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A Novel Approach to Neutrosophic Hypersoft Graphs with Properties

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Abstract. Neutrosophic hypersoft set is the combination of neutrosophic set and hypersoft set. It resolves the limitations of intuitionistic fuzzy sets and soft sets for the consideration of the degree of indeterminacy and multi-argument approximate function respectively. In this research article, a novel framework i.e. neutrosophic hypersoft graph, is formulated for handling neutrosophic hypersoft information by combining the concept of neutrosophic hypersoft sets with graph theory. Firstly, some of essential and fundamental notions of neutrosophic hypersoft graph are characterized with the help of numerical examples and graphical representation. Secondly, some set theoretic operations i.e. union, intersection and complement, are investigated with illustrative examples and pictorial depiction.

Keywords: Neutrosophic Set; Soft set; Hypersoft set; Neutrosophic soft graph; Neutrosophic hypersoft set; Neutrosophic hypersoft graph.

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1. Introduction

In different mathematical disciplines, fuzzy sets theory (FS-Theory) [1] and intuitionistic fuzzy set theory (IFS-Theory) [2] are considered apt mathematical modes to tackle several intricate problems involving various uncertainties. The former emphasizes on a certain object's degree of true belongingness from the initial sample space, while the latter emphasizes degree of true membership and degree of non-membership with the state of their interdependence. These theories portray some kind of inadequacy in terms of providing due status to a degree of indeterminacy. The implementation of neutrosophic set theory (NS-Theory) [3, 4] overcomes this impediment by taking into account not only the proper status of degree of indeterminacy but also the state of dependence. This theory is more adaptable and suitable for dealing with inconsistent data. Wang et el [5] conceptualized single-valued neutrosophic set in which truth membership degree, indeterminacy degree and falsity degree are restricted within unit closed interval. Many researchers [6]- [14] have been drawn to NS-Theory for further application in statistics, topological spaces, and the construction of some neutrosophic-like blended structures with other existing models for useful applications in decision making. Edalatpanah [15] studied a system of neutrosophic linear equations (SNLE) based on the embedding approach. He used (α, β, γ)-cut for transformation of SNLE into a crisp linear system. Kumar et al. [16] exhibited a novel linear programming approach for finding the neutrosophic shortest path problem (NSSPP) considering Gaussian valued neutrosophic number.

FS-Theory, IFS-Theory and NS-Theory have some kind of complexities which restrain them to solve problem involving uncertainty professionally. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. It demands a mathematical tool free of all such impediments to tackle such issues. This scantiness is resolved with the development of soft set theory (SS-Theory) [17] which is a new parameterized family of subsets of the universe of discourse. The researchers [18]- [27] studied and investigated some elementary properties, operations, laws and hybrids of SS-Theory with applications in decision making. The gluing concept of NS-Theory and SS-Theory, is studied in [28] to make the NS-Theory adequate with parameterized tool. In many real life situations, distinct attributes are further partitioned in disjoint attribute-valued sets but existing SS-Theory is insufficient for dealing with such kind of attribute-valued sets. Hypersoft set theory (HS-Theory) [29] is developed to make the SST in line with attribute-valued sets to tackle real life scenarios. HS-Theory is an extension of SS-Theory as it transforms the single argument function into a multi-argument function. Certain elementary properties, aggregation operations, laws, relations and functions of HS-Theory, are investigated by [30]- [32] for proper understanding and further utilization in different fields. The applications of HS-Theory in decision making is studied by [33]- [37] and the intermingling study of HS-Theory with complex sets, convex and concave sets is studied by [38,39]. Deli [40] characterized hybrid set structures under uncertainly parameterized hypersoft sets with theory and applications. Gayen et al. [41] analyzed some essential aspects of plithogenic hypersoft algebraic structures. They also investigated the notions and basic properties of plithogenic hypersoft subgroups ie plithogenic fuzzy hypersoft subgroup, plithogenic intuitionistic fuzzy hypersoft subgroup, plithogenic neutrosophic hypersoft subgroup. Saeed et al. [42,43] discussed decision making techniques for neutrosophic hypersoft mapping and complex multi-fuzzy hypersoft set. Rahman et al. [44–46] studied decision making applications based on neutrosophic parameterized hypersoft Set, fuzzy parameterized hypersoft set and rough hypersoft set. Ihsan et al. [47] investigated hypersoft expert set with application in decision making for the best

selection of product. The gluing concept of graph theory with uncertain environments like fuzzy, intuitionistic fuzzy, neutrosophic, fuzzy soft, intuitionistic fuzzy soft and neutrosophic soft set, is discussed and characterized by the authors [48]- [54]. Inspiring from above literature in general, and from [55], [56] in specific, new notions of neutrosophic hypersoft graph are conceptualized along with some elementary types, essential properties, aggregation operations and generalized typical results. The rest of the paper is organized as:
In section 2, some basic definitions and terminologies are presented. In section 3, the elementary notions of neutrosophic hypersoft graphs are discussed with properties and results. In section 4, some set theoretic operations of neutrosophic hypersoft graphs are presented with examples. In section 5, paper is summarized with future directions.

2. Preliminaries

Here some essential terms and definitions are recalled from existing literature.

Definition 2.1. [3]
A neutrosophic set $\mathcal{K}$ defined as $\mathcal{K} = \{(k, < \mathcal{M}_K(k), \mathcal{I}_K(k), \mathcal{N}_K(k) >) | k \in \mathcal{Z}\}$ such that $\mathcal{M}_K(k) : \mathcal{Z} \rightarrow ^{-}]0, 1[^{+}$, $\mathcal{I}_K(k) : \mathcal{Z} \rightarrow ^{-}]0, 1[^{+}$ and $\mathcal{N}_K(k) : \mathcal{Z} \rightarrow ^{-}]0, 1[^{+}$ where $\mathcal{M}_K(k)$ stands for membership, $\mathcal{N}_K(k)$ stands for non-membership and $\mathcal{I}_K(k)$ stands for indeterminacy under condition $^{-}0 \leq \mathcal{M}_K(k) + \mathcal{I}_K(k) + \mathcal{N}_K(k) \leq 3^{+}$.

Definition 2.2. [17]
A pair $(\Psi_M, \mathcal{W})$ is said to be soft set over $\mathcal{Z}$ (universe of discourse), where $\Psi_M : \mathcal{W} \rightarrow \mathcal{P}(\mathcal{Z})$ and $\mathcal{W}$ is a subset of set of attributes $\mathcal{X}$.

For more detail on soft set, see [18, 19].

Definition 2.3. [29]
A pair $(\xi_H, \mathcal{R})$ is said to be hypersoft set over $\mathcal{Z}$, where $\xi_H : \mathcal{R} \rightarrow \mathcal{P}(\mathcal{Z})$ and $\mathcal{R} = \mathcal{R}_1 \times \mathcal{R}_2 \times \mathcal{R}_3 \times ... \times \mathcal{R}_n$, $\mathcal{R}_i$ are disjoint attribute-valued sets corresponding to distinct attributes r_i respectively for $1 \leq i \leq n$.

Definition 2.4. [29]
A pair $(\zeta_N, \mathcal{U})$ is said to be neutrosophic hypersoft set over $\mathcal{Z}$ if $\zeta_N : \mathcal{U} \rightarrow \mathcal{P}(\mathcal{Z})$, where $\mathcal{P}(\mathcal{Z})$ is a collection of all neutrosophic subsets and $\mathcal{U} = \mathcal{U}_1 \times \mathcal{U}_2 \times \mathcal{U}_3 \times ... \times \mathcal{U}_n$, $\mathcal{U}_i$ are disjoint attribute-valued sets corresponding to distinct attributes u_i respectively for $1 \leq i \leq n$.

For more definitions and operations of hypersoft set, see [30–32].

Definition 2.5. [56]
Let $\mathcal{Q}$ and $\mathfrak{G}^{\star} = (\mathcal{V}, \mathcal{E})$ be a set of parameters and a simple graph respectively with $\mathcal{V}$ as set

of vertices and $\mathcal{E}$ as set of edges. Let $\mathcal{N}(\mathcal{V})$ be the set of all neutrosophic set in $\mathcal{V}$. By a neutrosophic soft graph (NS-Graph), we mean a 4-tuple $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ where $\mathbb{F} : \mathcal{Q} \to \mathcal{N}(\mathcal{V}), \mathbb{G} : \mathcal{Q} \to \mathcal{N}(\mathcal{V} \times \mathcal{V})$ given by

$$\mathbb{F}(\theta) = \mathbb{F}_\theta = \{\langle \nu, \mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\nu)\rangle, \nu \in \mathcal{V}\}$$

and

$$\mathbb{G}(\theta) = \mathbb{G}_\theta = \{\langle (\nu,\mu), \mathcal{T}_{\mathbb{F}_\theta}(\nu,\mu), \mathcal{I}_{\mathbb{F}_\theta}(\nu,\mu), \mathcal{F}_{\mathbb{F}_\theta}(\nu,\mu)\rangle, (\nu,\mu) \in \mathcal{V} \times \mathcal{V}\}$$

are neutrosophic sets over $\mathcal{V}$ and $\mathcal{V} \times \mathcal{V}$ respectively with

$$\mathcal{T}_{\mathbb{F}_\theta}(\nu,\mu) \le \min\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\}$$

$$\mathcal{I}_{\mathbb{F}_\theta}(\nu,\mu) \le \min\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\}$$

$$\mathcal{F}_{\mathbb{F}_\theta}(\nu,\mu) \ge \max\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\}$$

for all $(\nu,\mu) \in (\mathcal{V} \times \mathcal{V})$ and $\theta \in \mathcal{Q}$.

3. Neutrosophic Hypersoft Graphs

In this section, notions of neutrosophic hypersoft graph are characterized with some properties and examples.

