Neutrosophic bg-closed Sets and its Continuity

C.Maheswari¹ and S. Chandrasekar ²*

¹Department of Mathematics, Muthayammal College of Arts and Science, Rasipuram, Namakkal(DT), Tamil Nadu, India.
E-mail: mahi2gobi@gmail.com

²PG and Research Department of Mathematics, Arignar Anna Government Arts College, Namakkal(DT), Tamil Nadu, India.
E-mail: chandrumat@gmail.com

* Correspondence: chandrumat@gmail.com;

Abstract: Smarandache introduced and developed the new concept of Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama introduced Neutrosophic topological spaces by using the Neutrosophic crisp sets. Aim of this paper is we introduce and study the concepts Neutrosophic bg generalized closed sets and Neutrosophic bg generalized continuity in Neutrosophic topological spaces and its Properties are discussed details.

Keywords: Neutrosophic bg closed sets, Neutrosophic bg open sets, Neutrosophic bg continuity, Neutrosophic bg maps.

1. Introduction

Neutrosophic system plays important role in the fields of Information Systems, Computer Science, Artificial Intelligence, Applied Mathematics, Mechanics, decision making, Medicine, Management Science, and Electrical & Electronic, etc. Topology is a classical subject, as a generalization topological spaces many type of topological spaces introduced over the year. T Truth, F -Falsehood, I- Indeterminacy are three component of Neutrosophic sets. Neutrosophic topological spaces(N-T-S) introduced by Salama [22,23]etal., R.Dhavaseelan[10], Saied Jafari are introduced Neutrosophic generalized closed sets. Neutrosophic b closed sets are introduced by C.Maheswari[17] et al.Aim of this paper is we introduce and study about Neutrosophic b generalized closed sets and Neutrosophic b generalized continuity in Neutrosophic topological spaces and its properties and Characterization are discussed details.

2. Preliminaries

In this section, we recall needed basic definition and operation of Neutrosophic sets and its fundamental Results

Definition 2.1 [13] Let $\mathcal{X}$ be a non-empty fixed set. A Neutrosophic set $\mathcal{J}_1$ is a object having the form

$$\mathcal{J}_1 = \langle x, \mu_{\mathcal{J}_1}(x), \sigma_{\mathcal{J}_1}(x), \gamma_{\mathcal{J}_1}(x) : x \in \mathcal{X} \rangle,$$

where

- $\mu_{\mathcal{J}_1}(x)$-represents the degree of membership function
- $\sigma_{\mathcal{J}_1}(x)$-represents degree indeterminacy and then
- $\gamma_{\mathcal{J}_1}(x)$-represents the degree of non-membership function.

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Definition 2.2 [13]. Neutrosophic set $J_1^\ast = \{ x, \mu_{J_1}(x), \sigma_{J_1}(x), \gamma_{J_1}(x) : x \in X \}$ on $X$ and $\forall x \in X$

then complement of $J_1^\ast$ is $J_1^\ast C = \{ x, \mu_{J_1}(x), 1 - \sigma_{J_1}(x), 1 - \gamma_{J_1}(x) : x \in X \}$

Definition 2.3 [13]. Let $J_1^\ast$ and $J_2^\ast$ are two Neutrosophic sets, $\forall x \in X$

$J_1^\ast = \{ x, \mu_{J_1}(x), \sigma_{J_1}(x), \gamma_{J_1}(x) : x \in X \}$

$J_2^\ast = \{ x, \mu_{J_2}(x), \sigma_{J_2}(x), \gamma_{J_2}(x) : x \in X \}$

Then $J_1^\ast \subseteq J_2^\ast \iff \mu_{J_1}(x) \leq \mu_{J_2}(x), \sigma_{J_1}(x) \leq \sigma_{J_2}(x) \& \gamma_{J_1}(x) \geq \gamma_{J_2}(x)$

Definition 2.4 [13]. Let $X$ be a non-empty set, and Let $J_1^\ast$ and $J_2^\ast$ be two Neutrosophic sets are

$J_1^\ast = \{ x, \mu_{J_1}(x), \sigma_{J_1}(x), \gamma_{J_1}(x) : x \in X \}$, $J_2^\ast = \{ x, \mu_{J_2}(x), \sigma_{J_2}(x), \gamma_{J_2}(x) : x \in X \}$ Then

1. $J_1^\ast \cap J_2^\ast = \{ x, \min(\mu_{J_1}(x), \mu_{J_2}(x)), \max(\sigma_{J_1}(x), \sigma_{J_2}(x)), \max(\gamma_{J_1}(x), \gamma_{J_2}(x)) : x \in X \}$

2. $J_1^\ast \cup J_2^\ast = \{ x, \max(\mu_{J_1}(x), \mu_{J_2}(x)), \min(\sigma_{J_1}(x), \sigma_{J_2}(x)), \min(\gamma_{J_1}(x), \gamma_{J_2}(x)) : x \in X \}$

Definition 2.5 [23]. Let $X$ be non-empty set and $\tau_N$ be the collection of Neutrosophic subsets of $X$

satisfying the following properties:

1. $\forall 1_N \in \tau_N$

2. $T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$

3. $T_1 \in \tau_N$ for every $\{ T_i : i \in I \} \subseteq \tau_N$

Then the space $(X, \tau_N)$ is called a Neutrosophic topological space(N-T-S).

The element of $\tau_N$ are called Ne.OS (Neutrosophic open set)

and its complement is Ne.CS(Neutrosophic closed set)

Example 2.6. Let $X = \{ x \}$ and $\forall x \in X$

$A_1 = (x, \frac{6}{10}, \frac{6}{10}, \frac{5}{10})$, $A_2 = (x, \frac{5}{10}, \frac{7}{10}, \frac{9}{10})$

$A_3 = (x, \frac{6}{10}, \frac{7}{10}, \frac{5}{10})$, $A_4 = (x, \frac{5}{10}, \frac{6}{10}, \frac{9}{10})$

Then the collection $\tau_N = \{ 0_N, A_1, A_2, A_3, A_4, 1_N \}$ is called a N-T-S on $X$.

Definition 2.7. Let $(X, \tau_N)$ be a N-T-S and $J_1^\ast = \{ x, \mu_{J_1}(x), \sigma_{J_1}(x), \gamma_{J_1}(x) : x \in X \}$ be a Neutrosophic set in $X$. Then $J_1^\ast$ is said to be

1. Neutrosophic b closed set [17] (Ne.bCS) if $\text{Ne.cl}(\text{Ne.int}(J_1^\ast)) \cap \text{Ne.int}(\text{Ne.cl}(J_1^\ast)) \subseteq J_1^\ast$,

2. Neutrosophic a-closed set [7] (Ne.aCS) if $\text{Ne.cl}(\text{Ne.int}(\text{Int}(J_1^\ast))) \subseteq J_1^\ast$,

3. Neutrosophic pre-closed set [25] (Ne.Pre-CS) if $\text{Ne.cl}(\text{Ne.int}(J_1^\ast)) \subseteq J_1^\ast$,

4. Neutrosophic regular closed set [7] (Ne.RCS) if $\text{Ne.cl}(\text{Ne.int}(J_1^\ast)) = J_1^\ast$,

5. Neutrosophic semi closed set [7] (Ne.SCS) if $\text{Ne.int}(\text{Ne.cl}(J_1^\ast)) \subseteq J_1^\ast$,

6. Neutrosophic generalized closed set [10] (Ne.GCS) if $\text{Ne.cl}(J_1^\ast) \subseteq H$ whenever $J_1^\ast \subseteq H$ and $H$ is a Ne.OS,
7. Neutrosophic generalized pre closed set [17] (Ne.GPCS in short) if Ne.Pcl($J'_1$) $\subseteq$ H whenever $J'_1 \subseteq H$ and H is a Ne._OS.

