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# Neutrosophic Nano RW-Closed Sets in Neutrosophic Nano Topological Spaces

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***Abstract:*** The main objective of this study is to introduce a new class of closed sets namely Neutrosophic Nano RW-closed sets and Neutrosophic Nano RW-continuous functions in Neutrosophic Nano topological spaces. Some of its properties and interrelationship with some existing Neutrosophic nano closed sets have been discussed.

***Keywords:*** $N_N$RW-closed set, $N_N$RW-open set, $N_N$RW$T_{1/2}$ space, $N_N$RW-connected space, $N_N$RW-continuous, $N_N$RW-irresolute, $N_N$RW-open and $N_N$- closed maps.

## 1. Introduction

The theory of neutrosophic sets with three components namely, membership T (Truth), Indeterminacy I, and non-membership F (Falsehood), one of the interesting generalizations of theory of fuzzy sets and Intuitionistic fuzzy sets introduced by F.Smarandache [8]. In 2012, A.A. Salama and S.A. Alblowi [13] introduced and studied the theory of neutrosophic topological spaces. Since then several mathematicians contributed many papers to this area. Various results in ordinary topological spaces have been put in the neutrosophic setting, and also various departures have been observed. Neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data. The concept of nano topology explored by M. Lellis Thivagar et. al[11] can be described as a collection of nano approximations for which equivalence classes are building blocks. In 2018, M. Lellis Thivagar et. al. [12] introduced a new concept called as Neutrosophic Nano topology and discussed neutrosophic nano interior and neutrosophic nano closure.

In 2007, S.S. Benchalli and R.S. Wali [4] introduced RW-closed sets in topological spaces. The authors D. Savithiri and C. Janaki [15] introduced the concept of Neutrosophic RW-closed sets in Neutrosophic topological spaces. In this article we introduce Neutrosophic Nano RW-closed sets and discuss some of its properties.

## 2 PRELIMINARIES

The following recalls requisite ideas and preliminaries necessary in the sequel of our work.

**Definition 2.1:**[9] Let X be a non-empty fixed set a Neutrosophic set **(NS for short)** A is an object having the form A = < x, $\mu_A(x)$,$\sigma_A(x)$,$\gamma_A(x)$> , x∈X where $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element x∈X to the set A.

**Definition 2.2:[11]** Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let X ⊆ U.

(i) The lower approximation of X with respect to R is the set of all objects, which can be classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x\in U}\{(R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) – L_R(X)$.

**Remark 2.3:[11]**

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.

(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$.

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.

(iv) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.

(v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.

(vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$.

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$.

(ix) $U_RU_R(X) = L_RU_R(X) = U_R(X)$.

(x) $L_RL_R(X) = L_RU_R(X) = L_R(X)$.

**Definition 2.4:[11]** Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call (U, $\tau_R(X)$) as the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets.

**Definition 2.5:[12]** Let U be a non-empty set and R be an equivalence relation on U. Let S be a neutrosophic set in U with the membership function $\mu_S$, the indeterminacy function $\sigma_S$, and the non-membership function $\gamma_S$. The neutrosophic nano lower, neutrosophic nano upper approximation and neutrosophic nano boundary of S in the approximation (U,R) denoted by $\underline{N}(S), \overline{N}(S) and\ B(S)$ are respectively defined as follows:

(i) $\underline{N}(S) = \{\langle x, \mu_{\underline{R}(A)}(x),\ \sigma_{\underline{R}(A)}(x), \gamma_{\underline{R}(A)}(x)\rangle / y \in [x]_R, x \in U\}$.

(ii) $\overline{N}(S) = \{\langle x, \mu_{\overline{R}(A)}(x), \sigma_{\overline{R}(A)}(x), \gamma_{\overline{R}(A)}(x)\rangle /\ y \in [x]_R, x \in U\}$.

(iii) B(S) = $\overline{N}(S) - \underline{N}(S)$.

where $\mu_{\underline{R}(A)}(x) = \wedge_{y\in[x]_R} \mu_A(y),\ \sigma_{\overline{R}(A)}(x) = \wedge_{y\in[x]_R} \sigma_A(y), \gamma_{\underline{R}(A)}(x) = \vee_{y\in[x]_R} \gamma_A(y)$,

$$\mu_{\bar{R}(A)}(x) = \vee_{y\in[x]_R} \mu_A(y), \sigma_{\bar{R}(A)}(x) = \vee_{y\in[x]_R} \sigma_A(y), \gamma_{\bar{R}(A)}(x) = \wedge_{y\in[x]_R} \gamma_A(y).$$

**Definition 2.6:[12]** Let U be an universe, R be an equivalence relation on U and S be a neutrosophic set in U and if the collection $\tau_N(S)$ = {$0_N$, $1_N$, $\underline{N}(S), \overline{N}(S), B(S)$} forms a topology then it is said to be a neutrosophic nano topology. We call (U, $\tau_N(S)$) as the neutrosophic nano topological space **(Briefly NNTS)**. The elements of $\tau_N(S)$ are called as neutrosophic nano open **(In Short $N_N$O)** sets.

**Remark 2.7:[12]**$[\tau_N(S)]^C$ is called as dual neutrosophic nano topology of $\tau_N(S)$. The elements of $[\tau_N(S)]^C$ are called neutrosophic nano closed **(In Short $N_N$C)** sets.

**Remark 2.8:[12]** In neutrosophic nano topological space, the neutrosophic nano boundary cannot be empty. Since the difference between neutrosophic nano upper and neutrosophic nano lower approximations is defined as the maximum and minimum of the values in the neutrosophic sets.

**Proposition 2.9:[12]** Let U be a non-empty finite universe and S be a neutrosophic set on U. Then the following statements hold:

(i) The collection $\tau_N(S)$ = {$0_N$, $1_N$}, is the indiscrete neutrosophic nano topology on U.

(ii) If $\underline{N}(S) = \overline{N}(S) = B(S)$, then the neutrosophic nano topology, $\tau_N(S)$ = {$0_N$, $1_N$, $\underline{N}(S), B(S)$}.

(iii) If $\underline{N}(S) = B(S)$, then $\tau_N(S)$ = {$0_N$, $1_N$, $\underline{N}(S), \overline{N}(S)$} is a neutrosophic nano topology.

(iv) If $\overline{N}(S) = B(S)$, then $\tau_N(S)$ = {$0_N$, $1_N$, $\underline{N}(S), B(S)$}.

