Neutrosophic $\alpha^m$-continuity

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Abstract: In this paper, we introduce and study a new class of neutrosophic closed set called neutrosophic $\alpha^m$-closed set. In this respect, we introduce the concepts of neutrosophic $\alpha^m$-continuous, strongly neutrosophic $\alpha^m$-continuous, neutrosophic $\alpha^m$-irresolute and present their basic properties.

Keywords: Neutrosophic $\alpha^m$-closed set, neutrosophic $\alpha^m$-continuous, strongly neutrosophic $\alpha^m$-continuous, neutrosophic $\alpha^m$-irresolute.

1 Introduction

In 1965, Zadeh [21] studied the idea of fuzzy sets and its logic. Later, Chang [8] introduced the concept of fuzzy topological spaces. Atanassov [1] discussed the concepts of intuitionistic fuzzy set[[2],[3],[4]]. The concepts of strongly fuzzy continuous and fuzzy gc-irresolute are introduced by G. Balasubramanian and P. Sundaram [6]. The idea of $\alpha^m$-closed in topological spaces was introduced by M. Mathew and R. Parimelazhagan[16]. He also introduced and investigated, $\alpha^m$-continuous maps in topological spaces together with S. Jafari[17]. The concept of fuzzy $\alpha^m$-continuous function was introduced by R. Dhavaseelan[13]. After the introduction of the concept of neutrosophy and neutrosophic set by F. Smarandache [[19], [20]], the concepts of neutrosophic crisp set and neutrosophic crisp topological space were introduced by A. A. Salama and S. A. Alblowi[18]. In this paper, a new class of neutrosophic closed set called neutrosophic $\alpha^m$ closed set is studied. Furthermore, the concepts of neutrosophic $\alpha^m$-continuous, strongly neutrosophic $\alpha^m$-continuous, neutrosophic $\alpha^m$-irresolute are introduced and obtain some interesting properties. Throughout this paper neutrosophic topological spaces (briefly $NTS$) $(S_1, \xi_1), (S_2, \xi_2)$ and $(S_3, \xi_3)$ will be replaced by $S_1, S_2$ and $S_3$, respectively.
2 Preliminaries

Definition 2.1. [19] Let \( T, I, F \) be real standard or non standard subsets of \([0^-, 1^+]\), with \( sup_T = t_{sup}, inf_T = t_{inf} \)
\[
\begin{align*}
sup_T &= i_{sup}, \quad inf_T = i_{inf} \\
sup_F &= f_{sup}, \quad inf_F = f_{inf} \\
n - sup &= t_{sup} + i_{sup} + f_{sup} \\
n - inf &= t_{inf} + i_{inf} + f_{inf} 
\end{align*}
\]
\( T, I, F \) are neutrosophic components.

Definition 2.2. [19] Let \( S_1 \) be a non-empty fixed set. A neutrosophic set (briefly \( N \)-set) \( \Lambda \) is an object such that 
\[
\Lambda = \{x, \mu(x), \sigma(x), \gamma(x) : x \in S_1\}
\]
where \( \mu(x), \sigma(x) \) and \( \gamma(x) \) which represents the degree of membership function (namely \( \mu(x) \)), the degree of indeterminacy (namely \( \sigma(x) \)) and the degree of non-membership (namely \( \gamma(x) \)) respectively of each element \( x \in S_1 \) to the set \( \Lambda \).

Remark 2.3. [19]

1. An \( N \)-set \( \Lambda = \{x, \mu(x), \sigma(x), \gamma(x) : x \in S_1\} \) can be identified to an ordered triple \( \langle \mu, \sigma, \gamma \rangle \) in \([0^-, 1^+]\) on \( S_1 \).

2. In this paper, we use the symbol \( \Lambda = \langle \mu, \sigma, \gamma \rangle \) for the \( N \)-set \( \Lambda = \{x, \mu(x), \sigma(x), \gamma(x) : x \in S_1\} \).

