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NeutroGeometry & AntiGeometry are alternatives and generalizations

of the Non-Euclidean Geometries

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Abstract
In this paper we extend the NeutroAlgebra & AntiAlgebra to the geometric space, by founding the NeutroGeometry & AntiGeometry.
While the Non-Euclidean Geometries resulted from the total negation of only one specific axiom (Euclid's Fifth Postulate), the AntiGeometry results from the total negation of any axiom and even of more axioms from any geometric axiomatic system (Euclid's, Hilbert's, etc.), and the NeutroAxiom results from the partial negation of one or more axioms [and no total negation of no axiom] from any geometric axiomatic system.
Therefore, the NeutroGeometry and AntiGeometry are respectively alternatives and generalizations of the Non-Euclidean Geometries.
In the second part, we recall the evolution from Paradoxism to Neutrosophy, then to NeutroAlgebra & AntiAlgebra, afterwards to NeutroGeometry & AntiGeometry, and in general to NeutroStructure & AntiStructure that naturally arise in any field of knowledge.
At the end, we present applications of many NeutroStructures in our real world.

Keywords: Non-Euclidean Geometries, Euclidean Geometry, Lobachevski-Bolyai-Gauss Geometry, Riemannian Geometry, NeutroManifold, AntiManifold, NeutroAlgebra, AntiAlgebra, NeutroGeometry, AntiGeometry, NeutroAxiom, AntiAxiom, Partial Function, NeutroFunction, AntiFunction, NeutroOperation, AntiOperation, NeutroAttribute, AntiAttribute, NeutroRelation, AntiRelation, NeutroStructure, AntiStructure

1. Introduction

In our real world, the spaces are not homogeneous, but mixed, complex, even ambiguous. And the elements that populate them and the rules that act upon them are not perfect, uniform, or complete - but fragmentary and disparate, with unclear and conflicting information, and they do not apply in the same degree to each element.

The perfect, idealistic ones exist just in the theoretical sciences. We live in a multi-space endowed with a multi-structure [35]. Neither the space's elements nor the regulations that govern them are egalitarian, all of them are characterized by degrees of diversity and variance. The indeterminate (vague, unclear, incomplete, unknown, contradictory etc.) data and procedures are surrounding us.

That's why, for example, the classical algebraic and geometric spaces and structures were extended to more realistic spaces and structures [1], called respectively NeutroAlgebra & AntiAlgebra [2019] and respectively NeutroGeometry & AntiGeometry [1969, 2021], whose elements do not necessarily behave the same, while the operations and rules onto these spaces may only be partially (not totally) true.

While the Non-Euclidean Geometries resulted from the total negation of only one specific axiom (Euclid's Fifth Postulate), the AntiGeometry results from the total negation of any axiom and even of more axioms from any geometric axiomatic system (Euclid's five postulates, Hilbert's 20 axioms, etc.), and the NeutroAxiom results from the partial negation of one or more axioms [and no total negation of no axiom] from any geometric axiomatic system.
Therefore, the NeutroGeometry and AntiGeometry are respectively alternatives and generalizations of the Non-Euclidean Geometries.
In the second part, we recall the evolution from Paradoxism to Neutrosophy, then to NeutroAlgebra & AntiAlgebra, afterwards to NeutroGeometry & AntiGeometry, and in general to NeutroStructure & AntiStructure that naturally arise in any field of knowledge. At the end, we present applications of many NeutroStructures in our real world.

On a given space, a classical Axiom is totally (100%) true. While a NeutroAxiom is partially true, partially indeterminate, and partially false. Also, an AntiAxiom is totally (100%) false.

A classical Geometry has only totally true Axioms. While a NeutroGeometry is a geometry that has at least one NeutroAxiom and no AntiAxiom. Also, an AntiGeometry is a geometry that has at least one AntiAxiom.

Below we introduce, in the first part of this article, the construction of NeutroGeometry & AntiGeometry, together with the Non-Euclidean geometries, while in the second part we recall the evolution from paradoxism to neutrosophy, and then to NeutroAlgebra & AntiAlgebra, culminating with the most general form of NeutroStructure & AntiStructure in any field of knowledge.

A classical (100%) true statement on a given classical structure, may or may not be 100% true on its corresponding NeutroStructure or AntiStructure, it depends on the neutrosophication or antisophication procedures [1 – 24].

Further on, the neutrosophic triplet (Algebra, NeutroAlgebra, AntiAlgebra) was restrained or extended to all fuzzy and fuzzy extension theories (FET) triplets of the form (Algebra, Neutro$_{FET}$Algebra, Anti$_{FET}$Algebra), where FET may be: Fuzzy, Intuitionistic Fuzzy, Inconsistent Intuitionistic Fuzzy (Picture Fuzzy, Ternary Fuzzy), Pythagorean Fuzzy (Atanassov's Intuitionistic Fuzzy of second type), q-Rung Orthopair Fuzzy, Spherical Fuzzy, n-HyperSpherical Fuzzy, Refined Neutrosophic, etc.

