Neutrosophic $\mathcal{N}-$structures on strong Sheffer stroke non-associative MV-algebras

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Abstract. The aim of the study is to examine a neutrosophic $\mathcal{N}-$subalgebra, a neutrosophic $\mathcal{N}-$filter, level sets of these neutrosophic $\mathcal{N}-$structures and their properties on a strong Sheffer stroke non-associative MV-algebra. We show that the level set of neutrosophic $\mathcal{N}-$subalgebras on this algebra is its strong Sheffer stroke non-associative MV-subalgebra and vice versa. Then it is proved that the family of all neutrosophic $\mathcal{N}-$subalgebras of a strong Sheffer stroke non-associative MV-algebra forms a complete distributive lattice. By defining a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke non-associative MV-algebra, it is presented that every neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke non-associative MV-algebra is its neutrosophic $\mathcal{N}-$subalgebra but the inverse is generally not true, and some properties

Keywords: strong Sheffer stroke non-associative MV- algebra, filter, neutrosophic $\mathcal{N}-$subalgebra, neutrosophic $\mathcal{N}-$filter.

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1. Introduction

The concept of fuzzy sets which has the truth (t) (membership) function was introduced by L. Zadeh [29]. Since a positive meaning of information is explained by means of fuzzy theory, researchers desire to deal with a negative meaning of information. Thus, Atanassov introduced intuitionistic fuzzy sets [2] which are fuzzy sets with the falsehood (f) (nonmembership) function. Then, Smarandache introduced neutrosophic sets which are intuitionistic fuzzy sets with the indeteminacy/neutrality (i) function [26,27]. Accordingly, neutrosophic sets are defined on three components: $(t, i, f) : (truth,\ indeteminacy,\ falsehood)$ [32]. Specially, many scientists applied neutrosophic sets to the algebraic structures such as BCK/BCI-algebras, BE-algebras and semigroups [3, 4, 11–16, 24, 28, 30, 31].

Sheffer stroke, which is also called the NAND operator in computer science, was firstly introduced by H. M. Sheffer [25]. Since any axioms and formulas in Boolean algebras can be written only by using this operation [17], Sheffer stroke can be applied to many logical algebras such as orthoimplication algebras [1], ortholattices [5], Hilbert algebras [18]- [19], BL-algebras [23], UP-algebras [20] and BG-algebras [21]. Therefore, it is easier to control a logical system consisting of Sheffer stroke itself. Moreover, C. C. Chang introduced MV-algebras which are algebraic counterparts of Lukasiewicz many-valued logic [9, 10]. Then Chajda et al. introduced and improved non-associative MV-algebras (briefly, NMV-algebras) which are generalizations of MV-algebras [7,8]. Also, non-associative MV-algebras with Sheffer stroke [6] and their filters [22] are presented.

Basic definitions and notions about strong Sheffer stroke non-associative MV-algebras, $\mathcal{N}-$functions and neutrosophic $\mathcal{N}-$structures defined by the $\mathcal{N}-$functions on a nonempty universe X are presented. Then the concepts of a neutrosophic $\mathcal{N}-$subalgebra and a $(a, b, c)-$level set defined by $\mathcal{N}-$functions are given on strong Sheffer stroke non-associative MV-algebras. It is shown that the $(a, b, c)-$level set of a neutrosophic $\mathcal{N}-$subalgebra defined by $\mathcal{N}-$functions on strong Sheffer stroke non-associative MV-algebras is its strong Sheffer stroke non-associative MV-subalgebra and the inverse is true. In fact, we state that the family of all neutrosophic $\mathcal{N}-$subalgebras of this algebraic structure forms a complete distributive lattice. Some properties of neutrosophic $\mathcal{N}-$subalgebras of strong Sheffer stroke non-associative MV-algebras are analyzed. Also, it is investigated the images of the sequence under $\mathcal{N}-$functions on a strong Sheffer stroke non-associative MV-algebra. Besides, we examine that the case which $\mathcal{N}-$functions defining a neutrosophic $\mathcal{N}-$subalgebra of a strong Sheffer stroke non-associative MV-algebra are constant. After defining a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke non-associative MV-algebra by $\mathcal{N}-$functions, some features of $\mathcal{N}-$functions defining the neutrosophic $\mathcal{N}-$filter are studied. We propound that $(a, b, c)-$level set of a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke non-associative MV-algebra is its filter and that the subsets defined by $\mathcal{N}-$functions on a strong Sheffer stroke non-associative MV-algebra must be its filters so that a neutrosophic $\mathcal{N}-$structure on this algebra is a neutrosophic $\mathcal{N}-$filter. It is stated that every neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke non-associative MV-algebra is its neutrosophic $\mathcal{N}-$subalgebra while the inverse is usually not valid. In addition, new subsets of a strong Sheffer stroke non-associative MV-algebra are described by the $\mathcal{N}-$functions and certain elements in the algebra. We show that these subsets are filters of a strong Sheffer stroke non-associative MV-algebra for its neutrosophic $\mathcal{N}-$filter but the inverse does not mostly hold.

2. Preliminaries

In this section, we give basic definitions and notions about strong Sheffer stroke non-associative MV-algebras (briefly, strong Sheffer stroke NMV-algebras) and neutrosophic $\mathcal{N}-$structures.

Definition 2.1. [5] Let $\mathcal{A} = \langle A, | \rangle$ be a groupoid. The operation $|$ is said to be a *Sheffer stroke operation* if it satisfies the following conditions:

$(S1)$ $x|y = y|x$,
$(S2)$ $(x|x)|(x|y) = x$,
$(S3)$ $x|((y|z)|(y|z)) = ((x|y)|(x|y))|z$,
$(S4)$ $(x|((x|x)|(y|y)))|(x|((x|x)|(y|y))) = x$.

Definition 2.2. [6] A strong Sheffer stroke NMV-algebra is an algebra $(A, |, 1)$ of type $(2, 0)$ satisfying the identities for all $x, y, z \in A$:

$(n1)$ $x|y \approx y|x$,
$(n2)$ $x|0 \approx 1$,
$(n3)$ $(x|1)|1 \approx x$,
$(n4)$ $((x|1)|y)|y \approx ((y|1)|x)|x$,
$(n5)$ $(x|1)|((x|y)|1) \approx 1$,
$(n6)$ $x|(((((x|y)|y)|z)|z)|1) \approx 1$,

where 0 denotes the algebraic constant $1|1$.