8. Neutrosophic $\alpha$ generalized closed set [15] (Ne. G $\alpha$ CS in short) if Neu $\alpha$-cl($J'_1$) $\subseteq$ H whenever $J'_1 \subseteq H$ and H is a Ne.OS.

9. Neutrosophic generalized semi closed set [24] (Ne.GSCS in short) if Ne.Scl($J'_1$) $\subseteq$ H whenever $J'_1 \subseteq H$ and H is a Ne.OS.

10. Neutrosophic generalized $\alpha$ closed set [11] (Ne. G $\alpha$ CS in short) if Neu $\alpha$-cl($J'_1$) $\subseteq$ H whenever $J'_1 \subseteq H$ and H is a Ne. $\alpha$OS.

11. Neutrosophic semi generalized closed set [24] (Ne.SGC in short) if Ne.Scl($J'_1$) $\subseteq$ H whenever $J'_1 \subseteq H$ and H is a Ne.SOS.

**Definition 2.8.** Let $(X, \tau_X)$ be a N-T-S and $J'_1 = \{ (x, \mu_{J'_1}(x)), \sigma_{J'_1}(x), \gamma_{J'_1}(x) : x \in X \}$ be a Neutrosophic set in $X$. Then the Neutrosophic closure of $J'_1$ is Ne.Cl($J'_1$)=$\cap \{ H: H \text{ is a Ne.CS in } X \text{ and } J'_1 \subseteq H \}$.

**Definition 2.9.** Let $(X, \tau_X)$ be a N-T-S and $J'_1 = \{ (x, \mu_{J'_1}(x)), \sigma_{J'_1}(x), \gamma_{J'_1}(x) : x \in X \}$ be a Neutrosophic set in $X$. Then the Neutrosophic b closure of $J'_1$ (Ne.bcl($J'_1$) in short) and Neutrosophic b interior of $J'_1$ (Ne.bint($J'_1$) in short) are defined as Ne.bint($J'_1$)=$\cup \{ G/G \text{ is a Ne.bOS in } X \text{ and } G \subseteq J'_1 \}$, Ne.bcl($J'_1$)=$\cap \{ K/K \text{ is a Ne.bCS in } X \text{ and } J'_1 \subseteq K \}$.

**Proposition 2.10.** Let $(X, \tau_X)$ be any N-T-S. Let $J'_1$ and $J'_2$ be any two Neutrosophic sets in $(X, \tau_X)$. Then the Neutrosophic generalized b closure operator satisfies the following properties.

1. Ne.bcl(bN)=bN and Ne.bcl(1N) = 1N,
2. $J'_1 \subseteq$ Ne.bcl($J'_1$),
3. Ne.bint($J'_1$) $\subseteq$ $J'_1$,
4. If $J'_1$ is a Ne.bCS then $J'_1$ = Ne.bcl(Ne.bcl($J'_1$)),
5. $J'_1 \subseteq J'_2$ $\Rightarrow$ Ne.bcl($J'_1$) $\subseteq$ Ne.bcl($J'_2$),
6. $J'_1 \subseteq J'_2$ $\Rightarrow$ Ne.bint($J'_1$) $\subseteq$ Ne.bint($J'_2$).

**NEUTROSOPHIC b GENERALIZED CLOSED SETS**

In this part we introduce neutrosophic bG closed sets its properties are discussed.

**Definition 3.1.**

A Ne. set $J'_1$ in an NSTS $(X, \tau_X)$ is called Neutrosophic b generalized CS (briefly Ne.(b)GCS) iff Ne.bCl($J'_1$) $\subseteq$ $J'_2$ whenever $J'_1 \subseteq J'_2$ and $J'_2$ is Ne. (b)OS in $X$.

**Example 3.2.**

Let $X= \{ j_1, j_2 \}$, $\tau_X=\{ 0, J'_1, J'_2, 1 \}$, is a N.T.on $X$ where $J'_1 = (x_2, \left( \frac{2}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right))$.

Then the Neutrosophic set $J'_2 = (x_3, \left( \frac{7}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right))$ is a Ne.bGCS in $X$.

**Remark 3.3.**

A Ne. set $J'_1$ in an NSTS $(X, \tau_X)$ is called Ne.(b)generalized open (briefly Ne.(b)OS) if its compliment $J'_1^c$ is Ne.(b)CS.

**Theorem 3.4.**
Every Ne.-CS in \((\mathcal{X}, \mathcal{N}_1)\) is Ne.(bG)CS.

**Proof.**
Let \(J_1^*\) be a Ne.CS in NSTS \(\mathcal{X}\). Let \(J_1^* \subseteq J_2^*\), where \(J_2^*\) is Ne.(b)OS in \(\mathcal{X}\). Since \(J_1^*\) is Ne.CS it is Ne.(b)CS and so \(\text{NeuCl}(J_1^*) = \text{NeuCl}(J_2^*)\). Thus \(\text{Neu.bCl}(J_1^*) \subseteq J_2^*\). Hence \(J_1^*\) is Ne.(bG)CS.

**Example 3.5**
Let \(\mathcal{X} = \{j_1, j_2\}\), \(\mathcal{N}_2 = \{0, J_1^*, 1\}\), is a N.T.on \(\mathcal{X}\) where \(J_1^* = (x, \left(\frac{3}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right))\). Then the Neutrosophic set \(J_2^* = (x, \left(\frac{5}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right))\) is a Ne.bCS but not a Ne.CS in \(\mathcal{X}\).

**Theorem 3.6.**
Every Ne.(b)CS in \((\mathcal{X}, \mathcal{N}_1)\) is Ne.(bG)CS.

**Proof.**
Let \(J_1^*\) be a Ne.(b)CS in NSTS \(\mathcal{X}\). Let \(J_1^* \subseteq J_2^*\), where \(J_2^*\) is Ne.(b)OS in \(\mathcal{X}\). Since \(J_1^*\) is Ne.(b)CS, Ne.bCl\((J_1^*) = J_2^*\). Thus \(\text{Neu.bCl}(J_1^*) \subseteq J_2^*\). Hence \(J_1^*\) is Ne.(bG)CS.

**Example 3.7.** Let \(\mathcal{X} = \{j_1, j_2\}\), \(\mathcal{N}_2 = \{0, J_1^*, 1\}\), is a N.T.on \(\mathcal{X}\) where \(J_1^* = (x, \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right))\). Then the Neutrosophic set \(J_2^* = (x, \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right))\) is a Ne.bGCS but not a Ne.bCS in \(\mathcal{X}\).

