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Neutro-Sigma Algebras and Anti-Sigma Algebras

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Abstract: Neutro-algebra structures play a significant role in the neutrosophic theory. Especially, with the help of neutro-algebraic structures; neutrosophic theory makes a valuable addition to the classical theory. In this article, neutro-sigma algebras and anti-sigma algebras are obtained. Furthermore, basic properties and examples for neutro-sigma algebras and anti-sigma algebras are obtained and proved. Also, classical sigma algebra, neutro-sigma algebra and anti-sigma algebra are compared to each other. Neutro-sigma algebra is shown to have a more general structure with respect to classical sigma algebra and anti-sigma algebra. Thus, (T, I, F) components that constitute the neutrosophic theory are added into classical sigma algebra (without using neutrosophic sets) and a new structure is obtained. In addition, we show that a neutro-sigma algebra can be obtained from every classical sigma algebra and a neutro-sigma algebra can be obtained from every anti-sigma algebra.

Keywords: sigma algebra, neutro-algebra, anti-algebra, neutro-sigma algebra, anti-sigma algebra.

1 Introduction

We encounter many uncertainties in every moment of our lives. Often, classical mathematical logic is insufficient to get rid of these uncertainties. The reason is that when explaining a situation or a problem, it is not always possible to say that it is correct or certain. Neutrosophic logic and the concept of the neutrosophic set are defined in 1998 by Florentin Smarandache [1]. In the concept of neutrosophic logic and neutrosophic sets, there is a degree of membership T, a degree of uncertainty I, and a degree of non-membership F. These degrees are defined independently from each other. A neutrosophic value has the form (T, I, F). In other words, a situation is handled in neutrosophy according to its trueness, its indeterminacy, and its falsity. Thus, neutrosophic sets are a more general form of fuzzy sets [2] and intuitionistic fuzzy sets [3]. Several researchers have conducted studies on neutrosophic set theory [4-7]. Recently, Şahin and Uz studied multi-criteria decision-making applications based on set-valued generalized neutrosophic quadruple sets for law [8]; Şahin et al. obtained neutrosophic triplet partial g-metric spaces [9]; Kargın et al. introduced neutrosophic triplet m–Banach spaces [10]; Zhang et al. defined singular neutrosophic extended triplet groups and generalized groups [11]; Alhasan et al. studied neutrosophic reliability [12]; Mostafa et al. obtained hybridization between deep learning algorithms and neutrosophic theory in medical image processing [13]; Şahin et al. studied Hausdorff measures on generalized set-valued neutrosophic quadruple numbers and decision-making applications for the adequacy of online education [14].

Sigma algebra theory [15] has an important place in mathematics, especially in real analysis and probability theory. Sigma algebras have widespread use, especially in measurable functions and measurement theory. Also, Borel algebras are a widely used structure built on sigma algebras.

Florentin Smarandache defined neutro-structures and anti-structures in 2019 [16] and 2020 [17]. Similar to neutrosophic logic, an algebraic structure is divided into three regions: the set of elements that satisfy the conditions of the algebraic structure, the truth region A; the set of elements that do not meet the conditions of the algebraic structure, the uncertainty region Neutro-A, and the set of elements that do not meet the conditions of the algebraic structure, the inaccuracy region Anti-A. Thus, the structure of neutrosophic logic has been transferred into the structure of classical algebras, without using neutrosophic sets and neutrosophic numbers. Therefore, neutro-algebraic structures, which are more general structures than classical algebras, can be obtained. In addition, the region of the elements that do not satisfy any of the classical algebras is also taken as anti-algebraic structures. For this reason, many researchers have conducted studies on neutro-algebraic structures and anti-algebraic structures [18 - 21]. Recently, Smarandache studied the neutroalgebra [22]; Smarandache worked on neutro-algebraic structures and anti-algebraic structures [23]; Smarandache and Hamidi, defined neutro-bck-algebra [24]; Ibrahim and Agboola introduced neutro – vector spaces [25]; Şahin et al. studied neutro–G modules and anti–G modules [26]; Şahin et al. obtained neutro-topological space and anti-topological space [27]; Mirvakili et al. studied neutro-H v-semigroups [28]; Kargın and Şahin introduced neutro-law [29]; Hamidi and Smarandache defined single-valued neutro hyper BCK-Subalgebras [30].