Definition 2.4. [18] Let \( S_1 \neq \emptyset \) and the \( N \)-sets \( \Lambda \) and \( \Gamma \) be defined as 
\[
\Lambda = \{x, \mu(x), \sigma(x), \gamma(x) : x \in S_1\}, \quad \Gamma = \{x, \mu_\Gamma(x), \sigma_\Gamma(x), \gamma_\Gamma(x) : x \in S_1\}.
\]
Then

a. \( \Lambda \subseteq \Gamma \iff \mu(x) \leq \mu_\Gamma(x), \sigma(x) \leq \sigma_\Gamma(x) \) and \( \gamma(x) \geq \gamma_\Gamma(x) \) for all \( x \in S_1 \);

b. \( \Lambda = \Gamma \iff \Lambda \subseteq \Gamma \) and \( \Gamma \subseteq \Lambda \);

c. \( \bar{\Lambda} = \{x, \Gamma_\Lambda(x), \sigma_\Lambda(x), \mu_\Lambda(x) : x \in S_1\} \); [Complement of \( \Lambda \)]

d. \( \Lambda \cap \Gamma = \{x, \mu(x) \land \mu_\Gamma(x), \sigma(x) \land \sigma_\Gamma(x), \gamma(x) \lor \gamma_\Gamma(x) : x \in S_1\} \);

e. \( \Lambda \cup \Gamma = \{x, \mu(x) \lor \mu_\Gamma(x), \sigma(x) \lor \sigma_\Gamma(x), \gamma(x) \land \gamma_\Gamma(x) : x \in S_1\} \);

f. \( [\Lambda] = \{x, \mu(x), \sigma(x), 1 - \mu(x) : x \in S_1\} \).

g. \( \langle \Lambda \rangle = \{x, 1 - \Gamma_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) : x \in S_1\} \).

Definition 2.5. [10] Let \( \{\Lambda_i : i \in J\} \) be an arbitrary family of \( N \)-sets in \( S_1 \). Then

a. \( \bigcap \Lambda_i = \{x, \land \mu_{\Lambda_i}(x), \land \sigma_{\Lambda_i}(x), \lor \Gamma_{\Lambda_i}(x) : x \in S_1\} \);

b. \( \bigcup \Lambda_i = \{x, \lor \mu_{\Lambda_i}(x), \lor \sigma_{\Lambda_i}(x), \land \Gamma_{\Lambda_i}(x) : x \in S_1\} \).

In order to develop \( NTS \) we need to introduce the \( N \)-sets \( 0_N \) and \( 1_N \) in \( S_1 \) as follows:

Definition 2.6. [10] \( 0_N = \{x, 0, 0, 1 : x \in S_1\} \) and \( 1_N = \{x, 1, 1, 0 : x \in S_1\} \).

Definition 2.7. [10] A neutrosophic topology (briefly \( N \)-topology) on \( S_1 \neq \emptyset \) is a family \( \xi_1 \) of \( N \)-sets in \( S_1 \) satisfying the following axioms:

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(i) \( 0_N, 1_N \in \xi_1 \),
(ii) \( G_1 \cap G_2 \in T \) for any \( G_1, G_2 \in \xi_1 \),
(iii) \( \cup G_i \in \xi_1 \) for arbitrary family \( \{G_i \mid i \in \Lambda\} \subseteq \xi_1 \).

In this case the ordered pair \((S_1, \xi_1)\) or simply \( S_1 \) is called an \( NTS \) and each \( N \)-set in \( \xi_1 \) is called a neutrosophic open set (briefly \( N \)-open set). The complement \( \overline{\Lambda} \) of an \( N \)-open set \( \Lambda \) in \( S_1 \) is called a neutrosophic closed set (briefly \( N \)-closed set) in \( S_1 \).

