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The neutrosophic integrals by parts

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Abstract: The purpose of this article is to study the neutrosophic integrals by parts, all cases in which we can apply integration by parts are discussed, including solve the repeated and non-terminating functions like the product of trigonometric and exponential using rotary integrals. In addition, the Tabular method has been introduced in the computation of neutrosophic integrals, where the Tabular method is considered to be easier than the neutrosophic integrals by Parts method in finding the neutrosophic integrals. Where detailed examples were given to clarify each case.

Keywords: neutrosophic integrals by parts; tabular method; indeterminacy.

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1. Introduction

As an alternative to the existing logics, Smarandache proposed the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [3-13]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number, and found root index $n \geq 2$ of a neutrosophic real and complex number [2-4], studying the concept of the Neutrosophic probability [3-5], the Neutrosophic statistics [4][6], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1-8]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [9]. Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and right-hand side represented with triangular neutrosophic numbers [10]. Chakraborty used pentagonal neutrosophic number in networking problem, and Shortest Path Problem [11-12]. Y.Alhasan studied the concepts of neutrosophic complex numbers, the general exponential form of a neutrosophic complex, and the neutrosophic integrals and integration methods [7-14-24]. On the other hand, M.Abdel-Basset presented study in the science of neutrosophic about an approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number [15]. Also, neutrosophic sets played an important role in applied science such as health care, industry, and optimization [16-17-18-19]. Recently, there are increasing efforts to study the neutrosophic generalized structures and spaces such as refined neutrosophic modules, spaces, equations, and rings [21-22-23].

Integration is important in human life, and one of its most important applications is the calculation of area, size and arc length. In our reality we find things that cannot be precisely defined, and that contain an indeterminacy part.

Paper consists of 5 sections. In 1th section, provides an introduction, in which neutrosophic science review has given. In 2th section, some definitions and examples of neutrosophic real number neutrosophic indefinite integral and are discussed. The 3th section frames neutrosophic integration by parts, in which three cases were discussed, including solve the repeated and non-terminating functions like the product of trigonometric and exponential using rotary integrals. The 4th section The 4th section introduces the Tabular method to find the integrals by parts in the stats 1 and 2. In 5th section, a conclusion to the paper is given.

2. Preliminaries

2.1. Neutrosophic integration by substitution method [24]

Definition2.1.1

$Let\ \ f: D_f \subseteq R \to R_f \cup \{I\}$, to evaluate $\int f(x)dx$

Put: $x = g(u) \quad \Rightarrow \quad dx = \acute{g}(u)du$

By substitution, we get:

$$\int f(x)dx = \int f(u)\acute{g}(u)du$$

then we can directly integral it.

Theorme2.1.1:

If $\int f(x,I)dx = \varphi(x,I)\ \ then,$

$$\int f\big((a+bI)x+c+dI\big)\,dx = \left(\frac{1}{a} - \frac{b}{a(a+b)}I\right)\varphi\big((a+bI)x+c+dI\big) + C$$

where C is an indeterminate real constant, $a \neq 0\,, a \neq -b\ and\ b, c, d$ are real numbers, while $I\ =$ indeterminacy.

Theorme2.1.2:

Let $\ f: D_f \subseteq R \to R_f \cup \{I\}$ then:

$$\int \frac{\acute{f}(x,I)}{f(x,I)}dx = ln|f(x,I)| + C$$

where C is an indeterminate real constant (i.e. constant of the form $a + bI$, where a, b are real numbers, while $I\ =$ indeterminacy).

Theorme2.1.3:

Let $\ f: D_f \subseteq R \to R_f \cup \{I\}$, then:

$$\int \frac{\acute{f}(x,I)}{\sqrt{f(x,I)}}dx = 2\sqrt{f(x,I)} + C$$

where C is an indeterminate real constant (i.e. constant of the form $a + bI$, where a, b are real numbers, while $I\ =$ indeterminacy).

Theorme2.1.4:
$f: D_f \subseteq R \rightarrow R_f \cup \{I\}$, then:

$$\int [f(x,I)]^n \acute{f}(x)\, dx = \frac{[f(x,I)]^{n+1}}{n+1} + C$$

Where n is any rational number. C is an indeterminate real constant (i.e. constant of the form $a + bI$, where a, b are real numbers, while I = indeterminacy).