In the second section, we define sigma algebra [15] and give basic definitions of neutro-structures [22]. Also, we give definitions of neutro-topology and anti-topology [27]. In the third section, we define the neutro-sigma algebra and we obtain its basic properties. Also, we give similarities and differences between the classical sigma algebra and the neutro-sigma algebra. We show that neutro-sigma algebras have a more general structure with respect to classical sigma algebra and a neutro-sigma algebra can be obtained from every classic sigma algebra. In the fourth section, we define anti-sigma algebra and we give its basic features. Furthermore, we obtain similarities and differences between the classic sigma algebra and the anti-sigma algebra. Also, we show that a neutro-sigma algebra can be obtained from every anti-sigma algebra and neutro-sigma algebras have a more general structure with respect to anti-sigma algebras. In the last part, results and suggestions are given.

2 Preliminaries

Definition 1. [22] The Neutro-sophication of the Law (degree of well-defined, degree of indeterminacy, degree of outer-defined)

Let X be a non-empty set. * be binary operation. For at least a pair (x, y) ∈ (X, X), $x * y \in X$ (degree of well defined, corresponding in neutrosophy to Truth (T)) and for at least two pairs (a, b), (c, d) ∈ (X, X), [$a * b =$ indeterminate (degree of indeterminacy, corresponding in neutrosophy to Indeterminate (I)) or $c * d \notin X$ (degree of outer-defined, corresponding in neutrosophy to Falsehood (F)].

Property 2. [22] In neutro-algebra, the classical well-defined for the binary operation * is divided into three regions: degree of well-defined (T), degree of indeterminacy (I), and degree of outer-defined (F) similar to neutrosophic set and neutrosophic logic.

Definition 3. **[22] The Anti-sophication of the Law** (Total OuterFunction)

Let X be a non-empty set and let * be a binary operation. For all pairs (x, y) ∈ (X, X), $x * y \notin X$ (totally outer-defined)

Definition 4. [27] Let X be a non-empty set and $\mathcal{T}$ be a collection of subsets of X. If at least one of the following conditions {i, ii, iii} is satisfied, then $\mathcal{T}$ is called a neutro-topology on X and $(X, \mathcal{T})$ is called a neutro-topological space.

i) $[\emptyset \in \mathcal{T}, X \notin \mathcal{T}$ or $X \in \mathcal{T}, \emptyset \notin \mathcal{T}]$ or $[\emptyset, X \in_I \mathcal{T}]$

ii) For at least n elements $p_1, p_2, \ldots p_n \in \mathcal{T}$,

$$\bigcap_{i=1}^{n} p_i \in \mathcal{T}$$

and for at least n elements $q_1, q_2, \ldots q_n \in \mathcal{T}$, and for at least n elements $r_1, r_2, \ldots r_n \in \mathcal{T}$;

$$[(\cap_{i=1}^{n} q_i \notin \mathcal{T} \text{ or } (\cap_{i=1}^{n} r_i \in_I \mathcal{T})]$$

where n is finite.

iii) For at least n elements $p_1, p_2, \ldots p_n \in \mathcal{T}$,

$$\bigcup_{i \in I} p_i \in \mathcal{T}$$

and for at least n elements $q_1, q_2, \ldots q_n \in \mathcal{T}$, $r_1, r_2, \ldots r_n \in \mathcal{T}$;

$$[(\cup_{i \in I} q_i \notin \mathcal{T} \text{ or } (\cup_{i \in I} r_i \notin \mathcal{T})].$$

Definition 5. [27] Let X be a non-empty set and $\mathcal{T}$ be a collection of subsets of X. If the following conditions {Ai, Aii, Aiii} are satisfied, then $\mathcal{T}$ is called an anti-topology on X and $(X, \mathcal{T})$ is called an anti-topological space.

Ai) $\emptyset, X \notin \mathcal{T}$,

Aii) For all $q_1, q_2, \ldots\ q_n \in \mathcal{T}$, $(\bigcap_{i=1}^{n} q_i \notin \mathcal{T})$ where n is finite,

Aiii) For all $q_1, q_2, \ldots\ q_n \in \mathcal{T}$, $(\bigcup_{i \in I} q_i \notin \mathcal{T})$.

Definition 6. [15] Let S be a non-empty set and σ be a collection of subsets of S. If σ satisfies the following conditions, then σ is called a sigma algebra:

i) $\emptyset$ and S belongs to σ,

ii) For A $\in \sigma$, $A^c \in \sigma$,

iii) Any union of elements of σ belongs to σ,

iv) Any finite intersection of elements of σ belongs to σ.

