

In 2006, F.Smarandache introduced, for the first time, the degree of dependence (and consequently the degree of independence) between the components of the fuzzy set, and also between the components of the neutrosophic set. In 2016, the refined neutrosophic set was generalized to the degree of dependence or independence of subcomponents [22]. In neutrosophic set [22], if truth membership and falsity membership are 100% dependent and indeterminacy is 100% independent, that is $0 \leq u_A(x) + \zeta_A(x) + v_A(x) \leq 2$. Sometimes in real life, we face many problems which cannot be handled by using neutrosophic for example when $u_A(x) + \zeta_A(x) + v_A(x) > 2$. In such condition, a neutrosophic set has no ability to obtain any satisfactory result. To state this condition, we give an example: the truth membership, falsity membership and indeterminacy values are $\frac{8}{10}, \frac{5}{10}$ and $\frac{9}{10}$ respectively. This satisfies the condition that their sums exceeds 2 and are not presented to neutrosophic set. So, In Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] of condition is as their square sum does not exceeds 2. Here, T and F are dependent neutrosophic components and we make $u_A(x), v_A(x) as$ Pythagorean, then $\left(u_A(x)\right)^2 + \left(v_A(x)\right)^2 \leq 1$ with $u_A(x), v_A(x)$ in [0,1]. If $\zeta_A(x)$ is an Independent from them, then $0 \leq \zeta_A(x) \leq 1$. Then $0 \leq \left(u_A(x)\right)^2 + \left(\zeta_A(x)\right)^2 + \left(v_A(x)\right)^2 \leq 2$, with $u_A(x), \zeta_A(x), v_A(x)$ in [0,1]. We consider in general the degree of dependence between $u_A(x), \zeta_A(x), v_A(x)$ is 1 , hence $u_A(x), \zeta_A(x), v_A(x) \leq 3 - 1 = 2$.

Correlation coefficients are beneficial tools used to determine the degree of similarity between objects. The importance of correlation coefficients in fuzzy environments lies in the fact that these types of tools can feasibly be applied to problems of pattern recognition, MADM, medical diagnosis and clustering, etc. In other research, Ye[33] proposed three vector similarity measure for SNSs, an instance of SVNS and INS, includingthe Jaccard, Dice, and cosine similarity measures for SVNS and INSs, and applied them to multi-criteria decision-making problems with simplified neutrosophic information. Hanafy et al. [16] proposed the correlation coefficients of neutrosophic sets and studied some of their basic properties. Based on centroid method, Hanafy et al. [17], introduced and studied the concepts of correlation and correlation coefficient of neutrosophic sets and studied some of their properties.

Recently Bromi and Smarandache defined the Haudroff distance between neutrosophic sets and some similarity measures based on the distance such as; set theoretic approach and matching function to calculate the similarity degree between neutrosophic sets. In the same year, Broumi and Smarandache [11] also proposed the correlation coefficient between interval neutrsphic sets.

In this paper, we have to study the concept of Pythagorean neutrosophic set with T and F are neutrosophic components and also define the correlation measure of Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] and prove some of its properties. Then, using correlation of Pythagorean neutrosophic fuzzy set with T and F are dependent neutrosophic components [PNS] measure, the application of medical diagnosis is given.

Preliminaries

Definition 2.1 [1] Let E be a universe. An intuitionistic fuzzy set A on E can be defined as follows:

$$A = \{< x, u_A(x), v_A(x) >: x \in E\}$$

Where $u_A: E \to [0,1]$ and $v_A: E \to [0,1]$ such that $0 \leq u_A(x) + v_A(x) \leq 1$ for any $x \in E$. Where, $u_A(x)$ and $v_A(x)$ is the degree of membership and degree of non-membership of the element x, respectively.

Definition 2.2 [18, 24]
Let X be a non-empty set and I the unit interval [0, 1]. A Pythagorean fuzzy set S is an object having the form $A = \{(x, u_A(x), v_A(x)) : x \in X\}$ where the functions $u_A : X \to [0,1]$ and $v_A : X \to [0,1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P, and $0 \leq \left(u_A(x)\right)^2 + (v_A(x))^2 \leq 1$ for each $x \in X$.

Definition 2.3[15] Let X be a non-empty set (universe). A neutrosophic set A on X is an object of the form: $A = \{\left(x, u_A(x), \zeta_A(x), v_A(x)\right) : x \in X\}$,

Where $u_A(x), \zeta_A(x), v_A(x) \in [0,1], 0 \leq u_A(x) + \zeta_A(x) + v_A(x) \leq 2, for\ all\ x\ in\ X$. $u_A(x)$ is the degree of membership, $\zeta_A(x)$ is the degree of inderminancy and $v_A(x)$ is the degree of non-membership. Here $u_A(x)$ and $v_A(x)$ are dependent components and $\zeta_A(x)$ is an independent components.

Definition 2.4 Let X be a nonempty set and I the unit interval [0,1]. A neutrosophic set A and B of the form $A = \{(x, u_A(x), \zeta_A(x), v_A(x)) : x \in X\}$ and $B = \{(x, u_B(x), \zeta_B(x), v_B(x)) : x \in X\}$. Then

1) $A^C = \{\left(x, v_A(x), \zeta_A(x), u_A(x)\right) : x \in X\}$
2) $A \cup B = \{\left(x, \max\left(u_A(x), u_B(x)\right), \min\left(\zeta_A(x), \zeta_B(x)\right), \min(v_A(x), v_B(x))\right) : x \in X\}$
3) $A \cap B = \{(x, \min\left(u_A(x), u_B(x)\right), \max\left(\zeta_A(x), \zeta_B(x)\right), \max(v_A(x), v_B(x)) : x \in X\}$

3. Pythagorean Neutrosophic set with T and F are dependent neutrosophic components [PNS]:

Definition 3.1 Let X be a non-empty set (universe). A Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] A on X is an object of the form $A = \{\left(x, u_A(x), \zeta_A(x), v_A(x)\right) : x \in X\}$,

Where $u_A(x), \zeta_A(x), v_A(x) \in [0,1], 0 \leq \left(u_A(x)\right)^2 + \left(\zeta_A(x)\right)^2 + \left(v_A(x)\right)^2 \leq 2, for\ all\ x\ in\ X$. $u_A(x)$ is the degree of membership, $\zeta_A(x)$ is the degree of inderminancy and $v_A(x)$ is the degree of non-membership .Here $u_A(x)$ and $v_A(x)$ are dependent components and $\zeta_A(x)$ is an independent components.

Definition 3.2 Let X be a nonempty set and I the unit interval [0, 1]. A Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] A and B of the form

$A = \{(x, u_A(x), \zeta_A(x), v_A(x)) : x \in X\}$ and $B = \{(x, u_B(x), \zeta_B(x), v_B(x)) : x \in X\}$. Then
1) $A^C = \{\left(x, v_A(x), \zeta_A(x), u_A(x)\right) : x \in X\}$
2) $A \cup B = \{\left(x, \max\left(u_A(x), u_B(x)\right), \max\left(\zeta_A(x), \zeta_B(x)\right), \min(v_A(x), v_B(x))\right) : x \in X\}$
3) $A \cap B = \{(x, \max\left(u_A(x), u_B(x)\right), \max\left(\zeta_A(x), \zeta_B(x)\right), \min(v_A(x), v_B(x)) : x \in X\}$

Definition 3.3 Let X be a nonempty set and I the unit interval [0, 1]. A Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] A and B of the form
$A = \{(x, u_A(x), \zeta_A(x), v_A(x)) : x \in X\}$ and $B = \{(x, u_B(x), \zeta_B(x), v_B(x)) : x \in X\}$.
Then the correlation coefficient of A and B

$$\rho(A, B) = \frac{C(A, B)}{\sqrt{C(A, A). C(B, B)}} \qquad (1)$$

$$C(A,B)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$
$$C(A,A)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)$$

$$C(B,B)=\sum_{i=1}^{n}\left(\left(u_B(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_B(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_B(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$

Preposition 3.4 The defined correlation measure between PNS A and PNS B satisfies the following properties

(i) $0\leq\ \rho(A,B)\leq 1$

(ii) $\rho(A,B)=1\ if\ and\ only\ if\ A=B$

(iii) $\rho(A,B)=\ \rho(B,A)$.

Proof:

(i) $0\leq\ \rho(A,B)\leq 1$

As the membership, inderminate and non-membership functions of the PNS lies between 0 and 1, $\rho(A,B)$ also lies between 0 and 1.

We will prove $C(A,B)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$

$$=\left(\left(u_A(x_1)\right)^2.\left(u_B(x_1)\right)^2+\left(\zeta_A(x_1)\right)^2.\left(\zeta_B(x_1)\right)^2+\left(v_A(x_1)\right)^2.\left(v_B(x_1)\right)^2\right)+$$
$$\left(\left(u_A(x_2)\right)^2.\left(u_B(x_2)\right)^2+\left(\zeta_A(x_2)\right)^2.\left(\zeta_B(x_2)\right)^2+\left(v_A(x_2)\right)^2.\left(v_B(x_2)\right)^2\right)+\cdots+$$
$$\left(\left(u_A(x_n)\right)^2.\left(u_B(x_n)\right)^2+\left(\zeta_A(x_n)\right)^2.\left(\zeta_B(x_n)\right)^2+\left(v_A(x_n)\right)^2.\left(v_B(x_n)\right)^2\right)$$

By Cauchy-Schwarz inequality, $(x_1y_1+x_2y_2+\cdots+x_ny_n)^2\leq({x_1}^2+{x_2}^2+\cdots+{x_n}^2).({y_1}^2+{y_2}^2+\cdots+{y_n}^2)$, where $(x_1+x_2+\cdots+x_n)\in R^n\ and\ (y_1+y_2+\cdots+y_n)\in R^n$, we get

$$\left(C(A,B)\right)^2=\left(\left(u_A(x_1)\right)^4+\left(\zeta_A(x_1)\right)^4+\left(v_A(x_1)\right)^4\right)+\left(\left(u_A(x_2)\right)^4+\left(\zeta_A(x_2)\right)^4+\left(v_A(x_2)\right)^4\right)+\ldots+\left(\left(u_A(x_n)\right)^4+\left(\zeta_A(x_n)\right)^4+\left(v_A(x_n)\right)^4\right)$$

$$\times\left(\left(u_B(x_1)\right)^4+\left(\zeta_B(x_1)\right)^4+\left(v_B(x_1)\right)^4\right)+\left(\left(u_B(x_2)\right)^4+\left(\zeta_B(x_2)\right)^4+\left(v_B(x_2)\right)^4\right)+\cdots+\left(\left(u_B(x_n)\right)^4+\left(\zeta_B(x_n)\right)^4+\left(v_B(x_n)\right)^4\right)$$

$$=\left(\left(u_A(x_1)\right)^2.\left(u_A(x_1)\right)^2+\left(\zeta_A(x_1)\right)^2.\left(\zeta_A(x_1)\right)^2+\left(v_A(x_1)\right)^2.\left(v_A(x_1)\right)^2\right)$$
$$+\left(\left(u_A(x_2)\right)^2.\left(u_A(x_2)\right)^2+\left(\zeta_A(x_2)\right)^2.\left(\zeta_A(x_2)\right)^2+\left(v_A(x_2)\right)^2.\left(v_A(x_2)\right)^2\right)+\cdots+$$

$$\left(\left(u_A(x_n)\right)^2.\left(u_A(x_n)\right)^2+\left(\zeta_A(x_n)\right)^2.\left(\zeta_A(x_n)\right)^2+\left(v_A(x_n)\right)^2.\left(v_A(x_n)\right)^2\right)\times$$
$$\left(\left(u_B(x_1)\right)^2\left(u_B(x_1)\right)^2+\left(\zeta_B(x_1)\right)^2\left(\zeta_B(x_1)\right)^2+\left(v_B(x_1)\right)^2\left(v_B(x_1)\right)^2\right)+$$
$$\left(\left(u_B(x_2)\right)^2\left(u_B(x_2)\right)^2+\left(\zeta_B(x_2)\right)^2\left(\zeta_B(x_2)\right)^2+\left(v_B(x_2)\right)^2\left(v_B(x_2)\right)^2\right)+\cdots+$$
$$\left(\left(u_B(x_n)\right)^2\left(u_B(x_n)\right)^2+\left(\zeta_B(x_n)\right)^2+\left(v_B(x_n)\right)^2\left(v_B(x_n)\right)^2\right)$$

$$= C(A,A)\times C(B,B).$$

Therefore, $\left(C(A,B)\right)^2 \le C(A,A)\times C(B,B)$ and thus $\rho(A,B)\le 1$.

Hence we obtain the following propertity $0\le\ \rho(A,B)\le 1$

(ii) $\rho(A,B)=1\ if\ and\ only\ if\ A=B$

Let the two PNS A and B be equal (i.e A = B). Hence for any

$u_A(x_i)=u_B(x_i)$, $\zeta_A(x_i)=\zeta_B(x_i)$ and $v_A(x_i)=v_B(x_i)$,

Then $C(A,A)=C(B,B)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)$

And $C(A,B)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$

$$=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)=C(A,A)$$

Hence

$$\rho(A,B)=\frac{C(A,B)}{\sqrt{C(A,A).C(B,B)}}$$

$$=\frac{C(A,A)}{\sqrt{C(A,A).C(A,A)}}=1$$

Let the $\rho(A,B)=1$.Then, the unite measure is possible only if

$$\frac{C(A,B)}{\sqrt{C(A,A).C(B,B)}}=1$$

This refer that $u_A(x_i)=u_B(x_i)$, $\zeta_A(x_i)=\zeta_B(x_i)$ and $v_A(x_i)=v_B(x_i)$,

for all i. Hence A = B.

(iii) If $\rho(A,B)=\rho(B,A)$, it obvious that

$$\frac{C(A,B)}{\sqrt{C(A,A).C_{NPFS}(B,B)}}=\frac{C(A,B)}{\sqrt{C(A,A).C(B,B)}}=\rho(B,A)$$

as

$$C(A,B)=\sum_{i=1}^{n}\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$
$$=\sum_{i=1}^{n}\left(\left(u_B(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_B(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_B(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)$$
$$C(B,A)$$

Hence the proof.

Definition 3.5

Let A and B be two PNSs, then the correlation coefficient is defined as

$$\rho'(A,B)=\frac{C(A,B)}{max\{C(A,A).C(B,B)\}} \tag{2}$$

Theorem 3.6

The defined correlation measure between PNS A and PNS B satisfies the following properties

(i) $0\le\ \rho'(A,B)\le 1$

(ii) $\rho'(A,B)=1\ if\ and\ only\ if\ A=B$

(iii) $\rho'(A,B)=\ \rho'(B,A)$.

Proof: The property (i) and (ii) is straight forward, so omit here. Also $\rho'(A,B)\ge 0$ is evident. We now prove only $\rho'(A,B)\le 1$.

Since Theorem 3.4, we have $(C(A,B))^2\le C(A,A).C(B,B)$. Therefore, $C(A,B)\le max\{C(A,A),C(B,B)\}$ and thus $\rho'(A,B)\le 1$.

However, in many practical situations, the different set may have taken different weights, and thus, weight ω_i of the element $x_i\in X\ (i=1,2,\dots,n)$ should be taken into account. In the following, we develop a weighted correlation coefficient between PNSs. Let $\omega=\{\omega_1,\omega_2,\dots,\omega_n\}$ be the weight vector of the elements $x_i(i=1,2,\dots.,n)$ with $\omega_i\ge 0\ and\ \sum_{i=1}^{n}\omega_i=1$, then we have extended the above correlation coefficient $\rho(A,B)\ and\ \rho'(A,B)$ to weighted correlation coefficient as follows:

$$\rho''=\frac{C_\omega(A,B)}{\sqrt{C_\omega(A,A).C_\omega(B,B)}} \tag{3}$$

$$C_\omega(A,B)=\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$

$$C_\omega(A,A)=\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)$$

$$C_\omega(B,B)=\sum_{i=1}^{n}\omega_i\left(\left(u_B(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_B(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_B(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$

And

$$\rho''' = \frac{C_\omega(A,B)}{max\{C_\omega(A,A).C_\omega(B,B)\}} \quad (4)$$

$$= \frac{\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)}{max\left\{\begin{array}{l}\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right),\\ \sum_{i=1}^{n}\omega_i\left(\left(u_B(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_B(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_B(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)\end{array}\right\}}$$

It can be easy to verify that if $\omega = \left(\frac{1}{n},\frac{1}{n},\dots,\frac{1}{n}\right)^T$, then Equation (3) and (4) reduce that (1) and (2), respectively.

Theorem 3.7

Let $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of $x_i (i = 1,2, \dots, n)$ with $\omega_i \geq 0$ *and* $\sum_{i=1}^{n}\omega_i = 1$, then the weighted correlation coefficient between the PNSs A and B defined by Equation (3) satisfies:

(i) $0 \leq \rho''(A,B) \leq 1$

(ii) $\rho''(A,B) = 1$ *if and only if* $A = B$

(iii) $\rho''(A,B) = \rho''(B,A)$.

Proof:

The property (i) and (ii) are straight forward so omit here. Also $\rho''(A,B) \geq 0$ is evident so we need to show only $\rho''(A,B) \leq 1$.

Since,

$$C_\omega(A,B) = \sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)$$

$$= \omega_1\left(\left(u_A(x_1)\right)^2.\left(u_B(x_1)\right)^2+\left(\zeta_A(x_1)\right)^2.\left(\zeta_B(x_1)\right)^2+\left(v_A(x_1)\right)^2.\left(v_B(x_1)\right)^2\right)+$$
$$\omega_2\left(\left(u_A(x_2)\right)^2.\left(u_B(x_2)\right)^2+\left(\zeta_A(x_2)\right)^2.\left(\zeta_B(x_2)\right)^2+\left(v_A(x_2)\right)^2.\left(v_B(x_2)\right)^2\right)+\cdots+$$
$$\omega_n\left(\left(u_A(x_n)\right)^2.\left(u_B(x_n)\right)^2+\left(\zeta_A(x_n)\right)^2.\left(\zeta_B(x_n)\right)^2+\left(v_A(x_n)\right)^2.\left(v_B(x_n)\right)^2\right)$$

$$= \left(\sqrt{\omega_1}\left(u_A(x_1)\right)^2.\sqrt{\omega_1}\left(u_B(x_1)\right)^2+\sqrt{\omega_1}\left(\zeta_A(x_1)\right)^2.\sqrt{\omega_1}\left(\zeta_B(x_1)\right)^2+\sqrt{\omega_1}\left(v_A(x_1)\right)^2.\sqrt{\omega_1}\left(v_B(x_1)\right)^2\right)$$
$$+\left(\sqrt{\omega_2}\left(u_A(x_2)\right)^2.\sqrt{\omega_2}\left(u_B(x_2)\right)^2+\sqrt{\omega_2}\left(\zeta_A(x_2)\right)^2.\sqrt{\omega_2}\left(\zeta_B(x_2)\right)^2\right.$$
$$\left.+\sqrt{\omega_2}\left(v_A(x_2)\right)^2.\sqrt{\omega_2}\left(v_B(x_2)\right)^2\right)+\cdots+$$
$$\left(\sqrt{\omega_n}\left(u_A(x_n)\right)^2.\sqrt{\omega_n}\left(u_B(x_n)\right)^2+\sqrt{\omega_n}\left(\zeta_A(x_n)\right)^2.\sqrt{\omega_n}\left(\zeta_B(x_n)\right)^2+\right.$$
$$\left.\sqrt{\omega_n}\left(v_A(x_n)\right)^2.\sqrt{\omega_n}\left(v_B(x_n)\right)^2\right)$$

By using Cauchy-Schwarz inequality, we get

$$
\begin{aligned}
\left(C_\omega(A,B)\right)^2 \le\ & \left(\omega_1\left(u_A(x_1)\right)^2.\left(u_A(x_1)\right)^2+\left(\zeta_A(x_1)\right)^2.\left(\zeta_A(x_1)\right)^2+\left(v_A(x_1)\right)^2.\left(v_A(x_1)\right)^2\right)+\\
& \left(\omega_2\left(u_A(x_2)\right)^2.\left(u_A(x_2)\right)^2+\left(\zeta_A(x_2)\right)^2.\left(\zeta_A(x_2)\right)^2+\left(v_A(x_2)\right)^2.\left(v_A(x_2)\right)^2\right)+\\
& \dots+\left(\omega_n\left(u_A(x_n)\right)^2.\left(u_A(x_n)\right)^2+\left(\zeta_A(x_n)\right)^2.\left(\zeta_A(x_n)\right)^2+\left(v_A(x_n)\right)^2.\left(v_A(x_n)\right)^2\right)\times\\
& \left(\omega_1\left(u_B(x_1)\right)^2\left(u_B(x_1)\right)^2+\left(\zeta_B(x_1)\right)^2\left(\zeta_B(x_1)\right)^2+\left(v_B(x_1)\right)^2\left(v_B(x_1)\right)^2\right)+\\
& \left(\omega_2\left(u_B(x_2)\right)^2\left(u_B(x_2)\right)^2+\left(\zeta_B(x_2)\right)^2\left(\zeta_B(x_2)\right)^2+\left(v_B(x_2)\right)^2\left(v_B(x_2)\right)^2\right)\\
& +\cdots+\left(\omega_n\left(u_B(x_n)\right)^2\left(u_B(x_n)\right)^2+\left(\zeta_B(x_n)\right)^2\left(\zeta_B(x_n)\right)^2+\left(v_B(x_n)\right)^2\left(v_B(x_n)\right)^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&=\sum_{i=1}^{n}\omega_i\left(\left(u_A(x_i)\right)^2.\left(u_A(x_i)\right)^2+\left(\zeta_A(x_i)\right)^2.\left(\zeta_A(x_i)\right)^2+\left(v_A(x_i)\right)^2.\left(v_A(x_i)\right)^2\right)\times\\
&\quad \textstyle\sum_{i=1}^{n}\omega_i\left(\left(u_B(x_i)\right)^2.\left(u_B(x_i)\right)^2+\left(\zeta_B(x_i)\right)^2.\left(\zeta_B(x_i)\right)^2+\left(v_B(x_i)\right)^2.\left(v_B(x_i)\right)^2\right)\\
&=C_\omega(A,A)\times C_\omega(B,B)
\end{aligned}
$$

Therefore, $C_\omega(A,B)\le\sqrt{C_\omega(A,A)\times C_\omega(B,B)}$ and hence $0\le\ \rho''(A,B)\le 1$.

Theorem 3.8

The correlation coefficient of two PNSs A and B as defined in Equation (4), that is, $\rho'''(A,B)$ satisfies the same properties as those in Theorem 3.7

Proof: The proof of this theorem is similar to that of Theorem 3.6.

5. Application

In this section, we give some application of PNS in medical diagnosis problem using correlation measure.

Medical Diagnosis Problem

As medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult.In some practical problems, there is the possibility of each element having different truth membership , inderminate and false membership functions.The proposed correlation measure among the patients Vs. symptoms and symptoms Vs. diseases gives the proper medical diagnosis. Now, an example of a medical diagnosis will be presented

Example

Let P= $\{P_1,P_2,P_3\}$ be a set of patients, D= $\{Viral\ Fever, Malaria, Typhoid, Dengu\}$ be a set of diseases and S= $\{Temperature$, Headache, Cough, Joint pain$\}$ be a set of symptoms.

Table 1: M (the relation between Patient and Symptoms)

M	$Temperature$	Headache	Cough	Joint pain
P_1	(0.8,0.7,0.6)	(0.5,0.3,0.8)	(0.6,0.9,0.4)	(0.3,0.5,0.2)
P_2	(0.2,0.7,0.9)	(0.5,0.9,0.8)	(0.4,0.6,0.3)	(0.1,0.2,0.9)
P_3	(0.3,0.1,0.5)	(0.8,0.5,0.6)	(0.4,0.8,0.9)	(0.5,0.7,0.2)

Table 2: N (the relation between Symptoms and Diseases)

N	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
$Temperature$	(0.9,0.5,0.4)	(0.5,0.3,0.6)	(0.8,0.9,0.4)	(0.2,0.8,0.5)
Headache	(0.1,0.5,0.3)	(0.5,0.6,0.7)	(0.4,0.5,0.9)	(0.9,0.8,0.3)
Cough	(0.3,0.7,0.8)	(0.9,0.7,0.4)	(0.1,0.3,0.9)	(0.5,0.3,0.8)
Joint pain	(0.7,0.3,0.5)	(0.8,0.9,0.6)	(0.5,0.7,0.6)	(0.1,0.5,0.8)

Using Equations (1), we get the value of $\rho(A,B)$

Table 3: M and N (Correlation Measure)

M	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
P_1	**0.7670**	0.5363	0.5965	0.5446
P_2	0.4638	**0.6253**	0.4873	0.5434
P_3	0.4596	0.6606	0.6072	**0.7401**

Using Equations (2), we get the value of $\rho'(A,B)$

Table 4: M and N (Correlation Measure)

M	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
P_1	**0.6997**	0.5223	0.5786	0.5357
P_2	0.3670	**0.5292**	0.4358	0.5095
P_3	0.4269	0.6562	0.5784	**0.6729**

On the other hand, if we assign weights 0.10, 0.20, 0.30 and 0.40 respectively, then by applying correlation coefficient given in Equations (3) and (4), we can give the following values of the correlation coefficient:

Using Equations (3), we get the value of $\rho''(A,B)$

Table 5: M and N (Correlation Measure)

M	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
P_1	**0.7233**	0.6496	0.4527	0.4623
P_2	0.4390	**0.5469**	0.4758	0.4194
P_3	0.5123	0.6606	0.7229	**0.7638**

Using Equations (4), we get the value of $\rho'''(A,B)$

Table 6: M and N (Correlation Measure)

M	$Viral\ Fever$	$Malaria$	$Typhoid$	$Dengu$
P_1	**0.6936**	0.5324	0.4280	0.4039
P_2	0.2812	**0.5316**	0.4245	0.4084
P_3	0.4321	0.6154	0.6727	**0.7518**

The highest correlation measure from the Tables 3,4,5,6 gives the proper medical diagnosis. Therefore, patient P_1 suffers from Viral Fever, patient P_2 suffers from Malaria and patient P_3 suffers from Dengu. Hence, we can see from the above four kinds of correlation coefficient indices that the results are same.

Conclusion

In this paper, we found the correlation measure of Pythagorean neutrosophic set with T and F are neutrosophic components (PNS) and proved some of their basic properties. Based on that the present paper have extended the theory of correlation coefficient from and neutrosophic sets (NS) to the Pythagorean neutrosophic set with T and F are neutrosophic components in which the constraint condition of sum of membership, non-membership and indeterminacy be less than two has been relaxed. Illustrate examples have handle the situation where the existing correlation coefficient in NS environment fails. Also to deal with the situations where the elements in a set are correlative, a weighted correlation coefficients has been defined. We studied an application of correlation measure of Pythagorean neutrosophic set with T and F are neutrosophic components in medical diagnosis.

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Conflicts of Interest

The authors declare no conflict of interest.

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An Outranking Approach for MCDM-Problems with Neutrosophic Multi-Sets

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Abstract: In this paper, we introduced a new outranking approach for multi-criteria decision making (MCDM) problems to handle uncertain situations in neutrosophic multi environment. Therefore, we give some outranking relations of neutrosophic multi sets. We also examined some desired properties of the outranking relations and developed a ranking method for MCDM problems. Moreover, we describe a numerical example to verify the practicality and effectiveness of the proposed method.

Keywords: Single valued neutrosophic sets, neutrosophic multi-sets, outranking relations, decision making.

1. Introduction

Fuzzy set theory, intuitionistic fuzzy set theory and neutrosophic set theory is introduced by Zadeh [59], Atanassov [1] and Smarandache [28] to handle the uncertain, incomplete, indeterminate and inconsistent information, respectively. The above set theories have been applied to many different areas including real decision making problems [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 21, 22, 23, 24, 25, 26, 27, 32, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 58]. Also, several generalizations of the set theories made such as fuzzy multi-set theory [34, 35, 48], intuitionistic fuzzy multi-set theory [16, 31, 36, 37, 57] and n-valued refined neutrosophic set theory [29].

Another generalization of above theories that is relevant for our work is single valued neutrosophic refined (multi) set theory introduced by Ye [53, 56] which contain a few different values. A single valued neutrosophic multi set theory has truth-membership sequence $\left(\mu_A^1(t),\mu_A^2(t),\ldots,\mu_A^P(t)\right)$, indeterminacy membership sequence $\left(\nu_A^1(t),\nu_A^2(t),\ldots,\nu_A^P(t)\right)$ and falsity-membership sequence $\left(\omega_A^1(t),\omega_A^2(t),\ldots,\omega_A^P(t)\right)$ of element $t\in T$. Recently, the single valued neutrosophic multi set theory have attracted widely attention in [20, 33, 50, 51, 52, 54, 55]. The paper is organized as follows; In Section 2 we give some basic notions of neutrosophic sets and neutrosophic multi-sets. In Section 3, we first introduce outranking relations of neutrosophic multi-sets with proprieties. In Section 4, we propose an outranking approach for to solving the multi-criteria decision making problems based on neutrosophic multi-set information. In Section 5, we propose a selection example to validate the practicality. Finally, in Section 6, we conclude the paper.

2. Preliminaries

In this section, we present the basic definitions and results of neutrosophic set theory [28, 33] and neutrosophic multi (or refined) set theory [12, 53] that are useful for subsequent discussions.