**Definition 2.8.** [10] Let \( \Lambda \) be an \( N \)-set in an \( NTS \) \( S_1 \). Then

\[
Nint(\Lambda) = \bigcup \{G \mid G \text{ is an } N \text{-open set in } S_1 \text{ and } G \subseteq \Lambda\}
\]

is called the neutrosophic interior (briefly \( N \)-interior) of \( \Lambda \);

\[
Ncl(\Lambda) = \bigcap \{G \mid G \text{ is an } N \text{-closed set in } S_1 \text{ and } G \supseteq \Lambda\}
\]

is called the neutrosophic closure (briefly \( N \)-cl) of \( \Lambda \).

**Definition 2.9.** Let \( S_1 \neq \emptyset \). If \( r, t, s \) be real standard or non standard subsets of \( ]0^-, 1^+[ \) then the \( N \)-set \( x_{r,t,s} \) is called a neutrosophic point (briefly \( NP \)) in \( S_1 \) given by

\[
x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}
\]

for \( x_p \in S_1 \) is called the support of \( x_{r,t,s} \), where \( r \) denotes the degree of membership value, \( t \) denotes the degree of indeterminacy and \( s \) is the degree of non-membership value of \( x_{r,t,s} \).

Now we shall define the image and preimage of \( N \)-sets. Let \( S_1 \neq \emptyset \) and \( S_2 \neq \emptyset \) and \( \Omega : S_1 \to S_2 \) be a map.

**Definition 2.10.** [10]

(a) If \( \Gamma = \{(y, \mu_\Lambda(y), \sigma_\Lambda(y), \Gamma_\Lambda(y)) : y \in S_2\} \) is an \( N \)-set in \( S_1 \), then the pre-image of \( \Gamma \) under \( \Omega \), denoted by \( \Omega^{-1}(\Gamma) \), is the \( N \)-set in \( S_1 \) defined by

\[
\Omega^{-1}(\Gamma) = \{(x, \Omega^{-1}(\mu_\Lambda)(x), \Omega^{-1}(\sigma_\Lambda)(x), \Omega^{-1}(\Gamma_\Lambda)(x)) : x \in S_1\}.
\]

(b) If \( \Lambda = \{(x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x)) : x \in S_1\} \) is an \( N \)-set in \( S_1 \), then the image of \( \Lambda \) under \( \Omega \), denoted by \( \Omega(\Lambda) \), is the \( N \)-set in \( S_2 \) defined by

\[
\Omega(\Lambda) = \{(y, \Omega(\mu_\Lambda)(y), \Omega(\sigma_\Lambda)(y), (1 - \Omega(1 - \Gamma_\Lambda))(y)) : y \in S_2\}
\]

where

\[
\Omega(\mu_\Lambda)(y) = \begin{cases} \sup_{x \in \Omega^{-1}(y)} \mu_\Lambda(x), & \text{if } \Omega^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}
\]

\[
\Omega(\sigma_\Lambda)(y) = \begin{cases} \sup_{x \in \Omega^{-1}(y)} \sigma_\Lambda(x), & \text{if } \Omega^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}
\]

\[
(1 - \Omega(1 - \Gamma_\Lambda))(y) = \begin{cases} \inf_{x \in \Omega^{-1}(y)} \Gamma_\Lambda(x), & \text{if } \Omega^{-1}(y) \neq \emptyset, \\ 1, & \text{otherwise}, \end{cases}
\]

In what follows, we use the symbol \( \Omega_-(\Gamma_\Lambda) \) for \( 1 - \Omega(1 - \Gamma_\Lambda) \).