(v) The collection $\tau_N(S) = \{0_N, 1_N, \underline{N}(S), \overline{N}(S), B(S)\}$ is the discrete neutrosophic nano topology on U

**Definition 2.10:[12]** Let (U, $\tau_N$(S)) be NNTS and A = < x, $\mu_A(x),\sigma_A(x),\gamma_A(x)$, x ∈ U> be a NNS in X. Then the neutrosophic nano closure and neutrosophic nano interior of A are defined by

$N_N$Cl (A) = ∩ { K : K is a $N_N$CS in X and A ⊆ K }

$N_N$Int (A) = ∪ { G : G is a $N_N$OS in X and G ⊆ A }.

**Definition 2.11:[12]** A subset A of a neutrosophic nano topological space Let (U, $\tau_N$(S)) is said to be

(i) a neutrosophic nano pre closed **($N_N$pre-closed)** set if $N_N$Cl($N_N$Int(A)) ⊆ A.

(ii) a neutrosophic nano semi-closed **($N_N$semi-closed)** set if $N_N$Int($N_N$Cl(A)) ⊆ A.

(iii) a neutrosophic nano regular open **(In short $N_N$RO)** set if A =$N_N$Int($N_N$Cl(A)) and regular closed **(In short $N_N$RC)** set if A = $N_N$Cl($N_N$Int(A)) .

(iv) a neutrosophic regular semi open **(In short NRSO)** if there exists a NRO set U such that U ⊆ A ⊆ NCl(A)

(v) a neutrosophic nano α-closed **($N_N$α-closed)** set if $N_N$Cl($N_N$Int($N_N$Cl(A))) ⊆ A.

(vi) a neutrosophic nano g-closed **($N_N$ g-closed)** set if $N_N$Cl(A) ⊆ F whenever A ⊆ F and F is NNO in U.

**Definition 2.11:[6]** The difference between two neutrosophic nano sets A and B is defined as

A\B (S) = {x, min [($\mu_A(x)$, $\gamma_B(x)$)], min [($\sigma_A(x)$,1- $\sigma_B(x)$)], max [$\gamma_A(x)$, $\mu_B(x)$)].

**3. NEUTROSOPHIC NANO RW-CLOSED SETS**

**Definition 3.1:** A subset A of a neutrosophic nano topological space (U, $\tau_N$(S)) is called as neutrosophic nano regular weakly closed **(In short $N_N$RW-closed)** set, if $N_N$Cl(A) ⊆ V whenever A ⊆ V and V is a neutrosophic nano regular open in U.

**Definition 3.2:** The neutrosophic nano RW-closure and neutrosophic nano RW-interior of A are defined by

$N_N$RWCl (A) = ∩ { K : K is a $N_N$RWCS in X and A ⊆ K }

$N_N$RWInt (A) = ∪ { G : G is a $N_N$RWOS in X and G ⊆ A }.

**Definition 3.3:** (i) neutrosophic nano RG- Closed set **(shortly $N_N$RG – closed set)** of X if there exists a neutrosophic nano regular open set U such that $N_N$Cl(A) ⊆ U whenever A ⊆ U.

(ii) neutrosophic nano RWG- closed set **(shortly $N_N$RWG – closed set)** of X if there exists a neutrosophic nano regular open set U such that $N_N$Cl($N_N$Int(A)) ⊆ U whenever A ⊆ U.

(iii) neutrosophic nano W-closed set **(shortly NNW – closed set)** of X if there exists a neutrosophic nano semi-open set U such that NNCl(A) ⊆ U whenever A ⊆ U.

(iv) neutrosophic nano g-closed set **(shortly NNG – closed set)** of X if there exists a neutrosophic open set U such that NNCl(A) ⊆ U whenever A ⊆ U.

**Proposition 3.3:** (i) Every NN-closed set is NNRW-closed.

(ii) Every NN- regular closed set is NNRW-closed.

(iii) Every NN- πclosed set is NNRW-closed.

(iv) Every NNW-closed set is NNRW-closed.

**Proof:** Follows from [4].

The following example makes clear that the converse of the Proposition 3.3 need not be true.

**Example 3.4**: Let U = $\{p_1, p_2, p_3\}$ be the universe set and the equivalence relation U\R = $\{\{p_1, p\}, \{p_2\}\}$. Let $S = \left\{\langle\frac{p_1}{(0.1,0.2,0.3)}\rangle, \langle\frac{p_2}{(0.2,0.3,0.4)}\rangle, \langle\frac{p_3}{(0.1,0.6,0.4)}\rangle\right\}$ be a neutrosophic nano subset of U. Then $\overline{N}(S) = \left\{\langle\frac{p_1,p_3}{(0.1,0.6,0.3)}\rangle, \langle\frac{p_2}{(0.2,0.3,0.4)}\rangle\right\}$, $\underline{N}(S) = \left\{\langle\frac{p_1,p_3}{(0.1,0.2,0.4)}\rangle, \langle\frac{p_2}{(0.2,0.3,0.4)}\rangle\right\}$ and B $(S) = \left\{\langle\frac{p_1,p_3}{(0.1,0.6,0.3)}\rangle, \langle\frac{p_2}{(0.2,0.3,0.4)}\rangle\right\}$. So the neutrosophic nano topology τN = $\{0_N, 1_N, \underline{N}, B\}$ where the neutrosophic closed sets are τN$^C$ = $\{0_N, 1_N, \underline{N}^C, B^C\}$. Let $Q_1 = \left\{\langle\frac{p_1}{(0.2,0.1,0.3)}\rangle, \langle\frac{p_2}{(0.3,0.1,0.2)}\rangle, \langle\frac{p_3}{(0.1,0.2,0.3)}\rangle\right\}$, then $Q_1$ is NNRW-closed but it is not an NN-closed set in U. $Q_2 = \left\{\langle\frac{p_1}{(0.2,0.3,0.5)}\rangle, \langle\frac{p_2}{(0.3,0.6,0.5)}\rangle, \langle\frac{p_3}{(0.2,0.3,0.3)}\rangle\right\}$, $Q_2$ is NNRW-closed but it is neither NN Regular-closed nor NNπ-closed set and $Q_3 = \left\{\langle\frac{p_1}{(0.1,0.3,0.6)}\rangle, \langle\frac{p_2}{(0.2,0.6,0.6)}\rangle, \langle\frac{p_3}{(0.1,0.2,0.6)}\rangle\right\}$, then $Q_3$ is NNRW-closed but not NNW-closed set.