Definition 3.1. Let $\mathfrak{G}^\star = (\mathcal{V}, \mathcal{E})$ be a simple graph with $\mathcal{V}$ as set of vertices and $\mathcal{E}$ as set of edges and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, ..., \mathcal{Q}_n$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n$ with $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 \times ... \times \mathcal{Q}_n$. Let $\mathcal{N}(\mathcal{V})$ be the set of all neutrosophic set in $\mathcal{V}$. By a neutrosophic hypersoft graph (NHS-Graph), we mean a 4-tuple $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ where $\mathbb{F} : \mathcal{Q} \to \mathcal{N}(\mathcal{V}), \mathbb{G} : \mathcal{Q} \to \mathcal{N}(\mathcal{V} \times \mathcal{V})$ given by

$$\mathbb{F}(\theta) = \mathbb{F}_\theta = \{\langle \nu, \mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\nu)\rangle, \nu \in \mathcal{V}\}$$

and

$$\mathbb{G}(\theta) = \mathbb{G}_\theta = \{\langle (\nu,\mu), \mathcal{T}_{\mathbb{F}_\theta}(\nu,\mu), \mathcal{I}_{\mathbb{F}_\theta}(\nu,\mu), \mathcal{F}_{\mathbb{F}_\theta}(\nu,\mu)\rangle, (\nu,\mu) \in \mathcal{V} \times \mathcal{V}\}$$

are neutrosophic sets over $\mathcal{V}$ and $\mathcal{V} \times \mathcal{V}$ with

$$\mathcal{T}_{\mathbb{F}_\theta}(\nu,\mu) \le \min\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\}$$

$$\mathcal{I}_{\mathbb{F}_\theta}(\nu,\mu) \ge \min\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\}$$

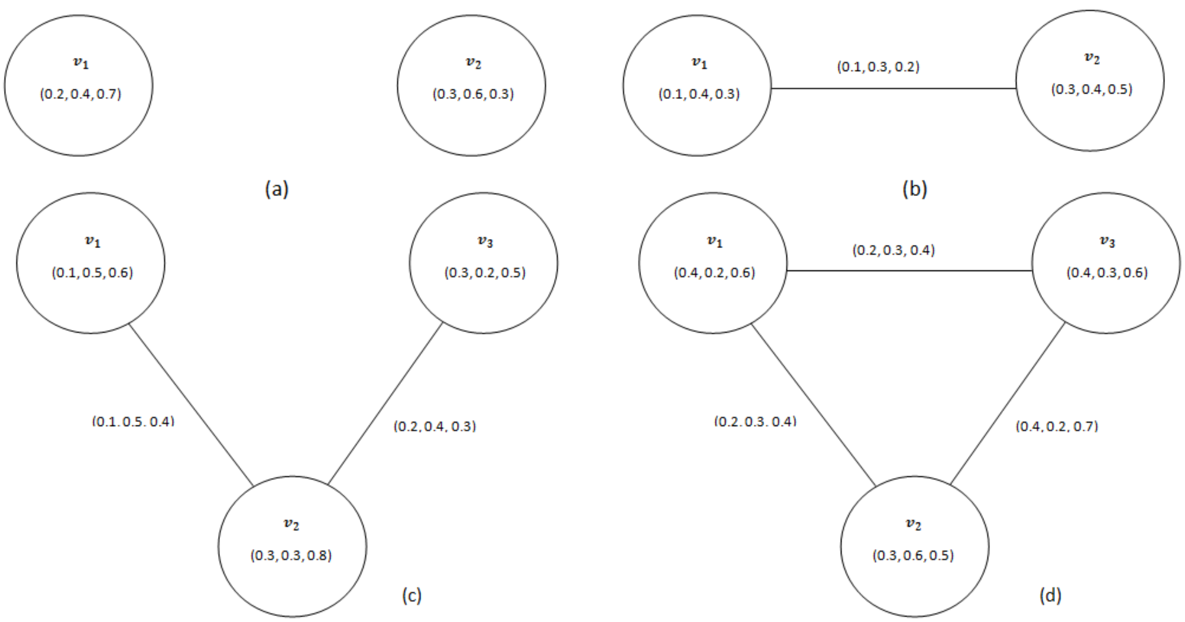
$$\mathcal{F}_{\mathbb{F}_\theta}(\nu,\mu) \ge \max\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\}$$

for all $(\nu,\mu) \in (\mathcal{V} \times \mathcal{V})$ and $\theta \in \mathcal{Q}$.
Note: The collection of all neutrosophic hypersoft graphs is denoted by Ω_{NHSG}.

TABLE 1. Tabular Representation of NHS-Graph $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$

$\mathbb{F}$	ν_1	ν_2	ν_3
θ_1	$(0.2, 0.4, 0.7)$	$(0.3, 0.6, 0.3)$	$(0, 0, 1)$
θ_2	$(0.1, 0.4, 0.3)$	$(0.3, 0.4, 0.5)$	$(0, 0, 1)$
θ_3	$(0.1, 0.5, 0.6)$	$(0.3, 0.3, 0.8)$	$(0.3, 0.2, 0.5)$
θ_4	$(0.4, 0.2, 0.6)$	$(0.3, 0.6, 0.5)$	$(0.4, 0.3, 0.6)$
$\mathbb{G}$	(ν_1, ν_2)	(ν_2, ν_3)	(ν_1, ν_3)
θ_1	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_2	$(0.1, 0.3, 0.2)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_3	$(0.1, 0.5, 0.4)$	$(0.2, 0.4, 0.3)$	$(0, 0, 1)$
θ_4	$(0.2, 0.3, 0.4)$	$(0.2, 0.5, 0.3)$	$(0.4, 0.2, 0.7)$

FIGURE 1. Graphical Representation of TABLE 1 with (a) $\mathcal{N}(\theta_1)$, (b) $\mathcal{N}(\theta_2)$, (c) $\mathcal{N}(\theta_3)$ and (d) $\mathcal{N}(\theta_4)$



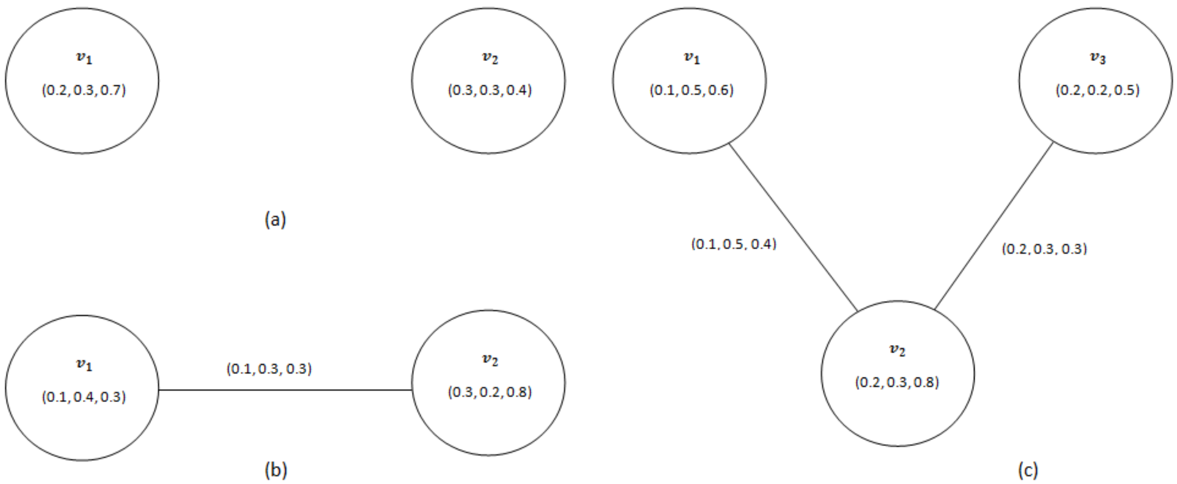
Example 3.2. Let $\mathfrak{G}^\star = (\mathcal{V}, \mathcal{E})$ be a simple graph with $\mathcal{V} = \{\nu_1, \nu_2, \nu_3\}$ and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, \alpha_3$ where $\mathcal{Q}_1 = \{\alpha_{11}, \alpha_{12}\}$, $\mathcal{Q}_2 = \{\alpha_{21}, \alpha_{22}\}$ and $\mathcal{Q}_3 = \{\alpha_{31}\}$. $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 = \{\theta_1, \theta_2, \theta_3, \theta_4\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V} \times \mathcal{V} \setminus \{(\nu_1, \nu_2), (\nu_2, \nu_3), (\nu_1, \nu_3)\}$. The tabular and graphical representation of NHS-Graph $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ are given in TABLE 1 and FIGURE 1 respectively.

Definition 3.3. A neutrosophic hypersoft graph $\mathfrak{G} = \left(\mathfrak{G}^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ is called a neutrosophic hypersoft subgraph of $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{A}, \mathbb{F}, \mathbb{G})$ if

TABLE 2. Tabular Representation of NHS-subgraph $\mathfrak{G} = \left(\mathfrak{G}^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$

$\mathbb{F}$	ν_1	ν_2	ν_3
θ_1	$(0.2, 0.3, 0.7)$	$(0.3, 0.3, 0.4)$	$(0, 0, 1)$
θ_2	$(0.1, 0.4, 0.3)$	$(0.3, 0.2, 0.8)$	$(0, 0, 1)$
θ_3	$(0.1, 0.5, 0.6)$	$(0.2, 0.3, 0.8)$	$(0.2, 0.2, 0.5)$
$\mathbb{G}$	(ν_1, ν_2)	(ν_2, ν_3)	(ν_1, ν_3)
θ_1	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_2	$(0.1, 0.3, 0.3)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_3	$(0.1, 0.5, 0.4)$	$(0.2, 0.3, 0.3)$	$(0, 0, 1)$

FIGURE 2. Graphical Representation of TABLE 2 with (a) $\mathcal{N}(\theta_1)$, (b) $\mathcal{N}(\theta_2)$ and (c) $\mathcal{N}(\theta_3)$



(1) $\mathcal{Q}^1 \subseteq \mathcal{Q}$
(2) $\mathbb{F}^1_\theta \subseteq f$ which means $\mathcal{T}_{\mathbb{F}^1_\theta}(\nu) \leq \mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\nu) \leq \mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\nu) \geq \mathcal{F}_{\mathbb{F}_\theta}(\nu)$.
(3) $\mathbb{G}^1_\theta \subseteq g$ which means $\mathcal{T}_{\mathbb{G}^1_\theta}(\nu) \leq \mathcal{T}_{\mathbb{G}_\theta}(\nu), \mathcal{I}_{\mathbb{G}^1_\theta}(\nu) \leq \mathcal{I}_{\mathbb{G}_\theta}(\nu), \mathcal{F}_{\mathbb{G}^1_\theta}(\nu) \geq \mathcal{F}_{\mathbb{G}_\theta}(\nu)$.

for all $\theta \in \mathcal{Q}^1$ and $\mathcal{Q}^1 = \mathcal{Q}_1 \times \mathcal{Q}_2 \times ... \times \mathcal{Q}_n$ where $\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_n$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, ..., \alpha_n$ respectively.