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Corollary 2.11. [10] Let $\Lambda$, $\Lambda_i (i \in J)$ be $N$-sets in $S_1$, $\Gamma$, $\Gamma_i (i \in K)$ be $N$-sets in $S_1$ and $\Omega : S_1 \rightarrow S_2$ a function. Then

(a) $\Lambda_1 \subseteq \Lambda_2 \Rightarrow \Omega(\Lambda_1) \subseteq \Omega(\Lambda_2)$,
(b) $\Gamma_1 \subseteq \Gamma_2 \Rightarrow \Omega^{-1}(\Gamma_1) \subseteq \Omega^{-1}(\Gamma_2)$,
(c) $\Lambda \subseteq \Omega^{-1}(\Omega(\Lambda))$ \{ If $\Omega$ is injective, then $\Lambda = \Omega^{-1}(\Omega(\Lambda))$ \},
(d) $\Omega(\Omega^{-1}(\Gamma)) \subseteq \Gamma$ \{ If $\Omega$ is surjective, then $\Omega(\Omega^{-1}(\Gamma)) = \Gamma$ \},
(e) $\Omega^{-1}(\bigcup \Gamma_j) = \bigcup \Omega^{-1}(\Gamma_j)$,
(f) $\Omega^{-1}(\bigcap \Gamma_j) = \bigcap \Omega^{-1}(\Gamma_j)$,
(g) $\Omega(\bigcup \Lambda_i) = \bigcup \Omega(\Lambda_i)$,
(h) $\Omega(\bigcap \Lambda_i) \subseteq \bigcap \Omega(\Lambda_i)$ \{ If $\Omega$ is injective, then $\Omega(\bigcap \Lambda_i) = \bigcap \Omega(\Lambda_i)$ \},
(i) $\Omega^{-1}(\{1\}) = 1_n$,
(j) $\Omega^{-1}(\{0\}) = 0_n$,
(k) $\Omega(\{1\}) = 1_n$, if $\Omega$ is surjective
(l) $\Omega(\{0\}) = 0_n$,
(m) $\overline{\Omega(\Lambda)} \subseteq \Omega(\overline{\Lambda})$, if $\Omega$ is surjective,
(n) $\Omega^{-1}(\overline{\Gamma}) = \overline{\Omega^{-1}(\Gamma)}$.

Definition 2.12. [11] An $N$-set $\Lambda$ in an $NTS (S_1, \xi_1)$ is called

1) a neutrosophic semiopen set (briefly $N$-semiopen) if $\Lambda \subseteq Ncl(Nint(\Lambda))$.
2) a neutrosophic $\alpha$ open set (briefly $N\alpha$-open set) if $\Lambda \subseteq Nint(Ncl(Nint(\Lambda)))$.
3) a neutrosophic preopen set (briefly $N$-preopen set) if $\Lambda \subseteq Nint(Ncl(\Lambda))$.
4) a neutrosophic regular open set (briefly $N$-regular open set) if $\Lambda = Nint(Ncl(\Lambda))$.
5) a neutrosophic semipre open or $\beta$ open set (briefly $N\beta$-open set) if $\Lambda \subseteq Ncl(Nint(Ncl(\Lambda)))$.

An $N$-set $\Lambda$ is called a neutrosophic semiclosed set, neutrosophic $\alpha$ closed set, neutrosophic preclosed set, neutrosophic regular closed set and neutrosophic $\beta$ closed set, respectively, if the complement of $\Lambda$ is an $N$-semiopen set, $N\alpha$-open set, $N$-preopen set, $N$-regular open set, and $N\beta$-open set, respectively.

Definition 2.13. [10] Let $(S_1, \xi_1)$ be an $NTS$. An $N$-set $\Lambda$ in $(S_1, \xi_1)$ is said to be a generalized neutrosophic closed set (briefly $g-N$-closed set) if $Ncl(\Lambda) \subseteq G$ whenever $\Lambda \subseteq G$ and $G$ is an $N$-open set. The complement of a generalized neutrosophic closed set is called a generalized neutrosophic open set (briefly $g-N$-open set).