**Proposition 3.5:** (i) Every NNRW-closed set is NNRG-closed.

(ii) Every NNRW-closed set is NNGPR-closed.

(iii) Every NNRW-closed set is NNRWG-closed.

**Proof:** Follows from [4].

The converse of the Proposition 3.4 need not be true.

**Example 3.6**: * Let U = $\{p_1, p_2, p_{3,}p_4, p_5\}$ be the universe set and the equivalence relation U\R = $\{\{p_1, p_3\}, \{p_2\}, \{p_4, p_5\}\}$. Let S $= \left\{\langle\frac{p_1}{(0.4,0.3,0.4)}\rangle, \langle\frac{p_2}{(0.5,0.3,0.5)}\rangle, \langle\frac{p_3}{(0.5,0.3,0.5)}\rangle, \langle\frac{p_4}{(0.6,0.3,0.1)}\rangle, \langle\frac{p_5}{(0.5,0.3,0.1)}\rangle\right\}$ be a neutrosophic nano subset of U $\overline{N}(S) = \left\{\langle\frac{p_1,p_3}{(0.5,0.3,0.2)}\rangle, \langle\frac{p_2}{(0.5,0.3,0.5)}\rangle, \langle\frac{p_4,p_5}{(0.6,0.3,0.1)}\rangle\right\}$, $\underline{N}(S) =$

$\left\{\langle\frac{p_1,p_3}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.5,0.3,0.1)}\rangle\right\}$ and B $(S) = \left\{\langle\frac{p_1,p_3}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.1,0.3,0.6)}\rangle\right\}$ . The neutrosophic nano topology τN = $\{0_N, 1_N, \underline{N}, \overline{N}, B\}$. Let $R_1 = \left\{\langle\frac{p_1}{(0.3,0.3,0.7)}\rangle,\langle\frac{p_2}{(0.2,0.3,0.6)}\rangle,\langle\frac{p_3}{(0.2,0.3,0.5)}\rangle,\langle\frac{p_4}{(0.1,0.2,0.7)}\rangle,\langle\frac{p_5}{(0.1,0.3,0.8)}\rangle\right\}$ . Then $R_1$ is both NNGPR-closed and NNRWG – closed but it is not an NNRW-closed.

* In example 3.4, let $R_2 = \left\{\langle\frac{p_1}{(0.3,0.7,0.5)}\rangle,\langle\frac{p_2}{(0.3,0.4,0.6)}\rangle,\langle\frac{p_3}{(0.2,0.5,0.5)}\rangle,\langle\frac{p_4}{(0.1,0.5,0.6)}\rangle,\langle\frac{p_5}{(0.1,0.6,0.7)}\rangle\right\}$ , then $R_2$ is NNRG-closed but not an NNRW-closed.

**Proposition 3.7:** The finite union of NNRW – closed subsets of U is also an NNRW – closed subset of U.

**Proof:** Assume that P and Q are NNRW –closed sets in U. Let R be an NNRSO set in X such that P ∪ Q ⊆ R. Then P ⊆ R and Q ⊆ R. Since P and Q are NNRW – closed sets, NNCl(P) ⊆ R and NNCl (Q) ⊆ R. Then NNCl(P∪Q) = NNCl(P) ∪ NNCl(Q) ⊆ R. Hence P ∪ Q is an NNRW – closed set in U.

**Remark 3.8:** The intersection of two NNRW-closed sets in (U, τN(S)) need not be an NNRW-closed set in U.

**Example 3.9:** Let U $=\{p_1, p_2, p_3, p_4, p_5\}$ be the universe set and the equivalence relation U\R = $\{\{p_1, p_3\}, \{p_2\}, \{p_4, p_5\}\}$. Let S= $\left\{\langle\frac{p_1}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_3}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4}{(0.6,0.3,0.1)}\rangle,\langle\frac{p_5}{(0.5,0.3,0.1)}\rangle\right\}$ be a neutrosophic nano subset of U$\overline{N}(S) = \left\{\langle\frac{p_1,p_3}{(0.5,0.3,0.2)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.6,0.3,0.1)}\rangle\right\}$, $\underline{N}(S) = \left\{\langle\frac{p_1,p_3}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.5,0.3,0.1)}\rangle\right\}$ and B$(S) = \left\{\langle\frac{p_1,p_3}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.1,0.3,0.6)}\rangle\right\}$. The neutrosophic nano topology τN = $\{0_N, 1_N, \underline{N}, \overline{N}, B\}$. $R_1 = \left\{\langle\frac{p_1}{(0.6,0.3,0.3)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.3)}\rangle,\langle\frac{p_3}{(0.5,0.2,0.3)}\rangle,\langle\frac{p_4}{(0.3,0.3,0.1)}\rangle,\langle\frac{p_5}{(0.4,0.4,0.1)}\rangle\right\}$, $R_2 = \left\{\langle\frac{p_1}{(0.2,0.3,0.5)}\rangle,\langle\frac{p_2}{(0.3,0.5,0.7)}\rangle,\langle\frac{p_3}{(0.2,0.3,0.5)}\rangle,\right.$ $\left.\langle\frac{p_4}{(0.1,0.5,0.6)}\rangle, \langle\frac{p_5}{(0.1,0.7,0.6)}\rangle\right\}$ . Then $R_1$ and $R_2$ are NNRW-closed sets but $R_1 \cap R_2$ is not an NNRW-closed set.

**Proposition 3.10:** If a subset A of U is NNRW – closed set in U, then NNCl(A)\A does not contain any non-empty neutrosophic nano regular semi-open set in U.

**Proof:** Suppose that A is an NNRW –closed set in U. We shall prove by contradiction. Let R be an NNRSO set such that NNCl(A)\A ⊃ R which implies R ⊆ U\A i.e., A ⊆ U\R. Since R is NNRSO, U\R is also NNRSO set in U. Since A is an NNRW – closed set, NNCl(A) ⊆ U\R. So R ⊆ U\NNCl(A) also R ⊆ NNCl(A) implies R = ϕ. Hence NNCl (A)\A does not contain any non-empty NNRSO set in U.

The converse of the Proposition 3.10 need not be true as shown in the following example.