1.1. Concept, NeutroConcept, AntiConcept

Let us consider on a given geometric space a *classical geometric concept* (such as: axiom, postulate, operator, transformation, function, theorem, property, theory, etc.).

We form the following geometric neutrosophic triplet:

Concept(1, 0, 0), *NeutroConcept*(T, I, F), *AntiConcept* (0, 0, 1),

where (T, I, F) ∉ {(1, 0, 0), (0, 0, 1)}.

{ Of course, we consider only the neutrosophic triplets (Concept, NeutroConcept, AntiConcept) that make sense in our everyday life and in the real world. }

Concept(1, 0, 0) means that the degree of truth of the concept is T = 1, I = 0, F = 0, or the Concept is 100% true, 0% indeterminate, and 0% false in the given geometric space.

NeutroConcept (T, I, F) means that the concept is T% true, I% indeterminate, and 0% false in the given geometric space, with (T, I, F) ∈ [0, 1], and (T, I, F) ∉ {(1, 0, 0), (0, 0, 1)}.

AntiConcept (0, 0, 1) means that T = 0, I = 0, and F = 1, or the Concept is 0% true, 0% indeterminate, and 100% false in the given geometric space.

1.2. Geometry, NeutroGeometry, AntiGeometry

We go from the neutrosophic triplet (*Algebra*, *NeutroAlgebra*, *AntiAlgebra*) to a similar neutrosophic triplet (*Geometry, NeutroGeometry, AntiGeometry*), in the same way.

Correspondingly from the algebraic structuires, with respect to the geometries, one has:

In the *classical (Euclidean) Geometry*, on a given space, all classical geometric *Concepts* are 100% true (i.e. true for all elements of the space).

While in a *NeutroGeometry*, on a given space, there is at least one *NeutroConcept* (and no *AntiConcept*).

In the *AntiGeometry*, on a given space, there is at least one *AntiConcept*.

1.3. Geometric NeutroSophication and Geometric AntiSophication

Similarly, as to the algebraic structures, using the process of *NeutroSophication* of a classical geometric structure, a *NeutroGeometry* is produced; while through the process of *AntiSophication* of a classical geometric structure produces an *AntiGeometry*.

Let *S* be a classical *geometric space*, and <A> be a *geometric concept* (such as: postulate, axiom, theorem, property, function, transformation, operator, theory, etc.). The <antiA> is the opposite of <A>, while <neutA> (also called <neutroA>) is the neutral (or indeterminate) part between <A> and <antiA>.

The neutrosophication tri-sections *S* into three subspaces:

- the first subspace, denoted just by <A>, where the *geometric concept* is totally true [degree of truth $T = 1$]; we denote it by *Concept(1,0,0).*

- the second subspace, denoted by <neutA>, where the *geometric concept* is partially true [degree of truth *T*], partially indeterminate [degree of indeterminacy *I*], and partially false [degree of falsehood *F*], denoted as *NeutroConcept(T,I,F)*, where $(T, I, F) \notin \{(1,0,0), (0,0,1)\}$;

- the third subspace, denoted by <antiA>, where the *geometric concept* is totally false [degree of falsehood *F* = 1], denoted by *AntiConcept(0,0,1).*

The three subspaces may or may not be disjoint, depending on the application, but they are exhaustive (their union equals the whole space *S*).

1.4. Non-Euclidean Geometries

1.4.1. The *Lobachevsky* (also known as *Lobachevsky-Bolyai-Gauss*) *Geometry*, and called *Hyperbolic Geometry*, is an AntiGeometry, because the Fifth Euclidean Postulate (in a plane, through a point outside a line, only one parallel can be drawn to that line) is 100% invalidated in the following AntiPostulate (first version) way: in a plane through a point outside of a line, there can be drawn infinitely many parallels to that line. Or (T, I, F) = (0, 0, 1).

1.4.2. The *Riemannian Geometry*, which is called *Elliptic Geometry*, is an AntiGeometry too, since the Fifth Euclidean Postulate is 100% invalidated in the following AntiPostulate (second version) way: in a place, through a point outside of a line, no parallel can be drawn to that line. Or (T, I, F) = (0, 0, 1).

1.4.3. The *Smarandache Geometries (SG)* are more complex [30 – 57]. Why this type of mixed non-Euclidean geometries, and sometimes partially Non-Euclidean and partially Euclidean? Because the real geometric spaces are not pure but hybrid, and the real rules do not uniformly apply to all space's elements, but they have degrees of diversity – applying to some geometrical concepts (point, line, plane, surface, etc.) in a smaller or bigger degree.