3 Neutro-Sigma Algebras

Definition 7. Let S be a non-empty set and σ be a collection of subsets of S. If at least one of the following conditions {i, ii, iii, iv} is satisfied, then σ is called a neutro-sigma algebra.

i) [$\emptyset \in \sigma, S \notin \sigma$ or $S \in \sigma, \emptyset \notin \sigma$] or [$\emptyset, S \in_I \sigma$].

ii) For at least one element

$$s_1 \in \sigma,\ {s_1}^c \in \sigma$$

and for at least two elements

$$t_1 \in \sigma,\ m_1 \in \sigma;\ [({t_1}^c \notin \sigma \text{ or } ({m_1}^c \in_I \sigma)].$$

iii) For at least n elements

$$s_1, s_2, \ldots\ s_n \in \sigma,\ \bigcap_{i=1}^{n} p_i \in \sigma$$

and for at least n elements $t_1, t_2, \ldots\ t_n \in \sigma$, $m_1, m_2, \ldots\ m_n \in \sigma$;

$$[(\bigcap_{i=1}^{n} t_i \notin \sigma \text{ or } (\bigcap_{i=1}^{n} m_i \in_I \sigma)],$$

where n is finite.

iv) For at least n elements

$$s_1, s_2, \dots\ s_n \in \sigma, \bigcup_{i \in I} p_i \in \sigma$$

and for at least n elements $t_1, t_2, \dots\ t_n \in \sigma, m_1, m_2, \dots\ m_n \in \sigma$;

$$[(\bigcup_{i \in I} t_i \notin \sigma \text{ or } (\bigcup_{i \in I} m_i \notin \sigma)].$$

We obtain Definition 7 using Definition 1 and Property 2.

Note 8: In this chapter, the symbol "$=_I$" will be used for situations where equality is uncertain. For example, if it is not certain whether "a" and "b" are equal, then it is denoted by a $=_I$ b.

Note 9: In this chapter, the symbol "$\in_I$" will be used for situations where the inclusion is not obvious. For example, if it is not certain whether "a" is a member of the set B, then it is denoted by a $\in_I$ B.

The notation in Note 8 and Note 9 is the same as in [27].

Corollary 10:

i) By condition {ii} of Definition 7, neutro-sigma algebras differ from neutro-topology (in Definition 4). In addition, every neutro-sigma algebra is a neutro-topology. However, the reverse is not always true.

ii) From Definition 7, the neutro-sigma algebras differ from the classical sigma algebras in Definition 6. Also, the neutro-sigma algebras are given as an alternative to the classical sigma algebras. However, for a neutro-sigma algebra, instead of the ones that are not met in Definition 7, classical sigma algebra conditions are valid.

Example 11. Let S $= \{k, l, m, n\}$ be a set and $\sigma = \{\emptyset, \{k\}, \{k, l\}, \{m, n\}, \{l, m\}\}$ be a collection of subsets of S. Then,

i) It is clear that S $\notin \sigma$ and $\emptyset \in \sigma$.

ii) Let $S_1 = \{k, l\}$ and $S_2 = \{k\}$. It is clear that

$${S_1}^c \in \sigma \text{ and } {S_2}^c \notin \sigma.$$

iii) Let $S_1 = \{k\}$, $S_2 = \{k, l\}$, $T_1 = \{m, n\}$ and $T_2 = \{l, m\}$. We obtain that

$$S_1 \cap S_2 \in \sigma \text{ and } T_1 \cap T_2 \notin \sigma.$$

iv) Let $S_1 = \{k, l\}$, $S_2 = \{m, n\}$, $T_1 = \{k\}$ and $T_2 = \{l, m\}$. We obtain that

$$S_1 \cup S_2 \in \sigma \text{ and } T_1 \cup T_2 \notin \sigma.$$

Hence, σ satisfies the conditions {i, ii, iii, iv} of Definition 7. Thus, σ is a neutro-sigma algebra.

Example 12. Let S $= \{k, l, m, n\}$ be a set and $\sigma = \{\emptyset, \text{S}, \{k\}, \{l\}, \{m\}, \{n\}, \{m, n\}, \{k, l\}\}$ be a collection of subsets of S. Then,

ii) Let $S_1 = \{m, n\}$ and $S_2 = \{l\}$. It is clear that

$$S_1{}^c \in \sigma \text{ and } S_2{}^c \notin \sigma.$$

iv) Let $S_1 = \{k\}$, $S_2 = \{l\}$, $T_1 = \{n\}$ and $T_2 = \{k, l\}$. We obtain that

$$S_1 \cup S_2 \in \sigma \text{ and } T_1 \cup T_2 \notin \sigma.$$

Thus, σ satisfies the conditions {ii, iv} of Definition 7. Hence σ is a neutro-sigma algebra.