**Example 3.11:** In example 3.9, in the neutrosophic nano topological space (U, τN(S)), let $A = \left\{\langle\frac{p_1}{(0.3,0.2,0.5)}\rangle,\langle\frac{p_2}{(0.3,0.2,0.6)}\rangle,\langle\frac{p_3}{(0.2,0.3,0.5)}\rangle,\langle\frac{p_4}{(0.1,0.2,0.7)}\rangle,\langle\frac{p_5}{(0.1,0.3,0.8)}\rangle\right\}$ , then NNCl(A)\A does not contain any non-empty NNRSO set, but A is not an NNRW-closed set in U.

**Corollary 3.12:** If a subset A of U is $N_N$RW – closed set in U, then $N_N$Cl(A)\A does not contain any non-empty neutrosophic nano regular- open set in U.

**Proof:** Follows from the Proposition 3.10 and the fact that every $N_N$RO set is $N_N$RSO in U.

**Proposition 3.13:** If A is $N_N$RO and $N_N$RW-closed, then A is $N_N$RC set and hence $N_N$-clopen.

**Proof:** Suppose A is $N_N$RO and $N_N$RW – closed. As every $N_N$RO set is $N_N$RSO and A ⊂ A, we have $N_N$Cl(A) ⊂ A. Also A ⊂ $N_N$Cl(A), thus $N_N$Cl(A) = A. Hence A is a $N_N$C set. Since A is $N_N$RO it is $N_N$O set. Now $N_N$Cl($N_N$Int(A) = $N_N$Cl(A) = A. Therefore A is $N_N$RC and Neutrosophic nano clopen.

**Proposition 3.14:** If A is an $N_N$RW – closed subset of U such that A ⊆ B ⊆ NCl(A), then B is an $N_N$RW – closed set in U.

**Proof:** Let A be an $N_N$RW – closed set of U such that A ⊆ B ⊆ $N_N$Cl(A). Let R be $N_N$RSO set of U such that B ⊆ R. Then A ⊆ R. Since A is $N_N$RW –closed set, we have $N_N$Cl(A) ⊆ R and $N_N$Cl(B) ⊆ $N_N$Cl($N_N$Cl(A)) ⊆ R. Therefore B is also an $N_N$RW – closed set in U.

The following example shows that the converse of the Proposition 3.13 need not be true.

**Example 3.15:** Let U = $\{n_1, n_2, n_3\}$ be the universe set and the equivalence relation U\R = $\{\{n_1, n_3\}, \{n_2\}\}$. Let $S = \left\{\langle\frac{x_1}{(0.1,0.2,0.3)}\rangle, \langle\frac{x_2}{(0.2,0.3,0.4)}\rangle, \langle\frac{x_3}{(0.1,0.6,0.4)}\rangle\right\}$ be a neutrosophic nano subset of U. Then $\overline{N}(S) = \left\{\langle\frac{x_1,x_3}{(0.1,0.6,0.3)}\rangle, \langle\frac{x_2}{(0.2,0.3,0.4)}\rangle\right\}$, $\underline{N}(S) = \left\{\langle\frac{x_1,x_3}{(0.1,0.2,0.4)}\rangle, \langle\frac{x_2}{(0.2,0.3,0.4)}\rangle\right\}$ and $B(S) = \left\{\langle\frac{x_1,x_3}{(0.1,0.6,0.3)}\rangle, \langle\frac{x_2}{(0.2,0.3,0.4)}\rangle\right\}$. So the neutrosophic nano topology $\tau_N = \{0_N, 1_N, \underline{N}, B\}$ and the neutrosophic closed sets are $\tau_N^C = \{0_N, 1_N, \underline{N}^C, B^C\}$. Let A = $\left\{\langle\frac{x_1}{(0.1,0.3,0.6)}\rangle, \langle\frac{x_2}{(0.2,0.6,0.6)}\rangle, \langle\frac{x_3}{(0.1,0.2,0.6)}\rangle\right\}$ and B = $\left\{\langle\frac{x_1}{(0.2,0.3,0.5)}\rangle, \langle\frac{x_2}{(0.3,0.6,0.5)}\rangle, \langle\frac{x_3}{(0.2,0.3,0.3)}\rangle\right\}$. Then A and B are $N_N$RW-closed sets in (U, $\tau_N$(S)), but A ⊂ B is not a subset of $N_N$Cl(A).

**Proposition 3.16:** Let A be an $N_N$RW-closed in (U, $\tau_N$(S)). Then A is $N_N$-closed if and only if $N_N$Cl(A)\A is $N_N$RSO.

**Proof:** Let A be an $N_N$-closed in (U, $\tau_N$(S)). Then $N_N$Cl(A)\A = ϕ which is $N_N$RSO.

Conversely, suppose $N_N$Cl(A)\A is $N_N$RSO in U. By hypothesis, A is $N_N$RW-closed implies $N_N$Cl(A)\A does not contain any non-empty $N_N$RSO in U. Then $N_N$Cl(A)\A = ϕ which implies that A is $N_N$-closed in U.

**Proposition 3.17:** If A is $N_N$RO and $N_N$RG closed, then A is $N_N$RW-closed in U.

**Proof:** Let A be an $N_N$RO and $N_N$RG-closed. Let Q be any $N_N$RSO set in U such that A ⊆ R. since A is $N_N$RO and $N_N$RG we have $N_N$Cl(A) ⊆ A ⊆ R. Therefore A is $N_N$RW-closed.

**Proposition 3.18:** If a subset A of a neutrosophic nano topological space U is both $N_N$RSO and $N_N$RW-closed, then it is $N_N$-closed.

**Proof:** Suppose A be a subset of a neutrosophic nano topological space U is both $N_N$RSO and $N_N$RW-closed. Then A ⊂ A and $N_N$Cl(A) ⊆ A which implies A is $N_N$-closed.

**Remark 3.19:** The concept of $N_N$RW-closed set is independent with the concepts of (i) $N_N$semi –closed (ii) $N_N$RW-preclosed (iii) $N_N\alpha$-closed (iv) $N_N$WG - closed sets which is shown by the following example.