2.2. Integrating products of neutrosophic trigonometric function [24]

I. $\int \sin^m(a+bI)x \cos^n(a+bI)x \;dx$, where m and n are positive integers.
To find this integral, we can distinguish the following two cases:

- Case $n\, is$ odd:
  - Split of $\cos(a+bI)x$
  - Apply $\cos^2(a+bI)x = 1 - \sin^2(a+bI)x$
  - We substitution $u = \sin(a+bI)x$
- Case $m\, is$ odd:
  - Split of $\sin(a+bI)x$
  - Apply $\sin^2(a+bI)x = 1 - \cos^2(a+bI)x$
  - We substitution $u = \cos(a+bI)x$

II. $\int \tan^m(a+bI)x \sec^n(a+bI)x \;dx$, where m and n are positive integers.
To find this integral, we can distinguish the following cases:

- Case $n\, is$ even:
  - Split of $\sec^2(a+bI)x$
  - Apply $\sec^2(a+bI)x = 1 + \tan^2(a+bI)x$
  - We substitution $u = \tan(a+bI)x$
- Case $m\, is$ odd:
  - Split of $\sec(a+bI)x \tan(a+bI)x$
  - Apply $\tan^2(a+bI)x = \mathrm{ses}^2(a+bI)x - 1$
  - We substitution $u = \mathrm{ses}(a+bI)x$
- Case m even and n odd:
  - Apply $\tan^2(a+bI)x = \mathrm{ses}^2(a+bI)x - 1$
  - We substitution $u = \mathrm{ses}(a+bI)x$ or $u = \tan(a+bI)x$, depending on the case.

III. $\int \cot^m(a+bI)x \csc^n(a+bI)x \;dx$, where m and n are positive integers.
To find this integral, we can distinguish the following cases:

- Case $n\, is$ even:
  - Split of $\csc^2(a+bI)x$
  - Apply $\csc^2(a+bI)x = 1 + \cot^2(a+bI)x$
  - We substitution $u = \cot(a+bI)x$
- Case $m\, is$ odd:

- Split of $\csc(a+bI)x\cot(a+bI)x$
- Apply $\cot^2(a+bI)x = \csc^2(a+bI)x - 1$
- We substitution $u = \csc(a+bI)x$

➢ Case m even and n odd:
- Apply $\cot^2(a+bI)x = \csc^2(a+bI)x - 1$
- We substitution $u = \csc(a+bI)x$ or $u = \cot(a+bI)x$, depending on the case.

2.3. Neutrosophic trigonometric identities [24]

1) $sin(a+bI)x\cos(c+dI)\,x = \frac{1}{2}\,[\sin(a+bI+c+dI) + \sin(a+bI-c-dI)]$

2) $cos(a+bI)x\sin(c+dI)\,x = \frac{1}{2}\,[\sin(a+bI+c+dI) - \sin(a+bI-c-dI)]$

3) $cos(a+bI)x\cos(c+dI)\,x = \frac{1}{2}\,[\cos(a+bI+c+dI) + \cos(a+bI-c-dI)]$

4) $sin(a+bI)x\sin(c+dI)\,x = \frac{-1}{2}\,[\cos(a+bI+c+dI) - \cos(a+bI-c-dI)]$

Where $a \neq c\ (not\ zero)\ and\ b, d$ are real numbers, while I = indeterminacy.

3. Neutrosophic integration by parts

There are integrals that cannot be evaluated by direct integration methods or by substitution, so in this current section we present a powerful tool called neutrosophic integration by parts. We have observed that every differentiation rule gives rise to a corresponding itegration rule. So, let:

$f: D_f \subseteq R \to R_f \cup \{I\}$ and $g: D_g \subseteq R \to R_g \cup \{I\}$

then, for the product rule:

$$\frac{d}{dx}[f(x)\ g(x)] = \acute{f}(x)g(x) + f(x)\acute{g}(x)$$

integrating both sides of this equation gives us:

$$\int \frac{d}{dx}[f(x)\ g(x)]\,dx = \int \acute{f}(x)g(x)\ dx + \int f(x)\acute{g}(x)\,dx$$

$$\int f(x)\acute{g}(x)\,dx = f(x)\ g(x) - \int \acute{f}(x)g(x)\ dx$$

it is usually convenient to write this using the notation:

$$u = f(x) \quad \Longrightarrow \quad du = \acute{f}(x)\,dx$$

$$v = g(x) \quad \Longrightarrow \quad dv = \acute{g}(x)\,dx$$

so the neutrosophic integration by parts algorithm becomes

$$\int u\,dv = u.v - \int v\,du$$

There are three cases of the neutrosophic integration by parts:

➢ state1: neutrosophic integration from the form:

$$\int (a+bI)x^n\, e^{(c+dI)x}\, dx$$

Where $c \neq 0$ *and* $c \neq -d$

To find this integral, we do the following:

Put $u=(a+bI)x^n \quad \Rightarrow \quad du = n(a+bI)x^{n-1}\, dx$

$dv = e^{(c+dI)x}\, dx \quad \Rightarrow \quad v = \frac{1}{c+dI} e^{(c+dI)x}$

Then apply:

$$\int u\, dv = u.v - \int v\, du$$

We get:

$$\int (a+bI)x^n\, e^{(c+dI)x}\, dx = \left(\frac{a}{c} + \frac{\mathrm{cb}-\mathrm{ad}}{\mathrm{c(c+d)}}.I\right)\left(x^n e^{(c+dI)x} - \int nx^{n-1}\, e^{(c+dI)x} dx\right) + C$$

We find the required integral by repeated the integration.

where C is an indeterminate real constant (i.e. constant of the form $e+kI$, where e,k are real numbers, while I = indeterminacy).

Example3.1

Find:

$$\int (3+2I)x\, e^{(2+4I)x}\, dx$$

Solution:

$u=(3+2I)x \quad \Rightarrow \quad du=(3+2I)dx$

$dv = e^{(2+4I)x}\, dx \quad \Rightarrow \quad v = \frac{1}{2+4I}\, e^{(2+4I)x}$

Then apply:

$$\int u\, dv = u.v - \int v\, du$$

We get:

$$\int (3+2I)x\, e^{(2+4I)x}\, dx = \left(\frac{3}{2} + \frac{4-12}{12}.I\right)\left(xe^{(2+4I)x} - \int e^{(2+4I)x}\, dx\right)$$

$$= \left(\frac{3}{2} - \frac{8}{12}.I\right)\left(xe^{(2+4I)x} - \frac{1}{2+4I} e^{(2+4I)x}\right)$$

$$= \left(\frac{3}{2} - \frac{2}{3}.I\right)\left(x - \frac{1}{2} + \frac{1}{3}I\right) e^{(2+4I)x} + C$$

- state2: neutrosophic integration from the form:

$$\int (a+bI)x^n \sin(c+dI)x\, dx \quad or \quad \int (a+bI)x^n \cos(c+dI)x\, dx$$

Where $c \neq 0$ *and* $c \neq -d$

To find the first integral, we do the following:

Put $u=(a+bI)x^n \quad \Rightarrow \quad du = n(a+bI)x^{n-1}\, dx$

$dv = \sin(c+dI)x\, dx \quad \Rightarrow \quad v = \frac{-1}{c+dI}\cos(c+dI)x$

Then apply:

$$\int u\,dv = u.v - \int v\,du$$

We get:

$$\int (a+bI)x^n \sin(c+dI)x\,dx$$
$$= \left(\frac{a}{c} + \frac{\mathrm{cb-ad}}{\mathrm{c(c+d)}}.I\right)\left(-x^n \sin(c+dI)x + \int nx^{n-1}\cos(c+dI)x\,dx\right) + C$$

We find the required integral by repeated the integration. By the same method we evaluate the second integral:

$$\int (a+bI)x^n \cos(c+dI)x\,dx$$

Example3.2

Find:

$$\int (1+I)x\ \sin(2-3I)x\,dx$$

Solution:

$$u = (1+I)x \quad \Longrightarrow \quad du = (1+I)dx$$

$$dv = \sin(2-3I)x\,dx \ \Longrightarrow \quad v = \tfrac{-1}{2-3I}\ \cos(2-3I)x$$

