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Neutrosophic Generalized Exponential Distribution with Application

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Abstract. The objective of this article is to create a Neutrosophic Generalized Exponential (NGE) distribution in the presence of uncertainty. It is possible to calculate the mean, variance, moments, and reliability expression of the NGE distribution. With the help of graphs, the nature of the distribution and the reliability and hazard functions are studied. To determine the NGE distribution's parameters, a maximum likelihood estimation technique is used. The performance of estimated parameters is further tested using simulations. Finally, an actual data set is examined to show how the NGE distribution works. According to a model validity test, the NGE distribution is superior to the existing neutrosophic distributions that can be found in the literature.

Keywords: Generalized exponential distribution; Neutrosophic; Indeterminacy; Maximum likelihood estimation; Simulation; Reliability.

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1. Introduction

Numerous researchers have started developing various studies based on Neutrosophic statistics in recent years. The original research on neutrosophic statistics was initiated by Smarandache [1]. This new area of research is a generalization of the fuzzy logic environment, and it is used in an uncertain environment. Due to its ability to administer sets of values in an interval form, neutrosophic statistics play a crucial role in statistics and other research fields. For more details about Neutrosophic statistics and its related works, please refer to [2–11].

The neutrosophic theory of probability is indispensable and has practical applications. This area of study has not received a great deal of attention. Some authors have focused more on the neutrosophic statistics approach and its applications in various fields in recent years. For more information about neutrosophic probability, see [12,13]. Patro and Smarandache [14] presented the neutrosophic statistical distribution, more problems, and more solutions. Alhabib et al. [9] studied some neutrosophic probability distributions by generalizing some classical probability distributions such as the Poisson distribution, exponential distribution, and uniform distribution to the neutrosophic type. Nayana et al. [15] created a new neutrosophic model using the DUS-Weibull transformation, while Alhasan and Smarandache [16] studied the neutrosophic Weibull distribution. Zeina and Hatip [17] developed the neutrosophic random variables. They studied various statistical properties and examples. Sherwani [18] studied neutrosophic beta distribution with properties and applications. The other application of neutrosophic statistics in various field like quality control, sampling plans, process capability analysis and social science indeterminacy environment studied by [19–22]. The neutrosophic theory has many applications in a variety of fields, such as the neutrosophic treatment of the static model, the integration of renewable energy using a variety of resources, such as photovoltaic panels and wind turbines, and COVID-19 and its Omicron mutation. In traditional mathematics, crispness is the most crucial prerequisite; however, in actual problems, ambiguous data are present. In order to solve these issues, mathematical concepts based on uncertainty must be used. Uncertainty modeling is something that many scientists and engineers are interested in because it helps them define and explain the useful information that is hidden in uncertain data. Although it is one of the most crucial tools and has practical applications, the neutrosophic probability theory has not gotten much attention. It has, however, been the subject of some studies. More studies have focused in recent years on various areas of neutrosophic statistics, including correlation, regression analysis, test procedures, probability distributions, etc.

The mentioned studies and literature reviews have motivated us to develop a neutrosophic generalized exponential distribution and its properties.

1.1. *Neutrosophic Approach*

Neutrosophic statistics is the generalization of classical statistics. We administer with specific or crumple values in classical statistics, but in neutrosophic statistics, the sample values are chosen from a population with an uncertain environment. In neutrosophic statistics, the information can be vague, imprecise, ambiguous, uncertain, incomplete, or even unknown. Neutrosophic numbers have a standard form based on classical statistics, which is given below.

$$X_N = E + I$$

Data is broken down into two parts, E and I, where E is the exact or determined data and I is the uncertain, inexact, or indeterminate part of the data. It is equivalent to $X_N \in [X_L, X_U]$. A subscript $_N$ is used to distinguish the neutrosophic random variable, for example, X_N.

1.2. *Generalized Exponential distribution*

The generalized exponential (GE) distribution is one of the most widely used and flexible distributions compared to the exponential, gamma, and Weibull distributions; see [23] for more details. The GE distribution has more applications in reliability analysis, hydrology, quality control and medical field etc, please refer [24–30].

If a continuous random variable X_i; $i = 1, 2, \ldots, n$ is followed by the generalized exponential distribution with shape parameter δ and scale parameter υ then its probability density function (p.d.f.) and cumulative distribution function are respectively given as follows:

$$f(x) = \frac{\delta}{\upsilon}\left(1 - \exp\{-\frac{x}{\upsilon}\}\right)^{\delta-1}\exp\{-\frac{x}{\upsilon}\}; \qquad x > 0,\ \delta > 0,\ \upsilon > 0, \tag{1}$$

and

$$F(x) = \left(1 - \exp\{-\frac{x}{\upsilon}\}\right)^{\delta}; \qquad x > 0,\ \delta > 0,\ \upsilon > 0. \tag{2}$$

2. **Neutrosophic Generalized Exponential distribution**

Let us assume that $X_{N_i} \in [X_L, X_U]$, $i = 1, 2, \ldots, n_N$ is neutrosophic random variable following the neutrosophic generalized exponential (NGE) distribution with neutrosophic shape parameter $\delta_N \in [\delta_L, \delta_U]$ and neutrosophic scale parameter $\upsilon_N \in [\upsilon_L, \upsilon_U]$. The neutrosophic probability density function (n.p.d.f.) of NGE distribution is given as follows:

$$f(x_N) = \frac{\delta_N}{\upsilon_N}\left(1 - \exp\{-\frac{x_N}{\upsilon_N}\}\right)^{\delta_N-1}\exp\{-\frac{x_N}{\upsilon_N}\}; \qquad x_N > 0,\ \delta_N > 0,\ \upsilon_N > 0 \tag{3}$$