We note that σ satisfies the classical sigma algebra conditions {i, iii}.

Corollary 13. In Example 11, σ is a neutro-sigma algebra, but σ is not a classical sigma algebra. In Example 12; σ is a neutro-sigma algebra but σ is not a classical sigma algebra. Thus, neutro-sigma algebras have a more general structure than classical sigma algebras.

Theorem 14. Let σ be a classical sigma algebra. Then, $\sigma - \emptyset$ is a neutro-sigma algebra.

Proof: Since σ is a classical sigma algebra, it is clear that S $\in \sigma$. Hence,

$$S^c = \emptyset \notin \sigma - \emptyset. \quad (1)$$

Also, since σ is a classical sigma algebra; we have, for all A $\in \sigma - \emptyset$

$$A^c \in \sigma - \emptyset. \quad (2)$$

Therefore, from (1) and (2); $\sigma - \emptyset$ satisfies the condition {ii} of Definition 7. Thus, $\sigma - \emptyset$ is a neutro-sigma algebra.

Also, $\sigma - \emptyset$ satisfies the condition {i} of Definition 7.

Theorem 15. Let σ be a classical sigma algebra. Then, $\sigma - S$ is a neutro-sigma algebra.

Proof: Since σ is a classical sigma algebra, it is clear that $\emptyset \in \sigma$. Hence,

$$\emptyset^c = S \notin \sigma - S . \tag{3}$$

Also, since σ is a classical sigma algebra; we obtain that for all $A \in \sigma - S$

$$A^c \in \sigma - S . \tag{4}$$

Therefore, from (3) and (4); $\sigma - S$ satisfies the condition {ii} of Definition 7. Thus, $\sigma - S$ is a neutro-sigma algebra.

Also, $\sigma - S$ satisfies the condition {i} in Definition 7.

Theorem 16. Let σ be a classical sigma algebra and A be a set such that $A^c \notin \sigma$. Then, $\sigma \cup A$ is a neutro-sigma algebra.

Proof: It is clear that since $A^c \notin \sigma$, we obtain

$$A^c \notin \sigma \cup A \tag{5}$$

Also, since σ is a classical sigma algebra; we obtain that for all $B \in \sigma$

$$B^c \in \sigma \tag{6}$$

Therefore, from (5) and (6); $\sigma \cup A$ satisfies the condition {ii} of Definition 7. Thus, $\sigma \cup A$ is a neutro-sigma algebra.

Theorem 17. Let σ be a classical sigma algebra and A be an element of σ. Then, $\sigma - A$ is a neutro-sigma algebra.

Proof: It is clear that since $A \in \sigma$, we obtain

$$(A^c)^c = A \notin \sigma - A. \tag{7}$$

Also, since σ is a classical sigma algebra; we obtain that for all $B \in \sigma - A$,

$$B^c \in \sigma - A. \tag{8}$$

Therefore, from (7) and (8); $\sigma - A$ satisfies the condition {ii} of Definition 7. Thus, $\sigma - A$ is a neutro-sigma algebra.

Corollary 18.

i) From Theorem 14, Theorem 15, Theorem 16, and Theorem 17, we obtain that a neutro-sigma algebra can be obtained from every classical sigma algebra.

ii) The classical sigma algebras do not satisfy the Theorem 14, Theorem 15, Theorem 16, and Theorem 17. However, neutro-sigma algebras satisfy Theorem 19 and Theorem 21.

Theorem 19. Let (σ_i) be a non-empty family of neutro-sigma algebras σ_i such that $\emptyset \in \sigma_i$, $S \notin \sigma_i$ $(i = 1, 2, \dots, n)$. Then, $\bigcup_{i=1}^{n}(\sigma_i)$ is a neutro-sigma algebra on S.