**Remark 3.8.**
(i) Every Ne. (b)CS is Ne.(Gb)CS.
(ii) Every Ne.(sG)CS is Ne.(bG)CS.
(iii) Every Ne.(Ga)CS is Ne.(bG)CS.

**Example 3.9.**
Let \(\mathcal{X} = \{j_1, j_2\}\), \(\mathcal{N}_2 = \{0, J_1^*, 1\}\), is a N.T.on \(\mathcal{X}\) where \(J_1^* = (x, \left(\frac{3}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right))\). Then the Neutrosophic set \(J_2^* = (x, \left(\frac{5}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right))\) is a Ne.GbCS but not Ne.(bG)CS in \(\mathcal{X}\).

**Example 3.10.**
Let \(\mathcal{X} = \{j_1, j_2\}\), \(\mathcal{N}_2 = \{0, J_1^*, 1\}\), is a N.T.on \(\mathcal{X}\) where \(J_1^* = (x, \left(\frac{2}{10}, \frac{5}{10}, \frac{8}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right))\). Then the Neutrosophic set \(J_2^* = (x, \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right))\) is a Ne.bGCS in \(\mathcal{X}\) and is not Ne.(sG)-CS.

**Diagram:**

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Theorem 3.11.

A Ne. set $J'_1$ of a NSTS $(X, N_\tau)$ is called Ne.(b)OS iff $J'_2 \subseteq \text{Ne.b.lnt}(J'_1)$, whenever $J'_2$ is Ne.(b)CS and $J'_2 \subseteq J'_1$.

Proof.

Suppose $J'_1$ is Ne.(b)OS in $X$. Then $J'_1$ is Ne.(b)CS in $X$. Let $J'_2$ be a Ne.(b)CS in $X$ such that $J'_2 \subseteq J'_1$. Then $J'_2 \subseteq J'_1$, $J'_2$ is Ne.(b)OS in $X$. Since $J'_2$ is Ne.(b)CS, Ne.b.Cl($J'_1^c$) $\subseteq J'_2^c$, which implies (Ne.b.lnt($J'_1$))$^c$ $\subseteq$ $J'_2^c$. Thus $J'_2 \subseteq \text{Ne.b.lnt}(J'_1)$.

Conversely, assume that $J'_2 \subseteq \text{Ne.b.lnt}(J'_1)$, whenever $J'_2 \subseteq J'_1$ and $J'_2$ is Ne.(b)CS in $X$. Then (Ne.b.lnt($J'_1$))$^c$ $\subseteq$ $J'_2^c \subseteq J'_3$, where $J'_3$ is Ne.(b)OS in $X$. Hence Ne.b.Cl($J'_1^c$) $\subseteq$ $J'_3$, which implies $J'_1^c$ is Ne.(b)CS. Therefore $J'_1^c$ is Ne.(b)OS.

Theorem 3.12.

If $J'_1$ is Ne.(b)CS in $(X, N_\tau)$ and $J'_2 \subseteq J'_1 \subseteq \text{Ne.b.Cl}(J'_1)$, then $J'_2$ is Ne.(b)CS in $(X, N_\tau)$.

Proof.

Let $J'_3$ be Ne.(b)OS in $X$ such that $J'_2 \subseteq J'_3$, then $J'_2 \subseteq J'_3$. Since $J'_1$ is a Ne.(b)CS in $X$, it follows that Ne.b.Cl($J'_1^c$) $\subseteq$ $J'_3$. Now $J'_2 \subseteq \text{Ne.b.Cl}(J'_1)$ implies Ne.b.Cl($J'_2$) $\subseteq$ Ne.b.Cl(Ne.b.Cl($J'_1$)) = Ne.b.Cl($J'_1$). Thus Ne.b.Cl($J'_2$) $\subseteq$ $J'_3$. Hence $J'_2$ is Ne.(b)CS in $X$.

Theorem 3.13.

If $J'_1$ is Ne.(b)OS in $(X, N_\tau)$ and Ne.b.lnt($J'_1$) $\subseteq$ $J'_2 \subseteq$ $J'_1$, then $J'_2$ is Ne.(b)OS in $(X, N_\tau)$.

Proof.

Let $J'_1$ be Ne.(b)OS and $J'_2$ be any Ne. set in $X$ such that Ne.b.lnt($J'_1$) $\subseteq$ $J'_2 \subseteq$ $J'_1$. Then $J'_1^c$ is Ne.(b)CS and $J'_1^c \subseteq J'_2^c \subseteq$ Ne.b.Cl($J'_1^c$). Then $J'_2$ is Ne.(b)CS. Hence $J'_2$ is Ne.(b)OS of $X$.

Theorem 3.14.

Finite intersection of Ne.(b)CSs is a Ne.(b)CS.

Proof.

Let $J'_1$ and $J'_2$ be Ne.(b)CSs in $X$. Let $\emptyset \subseteq J'_1 \cap J'_2$, where $\emptyset$ is Ne.(b)CS in $X$. Then $\emptyset \subseteq J'_1$ and $\emptyset \subseteq J'_2$. Since $J'_1$ and $J'_2$ are Ne.(b)CSs, $\emptyset \subseteq J'_1 = \text{Ne.b.lnt}(J'_1')$ and $\emptyset \subseteq J'_2 = \text{Ne.b.lnt}(J'_2)$, which implies $\emptyset \subseteq (\text{Ne.b.lnt}(J'_1')) \cap (\text{Ne.b.lnt}(J'_2))$. Hence $\emptyset \subseteq \text{Ne.b.lnt}(J'_1 \cap J'_2)$. Therefore $J'_1 \cap J'_2$ Ne.(b)CS in $X$.

Theorem 3.15.

A finite union of Ne.(b)OS is a Ne.(b)OS.

Proof.
Let \( J_1 \) and \( J_2 \) be Ne.(bG)OS in \( X \). Let \( J_1 \subseteq J_2 \subseteq K \), where \( K \) is Ne.(b)OS in \( X \). Then \( J_1 \subseteq K \) or \( J_2 \subseteq K \). Since \( J_1 \) and \( J_2 \) are Ne.(bG)OS, Ne.bCl(\( J_1 \))=\( J_1 \subseteq K \) or Ne.bCl(\( J_2 \))=\( J_2 \subseteq K \), which implies Ne.bCl(\( J_1 \))\subseteq Ne.bCl(\( J_2 \))\subseteq \( K \). Hence Ne.bCl(\( J_1 \cup J_2 \))\subseteq \( K \). Therefore \( J_1 \cup J_2 \) Ne.(bG)OS in \( X \).

However, union of two Ne.(bG)CSs need not be a Ne.(bG)CSs as shown in the following example.

**Example 3.16.**

Let \( X=\{j_1, j_2\}, \mathcal{N}_X=\{0, J_1, 1\} \), is a N.T on \( X \)

Where \( J_1 = \left\{ x, \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right), \left( \frac{8}{10}, \frac{5}{10}, \frac{2}{10} \right) \right\} \). Then Neutrosophic set \( J_1 = \left\{ x, \left( \frac{6}{10}, \frac{5}{10}, \frac{4}{10} \right), \left( \frac{8}{10}, \frac{5}{10}, \frac{2}{10} \right) \right\} \) is a are Ne.bGCSs but \( J_1 \cup J_2 \) is not a Ne.bGCS in \( X \).

**Theorem 3.17.**

If \( J_1 \) is Ne.(b)OS in \( (X, \mathcal{N}_X) \) and Ne.(b)CS, then \( J_1 \) is Ne.(b)CS in \( (X, \mathcal{N}_X) \).

