Neutrosophic $\mathcal{N}$-structures over UP-algebras

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Abstract: The notions of (special) neutrosophic $\mathcal{N}$-UP-subalgebras, (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-ideals, and (special) neutrosophic $\mathcal{N}$-strongly UP-ideals of UP-algebras are introduced, and several properties are investigated. Conditions for neutrosophic $\mathcal{N}$-structures to be (special) neutrosophic $\mathcal{N}$-UP-subalgebras, (special) neutrosophic $\mathcal{N}$-near UP-filters, (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-ideals, and (special) neutrosophic $\mathcal{N}$-strongly UP-ideals of UP-algebras are provided. Relations between (special) neutrosophic $\mathcal{N}$-UP-subalgebras (resp., (special) neutrosophic $\mathcal{N}$-near UP-filters, (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-ideals, (special) neutrosophic $\mathcal{N}$-strongly UP-ideals) and their level subsets are considered.

Keywords: UP-algebra; (special) neutrosophic $\mathcal{N}$-UP-subalgebra; (special) neutrosophic $\mathcal{N}$-near UP-filter; (special) neutrosophic $\mathcal{N}$-UP-filter; (special) neutrosophic $\mathcal{N}$-UP-ideal; (special) neutrosophic $\mathcal{N}$-strongly UP-ideal

1. Introduction

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [14], BCI-algebras [15], BCH-algebras [11], KU-algebras [28], SU-algebras [21] and others. They are strongly connected with logic. For example, BCI-algebras were introduced by Işeki [15] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Işeki [14, 15] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

The branch of the logical algebra, UP-algebras was introduced by Iampan [12] in 2017, and it is known that the class of KU-algebras [28] is a proper subclass of the class of UP-algebras. It has been examined by several researchers, for example, Somjanta et al. [32] introduced the notion of fuzzy sets in UP-algebras, the notion of intuitionistic fuzzy sets in UP-algebras was introduced by Kesorn et al. [22], Kaijää et al. [20] introduced the notions of anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras of UP-algebras, the notion of $Q$-fuzzy sets in UP-algebras was introduced by Tanamoon et al. [37], etc.

Neutrosophy provides a foundation for a whole family of new mathematical theories with the generalization of both classical and fuzzy counterparts. In a neutrosophic set, an element has three associated defining functions such as truth membership function ($T$), indeterminate membership
function \((I)\) and false membership function \((F)\) defined on a universe of discourse \(X\). These three functions are independent completely. The concept of neutrosophic logics was first introduced by Smarandache [31] in 1999. Jun et al. [16] introduced a new function, called a negative-valued function, and constructed \(N\)-structures in 2009. Khan et al. [23] discussed neutrosophic \(N\)-structures and their applications in semigroups in 2017. Jun et al. [17, 33] considered neutrosophic \(N\)-structures applied to BCK/BCI-algebras and neutrosophic commutative \(N\)-ideals in BCK-algebras in 2017. Jun et al. [19] studied neutrosophic positive implicative \(N\)-ideals in BCK-algebras in 2018. Abdel-Baset and his colleagues applied the notion of neutrosophic set theory in the new fields (see [1, 2, 3, 4, 5, 6, 27]). Jun and his colleagues applied the notion of neutrosophic set theory in BCK/BCI-algebras (see [8, 18, 24, 26, 35, 36]).

The remaining part of the paper is structured as follows: Section 2 gives some definitions and properties of UP-algebras. Section 3 introduces the notions of neutrosophic \(N\)-UP-subalgebras, neutrosophic \(N\)-near UP-filters, neutrosophic \(N\)-UP-filters, neutrosophic \(N\)-UP-ideals, and neutrosophic \(N\)-strongly UP-ideals of UP-algebras, and a level subset of a neutrosophic \(N\)-structure is proved in Section 4. Section 5 introduces the notions of special neutrosophic \(N\)-UP-subalgebras, special neutrosophic \(N\)-near UP-filters, special neutrosophic \(N\)-UP-filters, special neutrosophic \(N\)-UP-ideals, and special neutrosophic \(N\)-strongly UP-ideals of UP-algebras, and a level subset of a neutrosophic \(N\)-structure of special type is proved in Section 6. This paper has been finalized with that result.

### 2. Basic results on UP-algebras

Before we begin our study, we will give the definition of a UP-algebra.

**Definition 2.1** [12] An algebra \(X = (X, \cdot, 0)\) of type \((2, 0)\) is called a UP-algebra where \(X\) is a nonempty set, \(\cdot\) is a binary operation on \(X\), and 0 is a fixed element of \(X\) (i.e., a nullary operation) if it satisfies the following axioms:

1. **(UP-1)** \((\forall x, y, z \in X)((y \cdot z) \cdot (x \cdot y) = 0)\),
2. **(UP-2)** \((\forall x \in X)(0 \cdot x = x)\),
3. **(UP-3)** \((\forall x \in X)(x \cdot 0 = 0)\), and
4. **(UP-4)** \((\forall x, y \in X)(x \cdot y = 0, y \cdot x = 0) \Rightarrow x = y\).

From [12], we know that the notion of UP-algebras is a generalization of KU-algebras (see [28]).

**Example 2.2** [30] Let \(X\) be a universal set and let \(\Omega = P(X)\) where \(P(X)\) means the power set of \(X\). Let \(P_0(X) = \{A \in P(X) | \Omega \subseteq A\}\). Define a binary operation \(\cdot\) on \(P_0(X)\) by putting \(A \cdot B = B \cap (A^C \cup \Omega)\) for all \(A, B \in P_0(X)\) where \(A^C\) means the complement of a subset \(A\). Then \((P_0(X), \cdot, \Omega)\) is a UP-algebra and we shall call it the generalized power UP-algebra of type 1 with respect to \(\Omega\). Let \(P_1(X) = \{A \in P(X) | A \subseteq \Omega\}\). Define a binary operation \(*\) on \(P_1(X)\) by putting \(A * B = B \cup (A^C \cap \Omega)\) for all \(A, B \in P_1(X)\). Then \((P_1(X), *, \Omega)\) is a UP-algebra and we shall call it the generalized power UP-algebra of type 2 with respect to \(\Omega\). In particular, \((P(X), \cdot, \Omega)\) is a UP-algebra and we shall call it the power UP-algebra of type 1, and \((P(X), *, X)\) is a UP-algebra and we shall call it the power UP-algebra of type 2.

**Example 2.3** [9] Let \(N\) be the set of all natural numbers with two binary operations \(\circ\) and \(\cdot\) defined by

\[
\begin{align*}
(\forall x, y \in N) \left\{ x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases} \right. \quad \text{and} \quad (\forall x, y \in N) \left\{ x \cdot y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise} \end{cases} \right. 
\end{align*}
\]

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Then \((\mathbb{N}, \cdot, 0)\) and \((\mathbb{N}, \cdot, 0)\) are UP-algebras.

**Example 2.4** [25] Let \(X = \{0, 1, 2, 3, 4, 5\}\) be a set with a binary operation \(\cdot\) defined by the following Cayley table:

\[
\begin{array}{cccccc}
\cdot & 0 & 1 & 2 & 3 & 4 & 5 \\
0 & 0 & 1 & 2 & 3 & 4 & 5 \\
1 & 0 & 0 & 2 & 3 & 2 & 5 \\
2 & 0 & 1 & 0 & 3 & 1 & 5 \\
3 & 0 & 1 & 2 & 0 & 4 & 5 \\
4 & 0 & 0 & 0 & 3 & 0 & 5 \\
5 & 0 & 0 & 2 & 0 & 2 & 0 \\
\end{array}
\]

Then \((X, \cdot, 0)\) is a UP-algebra.

For more examples of UP-algebras, see [7, 13, 29, 30].

The following proposition is very important for the study of UP-algebras.

**Proposition 2.5** [12, 13] In a UP-algebra \(X = (X, \cdot, 0)\), the following properties hold:

1. \((\forall x \in X)(x \cdot e = 0)\),
2. \((\forall x, y \in X)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0)\),
3. \((\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0)\),
4. \((\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0)\),
5. \((\forall x, y \in X)(x \cdot (y \cdot x) = 0)\),
6. \((\forall x, y \in X)(y \cdot x \cdot x = 0 \Leftrightarrow x = y \cdot x)\),
7. \((\forall x, y \in X)(x \cdot (y \cdot y) = 0)\),
8. \((\forall a, x, y, z \in X)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0)\),
9. \((\forall a, x, y, z \in X)(((a \cdot x) \cdot (a \cdot y)) \cdot (x \cdot (y \cdot z)) = 0)\),
10. \((\forall x, y, z \in X)((x \cdot (y \cdot z)) \cdot (y \cdot z) = 0)\),
11. \((\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0)\),
12. \((\forall x, y, z \in X)((x \cdot (y \cdot y) \cdot (x \cdot (y \cdot z)) = 0)\),
13. \((\forall a, x, y, z \in X)((x \cdot (y \cdot y) \cdot (y \cdot (a \cdot z)) = 0)\).

On a UP-algebra \(X = (X, \cdot, 0)\), we define a binary relation \(\leq\) on \(X\) [12] as follows:

\((\forall x, y \in X)(x \leq y \Leftrightarrow x \cdot y = 0)\).

**Definition 2.6** [10, 12, 32] A nonempty subset \(S\) of a UP-algebra \((X, \cdot, 0)\) is called

1. a **UP-subalgebra** of \(X\) if \((\forall x, y \in S)(x \cdot y \in S)\).
2. a **near UP-filter** of \(X\) if
   (a) the constant \(0\) of \(X\) is in \(S\), and
   (b) \((\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S)\).
3. a **UP-filter** of \(X\) if
   (a) the constant \(0\) of \(X\) is in \(S\), and
   (b) \((\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S)\).
4. a **UP-ideal** of \(X\) if
   (a) the constant \(0\) of \(X\) is in \(S\), and
   (b) \((\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)\).
5. a **strongly UP-ideal** of \(X\) if
(a) the constant 0 of \( X \) is in \( S \), and
(b) \( \forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S) \).

Guntasow et al. [10] proved that the notion of UP-subalgebras is a generalization of near UP-filters, near UP-filters is a generalization of UP-filters, UP-filters is a generalization of UP-ideals, and UP-ideals is a generalization of strongly UP-ideals. Moreover, they also proved that a UP-algebra \( X \) is the only strongly UP-ideal of itself.

**Theorem 2.7** Let \( \mathcal{N} \) be a nonempty family of near UP-filters of a UP-algebra \( X = (X, \cdot, 0) \). Then \( \cap \mathcal{N} \) and \( \cup \mathcal{N} \) are near UP-filters of \( X \).

**Proof.** Clearly, \( 0 \in \mathcal{N} \) for all \( N \in \mathcal{N} \). Then \( 0 \in \cap \mathcal{N} \). Let \( x \in X \) and \( y \in \cap \mathcal{N} \). Then \( y \in \mathcal{N} \) for all \( N \in \mathcal{N} \). Since \( \cap \mathcal{N} \) is a near UP-filter of \( X \), we have \( x \cdot y \in \mathcal{N} \) for all \( N \in \mathcal{N} \) and so \( x \cdot y \in \cap \mathcal{N} \). Hence, \( \cap \mathcal{N} \) is a near UP-filter of \( X \). Since \( \cap \mathcal{N} \subseteq \cup \mathcal{N} \), we have \( 0 \in \cup \mathcal{N} \). Let \( x \in X \) and \( y \in \cup \mathcal{N} \). Then \( y \in \mathcal{N} \) for some \( N \in \mathcal{N} \). Since \( \cup \mathcal{N} \) is a near UP-filter of \( X \), we have \( x \cdot y \in \cup \mathcal{N} \). Hence, \( \cup \mathcal{N} \) is a near UP-filter of \( X \).