Example 3.4. Let $\mathfrak{G}^\star = (\mathcal{V}, E)$ be a simple graph with $\mathcal{V} = \{\nu_1, \nu_2, \nu_3\}$ and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3$ are disjoint attribute-valued sets corresponding to disjoint attributes $\alpha_1, \alpha_2, \alpha_3$ where $\mathcal{Q}_1 = \{\alpha_{11}, \alpha_{12}\}$, $\mathcal{Q}_2 = \{\alpha_{21}\}$ and $\mathcal{Q}_3 = \{\alpha_{31}\}$. $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 = \{\theta_1, \theta_2, \theta_3\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}$. The tabular and graphical representation of NHS-subgraph $\mathfrak{G} = \left(\mathfrak{G}^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ are given in TABLE 2 and FIGURE 2 respectively. In this graph, $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V} \times \mathcal{V} \setminus \{(\nu_1, \nu_2), (\nu_2, \nu_3), (\nu_1, \nu_3)\}$ and for all $\theta \in \mathcal{Q}$.

Definition 3.5. A neutrosophic hypersoft subgraph $\mathfrak{G} = \left(\mathfrak{G}^{\star}, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ is called a neutrosophic hypersoft spanning subgraph of $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ if $\mathbb{F}^1_{\theta}(\nu) = \mathbb{F}(\nu)$ for all $\nu \in \mathcal{V}, e \in \mathcal{Q}$ where $\mathcal{Q}^1 = \mathcal{Q}_1 \times \mathcal{Q}_2 \times ... \times \mathcal{Q}_n$ and $\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_n$ are disjoint attribute-valued sets corresponding to disjoint attributes $\alpha_1, \alpha_2, ..., \alpha_n$ respectively.

Definition 3.6. A strong neutrosophic hypersoft subgraph $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ is a neutrosophic hypersoft subgraph with condition that $\mathbb{G}_{\theta}(\nu, \mu) = \mathbb{F}_{\theta}(\nu) \cap \mathbb{F}_{\theta}(\mu)$ for $x, y \in \mathcal{V}$ and $e \in \mathcal{Q}$ such that $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 ... \times \mathcal{Q}_n$ and $\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_n$ are disjoint attribute-valued sets corresponding to disjoint attributes $\alpha_1, \alpha_2, ..., \alpha_n$ respectively.

4. Set Theoretic Operations of NHS-Graphs

In this section, some theoretic operations (i.e. union, intersection and complement) of neutrosophic hypersoft graph (NHS-Graphs) are investigated with suitable examples and results.

Definition 4.1. The union of two NHS-Graphs $\mathfrak{G}_1 = \left(\mathfrak{G}^{\star}_1, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$, $\mathfrak{G}_2 = \left(\mathfrak{G}^{\star}_2, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$, denoted by $\mathfrak{G}_1 \cup \mathfrak{G}_2$, is a NHS-Graph $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ such that $\mathcal{Q} = \mathcal{Q}^1 \cup \mathcal{Q}^2$. In this graph, the neutrosophic components for $\mathbb{F}$ are given as follows:

$$\mathcal{T}_{\mathbb{F}_{\theta}}(\nu) = \begin{cases} \mathcal{T}_{\mathbb{F}^1_{\theta}}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2 \end{cases} \\ \mathcal{T}_{\mathbb{F}^2_{\theta}}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 \cap \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1 \end{cases} \\ \max\left\{\mathcal{T}_{\mathbb{F}^1_{\theta}}(\nu), \mathcal{T}_{\mathbb{F}^2_{\theta}}(\nu)\right\} \left\{if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2\right. \\ 0,\, otherwise \end{cases}$$

and

$$\mathcal{I}_{\mathbb{F}_{\theta}}(\nu) = \begin{cases} \mathcal{I}_{\mathbb{F}^1_{\theta}}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2 \end{cases} \\ \mathcal{I}_{\mathbb{F}^2_{\theta}}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 \cap \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1 \end{cases} \\ \max\left\{\mathcal{I}_{\mathbb{F}^1_{\theta}}(\nu), \mathcal{I}_{\mathbb{F}^2_{\theta}}(\nu)\right\} \left\{if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2\right. \\ 0,\, otherwise \end{cases}$$

and

$$\mathcal{F}_{\mathbb{F}_\theta}(\nu)=\begin{cases} \mathcal{F}_{\mathbb{F}^1_\theta}(\nu)\begin{cases} if\,\theta\in\mathcal{Q}^1-\mathcal{Q}^2\,and\,\nu\in\mathcal{V}_1-\mathcal{V}_2\,or\\ if\,\theta\in\mathcal{Q}^1-\mathcal{Q}^2\,and\,\nu\in\mathcal{V}_1\cap\mathcal{V}_2\,or\\ if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,\nu\in\mathcal{V}_1-\mathcal{V}_2\end{cases}\\ \mathcal{F}_{\mathbb{F}^2_\theta}(\nu)\begin{cases} if\,\theta\in\mathcal{Q}^2-\mathcal{Q}^1\,and\,\nu\in\mathcal{V}_2-\mathcal{V}_1\,or\\ if\,\theta\in\mathcal{Q}^2-\mathcal{Q}^1\,and\,\nu\in\mathcal{V}_2\cap\mathcal{V}_1\,or\\ if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,\nu\in\mathcal{V}_2-\mathcal{V}_1\end{cases}\\ \min\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu),\mathcal{F}_{\mathbb{F}^2_\theta}(\nu)\right\}\left\{if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,\nu\in\mathcal{V}_1\cap\mathcal{V}_2\right.\\ 0,\,otherwise\end{cases}.$$

Also the neutrosophic components for $\mathbb{G}$ are given as follows:

$$\mathcal{T}_{\mathbb{G}_\theta}(\nu)=\begin{cases} \mathcal{T}_{\mathbb{G}^1_\theta}(\nu)\begin{cases} if\,\theta\in\mathcal{Q}^1-\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)-(\mathcal{V}_2\times\mathcal{V}_2)\ or\\ if\,\theta\in\mathcal{Q}^1-\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)\cap(\mathcal{V}_2\times\mathcal{V}_2)\ or\\ if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)-(\mathcal{V}_2\times\mathcal{V}_2)\end{cases}\\ \mathcal{T}_{\mathbb{G}^2_\theta}(\nu)\begin{cases} if\,\theta\in\mathcal{Q}^2-\mathcal{Q}^1\,and\,(\nu,\mu)\in(\mathcal{V}_2\times\mathcal{V}_2)-(\mathcal{V}_1\times\mathcal{V}_1)\ or\\ if\,\theta\in\mathcal{Q}^2-\mathcal{Q}^1\,and\,(\nu,\mu)\in(\mathcal{V}_2\times\mathcal{V}_2)\cap(\mathcal{V}_1\times\mathcal{V}_1)\ or\\ if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_2\times\mathcal{V}_2)-(\mathcal{V}_1\times\mathcal{V}_1)\end{cases}\\ \max\left\{\mathcal{T}_{\mathbb{G}^1_\theta}(\nu),\mathcal{T}_{\mathbb{G}^2_\theta}(\nu)\right\}\left\{if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)\cap(\mathcal{V}_2\times\mathcal{V}_2)\right.\\ 0,\,otherwise\end{cases}$$

and

$$\mathcal{I}_{\mathbb{G}_\theta}(\nu)=\begin{cases} \mathcal{I}_{\mathbb{G}^1_\theta}(\nu)\begin{cases} if\,\theta\in\mathcal{Q}^1-\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)-(\mathcal{V}_2\times\mathcal{V}_2)\ or\\ if\,\theta\in\mathcal{Q}^1-\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)\cap(\mathcal{V}_2\times\mathcal{V}_2)\ or\\ if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)-(\mathcal{V}_2\times\mathcal{V}_2)\end{cases}\\ \mathcal{I}_{\mathbb{G}^2_\theta}(\nu)\begin{cases} if\,\theta\in\mathcal{Q}^2-\mathcal{Q}^1\,and\,(\nu,\mu)\in(\mathcal{V}_2\times\mathcal{V}_2)-(\mathcal{V}_1\times\mathcal{V}_1)\ or\\ if\,\theta\in\mathcal{Q}^2-\mathcal{Q}^1\,and\,(\nu,\mu)\in(\mathcal{V}_2\times\mathcal{V}_2)\cap(\mathcal{V}_1\times\mathcal{V}_1)\ or\\ if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_2\times\mathcal{V}_2)-(\mathcal{V}_1\times\mathcal{V}_1)\end{cases}\\ \max\left\{\mathcal{I}_{\mathbb{G}^1_\theta}(\nu),\mathcal{I}_{\mathbb{G}^2_\theta}(\nu)\right\}\left\{if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)\cap(\mathcal{V}_2\times\mathcal{V}_2)\right.\\ 0,\,otherwise\end{cases}$$

and

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu)=\begin{cases} \mathcal{F}_{\mathbb{G}^1_\theta}(\nu)\begin{cases} if\,\theta\in\mathcal{Q}^1-\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)-(\mathcal{V}_2\times\mathcal{V}_2)\ or\\ if\,\theta\in\mathcal{Q}^1-\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)\cap(\mathcal{V}_2\times\mathcal{V}_2)\ or\\ if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)-(\mathcal{V}_2\times\mathcal{V}_2)\end{cases}\\ \mathcal{F}_{\mathbb{G}^2_\theta}(\nu)\begin{cases} if\,\theta\in\mathcal{Q}^2-\mathcal{Q}^1\,and\,(\nu,\mu)\in(\mathcal{V}_2\times\mathcal{V}_2)-(\mathcal{V}_1\times\mathcal{V}_1)\ or\\ if\,\theta\in\mathcal{Q}^2-\mathcal{Q}^1\,and\,(\nu,\mu)\in(\mathcal{V}_2\times\mathcal{V}_2)\cap(\mathcal{V}_1\times\mathcal{V}_1)\ or\\ if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_2\times\mathcal{V}_2)-(\mathcal{V}_1\times\mathcal{V}_1)\end{cases}\\ \min\left\{\mathcal{F}_{\mathbb{G}^1_\theta}(\nu),\mathcal{F}_{\mathbb{G}^2_\theta}(\nu)\right\}\left\{if\,\theta\in\mathcal{Q}^1\cap\mathcal{Q}^2\,and\,(\nu,\mu)\in(\mathcal{V}_1\times\mathcal{V}_1)\cap(\mathcal{V}_2\times\mathcal{V}_2)\right.\\ 0,\,otherwise\end{cases}.$$

Theorem 4.2. *If* $\mathfrak{G}_1,\mathfrak{G}_2\in\Omega_{NHSG}$ *then* $\mathfrak{G}_1\cup\mathfrak{G}_2\in\Omega_{NHSG}$.