Proposition 2.3. *[22] Let $(A, |, 1)$ be a strong Sheffer stroke NMV-algebra. Then the binary relation $\leq$ defined by*

$$x \leq y \text{ if and only if } x|(y|1) \approx 1$$

is a partial order on A. Hence, $(A, \leq)$ is a poset with the least element 0 and the greatest element 1.

Lemma 2.4. *[22] In a strong Sheffer stroke NMV-algebra $(A, |, 1)$, the following properties hold for all $x, y, z \in A$:*

(i) $x|(x|1) \approx 1$,
(ii) $x \leq y \Leftrightarrow y|1 \leq x|1$,
(iii) $y \leq x|(y|1)$,
(iv) $y|1 \leq x|y$,
(v) $x \leq (x|y)|y$,
(vi) $x \leq (((x|y)|y)|z)|z$,
(vii) $((x|y)|y)|y \approx x|y$,

$(viii)$ $x|1 \approx x|x$,
(ix) $x|(x|x) \approx 1$,
(x) $1|(x|x) \approx x$,
(xi) $x \leq y \Rightarrow y|z \leq x|z$,
(xii) $x|(y|1) \leq (y|(z|1))|((x|(z|1))|1)$,
$(xiii)$ $x|(y|1) \leq (z|(x|1))|((z|(y|1))|1)$,
(xiv) $x \leq y$ *and* $z \leq t$ *imply* $y|t \leq x|z$.

Definition 2.5. [22] A nonempty subset $F \subseteq A$ is called a filter of A if it satisfies the following properties:
$(S_f - 1)$ $1 \in F$,
$(S_f - 2)$ For all $x, y \in A$, $x|(y|1) \in F$ and $x \in F$ imply $y \in F$.

Lemma 2.6. *[22] A nonempty subset $F \subseteq A$ is a filter of A if and only if $1 \in F$ and $x \leq y$ and $x \in F$ imply $y \in F$.*

Definition 2.7. [11] $\mathcal{F}(X, [-1, 0])$ denotes the collection of functions from a set X to $[-1, 0]$ and a element of $\mathcal{F}(X, [-1, 0])$ is called a negative-valued function from X to $[-1, 0]$ (briefly, $\mathcal{N}-$function on X). An $\mathcal{N}-$structure refers to an ordered pair (X, f) of a set X and an $\mathcal{N}-$function f on X.

Definition 2.8. [16] A neutrosophic $\mathcal{N}-$structure over a nonempty universe X is defined by

$$X_N := \frac{X}{(T_N, I_N, F_N)} = \{\frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X\},$$

where T_N, I_N and F_N are $\mathcal{N}-$function on X, called the negative truth membership function, the negative indeterminacy membership function and the negative falsity membership function, respectively.

Every neutrosophic $\mathcal{N}-$structure X_N over X satisfies the condition

$$(\forall x \in X)(-3 \leq T_N(x) + I_N(x) + F_N(x) \leq 0).$$

3. Neutrosophic $\mathcal{N}-$structures

In this section, we give neutrosophic $\mathcal{N}-$subalgebras and neutrosophic $\mathcal{N}-$filters on strong Sheffer stroke NMV-algebras. Unless indicated otherwise, A states a strong Sheffer stroke NMV-algebra.

Definition 3.1. A neutrosophic $\mathcal{N}-$subalgebra A_N on a strong Sheffer stroke NMV-algebra A is a neutrosophic $\mathcal{N}-$structure of A satisfying the conditions

$$\min\{T_N(x), T_N(y)\} \leq T_N(x|(y|1)),$$

$$\max\{I_N(x), I_N(y)\} \geq I_N(x|(y|1))$$

and

$$\max\{F_N(x), F_N(y)\} \geq F_N(x|(y|1)),$$

for all $x, y \in A$.

Example 3.2. Consider a strong Sheffer stroke NMV-algebra A in which the set $A = \{0, u, v, 1\}$ and the Sheffer operation $|$ on A has the following Cayley table:

Table 1

$\vert$	0	u	v	1
0	1	1	1	1
u	1	v	1	v
v	1	1	u	u
1	1	v	u	0

A neutrosophic $\mathcal{N}-$structure

$$A_N = \{\frac{0}{(-0.79, -0.001, 0)}, \frac{u}{(-0.68, -0.72, -0.4)}, \frac{v}{(-0.68, -0.72, -0.4)}, \frac{1}{(0, -0.88, -1)}\}$$

on A is a neutrosophic $\mathcal{N}-$subalgebra of A.

Definition 3.3. Let A_N be a neutrosophic $\mathcal{N}-$structure on a strong Sheffer stroke NMV-algebra A and a, b, c be any elements of $[-1, 0]$ such that $-3 \leq a + b + c \leq 0$. For

$$T_N^a := \{x \in A : T_N(x) \geq a\},$$

$$I_N^b := \{x \in A : I_N(x) \leq b\}$$

and

$$F_N^c := \{x \in A : F_N(x) \leq c\},$$

the set

$$A_N(a, b, c) := \{x \in H : T_N(x) \geq a, I_N(x) \leq b \text{ and } F_N(x) \leq c\}$$

is called the $(a, b, c)-$level set of A_N. Moreover,

$$A_N(a, b, c) = T_N^a \cap I_N^b \cap F_N^c.$$

Definition 3.4. [22] A subset B of a strong Sheffer stroke NMV-algebra A is called a strong Sheffer stroke NMV-subalgebra of A if 1 of A is in B and $(B, |, 1)$ forms a strong Sheffer stroke NMV-algebra. Clearly, A itself and $\{1\}$ are strong Sheffer stroke NMV-subalgebras of A.