From Prof. Dr. Linfan Mao's arXiv.org paper *Pseudo-Manifold Geometries with Applications* [57], Cornell University, New York City, USA, 2006, https://arxiv.org/abs/math/0610307 :

> "A Smarandache geometry is a geometry which has at least one Smarandachely denied axiom (1969), i.e., an axiom behaves in at least two different ways within the same space, i.e., validated and invalided, or only invalided but in multiple distinct ways and a Smarandache n-manifold is a n-manifold that support a Smarandache geometry.
>
> Iseri provided a construction for Smarandache 2-manifolds by equilateral triangular disks on a plane and a more general way for Smarandache 2-manifolds on surfaces, called map geometries was presented by the author (…).
>
> However, few observations for cases of $n \geq 3$ are found on the journals. As a kind of Smarandache geometries, a general way for constructing dimensional n pseudo-manifolds are presented for any integer n

> ≥ 2 in this paper. Connection and principal fiber bundles are also defined on these manifolds. Following these constructions, nearly all existent geometries, such as those of Euclid geometry, Lobachevshy-Bolyai geometry, Riemann geometry, Weyl geometry, Kahler geometry and Finsler geometry, etc. are their sub-geometries."

Iseri ([34], [39 - 40]) has constructed some Smarandache Manifolds (S-manifolds) that topologically are piecewise linear, and whose geodesics have elliptic, Euclidean, and hyperbolic behavior. An SG geometry may exhibit one or more types of negative, zero, or positive curvatures into the same given space.

1.4.3.1) If at least one axiom is validated (partially true, T > 0) and invalidated (partially false, F > 0), and no other axiom is only invalidated (*AntiAxiom*), then this first class of SG geometry is a *NeutroGeometry*.

1.4.3.2) If at least one axiom is only invalidated (or F = 1), no matter if the other axioms are classical or *NeutroAxioms* or *AntiAxioms* too, then this second class of SG geometry is an *AntiGeometry*.

1.4.3.3) *The model of an SG geometry that is a NeutroGeometry:*

Bhattacharya [38] has constructed the following SG model:

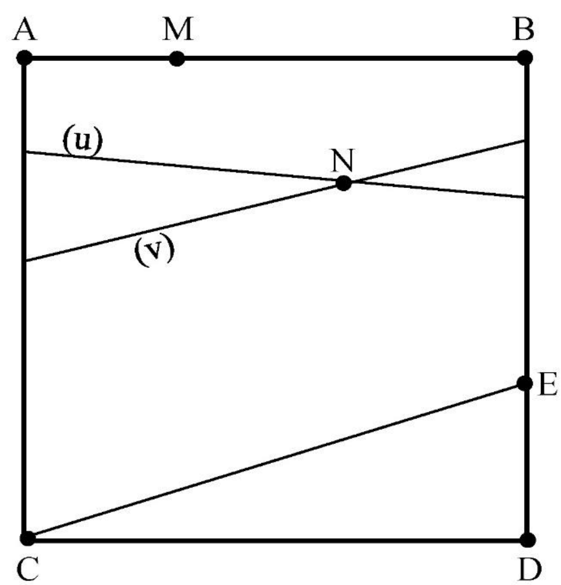

Fig. 1. Bhattacharya's Model for the SG geometry as a NeutroGeometry

The geometric space is a square ABCD, comprising all points inside and on its edges.

"Point" means the classical point, for example: A, B, C, D, E, N, and M.

"Line" means any segment of line connecting two points on the opposite square sides AC and BD, for example: AB, CD, CE, (u), and (v).

"Parallel lines" are lines that do not intersect.

Let us take a line CE and an exterior point N to it. We observe that there is an infinity of lines passing through N and parallel to CE [all lines passing through N and in between the lines (u) and (v) for example] – the *hyperbolic case*.

Also, taking another exterior point, D, there is no parallel line passing through D and parallel to CE because all lines passing through D intersects CE – the *elliptic case*.

Taking another exterior point $M \in AB$, then we only have one line AB parallel to CE, because only one line passes through the point M – the *Euclidean case*.

Consequently, the Fifth Euclidean Postulate is twice invalidated, but also once validated.

Being partially hyperbolic Non-Euclidean, partially elliptic Non-Euclidean, and partially Euclidean, therefore we have here a SG.

This is not a Non-Euclidean Geometry (since the Euclid's Fifth Postulate is not totally false, but only partially), but it is a NeutroGeometry.

Theorem 1.4.3.3.1

If a statement (proposition, theorem, lemma, property, algorithm, etc.) is (totally) true (degree of truth T = 1, degree of indeterminacy I = 0, and degree of falsehood F = 0) in the classical geometry, the statement may get any logical values (i.e. T, I, F may be any values in [0, 1]) in a NeutroGeometry or in an AntiGeometry

Proof.

The logical value the statement gets in a NeutroGeometry or in an AntiGeometry depends on what classical axioms the statement is based upon in the classical geometry, and how these axioms behave in the NeutroGeometry or AntiGeometry models.

Let's consider the below classical geometric proposition P(L1, L2, L3) that is 100% true:

> *In a 2D-Euclidean geometric space, if two lines L1 and L2 are parallel with the third line L3, then they are also parallel (i.e. L1 // L2).*

In Bhattacharya's Model of an SG geometry, this statement is partially true and partially false.

For example, in Fig. 1:

- degree of truth: the lines AB and (u) are parallel to the line CE, then AB is parallel to (u);
- degree of falsehood: the lines (u) and (v) are parallel to the line CE, but (u) and (v) are not parallel since they intersect in the point N.