**Proof.**

Let \( J_1 \) be Ne.(b)OS and Ne.(b)CS in \( X \). For \( J_1 \subseteq J_1 \), by definition Ne.bCl(\( J_1 \))\subseteq \( J_1 \).

But \( J_1 \subseteq Ne.bCl(\{J_1\}) \), which implies \( J_1 = Ne.bCl(J_1) \). Hence \( J_1 \) is Ne.(b)CS in \( X \).

**Definition 3.18.**

A NSTS \((X, \mathcal{N}_X)\) is called a Neutrosophic bT_1/2 space (in short Ne.(b)T^*_{1/2} space) if every Ne.(b)CS in \( X \) is Ne-.CS.

**Definition 3.19.**

A NSTS \((X, \mathcal{N}_X)\) is called a Neutrosophic bT_1/2 space (in short Ne.T_{1/2} space) if every Ne.(b)CS in \( X \) is Ne.(b)CS.

**Theorem 3.20.**

A NSTS \((X, \mathcal{N}_X)\) is Ne.(b)T_{1/2} space iff every Ne. set in \((X, \mathcal{N}_X)\) is both Ne.(b)OS and Ne.(b)OS.

**Proof.**

Let \( X \) be Ne.(b)T_{1/2} space and let \( J_1 \) be Ne.(b)OS in \( X \). Then \( J_1 \subseteq J_1 \), Ne.(b)CS in \( X \) Ne.(b)CS, so \( J_1 \subseteq J_1 \), Ne.(b)CS and hence \( J_1 \subseteq J_1 \). Conversely, let \( J_1 \) be Ne.(b)CS. Then \( J_1 \subseteq J_1 \), Ne.(b)OS which implies \( J_1 \subseteq J_1 \), Ne.(b)OS. Hence \( J_1 \subseteq J_1 \), Ne.(b)OS. Every Ne.(b)CS in \( X \) Ne.(b)CS. Therefore \( X \) is Ne.(b)T_{1/2} space.

**Theorem 3.21.**

A NSTS \((X, \mathcal{N}_X)\) is Ne.(b)T_{1/2} space iff every Ne. set in \((X, \mathcal{N}_X)\) is both Ne.OS and Ne.(b)OS.

**Remark 3.22.**

A NSTS \((X, \mathcal{N}_X)\) is

(i) Ne.(b)T_{1/2} space if every Ne.(b)OS in \( X \) is Ne.(b)OS.

(ii) Ne.(b)T_{1/2} space if \( \forall \) Ne.(b)OS in \( X \) is Ne-open.

**Remark 3.23.**

In a NSTS \((X, \mathcal{N}_X)\)

(i) Every Ne.T_{1/2} space is Ne.(b)T_{1/2}

(ii) Every Ne.(b)T_{1/2} space is Ne.(Gb)T_{1/2}

(iii) Every Ne.(b)T_{1/2} space is Ne.(Gb)T_{1/2}
4. Ne.(bG)-Continuous and Ne.(Gb)-closed mappings

In this section, Neutrosophic bg-CTS maps, Neutrosophic bg-irresolute maps, and Neutrosophic bg-homeomorphism in Neutrosophic topological spaces are introduced and studied.

**Definition 4.1.**
A mapping \( \varphi: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y) \) is said to be Neutrosophic b generalized Continuous (Ne.(bG)-CTS), if \( \varphi^{-1}(J^c) \) is Ne.(bG)CS in \( \mathcal{X} \), for every Neutrosophic-CS \( J_1^c \) in \( \mathcal{Y} \).

**Theorem 4.2.**
\( \varphi: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y) \) is Ne.(bG)-CTS iff the inverse image of each NSOS in \( \mathcal{Y} \) is Ne.(bG)OS in \( \mathcal{X} \).

**Proof.**
Let \( J^c_1 \) be a Ne.(bG)OS in \( \mathcal{Y} \). Then \( J^c_1 \) is Ne.(bG)CS in \( \mathcal{Y} \). Since \( \varphi \) is Ne.(bG)-CTS \( \varphi^{-1}(J^c_1) = (\varphi^{-1}(J^c_1))^c \) is Ne.(bG)CS in \( \mathcal{X} \). Thus \( \varphi^{-1}(J^c_2) \) is Ne.(bG)OS in \( \mathcal{X} \).

Converse, is obvious.

**Theorem 4.3.**
Every Ne.-CTS map is Ne.(bG)-CTS.

**Proof.**
Let \( \varphi: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y) \) be Ne.-CTS function. Let \( J^c_1 \) be a Ne. OS in \( \mathcal{Y} \). Since \( \varphi \) is Ne.-CTS, \( \varphi^{-1} \) Ne. OS in \( \mathcal{X} \). Mean while each Ne.OS is Ne.(bG)OS, \( \varphi^{-1} \) is Ne.(bG)OS in \( \mathcal{X} \). Therefore \( \varphi \) is Ne.(bG)-CTS.

**Example 4.4.**
Let \( \mathcal{X} = \{ j , j \} \), \( \mathcal{N}_x = \{ 0, J_1^c, 1 \} \), is a N.T.on \( \mathcal{X} \) \( \mathcal{N}_y = \{ 0, J_2^c, 1 \} \) on \( \mathcal{Y} \), then Then the Neutrosophic sets \( \mathcal{J}_1^c = (x, \left( \frac{2}{10}, \frac{5}{10}, \frac{8}{10} \right), \left( \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right)) \).

\( \mathcal{J}_2^c = (x, \left( \frac{7}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{5}{10}, \frac{6}{10}, \frac{4}{10} \right)) \) is a Ne.bGCS in \( \mathcal{X} \).

Identity mapping \( \varphi: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y) \) \( \varphi \) is Ne.(Gb)-CTS but not Ne.-CTS

**Definition 4.5.**
A mapping \( \varphi: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y) \) is said to be Neutrosophic b-generalized irresolute (briefly Ne.(bG)-irresolute), if \( \varphi^{-1}(J^c_1) \) is Ne.(bG)CS set in \( \mathcal{X} \), for each Ne.(bG) CS \( J_1^c \) in \( \mathcal{Y} \).

**Theorem 4.6.**
Every Ne.(bG)-irresolute map is Ne.(bG)-CTS.

**Proof.**
Let \( \varphi: \mathcal{X} \rightarrow \mathcal{Y} \) be Ne.(bG)-irresolute and let \( J^c_1 \) be Ne.-CS in \( \mathcal{Y} \). Since every Ne.-CS is Also Ne.(bG)CS, \( J^c_1 \) is Ne.(bG)CS in \( \mathcal{Y} \). Since \( \varphi: \mathcal{X} \rightarrow \mathcal{Y} \) is Ne.(bG)-irresolute, \( \varphi^{-1}(J^c_1) \) is Ne.(bG)CS. Thus inverse image of every Ne.CS in \( \mathcal{Y} \) is Ne.(bG)CS in \( \mathcal{X} \). Therefore the function \( \varphi: \mathcal{X} \rightarrow \mathcal{Y} \) is Ne.(bG)-CTS. The converse is not true.