Lemma 3.5. *Let B be a nonempty subset of a strong Sheffer stroke NMV-algebra A. Then B is a strong Sheffer stroke NMV-subalgebra of A if and only if $x|(y|1) \in B$, for all $x, y \in B$.*

Proof. Let B be a nonempty subset of a strong Sheffer stroke NMV-algebra A such that $x|(y|1) \in B$, for all $x, y \in B$. Then $1 \approx x|(x|1) \in B$ from Lemma 2.4 (i). Since $B \subseteq A$, $(B, |, 1)$ satisfies (n1)-(n6), for all $x, y, z \in B$. Thus, $(B, |, 1)$ is a strong Sheffer stroke NMV-subalgebra A.

Conversely, let B be a strong Sheffer stroke NMV-subalgebra of A. Since B states a strong Sheffer stroke NMV-algebra, it must be closed under the Sheffer operation $|$, that is, $x|y \in B$, for all $x, y \in B$. Hence, $x|(y|1) \in B$, for all $x, y \in B$. □

Theorem 3.6. *Let A_N be a neutrosophic $\mathcal{N}-$structure on a strong Sheffer stroke NMV-algebra A and a, b, c be any elements in $[-1, 0]$ such that $-3 \leq a + b + c \leq 0$. If A_N is a neutrosophic $\mathcal{N}-$subalgebra of A, then the nonempty level set $A_N(a, b, c)$ of A_N is a subalgebra of A.*

Proof. Let A_N be a neutrosophic $\mathcal{N}-$subalgebra of A and x, y be any elements in $A_N(a, b, c)$, for $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Then $T_N(x) \geq a, I_N(x) \leq b, F_N(x) \leq c$, $T_N(y) \geq a, I_N(y) \leq b$ and $F_N(y) \leq c$. Since

$$T_N(x|(y|1)) \geq \min\{T_N(x), T_N(y)\} \geq a,$$

$$I_N(x|(y|1))| \leq \max\{I_N(x), I_N(y)\} \leq b$$

and

$$F_N(x|(y|1)) \leq \max\{F_N(x), F_N(y)\} \leq c,$$

for all $x, y \in A$, it follows that $x|(y|1) \in T_N^a$, $x|(y|1) \in I_N^b$ and $x|(y|1) \in F_N^c$, which implies that $x|(y|1) \in T_N^a \cap I_N^b \cap F_N^c = A_N(a, b, c)$. Thus, $A_N(a, b, c)$ is a subalgebra of A by Lemma 3.5. □

Theorem 3.7. *Let A_N be a neutrosophic $\mathcal{N}-$structure on a strong Sheffer stroke NMV-algebra A and T_N^a, I_N^b and F_N^c be subalgebras of A, for all $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Then A_N is a neutrosophic $\mathcal{N}-$subalgebra of A.*

Proof. Let T_N^a, I_N^b and F_N^c be subalgebras of A, for all $a, b, c \in [-1, 0]$ with $-3 \leq a+b+c \leq 0$. Assume that x and y are any elements in A such that $u_1 = T_N(x|(y|1)) < \min\{T_N(x), T_N(y)\} = v_1$. If $a_0 = \frac{1}{2}(u_1 + v_1) \in [-1, 0)$, then $u_1 < a_0 < v_1$. So, $x, y \in T_N^{a_0}$ while $x|(y|1) \notin T_N^{a_0}$, which is a contradiction. Thus, $\min\{T_N(x), T_N(y)\} \leq T_N(x|(y|1))$, for all $x, y \in A$.

Suppose that x and y are any elements in A such that $u_2 = \max\{I_N(x), I_N(y)\} < I_N(x|(y|1)) = v_2$. If $b_0 = \frac{1}{2}(u_2 + v_2) \in [-1, 0)$, then $u_2 < b_0 < v_2$, which implies that $x, y \in I_N^{b_0}$ but $x|(y|1) \notin I_N^{b_0}$. This is a contradiction. Thus, $I_N(x|(y|1)) \leq \max\{I_N(x), I_N(y)\}$, for all $x, y \in A$.

Assume that x and y are any elements in A such that $v_3 = F_N(x|(y|1)) > \max\{F_N(x), F_N(y)\} = u_3$. If $c_0 = \frac{1}{2}(u_3 + v_3) \in [-1, 0)$, then $u_3 < c_0 < v_3$. Thus, $x, y \in F_N^{c_0}$ but $x|(y|1) \notin F_N^{c_0}$, which is a contradiction. Thereby, $\max\{F_N(x), F_N(y)\} \geq F_N(x|(y|1))$, for all $x, y \in A$.

Therefore, A_N is a neutrosophic $\mathcal{N}-$subalgebra of A. □

Theorem 3.8. *Let $\{A_{N_i} : i \in \mathbb{N}\}$ be a family of all neutrosophic $\mathcal{N}-$subalgebras of a strong Sheffer stroke NMV-algebra A. Then $\{A_{N_i} : i \in \mathbb{N}\}$ forms a complete distributive lattice.*

Proof. Let B be a nonempty subset of $\{A_{N_i} : i \in \mathbb{N}\}$. Since A_{N_i} is a neutrosophic $\mathcal{N}-$subalgebra of A, for all $A_{N_i} \in B$, it satisfies

$$\min\{T_N(x), T_N(y)\} \leq T_N(x|(y|1)),$$

$$I_N(x|(y|1)) \leq \max\{I_N(x), I_N(y)\}$$

and

$$F_N(x|(y|1)) \leq \max\{F_N(x), F_N(y)\},$$

for all $x, y \in A$. Then $\bigcap B$ satisfies these inequalities. Thus, $\bigcap B$ is a neutrosophic $\mathcal{N}-$subalgebra of A.

Let C be a family of all neutrosophic $\mathcal{N}-$subalgebras of A containing $\bigcup\{A_{N_i} : i \in \mathbb{N}\}$. Then $\bigcap C$ is also a neutrosophic $\mathcal{N}-$subalgebra of A.