Definition 2.14. [10] Let $(S_1, \xi_1)$ be an $NTS$ and $\Lambda$ be an $N$-set in $S_1$. Then the neutrosophic generalized closure (briefly $N-g-cl$) and neutrosophic generalized interior (briefly $N-g-Int$) of $\Lambda$ are defined by,

(i) $NGcl(\Lambda) = \bigcap \{ G : G \text{ is a } g-N\text{-closed set in } S_1 \text{ and } \Lambda \subseteq G \}$.
(ii) $NGint(\Lambda) = \bigcup \{ G : G \text{ is a } g-N\text{-open set in } S_1 \text{ and } \Lambda \supseteq G \}$.
3 Neutrosophic $\alpha^m$ continuous functions

Definition 3.1. An $N$-subset $\Lambda$ of an $NTS (S_1, \xi_1)$ is called neutrosophic $\alpha^m$-closed set (briefly $N\alpha^m$-closed set) if $Nint(Ncl(\Lambda)) \subseteq U$ whenever $\Lambda \subseteq U$ and $U$ is $N\alpha$-open.

Definition 3.2. An $N$-subset $\lambda$ of an $NTS (S_1, \xi_1)$ is called a neutrosophic $\alpha g$-closed set (briefly $N\alpha g$-closed set) if $\alpha Ncl(\Lambda) \subseteq U$ whenever $\lambda \subseteq U$ and $U$ is an $N\alpha$-open set in $S_1$.

Definition 3.3. An $N$-subset $\lambda$ of an $NTS (S_1, \xi_1)$ is called a neutrosophic $g\alpha$-closed set (briefly $Ng\alpha$-closed set) if $\alpha Ncl(\Lambda) \subseteq U$ whenever $\Lambda \subseteq U$ and $U$ is an $N$-open set in $S_1$.

Remark 3.4. In an $NTS (S_1, \xi_1)$, the following statements are true:

(i) Every $N$-closed set is an $Ng$-closed set.

(ii) Every $N$-closed set is an $N\alpha$-closed set.

Remark 3.5. In an $NTS (S_1, \xi_1)$, the following statements are true:

(i) Every $Ng$-closed set is an $N\alpha g$-closed set.

(ii) Every $N\alpha$-closed set is an $N\alpha g$-closed set.

(iii) Every $N\alpha g$-closed set is an $Ng\alpha$-closed set.

Remark 3.6. In an $NTS (S_1, \xi_1)$, the following statements are true:

(i) Every $N$-closed set is an $N\alpha^m$-closed set.

(ii) Every $N\alpha^m$-closed set is an $N\alpha$-closed set.

(iii) Every $N\alpha^m$-closed set is an $N\alpha g$-closed set.

(iv) Every $N\alpha^m$-closed set is an $Ng\alpha$-closed set.

Proof. (i) This follows directly from the definitions.

(ii) Let $\Lambda$ be an $N\alpha^m$-closed set in $S_1$ and $U$ a $N$-open set such that $\lambda \subseteq U$. Since every $N$-open set is an $N\alpha$-open set and $\Lambda$ is a $N\alpha^m$-closed set, $Nint(Ncl(\Lambda)) \subseteq (Nint(Ncl(\Lambda))) \cup (Ncl(Nint(\Lambda))) \subseteq U$. Therefore, $\Lambda$ is an $N\alpha$-closed set in $S_1$.

(iii) It is a consequence of (ii) and remark 3.5 (ii).

(iv) It is a consequence of (iii) and remark 3.5 (iii).

Proposition 3.7. The intersection of an $N\alpha^m$-closed set and an $N$-closed set is an $N\alpha^m$-closed set.