Definition 1 [28] let T be a universe. A neutrosophic set A over T is defined by

$$A = \left\{\left\langle t,\left(\mu_A(t),\nu_A(t),\omega_A(t)\right)\right\rangle, t\in T\right\}.$$

where $\mu_A(t)$, $\nu_A(t)$ and $\omega_A(t)$ are called truth-membership function, indeterminacy-membership function and falsity-membership function, respectively. They are respectively defined by

$$\mu_A(t): T \to \left]^-0,1^+\right[,\ \nu_A(t): T \to \left]^-0,1^+\right[,\ \omega_A(t): T \to \left]^-0,1^+\right[$$

such that $^-0 \le \mu_A(t) + \nu_A(t) + \omega_A(t) \le 3^+$.

Definition 2 [33] Let T be a universe. An single valued neutrosophic set (SVN-set) over T is a neutrosophic set over T, but the truth-membership function, indeterminacy-membership function and falsity-membership function are respectively defined by

$$\mu_A(t): T \to [0,1],\ \nu_A(t): T \to [0,1],\ \omega_A(t): T \to [0,1]$$

such that $0 \le \mu_A(t) + \nu_A(t) + \omega_A(t) \le 3$.

Definition 3 [53] Let T be a universe. A neutrosophic multiset set (Nms) $\mathcal{A}$ on T can be defined as follows:

$$\mathcal{A} = \{\prec t, \left(\mu_{\mathcal{A}}^1(t), \mu_{\mathcal{A}}^2(t), \dots \mu_{\mathcal{A}}^p(t)\right), \left(v_{\mathcal{A}}^1(t), v_{\mathcal{A}}^2(t), \dots v_{\mathcal{A}}^p(t)\right), \left(w_{\mathcal{A}}^1(t), w_{\mathcal{A}}^2(t), \dots w_{\mathcal{A}}^p(t)\right) \succ : t \in T\}$$

Where,

$$\mu_{\mathcal{A}}^1(t), \mu_{\mathcal{A}}^2(t), \dots \mu_{\mathcal{A}}^p(t): T \to [0,1],$$

$$v_{\mathcal{A}}^1(t), v_{\mathcal{A}}^2(t), \dots v_{\mathcal{A}}^p(t): T \to [0,1],$$

and

$$w_{\mathcal{A}}^1(t), w_{\mathcal{A}}^2(t), \dots w_{\mathcal{A}}^p(t): T \to [0,1]$$

such that

$$0 \le sup\mu_{\mathcal{A}}^i(t) + supv_{\mathcal{A}}^i(t) + supw_{\mathcal{A}}^i(t) \le 3$$

$(i = 1,2, \dots, P)$ and $\left(\mu_{\mathcal{A}}^1(t), \mu_{\mathcal{A}}^2(t), \dots, \mu_{\mathcal{A}}^p(t)\right), \left(v_{\mathcal{A}}^1(t), v_{\mathcal{A}}^2(t), \dots, v_{\mathcal{A}}^p(t)\right)$ *and* $\left(w_{\mathcal{A}}^1(t), w_{\mathcal{A}}^2(t), \dots, w_{\mathcal{A}}^p(t)\right)$ Is the truth-membership sequence, indeterminacy-membership sequence and falsity- membership sequence of the element u, respectively. Also, P is called the dimension (cardinality) of Nms $\mathcal{A}$, denoted $d(\mathcal{A})$. We arrange the truth- membership sequence in decreasing order but the corresponding indeterminacy- membership and falsity-membership sequence may not be in decreasing or increasing order.

The set of all Neutrosophic multisets on T is denoted by NMS(T).

Definition 4 [12, 53, 56] Let $A, B \in NMS(T)$. Then,

(1) $\mathcal{A}$ is said to be Nm-subset of $\mathcal{B}$ is denoted by $\mathcal{A} \widetilde{\subseteq} \mathcal{B}$ if $\mu_{\mathcal{A}}^i(t) \le \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) \ge v_{\mathcal{B}}^i(t)$, $w_{\mathcal{A}}^i(t) \ge w_{\mathcal{B}}^i(t)$, $\forall\, t \in T$ and $i = 1,2, \dots P$.

(2) $\mathcal{A}$ is said to be neutrosophic equal of $\mathcal{B}$ is denoted by $\mathcal{A} = \mathcal{B}$ if $\mu_{\mathcal{A}}^i(t) = \mu_{\mathcal{B}}^i(t)$, $v_{\mathcal{A}}^i(t) = v_{\mathcal{B}}^i(t)$, $w_{\mathcal{A}}^i(t) = w_{\mathcal{B}}^i(t)$, $\forall\, t \in T$ and $i = 1,2, \dots P$.

(3) The complement of $\mathcal{A}$ denoted by $\mathcal{A}^{\tilde{c}}$ and is defined by

$$\mathcal{A}^{\tilde{c}} = \prec t, \left(w_{\mathcal{A}}^1(t), w_{\mathcal{A}}^2(t), \dots, w_{\mathcal{A}}^p(t)\right), \left(v_{\mathcal{A}}^1(t), v_{\mathcal{A}}^2(t), \dots v_{\mathcal{A}}^p(t)\right), \left(\mu_{\mathcal{A}}^1(t), \mu_{\mathcal{A}}^2(t), \dots \mu_{\mathcal{A}}^p(t)\right) \succ : t \in T\}$$

(4) If $\mu^i_{\mathcal{A}}(t) = 0$ and $v^i_{\mathcal{A}}(t) = w^i_{\mathcal{A}}(t) = 1$ for all $t \in T$ and $i = 1,2, \dots P$, then $\mathcal{A}$ is called null ns-set and denoted by Φ.

(5) If $\mu^i_{\mathcal{A}}(t) = 1$ and $v^i_{\mathcal{A}}(t) = w^i_{\mathcal{A}}(t) = 0$ for all $t \in T$ and $i = 1,2, \dots P$, then $\mathcal{A}$ is called universal ns-set and denoted by $\tilde{T}$.

(6) The union of $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} \tilde{\cup} \mathcal{B} = \mathcal{C}$ and is defined by

$$\mathcal{C} = \{\prec t, \left(\mu^1_{\mathcal{C}}(t), \mu^2_{\mathcal{C}}(t), \dots \mu^p_{\mathcal{C}}(t)\right), \left(v^1_{\mathcal{C}}(t), v^2_{\mathcal{C}}(t), \dots v^p_{\mathcal{C}}(t)\right), \left(w^1_{\mathcal{C}}(t), w^2_{\mathcal{C}}(t), \dots w^p_{\mathcal{C}}(t)\right) \succ : t \in T\}$$

Where $\mu^i_{\mathcal{C}} = \mu^i_{\mathcal{A}}(t) \vee \mu^i_{\mathcal{B}}(t)$, $v^i_{\mathcal{C}} = v^i_{\mathcal{A}}(t) \wedge v^i_{\mathcal{B}}(t)$, $w^i_{\mathcal{C}} = w^i_{\mathcal{A}}(t) \wedge w^i_{\mathcal{B}}(t)$, $\forall t \in T$ and $i = 1,2, \dots P$.

(7) The intersection of $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} \tilde{\cap} \mathcal{B} = \mathcal{D}$ and is defined by

$$\mathcal{D} = \{\prec t, \left(\mu^1_{\mathcal{D}}(t), \mu^2_{\mathcal{D}}(t), \dots \mu^p_{\mathcal{D}}(t)\right), \left(v^1_{\mathcal{D}}(t), v^2_{\mathcal{D}}(t), \dots v^p_{\mathcal{D}}(t)\right), \left(w^1_{\mathcal{D}}(t), w^2_{\mathcal{D}}(t), \dots w^p_{\mathcal{D}}(t)\right) \succ : t \in T\}$$

where $\mu^i_{\mathcal{D}} = \mu^i_{\mathcal{A}}(t) \vee \mu^i_{\mathcal{B}}(t)$, $v^i_{\mathcal{D}} = v^i_{\mathcal{A}}(t) \wedge v^i_{\mathcal{B}}(t)$, $w^i_{\mathcal{D}} = w^i_{\mathcal{A}}(t) \wedge w^i_{\mathcal{B}}(t)$, $\forall\, t \in T$ and $i = 1,2, \dots P$.

(8) The addition of $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} \tilde{+} \mathcal{B} = \mathcal{U}_1$ and is defined by

$$\mathcal{U}_1 = \{\prec t, \left(\mu^1_{\mathcal{U}_1}(t), \mu^2_{\mathcal{U}_1}(t), \dots \mu^p_{\mathcal{U}_1}(t)\right), \left(v^1_{\mathcal{U}_1}(t), v^2_{\mathcal{U}_1}(t), \dots v^p_{\mathcal{U}_1}(t)\right), \left(w^1_{\mathcal{U}_1}(t), w^2_{\mathcal{U}_1}(t), \dots w^p_{\mathcal{U}_1}(t)\right) \succ : t \in T\}$$

where $\mu^i_{\mathcal{U}_1} = \mu^i_{\mathcal{A}}(t) + \mu^i_{\mathcal{B}}(t) - \mu^i_{\mathcal{A}}(t).\mu^i_{\mathcal{B}}(t)$, $v^i_{\mathcal{U}_1} = v^i_{\mathcal{A}}(t).v^i_{\mathcal{B}}(t)$, $w^i_{\mathcal{U}_1} = w^i_{\mathcal{A}}(t).w^i_{\mathcal{B}}(t)$ $\forall\, t \in T$ and $i = 1,2, \dots P$.

(9) The multiplication of $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} \tilde{x} \mathcal{B} = \mathcal{U}_2$ and is defined by

$$\mathcal{U}_2 = \{\prec t, \left(\mu^1_{\mathcal{U}_2}(t), \mu^2_{\mathcal{U}_2}(t), \dots \mu^p_{\mathcal{U}_2}(t)\right), \left(v^1_{\mathcal{U}_2}(t), v^2_{\mathcal{U}_2}(t), \dots v^p_{\mathcal{U}_2}(t)\right), \left(w^1_{\mathcal{U}_2}(t), w^2_{\mathcal{U}_2}(t), \dots w^p_{\mathcal{U}_2}(t)\right) \succ : t \in T\}$$

where $\mu^i_{\mathcal{U}_2} = \mu^i_{\mathcal{A}}(t).\mu^i_{\mathcal{B}}(t)$, $v^i_{\mathcal{U}_2} = v^i_{\mathcal{A}}(t) + v^i_{\mathcal{B}}(t) - v^i_{\mathcal{A}}(t).v^i_{\mathcal{B}}(t)$, $w^i_{\mathcal{U}_2} = w^i_{\mathcal{A}}(t) + w^i_{\mathcal{B}}(t) w^i_{\mathcal{A}}(t).w^i_{\mathcal{B}}(t)$ $\forall\, t \in T$ and $i = 1,2, \dots P$.

Here $\vee$, $\wedge, +, ., -$ denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.

Definition 5 [13] Let

$$\mathcal{A} = \{\prec t, \left(\mu^1_{\mathcal{A}}(t), \mu^2_{\mathcal{A}}(t), \dots \mu^p_{\mathcal{A}}(t)\right), \left(v^1_{\mathcal{A}}(t), v^2_{\mathcal{A}}(t), \dots v^p_{\mathcal{A}}(t)\right), \left(w^1_{\mathcal{A}}(t), w^2_{\mathcal{A}}(t), \dots w^p_{\mathcal{A}}(t)\right) \succ : t \in T\}$$

and

$$\mathcal{B} = \{\prec t, \left(\mu^1_{\mathcal{B}}(t), \mu^2_{\mathcal{B}}(t), \dots \mu^p_{\mathcal{B}}(t)\right), \left(v^1_{\mathcal{B}}(t), v^2_{\mathcal{B}}(t), \dots v^p_{\mathcal{B}}(t)\right), \left(w^1_{\mathcal{A}}(t), w^2_{\mathcal{A}}(t), \dots w^p_{\mathcal{A}}(t)\right) \succ : t \in T\}$$

and be two NMSs, then the normalized hamming distance between $\mathcal{A}$ and $\mathcal{B}$ can be defined as follows:

$$d_{NHD}(\mathcal{A}, \mathcal{B}\ ) = \frac{1}{3n.P} \sum_{j=1}^{P} \sum_{i=1}^{n} \left(\left|\mu^j_{\mathcal{A}}(t_i) - \mu^j_{\mathcal{B}}(t_i)\right| + \left|v^j_{\mathcal{A}}(t_i) - v^j_{\mathcal{B}}(t_i)\right| + \left|w^j_{\mathcal{A}}(t_i) - w^j_{\mathcal{B}}(t_i)\right| \right).$$

3. The Outranking Relations of Neutrosophic Multi-Sets

In this section, the binary relations between two neutrosophic refined sets that are based on ELECTRE by extending the studies in [22]. Some of it is quoted from [13, 22, 35, 49].

Definition 6 *Let* $\mathcal{A} = \{\prec t, \left(\mu^i_{\mathcal{A}}(t), v^i_{\mathcal{A}}(t), w^i_{\mathcal{A}}(t)\right) \succ : t \in T, (i = 1,2,3, \dots, p)\}$ *and* $\mathcal{B} = \{\prec t, \left(\mu^i_{\mathcal{B}}(t), v^i_{\mathcal{B}}(t), w^i_{\mathcal{B}}(t)\right) \succ : t \in T, (i = 1,2,3, \dots, p)\}$ *be two NMS on* T. *Then, the strong dominance relation, weak dominance relation, and indifference relation of NMS can be defined as follows:*

1. If $\mu_{\mathcal{A}}^i(t) \geq \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) < v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) < w_{\mathcal{B}}^i(t)$ or $\mu_{\mathcal{A}}^i(t) > \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) = v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) = w_{\mathcal{B}}^i(t), \forall t \in T$ and $i = 1,2,3, \ldots, p$. Then $\mathcal{A}$ strongly dominates $\mathcal{B}$ ($\mathcal{B}$ is strongly dominated by $\mathcal{A}$), denoted by $\mathcal{A} \succ_s \mathcal{B}$.
2. If $\mu_{\mathcal{A}}^i(t) \geq \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) \geq v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) < w_{\mathcal{B}}^i(t)$ or $\mu_{\mathcal{A}}^i(t) \geq \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) < v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) \geq w_{\mathcal{B}}^i(t), \forall t \in T$ and $i = 1,2,3, \ldots, p$. Then $\mathcal{A}$ weakly dominates $\mathcal{B}$ ($\mathcal{B}$ is weakly dominated by $\mathcal{A}$), denoted by$\mathcal{A} \succ_w \mathcal{B}$.
3. If $\mu_{\mathcal{A}}^i(t) = \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) = v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) = w_{\mathcal{B}}^i(t), \ \forall t \in T$ and $i = 1,2,3, \ldots, p$. Then $\mathcal{A}$ indifferent to $\mathcal{B}$, denoted by$\mathcal{A} \sim_l \mathcal{B}$.
4. If none of the relations mentioned above exist between $\mathcal{A}$ and $\mathcal{B}$ for any $t \in T$, then $\mathcal{A}$ and $\mathcal{B}$ are incomparable, denoted by $\mathcal{A} \perp \mathcal{B}$.

Proposition 7 *Let* $\mathcal{A} = \{< t, \left(\mu_{\mathcal{A}}^i(t), v_{\mathcal{A}}^i(t), w_{\mathcal{A}}^i(t)\right) >: t \in T, (i = 1,2,3, \ldots, p)\}$ *and* $\mathcal{B} = \{< t, \left(\mu_{\mathcal{B}}^i(t), v_{\mathcal{B}}^i(t), w_{\mathcal{B}}^i(t)\right) >: t \in T, (i = 1,2,3, \ldots, p)\}$ be two NMS on T, then the following properties can be obtained:

1. $If\ \mathcal{B} \subset \mathcal{A}, then\ \mathcal{A} \succ_s \mathcal{B};$
2. $If\ \mathcal{A} \succ_s \mathcal{B}, then\ If\ \mathcal{B} \subseteq \mathcal{A};$
3. $\mathcal{A} \sim_l \mathcal{B}\ if\ and\ only\ if\ \mathcal{A} = \mathcal{B}.$

Proof:
1. $If\ \mathcal{B} \subset \mathcal{A}$, then $\mu_{\mathcal{B}}^i(t) \leq \mu_{\mathcal{A}}^i(t), v_{\mathcal{B}}^i(t) \geq v_{\mathcal{A}}^i(t), w_{\mathcal{B}}^i(t) \geq w_{\mathcal{A}}^i(t), \forall t \in T$ and $i = 1,2,3, \ldots, p$. $\mathcal{A} \succ_s \mathcal{B}$ is definitely validated according to the strong dominance relation in Definition 6.

2. $\mathcal{A} \succ_s \mathcal{B}$ then based on Definition 6, $\mu_{\mathcal{A}}^i(t) \geq \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) < v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) < w_{\mathcal{B}}^i(t)$ *or* $\mu_{\mathcal{A}}^i(t) > \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) = v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) = w_{\mathcal{B}}^i(t), \forall t \in T$ *and* $i = 1,2,3, \ldots, p$. are realized. Then we have $\mathcal{B} \subseteq \mathcal{A}$.

3. *Necessity*: $\mathcal{A} \sim_l \mathcal{B} \Rightarrow \mathcal{A} = \mathcal{B}$. According to the indifference relation in Definition 6 it is known that $\mu_{\mathcal{A}}^i(t) = \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) = v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) = w_{\mathcal{B}}^i(t), \ \forall t \in T$ and $i = 1,2,3, \ldots, p$. Clearly $\mathcal{A} \subseteq \mathcal{A}$ and $\mathcal{B} \subseteq \mathcal{A}$ are achieved, then $\mathcal{A} = \mathcal{B}$.

Sufficiency: $\mathcal{A} = \mathcal{B} \Rightarrow \mathcal{A} \sim_l \mathcal{B}$. If $\mathcal{A} = \mathcal{B}$, then it is know that $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A}$, which means

$\mu_{\mathcal{B}}^i(t) \leq \mu_{\mathcal{A}}^i(t), v_{\mathcal{B}}^i(t) \geq v_{\mathcal{A}}^i(t), w_{\mathcal{B}}^i(t) \geq w_{\mathcal{A}}^i(t)\ or\ \mu_{\mathcal{A}}^i(t) = \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) = v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) = w_{\mathcal{B}}^i(t), \ \forall t \in T$ and $i = 1,2,3, \ldots, p$. are obtained. Due to the indifference relation in Definition 6, $\mathcal{A} \sim_l \mathcal{B}$ is definitely obtained.

Proposition 8 *Let* $\mathcal{A} = \{< t, \left(\mu_{\mathcal{A}}^i(t), v_{\mathcal{A}}^i(t), w_{\mathcal{A}}^i(t)\right) >: t \in T, (i = 1,2,3, \ldots, p)\}$, $\mathcal{B} = \{< t, \left(\mu_{\mathcal{B}}^i(t), v_{\mathcal{B}}^i(t), w_{\mathcal{B}}^i(t)\right) >: t \in T, (i = 1,2,3, \ldots, p)\}$ and $C = \{< t, \left(\mu_C^i(t), v_C^i(t), w_C^i(t)\right) >: t \in T, (i = 1,2,3, \ldots, p)\}$ be three NMS on T, if $\mathcal{A} \succ_s \mathcal{B}\ and\ \mathcal{B} \succ_s C$, then $\mathcal{A} \succ_s C$.

Proof: According to the strong dominance relation in Definition 6, if $\mathcal{A} \succ_s \mathcal{B}$, then $\mu_{\mathcal{A}}^i(t) \geq \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) < v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) < w_{\mathcal{B}}^i(t)$ or $\mu_{\mathcal{A}}^i(t) > \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) = v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) = w_{\mathcal{B}}^i(t), \forall t \in T$ and $i = 1,2,3, \ldots, p$.

if $\mathcal{B} \succ_s C$, then $\mu_{\mathcal{B}}^i(t) \geq \mu_C^i(t), v_{\mathcal{B}}^i(t) < v_C^i(t), w_{\mathcal{B}}^i(t) < w_C^i(t)$ *or* $\mu_{\mathcal{B}}^i(t) > \mu_C^i(t), v_{\mathcal{B}}^i(t) = v_C^i(t), w_{\mathcal{B}}^i(t) = w_C^i(t), \forall t \in T$ and $i = 1,2,3, \ldots, p$.

Therefore the further derivations are: If

$$\mu_{\mathcal{A}}^i(t) \geq \mu_{\mathcal{B}}^i(t), v_{\mathcal{A}}^i(t) < v_{\mathcal{B}}^i(t), w_{\mathcal{A}}^i(t) < w_{\mathcal{B}}^i(t), \ \ldots..(1)$$

$$\mu_{\mathcal{B}}^{i}(t) \geq \mu_{C}^{i}(t),\ v_{\mathcal{B}}^{i}(t) < v_{C}^{i}(t),\ w_{\mathcal{B}}^{i}(t) < w_{C}^{i}(t), \ldots\ (2)$$

from (1) and (2)

$$\mu_{\mathcal{A}}^{i}(t) \geq \mu_{C}^{i}(t), v_{\mathcal{A}}^{i}(t) < v_{C}^{i}(t), w_{\mathcal{A}}^{i}(t) < w_{C}^{i}(t),$$

then based on Definition 6 $\mathcal{A} \succ_s C$ is realized. If

$$\mu_{\mathcal{A}}^{i}(t) \geq \mu_{\mathcal{B}}^{i}(t), v_{\mathcal{A}}^{i}(t) < v_{\mathcal{B}}^{i}(t), w_{\mathcal{A}}^{i}(t) < w_{\mathcal{B}}^{i}(t),\ \ldots (3)$$
$$\mu_{\mathcal{B}}^{i}(t) > \mu_{C}^{i}(t),\ v_{\mathcal{B}}^{i}(t) = v_{C}^{i}(t),\ w_{\mathcal{B}}^{i}(t) = w_{C}^{i}(t), \ldots\ (4)$$

from (3) and (4)

$$\mu_{\mathcal{A}}^{i}(t) \geq \mu_{C}^{i}(t), v_{\mathcal{A}}^{i}(t) < v_{C}^{i}(t), w_{\mathcal{A}}^{i}(t) < w_{C}^{i}(t),$$

then based on Definition 6 $\mathcal{A} \succ_s C$ is achieved. If

$$\mu_{\mathcal{A}}^{i}(t) > \mu_{\mathcal{B}}^{i}(t), v_{\mathcal{A}}^{i}(t) = v_{\mathcal{B}}^{i}(t), w_{\mathcal{A}}^{i}(t) = w_{\mathcal{B}}^{i}(t),\ \ldots (5)$$
$$\mu_{\mathcal{B}}^{i}(t) \geq \mu_{C}^{i}(t),\ v_{\mathcal{B}}^{i}(t) < v_{C}^{i}(t),\ w_{\mathcal{B}}^{i}(t) < w_{C}^{i}(t), \ldots\ (6)$$

from (5) and (6)

$$\mu_{\mathcal{A}}^{i}(t) > \mu_{C}^{i}(t), v_{\mathcal{A}}^{i}(t) = v_{C}^{i}(t), w_{\mathcal{A}}^{i}(t) = w_{C}^{i}(t),$$

then based on Definition 6 $\mathcal{A} \succ_s C$ is obtained. If

$$\mu_{\mathcal{A}}^{i}(t) > \mu_{\mathcal{B}}^{i}(t), v_{\mathcal{A}}^{i}(t) = v_{\mathcal{B}}^{i}(t), w_{\mathcal{A}}^{i}(t) = w_{\mathcal{B}}^{i}(t),\ \ldots (7)$$
$$\mu_{\mathcal{B}}^{i}(t) > \mu_{C}^{i}(t),\ v_{\mathcal{B}}^{i}(t) = v_{C}^{i}(t),\ w_{\mathcal{B}}^{i}(t) = w_{C}^{i}(t), \ldots (8)$$

from (7) and (8)

$$\mu_{\mathcal{A}}^{i}(t) > \mu_{C}^{i}(t), v_{\mathcal{A}}^{i}(t) = v_{C}^{i}(t), w_{\mathcal{A}}^{i}(t) = w_{C}^{i}(t),$$

then based on Definition 6 $\mathcal{A} \succ_s C$ is realized. Therefore, if $\mathcal{A} \succ_s \mathcal{B}\ and\ \mathcal{B} \succ_s C$, then $\mathcal{A} \succ_s C$.

Proposition 9 *Let* $\mathcal{A} = \{\prec t, \left(\mu_{\mathcal{A}}^{i}(t), v_{\mathcal{A}}^{i}(t), w_{\mathcal{A}}^{i}(t)\right) \succ : t \in T, (i = 1,2,3, \ldots, p)\}$, $\mathcal{B} = \{\prec t, \left(\mu_{\mathcal{B}}^{i}(t), v_{\mathcal{B}}^{i}(t), w_{\mathcal{B}}^{i}(t)\right) \succ : t \in T, (i = 1,2,3, \ldots, p)\}$ and $C = \{\prec t, \left(\mu_{C}^{i}(t), v_{C}^{i}(t), w_{C}^{i}(t)\right) \succ : t \in T, (i = 1,2,3, \ldots, p)\}$ be three NMS on T, if $\mathcal{A} \sim_l \mathcal{B}\ and\ \mathcal{B} \sim_l C$, then $\mathcal{A} \sim_l C$.

Proof: Clearly, if $\mathcal{A} \sim_l \mathcal{B}\ and\ \mathcal{B} \sim_l C$, then $\mathcal{A} \sim_l C$ is surely validated.

Proposition 10 *Let* $\mathcal{A} = \{\prec t, \left(\mu_{\mathcal{A}}^{i}(t), v_{\mathcal{A}}^{i}(t), w_{\mathcal{A}}^{i}(t)\right) \succ : t \in T, (i = 1,2,3, \ldots, p)\}$, $\mathcal{B} = \{\prec t, \left(\mu_{\mathcal{B}}^{i}(t), v_{\mathcal{B}}^{i}(t), w_{\mathcal{B}}^{i}(t)\right) \succ : t \in T, (i = 1,2,3, \ldots, p)\}$ and $C = \{\prec t, \left(\mu_{C}^{i}(t), v_{C}^{i}(t), w_{C}^{i}(t)\right) \succ : t \in T, (i = 1,2,3, \ldots, p)\}$ be three NMS on $T = \{t_1, t_2, \ldots, t_n\}$, then the following results can be obtained.

1. $1 - irreflexivity:\ \forall \mathcal{A} \in NMSs, \mathcal{A} \nsucc_s \mathcal{A};$
   $2 - asymmetry: \forall \mathcal{A}, \mathcal{B}\ on\ NMSs; \mathcal{A} \succ_s \mathcal{B} \Rightarrow \mathcal{B} \nsucc_s \mathcal{A};$
   $3 - transitivity: \forall \mathcal{A}, \mathcal{B}, C\ on\ NMSs; \mathcal{A} \succ_s \mathcal{B}, \mathcal{B} \succ_s C, then\ \mathcal{A} \succ C.$

2. $4 - irreflexivity:\ \forall \mathcal{A} \in NMSs, \mathcal{A} \nsucc_w \mathcal{A};$
   $5 - asymmetry: \forall \mathcal{A}, \mathcal{B}\ on\ NMSs; \mathcal{A} \succ_w \mathcal{B} \Rightarrow \mathcal{B} \nsucc_w \mathcal{A};$
   $6 - non - transitivity: \exists \mathcal{A}, \mathcal{B}, C\ on\ NMSs; \mathcal{A} \succ_s \mathcal{B}, \mathcal{B} \succ_s C, then\ \mathcal{A} \succ C.$

3. $7 - reflexivity:\ \forall \mathcal{A} \in NMSs, \mathcal{A} \sim_l \mathcal{A};$
   $8 - symmetry: \forall \mathcal{A}, \mathcal{B}\ on\ NMSs; \mathcal{A} \sim_l \mathcal{B} \Rightarrow \mathcal{B} \sim_l \mathcal{A};$
   $9 - transitivity: \exists \mathcal{A}, \mathcal{B}, C\ on\ NMSs; \mathcal{A} \sim_l \mathcal{B}, \mathcal{B} \sim_l C, then\ \mathcal{A} \sim_l C.$

Example 11 1,2,4,5 and 6 are exemplified as follows.

1. If $\mathcal{A} = \langle (0.8, 0.5, \ldots, 0.6), (0.3, 0.1, \ldots, 0.5), (0.2, 0.3, \ldots, 0.4) \rangle$ is a NMSs, then $\mathcal{A} \nsucc_s \mathcal{A}$ can be obtained.
2. If $\mathcal{A} = \langle (0.5, 0.7, \ldots, 0.6), (0.2, 0.3, \ldots, 0.4), (0.1, 0.3, \ldots, 0.2) \rangle$ and $\mathcal{B} = \langle (0.4, 0.6, \ldots, 0.5), (0.3, 0.4, \ldots, 0.5), (0.2, 0.5, \ldots, 0.3) \rangle$ are two NMSs, then $\mathcal{A} \succ_s \mathcal{B}, but\ \mathcal{B} \nsucc_s \mathcal{A}$ is realized.

3. If $\mathcal{A} = \langle(0.7,0.4, \dots ,0.5), (0.4,0.2, \dots ,0.6), (0.3,0.3, \dots ,0.2)\rangle$ is a NMSs, then $\mathcal{A} \nsucc_w \mathcal{A}$ can be obtained.
4. If $\mathcal{A} = \langle(0.5,0.7, \dots ,0.6), (0.5,0.6, \dots ,0.4), (0.1,0.3, \dots ,0.2)\rangle$ and $\mathcal{B} = \langle(0.3,0.5, \dots ,0.6), (0.2,0.3, \dots ,0.1), (0.2,0.5, \dots ,0.3)\rangle$ are two NMSs, then $\mathcal{A} \succ_w \mathcal{B}, however\ \mathcal{B} \nsucc_w \mathcal{A}.$
5. If $\mathcal{A} = \langle(0.5,0.7, \dots ,0.6), (0.3,0.2, \dots ,0.4), (0.1,0.3, \dots ,0.2)\rangle,$
6. $\mathcal{B} = \langle(0.5,0.6, \dots ,0.4), (0.5,0.4, \dots ,0.6), (0.2,0.5, \dots ,0.3)\rangle$ and $C = \langle(0.4,0.3, \dots ,0.2), (0.6,0.5, \dots ,0.7), (0.3,0.6, \dots ,0.8)\rangle$ are three NMSs, then $\mathcal{A} \succ_w \mathcal{B}\ and\ \mathcal{B} \succ_w C$ are obtained, $\mathcal{A} \succ_w C$.