If $\bigwedge_{i\in\mathbb{N}} A_{N_i} = \bigcap_{i\in\mathbb{N}} A_{N_i}$ and $\bigvee_{i\in\mathbb{N}} A_{N_i} = \bigcap C$, then $(\{A_{N_i} : i \in \mathbb{N}\}, \bigvee, \bigwedge)$ forms a complete lattice. Moreover, it is distibutive by the definitions of $\bigvee$ and $\bigwedge$. □

Lemma 3.9. *If a neutrosophic $\mathcal{N}-$structure A_N on a strong Sheffer stroke NMV-algebra A is a neutrosophic $\mathcal{N}-$subalgebra of A, then $T_N(x) \leq T_N(1)$, $I_N(x) \geq I_N(1)$ and $F_N(x) \geq F_N(1)$, for all $x \in A$.*

Proof. Let a neutrosophic $\mathcal{N}-$structure A_N on a strong Sheffer stroke NMV-algebra A be a neutrosophic $\mathcal{N}-$subalgebra of A. By substituting $[y := x]$ in the inequalities in Definition 3.1, it is obtained from Lemma 2.4 (i) that

$$T_N(x) = \min\{T_N(x), T_N(x)\} \leq T_N(x|(x|1)) = T_N(1),$$

$$I_N(1) = I_N(x|(x|1)) \leq \max\{I_N(x), I_N(x)\} = I_N(x)$$

and

$$F_N(1) = F_N(x|(x|1)) \leq \max\{F_N(x), F_N(x)\} = F_N(x),$$

for all $x \in H$. □

The inverse of Lemma 3.9 does not hold in general.

Example 3.10. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Then a neutrosophic $\mathcal{N}-$structure

$$A_N = \{\frac{0}{(-0.8,-0.7,-0.02)}, \frac{u}{(-0.5,-0.4,-0.3)}, \frac{v}{(-0.2,-0.1,-0.11)}, \frac{1}{(0,-1,-0.6)}\}$$

on A is not a neutrosophic $\mathcal{N}-$subalgebra of A since

$$\max\{I_N(u), I_N(0)\} = \max\{-0.4,-0.7\} = -0.4 < -0.1 = I_N(v) = I_N(u|(0|1)).$$

Lemma 3.11. *Let A_N be a neutrosophic $\mathcal{N}-$subalgebra of a strong Sheffer stroke NMV-algebra A. If there exists a sequence $\{a_n\}$ on A such that*

$$\lim_{n\longrightarrow\infty} T_N(a_n) = 0,\ \lim_{n\longrightarrow\infty} I_N(a_n) = -1 \text{ and } \lim_{n\longrightarrow\infty} F_N(a_n) = -1,$$

then

$$T_N(1) = 0,\ I_N(1) = -1 \text{ and } F_N(1) = -1.$$

Proof. Let A_N be a neutrosophic $\mathcal{N}-$subalgebra of a strong Sheffer stroke NMV-algebra A. Suppose that there exists a sequence $\{a_n\}$ on A such that $\lim_{n\longrightarrow\infty} T_N(a_n) = 0$ and $\lim_{n\longrightarrow\infty} I_N(a_n) = -1 = \lim_{n\longrightarrow\infty} F_N(a_n)$. Since $T_N(a_n) \leq T_N(1)$, $I_N(a_n) \geq I_N(1)$ and $F_N(a_n) \geq F_N(1)$, for every $n \in \mathbb{N}$ from Lemma 3.9, it is obtained that

$$0 = \lim_{n\longrightarrow\infty} T_N(a_n) \leq \lim_{n\longrightarrow\infty} T_N(1) = T_N(1) \leq 0,$$
$$-1 \leq I_N(1) = \lim_{n\longrightarrow\infty} I_N(1) \leq \lim_{n\longrightarrow\infty} I_N(a_n) = -1$$

and

$$-1 \leq F_N(1) = \lim_{n\longrightarrow\infty} F_N(1) \leq \lim_{n\longrightarrow\infty} F_N(a_n) = -1.$$

Thus, $T_N(1) = 0$ and $I_N(1) = F_N(1) = -1$. □

Lemma 3.12. *A neutrosophic $\mathcal{N}-$subalgebra A_N of a strong Sheffer stroke NMV-algebra A satisfies $T_N(x) \leq T_N(x|(y|1))$, $I_N(x) \geq I_N(x|(y|1))$ and $F_N(x) \geq F_N(x|(y|1))$, for all $x, y \in A$ if and only if T_N, I_N and F_N are constant.*

Proof. Let A_N be a neutrosophic $\mathcal{N}-$subalgebra of a strong Sheffer stroke NMV-algebra A satisfying $T_N(x) \leq T_N(x|(y|1))$, $I_N(x) \geq I_N(x|(y|1))$ and $F_N(x) \geq F_N(x|(y|1))$, for all $x, y \in A$. Since $T_N(1) \leq T_N(1|(x|1)) = T_N((x|1)|1) = T_N(x)$, $I_N(1) \geq I_N(1|(x|1)) = I_N((x|1)|1) = I_N(x)$ and $F_N(1) \geq F_N(1|(x|1)) = F_N((x|1)|1) = F_N(x)$ from (n1) and (n3), it follows from Lemma 3.9 that $T_N(x) = T_N(1)$, $I_N(x) = I_N(1)$ and $F_N(x) = F_N(1)$, for all $x \in A$.

Conversely, every neutrosophic $\mathcal{N}-$subalgebra A_N of a strong Sheffer stroke NMV-algebra A satisfies $T_N(x) \leq T_N(x|(y|1))$, $I_N(x) \geq I_N(x|(y|1))$ and $F_N(x) \geq F_N(x|(y|1))$, for all $x, y \in A$ because T_N, I_N and F_N are constant. □

Definition 3.13. A neutrosophic $\mathcal{N}-$structure A_N on a strong Sheffer stroke NMV-algebra A is called a neutrosophic $\mathcal{N}-$filter of A if

$$\min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y) \leq T_N(1),$$

$$I_N(1) \leq I_N(y) \leq \max\{I_N(x|(y|1)), I_N(x)\}$$

and

$$F_N(1) \leq F_N(y) \leq \max\{F_N(x|(y|1)), F_N(x)\},$$

for all $x, y \in A$.