Proof: Since (σ_i) is a non-empty family of neutro-sigma algebras σ_i such that $\emptyset \in \sigma_i$, $S \notin \sigma_i$; it is clear that

$$\emptyset^c = S \notin \bigcup_{i=1}^{n}(\sigma_i) \,. \tag{9}$$

Also, since (σ_i) is a non-empty family of neutro-sigma algebras σ_i such that $\emptyset \in \sigma_i$, $S \notin \sigma_i$; we obtain that for all $B \in \bigcup_{i=1}^{n}(\sigma_i) - \emptyset$,

$$B^c \in \bigcup_{i=1}^{n}(\sigma_i) - \emptyset \,. \tag{10}$$

Therefore, from (9) and (10); $\bigcup_{i=1}^{n}(\sigma_i)$ satisfies the condition {ii} in Definition 7. Thus, $\bigcup_{i=1}^{n}(\sigma_i)$ is a neutro-sigma algebra.

Also, $\bigcup_{i=1}^{n}(\sigma_i)$ satisfies the condition {i} of Definition 7.

Example 20. Let $S = \{k, l, m, n\}$ be a set and $\sigma_1 = \big\{\emptyset, \{k\}, \{k, l\}, \{m, n\}, \{l, m\}\big\}$, $\sigma_2 = \big\{\emptyset, \{k\}, \{l\}, \{m\}, \{n\}, \{m, n\}, \{k, l\}\big\}$ be a collection of subsets of S. Then, from Example 12, σ_1 is a neutro-sigma algebra. Also, σ_2 is a neutro-sigma algebras since σ_2 satisfies conditions {i, ii, iv} of Definition 7.

Now, we show that

$$\sigma_1 \cup \sigma_2 = \{\emptyset, \{k\}, \{l\}, \{m\}, \{n\}, \{m, n\}, \{k, l\}, \{l, m\}\}$$

is a neutro-sigma algebra.

i) $\emptyset \in \sigma_1 \cup \sigma_2$ and $S \notin \sigma_1 \cup \sigma_2$.

ii) $\{m, n\}^c \in \sigma_1 \cup \sigma_2$ and $\{m\}^c \notin \sigma_1 \cup \sigma_2$.

iii) $\{k\} \cup \{l\} \in \sigma_1 \cup \sigma_2$ and $\{k, l\} \cup \{m, n\} \notin \sigma_1 \cup \sigma_2$.

iv) $\{k\} \cap \{k,l\} \in \sigma_1 \cup \sigma_2$ and $\{m,n\} \cap \{l,m\} \notin \sigma_1 \cup \sigma_2$.

Hence, $\sigma_1 \cup \sigma_2$ satisfies the conditions {i, ii, iii, iv} of Definition 7 and $\sigma_1 \cup \sigma_2$ is a neutro-sigma algebra.

Theorem 21. Let (σ_i) be a non-empty family of neutro-sigma algebras σ_i such that $\emptyset \notin \sigma_i$, $S \in \sigma_i$ $(i = 1, 2, \ldots, n)$. Then, $\bigcup_{i=1}^{n}(\sigma_i)$ is a neutro-sigma algebra.

Proof: Since (σ_i) is a non-empty family of neutro-sigma algebras σ_i such that $\emptyset \notin \sigma_i$, $S \in \sigma_i$, it is clear that

$$S^c = \emptyset \notin \bigcup_{i=1}^{n}(\sigma_i) \tag{11}$$

Also, since (σ_i) is a non-empty family of neutro-sigma algebras σ_i such that $\emptyset \in \sigma_i$, $S \notin \sigma_i$; we obtain that for all $B \in \bigcup_{i=1}^{n}(\sigma_i) - S$,

$$B^c \in \bigcup_{i=1}^{n}(\sigma_i) - S\,. \tag{12}$$

Therefore, from (11) and (12); $\bigcup_{i=1}^{n}(\sigma_i)$ satisfies the condition {ii} of Definition 7. Thus, $\bigcup_{i=1}^{n}(\sigma_i)$ is a neutro-sigma algebra.

Also, $\bigcup_{i=1}^{n}(\sigma_i)$ satisfies the condition {i} of Definition 7.

Example 22. Let $S = \{k,l,m,n\}$ be a set and $\sigma_1 = \{S, \{k\}, \{k,l\}, \{m,n\}, \{l,m\}\}$, $\sigma_2 = \{S, \{k\}, \{l\}, \{m\}, \{n\}, \{m,n\}, \{k,l\}\}$ be a collection of subsets of S. Then, σ_1 is a neutro-sigma algebra since σ_1 satisfies conditions {i, ii, iii, iv} of Definition 7. Also, σ_2 is a neutro-sigma algebras since σ_2 satisfies the conditions {i, ii, iv} of Definition 7.