1.4.3.4) The Model of a SG geometry that is an AntiGeometry

Let us consider the following rectangular piece of land PQRS,

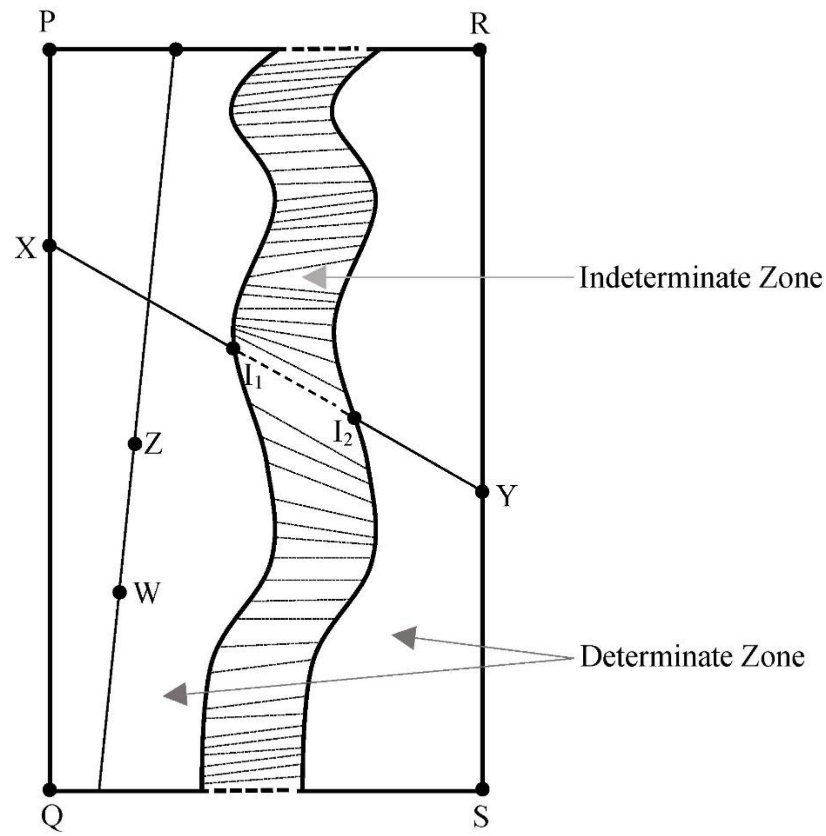

Fig. 2. Model for an SG geometry that is an AntiGeometry

whose middle (shaded) area is an indeterminate zone (a river, with swamp, canyons, and no bridge) that is impossible to cross over on the ground. Therefore, this piece of land is composed from a determinate zone and an indeterminate zone (as above).

"Point" means any classical (usual) point, for example: P, Q, R, S, X, Y, Z, and W that are determinate well-known (classical) points, and I_1, I_2 that are indeterminate (not well-known) points [in the indeterminate zone].

"Line" is any segment of line that connects a point on the side PQ with a point on the side RS. For example, PR, QS, XY. However, these lines have an indeterminate (not well known, not clear) part that is the indeterminate zone. On the other hand, ZW is not a line since it does not connect the sides PQ and RS.

The following geometric classical axiom: *through two distinct points there always passes one single line*, is totally (100%) denied in this model in the following two ways:

through any two distinct points, in this given model, either no line passes (see the case of ZW), or only one partially determinate line does (see the case of XY) - therefore no fully determinate line passes. Thus, this SG geometry is an AntiGeometry.

1.5. Manifold, NeutroManifold, AntiManifold

1.5.1. Manifold

The classical **Manifold** [29] is a topological space that, on the small scales, near each point, resembles the classical (Euclidean) Geometry Space [i.e. in this space there are only *classical Axioms (totally true)*]. Or each point has a neighborhood that is homeomorphic to an open unit ball of the Euclidean Space R^n (where R is the set of real numbers). Homeomorphism is a continuous and bijective function whose inverse is also continuous.

"In general, any object that is near 'flat' on the small scale is a manifold" [29].

1.5.2. NeutroManifold

The **NeutroManifold** is a topological space that, on the small scales, near each point, resembles the NeutroGeometry Space [i.e. in this space there is *at least a NeutroAxiom (partially true, partially indeterminate, and partially false) and no AntiAxiom*].

For example, Bhattacharya's Model for a SG geometry (Fig. 1) is a NeutroManifold, since the geometric space ABCD has a NeutroAxiom (i.e. the Fifth Euclidean Postulate, which is partially true and partially false), and no AntiAxiom.

1.5.3. AntiManifold

The Anti**Manifold** is a topological space that, on the small scales, near each point, resembles the AntiGeometry Space [i.e. in this space there is *at least one AntiAxiom (totally false)*].

For example, the Model for a SG geometry (Fig. 2) is an AntiManifold, since the geometric space PQRS has an AntiAxiom (i.e., through two distinct points there always passes a single line - which is totally false).