Example 3.14. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Then a neutrosophic $\mathcal{N}-$structure

$$A_N = \{\frac{0}{(-0.23, -0.3, -0.01)}, \frac{u}{(-0.02, -0.98, -0.11)},$$
$$\frac{v}{(-0.23, -0.3, -0.01)}, \frac{1}{(-0.02, -0.98, -0.11)}\}$$

on A is a neutrosophic $\mathcal{N}-$filter of A.

Lemma 3.15. *Every a neutrosophic $\mathcal{N}-$filter A_N of a strong Sheffer stroke NMV-algebra A satisfies that $x \leq y$ implies $T_N(x) \leq T_N(y)$, $I_N(x) \geq I_N(y)$ and $F_N(x) \geq F_N(y)$, for all $x, y \in A$.*

Proof. Let A_N be a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke NMV-algebra A and $x \leq y$. Then $x|(y|1) \approx 1$ from Proposition 2.3. Thus,

$$T_N(x) = \min\{T_N(1), T_N(x)\} = \min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y),$$

$$I_N(x) = \max\{I_N(1), I_N(x)\} = \max\{I_N(x|(y|1)), I_N(x)\} \geq I_N(y)$$

and

$$F_N(x) = \max\{F_N(1), F_N(x)\} = \max\{F_N(x|(y|1)), F_N(x)\} \geq F_N(y),$$

for any $x, y \in A$. □

The inverse of Lemma 3.15 is generally not true.

Example 3.16. Consider the neutrosophic $\mathcal{N}-$filter of A in Example 3.14. Then $v \nleq u$ when $-0.98 = I_N(u) \leq I_N(v) = -0.3$.

Lemma 3.17. *Let A_N be a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke NMV-algebra A. Then*

$$T_N((x|(y|1))|(z|1)) \leq T_N((x|(z|1))|((y|(z|1))|1)),$$

$$I_N((x|(y|1))|(z|1)) \geq I_N((x|(z|1))|((y|(z|1))|1)), \tag{1}$$

and

$$F_N((x|(y|1))|(z|1)) \geq F_N((x|(z|1))|((y|(z|1))|1)),$$

for all $x, y, z \in A$.

Proof. Let A_N be a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke NMV-algebra A. Since $(x|(y|1))|(z|1) \leq y|(z|1) \leq (x|(z|1))|((y|(z|1))|1)$ from Lemma 2.4 (iii) and (xi), it follows from Lemma 3.15 that

$$T_N((x|(y|1))|(z|1)) \leq T_N((x|(z|1))|((y|(z|1))|1)),$$

$$I_N((x|(y|1))|(z|1)) \geq I_N((x|(z|1))|((y|(z|1))|1))$$

and

$$F_N((x|(y|1))|(z|1)) \geq F_N((x|(z|1))|((y|(z|1))|1)),$$

for all $x, y, z \in A$. □

The inverse of Lemma 3.17 does not usually hold.

Example 3.18. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Then a neutrosophic $\mathcal{N}-$structure

$$A_N = \{\frac{0}{(-0.69, -0.12, 0)}, \frac{u}{(-0.58, -0.87, -0.22)}, \frac{v}{(-0.58, -0.87, -0.22)}, \frac{1}{(-0.14, -0.93, 0.96)}\}$$

on A satisfies the condition (1) in Lemma 3.17 but it is not a neutrosophic $\mathcal{N}-$filter of A since

$$\min\{T_N(u|(0|1)), T_N(u)\} = \min\{T_N(v), T_N(u)\} = -0.58 > -0.69 = T_N(0).$$

Lemma 3.19. *Let A_N be a neutrosophic $\mathcal{N}-$structure on a strong Sheffer stroke NMV-algebra A and a, b, c be any elements of $[-1, 0]$ with $-3 \leq a + b + c \leq 0$. If A_N is a neutrosophic $\mathcal{N}-$filter of A, then the nonempty subset $A_N(a, b, c)$ is a filter of A.*

Proof. Let A_N be a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke NMV-algebra A and $A_N(a, b, c) \neq \emptyset$ for $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Since $a \leq T_N(x) \leq T_N(1), b \geq I_N(x) \geq I_N(1)$ and $c \geq F_N(x) \geq F_N(1)$, for all $x \in A_N(a, b, c)$, we have $1 \in A_N(a, b, c)$. Let $x|(y|1), x \in A_N(a, b, c)$. Then $a \leq T_N(x)$, $I_N(x) \leq b$, $F_N(x) \leq c$, $a \leq T_N(x|(y|1))$, $I_N(x|(y|1)) \leq b$ and $F_N(x|(y|1)) \leq c$. Since

$$a \leq \min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y),$$

$$I_N(y) \leq \max\{I_N(x|(y|1)), I_N(x)\} \leq b$$

and

$$F_N(y) \leq \max\{F_N(x|(y|1)), F_N(x)\} \leq c,$$

for all $x, y \in A$, it is obtained $y \in A_N(a, b, c)$. Hence, $A_N(a, b, c)$ is a filter of A. □

Theorem 3.20. *Let A_N be a neutrosophic $\mathcal{N}-$structure on a strong Sheffer stroke NMV-algebra A and T_N^a, I_N^b, F_N^c be filters of A, for all $a, b, c \in [-1, 0]$ with $-3 \leq a+b+c \leq 0$. Then A_N is a neutrosophic $\mathcal{N}-$filter of A.*

Proof. Let A_N be a neutrosophic $\mathcal{N}-$structure on a strong Sheffer stroke NMV-algebra A and T_N^a, I_N^b, F_N^c be filters of A, for all $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Assume that $T_N(1) < T_N(x_0)$, $I_N(y_0) < I_N(1)$ and $F_N(z_0) < F_N(1)$. If $a_0 = \frac{1}{2}(T_N(1) + T_N(x_0))$, $b_0 = \frac{1}{2}(I_N(1) + I_N(y_0))$ and $c_0 = \frac{1}{2}(F_N(1) + F_N(z_0))$ in $[-1, 0)$, then $T_N(1) < a_0 < T_N(x_0)$, $I_N(1) > b_0 > I_N(y_0)$ and $F_N(1) > c_0 > F_N(z_0)$. Thus, $1 \notin T_N^{a_0}, 1 \notin I_N^{b_0}$ and $1 \notin F_N^{c_0}$, which contradict with $(S_f - 1)$. Hence, $T_N(x) \leq T_N(1)$, $I_N(x) \geq I_N(1)$ and $F_N(x) \geq F_N(1)$, for all $x \in A$. Suppose that x_1, x_2, x_3, y_1, y_2 and y_3 are any elements of A such that