Proof. Let $\Lambda$ be an $N\alpha^m$-closed set and $\Psi$ an $N$-closed set. Since $\Lambda$ is an $N\alpha^m$-closed set, $Nint(Ncl(\Lambda)) \subseteq U$ whenever $\Lambda \subseteq U$, where $U$ is an $N\alpha$-open set. To show that $\Lambda \cap \Psi$ is an $N\alpha^m$-closed set, it is enough to show that $Nint(Ncl(\Lambda \cap \Psi)) \subseteq U$ whenever $\Lambda \cap \Psi \subseteq U$, where $U$ is an $N\alpha$-open set. Let $M = S_1 - \Psi$. Then $\Lambda \subseteq U \cup M$. Since $M$ is an $N$-open set, $U \cup M$ is an $N\alpha$-open set and $\Lambda$ is an $N\alpha^m$-closed set, $Nint(Ncl(\Lambda)) \subseteq U \cup M$. Now, $Nint(Ncl(\Lambda \cap \Psi)) \subseteq N\alpha^m$-closed set, $Nint(Ncl(\Lambda)) \cap N\alpha^m$-closed set, $N\alpha^m$-closed set, $N\alpha^m$-closed set. This implies that $\Lambda \cap \Psi$ is an $N\alpha^m$-closed set.
Example 3.10. Let $\Lambda = \{ \xi \}$.

Definition 3.11. Let $\Lambda = \{ \xi \}$.

The following are the implications of an $\alpha$-set.

Remark 3.14. The following are the implications of an $\alpha$-set and the reverses are not true.
Definition 3.15. Let \((S_1, \xi_1)\) and \((S_2, \xi_2)\) be any two NTS.

1) A map \(\Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2)\) is called neutrosophic \(\alpha^m\)-continuous (briefly \(N\alpha^m\)-cont) if the inverse image of every \(N\)-closed set in \((S_2, \xi_2)\) is \(N\alpha^m\)-c-set in \((S_1, \xi_1)\).
   Equivalently if the inverse image of every \(N\)-open set in \((S_2, \xi_2)\) is \(N\alpha^m\)-open set in \((S_1, \xi_1)\).

2) A map \(\Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2)\) is called neutrosophic \(\alpha^m\)-irresolute (briefly \(N\alpha^m\)-I) if the inverse image of every \(N\alpha^m\)-c-set in \((S_2, \xi_2)\) is \(N\alpha^m\)-c-set in \((S_1, \xi_1)\).
   Equivalently if the inverse image of every \(N\alpha^m\)-open set in \((S_2, \xi_2)\) is \(N\alpha^m\)-open set in \((S_1, \xi_1)\).

3) A map \(\Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2)\) is called strongly neutrosophic \(\alpha^m\)-continuous (briefly \(SN\alpha^m\)-cont) if the inverse image of every \(N\alpha^m\)-c-set in \((S_2, \xi_2)\) is \(N\)-closed set in \((S_1, \xi_1)\).
   Equivalently if the inverse image of every \(N\alpha^m\)-open set in \((S_2, \xi_2)\) is \(N\)-open set in \((S_1, \xi_1)\).

Proposition 3.16. Let \((S_1, \xi_1)\) and \((S_2, \xi_2)\) be any two NTS. If \(\Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2)\) is NC, then it is \(N\alpha^m\)-cont.

Proof. Let \(\Lambda\) be any \(N\)-closed set in \((S_2, \xi_2)\). Since \(f\) is NC, \(\Omega^{-1}(\Lambda)\) is \(N\)-closed in \((S_1, \xi_1)\). Since every \(N\)-closed set is \(N\alpha^m\)-c-set, \(\Omega^{-1}(\Lambda)\) is \(N\alpha^m\)-c-set in \((S_1, \xi_1)\). Therefore \(\Omega\) is \(N\alpha^m\)-cont. \(\square\)

The converse of Proposition 3.16 need not be true as it is shown in the following example.

Example 3.17. Let \(S_1 = \{a, b, c\}\) and \(S_2 = \{a, b, c\}\). Define \(N\)-subsets \(E, F, G\) and \(D\) as follows
\[
E = \{x, (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.7}, \frac{b}{0.5}, \frac{c}{0.7})\}, \quad F = \{x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.8}, \frac{b}{0.5}, \frac{c}{0.6})\}, \quad G = \{x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6})\}, \quad D = \{x, (\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.3})\}. \]
The family \(\xi_1 = \{0_{S_1}, 1_{S_1}, E, F\}\) is an NT on \(S_1\) and \(\xi_2 = \{0_{S_2}, 1_{S_2}, G, D\}\) is an NT on \(S_2\). Thus \((S_1, \xi_1)\) and \((S_2, \xi_2)\) are NTS. Define \(\Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2)\) as \(\Omega(a) = a, \Omega(b) = c, \Omega(c) = b\). Clearly \(\Omega\) is \(N\alpha^m\)-cont but \(\Omega\) is not NC since \(\Omega^{-1}(D) \not\in \xi_1\) for \(D \in \xi_2\).