### 3. Neutrosophic \( \mathcal{N} \)-structures

We denote the family of all functions from a nonempty set \( X \) to the closed interval \([-1,0]\) of the real line by \( F(X,[-1,0]) \). An element of \( F(X,[-1,0]) \) is called a **negative-valued function** from \( X \) to \([-1,0]\) (briefly, \( \mathcal{N} \)-function on \( X \)). An ordered pair \((X,f)\) of \( X \) and an \( \mathcal{N} \)-function \( f \) on \( X \) is called an \( \mathcal{N} \)-structure.

A **neutrosophic \( \mathcal{N} \)-structure** over a nonempty universe of discourse \( X \) \([23]\) is defined to be the structure

\[
X_{\mathcal{N}} = \{(x, T_{\mathcal{N}}(x), I_{\mathcal{N}}(x), F_{\mathcal{N}}(x)) \mid x \in X\}
\]

where \( T_{\mathcal{N}}, I_{\mathcal{N}} \) and \( F_{\mathcal{N}} \) are \( \mathcal{N} \)-functions on \( X \) which are called the **negative truth membership function**, the **negative indeterminacy membership function** and the **negative falsity membership function** on \( X \), respectively.

For the sake of simplicity, we will use the notation \( X_{\mathcal{N}} \) or \( X_{\mathcal{N}} = (X, T_{\mathcal{N}}, I_{\mathcal{N}}, F_{\mathcal{N}}) \) instead of the neutrosophic \( \mathcal{N} \)-structure \([16]\).

**Definition 3.1** Let \( X_{\mathcal{N}} \) be a neutrosophic \( \mathcal{N} \)-structure over a nonempty set \( X \). The neutrosophic \( \mathcal{N} \)-structure \( \overline{X}_{\mathcal{N}} = (X, \overline{T}_{N}, \overline{I}_{N}, \overline{F}_{N}) \) defined by

\[
\begin{align*}
\overline{T}_{N}(x) &= -1 - T_{N}(x) \\
\overline{I}_{N}(x) &= -1 - I_{N}(x) \\
\overline{F}_{N}(x) &= -1 - F_{N}(x)
\end{align*}
\]

is called the **complement** of \( X_{\mathcal{N}} \) in \( X \).

**Remark 3.2** For all neutrosophic \( \mathcal{N} \)-structure \( X_{\mathcal{N}} \) over a nonempty set \( X \), we have \( X_{\mathcal{N}} = \overline{X}_{\mathcal{N}} \).

**Lemma 3.3** \([33]\) Let \( f \) be an \( \mathcal{N} \)-function on a nonempty set \( X \). Then the following statements hold:

1. \( \forall x, y \in X \) \((-1 - \max\{f(x), f(y)\} = \min\{-1 - f(x), -1 - f(y)\}) \), and
The following lemmas are easily proved

**Lemma 3.4** Let $f$ be an $N$-function on a nonempty set $X$. Then the following statements hold:

1. $(\forall x, y, z \in X)(\overline{f}(x) \geq \min(\overline{f}(y), \overline{f}(z)) \Rightarrow f(x) \leq \max(f(y), f(z)))$,

2. $(\forall x, y, z \in X)(\overline{f}(x) \leq \min(\overline{f}(y), \overline{f}(z)) \Rightarrow f(x) \geq \max(f(y), f(z)))$,

3. $(\forall x, y, z \in X)(\overline{f}(x) \geq \max(\overline{f}(y), \overline{f}(z)) \Rightarrow f(x) \leq \min(f(y), f(z)))$, and

4. $(\forall x, y, z \in X)(\overline{f}(x) \leq \max(\overline{f}(y), \overline{f}(z)) \Rightarrow f(x) \geq \min(f(y), f(z)))$.

In what follows, let $X$ denote a UP-algebra $(X, \cdot, 0)$ unless otherwise specified.

Now, we introduce the notions of neutrosophic $N$-UP-subalgebras, neutrosophic $N$-near UP-filters, neutrosophic $N$-UP-filters, neutrosophic $N$-UP-ideals, and neutrosophic $N$-strongly UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

**Definition 3.5** A neutrosophic $N$-structure $X_N$ over $X$ is called a **neutrosophic $N$-UP-subalgebra** of $X$ if it satisfies the following conditions:

$$
(\forall x, y \in X)(T_x(x \cdot y) \leq \max(T_x(x), T_x(y))) \quad (3.2)
$$

$$
(\forall x, y \in X)(I_x(x \cdot y) \geq \min(I_x(x), I_x(y))) \quad (3.3)
$$

$$
(\forall x, y \in X)(F_x(x \cdot y) \leq \max(F_x(x), F_x(y))) \quad (3.4)
$$

**Example 3.6** Let $X = \{0, 1, 2, 3, 4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$$
\begin{array}{cccc}
0 & 0 & 1 & 2 & 3 & 4 \\
0 & 0 & 1 & 2 & 3 & 4 \\
1 & 0 & 0 & 2 & 3 & 4 \\
2 & 0 & 0 & 0 & 3 & 4 \\
3 & 0 & 0 & 2 & 0 & 4 \\
4 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic $N$-structure $X_N$ over $X$ as follows:

- $T_x(0) = -0.8$, $I_x(0) = -0.3$, $F_x(0) = -0.8$,
- $T_x(1) = -0.6$, $I_x(1) = -0.7$, $F_x(1) = -0.8$,
- $T_x(2) = -0.4$, $I_x(2) = -0.8$, $F_x(2) = -0.7$,
- $T_x(3) = -0.1$, $I_x(3) = -0.5$, $F_x(3) = -0.5$,
- $T_x(4) = -0.2$, $I_x(4) = -0.9$, $F_x(4) = -0.3$.

Hence, $X_N$ is a neutrosophic $N$-UP-subalgebra of $X$.
Definition 3.7 A neutrosophic $N$-structure $X_N$ over $X$ is called a neutrosophic $N$-near UP-filter of $X$ if it satisfies the following conditions:

\[(\forall x \in X)(T_N(0) \leq T_N(x)),\]  
\[(\forall x \in X)(I_N(0) \geq I_N(x)),\]  
\[(\forall x \in X)(F_N(0) \leq F_N(x)),\]  
\[(\forall x, y \in X)(T_N(x \cdot y) \leq T_N(y)),\]  
\[(\forall x, y \in X)(I_N(x \cdot y) \geq I_N(y)),\]  
\[(\forall x, y \in X)(F_N(x \cdot y) \leq F_N(y)).\]  

Example 3.8 Let $X = \{0, 1, 2, 3, 4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 \\
0 & 0 & 1 & 2 & 3 & 4 \\
1 & 0 & 0 & 2 & 0 & 0 \\
2 & 0 & 1 & 0 & 3 & 1 \\
3 & 0 & 1 & 2 & 0 & 4 \\
4 & 0 & 0 & 0 & 3 & 0 \\
\end{array}
\]

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic $N$-structure $X_N$ over $X$ as follows:

$T_N(0) = -0.8, \ I_N(0) = -0.3, \ F_N(0) = -0.8,$

$T_N(1) = -0.6, \ I_N(1) = -0.7, \ F_N(1) = -0.6,$

$T_N(2) = -0.8, \ I_N(2) = -0.8, \ F_N(2) = -0.7,$

$T_N(3) = -0.1, \ I_N(3) = -0.5, \ F_N(3) = -0.5,$

$T_N(4) = -0.3, \ I_N(4) = -0.8, \ F_N(4) = -0.3.$

Hence, $X_N$ is a neutrosophic $N$-near UP-filter of $X$.

Definition 3.9 A neutrosophic $N$-structure $X_N$ over $X$ is called a neutrosophic $N$-UP-filter of $X$ if it satisfies the following conditions: (3.5), (3.6), (3.7), and

\[(\forall x, y \in X)(T_N(y) \leq \max\{T_N(x \cdot y), T_N(x)\}),\]  
\[(\forall x, y \in X)(I_N(y) \geq \min\{I_N(x \cdot y), I_N(x)\}),\]  
\[(\forall x, y \in X)(F_N(y) \leq \max\{F_N(x \cdot y), F_N(x)\}).\]  

Example 3.10 Let $X = \{0, 1, 2, 3, 4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 \\
0 & 0 & 1 & 2 & 3 & 4 \\
1 & 0 & 0 & 2 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 4 \\
3 & 0 & 1 & 2 & 0 & 4 \\
4 & 0 & 1 & 2 & 3 & 0 \\
\end{array}
\]

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic $N$-structure $X_N$ over $X$ as follows:

$T_N(0) = -0.9, \ I_N(0) = -0.2, \ F_N(0) = -0.8,$

$T_N(1) = -0.5, \ I_N(1) = -0.8, \ F_N(1) = -0.6,$

$T_N(2) = -0.2, \ I_N(2) = -0.6, \ F_N(2) = -0.3,$

$T_N(3) = -0.6, \ I_N(3) = -0.3, \ F_N(3) = -0.7,$

$T_N(4) = -0.8, \ I_N(4) = -0.7, \ F_N(4) = -0.8.$
\[ T_N(4) = -0.7, \quad I_N(4) = -0.3, \quad F_N(4) = -0.8. \]

Hence, \( X_N \) is a neutrosophic \( N \)-UP-filter of \( X \).

**Definition 3.11** A neutrosophic \( N \)-structure \( X_N \) over \( X \) is called a neutrosophic \( N \)-UP-ideal of \( X \) if it satisfies the following conditions: (3.5), (3.6), (3.7), and
\[
(\forall x, y, z \in X)(T_N(x \cdot z) \leq \max\{T_N(x \cdot (y \cdot z)), T_N(y)\}), \tag{3.14}
\]
\[
(\forall x, y, z \in X)(I_N(x \cdot z) \geq \min\{I_N(x \cdot (y \cdot z)), I_N(y)\}), \tag{3.15}
\]
\[
(\forall x, y, z \in X)(F_N(x \cdot z) \leq \max\{F_N(x \cdot (y \cdot z)), F_N(y)\}). \tag{3.16}
\]

**Example 3.12** Let \( X = \{0,1,2,3,4\} \) be a set with a binary operation \( \cdot \) defined by the following Cayley table:


| \( x \) | \( 0 \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) |
|--------|--------|--------|--------|--------|
| \( 0 \) | 0      | 0      | 0      | 0      | 0      |
| \( 1 \) | 0      | 1      | 0      | 3      | 2      |
| \( 2 \) | 0      | 1      | 0      | 3      | 1      |
| \( 3 \) | 0      | 1      | 2      | 0      | 4      |
| \( 4 \) | 0      | 0      | 0      | 3      | 0      |

Then \( (X, \cdot, 0) \) is a UP-algebra. We define a neutrosophic \( N \)-structure \( X_N \) over \( X \) as follows:
\[ T_N(0) = -0.8, \quad I_N(0) = -0.3, \quad F_N(0) = -0.8, \]
\[ T_N(1) = -0.5, \quad I_N(1) = -0.6, \quad F_N(1) = -0.8, \]
\[ T_N(2) = -0.4, \quad I_N(2) = -0.8, \quad F_N(2) = -0.7, \]
\[ T_N(3) = -0.1, \quad I_N(3) = -0.7, \quad F_N(3) = -0.5, \]
\[ T_N(4) = -0.2, \quad I_N(4) = -0.8, \quad F_N(4) = -0.3. \]

Hence, \( X_N \) is a neutrosophic \( N \)-UP-ideal of \( X \).