Proposition 11 [22] Let t_1 and t_2 be two actions, the performances for actions t_1 and t_2 be in the form of NMSs, and $P = s \cup w \cup l$ mean that "t_1 is at least as good as t_2", then four situations may arise:

1. t_1Pt_2 and not t_2Pt_1, that is $t_1 \succ_s t_2$ or $t_1 \succ_w t_2$;
2. t_2Pt_1 and not t_1Pt_2, that is $t_2 \succ_s t_1$ or $t_2 \succ_w t_1$;
3. $t_1Pt_2\ and\ t_2Pt_1$, that is $t_1 \sim_l t_2$;
4. not t_1Pt_2 and not t_2Pt_1, that is $t_1 \perp t_2$.

4. An outranking approach for MCDM with simplified neutrosophic multi-set information

In this section, we introduced an approach for a MCDM problem with neutrosophic multi-set information. Some of it is quoted from [22, 35, 49].

Definition 12 [15] Let $X = (x_1, x_2, \dots , x_n)$ be a set of alternatives, $C = (c_1, c_2, \dots , c_n)$ be the set of criteria, $w = (w_1, w, \dots , w_n)^T$ be the weight vector of the criterions $C_j (j = 1,2, \dots , n)$ such that $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$ and $Z_{ij} = \langle(\mu_{ij}^1 \mu_{ij}^2, \dots , \mu_{ij}^n), (v_{ij}^1 v_{ij}^2, \dots , v_{ij}^n), (w_{ij}^1 w_{ij}^2, \dots , w_{ij}^n)\rangle$ be the decision matrix in which the rating values of the alternatives in for NMSs. Then,

$$\left[Z_{ij}\right]_{m\times n} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{pmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mn} \end{pmatrix} \end{array}$$

is called an NMS-multi-criteria decision making matrix of the decision maker.

Definition 13 [22, 35] In multi-criteria decision making problems;

1. The cost-type criterion values can be transformed into benefit-type criterion values as follows:

$$\alpha_{ij} = \begin{cases} Z_{ij} & \text{for benefit criterion } C_j, \\ (Z_{ij})^c & \text{for benefit criterion } C_j, \qquad (i = 1,2, \dots , m; j = 1,2, \dots , n) \end{cases} \tag{9}$$

where $(Z_{ij})^c$ is complement of Z_{ij} as defined in Definition 4.

2. The concordance set of subscripts, which should satisfy the constraint $Z_{ij}PZ_{kj}$, is represented as:

$$O_{ik} = \{j:\ Z_{ij}PZ_{kj}\}\ (i, k = 1,2, \dots , m).$$

$Z_{ij}PZ_{kj}$ represents $Z_{ij} >_s Z_{kj}$ or $Z_{ij} >_w Z_{kj}$ or $Z_{ij} \backsim Z_{kj}$.

3. The concordance index h_{ik} between x_i and x_k is thus defined as follows:

$$h_{ik} = \sum_{j \in O_{ik}} w_j \tag{10}$$

Thus, the concordance matrix C is:

$$H = h_{ik} = \begin{pmatrix} - & h_{12} & \cdots & h_{1n} \\ h_{21} & - & \cdots & h_{2n} \\ \vdots & & \ddots & \vdots \\ h_{n1} & h_{n2} & \cdots & - \end{pmatrix}$$

In *H*; h_{ik} $(i \neq k)$ denote the degree to which the evaluations of x_i are at least as good as those of the competitorx_k, and the degree to which x_i is inferior to x_k decreases with increasing h_{ik}.

4. The discordance set of subscripts for criteria is given as;

$$G_{ik} = J - O_{ik}.$$

5. The discordance index $G(x_i\,;\,x_k)$ is represented as:

$$G_{ik} = \frac{\max_{j\in G_{ik}}\{d(Z_{ij}, Z_{kj})\}}{\max_{j\in J}\{d(Z_{ij}, Z_{kj})\}} \tag{11}$$

here $d(Z_{ij}, Z_{kj})$ denotes the normalized Hamming distance between Z_{ij} and Z_{kj} as defined in Definition 5.

Thus, the discordance matrix *D* is:

$$g_{ik} = \begin{pmatrix} - & g_{12} & \cdots & g_{1n} \\ g_{21} & - & \cdots & g_{2n} \\ \vdots & & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & - \end{pmatrix}$$

In *G*; g_{ik} $(i \neq k)$ denote the degree to which the evaluations of x_i are at least as good as those of the competitorx_k, and the degree to which x_i is inferior to x_k decreases with increasing g_{ik}.

6. To rank all alternatives, the net dominance index of x_k

$$h_{ik} = \sum_{i=1, i\neq k}^{n} h_{ik} - \sum_{i=1, i\neq k}^{n} h_{ki} \tag{12}$$

and the net disadvantage index of x_k is

$$g_{ik} = \sum_{i=1, i\neq k}^{n} g_{ik} - \sum_{i=1, i\neq k}^{n} g_{ki} \tag{13}$$

In here, h_k is the sum of the concordance indices between x_k and x_k $(i \neq k)$ minus the sum of the concordance indices between x_k $(i \neq k)$ and x_k, and reflects the dominance degree of the alternative x_k among the relevant alternatives. Meanwhile, g_k reflects the disadvantage degree of the alternative x_k among the relevant alternatives. Therefore, x_k obtains a greater dominance over the other alternatives that are being compared as h_k increases and g_k decreases.

Definition 14 [35] The ranking rules of two alternatives are

i. If $h_i < h_k$ and $g_i > g_k$ then x_k is superior to x_i, as denoted by $x_k \succ x_i$;

ii. If $h_i = h_k$ and $g_i = g_k$ then x_k is indifferent to x_i, as denoted by $x_k \sim x_i$;

i. if the relation between x_k and x_i does not belong to (i) or (ii);then x_k and x_i are incomparable; as denoted by $x_k \perp x_i$.

Now, we give an algorithm to develop a new approach as

Algorithm:

Step 1 Give the decision-making matrix $\left[Z_{ij}\right]_{m\times n}$; for decision;

Step 2 Compute the weighted normalized matrix as;

$$\left[\gamma_{ij}\right]_{m\times n} = \alpha_{ij} w_j \qquad i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n.$$

where w_j is the weight of the j th criterion with $\sum_{j=1}^{n} w_j = 1$.

Step 3 Find the concordance set of subscripts;

Step 4 Find the discordance set of subscripts;

Step 5 Compute the concordance matrix $H = (h_{ik})_{n\times n}$

Step 6 Compute the discordance matrix $G = (g_{ik})_{n\times n}$

Step 7. Compute the net dominance index of each alternative h_i (i=1,2,3,...,m)

Step 8. Compute the net disadvantage index of each alternative g_i (i=1,2,...,m)

Step 9. Rank all alternatives and select the best alternative.

5 Illustrative examples

In this section, we introduced an example for a MCDM problem with neutrosophic refined information. Some of it is quoted from [22, 35, 49].

Example 15 Assume that $X = (x_1, x_2, x_3, x_4)$ be a set of alternatives and $C = (c_1, c_2, c_3, c_4)$ be the set of criterions, $w = (0.1,0.3,0.2,0.4)^T$ be the weight vector of the criterions $C_j (j = 1,2, \dots, n)$. The four alternatives are to be evaluated under the above four criteria in the form of NMSs. Then,

Step 1. The decision matrix $\left[Z_{ij}\right]_{m\times n}$ is given as;

$$\begin{pmatrix}
\langle(0{:}1;\ 0{:}2;\ 0{:}4;\ 0{:}5);\ (0{:}6;\ 0{:}3;\ 0{:}5;\ 0{:}2);\ (0{:}2;\ 0{:}4;\ 0{:}5;\ 0{:}6)\rangle & \langle(0{:}3;\ 0{:}5;\ 0{:}7;\ 0{:}8);\ (0{:}4;\ 0{:}3;\ 0{:}6;\ 0{:}2);\ (0{:}1;\ 0{:}3;\ 0{:}5;\ 0{:}2)\rangle & \langle(0{:}2;\ 0{:}4;\ 0{:}5;\ 0{:}6);\ (0{:}3;\ 0{:}5;\ 0{:}2;\ 0{:}6);\ (0{:}1;\ 0{:}2;\ 0{:}5;\ 0{:}6)\rangle & \langle(0{:}1;\ 0{:}2;\ 0{:}4;\ 0{:}5);\ (0{:}2;\ 0{:}3;\ 0{:}5;\ 0{:}4);\ (0{:}1;\ 0{:}3;\ 0{:}7;\ 0{:}4)\rangle \\
\langle(0{:}3;\ 0{:}4;\ 0{:}6;\ 0{:}7);\ (0{:}2;\ 0{:}5;\ 0{:}1;\ 0{:}8);\ (0{:}3;\ 0{:}4;\ 0{:}6;\ 0{:}8)\rangle & \langle(0{:}2;\ 0{:}3;\ 0{:}4;\ 0{:}5);\ (0{:}1;\ 0{:}4;\ 0{:}3;\ 0{:}6);\ (0{:}2;\ 0{:}3;\ 0{:}4;\ 0{:}5)\rangle & \langle(0{:}4;\ 0{:}5;\ 0{:}7;\ 0{:}8);\ (0{:}1;\ 0{:}6;\ 0{:}2;\ 0{:}3);\ (0{:}1;\ 0{:}4;\ 0{:}3;\ 0{:}6)\rangle & \langle(0{:}3;\ 0{:}4;\ 0{:}5;\ 0{:}6);\ (0{:}3;\ 0{:}1;\ 0{:}2;\ 0{:}5);\ (0{:}3;\ 0{:}6;\ 0{:}8;\ 0{:}9)\rangle \\
\langle(0{:}1;\ 0{:}2;\ 0{:}5;\ 0{:}6);\ (0{:}1;\ 0{:}3;\ 0{:}5;\ 0{:}2);\ (0{:}1;\ 0{:}5;\ 0{:}7;\ 0{:}9)\rangle & \langle(0{:}1;\ 0{:}2;\ 0{:}6;\ 0{:}7);\ (0{:}3;\ 0{:}2;\ 0{:}5;\ 0{:}4);\ (0{:}1;\ 0{:}2;\ 0{:}5;\ 0{:}6)\rangle & \langle(0{:}3;\ 0{:}6;\ 0{:}8;\ 0{:}9);\ (0{:}2;\ 0{:}4;\ 0{:}1;\ 0{:}5);\ (0{:}2;\ 0{:}1;\ 0{:}3;\ 0{:}6)\rangle & \langle(0{:}1;\ 0{:}3;\ 0{:}4;\ 0{:}5);\ (0{:}1;\ 0{:}4;\ 0{:}6;\ 0{:}7);\ (0{:}1;\ 0{:}2;\ 0{:}6;\ 0{:}7)\rangle \\
\langle(0{:}2;\ 0{:}3;\ 0{:}4;\ 0{:}5);\ (0{:}3;\ 0{:}2;\ 0{:}4;\ 0{:}6);\ (0{:}2;\ 0{:}3;\ 0{:}5;\ 0{:}7)\rangle & \langle(0{:}3;\ 0{:}4;\ 0{:}6;\ 0{:}8);\ (0{:}2;\ 0{:}1;\ 0{:}3;\ 0{:}6);\ (0{:}4;\ 0{:}3;\ 0{:}2;\ 0{:}5)\rangle & \langle(0{:}1;\ 0{:}2;\ 0{:}4;\ 0{:}6);\ (0{:}1;\ 0{:}3;\ 0{:}7;\ 0{:}4);\ (0{:}3;\ 0{:}4;\ 0{:}6;\ 0{:}7)\rangle & \langle(0{:}2;\ 0{:}4;\ 0{:}5;\ 0{:}7);\ (0{:}2;\ 0{:}3;\ 0{:}5;\ 0{:}6);\ (0{:}3;\ 0{:}2;\ 0{:}4;\ 0{:}6)\rangle
\end{pmatrix}$$

Step 2. The weighted normalized matrix $\left\lfloor \gamma_{ij} \right\rfloor_{m\times n}$ is computed as;

$$\begin{pmatrix}
(0{:}7943;\ 0{:}8513;\ 0{:}9124;\ 0{:}9330);\ (0{:}0875;\ 0{:}0350;\ 0{:}0669;\ 0{:}0220);\ (0{:}0220;\ 0{:}0104;\ 0{:}0669;\ 0{:}0875) & (0{:}8865;\ 0{:}9330;\ 0{:}9649;\ 0{:}9779);\ (0{:}0498;\ 0{:}0350;\ 0{:}0875;\ 0{:}0620);\ (0{:}0104;\ 0{:}0350;\ 0{:}0669;\ 0{:}0220) & (0{:}8513;\ 0{:}9124;\ 0{:}9330;\ 0{:}9502);\ (0{:}0350;\ 0{:}0669;\ 0{:}0720;\ 0{:}0875);\ (0{:}0104;\ 0{:}0220;\ 0{:}0669;\ 0{:}0875) \\
(0{:}6968;\ 0{:}7596;\ 0{:}8579;\ 0{:}8985);\ (0{:}0647;\ 0{:}1877;\ 0{:}0311;\ 0{:}3829);\ (0{:}1014;\ 0{:}1420;\ 0{:}2403;\ 0{:}3829) & (0{:}6170;\ 0{:}6968;\ 0{:}7596;\ 0{:}8122);\ (0{:}0311;\ 0{:}1420;\ 0{:}1014;\ 0{:}2403);\ (0{:}0647;\ 0{:}1014;\ 0{:}1420;\ 0{:}1877) & (0{:}7596;\ 0{:}8122;\ 0{:}8985;\ 0{:}9352);\ (0{:}0311;\ 0{:}0203;\ 0{:}0647;\ 0{:}1014);\ (0{:}0311;\ 0{:}1420;\ 0{:}1014;\ 0{:}2403) \\
(0{:}6309;\ 0{:}7247;\ 0{:}8705;\ 0{:}9028);\ (0{:}2080;\ 0{:}0688;\ 0{:}1294;\ 0{:}0436);\ (0{:}2080;\ 0{:}1294;\ 0{:}2140;\ 0{:}3690) & (0{:}6309;\ 0{:}7247;\ 0{:}9028;\ 0{:}9311);\ (0{:}0188;\ 0{:}0436;\ 0{:}1294;\ 0{:}0971);\ (0{:}0208;\ 0{:}0436;\ 0{:}1294;\ 0{:}1674) & (0{:}7860;\ 0{:}9028;\ 0{:}9563;\ 0{:}9791);\ (0{:}0436;\ 0{:}0971;\ 0{:}0208;\ 0{:}1294);\ (0{:}0436;\ 0{:}0208;\ 0{:}0688;\ 0{:}1674) \\
(0{:}5253;\ 0{:}6178;\ 0{:}6931;\ 0{:}7578);\ (0{:}1329;\ 0{:}0853;\ 0{:}1848;\ 0{:}3068);\ (0{:}0853;\ 0{:}1329;\ 0{:}2421;\ 0{:}3822) & (0{:}6178;\ 0{:}6931;\ 0{:}8151;\ 0{:}9146);\ (0{:}0853;\ 0{:}0412;\ 0{:}1329;\ 0{:}3068);\ (0{:}1848;\ 0{:}1329;\ 0{:}0853;\ 0{:}2421) & (0{:}3981;\ 0{:}5253;\ 0{:}6931;\ 0{:}8151);\ (0{:}0412;\ 0{:}1329;\ 0{:}3822;\ 0{:}1848);\ (0{:}0412;\ 0{:}1329;\ 0{:}3822;\ 0{:}6018)
\end{pmatrix}$$

$$\left.\begin{matrix}(0{:}7943;\ 0{:}8513;\ 0{:}9124;\ 0{:}9330);\ (0{:}0220;\ 0{:}0350;\ 0{:}0669;\ 0{:}0498);\ (0{:}0104;\ 0{:}0350;\ 0{:}1134;\ 0{:}0498)\\ (0{:}6968;\ 0{:}7596;\ 0{:}8122;\ 0{:}8579);\ (0{:}1014;\ 0{:}0311;\ 0{:}0647;\ 0{:}1877);\ (0{:}1014;\ 0{:}2403;\ 0{:}2403;\ 0{:}4988)\\ (0{:}6309;\ 0{:}7860;\ 0{:}8325;\ 0{:}8705);\ (0{:}0228;\ 0{:}0971;\ 0{:}1674;\ 0{:}2140);\ (0{:}0208;\ 0{:}0436;\ 0{:}1674;\ 0{:}2140)\\ (0{:}5253;\ 0{:}6931;\ 0{:}7578;\ 0{:}8670);\ (0{:}1853;\ 0{:}1329;\ 0{:}2421;\ 0{:}3068);\ (0{:}0329;\ 0{:}0853;\ 0{:}1848;\ 0{:}3068)\end{matrix}\right)$$

Step 3. The concordance set is found as;

$$O_{12}=\{\ \};O_{21}=\{4\};O_{31}=\{\ \};O_{41}=\{\ \};O_{13}=\{1,2\};O_{23}=\{\ \};$$
$$O_{32}=\{\ \};O_{42}=\{\ \};O_{14}=\{4\};O_{24}=\{1,3\};O_{34}=\{1,2\};O_{43}=\{\ \}.$$

Step 4. The discordance set is found as;

$$G_{12}=\{1,2,3,4\};G_{21}=\{1,2,3\};G_{31}=\{1,2,3,4\};G_{41}=\{1,2,3,4\};O_{13}=\{1,2\};G_{23}=\{1,2,3,4\};$$
$$G_{32}=\{1,2,3,4\};G_{42}=\{1,2,3,4\};G_{14}=\{1,2,3\};G_{24}=\{2,4\};G_{34}=\{3,4\};G_{43}=\{1,2,3,4\}.$$

where $\{\ \}$ denotes "empty".

Step 5. The concordance is computed as;

$$H=\begin{pmatrix} - & 0 & 0.4 & 0.4\\ 0.4 & - & 0.4 & 0.3\\ 0 & 0 & - & 0.4\\ 0 & 0 & 0 & - \end{pmatrix}$$

Step 6. The discordance matrix is computed as;

$$G=\begin{pmatrix} - & 1 & 0.6612 & 1\\ 0.9958 & - & 1 & 0.5778\\ 1 & 1 & - & 1\\ 1 & 1 & 1 & - \end{pmatrix}$$

Step 7. The net dominance index of each alternative h_i (i=1,2,3,4) is computed as;

$$h_1=0.4, h_2=1.1, h_3=-0.4 \text{ and } h_4=-1.1, \Rightarrow h_4<h_3<h_1<h_2;$$

Step 8. The net disadvantage index of each alternative g_i (i=1,2,3,4) is computed as;

$$g_1=-0.3346, g_2=-0.428, g_3=0.3388 \text{ and } g_4=0.4242, \Rightarrow g_4>g_3>g_1>g_2.$$

Step 9. The final ranking is and the best alte $x_2 \succ x_1 \succ x_3 \succ x_4$ rnative is x_2.

6. Conclusions

This paper developed a multi-criteria decision making method for neutrosophic multi-sets based on these given the outranking relations. In further research, we will develop different methods and compare the different methods on neutrosophic multi-sets. The contribution of this study is that the proposed approach is simple and convenient with regard to computing, and effective in decreasing the loss of evaluative information. More effective decision methods of this proposes a new outranking approach will be investigated in the near future and applied these concepts to engineering, game theory, multi-agent systems, decision-making and so on.

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Neutrosophic Approach on Normed Linear Space

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Abstract: This paper proposed the idea of Neutrosophic norm in a linear space. An attempt has been made to find some related results in Neutrosophic normed linear space and study the Cauchy sequence and completeness in this structure.

Keywords: Linear space, Norm, Co-norm, Fuzzy Set, Fuzzy Norm, Neutrosophic norm, Neutrosophic normed linear space.

1. Introduction

This section gives the basic introduction about the present work starting with Literature survey, Scope and objective and chapter distribution.

1.1. Literature Survey:

The notion of normed linear space plays a major role in Functional Analysis. Dimension in normed linear space has attracted researchers to a greater extend. $G\ddot{a}$hler (1965) took effort in developing the structure of 2-normed linear space and n-normed linear space. Recently many researchers have engaged themselves in developing the theory of n-normed linear space. Zadeh (1965) [40], introduced fuzzy set in his pioneering work which is a remarkable theory to deal with uncertainty. He stated that a fuzzy set assigns a membership value to each element of a given crisp universe set from [0, 1]. This notion laid the foundation for a wide range usage of Mathematics and also applied to a great variety of real-life scenarios. Later Atanassov (1986) [11-13], focused intuitionistic fuzzy set, which is characterized by a membership function and non-membership function for each in the Universe and then Smarandache (1998-2005) [2 - 4] developed another idea called Neutrosophic set by adding an intermediate membership. Maji (2013) also dealt about this Neutrosophic concept. Felbin (1992) [19,20,21] assigned a fuzzy real number to each element of the linear space and introduction another idea of fuzzy norm on a linear space and also proved that a finite dimensional fuzzy normed linear space has a unique fuzzy norm on it up to fuzzy equivalence. Further in 1993 he discussed about the completion of fuzzy normed linear spaces and in 1993 he proved that any finite dimensional fuzzy normed linear space is necessarily complete.

Beg & Samanta (2003) [14 - 17] introduced a definition of fuzzy norm on a linear space. They also provided a decomposition theorem of fuzzy norms into a family of crisp norms and studied the properties of finite dimensional fuzzy normed linear spaces. This paper motivated Narayanan et.al to develop the theory of fuzzy n-normed linear space. Santhosh & Ramakrishnan (2011) [36] introduced the concepts of norm and inner product on fuzzy linear spaces over fuzzy fields.
Then Vijayabalaji (2008) [38, 39] et.al studied the idea of interval valued fuzzy n-normed linear spaces. Later Vijayabalaji (2007) et.al, Samanta (2009) et.al, and Issac (2012) [25] et.al dealt the

concepts of normed linear spaced with intuitionistic fuzzy settings. Recently Sandeep Kumar (2018) discussed some results on Interval valued intuitionistic fuzzy n-normed linear space.

1.2. Scope and Objective of the Present Investigation:

The present study is aimed to extend the structures of fuzzy normed linear space into Neutrosophic normed linear space. An attempt has been made to study some elegant results in this structure through Neutrosophic norm and analyze the Cauchy sequences on Neutrosophic Normed linear space. The paper is classified into the following sections: Section 1 shows the introduction and section 2 gives some basic definitions and properties of linear space, fuzzy set, t-norm, t-conorm , fuzzy normed linear space etc., Section 3 deals the Neutrosophic normed linear space and discussed their properties. Section 4, ends with concluding remarks and future scope of the study.

2. Preliminaries

This section recalls the basis definitions and results that are necessary for the present work.

Definition 2.1. [14] A linear space (or vector space) V over a field F consist of the following

1. A field F of scalars.
2. A set V of objects called vectors
3. A rule (or operation) called vector addition which associates with each pair of vectors, $u, v \in V$ a vector $u + v \in V$ called the sum of u and v in such a way that
    - Addition is commutative,
    - Addition is associative
    - There is unique vector in u in V called the zero vector, such that $u + 0 = u \;\forall u \in V$
    - For each vector $u \in V$, there is unique vectors $-u \in V$ such that $u + (-u) = 0$.
4. A rule (or operation) called scalars multiplication which associates with each scalar $a \in F$ and vector and $u \in V$ in such a way that
    - $1.u = u \;\forall u \in V$ and $1 \in F$
    - $ab(u) = a(bu) \;\forall a, b \in F$ and $\forall u \in V$
    - $a(u + v) = au + av \;\forall a \in F$ and $\forall u, v \in V$
    - $(a + b)u = au + bu \;\forall a, b \in F$ and $\forall u \in V$

It is denoted as $(V, +, \cdot)$ is a linear space.

Definition 2.2. [14]A nonnegative function on a linear vector space V , $\|\cdot\| : V \to [0, \infty)$ is called a norm if

1. $\| x \| = 0$ if and only if $x = 0$;
2. $\| x + y \| \leq \| x \| + \| y \|$ for all $x, y \in V$ (the triangular inequality)
3. $\| \alpha x \| = |\alpha| \, \| x \|$ for all $x \in V$ and $\alpha \in F$

Definition 2.3. [14]A normed linear space is a linear space V with a norm $\|\cdot\|_V$ on it.

Definition 2.4. [40] A fuzzy set A in X is defined as an object of the form $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x)$ is called the membership function of x in X which maps X to the unit interval $I = [0, 1]$.

Definition 2.5. [11]An intuitionistic fuzzy set A in a nonempty set X is defined as an objects of the form $A = \{(x, \mu_A(x), \vartheta_v(x)) : x \in X\}$ where the functions $\mu_A : X \to [0, 1]$ and $\vartheta_A : X \to [0, 1]$ defined the degree of membership and degree of non-membership of the element $x \in X$ respectively, and for $0 \leq \mu_v(x) + \vartheta_v(x) \leq 1 \;\forall x \in X$.

An ordinary fuzzy set A in X may be viewed as special intuitionistic fuzzy set with the non-membership function $\vartheta_A(x) = 1 - \mu_A(x)$.

Definition 2.6. Let [I] be the set of all closed sub intervals of the interval [0,1] and M = $[M_L, M_U] \in$ [I] where M_L and M_U are the lower extreme and upper extreme, respectively. For a set X, an IVFS (Interval Valued Fuzzy Set) A on X given by

$$A = \{\langle x, M_A(x) \rangle / \ x \in X\}$$

where the function M_A : X→ [0,1] defines the degree of membership of an element x on A, and $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ called an interval valued fuzzy number.

Definition 2.7. For a set X, an IVIFS (Interval Valued Intuitionistic Fuzzy Set) A on X is an objects having the form A = $\{\langle x, M_A(x), N_A(x)\rangle / \ x \in X\}$ where M_A : X→ [I] and N_A : X→ [I] represents the degree of membership and non-membership $0 \leq sup(M_A(x)) + sup(N_A(x)) \leq 1$ for every x ∈ X $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$
Hence A =$\{[M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\}$ is called IVIFS.

Definition 2.8. [14] Let X be a linear space over the field F (real or complex) and $*$ is a continuous t-norm. A fuzzy subset N on X × ℝ (R-set of all real numbers) is called a fuzzy norm on X if and only if for x,y ∈ X and c ∈ F,

(N1) ∀ t ∈ R with t ≤ 0, N(x,t) = 0
(N2) ∀ t ∈ R with t > 0 N(x,t) = 1, iff x = 0
(N3) t ∈ R, t > 0

$$N(cx,t) = N(x, \tfrac{t}{|c|}). \text{ If } ,c \neq 0$$

(N4) ∀ s,t ∈ R, x,y ∈ X,

$$N(x+y, t+s) \geq N(x,t)*N(y,s)$$

(N5) $\lim_{t\to\infty} N(x,t) = 1$.

The triplet (X,N,$*$) will be referred to as a fuzzy normed linear space.

Definition 2.9. [25] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ satisfies the following conditions:

1. $*$ is commutative and associative
2. $*$ is continuous
3. $a * 1 = a$, for all $a \in [0,1]$
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 2.10. A binary operation $\lozenge : [0,1] \times [0,1] \to [0,1]$ is continuous t-co-norm if $\lozenge$ satisfies the following conditions:

1. $\lozenge$ is commutative and associative
2. $\lozenge$ is continuous
3. $a \lozenge 0 = a$, for all $a \in [0,1]$
4. $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 2.11 Let $*$ be a continuous t-norm, $\lozenge$ be a continuous t-co-norm, and V be a linear space over the field F ($= R$ or C). An intuitionistic fuzzy norm or in short IFN on V is an object of the form $A = \{((x,t), N(x,t), M(x,t)) : (x,t) \in V \times \mathbb{R}^+$, where N, M are fuzzy sets on $V \times \mathbb{R}^+$,N denotes the degree of membership and M denotes the degree of non-membership $(x,t) \in V \times \mathbb{R}^+$ satisfying the following conditions:

1. $N(x,t) + M(x,t) \leq 1 \ \forall \ (x,t) \in V \times \mathbb{R}^+$
2. $N(x,t) > 0$
3. $N(x,t) = 1$ if and only if $x = 0$

4. $N(cx,t) = N\left(x, \frac{t}{|c|}\right), c \neq 0, c \in F$
5. $N(x,s) * N(y,t) \leq N(x+y, s+t)$
6. $N(x, \cdot)$ is non – decreasing function of $\mathbb{R}^+$ and $\lim_{t\to\infty} N(x,t) = 1$
7. $M(x,t) > 0$
8. $M(x,t) = 0$ if and only if $x = 0$
9. $M(cx,t) = M\left(x, \frac{t}{|c|}\right), c \neq 0, c \in F$
10. $M(x,s) \lozenge M(y,t) \geq M(x+y, s+t)$
11. $M(x, \cdot)$ is non – increasing function of $\mathbb{R}^+$ and $\lim_{t\to\infty} M(x,t) = 0$.

Then the quadruple $(V, A, *, \lozenge)$ will be referred as a intuitionistic fuzzy normed linear space.