**Example 3.20:** Let U = $\{n_1, n_2, n_3\}$ be the universe set. U\R = $\{\{n_1\}, \{n_2, n_3\}\}$ be an equivalence relation. Let $S = \left\{\langle\frac{n_1}{(0.1,0.2,0.3)}\rangle, \langle\frac{n_2}{(0.3,0.4,0.5)}\rangle, \langle\frac{n_3}{(0.6,0.4,0.1)}\rangle\right\}$ be a neutrosophic nano subset of U. Then $\overline{N}(S) = \left\{\langle\frac{n_1,n_3}{(0.1,0.2,0.3)}\rangle, \langle\frac{n_2}{(0.6,0.4,0.1)}\rangle\right\}$, $\underline{N}(S) = \left\{\langle\frac{n_1,n_3}{(0.1,0.2,0.3)}\rangle, \langle\frac{n_2}{(0.3,0.4,0.5)}\rangle\right\}$ and $B(S) = \left\{\langle\frac{n_1,n_3}{(0.1,0.2,0.3)}\rangle, \langle\frac{n_2}{(0.1,0.4,0.6)}\rangle\right\}$. So the neutrosophic nano topology $\tau_N = \{0_N, 1_N, \underline{N}, \overline{N}, B\}$. In the neutrosophic nano topology (U, $\tau_N$(S)),

- Let $A = \left\{\langle\frac{n_1}{(0.2,0.5,0.3)}\rangle, \langle\frac{n_2}{(0.1,0.5,0.6)}\rangle, \langle\frac{n_3}{(0.1,0.4,0.7)}\rangle\right\}$ and $B = \left\{\langle\frac{n_1}{(0.2,0.7,0.4)}\rangle, \langle\frac{n_2}{(0.5,0.6,0.4)}\rangle, \langle\frac{n_3}{(0.4,0.5,0.4)}\rangle\right\}$, then A is $N_N$semi-closed but not an $N_N$RW-closed and B is $N_N$RW-closed but it is not an $N_N$semi-closed.
- Let $C = \left\{\langle\frac{n_1}{(0.1,0.2,0.4)}\rangle, \langle\frac{n_2}{(0.3,0.3,0.6)}\rangle, \langle\frac{n_3}{(0.1,0.3,0.5)}\rangle\right\}$ and $D = \left\{\langle\frac{n_1}{(0.1,0.4,0.7)}\rangle, \langle\frac{n_2}{(0.1,0.6,0.7)}\rangle, \langle\frac{n_3}{(0.3,0.4,0.4)}\rangle\right\}$, then C is both $N_N$pre-closed set and $N_N$WG-closed but not an $N_N$RW-closed and D is $N_N$RW-closed but it is neither $N_N$pre-closed nor an $N_N$WG-closed sets.
- In example 3.8, in the topological space (U,$\tau_N$(S)), $E = \left\{\langle\frac{n_1}{(0.6,0.3,0.3)}\rangle, \langle\frac{n_2}{(0.5,0.3,0.3)}\rangle, \langle\frac{n_3}{(0.5,0.2,0.3)}\rangle, \langle\frac{n_4}{(0.3,0.3,0.1)}\rangle, \langle\frac{n_5}{(0.4,0.4,0.1)}\rangle\right\}$ and $F = \left\{\langle\frac{n_1}{(0.3,0.3,0.7)}\rangle, \langle\frac{n_2}{(0.2,0.3,0.6)}\rangle, \langle\frac{n_3}{(0.2,0.3,0.5)}\rangle, \langle\frac{n_4}{(0.1,0.2,0.7)}\rangle, \langle\frac{n_5}{(0.1,0.3,0.8)}\rangle\right\}$, E is $N_N$RW-closed set but not an $N_N\alpha$-closed set and F is $N_N\alpha$-closed but it is not an $N_N$RW-closed set.

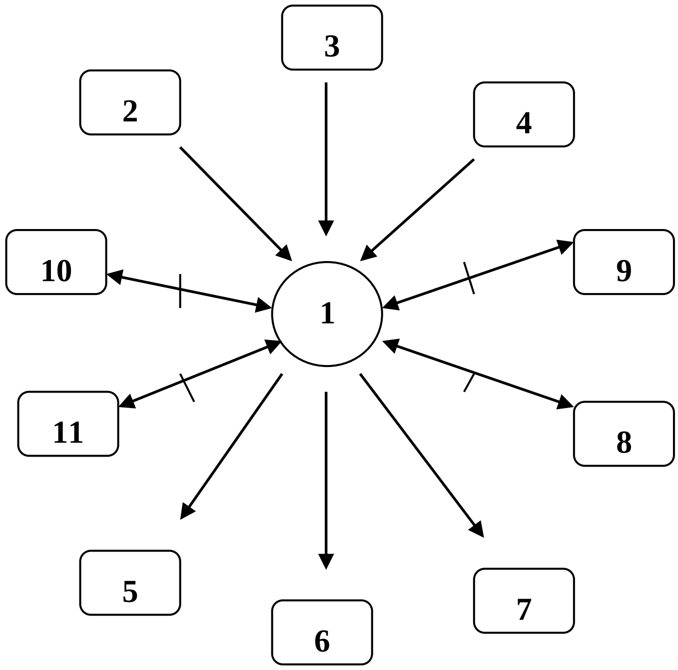
**Proposition 3.21:** If an $N_N$ subset A is both $N_N$-open and $N_N$G-closed in (U, $\tau_N$(S)), then it is $N_N$RW-closed in U.

**Proof:** Let A be $N_N$-open and $N_N$G-closed in U. Let A ⊂ U and U be an $N_N$RSO in U. Now, A ⊂ A. By hypothesis, $N_N$Cl(A) ⊂ U. Thus A is $N_N$RW-closed.

**Remark 3.22:** If A is both NN-open and NNRW-closed in U, then A need not be NNG-closed in general which is shown in the following example.

**Example 3.23:** In example 3.8, the $N_N$-open set B is $N_N$RW-closed but it is not an $N_N$G-closed set.

**The above discussions are implicated in the following diagram.**




1. $N_N$RW-closed 2. $N_N$-closed 3. $N_N$R-closed 4. $N_N\pi$-closed 5. $N_N$RG-closed
6. $N_N$RWG-closed 7. $N_N$GPR-closed 8. $N_N$semi-closed 9. $N_N$pre-closed
10. $N_N\alpha$-closed 11.$N_N$WG-closed.

**Proposition 3.24:** If a subset A of a neutrosophic nano topological space U is both $N_N$-open and $N_N$WG-closed, then it is $N_N$RW-closed.