**Definition 3.13** A neutrosophic \( N \)-structure \( X_N \) over \( X \) is called a neutrosophic \( N \)-strongly UP-ideal of \( X \) if it satisfies the following conditions: (3.5), (3.6), (3.7), and
\[
(\forall x, y, z \in X)(T_N((z \cdot y) \cdot (z \cdot x)) \leq T_N((z \cdot y) \cdot z)), \tag{3.17}
\]
\[
(\forall x, y, z \in X)(I_N((z \cdot y) \cdot (z \cdot x)) \geq I_N((z \cdot y) \cdot z)), \tag{3.18}
\]
\[
(\forall x, y, z \in X)(F_N((z \cdot y) \cdot (z \cdot x)) \leq F_N((z \cdot y) \cdot z)). \tag{3.19}
\]

**Example 3.14** Let \( X = \{0,1,2,3,4\} \) be a set with a binary operation \( \cdot \) defined by the following Cayley table:


| \( x \) | \( 0 \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) |
|--------|--------|--------|--------|--------|
| \( 0 \) | 0      | 1      | 2      | 3      | 4      |
| \( 1 \) | 1      | 1      | 2      | 1      | 4      |
| \( 2 \) | 2      | 2      | 1      | 4      | 0      |
| \( 3 \) | 3      | 3      | 4      | 0      | 2      |
| \( 4 \) | 4      | 4      | 0      | 2      | 3      |

Then \( (X, \cdot, 0) \) is a UP-algebra. We define a neutrosophic \( N \)-structure \( X_N \) over \( X \) as follows:
\[
\begin{cases}
T_N(x) = -1 \\
I_N(x) = -0.3 \\
F_N(x) = -0.7
\end{cases}
\]

Hence, \( X_N \) is neutrosophic \( N \)-strongly UP-ideal of \( X \).
**Definition 3.15** A neutrosophic $\mathcal{N}$-structure $X_N$ over $X$ is said to be constant if $X_N$ is a constant function from $X$ to $[-1, 0]^3$. That is, $T_N, I_N, \text{ and } F_N$ are constant functions from $X$ to $[-1, 0]$.

**Theorem 3.16** Every neutrosophic $\mathcal{N}$-UP-subalgebra of $X$ satisfies the conditions (3.5), (3.6), and (3.7).

**Proof.** Assume that $X_N$ is a neutrosophic $\mathcal{N}$-UP-subalgebra of $X$. Then for all $x \in X$, by Proposition 2.5 (1), (3.2), (3.3), and (3.4), we have

$$
T_N(x) = T_N(x \cdot x) \leq \max[T_N(x), T_N(x)] = T_N(x),
$$

$$
I_N(x) = I_N(x \cdot x) \geq \min[I_N(x), I_N(x)] = I_N(x),
$$

$$
F_N(x) = F_N(x \cdot x) \leq \max[F_N(x), F_N(x)] = F_N(x).
$$

Hence, $X_N$ satisfies the conditions (3.5), (3.6), and (3.7).

**Theorem 3.17** A neutrosophic $\mathcal{N}$-structure $X_N$ over $X$ is constant if and only if it is a neutrosophic $\mathcal{N}$-strongly UP-ideal of $X$.

**Proof.** Assume that $X_N$ is constant. Then for all $x \in X$, $T_N(x) = T_N(0), I_N(x) = I_N(0)$, and $F_N(x) = F_N(0)$ and so $T_N(0) \leq T_N(x), I_N(0) \geq I_N(x)$, and $F_N(0) \leq F_N(x)$. Next, for all $x, y, z \in X$,

$$
T_N(x) = T_N(0) = \max[T_N(0), T_N(0)] = \max[T_N((x \cdot y) \cdot (z \cdot x)), T_N(y)],
$$

$$
I_N(x) = I_N(0) = \min[I_N(0), I_N(0)] = \min[I_N((x \cdot y) \cdot (z \cdot x)), I_N(y)],
$$

$$
F_N(x) = F_N(0) = \max[F_N(0), F_N(0)] = \max[F_N((x \cdot y) \cdot (z \cdot x)), F_N(y)].
$$

Hence, $X_N$ is a neutrosophic $\mathcal{N}$-strongly UP-ideal of $X$.

Conversely, assume that $X_N$ is a neutrosophic $\mathcal{N}$-strongly UP-ideal of $X$. For any $x \in X$, by Proposition 2.5 (1), (UP-2), (UP-3), (3.17), (3.18), and (3.19), we have

$$
T_N(x) \leq \max[T_N((x \cdot 0) \cdot (x \cdot x)), T_N(0)] = \max[T_N(0), T_N(0)] = T_N(0),
$$

$$
I_N(x) \geq \min[I_N((x \cdot 0) \cdot (x \cdot x)), I_N(0)] = \min[I_N(0), I_N(0)] = \min[I_N(x, x), I_N(0)]
$$

$$
= \min[I_N(0), I_N(0)] = I_N(0),
$$

$$
F_N(x) \leq \max[F_N((x \cdot 0) \cdot (x \cdot x)), F_N(0)] = \max[F_N(0), F_N(0)] = \max[F_N(x, x), F_N(0)]
$$

Thus $T_N(x) = T_N(0), I_N(x) = I_N(0)$, and $F_N(x) = F_N(0)$ for all $x \in X$. Hence, $X_N$ is constant.

**Theorem 3.18** Every neutrosophic $\mathcal{N}$-strongly UP-ideal of $X$ is a neutrosophic $\mathcal{N}$-UP-ideal.

**Proof.** Assume that $X_N$ is a neutrosophic $\mathcal{N}$-strong UP-ideal of $X$. Then $X_N$ satisfies the conditions (3.5), (3.6), and (3.7). By Theorem 3.17, we have $X_N$ is constant. Then for all $x \in X$, $T_N(x) = T_N(0), I_N(x) = I_N(0)$, and $F_N(x) = F_N(0)$. By Proposition 2.5 (5), (UP-3), (3.5), (3.6), (3.7), (3.17), (3.18), and (3.19), we have

$$
T_N(x \cdot z) = \max[T_N((z \cdot y) \cdot (z \cdot (x \cdot z))), T_N(y)] = \max[T_N((z \cdot y) \cdot 0), T_N(y)] = \max[T_N(0), T_N(y)] = T_N(y)
$$

$$
\leq \max[T_N(x \cdot (y \cdot z)), T_N(y)],
$$

$$
I_N(x \cdot z) = \min[I_N((z \cdot y) \cdot (z \cdot (x \cdot z))), I_N(y)] = \min[I_N((z \cdot y) \cdot 0), I_N(y)] = \min[I_N(0), I_N(y)] = I_N(y)
$$

$$
\geq \min[I_N(x \cdot (y \cdot z)), I_N(y)],
$$

$$
F_N(x \cdot z) = \max[F_N((z \cdot y) \cdot (z \cdot (x \cdot z))), F_N(y)] = \max[F_N((z \cdot y) \cdot 0), F_N(y)] = \max[F_N(0), F_N(y)] = F_N(y)
$$

$$
\leq \max[F_N(x \cdot (y \cdot z)), F_N(y)].
$$
Hence, $X_N$ is a neutrosophic $N$-UP-ideal of $X$.

The following example show that the converse of Theorem 3.18 is not true.

**Example 3.19** Let $X = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\begin{array}{cccc}
0 & 1 & 2 & 3 \\
0 & 0 & 1 & 2 & 3 \\
1 & 0 & 0 & 2 & 3 \\
2 & 0 & 1 & 0 & 3 \\
3 & 0 & 1 & 2 & 0 \\
\end{array}$

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic $N$-structure $X_N$ over $X$ as follows:

$T_N(0) = -0.6$, $I_N(0) = -0.1$, $F_N(0) = -0.7$, 
$T_N(1) = -0.4$, $I_N(1) = -0.5$, $F_N(1) = -0.5$, 
$T_N(2) = -0.3$, $I_N(2) = -0.4$, $F_N(2) = -0.4$, 
$T_N(3) = -0.2$, $I_N(3) = -0.4$, $F_N(3) = -0.3$.

Hence, $X_N$ is a neutrosophic $N$-UP-ideal of $X$. Since $X_N$ is not constant, it follows from Theorem 3.17 that it is not a neutrosophic $N$-strongly UP-ideal of $X$.

**Theorem 3.20** Every neutrosophic $N$-UP-ideal of $X$ is a neutrosophic $N$-UP-filter.

**Proof.** Assume that $X_N$ is a neutrosophic $N$-UP-ideal of $X$. Then $X_N$ satisfies the conditions (3.5), (3.6), and (3.7). Next, let $x, y \in X$. By (UP-2), (3.14), (3.15), and (3.16), we have

$T_N(y) = T_N(0 \cdot y) \leq \max\{T_N(0 \cdot (x \cdot y)), T_N(x)\} = \max\{T_N(x \cdot y), T_N(x)\}$,

$I_N(y) = I_N(0 \cdot y) \geq \min\{I_N(0 \cdot (x \cdot y)), I_N(x)\} = \min\{I_N(x \cdot y), I_N(x)\}$,

$F_N(y) = F_N(0 \cdot y) \leq \max\{F_N(0 \cdot (x \cdot y)), F_N(x)\} = \max\{F_N(x \cdot y), F_N(x)\}$.

Hence, $X_N$ is a neutrosophic $N$-UP-filter of $X$.

The following example show that the converse of Theorem 3.20 is not true.

**Example 3.21** Let $X = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\begin{array}{cccc}
0 & 1 & 2 & 3 \\
0 & 0 & 1 & 2 & 3 \\
1 & 0 & 0 & 2 & 3 \\
2 & 0 & 1 & 0 & 2 \\
3 & 0 & 1 & 2 & 0 \\
\end{array}$

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic $N$-structure $X_N$ over $X$ as follows:

$T_N(0) = -0.7$, $I_N(0) = -0.1$, $F_N(0) = -0.9$, 
$T_N(1) = -0.6$, $I_N(1) = -0.5$, $F_N(1) = -0.8$, 
$T_N(2) = -0.3$, $I_N(2) = -0.4$, $F_N(2) = -0.5$, 
$T_N(3) = -0.2$, $I_N(3) = -0.4$, $F_N(3) = -0.3$.

Hence, $X_N$ is a neutrosophic $N$-UP-filter of $X$. Since $F_N(2 \cdot 3) = -0.3 > -0.8 = \max\{F_N(2 \cdot (1 \cdot 3)), F_N(1)\}$, we have $X_N$ is not a neutrosophic $N$-UP-ideal of $X$.

**Theorem 3.22** Every neutrosophic $N$-UP-filter of $X$ is a neutrosophic $N$-near UP-filter.
Proof. Assume that \( X_N \) is a neutrosophic \( N \)-UP-filter. Then \( X_N \) satisfies the conditions (3.5), (3.6), and (3.7). Next, let \( x, y \in X \). By Proposition 2.5 (5), (3.5), (3.6), (3.7), (3.11), (3.12), and (3.13), we have
\[
T_N(x \cdot y) \leq \max\{T_N(y \cdot (x \cdot y)), T_N(y)\} = \max\{T_N(0), T_N(y)\} = T_N(y),
\]
\[
I_N(x \cdot y) \geq \min\{I_N(y \cdot (x \cdot y)), I_N(y)\} = \min\{I_N(0), I_N(y)\} = I_N(y),
\]
\[
F_N(x \cdot y) \leq \max\{F_N(y \cdot (x \cdot y)), F_N(y)\} = \max\{F_N(0), F_N(y)\} = F_N(y).
\]
Hence, \( X_N \) is a neutrosophic \( N \)-near UP-filter of \( X \).

The following example show that the converse of Theorem 3.22 is not true.