**Example 4.7.**
Let \( \mathcal{X} = \{ j , j \} \), \( \mathcal{N}_x = \{ 0, J_1^c, 1 \} \), is a N.T.on \( \mathcal{X} \) \( \mathcal{N}_y = \{ 0, J_2^c, 1 \} \) on \( \mathcal{Y} \), then Then the Neutrosophic sets \( \mathcal{J}_1^c = (x, \left( \frac{4}{10}, \frac{5}{10}, \frac{7}{10} \right), \left( \frac{5}{10}, \frac{6}{10}, \frac{3}{10} \right)) \), and \( \mathcal{J}_2^c = (x, \left( \frac{8}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{4}{10}, \frac{6}{10}, \frac{2}{10} \right)) \).

Then Identity mapping \( \varphi: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y) \)

We have \( \mathcal{J}_3^c = (x, \left( \frac{2}{10}, \frac{5}{10}, \frac{9}{10} \right), \left( \frac{6}{10}, \frac{5}{10}, \frac{3}{10} \right)) \) is a Ne.(bG)-CTS maps but not Ne.(bG)-irresolute maps.

**Theorem 4.8.**
Every Ne.(bG)-CTS map is Ne.(Gb)-CTS.

**Proof.**

Clear from the fact that Ne.(bG)CS is Ne.(Gb)CS.

**Theorem 4.9.**

Let \(g: \mathcal{X} \rightarrow \mathcal{Y}, \xi: \mathcal{Y} \rightarrow \mathcal{Z}\) be two mappings. Then

(i) \(ξg\) is Ne.(bG)-CTS, if \(g\) is Ne.(bG)-CTS and \(ξ\) is Ne.-CTS.

(ii) \(ξg\) is Ne.(bG)-irresolute, if \(g\) and \(ξ\) are Ne.(bG)-irresolute.

(iii) \(ξg\) is Ne.(bG)-CTS if \(g\) is Ne.(bG)-irresolute and \(ξ\) is Ne.(bG)-CTS.

**Proof.**

(i) Let \(J_1^b\) be Ne.CS in \(\mathcal{X}\). Since \(ξ: \mathcal{Y} \rightarrow \mathcal{Z}\) is Neutrosophic CTS, by definition \(ξ^{-1}(J_1^b)\) is Ne.CS of \(\mathcal{Y}\).

Now \(g: \mathcal{X} \rightarrow \mathcal{Y}\) is Ne.(bG)-CTS so \(g^{-1}(ξ^{-1}(J_1^b)) = (ξ \circ g)^{-1}(J_1^b)\) is Ne.(bG)CS in \(\mathcal{X}\). Hence \(ξg: \mathcal{X} \rightarrow \mathcal{Z}\) is Ne.(bG)-CTS.

(ii) Let \(ξ\) be Ne.(bG)-irresolute and let \(J_1^b\) be Ne.(bG)CS subset in \(\mathcal{Y}\). Since \(ξ\) is Ne.(bG)-irresolute by definition, \(ξ^{-1}(J_1^b)\) is Ne.(bG)CS in \(\mathcal{Y}\). Also \(g: \mathcal{X} \rightarrow \mathcal{Y}\) is Ne.(bG)-irresolute, so \(g^{-1}(ξ^{-1}(J_1^b)) = (ξ \circ g)^{-1}(J_1^b)\) is Ne.(bG)CS. Thus \(ξ \circ g: \mathcal{X} \rightarrow \mathcal{Z}\) is Ne.(bG)-irresolute.

(iii) Let \(J_1^b\) be Ne.(b)-CS in \(\mathcal{X}\). Since \(ξ: \mathcal{Y} \rightarrow \mathcal{Z}\) is Ne.(bG)-CTS, \(ξ^{-1}(J_1^b)\) is Ne.(bG)CS in \(\mathcal{Y}\). Also \(g: \mathcal{X} \rightarrow \mathcal{Y}\) is Ne.(bG)CS in \(\mathcal{Y}\) and Ne.(b)CS in \(\mathcal{X}\). Hence \(g^{-1}(ξ^{-1}(J_1^b)) = (ξ \circ g)^{-1}(J_1^b)\) is Ne.(bG)CS in \(\mathcal{X}\). Thus \(ξ \circ g: \mathcal{X} \rightarrow \mathcal{Z}\) is Ne.(bG)-irresolute.

**Theorem 4.11.**

If \(g: (\mathcal{X}, \mathcal{N}_1) \rightarrow (\mathcal{Y}, \mathcal{N}_2)\) is Ne.(b)*-CTS and \(ξ: (\mathcal{Y}, \mathcal{N}_2) \rightarrow (\mathcal{Z}, \mathcal{N}_3)\) is Ne.(b)-CTS, then \(ξ \circ g: (\mathcal{X}, \mathcal{N}_1) \rightarrow (\mathcal{Z}, \mathcal{N}_3)\) is Ne.(b)CTS if \(\mathcal{Y}\) is Ne.(b)\(1/2\)-space.

**Proof.**

Suppose \(J_1^b\) is Ne.(b)-CS subset of \(\mathcal{X}\). Since \(ξ: \mathcal{Y} \rightarrow \mathcal{Z}\) is Ne.(b)CTS, \(ξ^{-1}(J_1^b)\) is Ne.(b)CS subset of \(\mathcal{Y}\). Now since \(\mathcal{Y}\) is Ne.(b)\(1/2\)-space, \(ξ^{-1}(J_1^b)\) is Ne.(b)-CS subset of \(\mathcal{Y}\). Also since \(g: \mathcal{X} \rightarrow \mathcal{Y}\) is Ne.(b)*-CTS, \(g^{-1}(ξ^{-1}(J_1^b)) = (ξ \circ g)^{-1}(J_1^b)\) is Ne.(b)-CS. Thus \(ξ \circ g: \mathcal{X} \rightarrow \mathcal{Z}\) is Ne.(b)-CTS.

**Theorem 4.12.**

Let \(g: (\mathcal{X}, \mathcal{N}_1) \rightarrow (\mathcal{Y}, \mathcal{N}_2)\) be Ne.(bG)-CTS. Then \(g\) is Ne.(b)-CTS if \(\mathcal{X}\) is Ne.(b)\(1/2\)-space.

**Proof.**

Let \(J_1^b\) be Ne.-CS in \(\mathcal{Y}\). Since \(g: \mathcal{X} \rightarrow \mathcal{Y}\) is Ne.(bG)CTS, \(g^{-1}(J_1^b)\) is Ne.(bG)CS subset in \(\mathcal{X}\). Since \(\mathcal{X}\) is Ne.(b)\(1/2\)-space, by hypothesis every Ne.(b)CS is Ne.(b)-CS. Hence \(g^{-1}(J_1^b)\) is Ne.(b)-CS subset in \(\mathcal{X}\). Therefore \(g: \mathcal{X} \rightarrow \mathcal{Y}\) is Ne.(b)-CTS.

**Theorem 4.13.**

Let \(g: (\mathcal{X}, \mathcal{N}_1) \rightarrow (\mathcal{Y}, \mathcal{N}_2)\) be onto Ne.(bG)-irresolute and Ne. b*CS. If \(\mathcal{X}\) is Ne.(b)\(1/2\)-space, then \((\mathcal{Y}, \mathcal{N}_2)\) is Ne.(b)\(1/2\)-space.