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2. Evolution from Paradoxism to Neutrosophy then to NeutroAlgebra/AntiAlgebra and now to NeutroGeometry/AntiGeometry

Below we recall and revise the previous foundations and developments that culminated with the introduction of NeutroAlgebra & AntiAlgebra as new field of research, extended then to NeutroStructure

& AntiStructure, and now particularized to NeutroGeometry & AntiGeometry that are extensions of the Non-Euclidean Geometries.

2.1. From Paradoxism to Neutrosophy

Paradoxism [58] is an international movement in science and culture, founded by Smarandache in 1980s, based on excessive use of antitheses, oxymoron, contradictions, and paradoxes. During three decades (1980-2020) hundreds of authors from tens of countries around the globe contributed papers to 15 international paradoxist anthologies.

In 1995, he extended the *paradoxism* (based on opposites) to a new branch of philosophy called *neutrosophy* (based on opposites and their neutral) [59], that gave birth to many scientific branches, such as: neutrosophic logic, neutrosophic set, neutrosophic probability, neutrosophic statistics, neutrosophic algebraic structures, and so on with multiple applications in engineering, computer science, administrative work, medical research, social sciences, etc.

Neutrosophy is an extension of Dialectics that have derived from the Yin-Yan Ancient Chinese Philosophy.

2.2. From Classical Algebraic Structures to NeutroAlgebraic Structures and AntiAlgebraic Structures

In 2019 Smarandache [1] generalized the classical Algebraic Structures to NeutroAlgebraic Structures (or NeutroAlgebras) {whose operations and axioms are partially true, partially indeterminate, and partially false} as extensions of Partial Algebra, and to AntiAlgebraic Structures (or AntiAlgebras) {whose operations and axioms are totally false} and on 2020 he continued to develop them [2,3,4].

The NeutroAlgebras & AntiAlgebras are a *new field of research*, which is inspired from our real world.

In classical algebraic structures, all operations are 100% well-defined, and all axioms are 100% true, but in real life, in many cases these restrictions are too harsh, since in our world we have things that only partially verify some operations or some laws.

By substituting Concept with Operation, Axiom, Theorem, Relation, Attribute, Algebra, Structure etc. respectively, into the above (Concept, NeutroConcept, AntiConcept), we get the below neutrosophic triplets:

2.3. Operation, NeutroOperation, AntiOperation

When we define an operation on a given set, it does not automatically mean that the operation is well-defined. There are three possibilities:

1) The operation is well-defined (also called inner-defined) for all set's elements [degree of truth T = 1] (as in classical algebraic structures; this is a classical Operation). Neutrosophically we write: Operation(1,0,0).

2) The operation if well-defined for some elements [degree of truth T], indeterminate for other elements [degree of indeterminacy I], and outer-defined for the other elements [degree of falsehood F], where (T,I,F) is different from (1,0,0) and from (0,0,1) (this is a NeutroOperation). Neutrosophically we write:

NeutroOperation(T,I,F).
3) The operation is outer-defined for all set's elements [degree of falsehood F = 1] (this is an AntiOperation). Neutrosophically we write: AntiOperation(0,0,1).

An operation * on a given non-empty set *S* is actually a *n-ary* function, for integer $n \geq 1$, $f : S^n \to S$.

2.4. Function, NeutroFunction, AntiFunction

Let *U* be a universe of discourse, *A* and *B* be two non-empty sets included in *U*, and *f* be a function: $f : A \to B$

Again, we have three possibilities:

1) The function is well-defined (also called inner-defined) for all elements of its domain *A* [degree of truth T = 1] (this is a classical Function), i.e. $\forall x \in A, f(x) \in B$. Neutrosophically we write: Function(1,0,0).
2) The function if well-defined for some elements of its domain, i.e. $\exists x \in A, f(x) \in B$ [degree of truth T], indeterminate for other elements, i.e. $\exists x \in A, f(x) = indeterminate$ [degree of indeterminacy I], and outer-defined for the other elements, i.e. $\exists x \in A, f(x) \notin B$ [degree of falsehood F], where (T,I,F) is different from (1,0,0) and from (0,0,1). This is a NeutroFunction. Neutrosophically we write: NeutroFunction(T,I,F).
3) The function is outer-defined for all elements of its domain *A* [degree of falsehood F = 1] (this is an AntiFunction), i.e. $\forall x \in A, f(x) \notin B$ (all function's values are outside of its codomain *B*; they may be outside of the universe of discourse too). Neutrosophically we write: AntiFunction(0,0,1).

2.5. NeutroFunction & AntiFunction vs. Partial Function

We prove that the NeutroFunction & AntiFunction are extensions and alternatives of the Partial Function.

Definition of Partial Function [60]

A function f: A→B is sometimes called a *total function*, to signify that f(a) is defined for every a ∈ A. If C is any set such that C ⊇ A then f is also a *partial function* from C to B.

Clearly if f is a function from A to B then it is a partial function from A to B, but a partial function need not be defined for every element of its domain. The set of elements of A for which f is defined is sometimes called the *domain of definition*.

From other sites, the Partial Function means: for any a ∈ A one has: f(a) ∈ B or f(a) = undefined.