Where $X_N \in [X_L, X_U]$, $\delta_N \in [\delta_L, \delta_U]$, $\upsilon_N \in [\upsilon_L, \upsilon_U]$. NGE distribution with neutrosophic shape parameter δ_N and neutrosophic scale parameter υ_N is denoted as NGED(δ_N, υ_N). NGE distribution is transformed into a neutrosophic exponential distribution with neutrosophic scale parameter $\upsilon_N \in [\upsilon_L, \upsilon_U]$ when NGED$(1, \upsilon_N)$. Figure 1 display the p.d.f. plots for different parametric values of NGE distribution.

The developed NGE distribution is more flexible on account of the different shapes of the density function. From Figure 1, The curves of p.d.f. show that the behavior of the curves exponentially diminishes and starts from the infinite point for $\delta_N < 1$. For $\delta_N = 1$, its behavior exponentially diminishes but starts from a specific point on the y-axis. The density curves show unimodal behavior for $\delta_N > 1$.

The cumulative distribution function (c.d.f.) of NGE distribution is

$$F(x_N) = \left(1 - \exp\{-\frac{x_N}{\upsilon_N}\}\right)^{\delta_N}; \qquad x_N > 0,\ \delta_N > 0,\ \upsilon_N > 0. \tag{4}$$

The survival function and hazard function of NGE distribution respectively expressed as

$$s(x_N) = 1 - \left(1 - \exp\{-\frac{x_N}{\upsilon_N}\}\right)^{\delta_N}, \tag{5}$$

and

$$h(x_N) = \frac{\frac{\delta_N}{\upsilon_N}\left(1 - \exp\{-\frac{x_N}{\upsilon_N}\}\right)^{\delta_N - 1}\exp\{-\frac{x_N}{\upsilon_N}\}}{1 - \left(1 - \exp\{-\frac{x_N}{\upsilon_N}\right)^{\delta_N}} \tag{6}$$

From Figure 2, it is interesting to note that the NGE distribution has variable shapes. The survival function and failure rate curves for various neutrosophic parametric values are presented in Figures 3 and 4. Form Figure 4, the failure rate of NGE distribution is a bathtub and increasing behavior, which is very important for analyzing data sets in various fields.

3. **Statistical Properties**

In this section, we reviewed some statistical characteristics of the NGE distribution.

The mean and variance values are respectively expressed as

$$\mu_N = \frac{1}{\upsilon_N}\left[\psi\left(\delta_N + 1\right) - \psi\left(1\right)\right], \tag{7}$$

and

$$\sigma_N^2 = \frac{1}{\upsilon_N^2}\left[\psi'\left(1\right) - \psi'\left(\delta_N + 1\right)\right]. \tag{8}$$

The expressions $\psi(\cdot)$ denotes the digamma function while $\psi'(\cdot)$ denotes a derivative of $\psi(\cdot)$. For details about classical GED moments, refer to [23]. The q^{th} quantile of NGE distribution is obtained as follows:

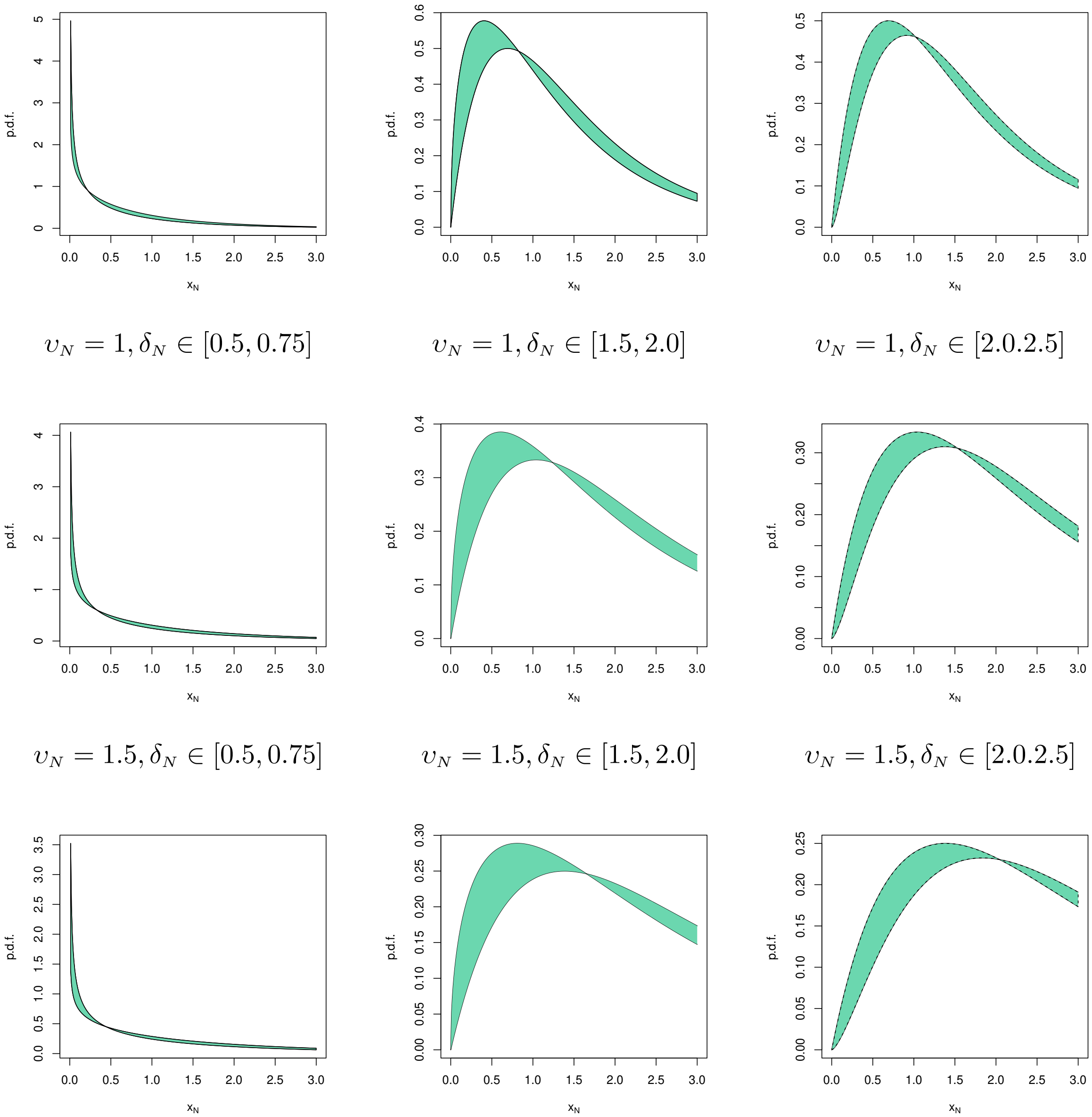
$$x_{Nq} = -\upsilon_N \ln\left(1 - \mathrm{q}_N^{\frac{1}{\delta_N}}\right). \tag{9}$$