**Proof.**

Let \(J_1^b\) be a Ne.(bG)CS in \(\mathcal{Y}\). Since \(g: \mathcal{X} \rightarrow \mathcal{Y}\) is Ne.(bG)irresolute, \(g^{-1}(J_1^b)\) is Ne.(bG)CS in \(\mathcal{X}\). As \(\mathcal{X}\) is Ne.(b)\(1/2\)-space, \(g^{-1}(J_1^b)\) is Ne.(b)CS in \(\mathcal{X}\). Also \(g: \mathcal{X} \rightarrow \mathcal{Y}\) is Ne. b*CS, \(g(g^{-1}(J_1^b))\) is Ne.(b)CS in \(\mathcal{Y}\). Since \(g: \mathcal{X} \rightarrow \mathcal{Y}\) is onto, \(g(g^{-1}(J_1^b)) = J_1^b\). Thus \(J_1^b\) is Ne.(b)CS in \(\mathcal{Y}\). Hence \((\mathcal{Y}, \mathcal{N}_2)\) is Ne.(b)\(1/2\)-space.

**Theorem 4.14.**

Let \(g: (\mathcal{X}, \mathcal{N}_1) \rightarrow (\mathcal{Y}, \mathcal{N}_2)\) be Ne.(bG)-CTS and \(ξ: (\mathcal{Y}, \mathcal{N}_2) \rightarrow (\mathcal{Z}, \mathcal{N}_3)\) be Ne.g-CTS. Then \(ξ \circ g\) is Ne.(bG)-CTS if \(\mathcal{Y}\) is Ne.\(1/2\)-space.

**Proof.**
Let $J_1^*$ be Ne.-CS in $\mathcal{J}$. Since $\zeta$ is Ne.g-CTS, $\zeta^{-1}(J_1^*)$ is Ne.g-CS in $\mathcal{Y}$. But $\mathcal{Y}$ is Ne.$T_{1/2}$ space and so $\zeta^{-1}(J_1^*)$ is Ne.-CS in $\mathcal{Y}$. Since $\varrho$ is Ne.(bG)-CTS $\varrho^{-1}(\zeta^{-1}(J_1^*)) = (\zeta \circ \varrho)^{-1}(J_1^*)$ is Ne.(bG)CS in $\mathcal{X}$. Hence $\zeta \circ \varrho$ Ne.(bG)-CTS.

**Theorem 4.15.**

If the bijective map $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is Ne.(b)*-open and Ne.(b)-irresolute, then $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is Ne.(bG)-irresolute.

**Proof.**

Let $J_1^*$ be a Ne.(bG)CS in $\mathcal{Y}$ and let $\varrho^{-1}(J_1^*) \subseteq J_2^*$ where $J_2^*$ is a Ne.(b)OS in $\mathcal{X}$. Clearly, $J_1^* \subseteq \varrho(J_2^*)$. Since $\varrho: \mathcal{X} \rightarrow \mathcal{Y}$ is Ne.(b)*-open map, $\varrho(J_2^*)$ is Ne.(b)-open in $\mathcal{Y}$ and $J_1^*$ is Ne.(bG)CS in $\mathcal{Y}$. Then Ne.$bCl(J_1)^* \subseteq \varrho(J_2^*)$, and thus $\varrho^{-1}(Ne.bCl(J_1)^*) \subseteq J_2^*$. Also $\varrho: \mathcal{X} \rightarrow \mathcal{Y}$ irresolute and Ne.$bCl(J_1)$ is a Ne.(b)-CS in $\mathcal{Y}$, then $\varrho^{-1}(Ne.bCl(J_1))$ is Ne.(b)CS in $\mathcal{X}$. Thus Ne.$bCl(\varrho^{-1}(J_1)) \subseteq Ne.bCl(\varrho^{-1}(Ne.bCl(J_1))) \subseteq Ne.bCl(J_1)$. So $\varrho^{-1}(J_1^*)$ is Ne.(bG)CS in $\mathcal{X}$. Hence $\varrho: \mathcal{X} \rightarrow \mathcal{Y}$ is Ne.(bG)-irresolute map.

**Definition 4.16.**

A mapping $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is said to be Neutrosophic bg-open (briefly Ne.(b)OS) if the image of every Ne.-OS in $\mathcal{X}$ is Ne.(b)OS in $\mathcal{Y}$.

**Definition 4.17.**

A mapping $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is said to be Neutrosophic bg-CS (briefly Ne.(b)CS) if the image of every Ne. CS in $\mathcal{X}$ is Ne.(bG)CS in $\mathcal{Y}$.

**Definition 4.18.**

A mapping $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is said to be Neutrosophic bg*-open (briefly Ne.(b)*-OS) if the image of every Ne.(b)OS in $\mathcal{X}$ is Ne.(bG)OS in $\mathcal{Y}$.

**Definition 4.19.**

A mapping $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is said to be Neutrosophic bg-CS (briefly Ne.(b)*-CS) if the image of every Ne.(b)CS in $\mathcal{X}$ is Ne.(bG)CS in $\mathcal{Y}$.

**Remark 4.20.**

(i) Every Ne.(b)*-CS mapping is Ne.(b)CS.

(ii) Every Ne.(b)*-CS mapping is Ne.(Gb)*-CS.

**Theorem 4.23.**

If $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is Ne-CS and $\zeta: (\mathcal{Y}, \mathcal{N}_y) \rightarrow (\mathcal{Z}, \mathcal{N}_z)$ is Ne.(bG)CS, then $\zeta \circ \varrho$ is Ne.(bG)CS.

**Proof.**

Let $J_1^*$ be a Ne.CS in $\mathcal{X}$. Then $\varrho(J_1^*)$ is Ne.CS in $\mathcal{Y}$. Since $\zeta: (\mathcal{Y}, \mathcal{N}_y) \rightarrow (\mathcal{Z}, \mathcal{N}_z)$ is Ne.(bG)CS, $\zeta(\varrho(J_1^*)) = (\zeta \circ \varrho)(J_1^*)$ is Ne.(bG)CS in $\mathcal{Z}$. Therefore $\zeta \circ \varrho$ is Ne.(bG)CS.

**Theorem 4.24.**

If $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is a Ne.(b)CS map and $\mathcal{Y}$ is Ne.(b)$T_{1/2}$ space, then $\varrho$ is a Ne.-CS.

**Proof.**

Let $J_1^*$ be a Ne.CS in $\mathcal{X}$. Then $\varrho(J_1^*)$ is Ne.(Gb)-CS in $\mathcal{Y}$, since $\varrho$ is Ne.(Gb)CS. Again since $\mathcal{Y}$ is Ne.(b)$T_{1/2}$ space, $\varrho(J_1^*)$ is Ne.-CS in $\mathcal{Y}$. Hence $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is a $\varrho$-CS.

**Theorem 4.25.**

If $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is a Ne.(b)CS map and $\mathcal{Y}$ is Ne.(b)$T_{1/2}$ space, then $\varrho$ is a Ne.(b)-CS map.

**Theorem 4.26.**

A mapping $\varrho: (\mathcal{X}, \mathcal{N}_x) \rightarrow (\mathcal{Y}, \mathcal{N}_y)$ is Ne.(bG)CS iff for each Ne. set $J_1^*$ in $\mathcal{Y}$, and Ne.OS $J_2^*$ such that $\varrho^{-1}(J_1^*) \subseteq J_2^*$, there is a Ne.(bG)OS $J_3^*$ of $\mathcal{Y}$ such that $J_1^* \subseteq J_3^*$ and $\varrho^{-1}(J_2^*) \subseteq J_3^*$.