Proof. Consider two NHS-Graphs $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ and $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ as defined in 3.1. Let $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ be the union of NHS-Graphs $\mathfrak{G}_1$ and $\mathfrak{G}_2$ where $\mathcal{Q} = \mathcal{Q}^1 \cup \mathcal{Q}^2$. Now let $\theta \in \mathcal{Q}^1 - \mathcal{Q}^2$ and $(\nu, \mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2)$, then

$$\begin{aligned} \mathcal{T}_{\mathbb{G}_\theta}(\nu,\mu) = \mathcal{T}_{\mathbb{G}^1_\theta}(\nu,\mu) &\leq \min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\} \\ &= \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\} \end{aligned}$$

so

$$\mathcal{T}_{\mathbb{G}_\theta}(\nu,\mu) \leq \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\}.$$

Also

$$\begin{aligned} \mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) = \mathcal{I}_{\mathbb{G}^1_\theta}(\nu,\mu) &\leq \min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\} \\ &= \min\left\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\right\} \end{aligned}$$

so

$$\mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) \leq \min\left\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\right\}.$$

Now

$$\begin{aligned} \mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) = \mathcal{F}_{\mathbb{G}^1_\theta}(\nu,\mu) &\geq \max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\} \\ &= \max\left\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\right\} \end{aligned}$$

so

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) \geq \max\left\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\right\}.$$

Similar results are obtained when $\theta \in \mathcal{Q}^1 - \mathcal{Q}^2$ and $(\nu, \mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2)$ or $\theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2$ and $(\nu, \mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2)$.
Now if $\theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2$ and $(\nu, \mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2)$ then

$$\begin{aligned} \mathcal{T}_{\mathbb{G}_\theta}(\nu,\mu) &= \max\left\{\mathcal{T}_{\mathbb{G}^1_\theta}(\nu,\mu), \mathcal{T}_{\mathbb{G}^2_\theta}(\nu,\mu)\right\} \\ &\leq \max\left\{\min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\}, \min\left\{\mathcal{T}_{\mathbb{F}^2_\theta}(\nu), \mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\} \\ &\leq \min\left\{\max\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\}, \max\left\{\mathcal{T}_{\mathbb{F}^2_\theta}(\nu), \mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\} \\ &= \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\}. \end{aligned}$$

Also

$$\begin{aligned} \mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) &= \max\left\{\mathcal{I}_{\mathbb{G}^1_\theta}(\nu,\mu), \mathcal{I}_{\mathbb{G}^2_\theta}(\nu,\mu)\right\} \\ &\leq \max\left\{\min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\}, \min\left\{\mathcal{I}_{\mathbb{F}^2_\theta}(\nu), \mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\} \\ &\leq \min\left\{\max\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\}, \max\left\{\mathcal{I}_{\mathbb{F}^2_\theta}(\nu), \mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\} \\ &= \min\left\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\right\}. \end{aligned}$$

In the same way

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) = \min\left\{\mathcal{F}_{\mathbb{G}^1_\theta}(\nu,\mu), \mathcal{F}_{\mathbb{G}^2_\theta}(\nu,\mu)\right\}$$

TABLE 3. Tabular Representation of NHS-Graph $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ according to Example 4.3

$\mathbb{F}$	ν_1	ν_2	ν_3
θ_1	$(0.2, 0.3, 0.4)$	$(0.3, 0.6, 0.8)$	$(0.3, 0.4, 0.5)$
θ_2	$(0.2, 0.4, 0.8)$	$(0.2, 0.3, 0.4)$	$(0.5, 0.7, 0.8)$
θ_3	$(0.6, 0.7, 0.8)$	$(0.4, 0.5, 0.7)$	$(0.7, 0.9, 0.9)$
$\mathbb{G}$	(ν_1, ν_2)	(ν_2, ν_3)	(ν_1, ν_3)
θ_1	$(0.2, 0.3, 0.6)$	$(0.2, 0.4, 0.9)$	$(0.2, 0.3, 0.8)$
θ_2	$(0.2, 0.3, 0.9)$	$(0.2, 0.2, 0.9)$	$(0.2, 0.3, 0.8)$
θ_3	$(0, 0, 1)$	$(0.3, 0.4, 0.9)$	$(0.2, 0.4, 0.9)$

$$\geq \min\left\{\max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\}, \max\left\{\mathcal{F}_{\mathbb{F}^2_\theta}(\nu), \mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$
$$\geq \max\left\{\min\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\}, \min\left\{\mathcal{F}_{\mathbb{F}^2_\theta}(\nu), \mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$
$$= \max\left\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\right\}.$$

Hence the union $\mathfrak{G} = \mathfrak{G}_1 \cup \mathfrak{G}_2$ is NHS-Graphs. □

Example 4.3. Let $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ be a neutrosophic hypersoft graph where $\mathfrak{G}_1^\star = (\mathcal{V}_1, \mathcal{E}_1)$ with $\mathcal{V}_1 = \{\nu_1, \nu_2, \nu_3\}$ and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, \alpha_3$ where $\mathcal{Q}_1 = \{\alpha_{11}\}$, $\mathcal{Q}_2 = \{\alpha_{21}\}$ and $\mathcal{Q}_3 = \{\alpha_{31}, \alpha_{32}, \alpha_{33}\}$. $\mathcal{Q}^1 = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 = \{\theta_1, \theta_2, \theta_3\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}^1$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V}_1 \times \mathcal{V}_1 \setminus \{(\nu_1, \nu_2), (\nu_2, \nu_3), (\nu_1, \nu_3)\}$. Its tabular representation is given in TABLE 3. Also let $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ be a neutrosophic hypersoft graph where $\mathfrak{G}_2^\star = (\mathcal{V}_2, \mathcal{E}_2)$ with $\mathcal{V}_2 = \{\nu_3, \nu_4, \nu_5\}$ and $\mathcal{Q}_3, \mathcal{Q}_4, \mathcal{Q}_5$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_3, \alpha_4, \alpha_5$ where $\mathcal{Q}_3 = \{\alpha_{31}, \alpha_{32}\}$, $\mathcal{Q}_4 = \{\alpha_{41}\}$, $\mathcal{Q}_5 = \{\alpha_{51}\}$. $\mathcal{Q}^2 = \mathcal{Q}_3 \times \mathcal{Q}_4 \times \mathcal{Q}_5 = \{\theta_2, \theta_4\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}^2$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V}_2 \times \mathcal{V}_2 \setminus \{(\nu_3, \nu_4), (\nu_4, \nu_5), (\nu_3, \nu_5)\}$. Its tabular representation is given in TABLE 4.
Now Let $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ be the union of two neutrosophic hypersoft graphs $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ and $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ where $\mathcal{Q} = \mathcal{Q}^1 \cup \mathcal{Q}^2$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V} \times \mathcal{V} \setminus \{(\nu_1, \nu_2), (\nu_1, \nu_3), (\nu_2, \nu_3), (\nu_3, \nu_4), (\nu_3, \nu_5), (\nu_4, \nu_5)\}$. Its (union of these two graphs) tabular representation is given in TABLE 5.

Definition 4.4. The intersection of two NHS-Graphs $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$, $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$, denoted by $\mathfrak{G}_1 \cap \mathfrak{G}_2$, is a NHS-Graph $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ such that $\mathcal{Q} =$

FIGURE 3. Graphical Representation of TABLE 3 with (a) $\mathcal{N}(\theta_1)$, (b) $\mathcal{N}(\theta_2)$ and (c) $\mathcal{N}(\theta_3)$

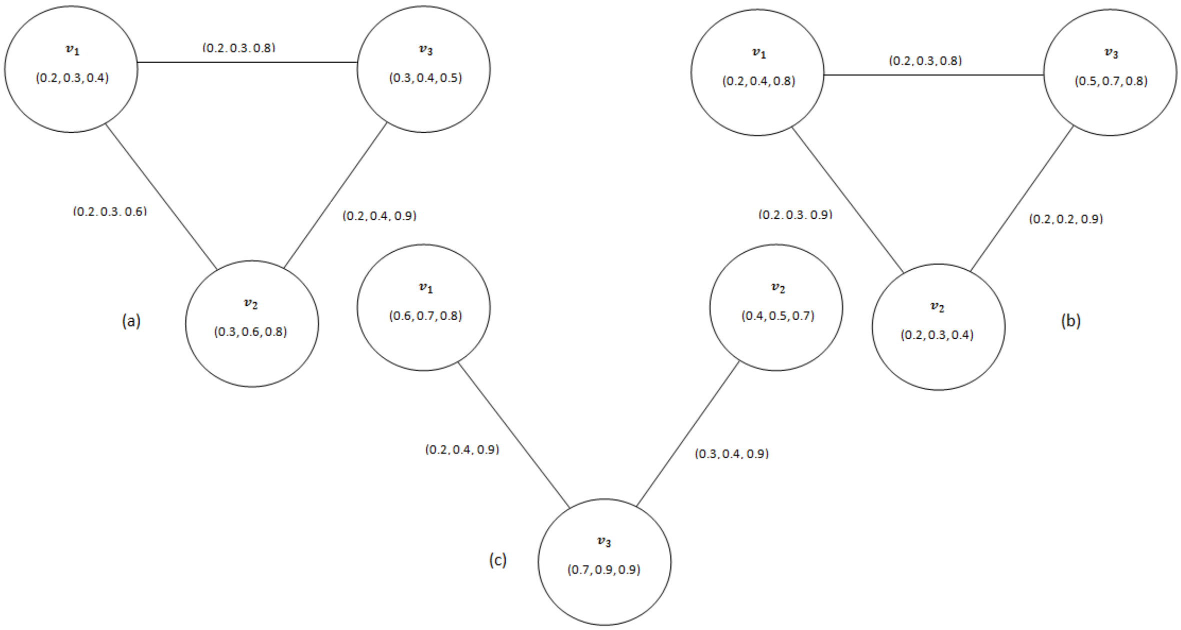


TABLE 4. Tabular Representation of NHS-Graph $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ according to Example 4.3