Now, we show that

$$\sigma_1 \cup \sigma_2 = \{S, \{k\}, \{l\}, \{m\}, \{n\}, \{m,n\}, \{k,l\}, \{l,m\}\}$$

is a neutro-sigma algebra.

i) $\emptyset \notin \sigma_1 \cup \sigma_2$ and $S \in \sigma_1 \cup \sigma_2$.

ii) $\{k,l\}^c \in \sigma_1 \cup \sigma_2$ and $\{l\}^c \notin \sigma_1 \cup \sigma_2$.

iii) $\{m\} \cup \{n\} \in \sigma_1 \cup \sigma_2$ and $\{k\} \cup \{l,m\} \notin \sigma_1 \cup \sigma_2$.

iv) $\{k\} \cap \{l\} \in \sigma_1 \cup \sigma_2$ and $\{m\} \cap \{l\} \notin \sigma_1 \cup \sigma_2$.

Hence, $\sigma_1 \cup \sigma_2$ satisfies the conditions {i, ii, iii, iv} of Definition 7 and $\sigma_1 \cup \sigma_2$ is a neutro-sigma algebra.

Corollary 23. The classical sigma algebras do not satisfy Theorem 19, Theorem 21, Example 20, and Example 22. However, neutro-sigma algebras satisfy Theorem 19, Theorem 21, Example 20, and Example 22.

4 Anti-Sigma Algebras

Definition 24. Let S be a non-empty set and σ be a collection of subsets of S. If the following conditions {i, ii, iii, iv} are satisfied, then σ is called an anti-sigma algebra on S.

i) $\emptyset$, S $\notin \sigma$,

ii) for all $S_i \in \sigma$, ${S_i}^c \notin \sigma$,

iii) for all $S_1, S_2, \ldots\ S_n \in \sigma$, $(\bigcap_{i=1}^{n} S_i \notin \sigma)$,

iv) for all $S_1, S_2, \ldots\ S_n \in \sigma$, $(\bigcup_{i\in I} S_i \notin \sigma)$.

Example 25. Let S $= \{k, l, m, n\}$ be a set and $\sigma = \big\{\{k\}, \{l\}, \{m\}\big\}$ be a collection of subsets of S. Then,

i) It is clear that $\emptyset \notin \sigma$ and S $\notin \sigma$.

ii) Let

$$S_1 = \{k\}\ , S_2 = \{l\}\ , S_3 = \{m\}.$$

Thus, we have

$$(S_1)^c \notin \sigma,\ (S_2)^c \notin \sigma,\ (S_3)^c \notin \sigma.$$

iii) Let

$$S_1 = \{k\}\ , S_2 = \{l\}\ , S_3 = \{m\}.$$

So,

$$\bigcap_{i=1}^{3} S_i \notin \sigma.$$

iv) Let

$$S_1 = \{k\}\ , S_2 = \{l\}\ , S_3 = \{m\}.$$

Then, we have

$$\bigcup_{i=1}^{3} S_i \notin \sigma.$$

Thus, σ satisfies the {i, ii, iii, iv} conditions of Definition 24. Therefore, σ is an anti-sigma algebra on S.

Corollary 26: In Example 25, σ is an anti-sigma algebra. But σ is not a neutro-sigma algebra or a classical sigma algebra. Thus, anti-sigma algebras are different from neutro-sigma algebras and classical sigma algebras.

Theorem 27. Let σ be an anti-sigma algebra on S, A and A^c be two sets such that

$$A \notin \sigma \text{ and } A^c \notin \sigma.$$

Then, $\sigma \cup A \cup A^c$ is a neutro-sigma algebra.

Proof: As $A \notin \sigma$ and $A^c \notin \sigma$, it is clear that

$$A \in \sigma \cup A \cup A^c \text{ and } A^c \in \sigma \cup A \cup A^c . \quad (13)$$

Also, since σ is an anti-sigma algebra; we obtain that for all $B \in \sigma$,

$$B^c \notin \sigma . \quad (14)$$

So, by (13) and (14); $\sigma \cup A \cup A^c$ satisfies the condition {ii} of Definition 7. Thus, $\sigma \cup A \cup A^c$ is a neutro-sigma algebra.

Theorem 28. Let σ be an anti-sigma algebra on S. Then, $\sigma \cup \emptyset \cup S$ is a neutro-sigma algebra.