$$v_1 = T_N(y_1) < \min\{T_N(x_1|(y_1|1)), T_N(x_1)\} = u_1,$$

$$u_2 = \max\{I_N(x_2|(y_2|1)), I_N(x_2)\} < I_N(y_2) = v_2,$$

and

$$u_3 = \max\{F_N(x_3|(y_3|1)), F_N(x_3)\} < F_N(y_3) = v_3.$$

If $a' = \frac{1}{2}(u_1 + v_1)$, $b' = \frac{1}{2}(u_2 + v_2)$ and $c' = \frac{1}{2}(u_3 + v_3)$ in $[-1, 0)$, then $v_1 < a' < u_1$, $u_2 < b' < v_2$ and $u_3 < c' < v_3$. So, $y_1 \notin T_N^{a'}$, $y_2 \notin I_N^{b'}$ and $y_3 \notin F_N^{c'}$ when $x_1|(y_1|1), x_1 \in T_N^{a'}$, $x_2|(y_2|1), x_2 \in I_N^{b'}$ and $x_3|(y_3|1), x_3 \in F_N^{c'}$. This is a contradiction. Thereby,

$$\min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y),$$

$$I_N(y) \leq \max\{I_N(x|(y|1)), I_N(x)\}$$

and

$$F_N(y) \leq \max\{F_N(x|(y|1)), F_N(x)\},$$

for all $x, y \in A$. Therefore, A_N is a neutrosophic $\mathcal{N}-$filter of A. □

Lemma 3.21. *Let A_N be a neutrosophic $\mathcal{N}-$structure on a strong Sheffer stroke NMV-algebra A. Then A_N is a neutrosophic $\mathcal{N}-$filter of A if and only if $z \leq y|(x|1)$ implies*

$$\min\{T_N(y), T_N(z)\} \leq T_N(x),$$

$$I_N(x) \leq \max\{I_N(y), I_N(z)\}$$

and

$$F_N(x) \leq \max\{F_N(y), F_N(z)\},$$

for all $x, y, z \in A$.

Proof. Let A_N be a neutrosophic $\mathcal{N}-$filter of A and x, y and z be any elements of A such that $z \leq y|(x|1)$. Since $T_N(z) \leq T_N(y|(x|1))$, $I_N(z) \geq I_N(y|(x|1))$ and $F_N(z) \geq F_N(y|(x|1))$ from Lemma 3.15, it follows that

$$\min\{T_N(y), T_N(z)\} \leq \min\{T_N(y|(x|1)), T_N(y)\} \leq T_N(x),$$

$$I_N(x) \leq \max\{I_N(y|(x|1)), I_N(y)\} \leq \max\{I_N(y), I_N(z)\}$$

and

$$F_N(x) \leq \max\{F_N(y|(x|1)), F_N(y)\} \leq \max\{F_N(y), F_N(z)\},$$

for all $x, y, z \in A$.

Conversely, suppose that A_N is a neutrosophic $\mathcal{N}-$structure on A such that $z \leq y|(x|1)$ implies

$$\min\{T_N(y), T_N(z)\} \leq T_N(x),$$

$$I_N(x) \leq \max\{I_N(y), I_N(z)\}$$

and

$$F_N(x) \leq \max\{F_N(y), F_N(z)\},$$

for all $x, y, z \in A$. Since $x \leq 1 \approx x|0 \approx x|(1|1)$ from (n2), it is obtained that $T_N(x) \leq T_N(1)$, $I_N(1) \leq I_N(x)$ and $F_N(1) \leq F_N(x)$, for all $x \in A$. Since $x \leq (x|(y|1))|(y|1)$ from Lemma 2.4 (v), we have

$$\min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y),$$

$$I_N(y) \leq \max\{I_N(x|(y|1)), I_N(x)\}$$

and

$$F_N(y) \leq \max\{F_N(x|(y|1)), F_N(x)\},$$

for all $x, y \in A$. Hence, A_N is a neutrosophic $\mathcal{N}-$filter of A. □

Theorem 3.22. *Every neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke NMV-algebra A is a neutrosophic $\mathcal{N}-$subalgebra of A.*

Proof. Let A_N be a neutrosophic $\mathcal{N}-$filter of A. Since

$$\begin{aligned} \min\{T_N(x), T_N(y)\} &\leq \min\{T_N(1), T_N(y)\} \\ &= \min\{T_N(((y|1)|1)|(((y|1)|x)|1)), T_N(y)\} \\ &= \min\{T_N(y|((x|(y|1))|1)), T_N(y)\} \\ &\leq T_N(x|(y|1)), \end{aligned}$$

and similarly,

$$I_N(x|(y|1)) \leq \max\{I_N(x), I_N(y)\}$$

and

$$F_N(x|(y|1)) \leq \max\{F_N(x), F_N(y)\},$$

from (n1), (n3) and (n5), it follows that A_N is a neutrosophic $\mathcal{N}-$subalgebra of A. □

The inverse of Theorem 3.22 does not usually hold.

Example 3.23. The neutrosophic $\mathcal{N}-$subalgebra A_N of A in Example 3.2. Then it is not a neutrosophic $\mathcal{N}-$filter of A since $\min\{T_N(u|(0|1)), T_N(u)\} = \min\{T_N(v), T_N(u)\} = -0.68 > -0.79 = T_N(0)$.

Definition 3.24. Let A be a strong Sheffer stroke NMV-algebra. Define

$$A_N^{x_t} := \{x \in A : T_N(x_t) \leq T_N(x)\},$$

$$A_N^{x_i} := \{x \in A : I_N(x) \leq I_N(x_i)\}$$

and

$$A_N^{x_f} := \{x \in A : F_N(x) \leq F_N(x_f)\},$$

for all $x_t, x_i, x_f \in A$. Obviously, $x_t \in A_N^{x_t}, x_i \in A_N^{x_i}$ and $x_f \in A_N^{x_f}$.