3. Neutrosophic Approach on Normed Linear Space

This section introduces the idea of Neutrosophic normed linear space using the notion of Neutrosophic set. Further, some result related to Cauchy sequence on Neutrosophic normed linear space are also dealt.

3.1 Neutrosophic Norm:

Here Neutrosophic norm is defined with suitable example. Further the convergence of sequence in NNLS and some properties also studied.

Definition 3.1. [33]Let S be a space of points (objects). A NS N on S is characterized by a truth-membership function ρ, an indeterminacy membership function ξ, and a falsity-membership function η, where $\rho(x), \xi(x) and\, \eta(x)$ and real standard and non-standard subset of $]^-0,1^+[$ i.e., ρ, ξ, η : X→ $]^-0,1^+[$. Thus the NS N over S is defined as:

$$N = \{< x, (\rho(x), \xi(x), \eta(x)) > | \ x \in S\}$$

On the same of $\rho(x), \xi(x) and\, \eta(x)$ there is no restriction and so $^-0 \leq sup\rho(x) + sup\xi(x) + sup\,\eta(x) \leq 3^+$. Here $1^+ = 1 + \epsilon$, where 1 is its standard part and ϵ its non-standard part. Also, $^-0 = 0 - \epsilon$where 0 is its standard part and ϵ its non-standard part.

From philosophical point of view, a NS takes the value from real standard or nonstandard subsets of$]^-0,1^+[$. But to practice in real scientific and engineering areas, it is difficult to use NS with value from real standard or nonstandard subset of$]^-0, 1^+[$. Hence, we consider the NS which takes the value from the subset of [0, 1].

Definition 3.2. Let V be a linear space field $F = (\mathbb{R}\ or\ \mathbb{C})$ and $*$ be a continuous t – norm, $\lozenge$ be a continuous t – co – norm. Then, a Neutrosophic subset $N : \langle \rho, \xi, \eta \rangle\ on\ V \times F$ is called a Neutrosophic norm on V if for $x, y \in V$ and $c \in F$ (c being scalar), if the following conditions hold.

1. $0 \leq \rho(x,t), \xi(x,t), \eta(x,t) \leq 1, \forall\ t \in R$
2. $0 \leq \rho(x,t) + \xi(x,t) + \eta(x,t) \leq 3, \forall\ t \in R$
3. $\rho(x,t) = 0$ with $t \leq 0$
4. $\rho(x,t) = 1\ with\ t > 0\ iff\ x = 0$, the null vector
5. $\rho(cx,t) = \rho\left(x, \frac{t}{|c|}\right), \forall\ c \neq 0, t > 0$
6. $\rho(x,s) * \rho(y,t) \leq \rho(x+y, s+t)\ \forall\ s, t \in R$
7. $\rho(x, \cdot)$ is continuous non – decreasing function for $t > 0$, $\lim_{t\to\infty} \rho(x,t) = 1$
8. $\xi(x,t) = 1\ with,\ t \leq 0$
9. $\xi(x,t) = 0\ with\ t > 0\ iff\ x = 0$, the null vector

10. $\xi(cx,t) = \xi\left(x, \frac{t}{|c|}\right), \forall\, c \neq 0, t > 0$
11. $\xi(x,s)\ \Diamond\ \xi(y,t) \geq \xi(x+y,\ s+t)\ \forall\, s, t \in R$
12. $\xi(x,\cdot)$ is a continuous non-increasing function for t > 0, $\lim_{t\to\infty} \xi(x,t) = 0$
13. $\eta(x,t) = 1\ with,$ t ≤ 0;
14. $\eta(x,t) = 0\ with\ t > 0\ iff\ x = 0$, the null vector;
15. $\eta(cx,t) = \eta\left(x, \frac{t}{|c|}\right), \forall\, c \neq 0, t > 0$
16. $\eta(x,s)\ \Diamond\ \eta(y,t) \geq \eta(x+y,\ s+t)\ \forall\, s, t \in R$
17. $\xi(x,\cdot)$ is a continuous non-increasing function for t > 0, $\lim_{t\to\infty} \eta(x,t) = 0$;

Further $(V, N, *, \Diamond)$ is Neutrosophic normed linear space (NNLS).

Example3.3.

Let $(V, \|\cdot\|)$ be a normed linear space. Take $a * \mathrm{b} = ab\ and\ a \Diamond b = a + b - ab$. Define,

$$\rho(\mathrm{x},\mathrm{t}) = \begin{cases} \frac{t}{t+||x||} & if\ \ t > ||x|| \\ 0 & otherwise. \end{cases}$$

$$\xi(\mathrm{x},\mathrm{t}) = \begin{cases} \frac{x}{t+||x||} & if\ \ t > ||x|| \\ 1 & otherwise. \end{cases}$$

$$\eta(\mathrm{x},\mathrm{t}) = \begin{cases} \frac{||x||}{t} & if\ \ t > ||x|| \\ 1 & otherwise. \end{cases}$$

, Then $(V, N, *, \Diamond)$ is an NNLS.

Proof:

All the conditions are obvious except the condition (6), (11), (16). For $s, t > 0$ because these are clearly true for $s, t \leq 0$.

Now, $\rho(x+y, s+t) - \rho(x,s) * \rho(y,t)$

$$= \frac{s+t}{s+t+||x+y||} - \frac{st}{(s+||x||)(t+||y||)}$$

$$\geq \frac{s+t}{s+t+||x+y||} - \frac{st}{(s+||x||)(t+||y||)}$$

$$= \{(s+t)(s+||x||)(t+||y||) - st(s+t+||x||+||y||)\}/\aleph$$

Where $\aleph = (s+t+||x||+||y||)(s+||x||)(t+||y||)$

$$= \{t^2||x||\, s^2||y|| + (s+t)||xy||\}/\aleph \ \geq 0.$$

Hence, $\rho(x,s) * \rho(y,t) \leq \rho(x+y, s+t), \forall\, s, t \in R$

$\xi(x,s)\ \Diamond\ \xi(y,t) - \xi(x+y, s+t)$

$$= \frac{||x||}{s+||x||} + \frac{||y||}{t+||y||} - \frac{||xy||}{(s+||x||)\ (t+||y||)} - \frac{x+y}{||x+y||+s+t}$$

$$= \frac{||xy|| + t||x|| + s||y||}{(s+||x||)\ (t+||y||)} - \frac{||x+y||}{||x+y||+s+t}$$

$$= \{(||x+y||+s+t)(t||x||+s||x||+||xy||) - ||x+y||(s+||x||)\ (t+||y||)\}/D$$

Where $D = (s+t+||x+y||)(s+||x||)(t+||y||)$

$$= \{(s+t)(t||x|| + s||y|| + ||xy||) - st||x+y||\}/D$$

$$\geq \{(s+t)(t||x|| + s||y|| + ||xy||) - st(||x|| + ||y||)\}/D$$

$$= \{t^2||x|| + s||y|| + (s+t)||xy||\}/D \geq 0.$$

Hence, $\xi(x,s) \lozenge \xi(y,t) \geq \xi(x+y, s+t), \forall s, t \in R$.

Finally $\eta(x,s) \lozenge \eta(y,t) \geq (x+y, s+t)$

$$= \frac{||x||}{s} + \frac{||y||}{t} - \frac{||xy||}{st} - \frac{||x+y||}{s+t}$$

$$= \frac{t||x|| + s||y|| - ||xy||}{st} - \frac{||x+y||}{s+t}$$

$$\geq \{s^2||y|| + t^2||x|| - (s+t)||xy||\}/st(s+t)$$

$$= \{s||y||(s - ||x||) + t||x||(t - ||y||)\} / st(s+t) \geq 0, (as\ s > ||x||, t > ||y||).$$

Thus, $\eta(x,s) \lozenge \eta(y,t) \geq (x+y, s+t), \forall s, t \in R$. This completes the proof.

Definition 3.4. Let $\{x_n\}$ be a sequence of points in a NNLS $(V, N, *, \lozenge)$. Then the sequence converges to a point$x \in V$ if and only if for given $r \in (0,1), t > 0$ there exist $n_0 \in N$ (the set of natural numbers) such that

$$\rho(x_n - x, t) > 1 - r, \xi(x_n - x, t) < r, \eta(x_n - x, t) < r, \forall n \geq n_0.$$

(or)

$$\lim_{n\to\infty} \rho(x_n - x, t) = 1, \lim_{n\to\infty} \xi(x_n - x, t) = 0, \lim_{n\to\infty} \eta(x_n - x, t) = 0, t \to \infty$$

Then the sequence $\{x_n\}$ is called a convergent sequence in the NNLS $(V, N, *, \lozenge)$.

Theorem 3.5.

If the sequence $\{x_n\}$ in a NNLS $(V, N, *, \lozenge)$ is convergent, then the point of convergence is unique.

Proof:

Let $\lim_{n\to\infty} x_n = x\ and\ \lim_{n\to\infty} x_n = y. for\ x \neq y.\ Then\ for\ s, t > 0,$

$$\lim_{n\to\infty} \rho(x_n - x, s) = 1, \lim_{n\to\infty} \xi(x_n - x, s) = 0, \lim_{n\to\infty} \eta(x_n - x, s) = 0, as\ s \to \infty\ and$$

$$\lim_{n\to\infty} \rho(x_n - x, t) = 1, \lim_{n\to\infty} \xi(x_n - x, t) = 0, \lim_{n\to\infty} \eta(x_n - x, t) = 0, as\ t \to \infty$$

Now,

$$\rho(x - y, s + t) = \rho(x - x_n + x_n - y, s + t) \leq \rho(x_n - x, s) * \rho(x_n - y, t)$$

Taking limit as $n \to \infty$ and for s, t $n \to \infty$,

$$\rho(x - y, s + t) \geq 1 * 1 = 1\ i.e., \rho(x - y, s + t) = 1$$

Further,

$$\xi(x - y, s + t) = \xi(x - x_n + x_n - y, s + t) \leq \xi(x_n - x, s) \lozenge \xi(x_n - y, t)$$

Taking limit as $n \to \infty$ and for s, t $n \to \infty$,

$$\xi(x - y, s + t) \leq 0 \lozenge 0 = 0 i.e., \xi(x - y, s + t) = 0$$

Similarly, $\eta(x-y,s+t)=0$

Hence, $x=y$ and this complete the proof.

Theorem 3.6.

In an NNLS $(V,N,*,\lozenge)$, if $\lim_{n\to\infty}(x_n)=x$ and $\lim_{n\to\infty}(y_n)=y$ then $\lim_{n\to\infty}(x_n+y_n)=x+y$

Proof:

Here, for $s,t>0$

$$\lim_{n\to\infty}\rho(x_n-x,s)=1, \lim_{n\to\infty}\xi(x_n-x,s)=0, \lim_{n\to\infty}\eta(x_n-x,s)=0, as\ s\to\infty\ and$$

$$\lim_{n\to\infty}\rho(y_n-y,t)=1, \lim_{n\to\infty}\xi(y_n-y,t)=0, \lim_{n\to\infty}\eta(y_n-y,t)=0, as\ t\to\infty.$$

Now, $\lim_{n\to\infty}\rho[(x_n+y_n)-(x+y),s+t)]=\lim_{n\to\infty}\rho[(x_n-x)+(y_n-y),s+t)]$,

$$\geq \lim_{n\to\infty}\rho(x_n-x,s)*\lim_{n\to\infty}\rho(y_n-y,t)[by\ (6) in\ Definition\ 3.2]$$

$$=1*1=1\ as\ s,t\to\infty$$

Hence $\lim_{n\to\infty}\rho[(x_n-y_n)-(x+y),s+t)]=1\ as, s,t\to\infty.\ Again$

$$\lim_{n\to\infty}\xi[(x_n+y_n)-(x+y),s+t)]=\lim_{n\to\infty}\xi[(x_n-x)+(y_n-y),s+t)]$$

$$\geq \lim_{n\to\infty}\xi(x_n-x,s)\lozenge\lim_{n\to\infty}\xi(y_n-y,t)\ [by\ (11) in\ Definition\ 3.2]$$

$$=0\lozenge 0=0\ as\ s,t\to\infty$$

$$So, \lim_{n\to\infty}\xi[(x_n+y_n)-(x+y),s+t)]=0\ as\ s,t\to\infty.$$

Similarly,

$$\lim_{n\to\infty}\eta[(x_n+y_n)-(x+y),s+t)]=0\ as\ s,t\to\infty. and\ this\ end\ the\ theorem.$$

Theorem 3.7.

If $\lim_{n\to\infty}x_n=x$ and $0\neq c\in F$, then $\lim_{n\to\infty}cx_n$ in an NNLS $(V,N,*,\lozenge)$.

Proof:

Here,

$$\lim_{n\to\infty}\rho(cx_n-cx,t)=\lim_{n\to\infty}\rho\left(x_n-x,\frac{t}{|c|}\right)=1, as\ \frac{t}{|c|}\to\infty.$$

$$\lim_{n\to\infty}\xi(cx_n-cx,t)=\lim_{n\to\infty}\xi\left(x_n-x,\frac{t}{|c|}\right)=1, as\ \frac{t}{|c|}\to\infty.$$

$$\lim_{n\to\infty}\eta(cx_n-cx,t)=\lim_{n\to\infty}\eta\left(x_n-x,\frac{t}{|c|}\right)=1, as\ \frac{t}{|c|}\to\infty.$$

Thus, the theorem is proved.

3.2. Completeness on Neutrosophic Normed Linear Space:

Here the Cauchy sequence in NNLS and complete NNLS are introduced. Further several structural characteristics of complete NNLS also studied. .

Definition 3.8. A sequence $\{x_n\}$ of points in an NNLS $(V,N,*,\lozenge)$ is said to be bounded for $r\in(0,1)$ and $t>0$. if the following hold:

$\rho(x_n, t) > 1 - r, \xi(x_n, t) < r, \eta(x_n, t) < r, \forall n \in N.(the\ set\ of\ all\ natural\ numbers\ ).$

Definition 3.9.

1. A sequence $\{x_n\}$ of points in an NNLS $(V, N,*,\Diamond)$.is said to be a Cauchy sequence if giver$\in (0,1), t > 0$ there exist $n_0 \in N$ $(the\ set\ of\ all\ natural\ numbers\ )\ such\ that$
$$\rho(x_n - x_m, t) > 1 - r, \xi(x_n - x_m, t) < r, \eta(x_n - x_m, t) < r \,\forall m, n \in n_0.$$
$$(or)$$
$$\lim_{n,m\to\infty} \rho(x_n - x_m, t) = 1, \lim_{n,m\to\infty} \xi(x_n - x_m, t) = 0, \lim_{n,m\to\infty} \eta(x_n - x_m, t) = 0, as\ t \to \infty$$

2. Let $\{x_n\}$ be Cauchy sequence of points in a normed linear space $(V, ||\bullet||)$. Then $\lim_{n,m\to\infty} ||x_n - x_m|| = 0$ hold.

Example 3.10. For $t > 0, let\ \rho(x, t) = \frac{t}{t+||x||}$, $\xi(x, t) = \frac{||x||}{t+||x||}$, $\eta(x, t) = \frac{||x||}{t}$. Then $(V, N,*,\Diamond)$ is an NNLS. Now,
$$\lim_{n,m\to\infty} \frac{t}{t + ||x_n - x_m||} = 1\,, \lim_{n,m\to\infty} \frac{||x_n - x_m||}{t + ||x_n - x_m||} = 0\,, \lim_{n,m\to\infty} \frac{||x_n - x_m||}{t} = 0$$
$$\lim_{n,m\to\infty} \rho(x_n - x_m, t) = 1, \lim_{n,m\to\infty} \xi(x_n - x_m, t) = 0, \lim_{n,m\to\infty} \eta(x_n - x_m, t) = 0, as\ t \to \infty$$

This shows that $\{x_n\}$ is a Cauchy sequence in the NNLS $(V, N,*,\Diamond)$.

Theorem 3.11. Every convergent sequence of points in a NNLS $(V, N,*,\Diamond)$ is a Cauchy sequence.

Proof:

Let $\{x_n\}$ be a convergent sequence of a points in a NNLS $(V, N,*,\Diamond)$ so that $\lim_{n\to\infty} x_n = x$. Then for $t > 0$,
$$\lim_{n,m\to\infty} \rho(x_n - x_m, t) = \lim_{n,m\to\infty} \rho(x_n - x_m + x - x, t) = \lim_{n,m\to\infty} \rho[(x_n - x) + (x - x_m), t]),$$
$$\geq \lim_{n\to\infty} \rho\left(x_n - x, \frac{t}{2}\right) = * \lim_{m\to\infty} \rho\left(x - x_m, \frac{t}{2}\right)\ [by\ (6) in\ Definition\ 3.2]$$
$$= \lim_{n\to\infty} \rho\left(x_n - x, \frac{t}{2}\right) = * \lim_{m\to\infty} \rho\left(x_m - x, \frac{t}{2}\right)\ [by\ (5) in\ Definition\ 3.2]$$
$$= 1 * 1 = 1\ \ as\ \ t \to \infty.$$
So , $\lim_{n,m\to\infty} \rho(x_n - x_m, t) = 1.$

Again $\lim_{n,m\to\infty} \xi(x_n - x_m, t) = \lim_{n,m\to\infty} \xi(x_n - x_m + x - x, t)$
$$= \lim_{n,m\to\infty} \xi[(x_n - x) + (x - x_m), t])$$
$$\geq \lim_{n\to\infty} \xi\left(x_n - x, \frac{t}{2}\right) = \Diamond \lim_{m\to\infty} \xi\left(x - x_m, \frac{t}{2}\right)\ [by\ (11)\ in\ Definition\ 3.2]$$
$$= \lim_{n\to\infty} \xi\left(x_n - x, \frac{t}{2}\right) = \Diamond \lim_{m\to\infty} \xi\left(x_m - x, \frac{t}{2}\right)\ [by\ (10)\ in\ Definition\ 3.2]$$
$$= 0 \Diamond 0 = 0\ \ as\ \ t \to \infty.$$
So $\lim_{n,m\to\infty} \xi(x_n - x_m, t) = 0$ and similarly $\lim_{n,m\to\infty} \eta(x_n - x_m, t) = 0$.

Hence,$\{x_n\}$ is a Cauchy Sequence.

Example 3.12. The following example will clarify that the inverse of the Theorem 3.11 may not be true. Let $R_1 = \left\{\frac{1}{n}\middle| n \in \mathrm{N}\right\}$ (the set of natural numbers) be a subset of real numbers and $||x|| = |x|$. With respect to the neutrosophic norm defined in Example.3.10, obviously (R,N,∗,◊) is an NNLS.
Now

$$\lim_{n,m\to\infty} \frac{t}{t + ||x_n - x_m||} = \lim_{n,m\to\infty} \frac{t}{t + \left|\frac{1}{n} - \frac{1}{m}\right|} = 1,$$

$$\lim_{n,m\to\infty} \frac{||x_n - x_m||}{t + ||x_n - x_m||} = \lim_{n,m\to\infty} \frac{\left|\frac{1}{n} - \frac{1}{m}\right|}{t + \left|\frac{1}{n} - \frac{1}{m}\right|} = 0,$$

$$and, \qquad \lim_{n,m\to\infty} \frac{||x_n - x_m||}{t} = \lim_{n,m\to\infty} \frac{\left|\frac{1}{n} - \frac{1}{m}\right|}{t} = 0,$$

Thus $\{x_n\}$ is a Cauchy Sequence of points in the NNLS (R, N,∗,◊). But

$$\lim_{n\to\infty} (x_n - x_k, t) = \lim_{n\to\infty} \frac{\left|\frac{1}{n} - \frac{1}{k}\right|}{t + \left|\frac{1}{n} - \frac{1}{k}\right|} \neq 0\,.$$

This shows that the Cauchy Sequence $\{x_n\}$ is not convergent in that NNLS.

Theorem 3.13. In an NNLS $(V, N,*,\Diamond)$, if $\{x_n\},\{y_n\}$ are Cauchy Sequence of vectors and $\{\lambda_n\}$ is Cauchy Sequence of scalars in an NNLS $(V, N,*,\Diamond)$, then $\{x_n + y_n\}\, and\, \{\lambda_n y_n\}$ are also Cauchy Sequence in NNLS $(V, N,*,\Diamond)$.

Proof:

For $t > 0$, we have,

$$\lim_{n,m\to\infty} \rho(x_n - x_m, t) = 1, \lim_{n,m\to\infty} \xi(x_n - x_m, t) = 0, \lim_{n,m\to\infty} \eta(x_n - x_m, t) = 0, as\ t \to \infty$$

And

$$\lim_{n,m\to\infty} \rho(y_n - y_m, t) = 1, \lim_{n,m\to\infty} \xi(y_n - y_m, t) = 0, \lim_{n,m\to\infty} \eta(y_n - y_m, t) = 0, as\ t \to \infty$$

$$\lim_{n,m\to\infty} \rho[(x_n + y_n) - (x_m + y_m), t)] = \lim_{n,m\to\infty} \rho[(x_n - x_m) + (y_n - y_m), t)]$$

$$\geq \lim_{n,m\to\infty} \rho\left(x_n - x_m, \frac{t}{2}\right) * \lim_{n,m\to\infty} \rho\left(y_n - y_m, \frac{t}{2}\right) = 1 * 1 = 1\ as\ t \to \infty$$

Hence, $\lim_{n,m\to\infty} \rho[(x_n + y_n) - (x_m + y_m), t)] = 1\ as\ t \to \infty$

$$\lim_{n,m\to\infty} \xi[(x_n + y_n) - (x_m + y_m), t)] = \lim_{n,m\to\infty} \xi[(x_n - x_m) + (y_n - y_m), t)]$$

$$\leq \lim_{n,m\to\infty} \xi\left(x_n - x_m, \frac{t}{2}\right) \Diamond \lim_{n,m\to\infty} \xi\left(y_n - y_m, \frac{t}{2}\right) = 0 \Diamond 0 = 0\ as\ t \to \infty$$

So, $\lim_{n,m\to\infty} \xi[(x_n + y_n) - (x_m + y_m), t)] = 0\ as\ t \to \infty$

Similarly,

$$\lim_{n,m\to\infty} \eta[(x_n + y_n) - (x_m + y_m), t)] = 0\ as\ t \to \infty$$

This ends the first part. For the next part,

$$\lim_{n,m\to\infty} \rho[(\lambda_m y_m - \lambda_n y_n), t] = \lim_{n,m\to\infty} \rho[(\lambda_m y_m - \lambda_n y_n) + (\lambda_m y_n - \lambda_m y_n), t]$$

$$= \lim_{n,m\to\infty} \rho[(\lambda_m(y_m - y_n) + y_n(\lambda_m - \lambda_n), t] \geq \lim_{n,m\to\infty} \rho[\left((y_m - y_n), \frac{t}{2|\lambda_m|}\right)] * \rho\left(y_n, \frac{t}{2|\lambda_m - \lambda_n|}\right)$$

Since $|\lambda_m - \lambda_n| \to 0$ $as\ m, n \to \infty, So\ |\lambda_m - \lambda_n| \neq 0$. Again $\{y_n\}$ being Cauchy sequence is bounded.

Hence , $\lim_{n,m\to\infty} \rho[(\lambda_m y_m - \lambda_n y_n), t] = 1\ as\ t \to \infty. Further,$

$$\lim_{n,m\to\infty} \xi[(\lambda_m y_m - \lambda_n y_n), t] = \lim_{n,m\to\infty} \xi[(\lambda_m y_m - \lambda_n y_n) + (\lambda_m y_n - \lambda_m y_n), t]$$

$$= \lim_{n,m\to\infty} \xi[(\lambda_m(y_m - y_n) + y_n(\lambda_m - \lambda_n), t] \leq \lim_{n,m\to\infty} \xi[\left((y_m - y_n), \frac{t}{2|\lambda_m|}\right)] \lozenge\, \xi\left(y_n, \frac{t}{2|\lambda_m - \lambda_n|}\right)$$

By similar argument, $\lim_{n,m\to\infty} \xi[(\lambda_m y_m - \lambda_n y_n), t] = 0\ as\ t \to \infty$ and finally,

$\lim_{n,m\to\infty} \eta[(\lambda_m y_m - \lambda_n y_n), t] = 0\ as\ t \to \infty$

Hence, the 2[nd] part is complete.

Definition 3.14. Let $(V, N,*,\lozenge)$ be a NNLS and $\triangle_V$ be the collection of all points on V. Then $(V, N,*,\lozenge)$ is said to be a complete NNLS if every Cauchy sequence of points in $\triangle_V$ converges to a point of $\triangle_V$.

Theorem 3.15. In an NNLS $(V, N,*,\lozenge)$, if every Cauchy sequence has a convergent subsequence then $(V, N,*,\lozenge)$ is a complete NNLS.

Proof: Let $\{x_{n_k}\}$ be a convergent subsequence of a Cauchy sequence $\{x_n\}$ in an NNLS $(V, N,*,\lozenge)$ such that $\{x_{n_k}\} \to x \in V$. Since $\{x_n\}$ be a Cauchy sequence in $(V, N,*,\lozenge)$, given $t > 0$

$$\lim_{n,k\to\infty} \rho\left(x_n - x_{n_k}, \frac{t}{2}\right) = 1, \lim_{n,k\to\infty} \xi\left(x_n - x_{n_k}, \frac{t}{2}\right) = 0, \lim_{n,k\to\infty} \eta\left(x_n - x_{n_k}, \frac{t}{2}\right) = 0, as\ t \to \infty$$

Again since $\{x_{n_k}\}$ converges to x, then

$$\lim_{n,k\to\infty} \rho\left(x_{n_k} - x, \frac{t}{2}\right) = 1, \lim_{n,k\to\infty} \xi\left(x_{n_k} - x, \frac{t}{2}\right) = 0, \lim_{n,k\to\infty} \eta\left(x_{n_k} - x, \frac{t}{2}\right) = 0, t \to \infty$$

Now,

$$\rho(x_n - x, t) = \rho(x_n - x_{n_k} + x_{n_k} - x, t) \geq \rho\left(x_n - x_{n_k}, \frac{t}{2}\right) * \rho\left(x_{n_k} - x, \frac{t}{2}\right).$$

It implies

$$\lim_{n\to\infty} \rho(x_n - x, t) = 1$$

Further,

$$\xi(x_n - x, t) = \xi(x_n - x_{n_k} + x_{n_k} - x, t) \leq \xi\left(x_n - x_{n_k}, \frac{t}{2}\right) \lozenge\, \xi\left(x_{n_k} - x, \frac{t}{2}\right).$$

It implies $\lim_{n\to\infty} \xi(x_n - x, t) = 0$.

It implies $\lim_{n\to\infty} \eta(x_n - x, t) = 0$.

This shows that x_n converges to $x \in V$ and thus the theorem is proved.

Theorem 3.16. In an NNLS $(V, N,*,\lozenge)$, every convergent sequence is a Cauchy sequence.

Proof: Let $\{x_n\}$ be a convergent sequence in the NNLS $(V, N,*,\lozenge)$ with $\lim_{n\to\infty} x_n = x$. Let $s, t \in \mathbb{R}^+$ and p = 1,2,3,…, we have

$$\rho(x_{n+p} - x_n, s + t) = \rho(x_{n+p} - x + x - x_n, s + t)$$

$$\geq \rho\left(x_{n+p} - x, s\right) * \rho(x - x_n, t)$$

$$= \rho\left(x_{n+p} - x, s\right) * \rho(x_n - x, t)$$

Taking limit, we have

$$\lim_{n\to\infty} \rho\left(x_{n+p} - x_n, s + t\right) \geq \lim_{n\to\infty} \rho\left(x_{n+p} - x, s\right) * \lim_{n\to\infty} \rho(x_n - x, t)$$
$$= 1 * 1 = 1$$

$$\lim_{n\to\infty} \rho\left(x_{n+p} - x_n, s + t\right) = 1 \; \forall s, t \to \infty \; and \; p = 1,2,3 \ldots.$$

Again,

$$\xi\left(x_{n+p} - x_n, s + t\right) \geq \xi(x_{n+p} - x + x - x_n, s + t)$$

$$\geq \xi\left(x_{n+p} - x, s\right) \lozenge \xi(x - x_n, t)$$

$$= \xi\left(x_{n+p} - x, s\right) \lozenge \xi(x_n - x, t)$$

Taking limit, we have

$$\lim_{n\to\infty} \xi\left(x_{n+p} - x_n, s + t\right) \geq \lim_{n\to\infty} \xi\left(x_{n+p} - x, s\right) \lozenge \lim_{n\to\infty} \xi(x_n - x, t)$$
$$= 0 \lozenge 0 = 0$$

$$\lim_{n\to\infty} \xi\left(x_{n+p} - x_n, s + t\right) = 0 \; \forall s, t \to \infty \; and \; p = 1,2,3 \ldots.$$

Similarly,

$$\lim_{n\to\infty} \eta\left(x_{n+p} - x_n, s + t\right) = 0 \; \forall s, t \to \infty \; and \; p = 1,2,3 \ldots.$$

Thus, $\{x_n\}$ is a Cauchy sequence in the NNLS $(V, N, *, \lozenge)$.

Theorem 3.17. Let $(V, N, *, \lozenge)$ be an NNLS, such that every Cauchy sequence in $(V, N, *, \lozenge)$has a convergent sebsequence. Then $(V, N, *, \lozenge)$ is complete.