Consequently, the median value is $x_{N(0.5)} = -\upsilon_N \ln\left(1 - 2^{\frac{-1}{\delta_N}}\right)$.

4. **Estimation of parameters**

In this section, using the method of maximum likelihood estimation (MLE) the parameters of NGE distribution are estimated. Let $X_{N1}, X_{N2}, \ldots, X_{Nn}$ be a neutrosophic random sample of size n taken from NGE distribution. The log-likelihood equation is given by

$$l\left(\delta_N, \upsilon_N\right) = \ln\left(L\right) = n\ln\left(\delta_N\right) - n\ln\left(\upsilon_N\right) - \sum_{i=1}^{n}\frac{x_{Ni}}{\upsilon_N} + \left(\delta_N - 1\right)\sum_{i=1}^{n}\ln\left(1 - \exp\{-\frac{x_{Ni}}{\upsilon_N}\}\right) \tag{10}$$



$\upsilon_N = 1, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 1, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 1, \delta_N \in [2.0.2.5]$

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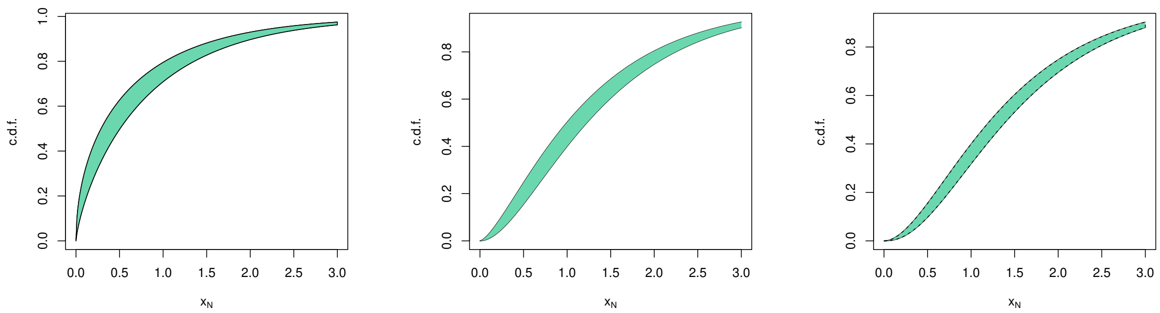
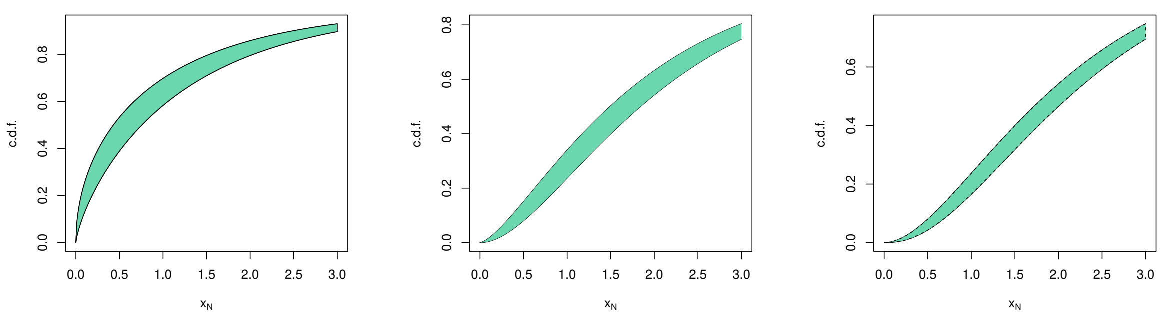
$\upsilon_N = 2, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 2, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 2, \delta_N \in [2.0.2.5]$