$\mathbb{F}$	ν_3	ν_4	ν_5
θ_2	$(0.3, 0.4, 0.5)$	$(0.2, 0.3, 0.5)$	$(0.5, 0.7, 0.8)$
θ_4	$(0.6, 0.8, 0.9)$	$(0.4, 0.7, 0.9)$	$(0.4, 0.5, 0.6)$
$\mathbb{G}$	(ν_3, ν_4)	(ν_4, ν_5)	(ν_3, ν_5)
θ_2	$(0.2, 0.3, 0.9)$	$(0.3, 0.4, 0.9)$	$(0, 0, 1)$
θ_4	$(0.2, 0.2, 0.9)$	$(0.3, 0.3, 0.9)$	$(0.3, 0.4, 0.9)$

TABLE 5. Tabular Representation of $\mathfrak{G} = \mathfrak{G}_1 \cup \mathfrak{G}_2$

$\mathbb{F}$	ν_1	ν_2	ν_3	ν_4	ν_5	
θ_1	$(0.2, 0.3, 0.4)$	$(0.3, 0.4, 0.5)$	$(0.3, 0.6, 0.8)$	$(0, 0, 1)$	$(0, 0, 1)$	
θ_2	$(0.2, 0.4, 0.8)$	$(0.2, 0.3, 0.4)$	$(0.3, 0.5, 0.5)$	$(0.2, 0.3, 0.4)$	$(0.5, 0.7, 0.8)$	
θ_3	$(0.6, 0.7, 0.8)$	$(0.4, 0.5, 0.7)$	$(0.7, 0.9, 0.9)$	$(0, 0, 1)$	$(0, 0, 1)$	
θ_4	$(0, 0, 1)$	$(0, 0, 1)$	$(0.6, 0.8, 0.9)$	$(0.4, 0.7, 0.9)$	$(0.4, 0.5, 0.6)$	
$\mathbb{G}$	(ν_1, ν_2)	(ν_1, ν_3)	(ν_2, ν_3)	(ν_3, ν_4)	(ν_3, ν_5)	(ν_4, ν_5)
θ_1	$(0.2, 0.3, 0.8)$	$(0.2, 0.3, 0.9)$	$(0.2, 0.4, 0.9)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_2	$(0.2, 0.3, 0.8)$	$(0.2, 0.3, 0.9)$	$(0.2, 0.2, 0.9)$	$(0.2, 0.3, 0.9)$	$(0.3, 0.4, 0.9)$	$(0, 0, 1)$
θ_3	$(0.2, 0.4, 0.9)$	$(0, 0, 1)$	$(0.3, 0.4, 0.9)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_4	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0.2, 0.2, 0.9)$	$(0.3, 0.3, 0.9)$	$(0.3, 0.4, 0.9)$

FIGURE 4. Graphical Representation of TABLE 4 with (a) $\mathcal{N}(\theta_2)$ and (b) $\mathcal{N}(\theta_4)$

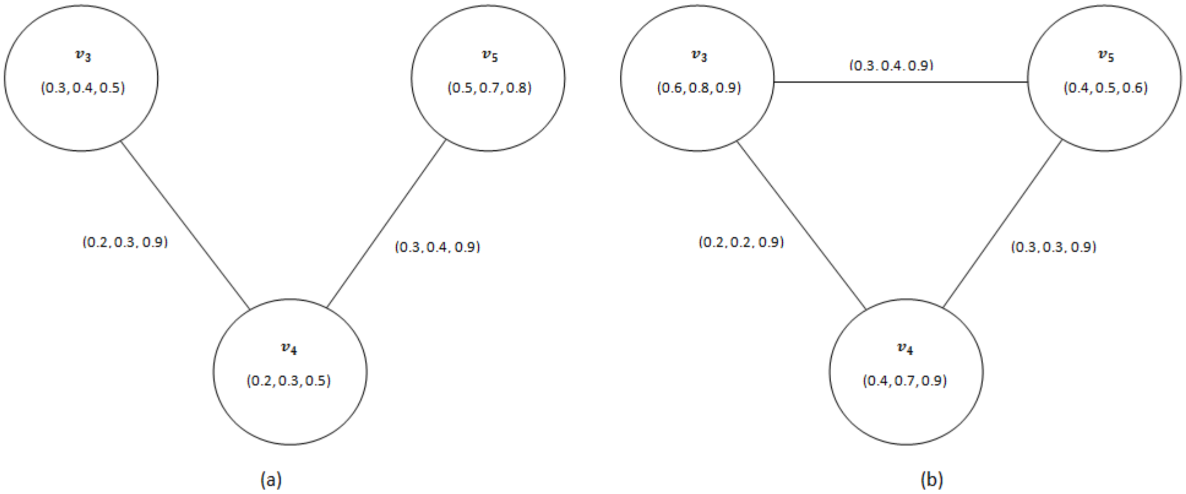
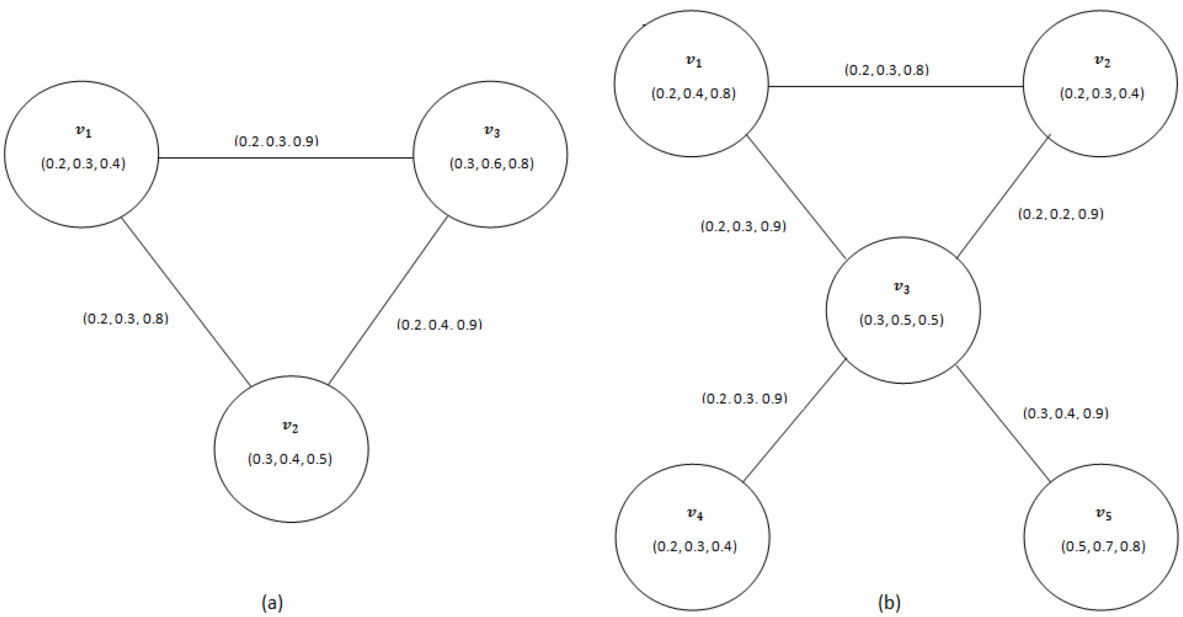


FIGURE 5. Graphical Representation of TABLE 5 with (a) $\mathcal{N}(\theta_1)$ and (b) $\mathcal{N}(\theta_2)$



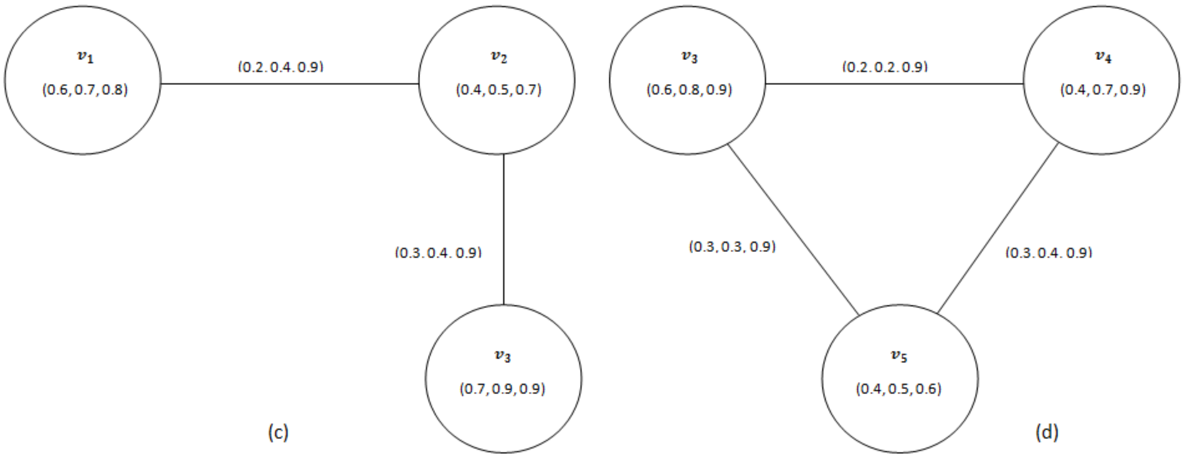
$\mathcal{Q}^1 \cap \mathcal{Q}^2, \mathcal{V} = \mathcal{V}_1 \cap \mathcal{V}_2$. In this graph, the neutrosophic components for $\mathbb{F}$ are given as follows:

$$\mathcal{T}_{\mathbb{F}_\theta} = \begin{cases} \mathcal{T}^1_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{T}^2_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \min\left\{\mathcal{T}^1_{\mathbb{F}_\theta}(\nu), \mathcal{T}^2_{\mathbb{F}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases},$$

and

$$\mathcal{I}_{\mathbb{F}_\theta} = \begin{cases} \mathcal{I}^1_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{I}^2_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \min\left\{\mathcal{I}^1_{\mathbb{F}_\theta}(\nu), \mathcal{I}^2_{\mathbb{F}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases},$$

FIGURE 6. Graphical Representation of TABLE 5 with (c) $\mathcal{N}(\theta_3)$ and (d) $\mathcal{N}(\theta_4)$



and

$$\mathcal{F}_{\mathbb{F}_\theta} = \begin{cases} \mathcal{F}^1_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{F}^2_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \max\left\{\mathcal{F}^1_{\mathbb{F}_\theta}(\nu), \mathcal{F}^2_{\mathbb{F}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases}.$$

The neutrosophic components for $\mathbb{G}$ are given as follows:

$$\mathcal{T}_{\mathbb{G}_\theta} = \begin{cases} \mathcal{T}^1_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{T}^2_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \min\left\{\mathcal{T}^1_{\mathbb{G}_\theta}(\nu), \mathcal{T}^2_{\mathbb{G}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases},$$

and

$$\mathcal{I}_{\mathbb{G}_\theta} = \begin{cases} \mathcal{I}^1_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{I}^2_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \min\left\{\mathcal{I}^1_{\mathbb{G}_\theta}(\nu), \mathcal{I}^2_{\mathbb{G}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases},$$

and

$$\mathcal{F}_{\mathbb{G}_\theta} = \begin{cases} \mathcal{F}^1_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{F}^2_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \max\left\{\mathcal{F}^1_{\mathbb{G}_\theta}(\nu), \mathcal{F}^2_{\mathbb{G}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases}.$$

Theorem 4.5. *If* $\mathfrak{G}_1, \mathfrak{G}_2 \in \Omega_{NHSG}$ *then* $\mathfrak{G}_1 \cap \mathfrak{G}_2 \in \Omega_{NHSG}$.