Comparison

i) "Partial" is mutually understood as there exist at least one element $a_1 \in A$ such that $f(a_1) \in B$, or the function is defined for at least one element (therefore T > 0).
Such restriction is released in the NeutroFunction and AntiFunction (where T is allowed to be 0).

Example 1.

Let's consider the set of positive integers $Z = \{1, 2, 3, \ldots\}$, included into the universe of discourse *R*, which is the set of real numbers. Let's define the function

$f_1 : Z \to Z$, $f_1(x) = \frac{x}{0}$, for all $x \in Z$.

Clearly, the function f_1 is *100%* undefined, therefore the indeterminacy I = 1, while T = 0 and F = 0.

Hence f_1 is a NeutroFunction, but not a Partial Function.

Example 2.

Let's take the set of odd positive integers $D = \{1, 3, 5, \ldots\}$, included in the universe of discourse *R*. Let's define the function $f_2 : D \to D$, $f_2(x) = \frac{x}{2}$, for all $x \in D$.

The function f_2 is *100%* outer-defined, since $\frac{x}{2} \notin D$ for all $x \in D$. Whence F = 1, T = 0, and I = 0. Hence this is an AntiFunction, but not a partial Function.

ii) The Partial Function does not catch ***all types of indeterminacies*** that are allowed in a NeutroFunction. Indeterminacies may occur with respect to: the function's domain, codomain, or relation that connects the elements in the domain with the elements in the codomain.

Example 3.

Let's consider the function $g: \{1, 2, 3, \ldots, 9, 10, 11\} \to \{12, 13, \ldots, 19\}$, about whom we only have vague, unclear information as below:

g(1 or 2) = 12, i.e. we are not sure if g(1) = 12 or g(2) = 12;
g(3) = 18 or 19, i.e. we are not sure if g(3) = 18 or g(3) = 19;
g(4 or 5 or 6) = 13 or 17;
g(7) = unknown;
g(unknown) = 14.
All the above values represent the function's degree of indeterminacy (I > 0).
g(10) = 20 that does not belong to the codomain; (outer-defined, or degree of falsehood F > 0);
g(11) = 15 that belongs to the codomain; (inner-defined, or degree of truth, hence T > 0).
Function *g* is a NeutroFunction (with I > 0, T > 0, F > 0), but not a Partial Function since such types of indeterminacies are not characteristic to it.

2.6. Axiom, NeutroAxiom, AntiAxiom

Similarly for an axiom, defined on a given set, endowed with some operation(s). When we define an axiom on a given set, it does not automatically mean that the axiom is true for all set's elements. We have three possibilities again:

1) The axiom is true for all set's elements (totally true) [degree of truth T = 1] (as in classical algebraic

structures; this is a classical Axiom). Neutrosophically we write: Axiom(1,0,0).

2) The axiom if true for some elements [degree of truth T], indeterminate for other elements [degree of indeterminacy I], and false for other elements [degree of falsehood F], where (T,I,F) is different from (1,0,0) and from (0,0,1) (this is NeutroAxiom). Neutrosophically we write NeutroAxiom(T,I,F).

3) The axiom is false for all set's elements [degree of falsehood F = 1](this is AntiAxiom). Neutrosophically we write AntiAxiom(0,0,1).

2.7. Theorem, NeutroTheorem, AntiTheorem

In any science, a classical Theorem, defined on a given space, is a statement that is 100% true (i.e. true for all elements of the space). To prove that a classical theorem is false, it is sufficient to get a single counter-example where the statement is false. Therefore, the classical sciences do not leave room for *partial truth* of a theorem (or a statement). But, in our world and in our everyday life, we have many more examples of statements that are only partially true, than statements that are totally true. The NeutroTheorem and AntiTheorem are generalizations and alternatives of the classical Theorem in any science.

Let's consider a theorem, stated on a given set, endowed with some operation(s). When we construct the theorem on a given set, it does not automatically mean that the theorem is true for all set’s elements. We have three possibilities again:

1) The theorem is true for all set's elements [totally true] (as in classical algebraic structures; this is a classical Theorem). Neutrosophically we write: Theorem(1,0,0).

2) The theorem if true for some elements [degree of truth T], indeterminate for other elements [degree of indeterminacy I], and false for the other elements [degree of falsehood F], where (T,I,F) is different from (1,0,0) and from (0,0,1) (this is a NeutroTheorem). Neutrosophically we write: NeutroTheorem(T,I,F).

3) The theorem is false for all set's elements (this is an AntiTheorem). Neutrosophically we write: AntiTheorem(0,0,1).

And similarly for (Lemma, NeutroLemma, AntiLemma), (Consequence, NeutroConsequence, AntiConsequence), (Algorithm, NeutroAlgorithm, AntiAlgorithm), (Property, NeutroProperty, AntiProperty), etc.

2.8. Relation, NeutroRelation, AntiRelation

1) A classical Relation is a relation that is true for all elements of the set (degree of truth T = 1). Neutrosophically we write Relation(1,0,0).