Example 3.23 Let \( X = \{0,1,2,3\} \) be a set with a binary operation \( \cdot \) defined by the following Cayley table:
\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
0 & 0 & 1 & 2 & 3 \\
1 & 0 & 0 & 1 & 3 \\
2 & 0 & 0 & 0 & 3 \\
3 & 0 & 1 & 1 & 0 \\
\end{array}
\]
Then \( (X, \cdot, 0) \) is a UP-algebra. We define a neutrosophic \( N \)-structure \( X_N \) over \( X \) as follows:
\[
T_N(0) = -0.9, \quad I_N(0) = -0.3, \quad F_N(0) = -0.8,
\]
\[
T_N(1) = -0.5, \quad I_N(1) = -0.7, \quad F_N(1) = -0.7,
\]
\[
T_N(2) = -0.2, \quad I_N(2) = -0.8, \quad F_N(2) = -0.6,
\]
\[
T_N(3) = -0.1, \quad I_N(3) = -0.5, \quad F_N(3) = -0.3.
\]
Hence, \( X_N \) is a neutrosophic \( N \)-near UP-filter of \( X \). Since \( I_N(2) = -0.8 < -0.7 = \min\{I_N(1-2), I_N(1)\} \), we have \( X_N \) is not a neutrosophic \( N \)-UP-filter of \( X \).

Theorem 3.24 Every neutrosophic \( N \)-near UP-filter of \( X \) is a neutrosophic \( N \)-UP-subalgebra.

Proof. Assume that \( X_N \) is a neutrosophic \( N \)-near UP-filter of \( X \). Then for all \( x, y \in X \), by (3.8), (3.9), and (3.10), we have
\[
T_N(x \cdot y) \leq T_N(y) \leq \max\{T_N(x), T_N(y)\},
\]
\[
I_N(x \cdot y) \geq I_N(y) \geq \min\{I_N(x), I_N(y)\},
\]
\[
F_N(x \cdot y) \leq F_N(y) \leq \max\{F_N(x), F_N(y)\}.
\]
Hence, \( X_N \) is a neutrosophic \( N \)-UP-subalgebra of \( X \).

The following example show that the converse of Theorem 3.24 is not true.

Example 3.25 Let \( X = \{0,1,2,3\} \) be a set with a binary operation \( \cdot \) defined by the following Cayley table:
\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
0 & 0 & 1 & 2 & 3 \\
1 & 0 & 0 & 1 & 2 \\
2 & 0 & 0 & 0 & 2 \\
3 & 0 & 0 & 0 & 0 \\
\end{array}
\]
Then \( (X, \cdot, 0) \) is a UP-algebra. We define a neutrosophic \( N \)-structure \( X_N \) over \( X \) as follows:
\[
T_N(0) = -0.8, \quad I_N(0) = -0.3, \quad F_N(0) = -0.8,
\]
\[
T_N(1) = -0.6, \quad I_N(1) = -0.6, \quad F_N(1) = -0.8,
\]
\[
T_N(2) = -0.4, \quad I_N(2) = -0.5, \quad F_N(2) = -0.7,
\]

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\[ T_N(3) = -0.1, \quad I_N(3) = -0.7, \quad F_N(3) = -0.5. \]
Hence, \( X_N \) is a neutrosophic \( \mathcal{N} \)-UP-subalgebra of \( X \). Since \( I_N(1\cdot2) = -0.6 < -0.5 = I_N(2) \), we have \( X_N \) is not a neutrosophic \( \mathcal{N} \)-near UP-filter of \( X \).

By Theorems 3.18, 3.20, 3.22, and 3.24 and Examples 3.19, 3.21, 3.23, and 3.25, we have that the notion of neutrosophic \( \mathcal{N} \)-UP-subalgebras is a generalization of neutrosophic \( \mathcal{N} \)-near UP-filters, neutrosophic \( \mathcal{N} \)-near UP-filters is a generalization of neutrosophic \( \mathcal{N} \)-UP-filters, neutrosophic \( \mathcal{N} \)-UP-filters is a generalization of neutrosophic \( \mathcal{N} \)-UP-ideals, and neutrosophic \( \mathcal{N} \)-UP-ideals is a generalization of neutrosophic \( \mathcal{N} \)-strongly UP-ideals. Moreover, by Theorem 3.17, we obtain that neutrosophic \( \mathcal{N} \)-strongly UP-ideals and constant neutrosophic \( \mathcal{N} \)-structures coincide.

**Theorem 3.26** If \( X_N \) is a neutrosophic \( \mathcal{N} \)-UP-subalgebra of \( X \) satisfying the following condition:

\[
(\forall x, y \in X) \left\{ \begin{array}{l}
T_N(x) \leq T_N(y) \\
I_N(x) \geq I_N(y) \\
F_N(x) \leq F_N(y)
\end{array} \right.
\]

then \( X_N \) is a neutrosophic \( \mathcal{N} \)-near UP-filter of \( X \).

**Proof.** Assume that \( X_N \) is a neutrosophic \( \mathcal{N} \)-UP-subalgebra of \( X \) satisfying the condition (3.20). By Theorem 3.16, we have \( X_N \) satisfies the conditions (3.5), (3.6), and (3.7). Next, let \( x, y \in X \).

**Case 1:** \( x \cdot y = 0 \).
Then, by (3.5), (3.6), and (3.7), we have
\[
T_N(x \cdot y) = T_N(0) \leq T_N(y), \quad I_N(x \cdot y) = I_N(0) \geq I_N(y), \quad F_N(x \cdot y) = F_N(0) \leq F_N(y).
\]

**Case 2:** \( x \cdot y \neq 0 \).
Then, by (3.2), (3.3), (3.4), and (3.20), we have
\[
T_N(x \cdot y) \leq \max{T_N(x), T_N(y)} = T_N(y), \quad I_N(x \cdot y) \geq \min{I_N(x), I_N(y)} = I_N(y), \quad F_N(x \cdot y) \leq \max{F_N(x), F_N(y)} = F_N(y).
\]

Hence, \( X_N \) is a neutrosophic \( \mathcal{N} \)-near UP-filter of \( X \).

**Theorem 3.27** If \( X_N \) is a neutrosophic \( \mathcal{N} \)-near UP-filter of \( X \) satisfying the following condition:

\[
T_N = I_N = F_N,
\]
then \( X_N \) is a neutrosophic \( \mathcal{N} \)-UP-filter of \( X \).

**Proof.** Assume that \( X_N \) is a neutrosophic \( \mathcal{N} \)-near UP-filter of \( X \) satisfying the condition (3.21). Then \( X_N \) satisfies the conditions (3.5), (3.6), and (3.7). Next, let \( x, y \in X \). Then, by (3.8), (3.9), and (3.21), we have
\[
\max{T_N(x \cdot y), T_N(x)} = \max{I_N(x \cdot y), I_N(x)} \leq \max{T_N(y), T_N(x)} = T_N(y),
\]
\[
\min{I_N(x \cdot y), I_N(x)} = \min{T_N(x \cdot y), T_N(x)} \leq \min{T_N(y), I_N(x)} = I_N(y),
\]
\[
\max{F_N(x \cdot y), F_N(x)} = \max{I_N(x \cdot y), F_N(x)} \geq \max{I_N(y), F_N(x)} = F_N(y).
\]

Hence, \( X_N \) is a neutrosophic \( \mathcal{N} \)-UP-filter of \( X \).

**Theorem 3.28** If \( X_N \) is a neutrosophic \( \mathcal{N} \)-UP-filter of \( X \) satisfying the following condition:

\[
(\forall x, y, z \in X) \left\{ \begin{array}{l}
T_N(y \cdot (x \cdot z)) = T_N(x \cdot (y \cdot z)) \\
I_N(y \cdot (x \cdot z)) = I_N(x \cdot (y \cdot z)) \\
F_N(y \cdot (x \cdot z)) = F_N(x \cdot (y \cdot z))
\end{array} \right.
\]

then \( X_N \) is a neutrosophic \( \mathcal{N} \)-UP-ideal of \( X \).
Proof. Assume that \( X_N \) is a neutrosophic \( N \)-UP-filter of \( X \) satisfying the condition (3.22). Then \( X_N \) satisfies the conditions (3.5), (3.6), and (3.7). Next, let \( x, y, z \in X \). Then, by (3.11), (3.12), (3.13), and (3.22), we have
\[
T_N(x \cdot z) \leq \max\{T_N(x \cdot (y \cdot z)), T_N(y)\} = \max\{T_N(x \cdot (y \cdot z)), T_N(y)\}, T_N
\]
\[
I_N(x \cdot z) \geq \min\{I_N(x \cdot (y \cdot z)), I_N(y)\} = \min\{I_N(x \cdot (y \cdot z)), I_N(y)\}, I_N
\]
\[
F_N(x \cdot z) \leq \max\{F_N(x \cdot (y \cdot z)), F_N(y)\} = \max\{F_N(x \cdot (y \cdot z)), F_N(y)\}, F_N
\]
Hence, \( X_N \) is a neutrosophic \( N \)-UP-ideal of \( X \).

Theorem 3.29 If \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the following condition:
\[
(\forall x, y, z \in X) \left\{ z \leq x \cdot y \Rightarrow \begin{align*}
T_N(z) &\leq \max\{T_N(x), T_N(y)\}, \\
I_N(z) &\geq \min\{I_N(x), I_N(y)\}, \\
F_N(z) &\leq \max\{F_N(x), F_N(y)\}
\end{align*} \right. \tag{3.23}
\]
then \( X_N \) is a neutrosophic \( N \)-UP-subalgebra of \( X \).

Proof. Assume that \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the condition (3.23). Let \( x, y \in X \). By Proposition 2.5 (1), we have \((x \cdot y) \cdot (x \cdot y) = 0\), that is, \( x \cdot y \leq x \cdot y \). It follows from (3.23) that
\[
T_N(x \cdot y) \leq \max\{T_N(x), T_N(y)\}, I_N(x \cdot y) \geq \min\{I_N(x), I_N(y)\}, F_N(x \cdot y) \leq \max\{F_N(x), F_N(y)\}.
\]
Hence, \( X_N \) is a neutrosophic \( N \)-UP-subalgebra of \( X \).

Theorem 3.30 If \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the following condition:
\[
(\forall x, y, z \in X) \left\{ z \leq x \cdot y \Rightarrow \begin{align*}
T_N(z) &\leq T_N(y), \\
I_N(z) &\geq I_N(y), \\
F_N(z) &\leq F_N(y)
\end{align*} \right. \tag{3.24}
\]
then \( X_N \) is a neutrosophic \( N \)-near UP-filter of \( X \).

Proof. Assume that \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the condition (3.24). Let \( x \in X \). By (UP-2) and Proposition 2.5 (1), we have \( 0 \cdot (x \cdot x) = 0 \), that is, \( 0 \leq x \cdot x \). It follows from (3.24) that \( T_N(0) \leq T_N(x), I_N(0) \geq I_N(x) \), and \( F_N(0) \leq F_N(x) \). Next, let \( x, y \in X \). By Proposition 2.5 (1), we have \((x \cdot y) \cdot (x \cdot y) = 0\), that is, \( x \cdot y \leq x \cdot y \). It follows from (3.24) that
\[
T_N(x \cdot y) \leq T_N(y), I_N(x \cdot y) \geq I_N(y), F_N(x \cdot y) \leq F_N(y).
\]
Hence, \( X_N \) is a neutrosophic \( N \)-near UP-filter of \( X \).

Theorem 3.31 If \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the following condition:
\[
(\forall x, y, z \in X) \left\{ z \leq x \cdot y \Rightarrow \begin{align*}
T_N(y) &\leq \max\{T_N(z), T_N(x)\}, \\
I_N(y) &\geq \min\{I_N(z), I_N(x)\}, \\
F_N(y) &\leq \max\{F_N(z), F_N(x)\}
\end{align*} \right. \tag{3.25}
\]
then \( X_N \) is a neutrosophic \( N \)-UP-filter of \( X \).