Example 3.25. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Let $T_N(0) = -0.113, T_N(u) = -0.12, T_N(v) = -0.13, T_N(1) = 0, I_N(0) = -0.21, I_N(u) = -0.22, I_N(v) = -0.23, I_N(1) = -1, F_N(0) = -0.31, F_N(u) = -0.32, F_N(v) = -0.33, F_N(1) = -0.34, x_t = u, x_i = v$ and $x_f = 0$. Then

$$A_N^{x_t} = \{x \in A : T_N(u) \leq T_N(x)\} = \{0, u, 1\},$$

$$A_N^{x_i} = \{x \in A : I_N(x) \leq I_N(v)\} = \{v, 1\}$$

and

$$A_N^{x_f} = \{x \in A : F_N(x) \leq F_N(0)\} = A.$$

Theorem 3.26. *Let x_t, x_i and x_f be any elements of a strong Sheffer stroke NMV-algebra A. If A_N is a neutrosophic $\mathcal{N}-$filter of A, then $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ are filters of A.*

Proof. Let A_N be a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke NMV-algebra A. Since $T_N(x_t) \leq T_N(1)$, $I_N(1) \leq I_N(x_i)$ and $F_N(1) \leq F_N(x_f)$, for any $x_t, x_i, x_f \in A$, we have $1 \in A_N^{x_t}$, $1 \in A_N^{x_i}$ and $1 \in A_N^{x_f}$. Let $x_1|(y_1|1), x_1 \in A_N^{x_t}$, $x_2|(y_2|1), x_2 \in A_N^{x_i}$ and $x_3|(y_3|1), x_3 \in A_N^{x_f}$. Then $T_N(x_t) \leq T_N(x_1|(y_1|1))$, $T_N(x_t) \leq T_N(x_1)$, $I_N(x_2|(y_2|1)) \leq I_N(x_i)$, $I_N(x_2) \leq I_N(x_i)$ and $F_N(x_3|(y_3|1)) \leq F_N(x_f)$, $F_N(x_3) \leq F_N(x_f)$. Since

$$T_N(x_t) \leq \min\{T_N(x_1|(y_1|1)), T_N(x_1)\} \leq T_N(y_1),$$

$$I_N(y_2) \leq \max\{I_N(x_2|(y_2|1)), I_N(x_2)\} \leq I_N(x_i)$$

and

$$F_N(y_3) \leq \max\{F_N(x_3|(y_3|1)), F_N(x_3)\} \leq F_N(x_f),$$

we get $y_1 \in A_N^{x_t}$, $y_2 \in A_N^{x_i}$ and $y_3 \in A_N^{x_f}$. Thus, $A_N^{x_t}$, $A_N^{x_i}$ and $A_N^{x_f}$ are filters of A. □

Example 3.27. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. For a neutrosophic $\mathcal{N}-$filter

$$A_N = \{\frac{0}{(-0.32, -0.29, -0.07)}, \frac{u}{(-0.32, -0.29, -0.07)}, \frac{v}{(-0.1, -0.78, -0.17)}, \frac{1}{(-0.1, -0.78, -0.17)}\}$$

of A, $x_t = u$, $x_i = v$ and $x_f = 1 \in A$, the subsets

$$A_N^{x_t} = \{x \in A : T_N(u) \leq T_N(x)\} = A,$$

$$A_N^{x_i} = \{x \in A : I_N(x) \leq I_N(v)\} = \{v, 1\}$$

and

$$A_N^{x_f} = \{x \in A : F_N(x) \leq F_N(1)\} = \{v, 1\}$$

of A are filterss of A.

Theorem 3.28. *Let x_t, x_i and x_f be any elements of a strong Sheffer stroke NMV-algebra A and A_N be a neutrosophic $\mathcal{N}-$structure on A.*

(a) *If $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ are filters of A, then*

$$T_N(x) \leq \min\{T_N(y|(z|1)), T_N(y)\} \Rightarrow T_N(x) \leq T_N(z),$$

$$I_N(x) \geq \max\{I_N(y|(z|1)), I_N(y)\} \Rightarrow I_N(x) \geq I_N(z) \tag{2}$$

and

$$F_N(x) \geq \max\{F_N(y|(z|1)), F_N(y)\} \Rightarrow F_N(x) \geq F_N(z),$$

for all $x, y, z \in A$.

(b) *If A_N satisfies the condition (3.2) and*

$$T_N(x) \leq T_N(1), \quad I_N(1) \leq I_N(x) \quad and \quad F_N(1) \leq F_N(x), \quad for\ all\ x \in A, \tag{3}$$

then $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ are filters of A, for all $x_t \in T_N^{-1},\ x_i \in I_N^{-1}$ and $x_f \in F_N^{-1}$.

Proof. Let A_N be a neutrosophic $\mathcal{N}-$structure on a strong Sheffer stroke NMV-algebra A.

(a) Let $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ be filters of A, for any $x_t, x_i, x_f \in A$, and x, y, z be any elements of A such that $T_N(x) \leq \min\{T_N(y|(z|1)), T_N(y)\}$, $I_N(x) \geq \max\{I_N(y|(z|1)), I_N(y)\}$ and $F_N(x) \geq \max\{F_N(y|(z|1)), F_N(y)\}$. Since $y|(z|1), y \in A_N^{x_t}$, $y|(z|1), y \in A_N^{x_i}$ and $y|(z|1), y \in A_N^{x_f}$, where $x_t = x_i = x_f = x$, it follows from $(S_f - 2)$ that $z \in A_N^{x_t}$, $z \in A_N^{x_i}$ and $z \in A_N^{x_f}$, where $x_t = x_i = x_f = x$. Thus, $T_N(x) \leq T_N(z)$, $I_N(z) \leq I_N(x)$ and $F_N(z) \leq F_N(x)$, for all $x, y, z \in A$.