**Proof:** Suppose a subset A of U is both NN-open and $N_N$WG-closed. Let $A \subset U$ and U is $N_N$RSO. Then $N_N$Cl ($N_N$Int(A)) = $A \subset A$, since A is NN-open. Hence $N_N$Cl(A) $\subset$ U implies that A is an $N_N$RW-closed in U.

**Definition 3.25:** A neutrosophic nano subset A of a neutrosophic nano topological space $(U,\tau_N(S))$ is called an $N_N$RW-open if and only if its complement $A^C$ is $N_N$RW-closed.

**Proposition 3.26:** An $N_N$ set A of a topological space $(U,\tau_N(S))$ is $N_N$RW-open if $F \subseteq$ NNInt(A) whenever F is $N_N$RSO and $F \subset A$.

**Proof:** Follows from the definition 3.1.

**Proposition 3.27:** Let A be an $N_N$RW-open set of neutrosophic nano topological space $(U,\tau_N(S))$ and $N_N$Int(A) $\subseteq B \subseteq A$. Then B is $N_N$RW-open.

**Proof:** Suppose that A is an $N_N$RW-open in U and $N_N$Int(A) $\subseteq B \subseteq A$ implies $A^c \subseteq B^c \subseteq N_N Cl(A^c)$. Since $A^c$ is $N_N$RW-closed, by Proposition 3.14, $B^c$ is $N_N$RW-closed. Hence B is $N_N$RW-open.

**Proposition 3.28:** Let $(U,\tau_N(S))$ be a neutrosophic nano topological space and $N_N$RSO(X) and $N_N$C(X) be the family of all $N_N$RSO sets and $N_N$C sets respectively. Then $N_N$RSO(X) $\subseteq$ $N_N$C(X) if and only if every neutrosophic nano set of U is $N_NRW$-closed.

**Proof: Necessity:** Suppose that $N_NRSO(X) \subseteq N_NC(X)$ and let A be an $N_N$ – set of U such that $A \subseteq R \in N_NRSO(X)$. Then $N_NCl(A) \subseteq N_NCl(R) = R$, by hypothesis. Hence $N_NCl(A) \subseteq R$ when $A \subseteq R$ and R is $N_NRSO$ which implies that A is $N_NRW$-closed.

**Sufficiency:** Assume that every neutrosophic nano set of U is $N_NRW$-closed. Let $R \in N_NRSO(X)$. Then since $R \subseteq R$ and R is $N_NRW$-closed, $N_NCl(R) \subseteq R$ then $R \in N_NCl(X)$. Therefore $N_NRSO(X) \subseteq N_NCl(X)$.

**Definition 3.29:** A neutrosophic nano topological space $(U,\tau_N(S))$ is called as $N_NRW$-connected if there is no proper $N_N$-subset of U which is both $N_NRW$-open $N_NRW$-closed.

**Proposition 3.30:** Every $N_NRW$-connected space is $N_N$-connected.

**Proof:** Let $(U,\tau_N(S))$ be an $N_NRW$-connected and suppose that $(U,\tau_N(S))$ is not $N_N$-connected. Then there exists a proper $N_N$-set A ($A \neq 0_N$, $A \neq 1_N$) such that A is both $N_N$-open and $N_N$-closed set. Since every $N_N$-open and $N_N$-closed set is $N_NRW$-open and $N_NRW$-closed, $(U,\tau_N(S))$ is not an $N_NRW$-connected which is a contradiction. This shows that U is $N_N$-connected.

**Proposition 3.31:** A $N_NT$ space is $N_NRW$- connected if and only if there exists no non-zero $N_NRW$- open sets A and B in X such that $A = B^C$.

**Proof: Necessity:** Suppose that A and B are $N_NRW$-open sets such that $A \neq 0_N \neq B$. and $A = B^C$. Since $B = A^C$, A is $N_NRW$-closed set and $B \neq 0_N$ implies $B^C \neq 1_N$, i.e., $A \neq 1_N$. Hence there exists a proper $N_N$ –set A which is both $N_NRW$-open and $N_NRW$-closed which is a contradiction to the fact that U is $N_NRW$-connected.

**Sufficiency:** Let $(U,\tau_N(S))$ be an $N_NTS$ and A is both $N_NRW$-open and $N_NRW$-closed set in U such that $0_N \neq A \neq 1_N$. Take $B = A^C$ implies that B is $N_NRW$-open and $A \neq 1_N \Rightarrow B = A^C \neq 0_N$ which is a contradiction. Hence there is no proper $N_N$-subset of U which is both $N_NRW$-open and $N_NRW$-closed. Therefore $N_NTS$ $(U,\tau_N(S))$ is $N_NRW$-connected.

**Definition 3.32:** A neutrosophic nano topological space$(U,\tau_N(S))$ is said to be an $N_NRWT_{1/2}$-space if every $N_NRW$-closed set in U is $N_N$-closed in U.

**Proposition 3.33:** A neutrosophic nano topological space $(U,\tau_N(S))$ is $N_NRWT_{1/2}$ space, then the following statements are equivalent:

(i) U is $N_NRW$-connected (ii) U is $N_N$-connected.

**Proof: (i) ⇒ (ii):** Follows from the Proposition 3.29.

**(ii) ⇒ (i):** Assume that U is $N_NRWT_{1/2}$-space, and $N_N$-connected. Suppose that U is not an $N_NRW$-connected, then there exists a proper $N_N$-set A which is both $N_NRW$-open and $N_NRW$-closed. Since $(U,\tau_N(S))$ is $N_NRWT_{1/2}$, A is both $N_N$-open and $N_N$-closed which is a contradiction to the fact that U is $N_N$-connected. This shows that U is $N_NRW$-connected.

## 4. $N_N$RW-CONTINUOUS FUNCTIONS

**Definition 4.1: (i)** A function f: (U,$\tau_N$(S)) → (V,$\tau_N$(T)) is said to be a neutrosophic nano RW-continuous **(In short $N_N$RW-continuous)** if the inverse image of $N_N$-closed set of V is $N_N$RW-closed in (U,$\tau_N$(S)).

**(ii)** A function f: (U,$\tau_N$(S)) → (V,$\tau_N$(T)) is said to be a neutrosophic nano RW-irresolute **(In short $N_N$RW-irresolute)** if the inverse image of $N_N$RW-closed set of V is $N_N$RW-closed in (U,$\tau_N$(S)).