Proof: Let $\{x_n\}$ be a Cauchy sequence in $(V, N, *, \lozenge)$ and $\{x_{n_k}\}$ be a subsequence of $\{x_n\}$ the converges to $x \epsilon V \; and \; t > 0$. Since $\{x_n\}$ is a Cauchy sequence in $(V, N, *, \lozenge)$, we have

$$\lim_{n,k\to\infty} \rho\left(x_n - x_k, \frac{t}{2}\right) = 1, \lim_{n,k\to\infty} \xi\left(x_n - x_k, \frac{t}{2}\right) = 0, \lim_{n,k\to\infty} \eta\left(x_n - x_k, \frac{t}{2}\right) = 0$$

Again since $\{x_{n_k}\}$ converges to x, we have

$$\lim_{k\to\infty} \rho\left(x_{n_k} - x, \frac{t}{2}\right) = 1, \lim_{k\to\infty} \xi\left(x_{n_k} - x, \frac{t}{2}\right) = 0, \lim_{n,k\to\infty} \eta\left(x_{n_k} - x, \frac{t}{2}\right) = 0.$$

Now,

$$\rho(x_n - x, t) = \rho(x_n - x_{n_k} + x_{n_k} - x, t)$$

$$\geq \rho\left(x_n - x_{n_k}, \frac{t}{2}\right) * \rho\left(x_{n_k} - x, \frac{t}{2}\right)$$

$$\lim_{n\to\infty} \rho(x_n - x, t) = 1$$

Again, we see that

$$\xi(x_n - x, t) = \xi(x_n - x_{n_k} + x_{n_k} - x, t)$$

$$\leq \xi\left(x_n - x_{n_k}, \frac{t}{2}\right) \lozenge \xi\left(x_{n_k} - x, \frac{t}{2}\right)$$

$$\lim_{n\to\infty} \xi(x_n - x, t) = 0$$

Similarly, $\lim_{n\to\infty} \eta(x_n - x, t) = 0$

Thus, $\{x_n\}$ converges to x in $(V, N, *, \Diamond)$ and hence is complete.

Theorem 3.18. Every finite dimensional NNLS satisfying the condition.

$$\left.\begin{matrix} a \Diamond a = a \\ a * a = a \end{matrix}\right\} \forall a \in [0,1] \ldots\ldots\ldots\ldots (1)$$

$\rho(x, t) > 0 \ \forall t > 0 \to x = 0$ ……………(2) is complete.

Proof: Let $(V, N, *, \Diamond)$ be a finite dimensional NNLS satisfying the condition (1) and (2). Also, let dim V =k and $e_1, e_2, \ldots, e_k$ be a basic of V.

Consider $\{x_n\}$ as an arbitrary Cauchy sequence in (V,A).

Let $x_n = \beta_1^{(n)} e_1 + \beta_2^{(n)} e_2 + \cdots + \beta_k^{(n)} e_k$ where $\beta_1^{(n)}, \beta_2^{(n)}, \ldots, \beta_k^{(n)}$suitable scalars are. Then by the same calculation, there exist $\beta_1, \beta_2, \ldots, \beta_k \in F$ such that the sequence $\{\beta_i^{(n)}\}_n$ converges to $\beta_i \ for\ i = 1,2,\ldots,k$. clearly $x = \rho(\sum_{i=1}^{k} \beta_i^{(n)} e_i \in V$

$$\begin{aligned} \rho(x_n - x, t) &= \rho(\sum_{i=1}^{k} \beta_i^{(n)} e_i - \sum_{i=1}^{k} \beta_i \, e_i, t) \\ &= \rho(\sum_{i=1}^{k} (\beta_i^{(n)} - \beta_i) \, e_i, t) \\ &\geq \rho\left((\beta_1^{(n)} - \beta_1) e_i, \frac{t}{k} \right) * \ldots * \ \rho\left((\beta_k^{(n)} - \beta_k) e_k, \frac{t}{k} \right) \\ &= \rho\left(e_1, \frac{t}{k|\beta_1^{(n)} - \beta_1|} \right) * \ldots * \rho\left(e_k, \frac{t}{k|\beta_k^{(n)} - \beta_k|} \right) \end{aligned}$$

Since $\lim_{n\to\infty} \frac{t}{k|\beta_i^{(n)} - \beta_i|} = \infty$, we see that $\lim_{n\to\infty} \rho\left(e_i, \frac{t}{k|\beta_i^{(n)} - \beta_i|} \right) = 1$

$\lim_{n\to\infty} \rho(x_n - x, t) \geq 1 * \ldots * 1 = 1 \ \forall \ t > 0$

$\lim_{n\to\infty} \rho(x_n - x, t) = 1 \ \forall \ t > 0.$

Again, for all $t > 0$

$$\begin{aligned} \xi(x_n - x, t) &= \xi(\sum_{i=1}^{k} \beta_i^{(n)} e_i - \sum_{i=1}^{k} \beta_i \, e_i, t) \\ &= \xi(\sum_{i=1}^{k} (\beta_i^{(n)} - \beta_i) \, e_i, t) \\ &\leq \xi\left((\beta_1^{(n)} - \beta_1) e_i, \frac{t}{k} \right) \Diamond \ldots \Diamond \ \xi\left((\beta_k^{(n)} - \beta_k) e_k, \frac{t}{k} \right) \\ &= \xi\left(e_1, \frac{t}{k|\beta_1^{(n)} - \beta_1|} \right) \Diamond \ldots \Diamond \ \xi\left(e_k, \frac{t}{k|\beta_k^{(n)} - \beta_k|} \right) \end{aligned}$$

Since $\lim_{n\to\infty} \frac{t}{k|\beta_i^{(n)} - \beta_i|} = \infty$, we see that $\lim_{n\to\infty} \xi\left(e_i, \frac{t}{k|\beta_i^{(n)} - \beta_i|} \right) = 0$

$\lim_{n\to\infty} \xi(x_n - x, t) \leq 0 \Diamond \ldots \Diamond \ 0 = 0 \ \forall \ t > 0$

$\lim_{n\to\infty} \xi(x_n - x, t) = 0 \ \forall \ t > 0.$

Similarly, Since $\lim_{n\to\infty} \frac{t}{k|\beta_i^{(n)}-\beta_i|} = \infty$, we see that $\lim_{n\to\infty} \eta\left(e_i, \frac{t}{k|\beta_i^{(n)}-\beta_i|}\right) = 0$

Thus, we see that $\{x_n\}$ is an arbitrary Cauchy Sequence that converges to x ∈ V, Hence the NNLS $(V, N,*,\Diamond)$ is complete.

Theorem 3.19. Let $(V, N,*,\Diamond)$be an NNLS satisfying the condition equation (1). Every Cauchy sequence in $(V, N,*,\Diamond)$ is bounded.

Proof: Let $\{x_n\}$ be a Cauchy sequence in the NNLS $(V, N,*,\Diamond)$. Then we have

$$\left.\begin{array}{l}\lim_{n\to\infty} \rho(x_{n+p} - x, t) = 1\\ \lim_{n\to\infty} \xi(x_{n+p} - x, t) = 0\\ \lim_{n\to\infty} \eta(x_{n+p} - x, t) = 0\end{array}\right\} \forall t > 0, p = 1,2,\ldots$$

Choose a fixed r_0 with $0 < r_0 < 1$. Now we see that

$$\lim_{n\to\infty} \rho(x_n - x_{n+p}, t) = 1 > r_0 \; \forall t > 0, p = 1,2,\ldots$$

For $t' > 0 \; \exists \; n_0 = n_0(t')$ such that $\rho(x_n - x_{n+p}, t') > r_0 \; \forall n \geq n_0,\; p = 1,2,\ldots$

Since, $\lim_{n\to\infty} \rho(x, t) = 1$, we have for each $x \epsilon t > 0$ such that

$$\rho(x_n, t) > r_0 \; \forall t > t_i, n = 1,2,\ldots$$

Let $t_0 = t' + max\{t_1, t_2, \ldots, t_{n_0}\}$ Then,

$$\rho(x_n, t_0) \geq \rho(x_n, t' + t_{n_0})$$

$$= \rho(x_n - x_{n_0} + x_{n_0}, t' + t_{n_0})$$

$$\geq \rho(x_n - x_{n_0}, t') * \rho(x_{n_0}, t_{n_0})$$

$$> \; r_0 * r_0 = r_0 \; \forall \; n \geq n_0$$

Thus, we have

$$\rho(x_n, t_0) > \; r_0 \; \forall \; n \geq n_0$$

$$Also, \quad \rho(x_n, t_0) \geq \rho(x_n, t_n) > \; r_0 \; \forall n = 1,2, \ldots, n_0$$

So, we have,

$$\rho(x_n, t_0) > \; r_0 \; \forall \; n = 1,2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$Now, \; \lim_{n\to\infty} \xi(x_n - x_{n+p}, t) = 0 < (1 - r_0) \; \forall t > 0, p = 1,2,\ldots$$

For $t' > 0 \; \exists \; n_0' = n_0'(t')$ such that $\xi(x_n - x_{n+p}, t') < (1 - r_0) \; \forall \; n \geq n_0',\; p = 1,2,\ldots$

Since, $\lim_{n\to\infty} \xi(x, t) = 0$, we have for each $x_i \; \exists \; t_i' > 0$ such that

$$\xi(x_n, t) < (1 - r_0) \; \forall t > \; t_i', n = 1,2,\ldots$$

Let $t_0' = t' + \; max\{t_1', t_2', \ldots, t_{n_0}'\}$ Then,

$$\xi(x_n, t_0') \leq \xi(x_n, t' + t_{n_0}')$$

$$= \xi\left(x_n - x_{n_0'} + x_{n_0'}, t' + t_{n_0}'\right)$$

$$\leq \xi\left(x_n - x_{n_0'}, t'\right) \Diamond \xi(x_{n_0'}, t_{n_0}')$$

$$< \ (1 - r_0) \lozenge (1 - r_0) = (1 - r_0) \, \forall \, n > n'_0$$

Thus, we have

$$\xi(x_n, t'_0) < (1 - r_0) \, \forall n > n'_0$$

$$Also, \qquad \xi(x_n, t'_0) \leq \xi(x_n, t'_n) < (1 - \ r_0) \ \ \forall n = 1,2, \dots, n'_0$$

So, we have,

$$\xi(x_n, t'_0) < (1 - \ r_0) \, \forall \, n = 1,2, \dots \dots \dots \dots \dots \dots .. (2)$$

Similarly, we prove

$$\eta(x_n, t'_0) < (1 - \ r_0) \, \forall \, n = 1,2, \dots \dots \dots \dots \dots \dots .. (3)$$

Let $t''_0 = \max\{t_0, t'_0\}$. Hence from (1),(2), and (3) we see that

$$\left.\begin{array}{r} \rho(x_n, t''_0) \qquad > \quad r_0 \\ \xi(x_n, t''_0) < (1 - r_0) \\ \eta(x_n, t''_0) < (1 - r_0) \end{array}\right\} \forall \, n = 1,2, \dots$$

This implies that $\{x_n\}$ is bounded in $(V, N, *, \lozenge)$.

4. Conclusion

4.1 Concluding Remarks:

The aim of the present work is to introduce a Neutrosophic norm on a linear space. Also, the convergence of sequence, characteristic of Cauchy sequence in NNLS (Neutrosophic normed linear space) have been studied here. These are illustrated by suitable examples. Their related properties and structural characteristic have been discussed.

4.2 Future Scope:

This studied provides the structure of NNLS (Neutrosophic normed linear space) on a NLS (Normed linear space) with help of NS (Neutrosophic Set). In future this study leads to the extension of the following ideas:

- Neutrosophic-n-Normed Linear Space
- Finite Dimensional Neutrosophic-n-Normed Linear Space
- Neutrosophic Metric Space

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Study of Imaginative Play in Children using Neutrosophic Cognitive Maps Model

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Abstract: This paper studies the imaginative play in young children using a model based on neutrosophic logic, viz, Neutrosophic Cognitive Maps (NCMs). NCMs are constructed with the help of expert opinion to establish relationships between the several concepts related with the imaginative play in children in the age group 1-10 years belonging to socially, economically and educationally backward groups. The NCMs are important in overcoming the hindrance posed by complicated and often imprecise nature of psychological or social data. Data was collected by video recording of children playing and the interpretations given by experts. Fifteen attributes / concepts related with children playing with the same toy were observed and according to experts several concepts were related and for some the relations between concepts were indeterminate, so it was appropriate to use NCMs. These NCMs were built using five expert's opinion and the hidden patterns of them happened to be a fixed point.

Keywords: Neutrosophic Cognitive Maps (NCMs) model; Dynamical system; Hidden patterns; Fixed point; Limit cycle; Child psychology; Imaginative play

1. Introduction

Imaginative play is role-play in which children are using their imagination to express something they have experienced or display what they like. It is an integral part for the development of social, cognitive and emotional well-being and language and thinking skills of children in the age group 1-10 years. It serves as a determinant of the imaginative capability and psychological development of the child. In this paper, we study the importance of imaginative play in children in the age group of 1 to 10 years using mathematical and computational models. This will help to qualitatively and quantitatively analyse the influence of imaginative play in the psychological development of a child.

In order to objectively study the influence of imaginative play in child development, we make use of Neutrosophic Cognitive Maps (NCMs) [1] model, a generalization of the Fuzzy Cognitive Maps (FCMs) models. The benefit of these tools lies in their ability to handle incomplete and/or conflicting information that gives the result as the hidden pattern which may be a fixed point or a limit cycle. They are also one of the most efficient and strongest AI technologies that can be used when the data in hand in not large. They work as combination of neural networks and neutrosophic logic.

Given the imprecise and subjective nature of our study, artificial intelligence is best suited for it. FCMs and NCMs are important tools in AI when the data is small [1-4] and with the help of these tools we propose a model for assessing the influence of imaginative play in a child's psychological development. The study begins with collecting data from various sources which is processed and transformed to NCMs models with the help of expert's opinion. Using these directed neutrosophic graphs [5] of the NCMs, a dynamical system is formed which acts as the mathematical model to determine the influence of imaginative play in child development.

2. Related Works

Fuzzy Cognitive Maps (FCMs) and Neutrosophic Cognitive Maps (NCMs) have found applications in several fields in their classical forms and have also been extended to suit other applications [1-2, 6-12]. The most fundamental application of FCMs and NCMs is to establish relationships between seemingly unrelated concepts. A cause-effect relationship has been established in the parameters determining interrelated dynamics in socio-political and psychological backgrounds. The FCMs and NCMs models have been used in social issues like untouchability, school dropouts, social aspects of migrant labourers living with HIV/AIDS [7, 11, 13] and so on. Hence using FCMs and NCMs in study of finding the cognitive and mental abilities of children in the age group of 1-10 will certainly yield a better result by relating the seemingly unrelated factors associated with child development. For this study we collected data by video recording of children playing with the toy phone and the interpretations were obtained from the experts. Using these experts NCMs models were constructed. Another important application of predictive capability of FCMs is to diagnose autism spectrum disorder [9]. However, they have not considered the indeterminacy concept involved in this study.

Diagnosis of language impairment in children using FCMs is another application of FCMs in the field of artificial intelligence [3]. The determinants of the disorder are assigned fuzzy weights and a qualitative and quantitative computer model is developed which gives accurate diagnosis. FCMs have played a significant role in development of IQ tests for AI-based systems [4]. This helps in establishing a relationship between IQ characteristics for AI system and analyze them objectively. FCMs have been used for opinion mining in [10].

NCMs have been used in the study of socio-economic model [8], problems of school dropouts [7], social stigma faced by people suffering with AIDS [6], psychological problems suffered by women with AIDS [11] and in medical diagnosis [12]. Neutrosophy has been used for studying several decision-making problems [14-17]

However, FCMs cannot asses when the problem under investigation is clouded under indeterminacy and incompleteness, under these situations NCMs is a better tool which can tackle them and yield a better solution. So, in this paper we use the NCMs model to study the imaginative play in children.

This paper is organized into six sections. Section one is introductory in nature. A literature survey and related works are mentioned in section two. Section three gives the necessary basic concepts to make the paper a self-contained one. Section four describes the problem in general and the concepts / attributes involved. Section five gives the NCMs model using five experts' opinion and the final section gives the conclusions based on our study.

3. Basic Concepts

This section describes the FCMs and NCMs to make the paper a self-contained one.

3.1. FCMs

The notion of Fuzzy Cognitive Maps (FCMs) which are fuzzy signed directed graphs with feedback are discussed and described [2]. The directed edge e_{ij} from causal concept C_i to concept C_j measures how much C_i causes C_j. The time varying concept function $C_i(t)$ measures the non negative occurrence of some fuzzy event, perhaps the strength of a political sentiment, historical trend or opinion about some topics like child labor or school dropouts etc. FCMs model the world as a collection of classes and causal relations between them. The edge e_{ij} takes values in the fuzzy causal interval $[-1,1]$ ($e_{ij} = 0$ indicates no causality, $e_{ij} > 0$ indicates causal increase; that C_j

increases as C_i increases and C_j decreases as C_i decreases and $e_{ij} < 0$ indicates causal decrease or negative causality C_j decreases as C_i increases or C_j, increases as C_i decreases. Simple FCMs have edge value in $\{-1,0,1\}$. Thus if causality occurs it occurs to maximal positive or negative degree. It is important to note that e_{ij} measures only absence or presence of influence of the node C_i on C_j but till now any researcher has not contemplated the indeterminacy of any relation between two nodes C_i and C_j. When we deal with unsupervised data, there are situations when no relation can be determined between some two nodes. So in this section we try to introduce the indeterminacy in FCMs, and we choose to call this generalized structure as Neutrosophic Cognitive Maps (NCMs). In our view this will certainly give a more appropriate result and also caution to the user about the risk of indeterminacy.

3.2. NCMs

Now we proceed on to define the concepts about NCMs [1]. For the notion of neutrosophic graphs refer [5].

Definition 3.1 *A Neutrosophic Cognitive Maps (NCMs) is a neutrosophic directed graph with concepts like policies, events etc. as nodes and causalities or indeterminates as edges. It represents the causal relationship between concepts. Let $C_1, C_2, \ldots, C_n$ denote n nodes, further we assume each node is a neutrosophic vector from the neutrosophic vector space V. So a node C_i will be represented by $(x_1, \ldots x_n)$ where x_k's are zero or one or I (I is the indeterminate) and $x_k = 1$ means that the node C_k is in the on state and $x_k = 0$ means the node is in the off state and $x_k = I$ means the nodes state is an indeterminate one at that time or in that situation. Let C_i and C_j denote the two nodes of the NCM. The directed edge from C_i to C_j denotes the causality of C_i on C_j called connections or relations. Every edge in the NCM is weighted with a number in the set $\{-1,0,1,I\}$. Let e_{ij} be the weight of the directed edge C_iC_j, $e_{ij} \in \{-1,0,1,I\}$. e_{ij} = 0 if C_i does not have any effect on C_j, $e_{ij} = 1$ if increase (or decrease) in C_i causes increase (or decreases) in C_j, $e_{ij} = -1$ if increase (or decrease) in C_i causes decrease (or increase) in C_j . $e_{ij} = I$ if the relation or effect of C_i on C_j is an indeterminate.*

NCMs with edge weight from $\{-1,0,1,I\}$ are called simple NCMs.

Let the neutrosophic matrix $N(E)$ be defined as $N(E)$ = (e_{ij}) where e_{ij} is the weight of the directed edge C_i C_j, where $e_{ij} \in \{0,1,-1,I\}$. N(E) is called the neutrosophic adjacency matrix of the NCMs.

Let $A = (a_1, a_2, \ldots, a_n)$ where $a_i \in \{0,1,I\}$. A is called the instantaneous state neutrosophic vector and it denotes the on-off-indeterminate state position of the node at an instant; $a_i = 0$ if a_i is off (no effect) $a_i = 1$ if a_i is on (has effect) $a_i = I$ if a_i is indeterminate(effect cannot be determined) for $i = 1,2, \ldots n$.

Let $\overline{C_1C_2}$, $\overline{C_2C_3}$, $\overline{C_3C_4}, \ldots, \overline{C_iC_j}$, be the edges of the NCMs. Then the edges form a directed cycle. A NCM is said to be cyclic if it possesses a directed cycle. A NCM is said to be acyclic if it does not possess any directed cycle. A NCM with cycles is said to have a feedback. When there is a feedback in the NCMs i.e. when the causal relations flow through a cycle in a revolutionary manner the NCMs is called a dynamical system.

Let $\overline{C_1C_2}$, $\overline{C_2C_3}$, $\overline{C_3C_4}, \ldots, \overline{C_{n-1}C_n}$ be a cycle, when C_i is switched on and if the causality flow through the edges of a cycle and if it again causes C_i, we say that the dynamical system goes round and round. This is true for any node C_i, for $i = 1,2, \ldots n$. The equilibrium state for this dynamical system is called the hidden pattern.

If the equilibrium state of a dynamical system is a unique state vector, then it is called a fixed point.

Consider the NCMs with C_1, $C_2, \ldots, C_n$ as nodes. For example let us start the dynamical system by switching on C_1. Let us assume that the NCMs settles down with C_1 and C_n on, i.e. the state vector remains as (1, 0,…, 0, 1) this neutrosophic state vector (1,0, …, 0, 1) is called the fixed point.

If the NCM settles with a neutrosophic state vector repeating in the form

$$A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_t \rightarrow A_{t+1} \rightarrow \ldots \rightarrow A_n \rightarrow A_t$$

Where A_i is the vector which is passed into a dynamical system *N(E)* repeatedly; $1 \leq i \leq n$ then this equilibrium is called a limit cycle of the NCM [1].

4. Description of the Problem

Here for the theme of imaginative play in children in the age group 1-10 years, the data is collected from nearby schools and an orphanage in Vellore, India. The play material supplied to them was just a play with a toy mobile phone that is to conduct imaginary talks which was video recorded. We recorded by video on phone separately we also recorded the comments made from observations of the expert. This data was analysed by a group of five experts and they gave the 15 concepts or attributes associated with the data, which formed the parameter or the concepts /attributes of our observation and is described the Table 1. The experts agreed on the point that the play material cannot be used as an attribute so the other 14 concepts can be used as attributes. However, the experts were given the liberty to use any number of concepts from the table and some of them used 8 of the concepts and some only 6 and others all the 14 of the concepts. They gave their directed neutrosophic graphs which gave the dynamical system and they worked with the attributes of their own choice which are described in the following section.

Based on expert's opinion and on the previous works [9, 3], the following have been considered as important parameters in assessing imaginative play capabilities in children. Each of these components will be used as attributes/nodes of the NCMs based on experts' opinion, the influence of these parameters is then mathematically determined by performing necessary operations and obtaining hidden pattern of the dynamical system.

Table 1. Concepts / Attributes of the NCMs

Concept	Concept Description
C_1	Imaginative Theme
C_2	Physical Movements
C_3	Gestures
C_4	Facial Expressions
C_5	Nature and Length of Social Interaction
C_6	Play Materials Used
C_7	Way Play Materials were Used
C_8	Verbalisation
C_9	Tone of Voice
C_{10}	Role Identification
C_{11}	Engagement Level
C_{12}	Eye Reaction
C_{13}	Cognitive Response
C_{14}	Grammar and Linguistics
C_{15}	Coherence

All the fifteen attributes or concepts happens to be self explanatory. Using these five experts work the NCMs models were construcuted.

5. NCMs in the analysis of the imaginative play in young children

We have described in the earlier section the method of data collection and the assignments of the fifteen concepts and their list is provided in the Table 1. Now we have five experts working with this problem taking some or all the attributes mentioned in the Table 1. The five experts are child psychologists, Montessori trained teachers and specialist in child psychology. However they wanted to remain anonymous.

The first expert wished to work with the concepts $C_2, C_3, C_4, C_8, C_9, C_{10}, C_{11}$, and C_{12}. Figure 1 represents the directed neutrosophic graph G_1 given by the first expert.

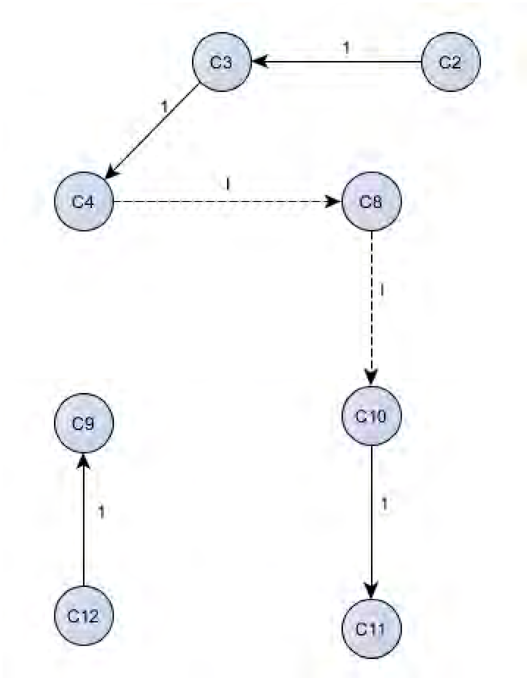

Figure 1. Directed Neutrosophic Graph *G*1

Let M_1 be the connection matrix associated with the directed graph G_1.

$$M_1 = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \begin{array}{cccccccc} C_2 & C_3 & C_4 & C_8 & C_9 & C_{10} & C_{11} & C_{12} \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} C_2 \\ C_3 \\ C_4 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \end{array} \end{array}$$

M_1 will serve as the dynamical system to find the effect of any state vector x on M_1. The state vectors $x \in \{(C_2, C_3, C_4, C_8, C_9, C_{10}, C_{11}, C_{12}); C_i \in \{0,1,\mathrm{I}\}; i = 2,3,4,8,9,10,11,12\}$. By default of notation we denote it by C_i's as we wish to record that the C_i's correspond to the attributes / concepts from the table and their on or off or indeterminate state. Let $x = (0,0,1,0,0,0,0,0)$ where only the concept C_4 that is facial expressions alone is in the on state and all other nodes are in the off state. The effect of x on the dynamical system M_1 is given by

$$x \circ M_1 = (0,0,0,I,0,0,0,0) \hookrightarrow (0,0,1,I,0,0,0,0) = x_1(say)$$

($\hookrightarrow$ symbol is used to denote the resultant vector that is thresholded and updated).

Now

$$x_1 \circ M_1 \hookrightarrow (0,0,1,I,0,I,0,0) = x_2(say)$$
$$x_2 \circ M_1 \hookrightarrow (0,0,1,I,0,I,I,0) = x_3(say)$$
$$x_3 \circ M_1 \hookrightarrow (0,0,1,I,0,I,I,0) = x_4(= x_3)$$

Thus the hidden pattern of the state vector x is a fixed point given by $x_4 = (0,0,1,I,0,I,I,0)$. Facial expression results in the indeterminate state of C_8, C_{10} and C_{11}; that is, role identification and engagement level respectively. That is according to this expert facial expression and its relation to verbalization, role identification and engagement level can not be determined as one can not find out exactly what the child imagines when he uses the phone. It can be an imitation of parents or others whom they have seen using it.

Next we find the effect of the on state of the two nodes C_{10} and C_{11} that is role identification and engagement level on the dynamical system M_1. Let $t = (0,0,0,0,0,1,1,0)$ be the state vector in which only the nodes C_{10} and C_{11} are in the on state. The effect of t on the dynamical system M_1 is given by

$$t \circ M_1 \hookrightarrow (0,0,0,0,0,1,1,0) = t_1(say)$$

This also results in a fixed point with no effect on the other concepts or attributes. So role identification and engagement level has no effect on the other nodes chosen by this expert for the study. Clearly when the child identifies the role it plays the engagement level is high and both the concepts are interdependent. We have just given these two state vectors but have worked with several such state vectors.

The second expert was interested to work with the attributes $C_1, C_4, C_5, C_7, C_{10}$ and C_{15} from Table 1. The neutrosophic directed graph G_2 given by him is as follows:

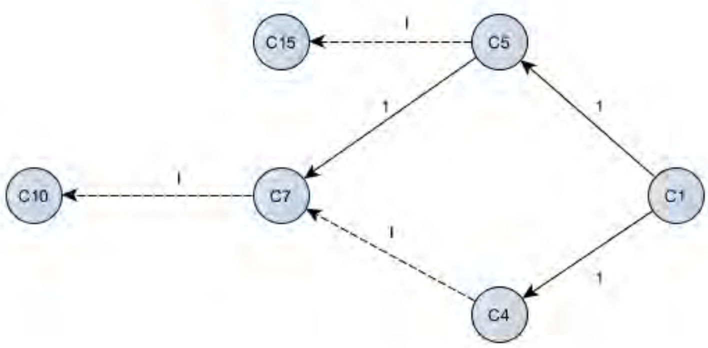

Figure 2. Directed Neutrosophic Graph G_2

Let M_2 be the connection matrix related with the graph G_2 which serves as the dynamical system.

$$M_2 = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \begin{array}{cccccc} C_1 & C_4 & C_5 & C_7 & C_{10} & C_{15} \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & I \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} C_1 \\ C_4 \\ C_5 \\ C_7 \\ C_{10} \\ C_{15} \end{array} \end{array}$$

Now the expert wishes to work with a state vector in which only the node C_4 is in the on state and all other nodes are in the off state.

Let $x = (0,1,0,0,0,0)$, the effect of x on the dynamical system M_2.

$$x \circ M_2 = (0,0,0,I,0,0) \hookrightarrow (0,1,0,I,0,0) = x_1(say)$$

$$x_1 \circ M_2 \hookrightarrow (0,1,0,I,I,0) = x_2(say)$$

$$x_2 \circ M_2 \hookrightarrow (0,1,0,I,I,0) = x_3(= x_2).$$

Thus the hidden pattern is a fixed point given by $x_2 = (0,1,0,I,I,0)$ that is the on state of facial expressions has indeterminate effect on C_7 and C_{10} that is the way play materials are used and role identification respectively. It is interesting to keep on record both the experts agree and arrive at the same conclusions.