Proof: Since σ is an anti-sigma algebra on S, for all $S_i \in \sigma$, we have ${S_i}^c \notin \sigma$.

$$S^c = \emptyset \in \sigma \cup \emptyset \cup S \text{ and } \emptyset^c = S \in \sigma \cup \emptyset \cup S.$$

Thus, $\sigma \cup \emptyset \cup S$ satisfies condition {ii} of Definition 7. Hence, $\sigma \cup \emptyset \cup S$ is a neutro-sigma algebra.

Also, $\sigma \cup \emptyset \cup S$ satisfies condition {iii, iv} of Definition 7.

In addition, in proof of Theorem 27; if we assume that

$$A = \emptyset \text{ and } A^c = S \text{ or } A = S \text{ and } A^c = \emptyset,$$

then $\emptyset$ and S satisfy Theorem 28.

Theorem 29. Let σ be an anti-sigma algebra on S and A be an element of σ. Then, $\sigma \cup A^c$ is a neutro-sigma algebra.

Proof: Since σ is an anti-sigma algebra on S, for all $S_i \in \sigma$, ${S_i}^c \notin \sigma$. However, it is clear that

$$A^c \in \sigma \cup A^c.$$

Thus, $\sigma \cup A^c$ satisfies condition {ii} of Definition 7. Hence, $\sigma \cup A^c$ is a neutro-sigma algebra.

Example 30. In Example 25, $\sigma = \{\{k\},\{l\},\{m\}\}$ is an anti-sigma algebra on S. By Theorem 27, $\sigma \cup \emptyset$ is a neutro-sigma algebra.

Example 31. In Example 25, $\sigma = \{\{k\},\{l\},\{m\}\}$ is an anti-sigma algebra on S. By Theorem 28, $\sigma \cup S$ is a neutro-sigma algebra.

Example 32. In Example 25, $\sigma = \{\{k\},\{l\},\{m\}\}$ is an anti-sigma algebra on S. By Theorem 29, $\sigma \cup \emptyset \cup S$ is a neutro-sigma algebra.

Example 33. In Example 25, $\sigma = \{\{k\},\{l\},\{m\}\}$ is an anti-sigma algebra on S. By Theorem 30,

$$\sigma \cup \{k\}^c = \{\{k\},\{l\},\{m\},\{l,m,n\}\}$$

is a neutro-sigma algebra.

Corollary 34.

i) From Theorem 27, Theorem 28, and Theorem 29, we obtain that a neutro-sigma algebra can be obtained from every anti-sigma algebra.

ii) Neutro-topology and anti-topology do not satisfy Theorem 27, Theorem 28, and Theorem 29. Thus, neutro-sigma algebras and anti-sigma algebras have a more general structure than neutro-topology and anti-topology.

5 Conclusions

In this study, a neutro-sigma algebra is defined and relevant basic properties are given. Similarities and differences between the classical sigma algebras and neutro-sigma algebras are discussed. We show that a neutro-sigma algebra can be obtained from every classical sigma algebra. In addition, we define anti-sigma algebras and we give corresponding basic properties. We discuss similarities and differences between the classical sigma algebra and anti- sigma algebras. Also, we show that a neutro-sigma algebra can be obtained from every anti-sigma algebra. In addition, we show that neutro-sigma algebras and anti-sigma algebras have a more general structure than neutro-topology and anti-topology. Thus, we add new structures to the neutro-algebra theory.

By using the definition of neutro-sigma algebras and anti-sigma algebras, researchers can define neutro-sigma measurable functions, anti-sigma measurable functions, neutro-Borel algebras, and anti-Borel algebras.

REFERENCES

1. F. Smarandache (1998) Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth, Amer. Research Press

2. Zadeh, L.,1965, Fuzzy Sets, *Inform and Control*, 8, 338-353

3. Atanassov, K. (1986) Intuitionistic fuzzy sets, *Fuzzy Sets Syst*., vol. 20, pp. 87–96

4. Aslan, C., Kargın, A., & Şahin, M. (2020). Neutrosophic modeling of Talcott Parsons's action and decision-making applications for it. *Symmetry*, *12*(7), 1166.

5. Şahin, M., Kargın, A., & Uz, M. S. (2021). Generalized Euclid Measures Based on Generalized Set Valued Neutrosophic Quadruple Numbers and Multi Criteria Decision Making Applications. *Neutrosophic Sets and Systems*, *47*, 573-600.

6. Şahin, M., Kargın, A., & Uz, M. S. (2020). Neutrosophic Triplet Partial Bipolar Metric Spaces. *Neutrosophic Sets and Systems*, *33*, 297-312.