Proof. Assume that \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the condition (3.25). Let \( x \in X \). By (UP-3), we have \( x \cdot (x \cdot 0) = 0 \), that is, \( x \leq x \cdot 0 \). It follows from (3.25) that
\[
T_N(0) \leq \max\{T_N(x), T_N(x)\}, I_N(0) \geq \min\{I_N(x), I_N(x)\} = I_N(x), \\
F_N(0) \leq \max\{F_N(x), F_N(x)\} = F_N(x).
\]
Next, let \( x, y \in X \). By Proposition 2.5 (1), we have \((x \cdot y) \cdot (x \cdot y) = 0\), that is, \( x \cdot y \leq x \cdot y \). It follows from (3.25) that
\[
T_n(y) \leq \max\{T_n(x \cdot y), T_n(x)\}, \quad I_n(y) \geq \min\{I_n(x \cdot y), I_n(x)\}, \quad F_n(y) \leq \max\{F_n(x \cdot y), F_n(x)\}.
\]
Hence, \( X_n \) is a neutrosophic \( N \)-UP-filter of \( X \).

**Theorem 3.32** If \( X_n \) is a neutrosophic \( N \)-structure over \( X \) satisfying the following condition:

\[
\forall x, y, z \in X \quad \begin{cases} a \leq x \cdot (y \cdot z) \Rightarrow \begin{bmatrix} T_n(x \cdot z) \leq \max\{T_n(a), T_n(y)\} \\ I_n(x \cdot z) \geq \min\{I_n(a), I_n(y)\} \\ F_n(x \cdot z) \leq \max\{F_n(a), F_n(y)\} \end{bmatrix} \end{cases}, \tag{3.26}
\]

then \( X_n \) is a neutrosophic \( N \)-UP-ideal of \( X \).

**Proof.** Assume that \( X_n \) is a neutrosophic \( N \)-structure over \( X \) satisfying the condition (3.26). Let \( x \in X \). By (UP-3), we have \( x \cdot (0 \cdot x) = 0 \), that is \( x \cdot (0 \cdot x) \leq 0 \). It follows from (3.26) and (UP-2) that
\[
T_n(0) = T_n(0 \cdot 0) \leq \max\{T_n(x), T_n(x)\} = T_n(x), \quad I_n(0) = I_n(0 \cdot 0) \geq \min\{I_n(x), I_n(x)\} = I_n(x),
\]
\[
F_n(0) = F_n(0 \cdot 0) \leq \max\{F_n(x), F_n(x)\} = F_n(x).
\]
Next, let \( x, y, z \in X \). By Proposition 2.5 (1), we have \((x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0\), that is \( x \cdot (y \cdot z) \leq x \cdot (y \cdot z) \). It follows from (3.26) that
\[
T_n(x \cdot z) \leq \max\{T_n(x \cdot (y \cdot z)), T_n(y)\}, \quad I_n(x \cdot z) \geq \min\{I_n(x \cdot (y \cdot z)), I_n(y)\},
\]
\[
F_n(x \cdot z) \leq \max\{F_n(x \cdot (y \cdot z)), F_n(y)\}.
\]
Hence, \( X_n \) is a neutrosophic \( N \)-UP-ideal of \( X \).

For any fixed numbers \( \alpha^-, \alpha^+, \beta^-, \beta^+, \gamma^-, \gamma^+ \in [-1,0] \) such that \( \alpha^- < \alpha^+ < \beta^- < \beta^+ < \gamma^- < \gamma^+ \) and a nonempty subset \( G \) of \( X \), a neutrosophic \( N \)-structure \( X_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] \) over \( X \) where \( T_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] \), \( I_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] \), and \( F_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] \) are \( N \)-functions on \( X \) which are given as follows:

\[
T_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (x) = \begin{cases} \alpha^- & \text{if } x \in G, \\ \alpha^+ & \text{otherwise,} \end{cases} \quad I_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (x) = \begin{cases} \beta^- & \text{if } x \in G, \\ \beta^+ & \text{otherwise,} \end{cases} \quad F_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (x) = \begin{cases} \gamma^- & \text{if } x \in G, \\ \gamma^+ & \text{otherwise.} \end{cases}
\]

**Lemma 3.33** If the constant \( 0 \) of \( X \) is in a nonempty subset \( G \) of \( X \), then a neutrosophic \( N \)-structure \( X_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] \) over \( X \) satisfies the conditions (3.5), (3.6), and (3.7).

**Proof.** If \( 0 \in G \), then \( T_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (0) = \alpha^- , \quad I_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (0) = \beta^- , \quad F_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (0) = \gamma^- \). Thus
\[
\begin{align*}
\forall x \in X \quad & \begin{bmatrix} T_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (x) = \alpha^- & \leq T_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (x) \\ I_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (x) = \beta^- & \geq I_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (x) \\ F_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (x) = \gamma^- & \leq F_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] (x) \end{bmatrix}
\end{align*}
\]
Hence, \( X_n[\alpha^- \beta^-, \beta^+, \gamma^- \gamma^+] \) satisfies the conditions (3.5), (3.6), and (3.7).
Lemma 3.34 If a neutrosophic $\mathbb{N}$-structure $X^G_{\alpha^+\beta^\gamma}$ over $X$ satisfies the condition (3.5) (resp., (3.6), (3.7)), then the constant $0$ of $X$ is in a nonempty subset $G$ of $X$.

Proof. Assume that the neutrosophic $\mathbb{N}$-structure $X^G_{\alpha^+\beta^\gamma}$ over $X$ satisfies the condition (3.5).

Then $T^G_{\alpha^+\beta^\gamma}(0) \leq T^G_{\alpha^+\beta^\gamma}(x)$ for all $x \in X$. Since $G$ is nonempty, there exists $g \in G$. Thus $T^G_{\alpha^+\beta^\gamma}(g) = \alpha^-$, so $T^G_{\alpha^+\beta^\gamma}(0) \leq T^G_{\alpha^+\beta^\gamma}(g) = \alpha^- \leq T^G_{\alpha^+\beta^\gamma}(0)$, that is, $T^G_{\alpha^+\beta^\gamma}(0) = \alpha^-$. Hence, $0 \in G$.

Theorem 3.35 A neutrosophic $\mathbb{N}$-structure $X^G_{\alpha^+\beta^\gamma}$ over $X$ is a neutrosophic $\mathbb{N}$-UP-subalgebra of $X$ if and only if a nonempty subset $G$ of $X$ is a UP-subalgebra of $X$.

Proof. Assume that $X^G_{\alpha^+\beta^\gamma}$ is a neutrosophic $\mathbb{N}$-UP-subalgebra of $X$. Let $x, y \in G$. Then $T^G_{\alpha^+\beta^\gamma}(x) = \alpha^- = T^G_{\alpha^+\beta^\gamma}(y)$. Thus, by (3.2), we have

$$T^G_{\alpha^+\beta^\gamma}(x \cdot y) \leq \max\{T^G_{\alpha^+\beta^\gamma}(x), T^G_{\alpha^+\beta^\gamma}(y)\} = \alpha^- \leq T^G_{\alpha^+\beta^\gamma}(x \cdot y)$$

and so $T^G_{\alpha^+\beta^\gamma}(x \cdot y) = \alpha^-$. Thus $x \cdot y \in G$. Hence, $G$ is a UP-subalgebra of $X$.

Conversely, assume that $G$ is a UP-subalgebra of $X$. Let $x, y \in X$.

Case 1: $x, y \in G$. Then $T^G_{\alpha^+\beta^\gamma}(x) = \alpha^- = T^G_{\alpha^+\beta^\gamma}(y)$, $I^G_{\alpha^+\beta^\gamma}(x) = \beta^+ = I^G_{\alpha^+\beta^\gamma}(y)$, $F^G_{\alpha^+\beta^\gamma}(x) = \gamma^- = F^G_{\alpha^+\beta^\gamma}(y)$.

Thus

$$\max\{T^G_{\alpha^+\beta^\gamma}(x), T^G_{\alpha^+\beta^\gamma}(y)\} = \alpha^- \quad \text{and} \quad \min\{I^G_{\alpha^+\beta^\gamma}(x), I^G_{\alpha^+\beta^\gamma}(y)\} = \beta^+ \quad \text{and} \quad \max\{F^G_{\alpha^+\beta^\gamma}(x), F^G_{\alpha^+\beta^\gamma}(y)\} = \gamma^-.$$

Since $G$ is a UP-subalgebra of $X$, we have $x \cdot y \in G$ and so $T^G_{\alpha^+\beta^\gamma}(x \cdot y) = \alpha^-, I^G_{\alpha^+\beta^\gamma}(x \cdot y) = \beta^+$, and $F^G_{\alpha^+\beta^\gamma}(x \cdot y) = \gamma^-$. Hence,

$$T^G_{\alpha^+\beta^\gamma}(x \cdot y) = \alpha^- \leq \alpha^+ = \max\{T^G_{\alpha^+\beta^\gamma}(x), T^G_{\alpha^+\beta^\gamma}(y)\}, \quad I^G_{\alpha^+\beta^\gamma}(x \cdot y) = \beta^+ \geq \beta^+ = \min\{I^G_{\alpha^+\beta^\gamma}(x), I^G_{\alpha^+\beta^\gamma}(y)\},$$

$$F^G_{\alpha^+\beta^\gamma}(x \cdot y) = \gamma^- \leq \gamma^- = \max\{F^G_{\alpha^+\beta^\gamma}(x), F^G_{\alpha^+\beta^\gamma}(y)\}.$$

Case 2: $x \notin G \lor y \notin G$. Then $T^G_{\alpha^+\beta^\gamma}(x) = \alpha^+ \lor T^G_{\alpha^+\beta^\gamma}(y) = \alpha^+$, $I^G_{\alpha^+\beta^\gamma}(x) = \beta^+ \lor I^G_{\alpha^+\beta^\gamma}(y) = \beta^+$, $F^G_{\alpha^+\beta^\gamma}(x) = \gamma^- \lor F^G_{\alpha^+\beta^\gamma}(y) = \gamma^-$. Thus

$$\max\{T^G_{\alpha^+\beta^\gamma}(x), T^G_{\alpha^+\beta^\gamma}(y)\} = \alpha^+ \quad \text{and} \quad \min\{I^G_{\alpha^+\beta^\gamma}(x), I^G_{\alpha^+\beta^\gamma}(y)\} = \beta^+ \quad \text{and} \quad \max\{F^G_{\alpha^+\beta^\gamma}(x), F^G_{\alpha^+\beta^\gamma}(y)\} = \gamma^-.$$

Therefore,
\[ T_N^{G^x_{[a^\alpha,\beta^\gamma]}}(x \cdot y) \leq \alpha^+ = \max\{T_N^{G^x_{[a^\alpha,\beta^\gamma]}}(x), T_N^{G^y_{[a^\gamma,\beta^\alpha]}}(y)\}, \quad I_N^{G^y_{[\beta^\alpha,\gamma^\beta]}}(x \cdot y) \geq \beta^+ = \min\{I_N^{G^y_{[\beta^\alpha,\gamma^\beta]}}(x), I_N^{G^x_{[\gamma^\beta,\alpha^\alpha]}}(y)\}, \]

\[ F_N^{G^y_{[\gamma^\beta,\alpha^\alpha]}}(x \cdot y) \leq \gamma^+ = \max\{F_N^{G^y_{[\gamma^\beta,\alpha^\alpha]}}(x), F_N^{G^x_{[\alpha^\alpha,\gamma^\beta]}}(y)\}. \]

Hence, \( X_N^{G^x_{[a^\alpha,\beta^\gamma]}} \) is a neutrosophic \( \mathcal{N} \)-UP-subalgebra of \( X \).

**Theorem 3.36** A neutrosophic \( \mathcal{N} \)-structure \( X_N^{G^x_{[a^\alpha,\beta^\gamma]}} \) over \( X \) is a neutrosophic \( \mathcal{N} \)-near UP-filter of \( X \) if and only if a nonempty subset \( G \) of \( X \) is a near UP-filter of \( X \).