(b) Let A_N be a neutrosophic $\mathcal{N}-$structure on A satisfying the conditions (2) and (3), for $x_t \in T_N^{-1}$, $x_i \in I_N^{-1}$ and $x_f \in F_N^{-1}$. Then $1 \in A_N^{x_t}$, $1 \in A_N^{x_i}$ and $1 \in A_N^{x_f}$ from the condition (3). Let $x_1|(y_1|1), x_1 \in X_N^{x_t}$, $x_2|(y_2|1), x_2 \in A_N^{x_i}$ and $x_3|(y_3|1), x_3 \in A_N^{x_f}$. Thus, $T_N(x_t) \leq T_N(x_1|(y_1|1))$, $T_N(x_t) \leq T_N(x_1)$, $I_N(x_2|(y_2|1)) \leq I_N(x_i)$, $I_N(x_2) \geq I_N(x_i)$ and $F_N(x_3|(y_3|1)) \leq F_N(x_f)$, $F_N(x_3) \leq F_N(x_f)$. Since

$$T_N(x_t) \leq \min\{T_N(x_1|(y_1|1)), T_N(x_1)\},$$

$$\max\{I_N(x_2|(y_2|1)), I_N(x_2)\} \leq I_N(x_i)$$

and

$$\max\{F_N(x_3|(y_3|1)), F_N(x_3)\} \leq F_N(x_f),$$

it follows from the condition (2) that $T_N(x_t) \leq T_N(y_1)$, $I_N(y_2) \leq I_N(x_i)$ and $F_N(y_3) \leq F_N(x_f)$. Hence, $y_1 \in A_N^{x_t}$, $y_2 \in A_N^{x_i}$ and $y_3 \in A_N^{x_f}$. Therefore, $A_N^{x_t}$, $A_N^{x_i}$ and $A_N^{x_f}$ are filters of A. □

Example 3.29. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Let $T_N(0) = T_N(v) = -1, T_N(u) = T_N(1) = 0, I_N(0) = I_N(v) = 0, I_N(u) = I_N(1) = -1, F_N(0) = F_N(v) = -0.71, F_N(u) = F_N(1) = -0.5$. Then the filters

$$A_N^{x_t} = A, A_N^{x_i} = \{u.1\} \text{ and } A_N^{x_f} = A$$

of A satisfy the condition (2) in Theorem 3.28, for $x_t = v, x_i = u$ and $x_f = 1 \in A$.

Moreover, let

$$A_N = \{\frac{0}{(-0.99, 0, -0.01)}, \frac{u}{(-0.99, 0, -0.01)}, \frac{v}{(-0.99, 0, -0.01)}, \frac{1}{(0, -1, -1)}\}$$

be a neutrosophic $\mathcal{N}-$structure on A satisfying the conditions (2) and (3) in Theorem 3.28. Then the subsets

$$A_N^{x_t} = \{x \in A : T_N(1) \leq T_N(x)\} = \{1\},$$

$$A_N^{x_i} = \{x \in A : I_N(x) \leq I_N(0)\} = A$$

and

$$A_N^{x_f} = \{x \in A : F_N(x) \leq F_N(u)\} = A$$

of A are filters of A, where $x_t = 1, x_i = 0$ and $x_f = u \in A$.

4. Conclusion

In this study, neutrosophic $\mathcal{N}-$structures defined by $\mathcal{N}-$functions on strong Sheffer stroke NMV-algebras have been investigated. Basic definitions and notions about strong Sheffer stroke NMV-algebras and neutrosophic $\mathcal{N}-$structures defined by $\mathcal{N}-$functions on a nonempty universe X are presented and then a neutrosophic $\mathcal{N}-$subalgebra and a $(a, b, c)-$ level set of a neutrosophic $\mathcal{N}-$structure are defined by the help of $\mathcal{N}-$functions on strong Sheffer stroke NMV-algebras. It is shown that the $(a, b, c)-$level set of a neutrosophic $\mathcal{N}-$subalgebra of a strong Sheffer stroke NMV-algebra is its strong Sheffer stroke NMV-subalgebra and vice versa. Also, it is proved that the family of all neutrosophic $\mathcal{N}-$subalgebras of this algebraic structure forms a complete distributive lattice. It is illustrated that every neutrosophic $\mathcal{N}-$subalgebra of a strong Sheffer stroke NMV-algebra satisfies $T_N(x) \leq T_N(1)$, $I_N(1) \leq I_N(x)$ and $F_N(1) \leq F_N(x)$, for all elements x in this algebra but a neutrosophic $\mathcal{N}-$structure on a strong Sheffer stroke NMV-algebra satisfying this property is generally not its neutrosophic $\mathcal{N}-$subalgebra. Besides, it is interpreted the images of the sequence under $\mathcal{N}-$functions on a strong Sheffer stroke NMV-algebra. Moreover, it is stated the case which $\mathcal{N}-$functions determining a neutrosophic $\mathcal{N}-$subalgebra of a strong Sheffer stroke NMV-algebra are constant. Then a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke NMV-algebra is defined via $\mathcal{N}-$functions and shown that the functions T_N, I_N and F_N defining the neutrosophic $\mathcal{N}-$filter satisfies $T_N(x) \leq T_N(y)$, $I_N(x) \geq I_N(y)$ and $F_N(x) \geq F_N(y)$ when $x \leq y$, but the inverse does not usually hold. It is demonstrated that $(a, b, c)-$level set of a neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke NMV-algebra is its filter. Indeed, it is given that the subsets defined by $\mathcal{N}-$functions on a strong Sheffer stroke NMV-algebra must be its filters so that a neutrosophic $\mathcal{N}-$structure on this algebra is a neutrosophic $\mathcal{N}-$filter. It is proved that every neutrosophic $\mathcal{N}-$filter of a strong Sheffer stroke NMV-algebra is its neutrosophic $\mathcal{N}-$subalgebra whereas the inverse is not true in general. Additionally, new three subsets $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ of a strong Sheffer stroke NMV-algebra are defined by $\mathcal{N}-$functions and any elements x_t, x_i and x_f of the algebra. We show that these subsets are filters of a strong Sheffer stroke NMV-algebra for its neutrosophic $\mathcal{N}-$filter but the inverse holds under special conditions.

In our future works, we wish to introduce new Sheffer stroke algebraic structures and investigate their neutrosophic $\mathcal{N}-$structures.

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