**Proposition 4.2:** A mapping f: (U,$\tau_N$(S)) → (V,$\tau_N$(T)) is $N_N$RW-continuous if and only if the inverse image of every $N_N$-open set of V is $N_N$RW-open in U.

**Proof:** It is obvious because $f^{-1}(A^c) = [f^{-1}(A)]^c$ for every $N_N$-set A of V.

**Proposition 4.3:** If f: (U,$\tau_N$(S)) → (V,$\tau_N$(T)) is $N_N$RW-continuous, then f($N_N$RWCl(A)) ⊆ $N_N$Cl(f(A)) for every $N_N$-set A of U.

**Proof:** Let A be an $N_N$-set of U. Then $N_N$Cl(f(A)) is an $N_N$-closed set of V. Since f is an $N_N$RW-continuous function, $f^{-1}$($N_N$Cl(f(A)) is $N_N$RW-closed in U. Clearly A ⊆ $f^{-1}$($N_N$Cl(f(A)). Therefore $N_N$RWCl(A) ⊆ $N_N$RWCl ($f^{-1}$($N_N$Cl f(A))) = $f^{-1}$($N_N$Cl(f(A)). Hence f($N_N$RWCl(A)) ⊆ $N_N$Cl(f(A)) for every $N_N$-set A of U.

**Proposition 4.4**: (i) Every $N_N$-continuous map is $N_N$RW-continuous.

(ii) Every $N_N$- regular continuous map is $N_N$RW-continuous.

(iii) Every $N_N$- π-continuous set is $N_N$RW-continuous.

(iv) Every $N_N$W-continuous map is $N_N$RW-continuous.

(v) Every $N_N$RW-irresolute map is $N_N$RW-continuous.

**Proof:** Obvious.

**Remark 4.4:** The following example makes clear that the converse of the Proposition 4.4 may not be true.

**Example 4.5:** Let U = $\{n_1, n_2, n_3\}$ = V be the universe sets. U\R₁ = $\{\{n_1\},\{n_2,n_3\}\}$ and U\R₂ = $\{\{n_1,n_3\},\{n_2\}\}$ be equivalence relations. Let $S_1 = \left\{\langle\frac{n_1}{(0.3,0.4,0.3)}\rangle,\langle\frac{n_2}{(0.6,0.3,0.1)}\rangle,\langle\frac{n_3}{(0.2,0.6,0.2)}\rangle\right\}$ , $S_2 = \left\{\langle\frac{n_1}{(0.1,0.2,0.3)}\rangle,\langle\frac{n_2}{(0.2,0.3,0.4)}\rangle,\langle\frac{n_3}{(0.1,0.6,0.4)}\rangle\right\}$ be a neutrosophic nano subsets of U. Then $\tau_N(S_1)$ = $\{0_N, \overline{N}(S_1), \underline{N}(S_1), B(S_1), 1_N\}$, $\tau_N(S_2)$ = $\{0_N, \overline{N}(S_2), B(S_2), 1_N\}$ be the neutrosophic nano topologies on U and V respectively. Define an identity map f: (U,$\tau_N$(S₁)) → (V.$\tau_N$(S₂)). Then f is $N_N$RW-continuous but is neither $N_N$-continuous nor $N_N$W-continuous. Similarly it's not an $N_N$R-continuous, $N_N$π -continuous and $N_N$RW-irresolute.

**Proposition 4.6:** (i) Every $N_N$RW-continuous map is $N_N$RG-continuous.

(ii) Every $N_N$RW- continuous map is $N_N$GPR- continuous.

(iii) Every $N_N$RW- continuous map is $N_N$RWG- continuous.

**Proposition 4.7:** If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is $N_N$RW-continuous and g: $(V,\tau_N(T)) \rightarrow (W,\tau_N(R))$ is $N_N$-continuous. Then g°f: f: $(U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is $N_N$RW-continuous.

**Proof:** Let A be an $N_N$-closed in W. Then $g^{-1}(A)$ is $N_N$-closed in V, because g is $N_N$-continuous. Therefore $(g°f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is $N_N$RW-closed in U. Hence g°f is $N_N$RW-continuous.

**Proposition 4.8:** If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is $N_N$RW-continuous and g: $(V,\tau_N(T)) \rightarrow (W,\tau_N(R))$ is $N_N$G-continuous and $(V,\tau_N(T))$ is $N_NT_{1/2}$ then g°f: $(U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is $N_N$RW-continuous.

**Proof:** Let A be an $N_N$-closed set in W, then $g^{-1}(A)$ is $N_N$G-closed in V. Since V is $N_NT_{1/2}$ then $g^{-1}(A)$ is $N_N$-closed in V. Hence, $(g°f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is $N_N$RW-closed in U. Hence g°f is $N_N$RW-continuous.

**Proposition 4.9:** If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is $N_N$RG - irresolute and g: $(V,\tau_N(V)) \rightarrow (W,\tau_N(R))$ is $N_N$RW-continuous, then g°f: $(U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is $N_N$RG-continuous.

**Proof:** Let A be an $N_N$-closed set in W, then $g^{-1}(A)$ is $N_N$RW-closed in V, since g is $N_N$RW-continuous. Every $N_N$RW-closed set is $N_N$RG-closed, $g^{-1}(V)$ is $N_N$RG-closed set in V. Then $(g°f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is NNRG-closed in U, by hypothesis. Hence g°f: $(U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is $N_N$RG-continuous.

**Proposition 4.10:** If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is $N_N$RW-continuous surjection and U is $N_N$RW-connected then V is $N_N$-connected.

**Proof:** Assume that V is not an $N_N$-connected space. Then there exists a proper $N_N$-subset F of V which is both $N_N$-open and $N_N$-closed. Therefore, by hypothesis, $f^{-1}(F)$ is a proper $N_N$-set of U which is both $N_N$RW-open and $N_N$RW-closed in U implies that U is not an $N_N$RW-connected which is a contradiction. This shows that V is $N_N$-connected.

**Definition 4.11:** (i) A mapping f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is said to be $N_N$RW-open map if the image of every $N_N$-open set of U is $N_N$RW-open set in V.

(ii) A mapping f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is said to be $N_N$RW-closed map if the image of every $N_N$-closed set of U is $N_N$RW-closed set in V.

**Proposition 4.12:** A mapping f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is $N_N$RW-open if and only if for every $N_N$-set A of U, $f(N_N Int(A)) \subseteq N_N RW\ Int(f(A))$.