**Proof.**


Suppose \( q \) is Ne.(b)CS map. Let \( J_1^* \) be a Ne. set of \( \mathfrak{g} \), and \( J_2^* \) be an Ne.OS of \( \mathfrak{x} \), such that \( q^{-1}(J_1^*) \subseteq J_2^* \). Then \( J_1^* = (J_2^*)^c \) is a Ne.(b)OS in \( \mathfrak{g} \) such that \( J_1^* \subseteq J_2^* \) and \( q^{-1}(J_1^*) \subseteq \mathfrak{g}^c \). Conversely, suppose that \( \mathfrak{g} \) is a Ne.CS of \( \mathfrak{x} \). Then \( q^{-1}(q(\mathfrak{g}))^c) \subseteq \mathfrak{g}^c \), and \( \mathfrak{g}^c \), is Ne.OS. By hypothesis, there is a Ne.(b)OS \( J_2^* \) of \( \mathfrak{g} \) such that \( (q(\mathfrak{g}))^c \subseteq J_2^* \) and \( q^{-1}(J_2^*) \subseteq \mathfrak{g}^c \). Therefore \( \mathfrak{g} \subseteq \left( q^{-1}(J_1^*) \right)^c \). Hence \( J_2^* \subseteq q(q(J_1^*)) \subseteq q \left( q^{-1}(J_2^*) \right)^c \subseteq \mathfrak{g}^c \), which implies \( q(\mathfrak{g}) = J_2^c \). Since \( J_2^c \) is Ne.(b)CS, \( q(\mathfrak{g}) \) is Ne.(b)CS and thus \( q \) is a Ne.(b)CS map.

**Theorem 4.27.**

If \( q : (\mathfrak{X}, N_\sigma) \rightarrow (\mathfrak{Y}, N_\sigma) \) and \( \zeta : (\mathfrak{Y}, N_\sigma) \rightarrow (3, N_\sigma) \) are Ne.(b)CS maps and \( \mathfrak{y} \) is Ne.(b)T1\(1/2 \) space, then \( \zeta \circ q : \mathfrak{x} \rightarrow \mathfrak{y} \) is Ne.(b)CS.

**Proof.** Let \( J_1^* \) be a Ne.-CS in \( \mathfrak{x} \). Since \( q : (\mathfrak{X}, N_\sigma) \rightarrow (\mathfrak{Y}, N_\sigma) \) is Ne.(b)CS, \( q(J_1^*) \) is Ne.(b)CS in \( \mathfrak{y} \). Now \( \mathfrak{y} \) is Ne.(b)T1\(1/2 \) space, so \( q(J_1^*) \) is Ne.-CS in \( \mathfrak{y} \). Also \( \zeta : \mathfrak{y} \rightarrow 3 \) is Ne.(b)CS, \( \zeta(q(J_1^*)) = (\zeta \circ q)(J_1^*) \) is Ne.(b)CS in \( 3 \). Therefore \( \zeta \circ q \) is Ne.(b)CS.

**Theorem 4.28.** If \( J_1^* \) is Ne.(b)CS in \( \mathfrak{x} \) and \( q : \mathfrak{x} \rightarrow \mathfrak{y} \) is bijective, Ne.(b)-irresolute and Ne.(b)CS, then \( q(J_1^*) \) is Ne.(b)CS in \( \mathfrak{y} \).

**Proof.**

Let \( q(J_1^*) \subseteq J_2^* \) where \( J_2^* \) is Ne.(b)OS in \( \mathfrak{y} \). Since \( q \) is Ne.(b)-irresolute, \( q^{-1}(J_2^*) \) is Ne.(b)OS containing \( J_1^* \). Hence \( q \circ \mathrm{bCl}(J_1^*) \subseteq q^{-1}(J_2^*) \) as \( J_1^* \) is Ne.(b)OS in \( \mathfrak{y} \). Since \( q \) is Ne.(b)CS, \( q \circ \mathrm{bCl}(J_1^*) \) is Ne.(b)CS contained in the Ne.(b)OS \( J_2^* \), which implies \( q \circ \mathrm{bCl}(q(\mathrm{bCl}(J_1^*))) \subseteq J_2^* \) and hence \( q \circ \mathrm{bCl}(q(J_1^*)) \subseteq J_2^* \). So \( q(J_1^*) \) is Ne.(b)CS in \( \mathfrak{y} \).

**Theorem 4.29.**

If \( q : (\mathfrak{X}, N_\sigma) \rightarrow (\mathfrak{Y}, N_\sigma) \) is Ne.(b)CS and \( \zeta : (\mathfrak{Y}, N_\sigma) \rightarrow (3, N_\sigma) \) is Ne.(b)*-CS, then \( \zeta \circ q \) is Ne.(b)CS.

**Proof.** Let \( J_1^* \) be Ne.CS in \( \mathfrak{x} \). Then \( q(J_1^*) \) is Ne.(b)CS in \( \mathfrak{y} \). Since \( \zeta : (\mathfrak{Y}, N_\sigma) \rightarrow (3, N_\sigma) \) is Ne.(b)*-CS. Thus \( \zeta(q(J_1^*)) = (\zeta \circ q)(J_1^*) \) is Ne.(b)CS in \( 3 \). Therefore \( \zeta \circ q \) is Ne.(b)CS. If \( q : (\mathfrak{X}, N_\sigma) \rightarrow (\mathfrak{Y}, N_\sigma) \) and \( \zeta : (\mathfrak{Y}, N_\sigma) \rightarrow (3, N_\sigma) \) are Ne.(b)*-CS maps, then \( \zeta \circ q : \mathfrak{x} \rightarrow 3 \) is Ne.(b)* CS.

**Theorem 4.30.**

Let \( q : (\mathfrak{X}, N_\sigma) \rightarrow (\mathfrak{Y}, N_\sigma), \zeta : (\mathfrak{Y}, N_\sigma) \rightarrow (3, N_\sigma) \) be two maps such that \( \zeta \circ q : \mathfrak{x} \rightarrow 3 \) is Ne.(b)CS.

(i) If \( q \) is Ne.-CTS and surjective, then \( \zeta \) is Ne.(b)CS.

(ii) If \( \zeta \) is Ne.(b)-irresolute and injective, then \( q \) is Ne.(b)CS.

**Proof.**

(i). Let \( \mathfrak{g} \) be Ne.CS in \( \mathfrak{y} \). Then \( q^{-1}(\mathfrak{g}) \) is Ne.CS in \( \mathfrak{X} \), as \( q \) is Ne.-CTS. Since \( \zeta \circ q \) is Ne.(b)CS map and \( q \) is surjective, \( (\zeta \circ q)(q^{-1}(\mathfrak{g})) = (\zeta \circ q)(\mathfrak{g}) \) is Ne.(b)CS in \( 3 \). Hence \( \zeta : \mathfrak{y} \rightarrow 3 \) is Ne.(b)CS.

(ii). Let \( \mathfrak{g} \) be a Ne. CS in \( \mathfrak{x} \). Then \( (\zeta \circ q)(\mathfrak{g}) \) is Ne.(b)CS in \( 3 \). Since \( \zeta \) is Ne.(b)-irresolute and injective \( \zeta^{-1}(\zeta \circ q)(\mathfrak{g}) = q(\mathfrak{g}) \) is Ne.(b)CS in \( \mathfrak{y} \). Hence \( q \) is a Ne.(b)CS.