If C_{15} alone is in on state we see the effect on the dynamical system M_2 has no influence for if $s = (0,0,0,0,0,1)$ then

$$s \circ M_2 \hookrightarrow (0,0,0,0,0,1) = s.$$

That is coherence has no influence on imaginative theme, facial expressions, nature and length of social interaction, way play materials are used and role identification. Evident from the fixed point resulting in s.

For usually a normal child with average IQ can not relate them however we found that majority of these children on whom we made the sample study belong to a poor and first generation learners background so in the task of using a phone, coherence can not play a role.

Next the 3^{rd} expert works with the nodes $C_2, C_3, C_4, C_8, C_9, C_{12}, C_{14}, C_{15}$. G_3 is the directed graph given by the expert.

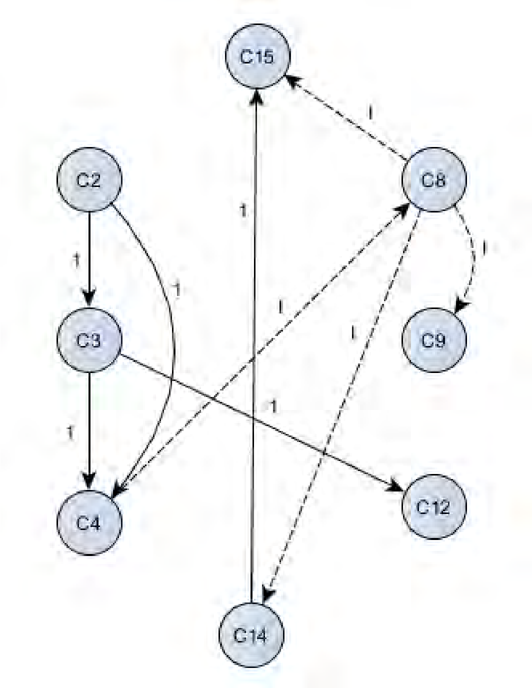

Figure 2. Directed Neutrosophic Graph G_3

Let M_3 be the connection matrix associated with the neutrosophic graph G_3.

$$M_3 = \begin{array}{c} \\ \end{array}\begin{array}{c} \begin{array}{cccccccc} C_2 & C_3 & C_4 & C_8 & C_9 & C_{12} & C_{14} & C_{15} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & I & I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ C_2 \\ C_3 \\ C_4 \\ C_8 \\ C_9 \\ C_{12} \\ C_{14} \\ C_{15} \end{array}$$

Let $m = (0,0,1,0,0,0,0,0)$ be the state vector where only the node C_4 is in the on state and all other nodes are in the off state.

The effect of m on the dynamical system M_3 is given in the following

$$m \circ M_3 = (0,0,1,I,0,0,0,0) = m_1(say)$$
$$m_1 \circ M_3 \hookrightarrow (0,0,1,I,I,0,I,I) = m_2(say)$$
$$m_2 \circ M_3 \hookrightarrow (0,0,1,I,I,0,I,I) = m_3(= m_2).$$

Thus the hidden pattern is a fixed point given by

$$m_2 = m_3 = (0,0,1,I,I,0,I,I).$$

Clearly the on state of C_4 node that is facial expression has indeterminate effect on verbalization - C_8, tone of voice - C_9, grammar, linguistics - C_{14} and coherence - C_{15}. Clearly the 3rd expert alone can not relate coherence he finds it is an indeterminate.

Let $n = (0,0,0,0,0,0,1,0)$ be the given state vector, to find the effect of n on M_3; Next we consider the only on state of the node C_{14} alone that is the child has grammar and linguistics in the on state and all other nodes are in the off state.

$$n \circ M_3 \hookrightarrow (0,0,0,0,0,0,1,1) = n_1(say)$$
$$n_1 \circ M_3 \hookrightarrow (0,0,0,0,,0,1,1) = n_2(= n_1).$$

The hidden pattern is a fixed point given by n_2. Clearly if the child has developed grammar and linguistics naturally the child would have developed coherence and vice versa.

The fourth expert wishes to work with 9 nodes, $C_2, C_3, C_4, C_5, C_7, C_8, C_9, C_{14}$ and C_{15} be the directed graph given by him.

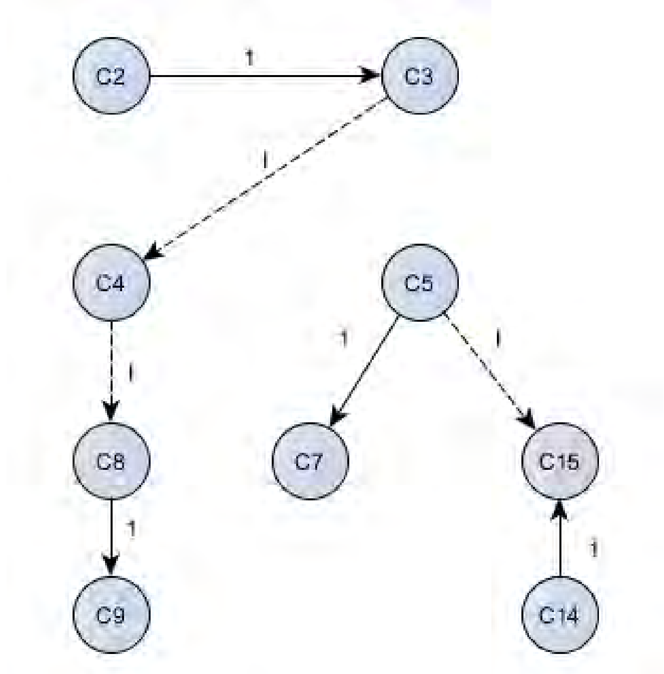

Figure 4. Directed Neutrosophic Graph G_4

Let M_4 be the connection matrix associated with the directed graph G_4 which will serve as the dynamical system for the neutrosophic directed graph G_4.

$$M_4 = \begin{array}{c} \begin{array}{ccccccccc} C_2 & C_3 & C_4 & C_5 & C_7 & C_8 & C_9 & C_{14} & C_{15} \end{array} \\ \left[\begin{array}{ccccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_7 \\ C_8 \\ C_9 \\ C_{14} \\ C_{15} \end{array}$$

The effect of the state vector $v = (0,0,1,0,0,0,0,0,0)$ where only the node C_4 is in the on state and all other nodes are in the off state. The effect of r on the dynamical system M_4 is given by

$$r \circ M_4 \hookrightarrow (0,0,1,0,0,I,0,0,0) = r_1(say)$$

$$r_1 \circ M_4 \hookrightarrow (0,0,1,0,0,I,I,0,0) = r_2(say)$$
$$r_2 \circ M_4 \hookrightarrow (0,0,1,0,0,I,I,0,0) = r_3(= r_2).$$

Thus the hidden pattern is a fixed point given by $r_2 = (0,0,1,0,0,I,I,0,0)$. The on state of facial expression makes on state C_8 and C_9 but both verbalization C_8 and tone of voice C_9 are in the indeterminate state only. That is facial expressions makes verbalization and tone of voice only to indeterminate state, rest of the states remain off. Next we study the effect of the state vector $z = (0,0,0,1,0,0,,0,0,0)$ on the dynamical system M_4. That is only the node C_5 nature and length of the social interaction is in the on state. All other nodes are in the off state. Effect of z on M_4 is as follows:

$$z \circ M_4 \hookrightarrow (0,0,0,1,1,0,0,0,I) = z_1(say)$$
$$z_1 \circ M_4 \hookrightarrow (0,0,0,1,1,0,0,0,I) = z_2(= z_1)$$

So the hidden pattern is the fixed point. On state of the concept nature and length of the social interaction makes on the node C_7 the way play materials are used but the coherence is in the indeterminate state, all other nodes remain unaffected.

Next expert wishes to work with all the 14 concepts barring the play materials used for study.

G_5 is the directed graph given by this expert. Let M_5 be the connections matrix which will serve as the dynamical system of the graph G_5.

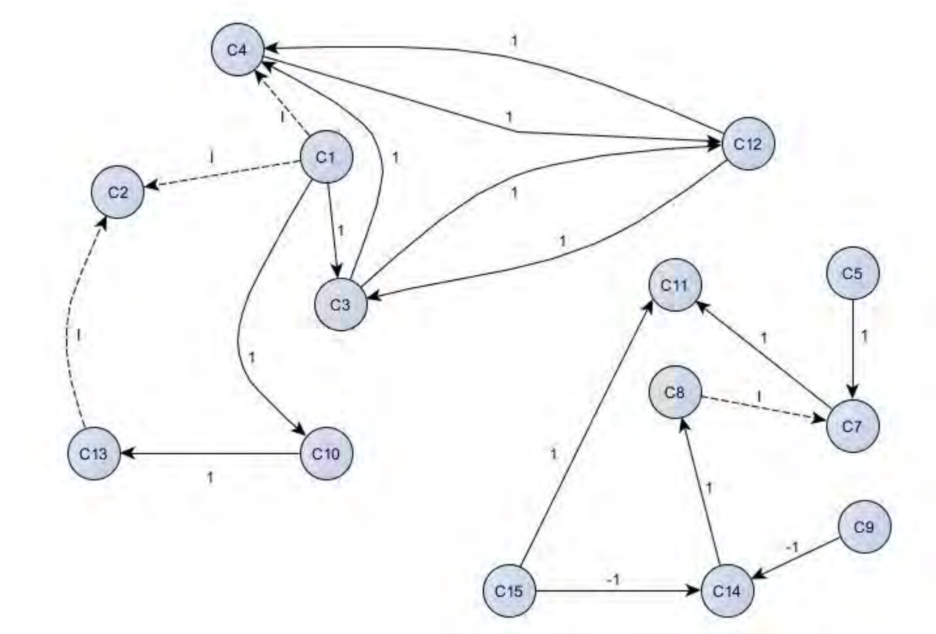

Figure 5. Directed Neutrosophic Graph G_5

$$M_5 = \begin{array}{c} \begin{array}{cccccccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_7 & C_8 & C_9 & C_{10} & C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \end{array} \\ \left[\begin{array}{cccccccccccccc}
0 & I & 1 & I & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0
\end{array}\right] \end{array} \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \end{array}$$

Let $p = (0,0,0,1,0,0,0,0,0,0,0,0,0,0)$ be the initial state vector in which only the node C_4 is in the on state all other nodes are in the off state. Effect of p on M_5 is given by

$$p \circ M_5 \hookrightarrow (0,0,0,1,0,0,0,0,0,0,1,0,0,0) = p_1(say)$$
$$p_1 \circ M_5 \hookrightarrow (0,0,0,1,0,0,0,0,0,0,1,0,0,0) = p_2(say)$$
$$p_2 \circ M_5 \hookrightarrow (0,0,1,1,0,0,0,0,0,0,1,0,0,0) = p_3(say)$$
$$p_3 \circ M_5 \hookrightarrow (0,0,1,1,0,0,0,0,0,0,1,0,0,0) = p_4(= p_3).$$

Thus the hidden pattern is a fixed point. This expert has taken all the 14 concepts, the on state of concept C_4 alone that is facial expressions makes on the states C_3 and C_{12} namely gestures and eye reaction respectively.

Next we study the effect of $w = (1,0,0,1,0,0,1,0,0,1,0,0,0,1)$ where C_1, C_4, C_8, C_{11} and C_{14}.

$$w \circ M_5 \hookrightarrow (1, I, 1,1,0,0,1,1,1,1,1,0,0,1) = w_1(say)$$
$$w_1 \circ M_5 \hookrightarrow (1, I, 1,1,0,1,1,1,1,1,1,0,1,1) = w_2(say)$$
$$w_2 \circ M_5 \hookrightarrow (1, I, 1,1,0,1,1,1,1,1,1,0,1,1) = w_3(= w_2)$$

Thus the hidden pattern of w is a fixed point and on state of the concepts C_1, C_4, C_8, C_{11} and C_{15} makes on all the states except C_5 nature and length of social interaction and C_{14}- grammar and linguistics and makes C_2 an indeterminate.

6. Conclusions

In this paper the authors have studied the imaginative play of children in the age group 1 to 10 years. We have taken these children from educationally, socially and economically backward classes. Study shows that the concepts C_1 to C_{15} are interrelated in a very special way. Further we saw that most children did not relate the facial expression with their verbal communication, in fact we could not determine it. For several, the coherence and the verbal communications or otherwise cannot be determined. For an 8-year old child started to talk to his elderly relative and ended up talking with a friend in less than 2 minutes of conversation. In fact, our study has authentically revealed that several concepts/relations cannot be determined. Further we felt for these children generally their overall ability was below average. Conclusions of each model for the state vectors under investigation are given along with the models. So, our future research would be to use the same toy phone and study the children of the same age group but from better socio-economic background and compare it with these children so that one can determine the ways to develop the first-generation learners.

Further for future research, we plan to adopt different Neutrosophic concepts [18-26] like Single Valued Neutrosophic Sets (SVNS), Double Valued Neutrosophic Sets (DVNS) and Triple Refined Indeterminate Neutrosophic sets (TRINS), Neutrosophic triplets and duplets in Cognitive models and study this problem.

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Validation of A Model for Knowledge Management in the Cocoa Producing Peasant Organizations of Vinces Using Neutrosophic Iadov Technique

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Abstract: The work departs with a model for knowledge management in the country productive organizations of cocoa of Vinces, in Ecuador. A model that is developed for the need to boost the correct management of knowledge and development of this type of entrepreneurship. The objective of the present work is to validate the qualitative aspects of the model using neutrosophy and the Iadov technique, due to that these techniques are appropriate for validating knowledge in different areas in the presence of uncertainty and indeterminacy. A final result is obtained that facilitates to calculate the index group satisfaction of the proposed model. The index of group satisfaction (GSI), in this case, is GSI =0.85. Results are positive, which validate the satisfaction with the model. Paper ends with conclusions and future works proposals.

Keywords: knowledge management, cocoa production, neutrosophic logic, Iadov

1. Introduction

The small and medium enterprises (SMEs) of Ecuador, have an impact of 40% in the gross domestic product and 60% in the generation of direct employment, according to Zúñiga Santillán, et al. [1]. These authors recognize that the main factors of the failure of the SMEs, they find in the limited knowledge on the official programs of support and information about sources of available public financings and the absence of competences.

Coincident with the before related authors, refer Messina and Hochsztain [2] that is important the level that possesses the SMEs and especially, the human capital, as for the knowledge, skills, and capacitances that can be converted in factors that induce to the success/failure. Other studies carried out in Ecuador recognize that the main influential elements to lean it take of decisions, are the ones not based in technical elements, nor in the registers took on the products that possess the SMEs.

It is shown in the studies of the before mentioned authors, faulty planning, organization and control of the labor process, about the matter Poveda Morales and Varna Hernández [3], outline the need for better implementation of knowledge management strategies and gaining institutional support [4]. On the other hand, Rodríguez and Gómez [5], recognize as factors of success of the SMEs such as human committed, competent capital, motivated with the business and with the dominion of management tools.

The development of the knowledge in the SMEs of Ecuador corresponds with the sustainable development and the exigencies that the state imposes in this sense [6]. Specifically, for the country productive organizations of cocoa of Vinces in Ecuador, where the economic and social development, requires the management of the knowledge generated [7], favoring:

- The support to takes empiric decisions
- The mechanisms to register historical results
- That the distribution of the work is carried out without the criterion of the managers
- The follow-up and control of the carried out work
- An improvement as for the external contracting pf adviser.

Other difficulties are recognized to keep the experiences of the region in the cultivation of the product, the conditions, and the particular properties of the area, transmit and formalize experiences and knowledge. The producers are developed in an environment that lacks activities that stimulate the management of human talent and knowledge, with impact on the organizational culture and productive results.

The management of human talent in the scientific literature defines the following mains steps: management of human resources, management of the human capital, management of the personnel. However, the fundamental thing is considered to the person or the human being as bearing integrity of the capacitance of work or the human capital, not as a means [8].

It is recognized that entrepreneurship must incorporate a philosophy of management that is based on the belief that the person could generalize the knowledge that generates. To center in the work position for the design of the systems of knowledge management.

It is essential to create the context that facilitated the peoples to acquire the capacitance and the motivation, as well as that, have the opportunity to involve in operation in which promotes collective apprenticeship [9] and it incorporates the organizational culture. In this sense, the effort of the national association of exporters highlights the cocoa producers [10].

The deficiencies and difficulties outlined, result in an exigency for the development of the human talent in the country productive organizations of cocoa of Vinces:

- Deciding the leaders of human talent and identify relevant knowledge
- Making good use of better experiences and transfers it.
- To motivate the personnel to explore and use knowledge
- Propitiating the innovation and the creation of values added in order to achieve competitiveness and sustainability.

Based on the documentary analysis, the literature consulted not recognize studies using knowledge management (KM) of these organizations. As for the KM that it has been effective, it has originated of enlarging interest, and it has been treated from different perspectives, as systems of information, organizational apprenticeship, strategic direction, and [11] innovations, accustomed is insufficient, for these undertakings.

In agreement with it before related, it is of highlighting that the models of knowledge management define in simplified form: symbolic and schematic the components that define it; to delimit someone of your dimensions; permitting an approximate sight; to describe processes and construct; finding one's bearings strategies; as well as to contribute essential data [12] is vital for the SMEs. Therefore, the KM model to boost the human talent in productive organizations of cocoa of Vinces, for later operationalization in specific procedures, contribute to keep the traditions (good practical in the historical conditions-make concrete of the territory), and at the same time to incorporate experiences, tools and knowledge to the increment of the productivity and the effectiveness of the process.

To verify the validity of the model that it is proposed neutrosophic Iadov technique is used. The Iadov technique constitutes an indirect form to study the users´ satisfaction [13]

This technique uses [14], the main criterion to formulate questions that validate the proposals, while the questions not related or complementary serve as an introduction and to get additional information about the proposal. The results of these form the "logical table of Iadov"[15, 16]. In this document, the satisfaction of the emitting actors and the beneficiaries of the strategy of development, are combined to form the receiving actors. The techniques of the criterion of user must be used as a form to evaluate the results in those cases in which the proposal is contextualized, immersed in the context and for finding the applicability of the result [17].

The degree of satisfaction- in satisfaction is a psychological state that it shows in the peoples as an expression of the interaction that moves between the positive poles and negative [17]. Neutrosophic Iadov allows to include indetermination and the importance of the user.

Recently, neutrosophy has been introduced as a theory for decision making [18]. The neutrosophic term means knowledge of the neutral thought and this neutral represents the main distinction between fuzzy and intuitionist logic [19]. The theory of neutrosophy introduces a new logic in which is estimated that each proposition has a true degree (t), indetermination degree (i) and a falsity degree (f) [20]. They have proposed many extensions of the classic methods of taking of decisions to treat the indetermination based on the theory of the neutrosophic as TOPSIS [19], DEMATEL [21], AHP [22] and VIKOR [23].

The original proposal of the Iadov method do not allow appropriate management of the indetermination. Another weakness is the impossibility of including users' importance. The introduction of the neutrosophy theory resolves the problems of indetermination that appear in the evaluations, being useful for capturing the neutrals or ambiguous positions of users [24]. Each idea tends to is neutralized, decreased, balanced for other ideas [25].

2. Materials and Methods

In the Iadov technique, questionnaires are used to decide the degree of satisfaction of the users with the proposal to measure the impact of the strategy of the investigator with a total of seven questions, three of those which are closed and four open, whose report is unknown for the subject [26]. These three ask about hidden sections relate through the "logical table of Iadov", that is to present adapted to investigation. The interrelation of the three questions shows the position of each user in the scale of satisfaction. This scale of satisfaction is expressed using SVN numbers [28]. The original definition of true value in the neutrosophic logic is presented as follow [27]:

It is N = {(T, I, F) : T, I, F ⊆ [0, 1]} a neutrosophic valuation as a mapped of a group from proportional formulae to N, and for each p sentence then:

$$v\,(p) = (T, I, F) \quad (1)$$

In order to make easy practical application to real-world, it was developed a proposal of single-valued neutrosophic sets (SVNS) allowing to use of linguistic variables [28, 29], this increase the interpretability of models and the use of the indetermination in practical problems.

Be X a universe of discourse. A SVNS A on X is an object of the form.

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (2)$$

Where, $uA(x)$: $X \rightarrow [0,1]$, $rA(x)$: $X \rightarrow [0,1]$ y $vA(x)$: $X \rightarrow [0,1]$, con $0 \leq uA(x)+ rA(x)+vA(x)$: ≤ 3 for all $x \in X$. The intervals (x), (x) and (x) denote the true, indeterminate and false membership of x in A, respectively. For motives of convenience, an SVN number could be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$, y + b + $c \leq 3$. The SVN numbers, that it is obtained, is of utility for the systems of decision making. To analyze the results, it establishes as a function of punctuation. To arrange the alternatives uses a function of [30] punctuation adapted

$$s(\mathrm{V}) = \mathrm{T} - \mathrm{F} - \mathrm{I} \quad (3)$$

In the case that the assessment corresponds to indeterminacy (I) a process of de-neutrosophication is developed [1]. In this case, I ∈ [-1, 1]. Lastly, we work with the average of the extreme values $I \in [0,1]$to obtain a single value.

$$\lambda([a_1, a_2]) = \frac{a_1 + a_2}{2} \quad (4)$$

Then, the results are aggregated. In this paper, the weighted average aggregation operator is proposed to calculate the group satisfaction index (GSI). The weighted average (WA) is extensively used [2, 3]. A WA operator has associated a vector of weights, V, with $v_i \in [0,1]$ and $\sum_1^n v_i = 1$, with the following form:

$$WA(a_1, .., a_n) = \sum_1^n v_i a_i \quad (5)$$

Where v_i represented the importance of expert i. This proposal allows dealing with indeterminacy and importance of users due to expertise or any other reason making Iadov method more practical [19].

3. Survey to Country Producers of Cocoa in Vinces

A model to promote the knowledge management of the country organizations producers of cocoa of Vinces, province Los Rios, Ecuador was proposed based on the study of a group of models of knowledge management, the legal framework and the particular properties of the sector by means of diagnosis.

The general procedure describes previously proposes five phases: build a work team, creation of the center of strategic information, allies and possibilities, implementation and measurement, and feedback. The conception integrates a series of tools as a methodological solution to the outlined scientific problem. The implementation permits the identification of the main deficiencies and related risks with the integral acting of the human talent and the generation of actions of improvement accordingly, as part of the continuous improvement.

A case study was developed for the validation of the model. A scale with individual expressions satisfaction and its corresponding score value is shown in Table 1.

Table 1. Scale satisfaction with SVN values.

Linguistic expression	SVN Number	Scoring
Clearly pleased	(1, 0, 0)	1
More pleased than unpleased	(1, 0.25, 0.25)	0.5
Not defined	I	0
More unpleased than pleased	(0.25, 0.25, 1)	-0.5
Clearly unpleased	(0,0,1)	-1
Contradictory	(1,0,1)	0

Table 2. The Iadov logical table

	Would you consider knowledge management without using the proposed model?								
	No			**I don´t know**			**yes**		
Do your expectations meet the application of the model for knowledge management?	If you could choose freely, a model for knowledge management, would you use the proposed model?								
	yes	I don´t know	No	yes	I don´t know	No	yes	I don´t know	No
Very pleased.	1 (6)	2 (1)	6	2	2	6	6	6	6
Partially pleased.	2	2	3	2	3 (1)	3	6	3	6
It's all the same to me	3	3	3	3	3	3	3	3	3
More unpleased than pleased.	6	3	6	3	4	4	3	4	4

Not pleased	6	6	6	6	4	4	6	4	5
I don´t know what to say	2	3	6	3	3	3	6	3	4

A sample of 21 specialists directed linked to the model were surveyed. The survey elaborated comprises 7 questions, three closed questions interspersed in four open questions, of which 1 fulfilled the introductory function and three functioned as reaffirmation and sustenance of objectivity of the user response.

In this case, the results are shown in Table 3.

Table 3. Results of the application to producers of cocoa in Vinces.

Expression	Total	%
Clearly pleased	6	75
More pleased than unpleased	1	12.5
Not defined	1	12.5
More unpleased than pleased	0	0
Clearly unpleased	0	0
Contradictory	0	0

The calculation of the score is carried out. In this case, it two initial user have more expertise with V= [0.2, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]. The final result of the index of group satisfaction (GSI) that the method portrays, in this case, is: GSI =0.85. Results are positive, show the satisfaction with the model, as displayed in Figure 1.

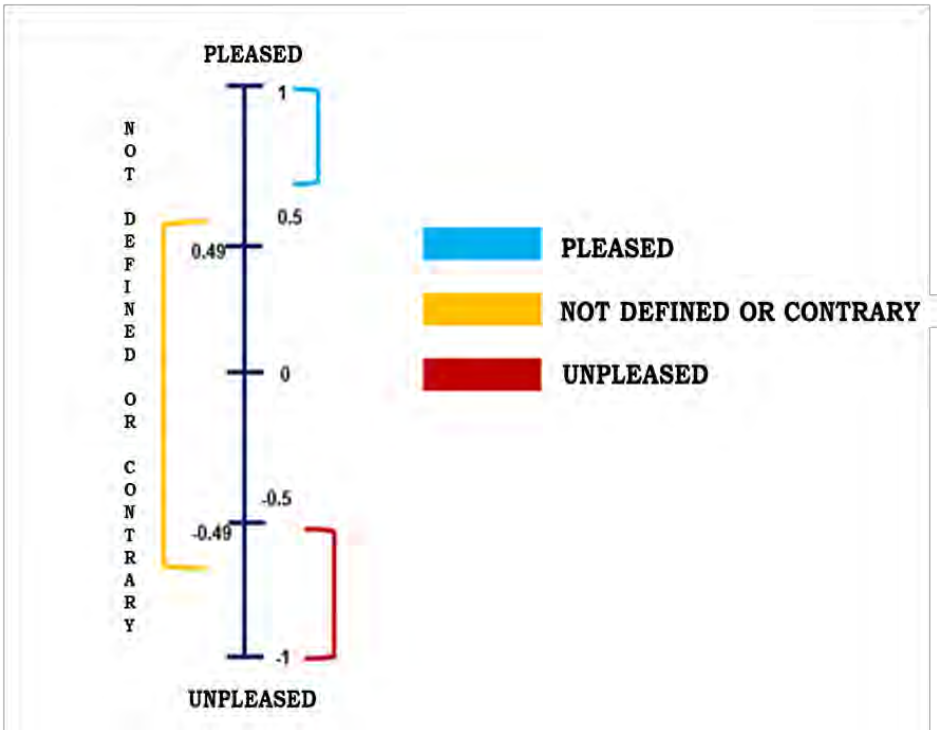

Figure 1. Scale with group satisfaction index.

The proposal to extend the Iadov method with SVN numbers making it easy to use and practical in applications for knowledge management model validation. The inclusions of indetermination allow a more robust and real-world compatible form to represent information in comparison with the typical application of Iadov. The inclusion of the WA operator improves the traditional method allowing to express the importance of the [34] sources of information o expertise of users. The real-world application of the proposal validates the model for knowledge management in the country productive organizations of cocoa of Vinces, Ecuador.

4. Conclusions (authors also should add some future directions points related to her/his research)

In this paper, the neutrosophic Iadov is used, which contributes to an appropriate method for the management of indeterminacy and for taking into account uncertainty in real-world problems and the importance of the users. The Iadov method with the inclusion of the neutrosophic analysis showed applicability and facility of use in the validation of the knowledge management model. Between the advantages concerning the original, it is that it can incorporate the indetermination in a more natural way. Another advantage is that allows the use of aggregation operators, which permits express the importance or the expertise of the users according to the experience or some other criterion.

The final result is of GSI = 0.85. Results that validate the satisfaction with the model for knowledge management in the cocoa producing peasant organizations of Vinces. Future works will concentrate on including the modeling of knowledge in the proposal trough neutrosophic cognitive mapping extending previous works from [35-38].

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Conflicts of Interest

The authors declare no conflict of interest.

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Neutrosophic Labeling Graph

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Abstract: In this paper, some new connectivity concepts in neutrosophic labeling graphs are portrayed. Definition of neutrosophic strong arc, neutrosophic partial cut node, Neutrosophic Bridge and block are introduced with examples. Also, neutrosophic labeling tree and partial intuitionistic fuzzy labeling tree is explored with interesting properties.

Keywords: neutrosophic graphs, neutrosophic labeling graphs, neutrosophic labeling tree, partial neutrosophic labeling tree.

1. Introduction

Fuzzy is a concept characterized by three basic criteria namely imprecision, uncertainty, and degrees of truthfulness of values. These criteria has been introduced by Zadeh in 1965 to give the detailed description for linguistic variables, representing size, age and temperature etc., used for system input and output. Once we collect the set of categories of the linguistic variables, it defines a fuzzy set along with the membership function developed for each member in that set. The membership function always takes values in the interval [0, 1] and this range is referred to as the membership grade or degree of membership. Intuitionistic fuzzy set, an extension of fuzzy set, has been introduced by Atanassov (1986). Intuitionistic fuzzy set has been found to be more efficient in dealing with vagueness and ambiguity. It is characterized by a membership function ($\mu_A(x)$) and a non-membership function ($\nu_A(x)$) with their sum being less than or equal to one ($\mu_A(x) + \nu_A(x) \leq 1$). This relaxes the enforced duality $\nu_A(x) = 1- \mu_A(x)$ from fuzzy set theory. Intuitionistic fuzzy set allows one to address the positive and negative side of an imprecise concept separately.