Proposition 3.18. Let \((S_1, \xi_1)\) and \((S_2, \xi_2)\) be any two neutrosophic NTS. If \(\Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2)\) is \(N\alpha^m\)-I, then it is \(N\alpha^m\)-cont.

Proof. Let \(\Lambda\) be an \(N\)-closed set in \((S_2, \xi_2)\). Since every \(N\)-closed set is \(N\alpha^m\)-c-set, \(\Lambda\) is \(N\alpha^m\)-c-set in \(S_2\). Since \(\Omega\) is \(N\alpha^m\)-I, \(\Omega^{-1}(\Lambda)\) is \(N\alpha^m\)-c-set in \((S_1, \xi_1)\). Therefore \(\Omega\) is \(N\alpha^m\)-cont. \(\square\)

The converse of Proposition 3.18 need not be true.

Example 3.19. Let \(S_1 = \{a, b, c\}\). Define the \(N\)-subsets \(E, F\) and \(G\) as follows
\[
E = \{x, (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6})\}, \quad F = \{x, (\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.5})\}, \quad G = \{x, (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.7}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.7}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6})\}. \]
Then \(\xi_1 = \{0_{S_1}, 1_{S_1}, E, F\}\) and \(\xi_2 = \{0_{S_1}, 1_{S_1}, C\}\) are \(N\)-topologies on \(S_1\). Define \(\Omega : (S_1, \xi_1) \rightarrow (S_1, \xi_2)\) as follows \(\Omega(a) = b, \Omega(b) = a, \Omega(c) = c\). Observe that \(\Omega\) is \(N\alpha^m\)-continuous. But \(\Omega\) is not \(N\alpha^m\)-I. Since \(D = \{x, (\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.6})\}\) is \(N\alpha^m\)-c-set in \((S_1, \xi_2)\), \(\Omega^{-1}(D)\) is not \(\alpha\)-c-set in \((S_1, \xi_1)\).

Proposition 3.20. Let \((S_1, \xi_1)\) and \((S_2, \xi_2)\) be any two NTS. If \(\Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2)\) is \(SN\alpha^m\)-I, then it is NC.

Proof. Let \(\Lambda\) be an \(N\)-closed set in \((S_2, \xi_2)\). Since every \(N\)-closed set is \(N\alpha^m\)-c-set. Since \(\Omega\) is \(SN\alpha^m\)-cont, \(\Omega^{-1}(\Lambda)\) is \(N\)-closed set in \((S_1, \xi_1)\). Therefore \(\Omega\) is NC. \(\square\)

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The converse of Proposition 3.20 need not be true.

**Example 3.21.** Let \( S_1 = \{a, b, c\} \). Define the \( N \)-subsets \( E,F \) and \( G \) as follows
\[
E = \{ x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}) \},
\]
\[
F = \{ x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}) \} \text{ and } G = \{ x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}) \}. \]
Then \( \xi_1 = \{0, 1, S_1, E, F\} \) and \( \xi_2 = \{0, 1, S_1, G\} \) are \( N \)-topologies on \( S_1 \). Define \( \Omega(S_1, \xi_1) \rightarrow (S_1, \xi_2) \) as follows \( \Omega(a) = \Omega(b) = a, \Omega(c) = c. \) \( \Omega \) is \( NC \) but \( \Omega \) is not \( S\alpha^m\)-cont. Since \( D = \{ x, (\frac{a}{0.05}, \frac{b}{0.05}, \frac{c}{0.05}), (\frac{a}{0.05}, \frac{b}{0.05}, \frac{c}{0.05}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}), \} \) is \( S\alpha^m\)-set in \( (S_1, \xi_2) \), \( \Omega^{-1}(D) \) is not \( N\)-closed set in \( (S_1, \xi_1) \).