**Proof: Necessity:** Let f be an $N_N$RW-open map and A is an $N_N$-open set in U, $N_N Int(A) \subseteq A$ which implies that $f(N_N Int(A)) \subseteq f(A)$. Since f is an $N_N$RW-open mapping, $f(N_N Int(A))$ is $N_N$RW-open set in V such that $f(N_N Int(A)) \subseteq f(A)$. Therefore $f(N_N Int(A)) \subseteq N_N RWInt\ f(A)$.

**Sufficiency:** Suppose that A is an $N_N$-open set of U. Then $f(A) = f(N_N Int(A) \subseteq N_N RWInt\ f(A)$. But $N_N RWInt\ (f(A)) \subseteq f(A)$. Consequently $f(A) = N_N RWInt(A)$ which implies that f(A) is an $N_N$RW-open set of V and hence f is an $N_N$RW-open map.

**Proposition 4.13:** A mapping f: $(U,\tau_N(S)) \to (V,\tau_N(T))$ is $N_N$RW-open if and only if for every neutrosophic nano set A of V and for each $N_N$-closed set B of U containing $f^{-1}(A)$ there is a $N_N$RW-closed set F of V such that $A \subseteq F$ and $f^{-1}(F) \subseteq B$.

**Proof: Necessity:** Suppose that f is $N_N$RW-open map. Let A be a $N_N$-closed set of V and B be a $N_N$C set of U such that $f^{-1}(A) \subseteq B$. Then $F = f^{-1}(B^c)^c$ is a $N_N$RW- closed set of V such that $f^{-1}(F) \subseteq B$.

**Sufficiency:** Let F be a $N_N$O set of U. Then $f^{-1}(f(F))^c \subseteq F^c$ and $F^c$ is a $N_N$C set in X. By hypothesis there is an $N_N$RW- closed set G of V such that $(f(F))^c \subseteq G$ and $f^{-1}(G) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(G))^c$. Hence $G^c \subseteq f(F) \subseteq f((f^{-1}(G))^c) \subseteq G^c$ i.e., $f(F) = G^c$ which is $N_N$RW-open in V and thus f is $N_N$RW-open map.

**Proposition 4.14:** If a mapping f: $(U,\tau_N(S)) \to (V,\tau_N(T))$ is $N_N$RW-open, then $N_N Int(f^{-1}(G)) \subseteq f^{-1}(N_N RWInt(G))$ for every neutrosophic nano set G of Y.

**Proof:** Let G be neutrosophic nano set of V. Then $N_N Intf^{-1}(G)$ is a $N_N$O set in U. Since f is $N_N$RW – open $f(N_N Intf^{-1}(G)) \subseteq N_N RWInt(f(f^{-1}(G)) \subseteq N_N RWInt(G))$. Thus $N_N Int(f^{-1}(G)) \subseteq f^{-1}(N_N RWInt(G))$.

**Proposition 4.15:** A mapping f: $(U,\tau_N(S)) \to (V,\tau_N(T))$ is $N_N$RW-closed if and only if for every neutrosophic nano set A of V and for each $N_N$O set B of U containing $f^{-1}(A)$ there is a $N_N$RW -open set F of V such that $A \subseteq F$ and $f^{-1}(F) \subseteq B$.

**Proof: Necessity:** Suppose that f is $N_N$RW -closed map. Let A be a $N_N$C set of V and B be a $N_N$O set of U such that $f^{-1}(A) \subseteq B$. Then $F = V \setminus f^{-1}(B^c)$ is a $N_N$RW –open set of V such that $f^{-1}(F) \subseteq B$.

**Sufficiency:** Let F be a $N_N$C set of X. Then $f^{-1}(f(F))^c \subseteq F^c$ and $F^c$ is a $N_N$O set in U. By hypothesis there is an $N_N$RW - open set R of V such that $(f(F))^c \subseteq R$ and $f^{-1}(R) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(R))^c$. Hence $R^c \subseteq f(F) \subseteq f((f^{-1}(R))^c) \subseteq R^c$ i.e., $f(F) = R^c$ which is $N_N$RW -closed in V. Thus f is $N_N$RW -closed map.

**Proposition 4.16:** If f: $(U,\tau_N(S)) \to (V,\tau_N(T))$ is NN-almost irresolute and $N_N$RW-closed map. If A is $N_N$RW-closed set of U, then f(A) is $N_N$RW-closed in V.

**Proof:** Let $f(A) \subseteq R$ where R is an $N_N$RSO set of V. since f is an $N_N$-almost irresolute, $f^{-1}(R)$ is an $N_N$SO set of U such that $A \subseteq f^{-1}(R)$. Since A is $N_N$W-closed set of U which implies that $N_N Cl(A) \subseteq f^{-1}(R) \Rightarrow f(N_N Cl(A)) \subseteq R$, i.e., $N_N Cl(f(N_N Cl(A)) \subseteq R$. Therefore $N_N Cl(f(A)) \subseteq R$ whenever $f(A) \subseteq R$ where R is an $N_N$RSO set of V. Hence f(A) is an $N_N$RW-closed set of V.

**Proposition 4.17:** If f: $(U,\tau_N(S)) \to (V,\tau_N(T))$ is $N_N$-closed and g: $(V,\tau_N(T)) \to (W,\tau_N(R))$ is $N_N$RW-closed then $g^\circ f$: $(U,\tau_N(S)) \to (W,\tau_N(R))$ is $N_N$RW-closed.

**Proof:** Let F be an $N_N$-closed set of neutrosophic nano topological space $(U, _N(S))$. Then f(F) is an $N_N$-closed set of $(V,\tau_N(T))$. By hypothesis, $g^\circ f(F) = g(f(F))$ is an $N_N$RW-closed set in $N_N$-topological space W. Thus $g^\circ f$: $(U,\tau_N(S)) \to (W,\tau_N(R))$ is $N_N$RW-closed.

**Conclusions:** In this article, the authors have introduced and studied the concepts such as, Neutrosophic nano RW- closed set, $N_N$RW-open set, $N_N RWT_{1/2}$ space, $N_N$RW-connected space, $N_N$RW-continuous, $N_N$RW-irresolute, $N_N$RW-open and $N_N$- closed maps. In future it can be extended to some new forms of continuous functions and homeomorphisms.

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