Then apply:

$$\int u\,dv = u.v - \int v\,du$$

We get:

$$\int (1+I)x\sin(2-3I)x\ dx = \left(\frac{1}{2} + \frac{-3-2}{-2}.I\right)\left(-x\cos(2-3I)x + \int \cos(2-3I)x\ dx\right)$$

$$= \left(\frac{3}{2} + \frac{1}{2}.I\right)\left(-x\cos(2-3I)x + \frac{1}{2-3I}\sin(2-3I)x\right)$$

$$= \left(\frac{3}{2} + \frac{1}{2}.I\right)\left(-x\cos(2-3I)x + \left(\frac{1}{2} - \frac{3}{2}I\right)\sin(2-3I)x\right) + C$$

- state3: neutrosophic integration from the form:

$$\int e^{(a+bI)x}\sin(c+dI)x\,dx \quad or \quad \int e^{(a+bI)x}\cos(c+dI)x\,dx$$

Where $c \neq 0\ and\ c \neq -d$

To find the first integral, we do the following:

Put $\quad u = e^{(a+bI)x} \quad \Longrightarrow \quad du = (a+bI)e^{(a+bI)x}\,dx$

$$dv = \sin(c+dI)x\,dx \quad \Longrightarrow \quad v = \tfrac{-1}{c+dI}\cos(c+dI)x$$

Then apply:

$$\int u\,dv = u.v - \int v\,du$$

We get:

$$\int e^{(a+bI)x}\sin(c+dI)x\,dx$$

$$= \left(\frac{-1}{c+dI}\right) e^{(a+bI)x}\cos(c+dI)x + \left(\frac{a+bI}{c+dI}\right)\int e^{(a+bI)x}\cos(c+dI)x\,dx \quad (*)$$

By using integration by parts again to evaluate:

$$\int e^{(a+bI)x}\cos(c+dI)x\,dx$$

Put $u = e^{(a+bI)x} \quad \Longrightarrow \quad du = (a+bI)e^{(a+bI)x}\,dx$

$dv = \cos(c+dI)x\,dx \quad \Longrightarrow \quad v = \frac{1}{c+dI}\sin(c+dI)x$

We get:

$$\int e^{(a+bI)x}\cos(c+dI)x\,dx = \left(\frac{-1}{c+dI}\right) e^{(a+bI)x}\sin(c+dI)x + \left(\frac{a+bI}{c+dI}\right)\int e^{(a+bI)x}\sin(c+dI)x\,dx$$

By substitution in $(*)$:

$$\int e^{(a+bI)x}\sin(c+dI)x\,dx = \left(\frac{-1}{c+dI}\right) e^{(a+bI)x}\cos(c+dI)x + \left(\frac{a+bI}{c+dI}\right)\left(\left(\frac{-1}{c+dI}\right) e^{(a+bI)x}\sin(c+dI)x + \left(\frac{a+bI}{c+dI}\right)\int e^{(a+bI)x}\sin(c+dI)x\,dx\right)$$

$$= \left(\frac{-1}{c+dI}\right) e^{(a+bI)x}\cos(c+dI)x - \left(\frac{a+bI}{(c+dI)^2}\right) e^{(a+bI)x}\sin(c+dI)x + \left(\frac{a+bI}{c+dI}\right)^2\int e^{(a+bI)x}\sin(c+dI)x\,dx$$

$$\Longrightarrow \left(1-\left(\frac{a+bI}{c+dI}\right)^2\right)\int e^{(a+bI)x}\sin(c+dI)x\,dx = \left(\frac{-1}{c+dI}\right) e^{(a+bI)x}\cos(c+dI)x - \left(\frac{a+bI}{(c+dI)^2}\right) e^{(a+bI)x}\sin(c+dI)x$$

$$\Longrightarrow \int e^{(a+bI)x}\sin(c+dI)x\,dx = \left(\frac{(c+dI)^2}{(c+dI)^2-(a+bI)^2}\right)\left(\left(\frac{-1}{c+dI}\right) e^{(a+bI)x}\cos(c+dI)x - \left(\frac{a+bI}{(c+dI)^2}\right) e^{(a+bI)x}\sin(c+dI)x + C\right)$$

By the same method we evaluate the second integral:

$$\int e^{(a+bI)x}\cos(c+dI)x\,dx$$

Example3.2

Find:

$$\int e^{(1+I)x}\cos(2+I)x\,dx$$

Solution:

Put $u = e^{(1+I)x} \quad \Rightarrow \quad du = (1+I)e^{(1+I)x}\,dx$

$dv = \cos(2+I)x dx \quad \Rightarrow \quad v = \frac{1}{2+I}\sin(2+I)x$

T hen apply:

$$\int u\,dv = u.v - \int v\,du$$

We get:

$$\int e^{(1+I)x}\cos(2+I)x\,dx = \frac{1}{2+I}e^{(1+I)x}\sin(2+I)x - \left(\frac{1+I}{2+I}\right)\int e^{(1+I)x}\sin(2+I)x\ dx \quad (*)$$

By using integration by parts again to evaluate:

$$\int e^{(1+I)x}\sin(2+I)x\ dx$$

Put $u = e^{(1+I)x} \quad \Rightarrow \quad du = (1+I)e^{(1+I)x}\,dx$

$dv = \sin(2+I)x\,dx \quad \Rightarrow \quad v = \frac{-1}{2+I}\cos(2+I)x$

We get:

$$\int e^{(1+I)x}\sin(2+I)x\,dx = \frac{-1}{2+I}e^{(1+I)x}\cos(2+I)x + \left(\frac{1+I}{2+I}\right)\int e^{(1+I)x}\cos(2+I)x\,dx$$

By substitution in $(*)$:

$$\int e^{(1+I)x}\cos(2+I)x\,dx = \frac{1}{2+I}e^{(1+I)x}\sin(2+I)x - \left(\frac{1+I}{2+I}\right)\left(\frac{-1}{2+I}e^{(1+I)x}\cos(2+I)x + \left(\frac{1+I}{2+I}\right)\int e^{(1+I)x}\cos(2+I)x\,dx\right)$$

$$= \frac{1}{2+I}e^{(1+I)x}\sin(2+I)x + \frac{1+I}{(2+I)^2}e^{(1+I)x}\cos(2+I)x - \left(\frac{1+I}{2+I}\right)^2\int e^{(1+I)x}\cos(2+I)x\,dx$$

$$\Rightarrow \left(1 + \left(\frac{1+I}{2+I}\right)^2\right)\int e^{(1+I)x}\cos(2+I)x\,dx = \frac{1}{2+I}e^{(1+I)x}\sin(2+I)x + \frac{1+I}{4+5I}e^{(1+I)x}\cos(2+I)x$$

$$\Rightarrow \int e^{(1+I)x}\cos(2+I)x\,dx = \left(\frac{4+5I}{5+8I}\right)\left(\frac{1}{2+I}e^{(1+I)x}\sin(2+I)x + \frac{1+I}{4+5I}e^{(1+I)x}\cos(2+I)x\right)$$

$$= \left(\frac{4}{5} - \frac{7}{65}I\right)\left(\left(\frac{1}{2} - \frac{1}{6}I\right)e^{(1+I)x}\sin(2+I)x + \left(\frac{1}{4} - \frac{1}{36}I\right)e^{(1+I)x}\cos(2+I)x\right) + C$$

- state3: neutrosophic integration from the form:

$$\int (a+bI)x^n \sin(c+dI)x\,dx \quad or \quad \int (a+bI)x^n \cos(c+dI)x\,dx$$

Where $c \neq 0\ and\ c \neq -d$

To find the first integral, we do the following:

Put $u = (a + bI)x^n \quad \Longrightarrow \quad du = n(a + bI)x^{n-1}\, dx$

$dv = \sin(c + dI)x\, dx \quad \Longrightarrow \quad v = \frac{-1}{c+dI}\cos(c + dI)x$

Then apply:

$$\int u\, dv = u.v - \int v\, du$$

We get:

$$\int (a + bI)x^n \sin(c + dI)x\, dx$$
$$= \left(\frac{a}{c} + \frac{\text{cb} - \text{ad}}{\text{c(c + d)}}.I\right)\left(-x^n \sin(c + dI)x + \int nx^{n-1}\cos(c + dI)x\, dx\right) + C$$