2) A NeutroRelation is a relation that is true for some of the elements (degree of truth T), indeterminate for other elements (degree of indeterminacy I), and false for the other elements (degree of falsehood F). Neutrosophically we write Relation(T,I,F), where (T,I,F) is different from (1,0,0) and (0,0,1).

3) An AntiRelation is a relation that is false for all elements (degree of falsehood F = 1). Neutrosophically we write Relation(0,0,1).

2.9. Attribute, NeutroAttribute, AntiAttribute

1) A classical Attribute is an attribute that is true for all elements of the set (degree of truth T = 1). Neutrosophically we write Attribute(1,0,0).

2) A NeutroAttribute is an attribute that is true for some of the elements (degree of truth T), indeterminate for other elements (degree of indeterminacy I), and false for the other elements (degree of falsehood F). Neutrosophically we write Attribute(T,I,F), where (T,I,F) is different from (1,0,0) and (0,0,1).

3) An AntiAttribute is an attribute that is false for all elements (degree of falsehood F = 1). Neutrosophically we write Attribute(0,0,1).

2.10. Algebra, NeutroAlgebra, AntiAlgebra

1) An algebraic structure who's all operations are well-defined and all axioms are totally true is called a classical Algebraic Structure (or Algebra).

2) An algebraic structure that has at least one NeutroOperation or one NeutroAxiom (and no AntiOperation and no AntiAxiom) is called a NeutroAlgebraic Structure (or NeutroAlgebra).

3) An algebraic structure that has at least one AntiOperation or one Anti Axiom is called an AntiAlgebraic Structure (or AntiAlgebra).

Therefore, a neutrosophic triplet is formed: <Algebra, NeutroAlgebra, AntiAlgebra>, where "Algebra" can be any classical algebraic structure, such as: a groupoid, semigroup, monoid, group, commutative group, ring, field, vector space, BCK-Algebra, BCI-Algebra, etc.

2.11. Algebra, Neutro$_{FET}$Algebra, Anti$_{FET}$Algebra

The neutrosophic triplet (Algebra, NeutroAlgebra, AntiAlgebra) was further on restrained or extended to all fuzzy and fuzzy extension theories (FET), making triplets of the form: (Algebra, Neutro$_{FET}$Algebra, Anti$_{FET}$Algebra), where FET may be: Fuzzy, Intuitionistic Fuzzy, Inconsistent Intuitionistic Fuzzy (Picture Fuzzy, Ternary Fuzzy), Pythagorean Fuzzy (Atanassov's Intuitionistic Fuzzy of second type), q-Rung Orthopair Fuzzy, Spherical Fuzzy, n-HyperSpherical Fuzzy, Refined Neutrosophic, etc. See several examples below.

2.11.1. The Intuitionistic Fuzzy Triplet (Algebra, Neutro$_{IF}$Algebra, Anti$_{IF}$Algebra)

Herein "IF" stands for intuitionistic fuzzy.

When Indeterminacy (I) is missing, only two components remain, T and F.

1) The Algebra is the same as in the neutrosophic environment, i.e. a classical Algebra where all operations are totally well-defined and all axioms are totally true (T = 1, F = 0).
2) The Neutro$_{IF}$Algebra means that at least one operation or one axiom is partially true (degree of truth T) and partially false (degree of partially falsehood F), with $T, F \in [0,1], 0 \le T + F \le 1$, with $(T, F) \neq (1,0)$ that represents the classical Axiom, and $(T, F) \neq (0,1)$ that represents the Anti$_{IF}$Axiom,

and no Anti$_{IF}$Operation (operation that is totally outer-defined) and no Anti$_{IF}$Axiom.
3) The Anti$_{IF}$Algebra means that at least one operation or one axiom is totally false (T = 0, F = 1), no matter how the other operations or axioms are.

Therefore, one similarly has the triplets: (Operation, Neutro$_{IF}$Operation, Anti$_{IF}$Operation) and (Axiom, Neutro$_{IF}$Axiom, Anti$_{IF}$Axiom).

2.11.2. The Fuzzy Triplet (Algebra, Neutro$_{Fuzzy}$Algebra, Anti$_{Fuzzy}$Algebra)

When the Indeterminacy (I) and the Falsehood (F) are missing, only one component remains, T.

1) The Algebra is the same as in the neutrosophic environment, i.e. a classical Algebra where all operations are totally well-defined and all axioms are totally true (T = 1).
2) The Neutro$_{Fuzzy}$Algebra means that at least one operation or one axiom is partially true (degree of truth T), with $T \in (0,1)$,
   and no Anti$_{Fuzzy}$Operation (operation that is totally outer-defined) and no Anti$_{Fuzzy}$Axiom.
3) The Anti$_{IF}$Algebra means that at least one operation or one axiom is totally false (F = 1), no matter how the other operations or axioms are.

Therefore, one similarly has the triplets: (Operation, Neutro$_{Fuzzy}$Operation, Anti$_{Fuzzy}$Operation) and (Axiom, Neutro$_{Fuzzy}$Axiom, Anti$_{Fuzzy}$Axiom).