Proof. Consider two NHS-Graphs $\mathfrak{G}_1 = \left(\mathfrak{G}^\star_1, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ and $\mathfrak{G}_2 = \left(\mathfrak{G}^\star_2, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ as defined in 3.1. Let $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ be the intersection of NHS-Graphs $\mathfrak{G}_1$ and $\mathfrak{G}_2$ where $\mathcal{Q} = \mathcal{Q}^1 \cup \mathcal{Q}^2$ and $\mathcal{V} = \mathcal{V}_1 \cap \mathcal{V}_2$. Let $\theta \in \mathcal{Q}^1 - \mathcal{Q}^2$ then

$$\mathcal{T}_{\mathbb{G}_\theta}(\nu, \mu) = \mathcal{T}_{\mathbb{G}^1_\theta}(\nu, \mu)$$

$$\leq \min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\}$$

$$= \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\}$$

so

$$\mathcal{T}_{\mathbb{G}_\theta}(\nu,\mu) \leq \min\{\mathcal{T}_{\mathbb{F}_\theta}(\nu),\mathcal{T}_{\mathbb{F}_\theta}(\mu)\}$$

Also

$$\begin{aligned}\mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) &= \mathcal{I}_{\mathbb{G}^1_\theta}(\nu,\mu)\\ &\leq \min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu),\mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\}\\ &= \min\{\mathcal{I}_{\mathbb{F}_\theta}(\nu),\mathcal{I}_{\mathbb{F}_\theta}(\mu)\}\end{aligned}$$

so

$$\mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) \leq \min\{\mathcal{I}_{\mathbb{F}_\theta}(\nu),\mathcal{I}_{\mathbb{F}_\theta}(\mu)\}$$

Now

$$\begin{aligned}\mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) &= \mathcal{F}_{\mathbb{G}^1_\theta}(\nu,\mu)\\ &\geq \max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu),\mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\}\\ &= \max\{\mathcal{F}_{\mathbb{F}_\theta}(\nu),\mathcal{F}_{\mathbb{F}_\theta}(\mu)\}\end{aligned}$$

so

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) \geq \max\{\mathcal{F}_{\mathbb{F}_\theta}(\nu),\mathcal{F}_{\mathbb{F}_\theta}(\mu)\}$$

Similar results are obtained when $\theta \in \mathcal{Q}^2 - \mathcal{Q}^1$

Now if $\theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2$ then

$$\begin{aligned}\mathcal{T}_{\mathbb{G}_\theta}(\nu,\mu) &= \min\left\{\mathcal{T}_{\mathbb{G}^1_\theta}(\nu,\mu),\mathcal{T}_{\mathbb{G}^2_\theta}(\nu,\mu)\right\}\\ &\leq \min\left\{\min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu),\mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\},\min\left\{\mathcal{T}_{\mathbb{F}^2_\theta}(\nu),\mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}\\ &\leq \min\left\{\min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu),\mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\},\min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu),\mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}\\ &= \min\{\mathcal{T}_{\mathbb{F}_\theta}(\nu),\mathcal{T}_{\mathbb{F}_\theta}(\mu)\}\end{aligned}$$

Also

$$\begin{aligned}\mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) &= \min\left\{\mathcal{I}_{\mathbb{G}^1_\theta}(\nu,\mu),\mathcal{I}_{\mathbb{G}^2_\theta}(\nu,\mu)\right\}\\ &\leq \min\left\{\min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu),\mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\},\min\left\{\mathcal{I}_{\mathbb{F}^2_\theta}(\nu),\mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}\\ &\leq \min\left\{\min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu),\mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\},\min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu),\mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}\\ &= \min\{\mathcal{I}_{\mathbb{F}_\theta}(\nu),\mathcal{I}_{\mathbb{F}_\theta}(\mu)\}\end{aligned}$$

In the same way

$$\begin{aligned}\mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) &= \max\left\{\mathcal{F}_{\mathbb{G}^1_\theta}(\nu,\mu),\mathcal{F}_{\mathbb{G}^2_\theta}(\nu,\mu)\right\}\\ &\geq \max\left\{\max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu),\mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\},\max\left\{\mathcal{F}_{\mathbb{F}^2_\theta}(\nu),\mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}\\ &\geq \max\left\{\max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu),\mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\},\max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu),\mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}\\ &= \max\{\mathcal{F}_{\mathbb{F}_\theta}(\nu),\mathcal{F}_{\mathbb{F}_\theta}(\mu)\}\end{aligned}$$

Hence the intersection $\mathfrak{G} = \mathfrak{G}_1 \cap \mathfrak{G}_2$ is NHS-Graphs. □

TABLE 6. Tabular Representation of NHS-Graph $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ according to Example 4.6

$\mathbb{F}$	ν_1	ν_2	ν_3
θ_1	$(0.2, 0.3, 0.4)$	$(0.3, 0.5, 0.6)$	$(0.2, 0.6, 0.8)$
θ_2	$(0.3, 0.4, 0.8)$	$(0.5, 0.7, 0.8)$	$(0.4, 0.5, 0.7)$
$\mathbb{G}$	(ν_1, ν_2)	(ν_2, ν_3)	(ν_1, ν_3)
θ_1	$(0.2, 0.2, 0.7)$	$(0.2, 0.4, 0.9)$	$(0.2, 0.2, 0.9)$
θ_2	$(0.3, 0.4, 0.8)$	$(0.4, 0.5, 0.9)$	$(0.3, 0.4, 0.8)$

TABLE 7. Tabular Representation of NHS-Graph $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ according to Example 4.6

$\mathbb{F}$	ν_2	ν_3	ν_4
θ_2	$(0.4, 0.6, 0.7)$	$(0.5, 0.6, 0.9)$	$(0.3, 0.5, 0.7)$
θ_3	$(0.3, 0.5, 0.6)$	$(0.2, 0.6, 0.8)$	$(0.2, 0.3, 0.7)$
$\mathbb{G}$	(ν_2, ν_3)	(ν_3, ν_4)	(ν_2, ν_4)
θ_2	$(0.2, 0.2, 0.7)$	$(0.2, 0.4, 0.9)$	$(0.2, 0.2, 0.9)$
θ_3	$(0.3, 0.4, 0.8)$	$(0.4, 0.5, 0.9)$	$(0.3, 0.4, 0.8)$

Example 4.6. Let $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ be a neutrosophic hypersoft graph where $\mathfrak{G}_1^\star = (\mathcal{V}_1, \mathcal{E}_1)$ with $\mathcal{V}_1 = \{\nu_1, \nu_2, \nu_3\}$ and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, \alpha_3$ where $\mathcal{Q}_1 = \{\alpha_{11}\}$, $\mathcal{Q}_2 = \{\alpha_{21}\}$ and $\mathcal{Q}_3 = \{\alpha_{31}, \alpha_{32}\}$. $\mathcal{Q}^1 = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 = \{\theta_1, \theta_2\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}^1$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V}_1 \times \mathcal{V}_1 \setminus \{(\nu_1, \nu_2), (\nu_2, \nu_3), (\nu_1, \nu_3)\}$. Its tabular and graphical representation are given in TABLE 6 and FIGURE 7 respectively. Also let $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ be a neutrosophic hypersoft graph where $\mathfrak{G}_2^\star = (\mathcal{V}_2, \mathcal{E}_2)$ with $\mathcal{V}_2 = \{\nu_2, \nu_3, \nu_4\}$ and $\mathcal{Q}_2, \mathcal{Q}_3, \mathcal{Q}_4$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_2, \alpha_3, \alpha_4$ where $\mathcal{Q}_2 = \{\alpha_{21}\}$, $\mathcal{Q}_3 = \{\alpha_{31}, \alpha_{32}\}$, $\mathcal{Q}_4 = \{\alpha_{41}\}$. $\mathcal{Q}^2 = \mathcal{Q}_2 \times \mathcal{Q}_3 \times \mathcal{Q}_4 = \{\theta_2, \theta_3\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}^2$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V}_2 \times \mathcal{V}_2 \setminus \{(\nu_2, \nu_3), (\nu_3, \nu_4), (\nu_2, \nu_4)\}$. Its tabular and graphical representation are given in TABLE 7 and FIGURE 8 respectively.

Now Let $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ be the intersection of two neutrosophic hypersoft graphs $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ and $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ where $\mathcal{Q} = \mathcal{Q}^1 \cap \mathcal{Q}^2$.Its (intersection of these two NHS-graphs) tabular and graphical representation are given in TABLE 8 and FIGURE 9 respectively.