We find the required integral by repeated the integration. By the same method we evaluate the second integral:

$$\int (a + bI)x^n \cos(c + dI)x\, dx$$

Example3.3

Find:

$$\int (1 + I)x \ \sin(2 - 3I)x\, dx$$

Solution:

$$u = (1 + I)x \quad \Longrightarrow \quad du = (1 + I)dx$$

$$dv = \ \sin(2 - 3I)x\, dx \ \Longrightarrow \quad v = \frac{-1}{2-3I} \ \cos(2 - 3I)x$$

Then apply:

$$\int u\, dv = u.v - \int v\, du$$

We get:

$$\int (1 + I)x \sin(2 - 3I)x \ dx = \left(\frac{1}{2} + \frac{-3 - 2}{-2}.I\right)\left(-x \cos(2 - 3I)x + \int \ \cos(2 - 3I)x \ dx\right)$$

$$= \left(\frac{3}{2} + \frac{1}{2}.I\right)\left(-x \cos(2 - 3I)x + \frac{1}{2 - 3I} \sin(2 - 3I)x\right)$$

$$= \left(\frac{3}{2} + \frac{1}{2}.I\right)\left(-x \cos(2 - 3I)x(c + dI)x + \left(\frac{1}{2} - \frac{3}{2}I\right)\sin(2 - 3I)x\right) + C$$

- state4: neutrosophic integration from the form:

$$\int (a + bI)x^n \ln(c + dI)x\, dx\,, \ \ n \neq 1$$

To find the first integral, we do the following:

Put $u = ln(c + dI)x \quad \Longrightarrow \quad du = \frac{1}{x}\, dx$

$dv = (a + bI)x^n\, dx \quad \Longrightarrow \quad v = \frac{a+bI}{n+1} x^{n+1}$

Then apply:

$$\int u\, dv = u.v - \int v\, du$$

We get:

$$\int (a + bI)x^n\, ln(c + dI)x\, dx = \frac{a + bI}{n + 1} x^{n+1} \ln|(c + dI)|x + \int \frac{a + bI}{n + 1} x^n\, dx$$

$$= \frac{a + bI}{n + 1} x^{n+1} \ln(c + dI)x + \frac{a + bI}{(n + 1)^2} x^{n+1} + C$$

Example3.4

Find:

$$\int (7 + 4I)x\;\; ln(6 + 3I)x\, dx$$

Solution:

Put $u = ln(6 + 3I)x \quad \Longrightarrow \quad du = \frac{1}{x}\, dx$

$dv = (7 + 4I)x\, dx \quad \Longrightarrow \quad v = \frac{7+4I}{2} x^2$

Then apply:

$$\int u\, dv = u.v - \int v\, du$$

We get:

$$\int (7 + 4I)x\;\; ln\,(6 + 3I)x\, dx = \frac{7 + 4I}{2} x^2 ln(6 + 3I)x - \frac{7 + 4I}{2} \int x\, dx$$
$$= \frac{7 + 4I}{2} x^2 ln(6 + 3I)x - \frac{7 + 4I}{2}\frac{x^2}{2}$$

$$= \left(\frac{7}{2} + 2I\right)\left(x^2 ln(6 + 3I)x - \frac{x^2}{2}\right) + C$$

Remark:

To find the following integrals:

$$\int (a + bI)x^n\, sin^{-1}(c + dI)x\, dx,\;\; \int (a + bI)x^n\, cos^{-1}(c + dI)x\, dx,$$
$$\int (a + bI)x^n\, tan^{-1}(c + dI)x\, dx \quad n \neq 1$$

We are following the same state4, whereas:

Put

$u = sin^{-1}(c + dI)x \;\; or \;\; cos^{-1}(c + dI)x \;\; or \;\; tan^{-1}(c + dI)x, \quad and \;\; dv = (a + bI)x^n\, dx$

Example3.5

Find:

$$\int (4 + I)x\, tan^{-1}(2 + 5I)x\, dx$$

Solution:

Put

$$u = tan^{-1}(2 + 5I)x \quad \Longrightarrow \quad du = \frac{2 + 5I}{1 + (2 + 5I)^2 x^2}\, dx$$

$$dv = (4+I)x\,dx \qquad \Rightarrow \qquad v = \frac{4+I}{2}x^2$$

We get:

$$\int (4+I)x\,tan^{-1}(2+5I)x\,dx = \frac{4+I}{2}x^2\,tan^{-1}(2+5I)x - \frac{8+27I}{2}\int \frac{x^2}{1+(4+45I)x^2}\,dx$$

$$= \frac{4+I}{2}x^2\,tan^{-1}(2+5I)x - \frac{8+27I}{2}\int \left(\frac{1}{4+45I} - \frac{1}{4+45I}\,\frac{1}{1+(4+45I)x^2}\right)dx$$

$$= \frac{4+I}{2}x^2\,tan^{-1}(2+5I)x - \frac{8+27I}{2}\left(\frac{1}{4+45I}x - \frac{2+5I}{4+45I}\;tan^{-1}(2+5I)x\right) + C$$

$$= \left(2+\frac{1}{2}I\right)x^2\,tan^{-1}(2+5I)x - \frac{8+27I}{2}\left(\left(\frac{1}{4}-\frac{45}{196}\right)x - \left(\frac{2}{4}-\frac{70}{196}I\right)\;tan^{-1}(2+5I)x\right) + C$$

4.Tabular method to find the integrals by parts in the stats 1 and 2

- Differentiate the polynomial function, and we repeat that until we get to zero.
- Integral the second function, and repeat that until we get to the zero that we got from the differentiation.
- Arrange the products of the derivatives in one column, and the products of the integrals in another column corresponding to it.
- Draw an arrow from each entry in the first column to the entry that is one row down in the second column.
- Label the arrows with alternating + and − signs, starting with a +.
- For each arrow, form the product of the expressions at its tip and tail and then multiply that product by + or − in accordance with the sign on the arrow.

Example4.1

Find the following integral by using tabular method:

$$\int \left((3+I)x^2+2x\right)e^{(2-4I)x}\,dx$$

Solution:

derivation	integration
$(+)\ (3+I)x^2+2x$	$e^{(2-4I)x}$
$(-)\ (3+I)x+2$	$\frac{1}{2-4I}e^{(2-4I)x}$
$(+)\ (3+I)$	$\frac{1}{4}e^{(2-4I)x}$
0	$\frac{1}{8-16I}e^{(2-4I)x}$

Hence:

$$\int \left((3+I)x^2+2x\right)e^{(2-4I)x}\,dx$$

$$= \left((3+I)x^2+2x\right)\frac{1}{2-4I}e^{(2-4I)x} - \left((3+I)x+2\right)\frac{1}{2}e^{(2-4I)x} + \frac{3+I}{8-16I}e^{(2-4I)x}$$

$$= \left((3+I)x^2+2x\right)\left(\frac{1}{2}-I\right)e^{(2-4I)x} - \left((3+I)x+2\right)\frac{1}{4}e^{(2-4I)x} + \left(\frac{3}{8}-\frac{5}{8}\right)e^{(2-4I)x} + C$$

Example4.2

Find the following integral by using tabular method:

$$\int \left((3+I)x^2+2x\right) cos(2-4I)x\, dx$$

Solution:

derivation	integration
$(+)\ (3+I)x^2+2x$	$cos(2-4I)x$
$(-)\ (3+I)x+2$	$\frac{1}{2-4I} sin(2-4I)x$
$(+)\ (3+I)$	$\frac{-1}{4} cos(2-4I)x$
0	$\frac{-1}{8-16I} sin(2-4I)x$

Hence:

$$\int \left((3+I)x^2+2x\right) e^{(2-4I)x}\, dx$$
$$= \left((3+I)x^2+2x\right)\frac{1}{2-4I}e^{(2-4I)x} - \left((3+I)x+2\right)\frac{1}{2}e^{(2-4I)x} - \frac{3-I}{8-16I}e^{(2-4I)x}$$

$$= \left((3+I)x^2+2x\right)\left(\frac{1}{2}-I\right) sin(2-4I)x + \left((3+I)x+2\right)\frac{1}{4}cos(2-4I)x - \left(\frac{3}{8}-\frac{5}{8}\right) sin(2-4I)x + C$$

5. Conclusions

Integrals are important in our life, as they facilitate many mathematical operations in our reality, and this is what led us to study the neutrosophic integrals by parts, and the tabular method, which is considered easier than the neutrosophic integrals by parts for some neutrosophic integrals. This paper is considered an introduction to the applications in neutrosophic integrals.

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