2.12. Structure, NeutroStructure, AntiStructure in any field of knowledge

In general, by NeutroSophication, Smarandache extended any classical *Structure*, in no matter what field of knowledge, to a *NeutroStructur*e, and by AntiSophication to an *AntiStructure*.

i) A classical Structure, in any field of knowledge, is composed of: a non-empty space, populated by some elements, and both (the space and all elements) are characterized by some relations among themselves (such as: operations, laws, axioms, properties, functions, theorems, lemmas, consequences, algorithms, charts, hierarchies, equations, inequalities, etc.), and by their attributes (size, weight, color, shape, location, etc.).

Of course, when analysing a structure, it counts with respect to what relations and what attributes we do it.

ii) A NeutroStructure is a structure that has at least one NeutroRelation or one NeutroAttribute, and no AntiRelation and no AntiAttribute.

iii) An AntiStructure is a structure that has at least one AntiRelation or one AntiAttribute.

2.13. Almost all real Structures are NeutroStructures

The Classical Structures in science mostly exist in theoretical, abstract, perfect, homogeneous, idealistic spaces - because in our everyday life almost all structures are NeutroStructures, since they are neither perfect nor applying to the whole population, and not all elements of the space have the same relations and same attributes in the same degree (not all elements behave in the same way).

The indeterminacy and partiality, with respect to the space, to their elements, to their relations or to their attributes are not taken into consideration in the Classical Structures. But our Real World is full of structures with indeterminate (vague, unclear, conflicting, unknown, etc.) data and partialities.

There are exceptions to almost all laws, and the laws are perceived in different degrees by different people.

2.14. Applications of NeutroStructures in our Real World

(i) In the Christian society the marriage law is defined as the union between a male and a female (*degree of truth*).

But, in the last decades, this law has become less than 100% true, since persons of the same sex were allowed to marry as well (*degree of falsehood*).

On the other hand, there are transgender people (whose sex is indeterminate), and people who have changed the sex by surgical procedures, and these people (and their marriage) cannot be included in the first two categories (*degree of indeterminacy*).

Therefore, since we have a NeutroLaw (with respect to the Law of Marriage) we have a Christian NeutroStructure.

(ii) In India, the law of marriage is not the same for all citizen: Hindi religious men may marry only one wife, while the Muslims may marry up to four wives.

(iii) Not always the difference between good and bad may be clear, from a point of view a thing may be good, while from another point of view bad. There are things that are partially good, partially neutral, and partially bad.

(iv) The laws do not equally apply to all citizens, so they are NeutroLaws. Some laws apply to some degree to a category of citizens, and to a different degree to another category. As such, there is an American folkloric joke: All people are born equal, but some people are more equal than others!

- There are powerful people that are above the laws, and other people that benefit of immunity with respect to the laws.

- For example, in the court of law, privileged people benefit from better defense lawyers than the lower classes, so they may get a lighter sentence.

- Not all criminals go to jail, but only those caught and proven guilty in the court of law. Nor the criminals that for reason of insanity cannot stand trail and do not go to jail since they cannot make a difference between right and wrong.

- Unfortunately, even innocent people went and may go to jail because of sometimes jurisdiction mistakes...

- The Hypocrisy and Double Standard are widely spread: some regulation applies to some people, but not to others!

(v) Anti-Abortion Law does not apply to all pregnant women: the incest, rapes, and women whose life is threatened may get abortions.

(vi) Gun-Control Law does not apply to all citizen: the police, army, security, professional hunters are allowed to bear arms.

Etc.

Conclusion

In this paper we have extended the Non-Euclidean Geometries to NeutroGeometry (a geometric space that has at least one NeutroAxiom and no AntiAxiom) and to AntiGeometry (a geometric space that has at least one AntiAxiom) similarly to the NeutroAlgebras and AntiAlgebras.

A NeutroAxiom is an axiom that is partially true, partially indeterminate, and partially false in the same space. While the AntiAxiom is an axiom that is totally false in the given space.

While the Non-Euclidean Geometries resulted from the total negation of only one specific axiom (Euclid's Fifth Postulate), the AntiGeometry (1969) results from the total negation of any axiom and even of more axioms from any geometric axiomatic system (Euclid's, Hilbert's, etc.), and the NeutroGeometry results from the partial negation of one or more axioms [and no total negation of no axiom] from any geometric axiomatic system.
Therefore, the NeutroGeometry and AntiGeometry are respectively alternatives and generalizations of the Non-Euclidean Geometries.
In the second part, we recall the evolution from Paradoxism to Neutrosophy, then to NeutroAlgebra & AntiAlgebra, afterwards to NeutroGeometry & AntiGeometry, and in general to NeutroStructure & AntiStructure that naturally arise in any field of knowledge.
At the end, we present applications of many NeutroStructures in our real world.

Further on, we have recalled and reviewed the evolution from Paradoxism to Neutrosophy, and from the classical algebraic structures to NeutroAlgebra and AntiAlgebra structures, and in general to the NeutroStructure and AntiStructure in any field of knowledge. Then many applications of NeutroStructures from everyday life were presented.

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