7. Kargın, A., Dayan, A., & Şahin, N. M. (2021). Generalized Hamming Similarity Measure Based on Neutrosophic Quadruple Numbers and Its Applications to Law Sciences. *Neutrosophic Sets and Systems*, 40(1), 4.

8. Şahin N. M., Uz M. S. (2021) Multi-criteria Decision-making Applications Based on Set Valued Generalized Neutrosophic Quadruple Sets for Law, *International Journal of Neutrosophic Science*, Vol. 17 , No. 1, 41-60

9. Sahin, M., Kargın, A., & Yucel, M. (2020). Neutrosophic Triplet Partial g-Metric Spaces. *Neutrosophic Sets and Systems*, *33*, 116-135.

10. Kargın, A., Dayan, A., Yıldız, İ., & Kılıç, A. (2020). Neutrosophic Triplet m–Banach Spaces. *Neutrosophic Sets and Systems*, *38*(1), 25.

11. Zhang, X., Wang, X., Smarandache, F., Jaíyéolá, T. G., & Lian, T. (2019). Singular neutrosophic extended triplet groups and generalized groups. *Cognitive Systems Research*, *57*, 32-40.

12. Alhasan, K. F., Salama, A. A., & Smarandache, F. (2021). Introduction to Neutrosophic Reliability Theory, *International Journal of Neutrosophic Science*, 15-1, 52-61,

13. Mostafa, N. N., Ahmed, K., & El-Henawy, I. (2021). Hybridization between deep learning algorithms and neutrosophic theory in medical image processing: A survey. *Neutrosophic Sets and Systems*, *45*, 378-401.

14. Şahin, S., Kargın, A., & Yücel, M. (2021). Hausdorff Measures on Generalized Set Valued Neutrosophic Quadruple Numbers and Decision Making Applications for Adequacy of Online Education. *Neutrosophic Sets and Systems*, 40(1), 6.

15. Wang, Z., Klir, G. J., Fuzzy Measure Theory, New York-Binghamton, 1989

16. Smarandache F. (2019) Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures, in Advances of Standard and Nonstandard Neutrosophic Theories, Pons Publishing House Brussels, Belgium, Ch. 6, 240-265

17. Rezaei A, Smarandache F. (2020) The Neutrosophic Triplet of BI-algebras, *Neutrosophic Sets and Systems*, 33, 313-321

18. Şahin, M., & Kargın, A. (2019). Neutrosophic Triplet Metric Topology. *Neutrosophic Sets and Systems*, *27*(1), 14.

19. Agboola, A.A.A. (2020) Introduction to NeutroRings, *International Journal of Neutrosophic Science*, 7(2), 62-73

20 Sahin, M., & Kargın, A. (2018). Neutrosophic Triplet Normed Ring Space. *Neutrosophic Sets and Systems*, *21*, 20-28.

21. Rezaei, A. and Smarandache, F. (2020) On Neutro-BE-algebras and Anti-BE-algebras (revisited), *International Journal of Neutrosophic Science*, 4(1), 8-15

22. Smarandache, F. (2020) NeutroAlgebra is a Generalization of Partial Algebra, *International Journal of Neutrosophic Science*, 2(1), 08-17

23. Smarandache, F., (2020) Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures (revisited), *Neutrosophic Sets and Systems*, vol. 31, 1-16

24. Smarandache, F., & Hamidi, M. (2020). Neutro-bck-algebra. *International Journal of Neutrosophic Science*, 8(2), 110.

25. Ibrahim, M. A., & Agboola, A. A. A. (2020). NeutroVectorSpaces I. *Neutrosophic Sets and Systems*, 36, 328-351.

26. Şahin, M., Kargın, A., & Smarandache, F. (2021). Neutro–G Modules and Anti–G Modules. NeutroAlgebra Theory Volume I, 50.

27. Şahin, M., Kargın, A., & Yücel, M. (2021). Neutro-Topological Space and Anti-Topological Space. NeutroAlgebra Theory Volume I, 16.

28. Mirvakili, S., Smarandache, F., & Rezaei, A. On neutro-H v-semigroups. *Journal of Intelligent & Fuzzy Systems*, (Preprint), 1-11.

29. Kargın, A., & Şahin, N. M. (2021) Neutro-law, NeutroAlgebra Theory Volume I, 198.

30. Hamidi, M., & Smarandache, F. (2021) Single-Valued Neutro Hyper BCK-Subalgebras. *Journal of Mathematics*, *2021*.31.

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