**Proof.** Assume that \( X_N^{G^x_{[a^\alpha,\beta^\gamma]}} \) is a neutrosophic \( \mathcal{N} \)-near UP-filter of \( X \). Since \( X_N^{G^x_{[a^\alpha,\beta^\gamma]}} \) satisfies the condition (3.5), it follows from Lemma 3.34 that \( 0 \in G \). Next, let \( x \in X \) and \( y \in G \).

Then \( T_N^{G^x_{[a^\alpha]}}(y) = \alpha^+ \). Thus, by (3.8), we have \( T_N^{G^x_{[a^\alpha]}}(x \cdot y) \leq T_N^{G^y_{[a^\gamma]}}(y) = \alpha^+ \leq T_N^{G^y_{[a^\gamma]}}(x \cdot y) \)

and so \( T_N^{G^x_{[a^\alpha]}}(x \cdot y) = \alpha^+ \). Thus \( x \cdot y \in G \). Hence, \( G \) is a near UP-filter of \( X \).

Conversely, assume that \( G \) is a near UP-filter of \( X \). Since \( 0 \in G \), it follows from Lemma 3.33 that \( X_N^{G^x_{[a^\alpha,\beta^\gamma]}} \) satisfies the conditions (3.5), (3.6), and (3.7). Next, let \( x, y \in X \).

**Case 1:** \( y \in G \). Then \( T_N^{G^x_{[a^\alpha]}}(y) = \alpha^+, I_N^{G^y_{[\beta^\gamma]}}(y) = \beta^+, \) and \( F_N^{G^y_{[\gamma^\beta]}}(y) = \gamma^+ \). Since \( G \) is a near UP-filter of \( X \), we have \( x \cdot y \in G \) and so \( T_N^{G^x_{[a^\alpha]}}(x \cdot y) = \alpha^+, I_N^{G^y_{[\beta^\gamma]}}(x \cdot y) = \beta^+, \) and \( F_N^{G^y_{[\gamma^\beta]}}(x \cdot y) = \gamma^+ \).

Thus

\[ T_N^{G^x_{[a^\alpha]}}(x \cdot y) = \alpha^- \leq \alpha^+ = T_N^{G^y_{[a^\gamma]}}(y), \quad I_N^{G^y_{[\beta^\gamma]}}(x \cdot y) = \beta^+ \leq \beta^+ = I_N^{G^x_{[\gamma^\beta]}}(y), \]

\[ F_N^{G^y_{[\gamma^\beta]}}(x \cdot y) = \gamma^- \leq \gamma^+ = F_N^{G^x_{[\alpha^\alpha]}}(y). \]

**Case 2:** \( y \not\in G \). Then \( T_N^{G^x_{[a^\alpha]}}(y) = \alpha^+, I_N^{G^y_{[\beta^\gamma]}}(y) = \beta^-, \) and \( F_N^{G^y_{[\gamma^\beta]}}(y) = \gamma^+ \). Thus

\[ T_N^{G^x_{[a^\alpha]}}(x \cdot y) \leq \alpha^+ = T_N^{G^y_{[a^\gamma]}}(y), \quad I_N^{G^y_{[\beta^\gamma]}}(x \cdot y) \geq \beta^- = I_N^{G^x_{[\gamma^\beta]}}(y), \quad F_N^{G^y_{[\gamma^\beta]}}(x \cdot y) \leq \gamma^- = F_N^{G^x_{[\alpha^\alpha]}}(y). \]

Hence, \( X_N^{G^x_{[a^\alpha,\beta^\gamma]}} \) is a neutrosophic \( \mathcal{N} \)-near UP-filter of \( X \).

**Theorem 3.37** A neutrosophic \( \mathcal{N} \)-structure \( X_N^{G^x_{[a^\alpha,\beta^\gamma]}} \) over \( X \) is a neutrosophic \( \mathcal{N} \)-UP-filter of \( X \) if and only if a nonempty subset \( G \) of \( X \) is a UP-filter of \( X \).

**Proof.** Assume that \( X_N^{G^x_{[a^\alpha,\beta^\gamma]}} \) is a neutrosophic \( \mathcal{N} \)-UP-filter of \( X \). Since \( X_N^{G^x_{[a^\alpha,\beta^\gamma]}} \) satisfies the condition (3.5), it follows from Lemma 3.34 that \( 0 \in G \). Next, let \( x, y \in X \) be such that \( x \cdot y \in G \) and \( x \in G \). Then \( T_N^{G^x_{[a^\alpha]}}(x \cdot y) = \alpha^- = T_N^{G^y_{[a^\gamma]}}(x) \). Thus, by (3.11), we have

\[ T_N^{G^x_{[a^\alpha]}}(x \cdot y) = \alpha^- = T_N^{G^y_{[a^\gamma]}}(x) \].
\[ T_N^{G}[\alpha^*](y) \leq \max\{ T_N^{G}[\alpha^*](x \cdot y), T_N^{G}[\alpha^*](x) \} = \alpha^* \leq T_N^{G}[\alpha^*](y) \]

and so \( T_N^{G}[\alpha^*](y) = \alpha^* \). Thus \( y \in G \). Hence, \( G \) is a UP-filter of \( X \).

Conversely, assume that \( G \) is a UP-filter of \( X \). Since \( 0 \in G \), it follows from Lemma 3.33 that \( X_N^{G}[\alpha, \beta, \gamma] \) satisfies the conditions (3.5), (3.6), and (3.7). Next, let \( x, y \in X \).

**Case 1:** \( x \cdot y \in G \) and \( x \in G \). Then
\[
T_N^{G}[\alpha^*](x \cdot y) = \alpha^* = T_N^{G}[\alpha^*](x), \quad I_N^{G}[\beta^*](x \cdot y) = \beta^* = I_N^{G}[\beta^*](x), \quad F_N^{G}[\gamma^*](x \cdot y) = \gamma^* = F_N^{G}[\gamma^*](x).
\]

Since \( G \) is a UP-filter of \( X \), we have \( y \in G \) and so \( T_N^{G}[\alpha^*](y) = \alpha^* \), \( I_N^{G}[\beta^*](y) = \beta^+ \), and \( F_N^{G}[\gamma^*](y) = \gamma^- \). Thus
\[
T_N^{G}[\alpha^*](y) = \alpha^* \leq \alpha^* = \max\{ T_N^{G}[\alpha^*](x \cdot y), T_N^{G}[\alpha^*](x) \} = \alpha^*, \quad I_N^{G}[\beta^*](y) = \beta^+ \geq \beta^+ = \min\{ I_N^{G}[\beta^*](x \cdot y), I_N^{G}[\beta^*](x) \}, \quad F_N^{G}[\gamma^*](y) = \gamma^- = \max\{ F_N^{G}[\gamma^*](x \cdot y), F_N^{G}[\gamma^*](x) \}.
\]

Thus
\[
\max\{ T_N^{G}[\alpha^*](x \cdot y), T_N^{G}[\alpha^*](x) \} = \alpha^*, \quad \min\{ I_N^{G}[\beta^*](x \cdot y), I_N^{G}[\beta^*](x) \} = \beta^+, \quad \max\{ F_N^{G}[\gamma^*](x \cdot y), F_N^{G}[\gamma^*](x) \} = \gamma^-.
\]

Therefore,
\[
T_N^{G}[\alpha^*](y) \leq \alpha^* = \max\{ T_N^{G}[\alpha^*](x \cdot y), T_N^{G}[\alpha^*](x) \}, \quad I_N^{G}[\beta^*](y) \geq \beta^+ = \min\{ I_N^{G}[\beta^*](x \cdot y), I_N^{G}[\beta^*](x) \}, \quad F_N^{G}[\gamma^*](y) \leq \gamma^- = \max\{ F_N^{G}[\gamma^*](x \cdot y), F_N^{G}[\gamma^*](x) \}.
\]

Hence, \( X_N^{G}[\alpha, \beta, \gamma] \) is a neutrosophic \( N \)-UP-filter of \( X \).

**Theorem 3.38** A neutrosophic \( N \)-structure \( X_N^{G}[\alpha, \beta, \gamma] \) over \( X \) is a neutrosophic \( N \)-UP-ideal of \( X \) if and only if a nonempty subset \( G \) of \( X \) is a UP-ideal of \( X \).

**Proof.** Assume that \( X_N^{G}[\alpha, \beta, \gamma] \) is a neutrosophic \( N \)-UP-ideal of \( X \). Since \( X_N^{G}[\alpha, \beta, \gamma] \) satisfies the condition (3.5), it follows from Lemma 3.34 that \( 0 \in G \). Next, let \( x, y, z \in X \) be such that \( x \cdot (y \cdot z) \in G \) and \( y \in G \). Then \( T_N^{G}[\alpha^*](x \cdot (y \cdot z)) = \alpha^* = T_N^{G}[\alpha^*](y) \). Thus, by (3.17), we have

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\[ T_N^{G_a}(x \cdot z) \leq \max\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\} = \alpha^r \leq T_N^{G_a}(x \cdot z) \]

and so \( T_N^{G_a}(x \cdot z) = \alpha^r \). Thus \( x \cdot z \in G \). Hence, \( G \) is a UP-ideal of \( X \).

Conversely, assume that \( G \) is a UP-ideal of \( X \). Since \( 0 \in G \), it follows from Lemma 3.33 that \( X^{G_a, \beta, \gamma} \) satisfies the conditions (3.5), (3.6), and (3.7). Next, let \( x, y, z \in X \).

**Case 1:** \( x \cdot (y \cdot z) \in G \) and \( y \in G \). Then

\[
T_N^{G_a}(x \cdot (y \cdot z)) = \alpha^r = T_N^{G_a}(x \cdot (y \cdot z))(y), \quad T_N^{G_a}(x \cdot (y \cdot z)) = \beta^r = T_N^{G_a}(x \cdot (y \cdot z))(y), \quad T_N^{G_a}(x \cdot (y \cdot z)) = \gamma^r = T_N^{G_a}(x \cdot (y \cdot z))(y). \]

Thus

\[
\max\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\} = \alpha^r, \quad \min\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\} = \beta^r, \quad \max\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\} = \gamma^r. \]

Since \( G \) is a UP-ideal of \( X \), we have \( x \cdot z \in G \) and so \( T_N^{G_a}(x \cdot z) = \alpha^r, T_N^{G_a}(x \cdot z) = \beta^r, \) and \( T_N^{G_a}(x \cdot z) = \gamma^r \). Thus

\[
T_N^{G_a}(x \cdot z) = \alpha^r \leq \alpha^r = \max\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\}, \quad T_N^{G_a}(x \cdot z) = \beta^r \geq \beta^r = \min\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\}, \quad T_N^{G_a}(x \cdot z) = \gamma^r \leq \gamma^r = \max\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\}. \]

**Case 2:** \( x \cdot (y \cdot z) \not\in G \) or \( y \not\in G \). Then

\[
T_N^{G_a}(x \cdot (y \cdot z)) = \alpha^r \quad \text{or} \quad T_N^{G_a}(x \cdot (y \cdot z))(y) = \alpha^r, \quad T_N^{G_a}(x \cdot (y \cdot z)) = \beta^r \quad \text{or} \quad T_N^{G_a}(x \cdot (y \cdot z))(y) = \beta^r, \quad F_N^{G_a}(x \cdot (y \cdot z)) = \gamma^r \quad \text{or} \quad F_N^{G_a}(x \cdot (y \cdot z))(y) = \gamma^r. \]

Thus

\[
\max\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\} = \alpha^r, \quad \min\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\} = \beta^r, \quad \max\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\} = \gamma^r. \]

Therefore,

\[
T_N^{G_a}(x \cdot z) \leq \alpha^r = \max\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\}, \quad T_N^{G_a}(x \cdot z) \geq \beta^r = \min\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\}, \quad T_N^{G_a}(x \cdot z) \leq \gamma^r = \max\{ T_N^{G_a}(x \cdot (y \cdot z)), T_N^{G_a}(y)\}. \]
Hence, $X_N^{G}[a^{-,\beta^{-,\gamma^{-}}}]$ is a neutrosophic $N$-UP-ideal of $X$.