Neutrosophic set is simply an extension of intuitionistic fuzzy set and fuzzy set. This concept came into existence when Floretic Smarandache, the professor of mathematics from university of New Mexico, proposed a paper in 1998 [26, 27]. He characterized the Neutrosophic set by using 3 values namely a truth-membership degree, an indeterminacy-membership degree and a falsity membership degree, whose sum lies between 0 and 3. This concept has been successfully applied to many fields such as medical diagnosis problem, decision making problem, etc. The graphical representation of fuzzy set was developed by Rosenfeld in1973. This induces several graphical concepts based of fuzzy-graph logics. Ansari in 2013 extended the fuzzy logic to neutrosophic logic and also developed neutrosophication of fuzzy models. In 2016, Rajab Ali Borzooei defined some basic concepts in fuzzy labeling graph and in 2017, Akram and shahzadi introduced the neutrosophic graph. Recently many applications of neutrosophic sets were developed by Abdel Basset [1-6] and Broumi [14-22].

In this paper, we extend the fuzzy- graph logics by introducing the neutrosophic labeling graphs which has a scope in the entire real world field which involves decision making problems. The new criteria that define neutrosophic labeling tree were introduced.

2. Preliminaries

Definition 2.1: A neutrosophic graph is of the form $G* = (V, \sigma, \mu)$ where $\sigma = (T_1, I_1, F_1)$

and $\mu = (T_2, I_2, F_2)$

(i) $V = \{v_1, v_2, v_3, \cdots, v_n\}$ such that $T_1: V \to [0, 1]$, $I_1: V \to [0, 1]$ and $F_1 : V \to [0, 1]$ denote the degree of truth-membership function, indeterminacy-membership function and falsity-membership function of the vertex $v_i \in V$ respectively, and $0 \leq T_1(v) + I_1(v) + F_1(v) \leq 3$ $\forall$ $v_i \in V$ (i=1, 2, 3….n).

(ii) $T_2 : V \times V \to [0, 1]$, $I_2 : V \times V \to [0, 1]$ and $F_2 : V \times V \to [0, 1]$, where $T_2(v_i, v_j)$, $I_2(v_i, v_j)$ and $F_2(v_i, v_j)$ denote the degree of truth-membership function, indeterminacy membership function and falsity-membership function of the edge (v_i, v_j) respectively such that for every (v_i, v_j),

$T_2(v_i, v_j) \leq \min \{T_1(v_i), T_1(v_j)\}$,

$I_2(v_i, v_j) \leq \min \{I_1(v_i), I_1(v_j)\}$,

$F_2(v_i, v_j) \leq \max \{F_1(v_i), F_1(v_j)\}$, and $0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$.

Example 2.2: Let $G^* = (V, \sigma, \mu)$ be an neutrosophic graph, where $\sigma = (T_1(v), I_1(v), F_1(v))$,

$\mu = (T_2(v_i, v_j), I_2(v_i, v_j), F_2(v_i, v_j))$. Let the vertex set be $V= \{v_1, v_2, v_3, v_4, v_5\}$ and $\sigma(v_1) = (0.5,0.3,0.4)$, $\sigma(v_2) = (0.2,0.2,0.6)$, $\sigma(v_3) = (0.6,0.45,0.3)$, $\sigma(v_4) = (0.4,0.8,0.35)$, $\sigma(v_5) = (0.4,0.6,0.5)$, $\mu(v_1, v_2) = (0.1,0.2,0.5)$, $\mu(v_2, v_3) = (0.15,0.1,0.5)$, $\mu(v_3, v_4) = (0.3,0.35,0.3)$, $\mu(v_4, v_5) = (0.35,0.5,0.45)$ $\mu(v_5, v_1) = (0.4,0.2,0.4)$, $\mu(v_5, v_2) = (0.15,0.15,0.4)$, $\mu(v_1, v_4) = (0.3,0.25,0.3)$, $\mu(v_4, v_2) = (0.05,0.1,0.4)$.

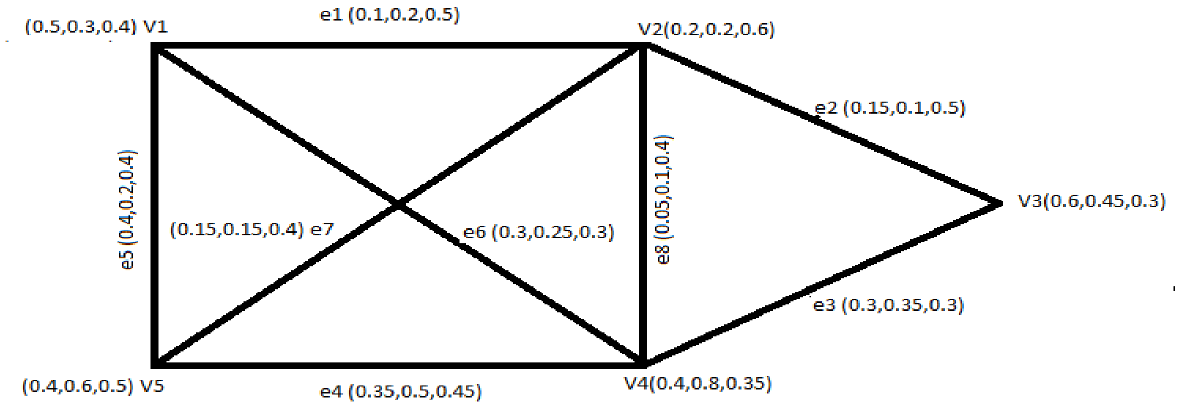

Fig 1: **NEUTROSOPHIC GRAPH**

3. Neutrosophic labeling graph

In this section we introduce neutrosophic labeling graph, neutrosophic labeling subgraph, connectedness in neutrosophic labeling graph, neutrosophic partial cut node and neutrosophic partial bridge and investigated some of the properties with suitable examples.

Definition 3.1: A neutrosophic graph $G^* = (V, \sigma, \mu)$ is said to be an neutrosophic labeling graph if $T_1 : V \to [0, 1]$, $I_1 : V \to [0, 1]$ $F_1 : V \to [0, 1]$ and $T_2 : V \times V \to [0, 1]$, $I_2 : V \times V \to [0, 1]$, $F_2 : V \times V \to [0, 1]$ is bijective such that truth-membership function, indeterminacy-membership function and falsity-membership of the vertices and edges are distinct and for every edges (v_i, v_j),

$T_2(v_i, v_j) \leq \min\{T_1(v_i), T_1(v_j)\}$,

$I_2(v_i, v_j) \leq \min\{I_1(v_i), I_1(v_j)\}$,

$F_2(v_i, v_j) \leq \max\{F_1(v_i), F_1(v_j)\}$, and

$0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$

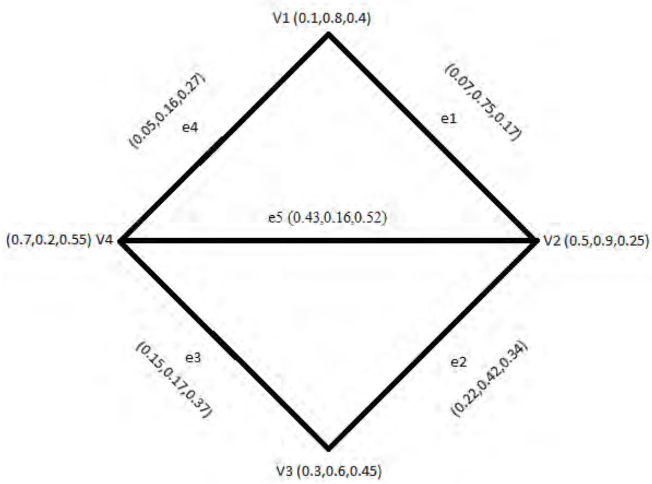

Fig.2 NEUTROSOPHIC LABELING GRAPH

Example 3.2: In the above figure 2, all the vertices and edges have distinct values for membership, indeterminacy and falsity. Therefore σ , I and μ are one to one and onto functions.

Definition 3.3: Neutrosophic labeling graph R= (V, α, β) where $\alpha = (\alpha_1(c), \alpha_2(c), \alpha_3(c))$ and $\beta = (\beta_1(c,d), \beta_2(c,d), \beta_3(c,d))$ is called an neutrosophic labeling subgraph of $G^* = (V, \sigma, \mu)$ where $\sigma = (T_1(c), I_1(c), F_1(c))$ and $\mu = (T_2(c,d), I_2(c,d), F_2(c,d))$, if $\alpha_1(c) \leq T_1(c)$, $\alpha_2(c) \leq I_1(c)$, $\alpha_3(c) \geq F_1(c)$ for all $c \in V$ and $\beta_1(c,d) \leq T_2(c,d)$, $\beta_2(c,d) \leq I_2(c,d)$, $\beta_3(c,d) \leq F_2(c,d)$ for all edges (c,d).

Theorem 3.4: If R=(V, α, β) is an neutrosophic labeling subgraph of $G* = (V, \sigma, \mu)$, then

$\beta_1^{\infty}(c,d) \leq T_2^{\infty}(c,d)$, $\beta_2^{\infty}(c,d) \leq I_2^{\infty}(c,d)$, $\beta_3^{\infty}(c,d) \geq F_2^{\infty}(c,d)$, for all c,d ∈ V.

Proof: Let $G* = (V, \sigma, \mu)$ be any neutrosophic labeling graph and R = (v, α, β) be its subgraph. Let (c,d) be any path in G* then its strength be $((T_2^{\infty}(c,d), I_2^{\infty}(c,d), F_2^{\infty}(c,d))$. Since R in a subgraph of G* ,then $\alpha_1(c) \leq T_1(c)$, $\beta_1(c,d) \leq T_2(c,d)$, $\alpha_2(c) \leq I_1(c)$, $\beta_2(c,d) \leq I_2(c,d)$, $\alpha_3(c) \geq F_1(c)$ and $\beta_3(c,d) \geq F_2(c,d)$, which implies that $\beta_1^{\infty}(c,d) \leq T_2^{\infty}(c,d)$, $\beta_2^{\infty}(c,d) \leq I_2^{\infty}(c,d)$, $\beta_3^{\infty}(c,d) \geq F_2^{\infty}(c,d)$, for all c,d ∈ V.

Theorem 3.5: The union of any two neutrosophic labeling graph $G^* = (V^1, \sigma_1, \mu_1)$ and $G^{**} = (V^{11}, \sigma_2, \mu_2)$ where $\sigma_1 = (T_1(c), I_1(c), F_1(c))$, $\mu_1 = (T_2(c,d), I_2(c,d), F_2(c,d))$, $\sigma_2 = (T_3(c), I_3(c), F_3(c))$, $\mu_2 = (T_4(c,d), I_4(c,d), F_4(c,d))$, is also an neutrosophic labeling graph, if the Truth membership, Indeterminacy, Falsity membership values of the edges between G^* and G^{**} are distinct.

Proof: Let $G^* = (V^1, \sigma_1, \mu_1)$ and $G^{**} = (V^{11}, \sigma_2, \mu_2)$ be any two neutrosophic labeling graph such that, the Truth membership, Indeterminacy, Falsity membership values of the edges between G^* and G^{**} are distinct and $G = (V, \sigma, \mu)$, where $\sigma = (\sigma_M, \sigma_I, \sigma_N)$ and $\mu = (\mu_M, \mu_I, \mu_N)$, be the union of two neutrosophic labeling graph G^* and G^{**}.

To prove: G is a Neutrosophic labeling graph.

Now,

For Truth membership values $\sigma_M(c) = \begin{cases} T_1(c), & if\ c \in V^1 - V^{11} \\ T_3(c), & if\ c \in V^{11} - V^1 \\ T_1(c) \vee T_3(c), & if\ c \in V^1 \bigcap V^{11} \end{cases}$

For Indeterminacy values $\sigma_I(c) = \begin{cases} I_1(c), & if\ c \in V^1 - V^{11} \\ I_3(c), & if\ c \in V^{11} - V^1 \\ I_1(c) \vee I_3(c), & if\ c \in V^1 \bigcap V^{11} \end{cases}$

For Falsity membership values $\sigma_F(u) = \begin{cases} F_1(u), & if\ u \in V^1 - V^{11} \\ F_3(u), & if\ u \in V^{11} - V^1 \\ F_1(u) \wedge F_3(u), & if\ u \in V^1 \bigcap V^{11} \end{cases}$

Similarly,

For Truth membership values $\mu_M(c,d) = \begin{cases} T_2(c,d), & if\ (c,d) \in E^1 - E^{11} \\ T_4(c,d), & if\ (c,d) \in E^{11} - E^1 \\ T_2(c,d) \vee T_4(c,d), & if\ (c,d) \in E^1 \bigcap E^{11} \end{cases}$

For Indeterminacy values $\mu_I(c,d) = \begin{cases} I_2(c,d), & if\ (c,d) \in E^1 - E^{11} \\ I_4(c,d), & if\ (c,d) \in E^{11} - E^1 \\ I_2(c,d) \vee I_4(c,d), & if\ (c,d) \in E^1 \bigcap E^{11} \end{cases}$

For Falsity membership values $\mu_F(c,d) = \begin{cases} F_2(c,d), & if\ (c,d) \in E^1 - E^{11} \\ F_4(c,d), & if\ (c,d) \in E^{11} - E^1 \\ F_2(c,d) \wedge F_4(c,d), & if\ (c,d) \in E^1 \bigcap E^{11} \end{cases}$

Thus the Truth membership, Indeterminacy and Falsity membership values of the vertices and edges are distinct. Hence, $G = (V, \sigma, \mu)$ is a Neutrosophic labeling graph.

Definition 3.6: Let $G^* = (V, \sigma, \mu)$ be an neutrosophic labeling graph. The strength of the path P of n edges e_i for i = 1,2,……,n is denoted by S(P) = (S_1(P), S_2(P), S_3(P)) and denoted by $S_1(P) = \min_{1 \le i \le n} T_2(e_i)$, $S_2(R) = \min_{1 \le i \le n} I_2(e_i)$ and $S_3(R) = \max_{1 \le i \le n} F_2(e_i)$.

Definition 3.7: Let G = (V, σ, μ) be a neutrosophic labeling graph. Then for a pair of vertices c,d ∈ V, the strength of connectedness, denoted by $CONN_G(c,d) = (CONN_{1G}(c,d), CONN_{2G}(c,d), CONN_{3G}(c,d))$ and is defined as

$CONN_{1G}(c,d) = \max\{S_1(P)\}$, $CONN_{2G}(c,d) = \max\{S_1(P)\}$ and $CONN_{3G}(c,d) = \min\{S_2(P)\}$, where P is a path connecting the vertices c,d in G. If c and d are isolated vertices of G, then $CONN_G(c,d) = (0, 0)$.

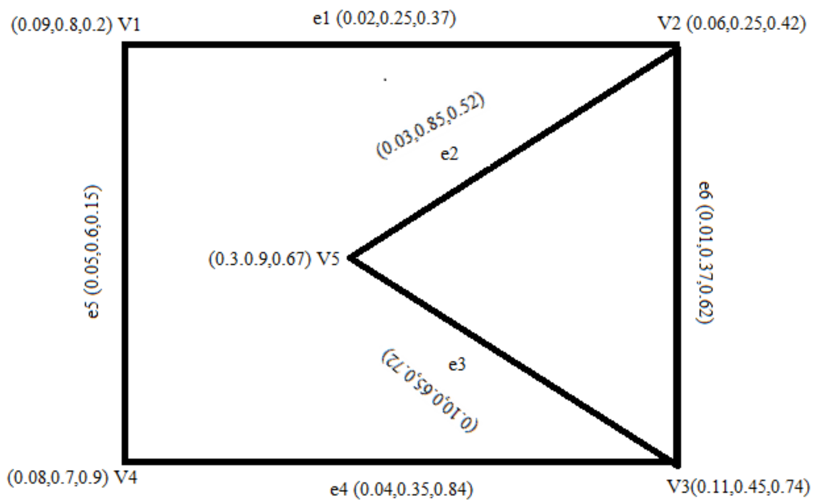

Fig. 3 **CONNECTEDNESS IN NEUTROSOPHIC LABELING GRAPH**

Example 3.8: Figure 3 is an example of neutrosophic labeling graph G having $CONN_G(v_1, v2) = (0.02, 0.75, 0.37)$, $CONN_G(v_1, v_3) = (0.04, 0.6, 0.62)$, $CONN_G(v_1, v_5) = (0.04, 0.65, 0.52)$ and so on.

Proposition 3.9: Let G be an neutrosophic labeling graph and R is an neutrosophic labeling subgraph of G. Then for every pair of vertices c,d ∈ V, we have $CONN_{1R}(c,d) \leq CONN_{1G}(c,d)$, $CONN_{2R}(c,d) \leq CONN_{2G}(c,d)$ and $CONN_{3R}(c,d) \geq CONN_{3G}(c,d)$.

Definition 3.10: If $S_1(P) = CONN_{1G}(c,d)$ $S_2(P) = CONN_{2G}(c,d)$ and $S_3(P) = CONN_{3G}(c,d)$, where P is a path connecting the vertices c,d in the neutrosophic labeling graph G then P is called the strongest path connecting c, d in G.

Definition 3.11: Let G be an neutrosophic labeling graph. A node z is called a neutrosophic partial cut node (Neu p-cut node) of G if there exists a pair of nodes c,d ∈ G such that c $\neq$ d $\neq$ z and $CONN_{1(G-z)}(c,d) < CONN_{1G}(c,d)$, $CONN_{2(G-z)}(c,d) < CONN_{2G}(c,d)$ and $CONN_{3(G-z)}(c,d) > CONN_{3G}(c,d)$

A neutrosophic partial block (Neu p-block) is a neutrosophic labeling graph which is connected and does not contain any Neu p-cut nodes in it.

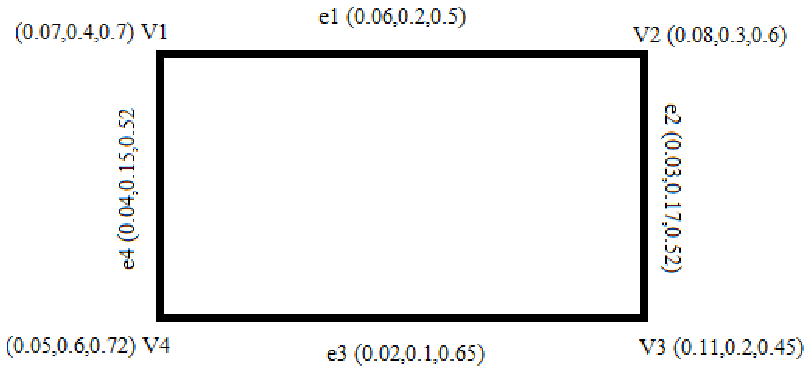

Fig.4 **NEUTROSOPHIC LABELING GRAPH**

Example 3.12 : Let G be an neutrosophic labeling graph, which is shown in above Figure 4.
Node v_1 is a neutrosophic partial cut node, since
$CONN_{1(G-v_1)}(v_2, v_4) = 0.02 < 0.04 = CONN_{1G}(v_2, v_4)$,
$CONN_{2(G-v_1)}(v_2, v_4) = 0.1 < 0.15 = CONN_{2G}(v_2, v_4)$ and
$CONN_{3(G-v_1)}(v_2, v_4) = 0.65 > 0.55 = CONN_{3G}(v_2, v_4)$.
Similarly, Node v_2 is a neutrosophic partial cut node, since,
$CONN_{1(G-v_2)}(v_1, v_3) = 0.02 < 0.03 = CONN_{1G}(v_1, v_3)$,
$CONN_{2(G-v_2)}(v_1, v_3) = 0.1 < 0.17 = CONN_{2G}(v_1, v_3)$ and
$CONN_{1(G-v_2)}(v_1, v_3) = 0.65 > 0.52 = CONN_{3G}(v_1, v_3)$.

Definition 3.13: Let G be an neutrosophic labeling graph. An arc e = (c,d) is called neutrosophic partial bridge (Neu p- bridge) if $CONN_{1(G-e)}(c,d) < CONN_{1G(c,d)}$, $CONN_{1(G-e)}(c,d) < CONN_{1G(c,d)}$ and $CONN_{3(G-e)}(c,d) > CONN_{3G(c,d)}$.

A neutrosophic p-bridge is said to be a neutrosophic partial bond (Neu p-bond) if $CONN_{1(G-e)}(x, y) < CONN_{1G(x, y)}$, $CONN_{2(G-e)}(x, y) < CONN_{2G(x, y)}$, $CONN_{3(G-e)}(x, y) > CONN_{3G(x, y)}$ with at least one of x or y different from both u and v and is said to be a neutrosophic partial cut bond (p-cut bond) if both x or y are different from u and v.

Example 3.14 : In the Figure 4, for all arcs except the arc (v_4, v_3) are neutrosophic partial bridge. In specific particular, arc (v_2, v_3) is a neutrosophic partial cut bond, since
$CONN_{1(G-(v2,v3))}(v_3, v_4) = 0.03 < 0.06 = CONN_{1G(v3, v4)}$, $CONN_{2(G-(v2,v3))}(v_3, v_4) = 0.03 < 0.06 = CONN_{2G(v3, v4)}$ and $CONN_{3(G-(v2,v3))}(v_3, v_4) = 0.55 > 0.5 = CONN_{3G(v3, v4)}$.

4. Types of Arcs in a Neutrosophic Labeling Graph

In this section we discussed some types of neutrosophic α strong, δ strong, β strong arcs.

Definition 4.1: If all the arcs of cycle C in the neutorsophic labeling graph G are strong, then C is called the strong cycle in G.

Definition 4.2: An arc (n,m) of G is called neutrosophic α strong if $T_2(c,d) > CONN_{1(G-(n,m))}(n,m)$, $I_2(c,d) > CONN_{2(G-(n,m))}(n,m)$ and $F_2(c,d) < CONN_{3(G-(n,m))}(n,m)$

Definition 4.3: An arc (n,m) of G is called neutrosophic δ strong if $T_2(c,d) < CONN_{1(G-(n,m))}(n,m)$, $I_2(c,d) < CONN_{2(G-(n,m))}(n,m)$ and $F_2(c,d) > CONN_{3(G-(n,m))}(n,m)$

Definition 4.4: An arc (n,m) of G is called neutrosophic β strong if $T_2(c,d) = CONN_{1(G-(n,m))}(n,m)$, $I_2(c,d) = CONN_{2(G-(n,m))}(n,m)$ and $F_2(c,d) = CONN_{3(G-(n,m))}(n,m)$

Definition 4.5: An n-m path P in G is called a strong n-m path if all the arcs of P are strong. In particular, if all the arcs of P are neutrosophic α-strong, then P is called neutrosophic α strong path. Obviously, An arc (n,m) is strong if it is neutrosophic α-strong, if (n,m) is strong arc, then n and m are said to be strong neighbors of each other.

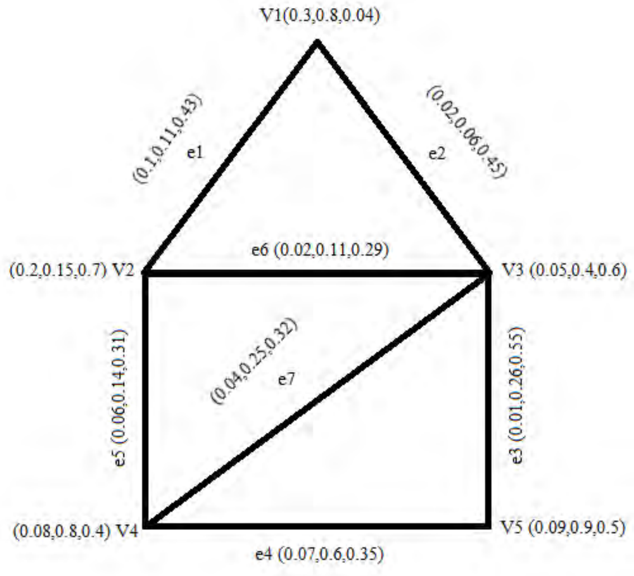

Fig.5 **NEUTROSOPHIC LABELING GRAPH**

Example 4.6: In the above figure 5, the arcs (V_1, V_2), (V_2, V_4), (V_4, V_5) are neutrosophic α strong, the arc (V_3, V_4) is neutrosophic δ strong, the arcs (V_1, V_3) is neutrosophic β strong and $P = V_1V_2V_4V_5$ is a neutrosophic α strong path.

Theorem 4.7. Let G be a connected neutrosophic labeling graph and let r and s be any two nodes in G. Then there exists a strong path from c to do.

Proof.

Assume that $G = (V, \sigma, \mu)$ is a connected neutrosophic labeling graph. Let r and s be any two nodes of G. If the arc (r, s) is strong, then there is nothing to prove. Otherwise, either (r, s) is a δ arc or there exist a path of length more than one from r to s.

In the first case, we can find a path P (say) such that $S_1(P) > T_2(r,s)$, $S_2(P) > I_2(r,s)$ and $S_3(P) < F_2(r,s)$ In either case, the path from c to d of length more than one. If some arc on this path is not strong, replace it by a path having more strength. Hence P is a path from r to s, whose arcs are strong and thus P is a strong path from r to s.

Theorem 4.8: A connected neutrosophic labeling graph G is a neutrosophic partial block if and only if any two nodes x, y ∈ V such that (x y) is not neutrosophic α strong are joined by two internally disjoint strongest path.

Proof:

Suppose that G is a neutrosophic partial block. Let x, y ∈ V such that (x, y) is not neutrosophic α strong arc. Now, we shall prove that there exist two internally disjoint strongest x–y paths. If not, i.e there exist exactly one internally disjoint strongest x-y path in G. Since (x, y) is not α strong, length of all strongest x - y path must be at least two. Also for all strongest x - y paths in G, there must be a common vertex. Let z be such node in G. Then $CONN_{1(G-z)}(x, y) > CONN_{1G(x, y)}$, $CONN_{2(G-z)}(x, y) > CONN_{2G(x, y)}$ and $CONN_{3(G-z)}(x, y) < CONN_{3G(x, y)}$, which contradict the fact that G has no P-cut nodes. Hence there exist two internally disjoint strongest x - y paths.

Conversely, let any two nodes of G are joined by two internally disjoint strongest paths. Let w be a node in G. For any pair of nodes c,d $\in$ V such that u $\neq$ v $\neq$ w, there always exists a strongest path not containing w. So, we cannot be a neutrosophic p-cut node. Hence G is a neutrosophic partial block.

5. Neutrosophic Labeling Tree

In this section we define neutrosophic labeling tree as follows

Definition 5.1: A graph $G^* = (V, \sigma, \mu)$ where σ (v)= (T_1(r), I_1(r), F_1(r)) and μ = (T_2(r,s) , I_2(r,s), F_2(r,s)) is said to be neutrosophic labeling tree, if it has neutrosophic labeling graph and an neutrosophic spanning subgraph M= (V, α, β) where α(r)= (α_1(r), α_2(r), α_3(r)) and β= (β_1(r,s), β_2(r,s), β_3(r,s)) which is a tree, where for all arcs (r, s) not in $T_2(r,s) < \beta_1^\infty(r,s)$, $I_2(r,s) < \beta_2^\infty(r,s)$, $F_2(r,s) > \beta_3^\infty(r,s)$.

Theorem 5.2: If $G^* = (V, \sigma, \mu)$ is a neutrosophic labeling tree, then the arcs of neutrosophic spanning subgraph M= (V, α, β) are neutrosophic bridges of G^*.

Proof: Let $G^* = (V, \sigma, \mu)$ be a neutrosophic labeling tree and M= (V, α, β) be its spanning subgraph. Let (r, s) be an arc in M. Then $\beta_1^\infty(r,s) < T_2(r,s) \leq T_2^\infty(c,d)$, $\beta_2^\infty(r,s) < I_2(r,s) \leq I_2^\infty(r,s)$, $\beta_3^\infty(r,s) > F_2(r,s) \geq F_2^\infty(r,s)$, which implies that the arc (r, s) is an neutrosophic bridge of G^*. Since the arc (r, s) is an arbitrary, then the arcs of M are the neutrosophic bridges of G^*.

Theorem 5.3: Every neutrosophic labeling graph is a neutrosophic labeling tree.

Proof: Let $G^* = (V, \sigma, \mu)$ be a neutrosophic labeling graph. Since is μ is bijective, each and every vertex of G^* will have at least one arc as neutrosophic bridge. Therefore, the spanning subgraph M will exist, such that whose arcs are neutrosophic bridges. Hence, by above theorem 5.2, every neutrosophic labeling graph is an neutrosophic labeling tree.

6. Partial Neutrosophic Labeling Tree

Finally, we define partial neutrosophic labeling tree and discussed some of the properties.

Definition 6.1: A connected neutrosophiclabeling graph $G^* = (V, \sigma, \mu)$ is called a partial neutrosophic labeling tree if G^* has a spanning subgraph M= (V, α, β) which is a tree, where for all arc (r, s) of G^* which are not in M, $CONN_{1G}(r,s) > T_2(r,s)$, $CONN_{2G}(r,s) > I_2(r,s)$ and $CONN_{3G}(r,s) < F_2(r,s)$.

If all the components of disconnected graph G^* satisfies above condition, then G^* is called a partial forest.