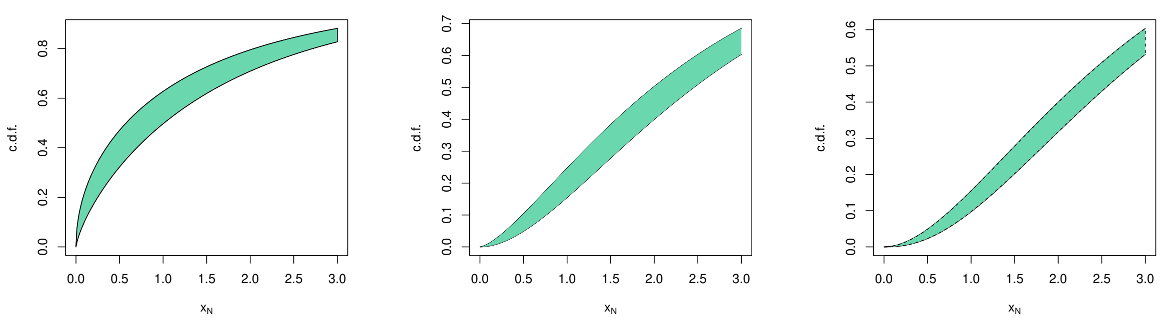
FIGURE 1. The p.d.f. plots of NGE distribution



$\upsilon_N = 1, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 1, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 1, \delta_N \in [2.0.2.5]$

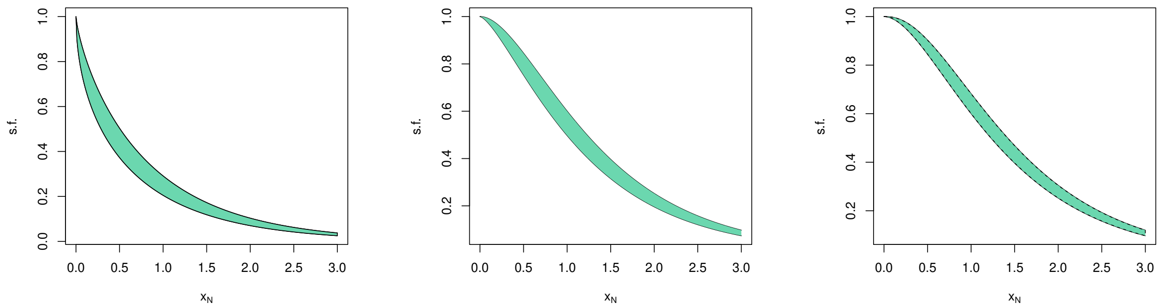


$\upsilon_N = 1.5, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 1.5, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 1.5, \delta_N \in [2.0.2.5]$

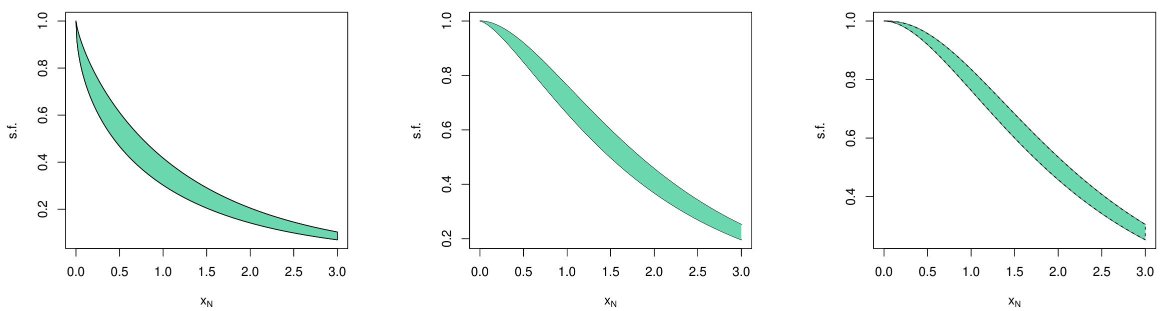


$\upsilon_N = 2, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 2, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 2, \delta_N \in [2.0.2.5]$

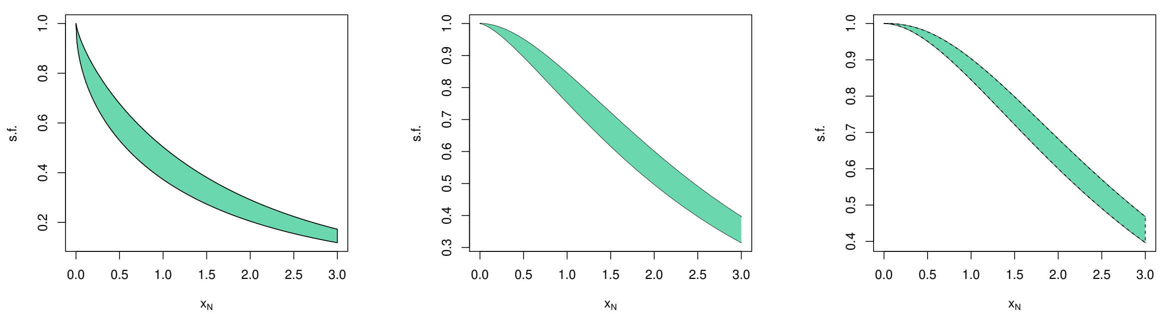
FIGURE 2. The c.d.f. plots of NGE distribution