**Theorem 3.39** A neutrosophic $N$-structure $X_N^{G}[a^{-,\beta^{-,\gamma^{-}}}]$ over $X$ is a neutrosophic $N$-strongly UP-ideal of $X$ if and only if a nonempty subset $G$ of $X$ is a strongly UP-ideal of $X$.

**Proof.** Assume that $X_N^{G}[a^{-,\beta^{-,\gamma^{-}}}]$ is a neutrosophic $N$-strongly UP-ideal of $X$. By Theorem 3.17, we have $X_N^{G}[a^{-,\beta^{-,\gamma^{-}}}]$ is constant, that is, $T_N^{G}[a^{-}]$ is constant. Since $G$ is nonempty, we have $T_N^{G}[a^{-}](x) = \alpha^{-}$ for all $x \in X$. Thus $G = X$. Hence, $G$ is a strongly UP-ideal of $X$.

Conversely, assume that $G$ is a strongly UP-ideal of $X$. Then $G = X$, so

\[
\begin{align*}
T_N^{G}[a^{-}](x) &= \alpha^{-} \\
(I_N^{G}[\beta^{-}](x)) &= \beta^{+} \\
(F_N^{G}[\gamma^{-}](x)) &= \gamma^{-}
\end{align*}
\]

Thus $T_N^{G}[a^{-}], I_N^{G}[\beta^{-}], \text{and } F_N^{G}[\gamma^{-}]$ are constant, that is, $X_N^{G}[a^{-,\beta^{-,\gamma^{-}}}]$ is constant. By Theorem 3.17, we have $X_N^{G}[a^{-,\beta^{-,\gamma^{-}}}]$ is a neutrosophic $N$-strongly UP-ideal of $X$.

**4. Level subsets of a neutrosophic $N$-structure**

In this section, we discuss the relationships among neutrosophic $N$-UP-subalgebras (resp., neutrosophic $N$-near UP-filters, neutrosophic $N$-UP-filters, neutrosophic $N$-UP-ideals, neutrosophic $N$-strongly UP-ideals) of UP-algebras and their level subsets.

**Definition 4.1** [34] Let $f$ be an $N$-function on a nonempty set $X$. For any $t \in [-1, 0]$, the sets

\[
U(f; t) = \{x \in X \mid f(x) \geq t\}, \quad L(f; t) = \{x \in X \mid f(x) \leq t\}, \quad E(f; t) = \{x \in X \mid f(x) = t\}
\]

are called an upper $t$-level subset, a lower $t$-level subset, and an equal $t$-level subset of $f$, respectively.

**Theorem 4.2** A neutrosophic $N$-structure $X_N$ over $X$ is a neutrosophic $N$-UP-subalgebra of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1, 0]$, the sets $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are UP-subalgebras of $X$ if $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are nonempty.

**Proof.** Assume that $X_N$ is a neutrosophic $N$-UP-subalgebra of $X$. Let $\alpha, \beta, \gamma \in [-1, 0]$ be such that $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are nonempty.

Let $x, y \in L(T_N; \alpha)$. Then $T_N(x) \leq \alpha$ and $T_N(y) \leq \alpha$, so $\alpha$ is an upper bound of $\{T_N(x), T_N(y)\}$. By (3.2), we have $T_N(x \cdot y) \leq \max\{T_N(x), T_N(y)\} \leq \alpha$. Thus $x \cdot y \in L(T_N; \alpha)$.

Let $x, y \in U(I_N; \beta)$. Then $I_N(x) \geq \beta$ and $I_N(y) \geq \beta$, so $\beta$ is a lower bound of $\{I_N(x), I_N(y)\}$. By (3.3), we have $I_N(x \cdot y) \geq \min\{I_N(x), I_N(y)\} \geq \beta$. Thus $x \cdot y \in U(I_N; \beta)$.

Let $x, y \in L(F_N; \gamma)$. Then $F_N(x) \leq \gamma$ and $F_N(y) \leq \gamma$, so $\gamma$ is an upper bound of $\{F_N(x), F_N(y)\}$. By (3.4), we have $F_N(x \cdot y) \leq \min\{F_N(x), F_N(y)\} \leq \gamma$. Thus $x \cdot y \in L(F_N; \gamma)$.

Hence, $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are UP-subalgebras of $X$. 
Conversely, assume that for all \( \alpha, \beta, \gamma \in [-1,0] \), the sets \( L(T_N; \alpha), U(I_N; \beta) \), and \( L(F_N; \gamma) \) are UP-subalgebras of \( X \) if \( L(T_N; \alpha), U(I_N; \beta) \), and \( L(F_N; \gamma) \) are nonempty.

Let \( x, y \in X \). Then \( T_N(x), T_N(y) \in [-1,0] \). Choose \( \alpha = \max\{T_N(x), T_N(y)\} \). Thus \( T_N(x) \leq \alpha \) and \( T_N(y) \leq \alpha \), so \( x, y \in L(T_N; \alpha) \neq \emptyset \). By assumption, we have \( L(T_N; \alpha) \) is a UP-subalgebra of \( X \) and so \( x \cdot y \in L(T_N; \alpha) \). Thus \( T_N(x \cdot y) \leq \alpha = \max\{T_N(x), T_N(y)\} \).

Let \( x, y \in X \). Then \( I_N(x), I_N(y) \in [-1,0] \). Choose \( \beta = \min\{I_N(x), I_N(y)\} \). Thus \( I_N(x) \geq \beta \) and \( I_N(y) \geq \beta \), so \( x, y \in U(I_N; \beta) \neq \emptyset \). By assumption, we have \( U(I_N; \beta) \) is a UP-subalgebra of \( X \) and so \( x \cdot y \in U(I_N; \beta) \). Thus \( I_N(x \cdot y) \leq \beta = \min\{I_N(x), I_N(y)\} \).

Let \( x, y \in X \). Then \( F_N(x), F_N(y) \in [-1,0] \). Choose \( \gamma = \max\{F_N(x), F_N(y)\} \). Thus \( F_N(x) \leq \gamma \) and \( F_N(y) \leq \gamma \), so \( x, y \in L(F_N; \gamma) \neq \emptyset \). By assumption, we have \( L(F_N; \gamma) \) is a UP-subalgebra of \( X \) and so \( x \cdot y \in L(F_N; \gamma) \). Thus \( F_N(x \cdot y) \leq \gamma = \max\{F_N(x), F_N(y)\} \).

Therefore, \( X_N \) is a neutrosophic \( N \)-UP-subalgebra of \( X \).

**Theorem 4.3** A neutrosophic \( N \)-structure \( X_N \) over \( X \) is a neutrosophic \( N \)-near UP-filter of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [-1,0] \), the sets \( L(T_N; \alpha), U(I_N; \beta) \), and \( L(F_N; \gamma) \) are near UP-filters of \( X \) if \( L(T_N; \alpha), U(I_N; \beta) \), and \( L(F_N; \gamma) \) are nonempty.

**Proof.** Assume that \( X_N \) is a neutrosophic \( N \)-near UP-filter of \( X \). Let \( \alpha, \beta, \gamma \in [-1,0] \) be such that \( L(T_N; \alpha), U(I_N; \beta) \), and \( L(F_N; \gamma) \) are nonempty.

Let \( x \in L(T_N; \alpha) \). Then \( T_N(x) \leq \alpha \). By (3.5), we have \( T_N(0) \leq T_N(x) \leq \alpha \). Thus \( 0 \in L(T_N; \alpha) \). Next, let \( x \in X \) and \( y \in L(T_N; \alpha) \). Then \( T_N(y) \leq \alpha \). By (3.8), we have \( T_N(x \cdot y) \leq T_N(y) \leq \alpha \). Thus \( x \cdot y \in L(T_N; \alpha) \).

Let \( x \in U(I_N; \beta) \). Then \( I_N(x) \geq \beta \). By (3.6), we have \( I_N(0) \geq I_N(x) \geq \beta \). Thus \( 0 \in U(I_N; \beta) \). Next, let \( x \in X \) and \( y \in U(I_N; \beta) \). Then \( I_N(y) \geq \beta \). By (3.9), we have \( I_N(x \cdot y) \geq I_N(y) \geq \beta \). Thus \( x \cdot y \in U(I_N; \beta) \).

Let \( x \in L(F_N; \gamma) \). Then \( F_N(x) \leq \gamma \). By (3.7), we have \( F_N(0) \leq F_N(x) \leq \gamma \). Thus \( 0 \in L(F_N; \gamma) \). Next, let \( x \in X \) and \( y \in L(F_N; \gamma) \). Then \( F_N(y) \leq \gamma \). By (3.10), we have \( F_N(x \cdot y) \leq F_N(y) \leq \gamma \). Thus \( x \cdot y \in L(F_N; \gamma) \).

Hence, \( L(T_N; \alpha), U(I_N; \beta) \), and \( L(F_N; \gamma) \) are near UP-filters of \( X \).

Conversely, assume that for all \( \alpha, \beta, \gamma \in [-1,0] \), the sets \( L(T_N; \alpha), U(I_N; \beta) \), and \( L(F_N; \gamma) \) are near UP-filters of \( X \) if \( L(T_N; \alpha), U(I_N; \beta) \), and \( L(F_N; \gamma) \) are nonempty.

Let \( x \in X \). Then \( T_N(x) \in [-1,0] \). Choose \( \alpha = T_N(x) \). Thus \( T_N(x) \leq \alpha \), so \( x \in L(T_N; \alpha) \neq \emptyset \). By assumption, we have \( L(T_N; \alpha) \) is a near UP-filter of \( X \) and so \( 0 \in L(T_N; \alpha) \). Thus \( T_N(0) \leq \alpha = T_N(x) \). Next, let \( x, y \in X \). Then \( T_N(y) \in [-1,0] \). Choose \( \alpha = T_N(y) \). Thus \( T_N(y) \leq \alpha \), so \( y \in L(T_N; \alpha) \neq \emptyset \). By assumption, we have \( L(T_N; \alpha) \) is a near UP-filter of \( X \) and so \( x \cdot y \in L(T_N; \alpha) \). Thus \( T_N(x \cdot y) \leq \alpha = T_N(x) \).

Let \( x \in X \). Then \( I_N(x) \in [-1,0] \). Choose \( \beta = I_N(x) \). Thus \( I_N(x) \geq \beta \), so \( x \in U(I_N; \beta) \neq \emptyset \). By assumption, we have \( U(I_N; \beta) \) is a near UP-filter of \( X \) and so \( 0 \in U(I_N; \beta) \). Thus \( I_N(0) \geq \beta = I_N(x) \). Next, let \( x, y \in X \). Then \( I_N(y) \in [-1,0] \). Choose \( \beta = I_N(y) \). Thus \( I_N(y) \geq \beta \), so \( y \in U(I_N; \beta) \neq \emptyset \). By assumption, we have \( U(I_N; \beta) \) is a near UP-filter of \( X \) and so \( x \cdot y \in U(I_N; \beta) \). Thus \( I_N(x \cdot y) \geq \beta = I_N(x) \).

Let \( x \in X \). Then \( F_N(x) \in [-1,0] \). Choose \( \gamma = F_N(x) \). Thus \( F_N(x) \leq \gamma \), so \( x \in L(F_N; \gamma) \neq \emptyset \). By assumption, we have \( L(F_N; \gamma) \) is a near UP-filter of \( X \) and so \( 0 \in L(F_N; \gamma) \). Thus
Let $F_n(0) \leq \gamma = F_n(x)$. Next, let $x, y \in X$. Then $F_n(y) \in [-1,0]$. Choose $\gamma = F_n(y)$. Thus $F_n(\gamma) \leq \gamma$, so $y \in L(F_n; \gamma) \neq \emptyset$. By assumption, we have $L(F_n; \gamma)$ is a near UP-filter of $X$ and so $x \cdot y \in L(F_n; \gamma)$. Thus $F_n(x \cdot y) \leq \gamma = F_n(y)$.