**Theorem 4.31.**

Let \( q : (\mathfrak{X}, N_\sigma) \rightarrow (\mathfrak{Y}, N_\sigma), \zeta : (\mathfrak{Y}, N_\sigma) \rightarrow (3, N_\sigma) \) be two maps such that \( \zeta \circ q : \mathfrak{x} \rightarrow 3 \) is Ne.(b)*-CS map.

(i) If \( q \) is Ne.(b)-CTS and surjective, then \( \zeta \) is Ne.(b)CS.

(ii) If \( \zeta \) is Ne.(b)-irresolute and injective, then \( q \) is Ne.(b)*-CS.

**Theorem 4.32.** Let \( q : (\mathfrak{X}, N_\sigma) \rightarrow (\mathfrak{Y}, N_\sigma) \) then the following statements are equivalent:

(i) \( q \) is Ne.(b)-irresolute.

(ii) for every Ne.(b)CS \( J_1^* \) in \( \mathfrak{y} \), \( q^{-1}(J_1^*) \) is Ne.(b)CS in \( \mathfrak{x} \).
Proof. (i)⇒ (ii) Obvious.
(ii)⇒ (i) Let \( \mathcal{J}_1 \) be a Ne.(b)CS in \( \mathcal{Y} \) which implies \( \mathcal{J}_1^{*^+} \) is Ne.(b)OS in \( \mathcal{Y} \). \( q^{-1}(\mathcal{J}_1^{*^+}) \) is Ne.(b)open in \( \mathcal{X} \) implies \( q^{-1}(\mathcal{J}_1) \) is Ne.(b)CS in \( \mathcal{X} \). Hence \( q \) is Ne.(b)-irresolute.

Neutrosophic bg-homeomorphism and Neutrosophic bg*-homeomorphism are defined as follows.

**Definition 4.33.**
A mapping \( q : (\mathcal{X}, \mathcal{N}_r) \to (\mathcal{Y}, \mathcal{N}_s) \) is called Neutrosophic bg-homeomorphism (briefly Ne.(b)-homeomorphism) if \( q \) and \( q^{-1} \) are Ne.(b)CTS.

**Definition 4.34.**
A mapping \( q : (\mathcal{X}, \mathcal{N}_r) \to (\mathcal{Y}, \mathcal{N}_s) \) is called Neutrosophic bg*-homeomorphism (briefly Ne.(b)G*-homeomorphism) if \( q \) and \( q^{-1} \) are Ne.(b)irresolute.

**Theorem 4.35.**
Every Ne.-homeomorphism is Ne.(b)-homeomorphism.

The converse of the above theorem need not be true as seen from the following example.

**Example 4.36.**
Let \( \mathcal{X} = \{ j_1, j_2 \} = \mathcal{Y}, \mathcal{N}_r = \{ 0, j_1', 1 \} \) is a N.T on \( \mathcal{X} \). \( \mathcal{N}_s = \{ 0, j_2', 1 \} \) on \( \mathcal{Y} \).

Then Neutrosophic sets

\[ J_1' = (\alpha, (\frac{10}{10}, \frac{5}{10}, \frac{0}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{2}{10})) \]

\[ J_2' = (\alpha, (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{6}{10}, \frac{6}{10}, \frac{4}{10})) \]

\[ J_3' = (\alpha, (\frac{4}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{4}{10})) \]

Define mapping \( q : (\mathcal{X}, \mathcal{N}_r) \to (\mathcal{Y}, \mathcal{N}_s) \) by \( q(j_1) = j_1 \) and \( q(j_2) = j_2 \).

Then \( q \) is Ne.(b)-homeomorphism but not Ne.-homeomorphism.

**Theorem 4.37.**
Every Ne.(b)G*-homeomorphism is Ne.(b)-homeomorphism.

**Proof.**
Let \( q : (\mathcal{X}, \mathcal{N}_r) \to (\mathcal{Y}, \mathcal{N}_s) \) be Ne.(b)G*-homeomorphism. Then \( q \) and \( q^{-1} \) are Ne.(b)-irresolute mappings. By theorem 4.7 \( q \) and \( q^{-1} \) are Ne.(b)-CTS. Hence \( q : (\mathcal{X}, \mathcal{N}_r) \to (\mathcal{Y}, \mathcal{N}_s) \) is Ne.(b)(G)-homeomorphism.

**Theorem 4.38.**
If \( q : (\mathcal{X}, \mathcal{N}_r) \to (\mathcal{Y}, \mathcal{N}_s) \) is Ne.(b)-homeomorphism and \( \zeta : (\mathcal{Y}, \mathcal{N}_s) \to (\mathcal{Z}, \mathcal{N}_t) \) is Ne.(b)G*-homeomorphism and \( \mathcal{Y} \) is Ne.(b)T\( \frac{1}{2} \)-space, then \( \zeta \circ q : \mathcal{X} \to \mathcal{Z} \) is Ne.(b)-homeomorphism.

**Proof.**
To show that \( \zeta \circ q \) and \( (\zeta \circ q)^{-1} \) are Ne.(b)G*-CTS. Let \( \mathcal{J}_1' \) be a Ne.OS in \( \mathcal{Z} \). Since \( \zeta : \mathcal{Y} \to \mathcal{Z} \) is Ne.(b)G*-CTS, \( \zeta^{-1}(\mathcal{J}_1') \) is Ne.(b)open in \( \mathcal{Y} \). Then \( \zeta^{-1}(\mathcal{J}_1') \) is a Ne.-open in \( \mathcal{Y} \) as \( \mathcal{Y} \) is Ne.(b)T\( \frac{1}{2} \)-space. Also since \( q : \mathcal{X} \to \mathcal{Y} \) is Ne.(b)-CTS, \( q^{-1}(\zeta^{-1}(\mathcal{J}_1')) = q^{-1}(\zeta^{-1}(\mathcal{J}_1')) \) is Ne.(b)-open in \( \mathcal{X} \).

Therefore \( \zeta \circ q \) is Ne.(b)G*-CTS. Again, let \( \mathcal{J}_1' \) be a Ne.OS in \( \mathcal{X} \). Since \( q^{-1} : \mathcal{Y} \to \mathcal{X} \) is Ne.(b)G*-CTS, \( (q^{-1})^{-1}(\mathcal{J}_1') \) is Ne.(b)G*-OS in \( \mathcal{Y} \). And so \( q^{-1}(\mathcal{J}_1') \) is Ne.-open in \( \mathcal{Y} \) since \( \mathcal{Y} \) is Ne.(b)T\( \frac{1}{2} \)-space.

Also since \( \zeta^{-1} : \mathcal{Y} \to \mathcal{Z} \) is Ne.(b)G*-CTS, \( (\zeta^{-1})^{-1}(q^{-1}(\mathcal{J}_1')) = \zeta(q^{-1}(\mathcal{J}_1')) = (\zeta \circ q)(\mathcal{J}_1') \) is Ne.(b)-open in \( \mathcal{Z} \).

Therefore \( (\zeta \circ q)^{-1} \) is Ne.(b)-CTS. Thus \( \zeta \circ q \) is Ne.(b)-homeomorphism.

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