**Proposition 3.22.** Let \((S_1, \xi_1), (S_2, \xi_2)\) and \((S_3, \xi_3)\) be any three \( NTS \). Suppose \( \Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2), \Xi : (S_2, \xi_2) \rightarrow (S_3, \xi_3) \) are maps. Assume \( \Omega \) is \( S\alpha^m\)-I and \( \Xi \) is \( S\alpha^m\)-cont, then \( \Xi \circ \Omega \) is \( S\alpha^m\)-cont.

**Proof.** Let \( \Lambda \) be an \( N \)-closed set in \((S_3, \xi_3)\). Since \( \Xi \) is \( S\alpha^m\)-cont, \( \Xi^{-1}(\Lambda) \) is \( S\alpha^m\)-set in \((S_2, \xi_2)\). Since \( \Omega \) is \( S\alpha^m\)-I, \( \Omega^{-1}(\Xi^{-1}(\Lambda)) \) is \( S\alpha^m\)-closed in \((S_1, \xi_1)\). Thus \( \Xi \circ \Omega \) is \( S\alpha^m\)-cont.

**Proposition 3.23.** Let \((S_1, \xi_1), (S_2, \xi_2)\) and \((S_3, \xi_3)\) be any three \( NTS \). Let \( \Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2) \) and \( \Xi : (S_2, \xi_2) \rightarrow (S_3, \xi_3) \) be maps such that \( \Omega \) is \( S\alpha^m\)-cont and \( \Xi \) is \( S\alpha^m\)-cont, then \( \Xi \circ \Omega \) is \( NC \).

**Proof.** Let \( \Lambda \) be an \( N \)-c-set in \((S_3, \xi_3)\). Since \( \Xi \) is \( S\alpha^m\)-cont, \( \Xi^{-1}(\Lambda) \) is \( S\alpha^m\)-c-set in \((S_2, \xi_2)\). Moreover, since \( \Omega \) is \( S\alpha^m\)-cont, \( \Omega^{-1}(\Xi^{-1}(\Lambda)) \) is \( N \)-closed in \((S_1, \xi_1)\). Thus \( \Xi \circ \Omega \) is \( NC \).

**Proposition 3.24.** Let \((S_1, \xi_1), (S_2, \xi_2)\) and \((S_3, \xi_3)\) be any three \( NTS \). Let \( \Omega : (S_1, \xi_1) \rightarrow (S_2, \xi_2) \) and \( \Xi : (S_2, \xi_2) \rightarrow (S_3, \xi_3) \) be two maps. Assume \( \Omega \) and \( \Xi \) are \( S\alpha^m\)-I, then \( \Xi \circ \Omega \) is \( S\alpha^m\)-I.

**Proof.** Let \( \Lambda \) be an \( S\alpha^m\)-c-set in \((S_3, \xi_3)\). Since \( \Xi \) is \( S\alpha^m\)-I, \( \Xi^{-1}(\Lambda) \) is \( S\alpha^m\)-c-set in \((S_2, \xi_2)\). Since \( \Omega \) is \( S\alpha^m\)-I, \( \omega^{-1}(\Xi^{-1}(\Lambda)) \) is an \( S\alpha^m\)-c-set in \((S_1, \xi_1)\). Thus \( \Xi \circ \Omega \) is \( S\alpha^m\)-I.

4 Conclusions

In this paper, a new class of neutrosophic closed set called neutrosophic \( \alpha^m \) closed set is introduced and studied. Furthermore, the basic properties of neutrosophic \( \alpha^m\)-continuity are presented with some examples.

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