$\upsilon_N = 1, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 1, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 1, \delta_N \in [2.0.2.5]$

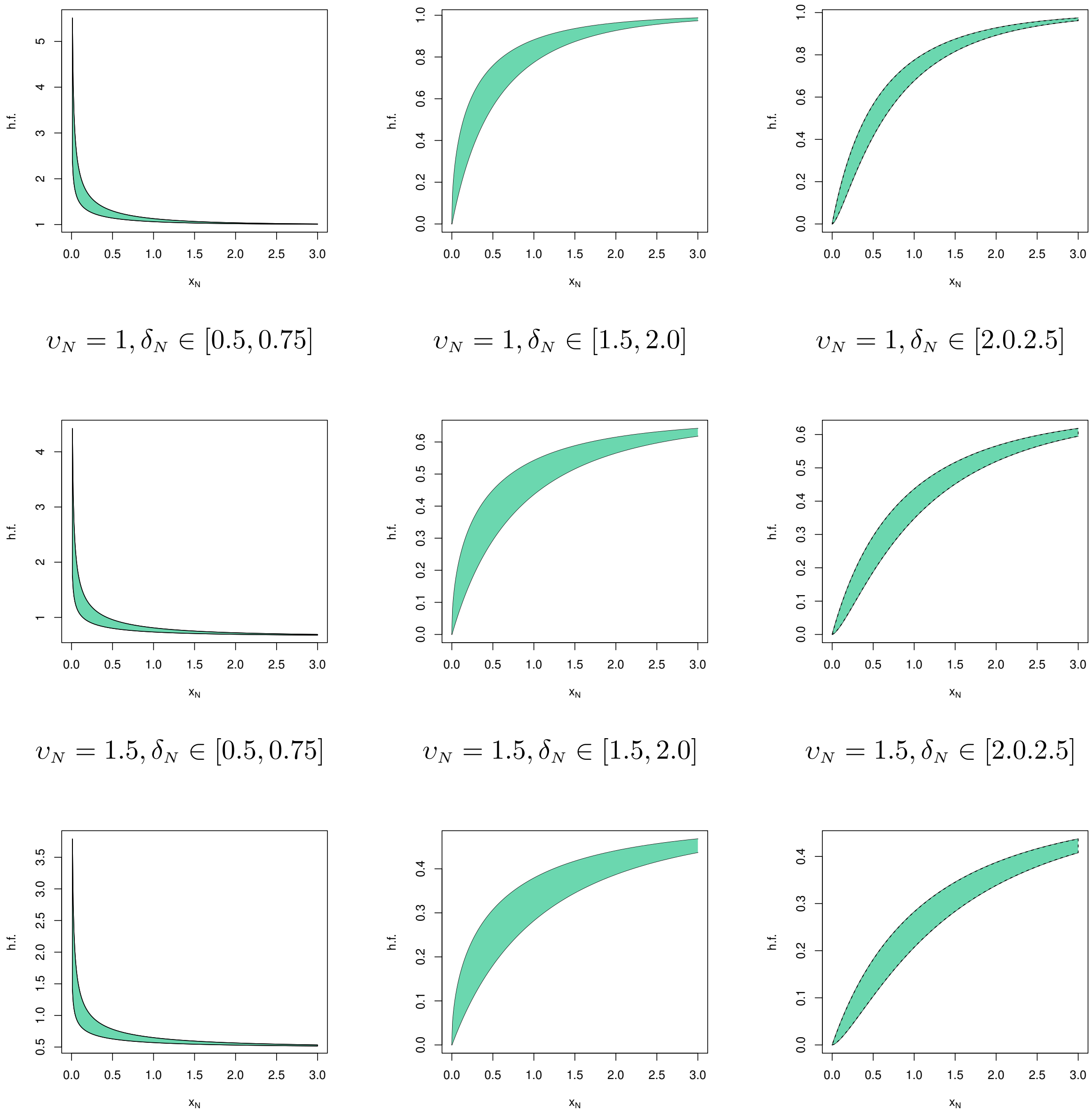


$\upsilon_N = 1.5, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 1.5, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 1.5, \delta_N \in [2.0.2.5]$



$\upsilon_N = 2, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 2, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 2, \delta_N \in [2.0.2.5]$

FIGURE 3. The survival function plots of NGE distribution



$\upsilon_N = 1, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 1, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 1, \delta_N \in [2.0.2.5]$

$\upsilon_N = 1.5, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 1.5, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 1.5, \delta_N \in [2.0.2.5]$

$\upsilon_N = 2, \delta_N \in [0.5, 0.75]$ $\upsilon_N = 2, \delta_N \in [1.5, 2.0]$ $\upsilon_N = 2, \delta_N \in [2.0.2.5]$