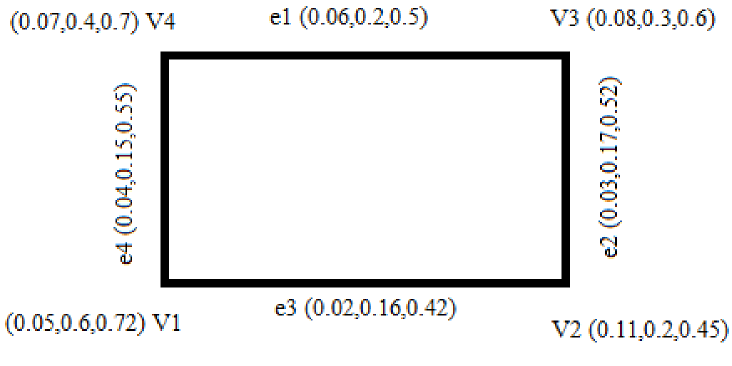

Fig.6 **PARTIAL NEUTROSOPHIC LABELING TREE**

Example 6.2: If we remove the arc (v_1, v_2) figure 6, we will get a spanning tree M. Also for the arc (v_1, v_2), $CONN_{1G}$ (v_1, v_2) = 0.03 > 0.02 = T_1 (v_1, v_2), CONN2G (v_1, v_2) = 0.16 > 0.15 = I_1 (v_1, v_2), and $CONN_{3G}$ (v_1, v_2) = 0.42 < 0.55 = F_1 (v_1, v_2). Thus figure 6 is an example of partial neutrosophic labeling tree.

Theorem 6.3: Let G^* = (V, σ, μ) be a connected neutrosophic labeling graph. Then the necessary and sufficient condition for G^* to be a neutrosophic partial tree is that , for any cycle C in G^*, there must exists an arc γ = (r, s) such that $T_2(\gamma) < CONN_{1(G^*-\gamma)}(r, s)$, $I_2(\gamma) < CONN_{2(G^*-\gamma)}(r, s)$ and $F_2(\gamma) > CONN_{3(G^*-\gamma)}(r, s)$, where $G^*- \gamma$ is the subgraph of G^* obtained by deleting the arc γ from G^*.

Proof: Assume that G^* is a connected neutrosophic labeling graph. If G^* has no cycle, then G^* itself behave as a partial tree.

If G^* has a cycle C and let γ = (r,s) be an arc of C with minimum weightage for truth membership, indeterminacy and maximum weightage for falsity membership in G^*. Now, remove the arc γ = (r,s) from G^* and continue this process until we get a tree M which is the subgraph of G^*.

The arcs deleted in each process were stronger than the one which removed preceding process. Since M is a tree and the arc γ = (r, s) having minimum membership value, minimum indeterminacy and maximum falsity membership value from the arcs of a cycle in G^* does not belongs to M, we can conclude that there exists a path from r to s whose membership value greater than T_2(r, s), indeterminacy value greater than I_2(r, s) and falsity membership value less than F_2(r, s), and that does not involve (r, s) or any arcs deleted prior to it. It contains only the arcs of M. Thus G^* is a partial neutrosophic labeling tree.

Conversely, if G^* is a partial neutrosophic labeling tree and P is cycle, then some arc γ = (r, s) of P does not belong to M. Thus by definition we have $T_2(\gamma) < CONN_{1(G^*-\gamma)}(r, s)$, $I_2(\gamma) < CONN_{2(G^*-\gamma)}(r, s)$ and $F_2(\gamma) > CONN_{3(G^*-\gamma)}(r, s)$.

Theorem 6.4: Between any two nodes of G^*, If there exist at most one strongest path, then G^* must be a partial forest.

Proof:

Assume that G^* is not a partial forest. Then G^* must contain a cycle C such that $T_2(r, s) \geq CONN_{1G}(r, s)$, $I_2(r, s) \geq CONN_{2G}(r, s)$ and $F_2(r, s) \leq CONN_{3G}(r, s)$ for all arcs γ = (r, s) of the cycle C. Thus, γ = (r, s) is the strongest path from r to s. If we choose (r, s) to be a weakest arc of C, it follows that the rest of the cycle C is also a strongest path from r to s, which is a contradiction. Hence, G^* must be a partial forest.

Theorem 6.5: If G^* is a not a tree but partial tree, then has G^* at least one arc $\gamma = (r, s)$ for which $T_2(r, s) < CONN_{1G}(r, s)$, $I_2(r, s) < CONN_{2G}(r, s)$ and $F_2(r, s) > CONN_{3G}(r, s)$.

Proof:

Assume that G^* is a partial tree, then by definition of partial tree, G^* must contain a spanning tree M such that $T_2(r, s) < CONN_{1G}(r, s)$, $I_2(r, s) < CONN_{2G}(r, s)$ and $F_2(r, s) > CONN_{3G}(r, s)$, for all arcs $\gamma = (r, s)$ not in M. Thus has G^* at least one arc $\gamma = (r, s)$ (since G^* is not a tree), which satisfies the above condition.

Theorem 6.6: If M is the spanning tree of the partial tree G^*, then the arcs of M are the partial bridges of G^*.

Proof:

Let $\gamma = (r, s)$ be an arc in M. Since, M is a spanning tree, this arc γ form a unique path between the nodes r and s in M.

If G^* has no other paths between r and s, then clearly $\gamma = (r, s)$ is a bridge of G^* and hence it is a partial bridge of G^*.

On the other hand, if P is a path connecting r and s in G^*, then P must contain an arc $\gamma = (r, s)$ which is not in M such that $T_2(r, s) < CONN_{1G}(r, s)$, $I_2(r, s) < CONN_{2G}(r, s)$ and $F_2(r, s) > CONN_{3G}(r, s)$. Then $\gamma = (r, s)$ is not a weakest arc of any cycle in G^* and hence (r, s) is a partial bridge.

7. Conclusion

Connectivity concepts are the major key in neutrosophic graph problems. This paper presented new connectivity concepts in neutrosophic labeling graphs. Definition of neutrosophic strong arc, neutrosophic partial cut node, Neutrosophic Bridge and block based on connectivity concepts of intuitionistic fuzzy graph was introduced. The neutrosophic labeling tree and partial neutrosophic labeling tree concepts were established with interesting properties on them. We extended our research work to bipolar neutrosophic graph, covering problem on neutrosophic graphs, Chromatic number in neutrosophic graphs, Colouring of neutrosophic graphs.

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Conflicts of Interest

The authors declare no conflict of interest.

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An Approach for Study of Traffic Congestion Problem Using Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps-the Case of Indian Traffic

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Abstract: The aim of this paper is to find the reasons for traffic congestion problem and its solution using Neutrosophic Cognitive Maps (NCMs) and Fuzzy Cognitive Maps (FCMs). Fuzzy theory only measures the grade of membership but fuzzy theory has failed to characteristic the perception when the relations between concepts in problems are indeterminate. Addition of concepts of indeterminate situation with fuzzy logic forms the neutrosophic logic. Since, some of the reasons for traffic congestions are indeterminate we use Neutrosophic Cognitive Maps to find a solution. The discussion is based on Indian road scenario.

Keywords: Fuzzy Cognitive Maps; Neutrosophic Cognitive Maps; Traffic congestion problem; Connection matrix.

1. Introduction

Road traffic congestion is a main problem in most of the cities in India, particularly in developing regions resulting in unexpected waiting time, fuel wastage and unnecessary tension. Congestion in the cities has increased considerably over the previous 10 years because of increase in no of private vehicles in the road. As a result of traffic congestion, people are suffering economically, physically and even mentally. Identification of traffic congestion is the initial step and essential guidance for selecting appropriate measures. In this paper, our goal is to determine the main reasons for traffic congestion using Neutrosophic Cognitive Maps(NCMs) which is an extension of Fuzzy Cognitive Maps (FCMs) with an inclusion of indeterminacy. FCMs mainly find the relationship/non-relationship between two nodes or concepts but fail to find the relation between two conceptual nodes when the relationship is an indeterminate one. FCMs are suitable when the data is unsupervised. Both FCM and NCM are based on the opinion of experts.

The reason for using NCMs to identify the main reason for traffic congestion is that some of the concepts in traffic are indeterminate. For instance, political leaders visit, unannounced meetings in the main road, sudden diversions due to heavy downpour are some of the concepts are indeterminate reasons for the traffic in India. In this paper we will mathematically find the main reasons for traffic congestions and we will give some realistic possible suggestions based on the results of FCMs and NCMs to control the traffic. This paper is structured in eight sections. The background and motivation of this study is discussed in section 2. The fundamental concepts of

FCMs and NCMs are given in section 3. In Section 4 an experimental example is detailed. Then, in fifth section the comparison of expert's opinion is analysed and in Section 6 conclusions are exposed. Finally in the seventh section suggestions are given to reduce the traffic congestion based on the conclusion of NCMs and FCMs.

2. Background and Motivation

Zadeh [26] introduced the concept of fuzzy set theory in 1965. In crisp set, membership function μ_A maps the set of all elements in the universal set $'X'$ to the set $\{0, 1\}$, whereas in fuzzy set each element in $'X'$ is mapped to the set [0,1] by the membership function μ_A. Fuzzy set is 'vague boundary set' when compared with crisp set. Table.1 helps to understand the basic concepts of fuzzy set and neutrosophic set in a better way.

Table 1: Comparison of Fuzzy set and Neutrosophic set

Fuzzy set	Neutrosophic set
Fuzzy set gives only the degree of membership of an element $x \in A$.	The Neutrosophic set gives the degrees of membership, indeterminacy, and non-membership of the element $x \in A$.
Example:$\mu(0.3) \in A$ means probability of 30% $'x'$ belong to the set A.	Example: $\mu(0.5,0.3,0.2) \in A$ means probability of 50% $'x'$ belong to the set A 20% $'x'$ is not in A and 30% is undecided. Also we say 50% there will be a traffic tomorrow, 20% no traffic and 30% is indeterminate.
In more practical example, we say there will be a chance of 30% traffic tomorrow in the city. Here the degree of non-membership funcion is not discussed.	
Max,Min operations in Fuzzy sets	Operations are entirely different.
Example: For any two fuzzy sets A and B in X their union is defined by the membership function $\mu_{A\cup B} = \max(\mu_A(x), \mu_B(x)) \forall x \in X$.	Example:For any two neutrosophic sets A and B, $\mu(T_1, I_1, F_1) \in A$ and $\mu(T_2, I_2, F_2) \in B$ $then$ $\mu((T_1 + T_2) - (T_1 * T_2)), (I_1 + I_2) - (I_1 * I_2), (F_1 + F_2) - (F_1 * F_2)) \in A \cup B$.
In fuzzy theory,fuzzy numbers are used. Example:Triangular fuzzy number,trapezoidal fuzzy etc.	In neutrosophic theory,neutrosophic numbers are used denoted by $a + Ib$ where $a, b \in R$. Example: Trapezoidal neutrosophic number.

FCM is a combination of fuzzy logic and cognitive mapping. Fuzzy cognitive map was introduced by Bart kosko [11] in 1965 as an extension of cognitive maps, powerful equipment for modelling of dynamical systems. As a data representation and logic technique, it depicts a system in a structure that corresponds strongly to the way humans observe it.

Due to its simplicity, FCM was applied to many diverse scientific areas including medicine [16,22],software engineering [21], transportation [24] and so on. Many methods of FCM modelling and/or extension of FCM for modelling dynamical systems have been proposed in [4,5,6,7,8,9,14,15,17,19,22,.23]. Smarandache and Vasantha Kandasamy W.B[25] introduced the concept of indefinite statistics called Neutrosophic Cognitive Maps (NCMs) as generalizations of FCMs. Like FCMs, NCMs also many applications in practical life. We listed few here. Abdel-Basset et al [1] used NCMs to solve the transition difficulties of IoT-based enterprises. Real time applications of NCMs is given in [2,3,12,13,20]. Kalaichelvi et al[10] used NCMs to identify the problems faced by girl students who got married during the period of study. In another applications, Rahunathan Anitha et al. [18] used NCMs for raga classifications using musical features. This is the first approach used NCMs in transportation field.

3. Fundamental concepts of FCMs and NCMs

A directed graph representing concepts like policies, events etc as nodes and causalities as edges is FCM denoted as $(C_{1,}C_2, C_3 \dots C_n)$. The edge weights between the concepts denote the causal relationship between them. Weight $e_{ij} = 1$ denotes increase (or decrease) leading to a corresponding increase (or decrease) in the other. Weight $e_{ij} = -1$ means vice versa; weight $e_{ij} = 0$ means no relation between them. Thus edge weight is from the set $\{0,1,-1\}$. Weights of the directed edges are denoted by the connection matrix $M = (e_{ij})$, with diagonal entries as zero. The indeterminacy between the concepts cannot be captured by FCMs. In such circumstances Neutrosophic Cognitive Map (NCM) can be used. NCM is similar to FCM; $e_{ij} = I$ if the relation or effect of C_i on C_j is an indeterminate. Dotted lines denote indeterminacy of an edge between two vertices. The neutrosophic adjacency matrix is $N(E)$. To derive conclusions from the FCM, the instantaneous behaviour of each node is given as an input vector $A = (a_1, a_2, \cdots, a_n)$ where $a_i \in \{0,1\}$; 0 represents OFF and 1 represents ON position. The hidden pattern is the equilibrium state of the FCM. If the equilibrium state is a unique state vector, then is called fixed point. The dynamical system goes round and round when the causality flows through the edges like a cycle starting with concept $\mathrm{C_i}$ and ending at $\mathrm{C_i}$ when $\mathrm{C_i}$ is switched ON.

In order to find the hidden pattern, the instantaneous input vector $A_1 = (a_1, a_2, \cdots, a_n)$ is passed into a dynamical system i.e. FCM or NCM. This is done by multiplying A with matrix E or $N(E)$. Let us consider $N(E)$. Let. $A * N(E) = (b_1, b_2, \dots, b_n)$. With the threshold operation, b_i is replaced by 1 if $b_i > k \; and \; b_i \; by \; 0 \; b_i < k$ (k-a suitable positive integer) and b_i by I if b_i is not an integer. This vector is further updated by making the corresponding entries as 1 for the concepts in the ON position of the input. The resultant vector after thresholding and updating is A_2. This procedure is repeated till we get a limit cycle or a fixed point.

The pseudo code for the Traffic Congestion Problem is

- Collect the concepts (nodes) for the Traffic congestion problem.
- Obtain the connection square matrix E ,$N(E)$ and the corresponding graph, neutrosophic graph through expert opinion.
- Set the concept C_i (i=1, 2, 3,…, n) in ON-State.
- Multiply C_i (i=1, 2, 3,…, n) with E , $N(E)$ and threshold value is calculated by assigning 1 to the first state and for the values > 0 to get C_2.
- Multiply C_2 with E ,$N(E)$ and repeat the procedure to get the fixed point.
- Similarly proceed the above process for the remaining state vector and find the hidden pattern and the indeterminacy in the traffic congestion problem.

Both FCM and NCM are based on experts' opinion. To avoid biasness, it is essential to consider more than one expert. Now we will see the difference between the FCMs and NCMs in Table 2.

Table 2: Comparison of Neutrosophic cognitive maps and Fuzzy cognitive maps

Neutrosophic Cognitive Maps	Fuzzy Cognitive Maps
In neutrosophic cognitive maps we have the possibility to consider that the relation between two vertices is indeterminate (unknown), denoted by "I".	We are not having such concepts in fuzzy cognitive maps.
NCMs cannot be applied for all unsupervised data. NCM has meaning only when the relation between at least two concepts C_i and C_j are indeterminate.	Fuzzy cognitive maps are applicable to all unsupervised datas.
Neutrosophic graphs have the values (T, I, F) for vertices and for edges in which the indeterminacy is denoted by dotted lines [20]; whereas NCMs are directed neutrosophic graphs with the weights of the edges are from the set $\{-1,0,1,I\}$.	Fuzzy cognitive maps are directed fuzzy graphs with the edge set belong to {-1,0,1}.

Let M_1 and M_2 be any two FCMs or NCMs working on the same set of concepts. We consider a state vector $X = (a_1, a_2, \dots a_n)$ where $a_i \in \{0,1,I\}$. Let the resultant of X on M_1 and M_2 be Y_1 and Y_2. The Kosko-Hamming distance between them is denoted by $d_k(Y_1, Y_2)$. Using the definition of Kosko-Hamming distance we can find how far two experts have the same opinion or differ upon a given consequential state vector. By this comparison, one can get the variation or the maximum deviated state vectors for a particular concept which can be specially analysed to identify the cause of such variation.

4. Description of the traffic congestion problem

India is a country which is one of the major non-lane road network in the world. The traffic congestions are frequent problem in India. India is one of the quick developing country in the world which have the peak density of public and private vehicles. It is very hard to maintain traffic in India. High traffic congestion problem is the consequence of variable expected and unexpected factors. In this paper we list all the reasons for the traffic congestion problems and we identity the main reasons to control the traffic using FCMs and NCMs. The concepts for the traffic congestion problem are identified. The connection matrices for FCM and NCM are constructed based on the experts opinion.

The different reasons considered to study the traffic congestion problem are:

$C_1 -$ Traffic congestion

$C_2 -$ Increase in no number of private vehicles in the road

$C_3 -$ Damage of roads (construction of drainages, metro train)

C_4 −Present roadwidth conditions (depending on the number of vehicles the road width is not expanded)

$C_5 -$ Special occurrences (such as religious functions, special road meetings, dharnas etc)

$C_6 -$ Sudden signal failure

$C_7 -$ Vehicle parking in main road (due to increase in vehicles and non-availability of parking facilities).

$C_8 -$ Accidents

C_9 −Inadequate enforcement of traffic rules.

The above nine main reasons for the traffic congestion problem we considered for our study. In Figure 1 we give the directed graph as well as the connection square matrix E by the first expert's opinion.

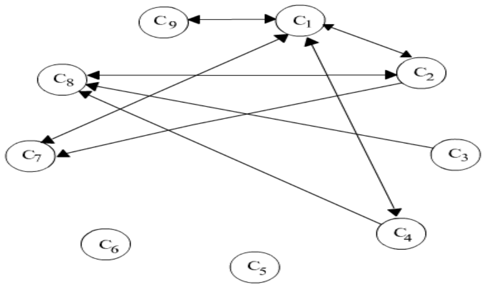

Figure-1: Directed graph given by the first Expert for the traffic congestion problem.

The connection square matrix E to the above directed graph is given below:

$$E = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{1}$$

Case-1: Suppose we take the state vector $A_1 = (1,0,0,0,0,0,0,0,0)$ in ON State. We will see the effect of A_1 on E.

$$\begin{aligned} A_1E &= (0,1,0,1,0,0,1,0,1) \\ &\to (1,1,0,1,0,0,1,0,1) \\ &= A_2. \end{aligned} \tag{2}$$

$$\begin{aligned} A_2E &= (4,1,0,1,0,0,2,2,1) \\ &\to (1,1,0,1,0,0,1,1,1) \\ &= A_3 \end{aligned} \tag{3}$$

$$\begin{aligned} A_3E &= (4,1,0,1,0,0,2,2,1) \\ &\to (1,1,0,1,0,0,1,1,1) \\ &= A_4 = A_3. \end{aligned} \tag{4}$$

For the traffic congestion problem, now we allow the first expert to give answers regarding the indeterminance between the nodes. Because NCM handles the indeterminance, the expert of the model can give suitable careful demonstration while implementing the results of the model. Using the concept of indeterminacy and based on the first experts opinion we get the following neutrosophic directed graph given in Figure-2.

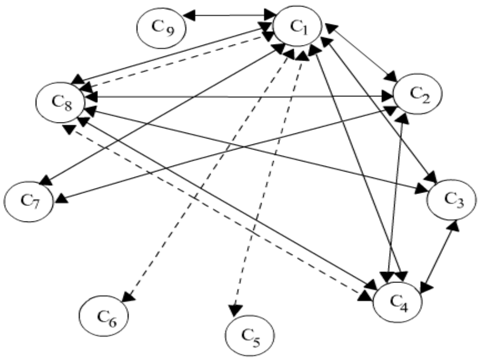

Figure-2 Neutrosophic Directed graph given by the first Expert for the traffic congestion problem.

The corresponding neutrosophic adjacency matrix N(E) related to the above neutrosophic directed graph is given below:

$$N(E) = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & I & I & 1 & I & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 1 & 1 & I & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad (5)$$

Case-2: Now we find the effect of $A_1 = (1,0,0,0,0,0,0,0,0)$ in ON state on $N(E)$.

$$\begin{aligned} A_1N(E) &= (0,1,0,1,0,0,1,0,1) \\ &\to (1,1,0,0,I,I,1,I,1) \\ &= A_2. \end{aligned} \quad (6)$$

$$\begin{aligned} A_2N(E) &= (3+3I^2, 2+I, I, -1+I, I, I, 2, 1+I, 1) \\ &= (3+3I, 1, I, 0, I, I, 1,1,1) \\ &\to (1,1,I,0,I,I,1,1,1) \\ &= A_3. \end{aligned} \quad (7)$$

$$\begin{aligned} A_3N(E) &= (3+2I+2I^2, 3,1, -1+I, I, I, 2, 1+2I, 1) \\ &= (3+2I+2I, 3,1, -1+I, I, I, 2, 1+2I, 1) \\ &= (3+4I, 3,1, -1+I, I, I, 1+2I, 1) \\ &\to (1,1,1,0,I,I,2,1+2I,1) \\ &= A_4. \end{aligned} \quad (8)$$

$$\begin{aligned} A_4N(E) &= (4+I+2I^2, 3,1, -1+I, I, I, 2, +I, 2+I, 1) \\ &= (4+I+2I, 3,1, -1+I, I, I, 2,2+I, 1) \\ &= (4+3I, 3,1, -1+I, I, I, 2,2+I, 1) \\ &\to (1,1,1,0,I,I,1,1,1) \\ &= A_5 = A_4. \end{aligned} \quad (9)$$

Next,based on the opinion of second expert FCM model is constructed. Let us consider the second experts directed graph given in Figure-3 and the connection matrix of the FCM of the traffic congestion problem with the same set of attributes.

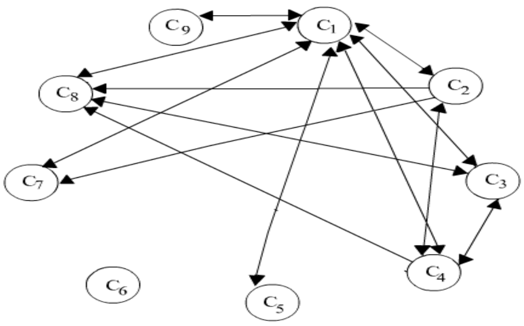

Figure-3: Directed graph given by the second Expert for the traffic congestion problem.

The connection square matrix E_1 to the above directed graph is given below:

$$E_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & -1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad (10)$$

Case-3: Take $A_1 = (1,0,0,0,0,0,0,0,0)$ the effect of A_1 on the system E_1 is

$$\begin{aligned} A_1E_1 &= (0,1,0,1,0,0,1,0,1) \\ &\rightarrow (1,1,1,0,1,0,1,1,1) \\ &= A_2. \end{aligned} \qquad (11)$$

$$\begin{aligned} A_2E_1 &= (6,1,1,-1,1,0,2,3,1) \\ &\rightarrow (1,1,1,0,1,0,1,1,1) \\ &= A_3 = A_2. \end{aligned} \qquad (12)$$

Now the second expert is permitted to give his opinion including indeterminacy. The neutrosophic directed graph is drawn using this opinion given in the Figure-4.

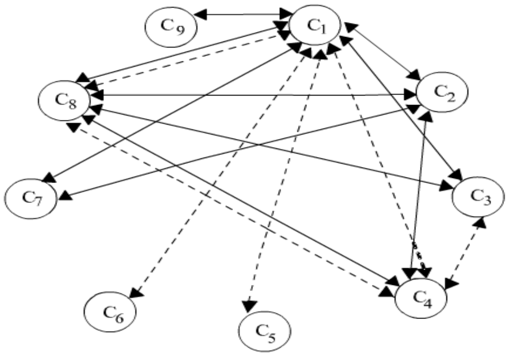

Figure-4 Neutrosophic Directed graph given by the second Expert for the traffic congestion problem.

The corresponding neutrosophic connection matrix is as follows:

$$
N(E_1) = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{array}
\begin{array}{c} \begin{array}{ccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \end{array} \\
\begin{bmatrix}
0 & 1 & 1 & 0 & I & I & 1 & I & 1 \\
1 & 0 & 0 & -1 & 0 & 0 & I & 1 & 0 \\
1 & 0 & 0 & I & 0 & 0 & 0 & 1 & 0 \\
I & -1 & 0 & 0 & 0 & 0 & 0 & I & 0 \\
I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
I & 1 & 1 & I & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \end{array} \tag{13}
$$

Case-4 Suppose $A_1 = (1,0,0,0,0,0,0,0,0)$ is the state vector whose effect on the neutrosophic system $N(E_1)$ is to be considered.

$$
\begin{aligned}
A_1N(E_1) &= (0,1,1,0,I,I,1,I,1) \\
&\to (1,1,1,0,I,I,1,I,1) \\
&= A_2. && (14) \\
A_2N(E_1) &= (4+3I^2, 1+2I, 1+I, -1+I, I, I, 2, 1+I, 1) \\
&= (4+3I, 1+2I, 1+I, -1+I, I, I, 2, 1+I, 1) \\
&\to (1,1,1,0,I,I,1,1,1) \\
&= A_3. && (15) \\
A_3N(E_1) &= (4+I+I^2, 2+I, 1+I, -1+2I, I, I, 2, 2+I, I) \\
&= (4+I+I, 2+I, 1+I, -1+2I, I, I, 2, 2+I, 1) \\
&\to (1,1,1,0,I,I,1,1,1) \\
&= A_4 = A_3. && (16)
\end{aligned}
$$

5. Comparison of experts opinion

We now give the Kosko-Hamming distance function for the FCMs between the hidden pattern given by the two experts for the $A_i's$ where $A_1 = (1,0,0,0,0,0,0,0,0), A_2 = (0,1,0,0,0,0,0,0,0), \ldots, A_9 = (0,0,0,0,0,0,0,0,1)$. We tabulate them in table 3.

Table 3: Expert's opinion comparison for FCMs

$A_i's$	Hidden pattern given by E	Hidden pattern given by E_1	$d(E, E_1)$
(1,0,0,0,0,0,0,0,0)	(1,1,0,1,0,0,1,1,1)	(1,1,1,0,1,0,1,1,1)	4
(0,1,0,0,0,0,0,0,0)	(1,1,0,1,0,0,1,1,1)	(1,1,1,0,1,0,1,1,1)	4
(0,0,1,0,0,0,0,0,0)	(1,1,1,1,0,0,1,1,1)	(1,1,1,0,1,0,1,1,1)	2
(0,0,0,1,0,0,0,0,0)	(1,1,0,1,0,0,1,1,1)	(0,0,0,1,0,0,0,1,0)	4
(0,0,0,0,1,0,0,0,0)	(0,0,0,0,0,1,0,0,0)	(1,0,0,0,1,0,0,0,0)	2
(0,0,0,0,0,1,0,0,0)	(0,0,0,0,0,0,1,0,0)	(0,0,0,0,0,1,0,0,0)	2
(0,0,0,0,0,0,1,0,0)	(1,1,0,1,0,0,1,1,1)	(1,1,1,0,1,0,1,1,1)	3
(0,0,0,0,0,0,0,1,0)	(1,1,0,1,0,0,1,1,1)	(1,1,1,0,1,0,1,1,1)	3
(0,0,0,0,0,0,0,0,1)	(1,1,0,1,0,0,1,1,1)	(1,1,1,0,1,0,1,1,1)	3

Clearly from the table for the FCMs we see the experts do not agree upon the resultants and the deviations in most of the places are large. Let us compare the two experts' opinion using NCM on the same problem. From case-3 and case-4 we are getting $(1,1,1,0,I,I,1,1,1)$ as the fixed point. The Kosko-Hamming distance is 0. So both the experts have the same opinion. Simply the preface of the Kosko-Hamming distance function can give such fine results and yield of such experts' comparison. By this process we can find the experts nearness or distance.

6. Conclusion

From Case-1, the result $(1,1,0,1,0,0,1,1,1)$ is the fixed point given by FCM. According to this expert, the traffic congestion problem flourishes mainly with Increase in number of private vehicles, present road width conditions, vehicle parking in the main road, accidents, inadequate enforcement of traffic rules causes traffic congestion problem but damage of roads, special occurrences and sudden signal failures are absent in such a scenario.

From Case-3, we are getting $(1,1,1,0,1,0,1,1,1)$ as the fixed point by FCMs. According to this expert opinion the Damage of roads and Sudden signal failures are not the consequences for the traffic congestion problem.

From Case-2 and Case-4, we are getting the same fixed point is $(1,1,1,0,I,I,1,1,1)$ by NCMs. According to the two experts, the increase or the on state of the traffic congestion problem increases with Increase in number of private vehicles, Present road width conditions, Vehicle parking in the main road, Accidents, Inadequate enforcement of traffic rules and other factors such as Special occurrences and Sudden signal failure are indeterminate.

7. Some suggestions to reduce traffic congestion using FCMs and NCMs:

From the above conclusions of FCMs and NCMs from case-1 and case-3 we observe that increase in number of private vehicles is the main reason for the traffic congestion problem because at present we observe that most of the people having own car use them to reach even a small distance. A car can occupy minimum capacity of 4 people but, mostly only one person uses the car and occupy additional space on the main road. Further, 30 cars placed in a row it will engage atleast half kilometer on a single lane whereas, if 60 people travel in public transport, then it leads to less vehicles on the road and less pollution as well. So encouraging public transport reduces traffic congestion problem in most of the cities. It is suggested that Government can take action to run the buses frequently particularly in the peak hours. Carpooling and introducing flying trains all over the city are also the best options to reduce the traffic congestion.

According to the result of FCMs and NCMs recognising vehicle parking control as a powerful tool in combating traffic congestion. Develop multi-level parking at major traffic generating locations with (or without) private participation. Construct multilevel parking facility at all critical sub-urban railway stations, metro railway stations, all critical bus terminals and mainly in shopping complexes. Establish the idea of community parking. Use the bottom space of flyovers for parking. Finally Government must take necessary action atleast not to decrease the present road width conditions for the free flow of traffic.

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