Definition 4.7. The compliment $\overline{\mathfrak{G}} = \left(\overline{\mathfrak{G}^\star}, \overline{\mathcal{Q}}, \overline{\mathbb{F}}, \overline{\mathbb{G}}\right)$ of strong neutrosophic hypersoft subgraph $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ with $\mathbb{G}_\theta(\nu, \mu) = \mathbb{F}_\theta(\nu) \cap \mathbb{F}_\theta(\mu)$ where

FIGURE 7. Graphical Representation of TABLE 6with (a) $\mathcal{N}(\theta_1)$ and (b) $\mathcal{N}(\theta_2)$

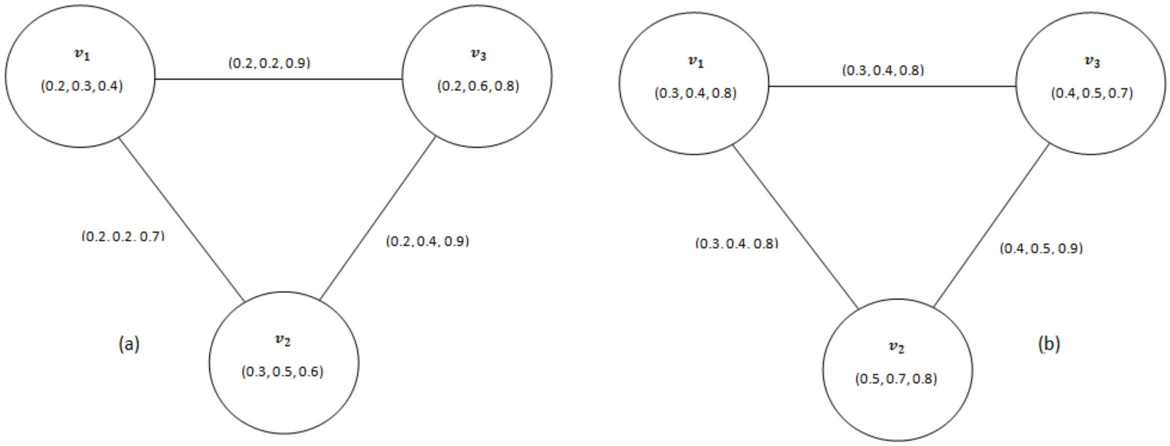


FIGURE 8. Graphical Representation of TABLE 7with (a) $\mathcal{N}(\theta_2)$ and (b) $\mathcal{N}(\theta_3)$

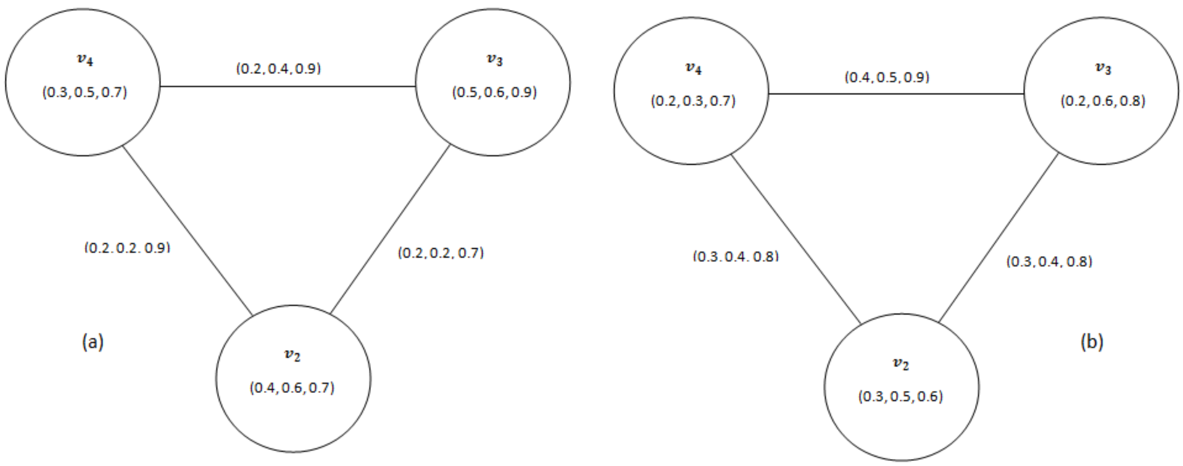


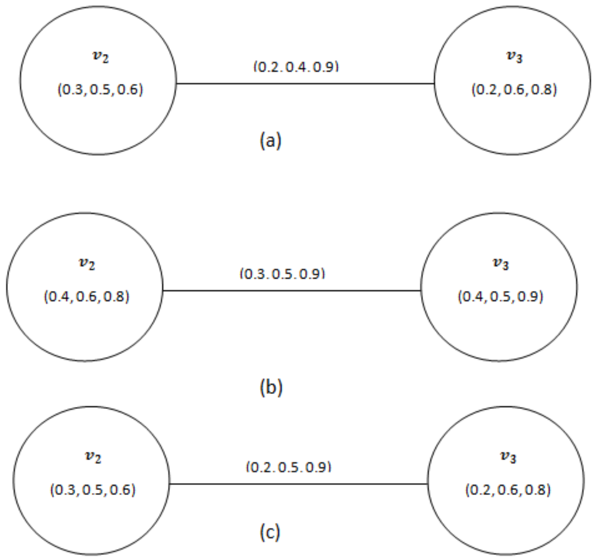
TABLE 8. Tabular Representation of NHS-Graph $\mathfrak{G} = \mathfrak{G}_1 \cap \mathfrak{G}_2$

$\mathbb{F}$	ν_2	ν_3
θ_1	$(0.3, 0.5, 0.6)$	$(0.2, 0.6, 0.8)$
θ_2	$(0.4, 0.6, 0.8)$	$(0.4, 0.5, 0.9)$
θ_3	$(0.3, 0.5, 0.6)$	$(0.2, 0.6, 0.8)$
$\mathbb{G}$	(ν_2, ν_3)	
θ_1	$(0.2, 0.4, 0.9)$	
θ_2	$(0.3, 0.5, 0.9)$	
θ_3	$(0.2, 0.5, 0.9)$	

(1) $\overline{\mathcal{Q}} = \mathcal{Q}$

(2) $\overline{\mathcal{T}_{\mathbb{F}_\theta}(\nu)} = \mathcal{T}_{\mathbb{F}_\theta}(\nu), \overline{\mathcal{I}_{\mathbb{F}_\theta}(\nu)} = \mathcal{I}_{\mathbb{F}_\theta}(\nu), \overline{\mathcal{F}_{\mathbb{F}_\theta}(\nu)} = \mathcal{F}_{\mathbb{F}_\theta}(\nu)$ for all $\nu \in \mathcal{V}$

FIGURE 9. Graphical Representation of TABLE 8 with (a) $\mathcal{N}(\theta_1)$, (b) $\mathcal{N}(\theta_2)$ and (c) $\mathcal{N}(\theta_3)$



(3)
$$\overline{\mathcal{T}_{\mathbb{F}_\theta}(\nu,\mu)} = \begin{cases} \min\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\} & if\ \mathcal{T}_{\mathbb{G}_\theta}(\nu,\mu) = 0 \\ 0 & otherwise \end{cases}$$
$$\overline{\mathcal{I}_{\mathbb{F}_\theta}(\nu,\mu)} = \begin{cases} \min\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\} & if\ \mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) = 0 \\ 0 & otherwise \end{cases}$$
$$\overline{\mathcal{F}_{\mathbb{F}_\theta}(\nu,\mu)} = \begin{cases} \max\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\} & if\ \mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) = 0 \\ 0 & otherwise \end{cases}.$$

5. Conclusions

In this study, a gluing concept of neutrosophic hypersoft set and graph theory is characterized. Some of elementary properties, types, operations and results are generalized under neutrosophic hypersoft set environment. Future work may include the extension of this study for the following structures and fields:

- Interval valued neutrosophic hypersoft set
- Neutrosophic parameterized hypersoft set
- m-polar neutrosophic hypersoft set
- Decision making problems
- New kinds of graphs
- Energies of graph

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References

1. Zadeh, L. Fuzzy sets. *Information and control* **1965**, *8*(3), 338-353. http://doi.org/10.1016/S0019-9958(65)90241-X
2. Atanassov, K. Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems* **1986**, *20*, 87-96. http://doi.org/10.1016/S0165-0114(86)80034-3
3. Smarandache, F. *Neutrosophy, neutrosophic probability, set, and logic*, analytic synthesis and synthetic analysis. Rehoboth, American Research Press 1998.
4. Smarandache, F. Neutrosophic set- a generalization of intuitionistic fuzzy set. *Granular Computing, IEEE, International Conference* **2006**, 38-42. http://doi.org/10.1109/GRC.2006.1635754
5. Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. Single valued neutrosophic sets. *Multissp. Multistruct.* **2010**, *4*, 410-413
6. Smarandache, F. n-Valued refined neutrosophic logic and its applications in Physics. *Progress in Physics* **2013**, *4* , 143-146. http://doi.org/10.5281/zenodo.49149
7. Smarandache, F. Operators on single valued neutrosophic sets, neutrosophic undersets, and neutrosophic offsets. *Journal of Mathematics and Informatics* **2016**, *5* , 63-67. http://doi.org/10.5281/zenodo.57412
8. Smarandache, F. Neutrosophic overset, neutrosophic underset, neutrosophic offset, similarly for neutrosophicover-/under-/offlogic, probability, and statistic. Pons Editions, Brussels, 2016. http://doi.org/10.5281/zenodo.57410
9. Pramanik, S.; Dey, P. P.; Smarandache, F. Correlation coefficient measures of interval bipolar neutrosophic sets for solving multi-attribute decision making problems. *Neutrosophic Sets and Systems* **2018**, *19* , 70-79. http://doi.org/10.5281/zenodo.1235151.
10. Broumi, S.; Smarandache, F. Intuitionistic Neutrosophic Soft Set. *Journal of Information and Computing Science* **2013**, *8*(2) , 130-140. http://doi.org/10.5281/zenodo.2861554.
11. Broumi, S. Generalized Neutrosophic Soft Set. *International Journal of Computer Science, Engineering and Information Technology* **2013**, *3*(2) , 17-30. http://doi.org/10.5121/ijcseit.2013.3202.
12. Broumi, S.; Deli, I.; Smarandache, F. Relations on Interval Valued Neutrosophic Soft Sets. *Journal of New Results in Science* **2014**, *5* , 1-20. http://doi.org/10.5281/zenodo.30306.
13. Deli, I. Interval-valued neutrosophic soft sets and its decision making. *International Journal of Machine Learning and Cybernetics* **2017**, *8*(2) , 665-676. http://doi.org/10.1007/s13042-015-0461-3.
14. Kharal, A. A Neutrosophic Multicriteria Decision Making Method. *New Mathematics and Natural Computation* **2014**, *10*(2), 143-162. http://doi.org/10.1142/S1793005714500070
15. Edalatpanah, S. A. Systems of Neutrosophic Linear Equations. *Neutrosophic Sets and Systems* **2020**, *33*, 92-104. http://doi.org/10.5281/zenodo.3782826.
16. Kumar, R.; Edalatpanah, S. A.; Jha, S.,; Singh, R. A novel approach to solve gaussian valued neutrosophic shortest path problems. *International Journal of Engineering and Advanced Technology* **2019**, *8*(3), 347-353. http://doi.org/10.35940/ijeat.E1049.0585C19.
17. Molodtsov, D. Soft Set Theory - First Results. *Computers and Mathematics with Applications* **1999**, *37*, 19-31. http://doi.org/10.1016/S0898-1221(99)00056-5
18. Maji. P.K.; Biswas R.; Roy, A. R. Soft Set Theory. *Computers and Mathematics with Applications* **2003**, *45*, 555-562. http://doi.org/10.1016/S0898-1221(03)00016-6
19. Maji, P. K.; Biswas, R.; Roy, A. R. Fuzzy soft sets. *Journal of Fuzzy Mathematics* **2001**, *9*(3), 589-602.
20. Pei, D.; Miao, D. From soft set to information system. *In international conference of granular computing IEEE* **2005**, *2* , 617-621. http://doi.org/10.1109/GRC.2005.1547365
21. Ali, M. I.; Feng, F.; Liu, X.; Min, W. K.; Sabir, M. On some new operations in soft set theory. *Computers and Mathematics with Applications 2009*, *57* , 1547-1553. http://doi.org/10.1016/j.camwa.2008.11.009