FIGURE 4. The hazard function plots of NGE distribution

The MLEs of δ_N and υ_N are denoted as $\hat{\delta}_N \in \left[\hat{\delta}_L, \hat{\delta}_U\right]$ and $\hat{\upsilon}_N \in [\hat{\upsilon}_L, \hat{\upsilon}_U]$ respectively, and are obtained by maximizing the equation (10). Thus $\hat{\delta}_N$ and $\hat{\upsilon}_N$ are the solutions of the following two derivative equations

$$\frac{\partial l\left(\delta_N, \upsilon_N\right)}{\partial \delta_N} = \frac{n}{\delta_N} + \sum_{i=1}^{n} \ln\left(1 - \exp\{-\frac{x_{Ni}}{\upsilon_N}\}\right) = 0 \tag{11}$$

and

$$\frac{\partial l\left(\delta_N, \upsilon_N\right)}{\partial \upsilon_N} = \frac{-n}{\upsilon_N} + \sum_{i=1}^{n} \frac{x_{Ni}}{\upsilon_N^2} - \frac{(\delta_N - 1)}{\upsilon_N^2} \sum_{i=1}^{n} \frac{x_{Ni} \exp\{-\frac{x_{Ni}}{\upsilon_N}\}}{\left(1 - \exp\{-\frac{x_{Ni}}{\upsilon_N}\}\right)} = 0 \tag{12}$$

or simply,

$$\frac{\partial l\left(\delta_N, \upsilon_N\right)}{\partial \upsilon_N} = -n\upsilon_N + \sum_{i=1}^{n} x_{Ni} - (\delta_N - 1) \sum_{i=1}^{n} \frac{x_{Ni} \exp\{-\frac{x_{Ni}}{\upsilon_N}\}}{\left(1 - \exp\{-\frac{x_{Ni}}{\upsilon_N}\}\right)} = 0. \tag{13}$$

Solving Eq. (11) results in

$$\hat{\delta}_N\left(\upsilon_N\right) = \frac{-n}{\sum\limits_{i=1}^{n} \ln\left(1 - \exp\{-\frac{x_{Ni}}{\upsilon_N}\}\right)}. \tag{14}$$

The estimator $\hat{\nu}_N$ is calculated by substituting $\hat{\delta}_N$ value in Eq. (12), which results in an expression in terms of υ_N as

$$-n\upsilon_N + \sum_{i=1}^{n} x_{Ni} + \frac{n}{\sum\limits_{i=1}^{n} \ln\left(1 - \exp\{-\frac{x_{Ni}}{\upsilon_N}\}\right)} \left[\sum_{i=1}^{n} \frac{x_{Ni} \exp\{-\frac{x_{Ni}}{\upsilon_N}\}}{\left(1 - \exp\{-\frac{x_{Ni}}{\upsilon_N}\}\right)}\right] + \sum_{i=1}^{n} \frac{x_{Ni} \exp\{-\frac{x_{Ni}}{\upsilon_N}\}}{\left(1 - \exp\{-\frac{x_{Ni}}{\upsilon_N}\}\right)} = 0 \tag{15}$$

Hence, MLE of υ_N say $\hat{\upsilon}_N$ is an iterative solution of equation (15). After finding $\hat{\upsilon}_N$ by iterative solution, we can substitute in Eq. (14) to get the MLE of $\hat{\delta}_N$.

5. **Justification of Estimation with Simulation**

To study the performance of the proposed NGE distribution model, a simulation study is carried out. The accomplishment of NGE distribution estimated parameters and their performance are expressed as neutrosophic average estimates (AEs), neutrosophic average biased (Avg. Biases), and neutrosophic measure square error (MSEs) using simulation investigation. The simulation results of average Bias and MSE are summarized in Tables 1-4. It is noticed from the tables that the average Bias and MSE decrease when the size of the sample increases, as expected. According to Tables 1-4, Bias of shape parameters is negative and the scale parameter is positive at different values of shape parametric and scale parametric values.

Table 1. $\upsilon_N = [1,1]$, $\delta_N = [1,3]$

	AEs		Avg. Biases		MSEs	
	$\hat{\upsilon}_N$	$\hat{\delta}_N$	$\hat{\upsilon}_N$	$\hat{\delta}_N$	$\hat{\upsilon}_N$	$\hat{\upsilon}_N$
30	0.9781	[1.1008,3.4551]	-0.0219	[0.1008,0.4551]	0.2137	[0.3258,1.3166]
50	0.9866	[1.0558,3.2537]	-0.0133	[0.0558,0.2537]	0.1645	[0.2149,0.8673]
100	0.9929	[1.0267,3.1265]	-0.0071	[0.0267,0.1265]	0.1168	[0.1410,0.5542]
200	0.9958	[1.0131,3.0610]	-0.0041	[0.0131,0.0610]	0.0825	[0.0948,0.3640]
500	0.9987	[1.0048,3.0231]	-0.0012	[0.0048,0.0231]	0.0524	[0.0589,0.2294]
1000	0.9995	[1.0024,3.0112]	-0.0005	[0.0024,0.0112]	0.0367	[0.0419,0.1572]

Table 2. $\upsilon_N = [1,1]$, $\delta_N = [0.5, 0.75]$

	AEs		Avg. Biases		MSEs	
	$\hat{\upsilon}_N$	$\hat{\delta}_N$	$\hat{\upsilon}_N$	$\hat{\delta}_N$	$\hat{\upsilon}_N$	$\hat{\delta}_N$
30	0.9775	[0.5397,0.8185]	-0.0225	[0.0397,0.0685]	0.2761	[0.1363,0.2205]
50	0.9862	[0.5225,0.7888]	-0.0138	[0.0225,0.0388]	0.2131	[0.0942,0.1524]
100	0.9925	[0.5108,0.7692]	-0.0075	[0.0108,0.0192]	0.1516	[0.0625,0.1008]
200	0.9955	[0.5053,0.7591]	-0.0045	[0.0053,0.0091]	0.1071	[0.0424,0.0676]
500	0.9987	[0.5018,0.7538]	-0.0012	[0.0018,0.0038]	0.0682	[0.0264,0.0434]
1000	1.0000	[0.5011,0.7514]	0.0000	[0.0011,0.0014]	0.0477	[0.0189,0.0301]

Table 3. $\upsilon_N = [0.5, 0.75]$, $\delta_N = [1,1]$

	AEs		Avg. Biases		MSEs	
	$\hat{\alpha}_N$	$\hat{\delta}_N$	$\hat{\upsilon}_N$	$\hat{\delta}_N$	$\hat{\upsilon}_N$	$\hat{\delta}_N$
30	[0.4877,0.7362]	1.1005	[-0.0123,-0.0138]	0.1005	[0.1196,0.1783]	0.3204
50	[0.4928,0.7412]	1.0562	[-0.0072,-0.0088]	0.0562	[0.0927,0.1364]	0.2154
100	[0.497,0.7436]	1.0274	[-0.003,-0.0064]	0.0274	[0.0652,0.0982]	0.1415
200	[0.4981,0.7465]	1.0132	[-0.0019,-0.0035]	0.0132	[0.0461,0.0692]	0.0948
500	[0.4992,0.7495]	1.0051	[-0.0008,-0.0005]	0.0051	[0.0292,0.0443]	0.0598
1000	[0.4997,0.7497]	1.0025	[-0.0003,-0.0003]	0.0025	[0.0205,0.031]	0.0419

6. **Application**

A realistic attempt of NGE distribution model is studied with help a real data in this section. The Parameter estimates along with the values of AIC (Akaike's Information criteria), BIC (Bayesian Information criteria) and KS (Kolmogorov–Smirnov) statistic are provided for comparision neutrosophic normal distribution (NND), neutrosophic gamma distribution (NGD), neutrosophic Weibull distribution (NWD), neutrosophic Rayleigh distribution (NRD),

Table 4. $\upsilon_N = [0.5, 0.75]$, $\delta_N = [1, 3]$

	AEs		Avg. Biases		MSEs	
	$\hat{\alpha}_N$	$\hat{\delta}_N$	$\hat{\upsilon}_N$	$\hat{\delta}_N$	$\hat{\upsilon}_N$	$\hat{\delta}_N$
30	[0.4877,0.7356]	[1.1008,3.4551]	[-0.0123,-0.0144]	[0.1008,0.4551]	[0.1196,0.1411]	[0.3258,1.3166]
50	[0.4928,0.7408]	[1.0558,3.2537]	[-0.0072,-0.0092]	[0.0558,0.2537]	[0.0927,0.1079]	[0.2149,0.8673]
100	[0.497,0.7439]	[1.0267,3.1264]	[-0.0030,-0.0061]	[0.0267,0.1264]	[0.0652,0.0776]	[0.1410,0.5542]
200	[0.4981,0.7467]	[1.0131,3.061]	[-0.0019,-0.0033]	[0.0131,0.061]	[0.0461,0.0546]	[0.0948,0.3640]
500	[0.4992,0.7494]	[1.0048,3.0231]	[-0.0008,-0.0006]	[0.0048,0.0231]	[0.0292,0.0349]	[0.0589,0.2294]
1000	[0.4997,0.7497]	[1.0024,3.0112]	[-0.0003,-0.0003]	[0.0024,0.0112]	[0.0205,0.0244]	[0.0419,0.1572]

neutrosophic exponential distribution (NED) and neutrosophic generalized exponential distribution (NGED).

6.1. *Example 1*

The data set reported in Table 5 attempted is related to remission time in months of 128 cancer patients. The remission times data was originally studied and reported in [31] from bladder cancer research. Under a neutrosophic environment, the remission periods data set is used by [32] to model the neutrosophic exponential distribution.

Based on their study remission periods of cancer patients is well fitted to NED. We use the same data set for the illustration of NGE distribution. Actively, data are the crumple observations, whereas to demonstrate the model, consider them as ambiguous sample observations for specified cancer patients. The developed NGE distribution parameters are estimated based on uncertainties of remission periods of cancer patients. The results in Table 6 shows that NGED is more effective to investigate the properties of uncertainties of remission periods of cancer patients neutrosophic data than the NED.

6.2. *Example 2*

To demonstrate a real example here we considered an rough population compactness of few villages in rural USA. This data is taken from [33] and they studied for neutrosophic W/S test based on the data follows to neutrosophic normal distribution. This data consists of the population of 17 villages in USA and their neutrosophic data, which is reproduced in Table 7 for ready reference. The results in Table 8 also shows that NGED is more suitable to fit the data than the NED.

7. **Conclusions**

In this article, a generalization exponential distribution is developed under neutrosophic statistics environment. Very few researchers have studied probability distributions based on

Table 5. Remission periods of 128 cancer patients.