22. Babitha, K. V.; Sunil, J. J. Soft set relations and functions. *Computers and Mathematics with Applications* **2010**, *60*, 1840-1849. http://doi.org/10.1016/j.camwa.2010.07.014
23. Babitha, K. V.; Sunil, J. J. Transitive closure and ordering in soft set. *Computers and Mathematics with Applications* **2010**, *61* 2235-2239. http://doi.org/10.1016/j.camwa.2011.07.010
24. Sezgin, A.; Atagün A. O. On operations of soft sets. *Computers and Mathematics with Applications* **2011**, *61*(5), 1457-1467. http://doi.org/10.1016/j.camwa.2011.01.018
25. Ge, X.; Yang, S. Investigations on some operations of soft sets. *World Academy of Science Engineering and Technology* **2011**, *75*, 1113-1116.
26. Li, F. Notes on soft set operations. *ARPN Journal of systems and softwares* **2011**, 205-208.
27. Maji, P. K.; Biswas, R.; Roy, A. R. Intuitionistic fuzzy soft sets. *The Journal of Fuzzy Mathematics* **2001**, *9*(3), 677-692.
28. Maji, P. K. Neutrosophic Soft Set. *Annals of Fuzzy Mathematics and Informatics* **2013**, *5*(1), 2287-623.
29. Smarandache, F. Extension of Soft Set of Hypersoft Set, and then to Plithogenic Hypersoft Set. *Neutrosophic Sets and Systems* **2018**, *22* , 168-170. http://doi.org/10.5281/zenodo.2838716
30. Saeed, M.; Ahsan, M.; Siddique, M.k.; Ahmad, M.R. A Study of the Fundamentals of Hypersoft Set Theory. *International Journal of Scientific and Engineering Research* **2020**, *11*(1), 320-329.
31. Saeed, M.; Rahman, A. U.; Ahsan, M.; Smarandache, F. An Inclusive Study on Fundamentals of Hypersoft Set, In Theory and Application of Hypersoft Set, Pons Publishing House, Brussel, 2021, 1-23.
32. Abbas, F.; Murtaza, G.; Smarandache, F. (2020). Basic operations on hypersoft sets and hypersoft points. *Neutrosophic Sets and Systems* **2020**, *35*, 407-421. http://doi.org/10.5281/zenodo.3951694
33. Saqlain, M.; Jafar, N.; Moin, S.; Saeed, M.; Broumi, S. Single and Multi-valued Neutrosophic Hypersoft Set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets. *Neutrosophic Sets and Systems* **2020**, *32*, 317-329. http://doi.org/10.5281/zenodo.3723165
34. Saqlain, M.; Moin, S.; Jafar, N.; Saeed, M.; Smarandache, F. Aggregate Operators of Neutrosophic Hypersoft Sets. *Neutrosophic Sets and Systems* **2020**, 32, 294-306. http://doi.org/10.5281/zenodo.3723155
35. Saqlain, M.; Saeed, M.; Ahmad, M.R.; Smarandache, F. Generalization of TOPSIS for Neutrosophic Hypersoft Sets using Accuracy Function and its Application. *Neutrosophic Sets and Systems* **2020**, *27*, 131-137. http://doi.org/10.5281/zenodo.3275533
36. Martin, N.; Smarandache, F. Concentric Plithogenic Hypergraph based on Plithogenic Hypersoft Sets A Novel Outlook. *Neutrosophic Sets and Systems* **2020**, *33*, 78-91. http://doi.org/10.5281/zenodo.3782824
37. Rahman, A. U.; Saeed, M.; Alodhaibi, S.S.; Abd, H. Decision Making Algorithmic Approaches Based on Parameterization of Neutrosophic Set under Hypersoft Set Environment with Fuzzy, Intuitionistic Fuzzy and Neutrosophic Settings. *CMES-Computer Modeling in Engineering & Sciences* **2021**, *128*(2), 743-777. http://doi.org/10.32604/cmes.2021.016736
38. Rahman, A. U.; Saeed, M.; Smarandache, F.; Ahmad, M. R. Development of Hybrids of Hypersoft Set with Complex Fuzzy Set, Complex Intuitionistic Fuzzy set and Complex Neutrosophic Set. *Neutrosophic Sets and Systems* **2020**, 38, 335-354. http://doi.org/10.5281/zenodo.4300520
39. Rahman, A. U.; Saeed, M.; Smarandache, F. Convex and Concave Hypersoft Sets with Some Properties. *Neutrosophic Sets and Systems* **2020**, 38, 497-508. http://doi.org/10.5281/zenodo.4300580
40. Deli, I. Hybrid set structures under uncertainly parameterized hypersoft sets, Theory and applications. In Theory and Application of Hypersoft Set, Pons Publishing House, Brussel, 2021, 24-49.
41. Gayen, S.; Smarandache, F.; Jha, S.; Singh, M. K.; Broumi, S.; Kumar, R. Introduction to Plithogenic Hypersoft Subgroup. *Neutrosophic Sets and Systems 2020*, *33*, 14. http://doi.org/10.5281/zenodo.3782897
42. Saeed, M.; Ahsan, M.; Saeed, M. H.; Mehmood, A.; Abdeljawad, T. An Application of Neutrosophic Hypersoft Mapping to Diagnose Hepatitis and Propose Appropriate Treatment. *IEEE Access* **2021**, *9*, 70455-70471. http://doi.org/10.1109/ACCESS.2021.3077867

43. Saeed, M.; Ahsan, M.; Abdeljawad, T. A Development of Complex Multi-Fuzzy Hypersoft Set With Application in MCDM Based on Entropy and Similarity Measure. *IEEE Access* **2021**, *9*, 60026-60042. 10.1109/ACCESS.2021.3073206
44. Rahman, A.U.; Saeed, M.; Dhital, A. Decision Making Application Based on Neutrosophic Parameterized Hypersoft Set Theory. *Neutrosophic Sets and Systems* **2021**, *41*, 1-14. http://doi.org/10.5281/zenodo.4625665
45. Rahman, A.U.; Saeed, M.; Zahid, S. Application in Decision Making Based on Fuzzy Parameterized Hypersoft Set Theory. *Asia Mathematika 2021*, *5*(1), 19-27. http://doi.org/10.5281/zenodo.4721481
46. Rahman, A.U.; Hafeez, A.; Saeed, M.; Ahmad, M.R.; Farwa, U. Development of Rough Hypersoft Set with Application in Decision Making for the Best Choice of Chemical Material, In Theory and Application of Hypersoft Set, Pons Publication House, Brussel, 2021, 192-202. http://doi.org/10.5281/zenodo.4743367
47. Ihsan,M.; Rahman, A.U.; Saeed, M. Hypersoft Expert Set With Application in Decision Making for Recruitment Process. *Neutrosophic Sets and Systems* **2021**, *42*, 191-207. http://doi.org/10.5281/zenodo.4711524
48. Thumbakara, R. K.; George, B. Soft graphs. *General Mathematics Notes* **2014**, *21*(2), 75-86.
49. Mohinta, S.; Samanta, T. K. An introduction to fuzzy soft graph. *Mathematica Moravica* **2015**, *19*(2), 35-48. https://doi.org/10.5937/matmor1502035m
50. Shahzadi, S.; Akram, M. Intuitionistic fuzzy soft graphs with applications. *Journal of Applied Mathematics and Computing* **2017**, *55*(1), 369-392. https://doi.org/10.1007/s12190-016-1041-8
51. Shyla, A. M.; Varkey, T. M. Intuitionistic Fuzzy soft graph. *International Journal of Fuzzy Mathematical Archive* **2016**, *11*(2), 63-77.
52. Akram, M.; Nawaz, S. On fuzzy soft graphs. *Italian journal of pure and applied mathematics* **2015**, *34*, 497-514.
53. Al-Masarwah, A.; Qamar, M. A. Some new concepts of fuzzy soft graphs. *Fuzzy Information and Engineering* 2016, *8*(4), 427-438. https://doi.org/10.1016/j.fiae.2017.01.003
54. Akram, M.; Malik, H. M.; Shahzadi, S.; Smarandache, F. Neutrosophic soft rough graphs with application. *Axioms* **2018**, *7*(1), 14. https://doi.org/10.3390/axioms7010014
55. Akram, M.; Shahzadi, S. Neutrosophic soft graphs with application. *Journal of Intelligent and Fuzzy Systems* **2017**, *32*(1), 841-858. https://doi.org/10.3233/jifs-16090
56. Shah, N.; Hussain, A. Neutrosophic soft graphs. *Neutrosophic Sets and Systems* **2016**, *11*, 31-44. https://doi.org/10.5281/zenodo.571574

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