Remission times								
0.08	2.09	3.48	4.87	6.94	8.66	13.11	23.63	0.2
2.23	3.52	4.98	6.97	9.02	13.29	0.4	2.26	3.57
5.06	7.09	9.22	13.8	25.74	0.5	2.46	3.64	5.09
$[7.26, 8.2]$	9.47	14.24	25.82	0.51	2.54	3.7	5.17	7.28
9.74	14.76	$[5.3, 7.1]$	0.81	2.62	3.82	5.32	7.32	10.06
$[12, 14.77]$	32.15	2.64	3.88	5.32	7.39	10.34	14.83	34.26
0.9	2.69	4.18	5.34	7.59	10.66	15.96	36.66	1.05
2.69	4.23	5.41	7.62	10.75	16.62	43.01	1.19	2.75
4.26	5.41	7.63	$[15, 17.2]$	46.12	1.26	2.83	4.33	5.49
7.66	11.25	17.14	$[75.02, 81]$	1.35	2.87	5.62	7.87	11.64
17.36	1.4	3.02	4.34	5.71	7.93	11.79	18.1	1.46
4.4	5.85	8.26	11.98	19.13	1.76	3.25	4.5	6.25
8.37	12.02	$[1.5, 3.2]$	3.31	4.51	6.54	$[7.5, 8.2]$	12.03	20.28
2.02	3.36	6.76	12.07	21.73	2.07	3.36	6.93	8.65
12.63	22.69							

Table 6. Estimates and Goodness-of-fit statistics for data set 1.

Model	Parameter	Estimates	LogLikelihood	AIC	BIC	KS
NND	μ	[9.1196,9.2453]	[-478.1315,-482.0838]	[960.263,968.1676]	[975.6711,983.5758]	[0.1899,0.1941]
	σ	[10.1397,10.4577]				
NGD	shape	[1.1896,1.1884]	[-409.7832,-411.5487]	[823.5665,827.0975]	[838.9746,842.5056]	[0.0757,0.0769]
	scale	[7.6658,7.7796]				
NWD	shape	[1.0553,1.0519]	[-410.5979,-412.3855]	[825.1958,828.7710]	[840.6039,844.1791]	[0.0716,0.0737]
	scale	[9.3370,9.4544]				
NRD	υ	[9.6432,9.8702]	[-486.1404,-490.4138]	[976.2808,984.8275]	[991.6890,1000.2360]	[0.3544,0.3542]
NED	υ	[0.1096,0.1081]	[-410.9358,-412.6880]	[825.8715,829.3760]	[841.2796,844.7842]	[0.0815,0.0869]
NGED	υ	[7.9506,8.0568]	[-409.4565,-411.2037]	[822.9129,826.4074]	[838.3210,841.8155]	[0.0752,0.0759]
	δ	[1.2390,1.2397]				

neutrosophic statistics. The mathematical properties of the developed neutrosophic generalization exponential distribution are studied. The nature of the distribution is studied through various neutrosophic parametric combinations. Using the maximum likelihood method the parameters are estimated. A simulation study is carried out under neutrosophic environment. The average Bias and MSE decrease as the sample size increases, as expected. Finally, the application of the proposed NGE distribution is presented through real data sets. A comparative study with other distributions is also done based real data sets. Based on real data examples, we conclude that the NGE distribution furnishes better performance over existing

TABLE 7. Neutrosophic population density of some villages in the USA

Villages	Population density	Villages	Population density
Aranza	[4.13,4.14]	Charapan	[5.10,5.12]
Corupo	[4.53,4.55]	Comachuen	[5.25,5.27]
San Lorenzo	[4.69,4.70]	Pichataro	[5.36,5.38]
Cheranatzicurin	[4.76,4.78]	Quinceo	[5.94,5.96]
Nahuatzen	[4.77,4.79]	Nurio	[6.06,6.08]
Pomacuaran	[4.96,4.98]	Turicuaro	[6.19,6.21]
Servina	[4.97,4.99]	Urapicho	[6.30,6.32]
Arantepacua	[5.00,5.06]	Capacuaro	[7.73,7.98]
Cocucho	[5.04,5.06]		

TABLE 8. Estimates and Goodness-of-fit statistics for data set 2.

Model	Parameter	Estimates	LogLikelihood	AIC	BIC	KS
NND	μ	[5.3400,5.3723]	[-21.1554,-21.9577]	[46.3107,47.9155]	[53.6436,55.2483]	[0.2007,0.2024]
	σ	[0.8398,0.8804]				
NGD	shape	[40.4254,37.2310]	[-20.2481,-20.9136]	[44.4962,45.8272]	[51.8290,53.1600]	[0.1821,0.1816]
	scale	[0.1320,0.1442]				
NWD	shape	[5.8980,5.5773]	[-23.0417,-23.9417]	[50.0834,51.8834]	[57.4162,59.2162]	[0.2097,0.2115]
	scale	[5.7143,5.7621]				
NRD	υ	[3.8223,3.8495]	[-34.3024,-34.4533]	[72.6049,72.9067]	[79.9377,80.2395]	[0.4457,0.4439]
NED	υ	[0.1873,0.1861]	[-45.47884,-45.5815]	[94.9577,95.1630]	[102.2905,102.4959]	[0.5386,0.5373]
NGED	υ	[0.6067,0.6187]	[-18.7673,-19.1981]	[41.5345,42.3961]	[48.8674,49.7290]	[0.1443,0.1466]
	δ	[3630.608,3209.943]				

distributions. This article develops a generalized exponential distribution inside a neutrosophic statistical framework. The study of probability distributions based on neutrosophic statistics is quite uncommon. The generated neutrosophic generalization exponential distribution's mathematical characteristics are investigated. The distribution's nature is investigated using a variety of neutrosophic parametric combinations. The parameters are computed using the maximum likelihood approach. Simulation research is conducted in a neutrosophic setting. When expected, as the sample size grows, the average bias and MSE drop. The use of the suggested NGE distribution is then shown using actual data sets. Based on actual data sets, a comparison study with different distributions is also conducted. We draw the conclusion that the NGE distribution offers superior performance over current distributions based on studied instances.

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