Therefore, $X_n$ is a neutrosophic $N$-near UP-filter of $X$.

**Theorem 4.4** A neutrosophic $N$-structure $X_n$ over $X$ is a neutrosophic $N$-UP-filter of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, the sets $L(T_n; \alpha), U(I_n; \beta)$, and $L(F_n; \gamma)$ are UP-filters of $X$ if $L(T_n; \alpha), U(I_n; \beta)$, and $L(F_n; \gamma)$ are nonempty.

**Proof.** Assume that $X_n$ is a neutrosophic $N$-UP-filter of $X$. Let $\alpha, \beta, \gamma \in [-1,0]$ be such that $L(T_n; \alpha), U(I_n; \beta)$, and $L(F_n; \gamma)$ are nonempty.

Let $x \in L(T_n; \alpha)$. Then $T_n(\alpha) \leq \alpha$. By (3.5), we have $T_n(0) \leq T_n(\alpha) \leq \alpha$. Thus $0 \in L(T_n; \alpha)$. Next, let $x, y \in X$ be such that $x \cdot y \in L(T_n; \alpha)$ and $x \in L(T_n; \alpha)$. Then $T_n(x \cdot y) \leq \alpha$ and $T_n(x) \leq \alpha$, so $\alpha$ is an upper bound of $\{T_n(x \cdot y), T_n(x)\}$. By (3.11), we have $T_n(y) \leq \max\{T_n(x \cdot y), T_n(x)\} \leq \alpha$. Thus $y \in L(T_n; \alpha)$.

Let $x \in U(I_n; \beta)$. Then $I_n(\beta) \geq \beta$. By (3.5), we have $I_n(0) \geq I_n(\beta) \geq \beta$. Thus $0 \in U(I_n; \beta)$. Next, let $x, y \in X$ be such that $x \cdot y \in U(I_n; \beta)$ and $x \in U(I_n; \beta)$. Then $I_n(x \cdot y) \geq \beta$ and $I_n(x) \geq \beta$, so $\beta$ is a lower bound of $\{I_n(x \cdot y), I_n(x)\}$. By (3.12), we have $I_n(y) \geq \min\{I_n(x \cdot y), I_n(x)\} \geq \beta$ Thus $y \in U(I_n; \beta)$.

Conversely, assume that for all $\alpha, \beta, \gamma \in [-1,0]$, the sets $L(T_n; \alpha), U(I_n; \beta)$, and $L(F_n; \gamma)$ are UP-filters of $X$ if $L(T_n; \alpha), U(I_n; \beta)$, and $L(F_n; \gamma)$ are nonempty.

Let $x \in X$. Then $T_n(x) \in [-1,0]$. Choose $\alpha = T_n(x)$. Thus $T_n(\alpha) \leq \alpha$, so $x \in L(T_n; \alpha) \neq \emptyset$. By assumption, we have $L(T_n; \alpha)$ is a UP-filter of $X$ and so $0 \in L(T_n; \alpha)$. Thus $T_n(0) \leq \alpha = T_n(x)$. Next, let $x, y \in X$. Then $T_n(x \cdot y), T_n(x) \in [-1,0]$. Choose $\alpha = \max\{T_n(x \cdot y), T_n(x)\}$. Thus $T_n(x \cdot y) \leq \alpha$ and $T_n(x) \leq \alpha$, so $x \cdot y \in L(T_n; \alpha) \neq \emptyset$. By assumption, we have $L(T_n; \alpha)$ is a UP-filter of $X$ and so $y \in L(T_n; \alpha)$. Thus $T_n(y) \leq \alpha = \max\{T_n(x \cdot y), T_n(x)\}$.

Let $x \in X$. Then $I_n(x) \in [-1,0]$. Choose $\beta = I_n(x)$. Thus $I_n(\beta) \geq \beta$, so $x \in U(I_n; \beta) \neq \emptyset$. By assumption, we have $U(I_n; \beta)$ is a UP-filter of $X$ and so $0 \in U(I_n; \beta)$. Thus $I_n(0) \geq \beta = I_n(x)$. Next, let $x, y \in X$. Then $I_n(x \cdot y), I_n(x) \in [-1,0]$. Choose $\beta = \min\{I_n(x \cdot y), I_n(x)\}$. Thus $I_n(x \cdot y) \geq \beta$ and $I_n(x) \geq \beta$, so $x \cdot y \in U(I_n; \beta) \neq \emptyset$. By assumption, we have $U(I_n; \beta)$ is a UP-filter of $X$ and so $y \in U(I_n; \beta)$. Thus $I_n(y) \geq \beta = \min\{I_n(x \cdot y), I_n(x)\}$.

Let $x \in X$. Then $F_n(x) \in [-1,0]$. Choose $\gamma = F_n(x)$. Thus $F_n(\gamma) \leq \gamma$, so $x \in L(F_n; \gamma) \neq \emptyset$. By assumption, we have $L(F_n; \gamma)$ is a UP-filter of $X$ and so $0 \in L(F_n; \gamma)$. Thus $F_n(0) \leq \gamma = F_n(x)$. Next, let $x, y \in X$. Then $F_n(x \cdot y), F_n(x) \in [-1,0]$. Choose $\gamma = \max\{F_n(x \cdot y), F_n(x)\}$. Thus $F_n(x \cdot y) \leq \gamma$ and $F_n(x) \leq \gamma$, so $x \cdot y \in L(F_n; \gamma) \neq \emptyset$. By assumption, we have $L(F_n; \gamma)$ is a UP-filter of $X$ and so $y \in L(F_n; \gamma)$. Thus $F_n(y) \leq \gamma = \max\{F_n(x \cdot y), F_n(x)\}$.

Therefore, $X_n$ is a neutrosophic $N$-UP-filter of $X$. 

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**Theorem 4.5** A neutrosophic $N$-structure $X_N$ over $X$ is a neutrosophic $N$-UP-ideal of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, the sets $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are UP-ideals of $X$ if $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are nonempty.

**Proof.** Assume that $X_N$ is a neutrosophic $N$-UP-ideal of $X$. Let $\alpha, \beta, \gamma \in [-1,0]$ be such that $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are nonempty.

Let $x \in L(T_N; \alpha)$. Then $T_N(x) \leq \alpha$. By (3.5), we have $T_N(0) \leq T_N(x) \leq \alpha$. Thus $0 \in L(T_N; \alpha)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in L(T_N; \alpha)$ and $y \in L(T_N; \alpha)$. Then $T_N(x \cdot (y \cdot z)) \leq \alpha$ and $T_N(y) \leq \alpha$, so $\alpha$ is an upper bound of $\{T_N(x \cdot (y \cdot z)), T_N(y)\}$. By (3.14), we have $T_N(x \cdot (y \cdot z)) \leq \max\{T_N(x \cdot (y \cdot z)), T_N(0)\} \leq \alpha$. Thus $x \cdot z \in L(T_N; \alpha)$.

Let $x \in U(I_N; \beta)$. Then $I_N(x) \geq \beta$. By (3.5), we have $I_N(0) \geq I_N(x) \geq \beta$. Thus $0 \in U(I_N; \beta)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U(I_N; \beta)$ and $y \in U(I_N; \beta)$. Then $I_N(x \cdot (y \cdot z)) \geq \beta$ and $I_N(y) \geq \beta$, so $\beta$ is a lower bound of $\{I_N(x \cdot (y \cdot z)), I_N(y)\}$. By (3.15), we have $I_N(x \cdot (y \cdot z)) \geq \min\{I_N(x \cdot (y \cdot z)), I_N(0)\} \geq \beta$. Thus $x \cdot z \in U(I_N; \beta)$.

Let $x \in L(F_N; \gamma)$. Then $F_N(x) \leq \gamma$. By (3.5), we have $F_N(0) \leq F_N(x) \leq \gamma$. Thus $0 \in L(F_N; \gamma)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in L(F_N; \gamma)$ and $y \in L(F_N; \gamma)$. Then $F_N(x \cdot (y \cdot z)) \leq \gamma$ and $F_N(y) \leq \gamma$, so $\gamma$ is an upper bound of $\{F_N(x \cdot (y \cdot z)), F_N(y)\}$. By (3.16), we have $F_N(x \cdot (y \cdot z)) \leq \max\{F_N(x \cdot (y \cdot z)), F_N(0)\}$.

Hence, $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are UP-ideals of $X$.

Conversely, assume that for all $\alpha, \beta, \gamma \in [-1,0]$, the sets $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are UP-ideals of $X$ if $L(T_N; \alpha), U(I_N; \beta)$, and $L(F_N; \gamma)$ are nonempty.

Let $x \in X$. Then $T_N(x) \in [-1,0]$. Choose $\alpha = T_N(x)$. Thus $T_N(x) \leq \alpha$, so $x \in L(T_N; \alpha) \neq \emptyset$. By assumption, we have $L(T_N; \alpha)$ is a UP-ideal of $X$ and so $0 \in L(T_N; \alpha)$. Thus $T_N(0) \leq \alpha = T_N(x)$. Next, let $x, y, z \in X$. Then $T_N(x \cdot (y \cdot z)), T_N(y) \in [-1,0]$. Choose $\alpha = \max\{T_N(x \cdot (y \cdot z)), T_N(y)\}$. Thus $T_N(x \cdot (y \cdot z)) \leq \alpha$ and $T_N(y) \leq \alpha$, so $x \cdot (y \cdot z) \cdot y \in L(T_N; \alpha) \neq \emptyset$. By assumption, we have $L(T_N; \alpha)$ is a UP-ideal of $X$ and so $x \cdot z \in L(T_N; \alpha)$. Thus $T_N(x \cdot z) \leq \alpha = \max\{T_N(x \cdot (y \cdot z)), T_N(y)\}$.

Let $x \in X$. Then $I_N(x) \in [-1,0]$. Choose $\beta = I_N(x)$. Thus $I_N(x) \geq \beta$, so $x \in U(I_N; \beta) \neq \emptyset$. By assumption, we have $U(I_N; \beta)$ is a UP-ideal of $X$ and so $0 \in U(I_N; \beta)$. Thus $I_N(0) \geq \beta = I_N(x)$. Next, let $x, y, z \in X$. Then $I_N(x \cdot (y \cdot z)), I_N(y) \in [-1,0]$. Choose $\beta = \min\{I_N(x \cdot (y \cdot z)), I_N(y)\}$. Thus $I_N(x \cdot (y \cdot z)) \geq \beta$ and $I_N(y) \geq \beta$, so $x \cdot (y \cdot z), y \in U(I_N; \beta) \neq \emptyset$. By assumption, we have $U(I_N; \beta)$ is a UP-ideal of $X$ and so $x \cdot z \in U(I_N; \beta)$. Thus $I_N(x \cdot z) \geq \beta = \min\{I_N(x \cdot (y \cdot z)), I_N(y)\}$.

Let $x \in X$. Then $F_N(x) \in [-1,0]$. Choose $\gamma = F_N(x)$. Thus $F_N(x) \leq \gamma$, so $x \in L(F_N; \gamma) \neq \emptyset$. By assumption, we have $L(F_N; \gamma)$ is a UP-ideal of $X$ and so $0 \in L(F_N; \gamma)$. Thus $F_N(0) \leq \gamma = F_N(x)$. Next, let $x, y, z \in X$. Then $F_N(x \cdot (y \cdot z)), F_N(y) \in [-1,0]$. Choose $\gamma = \max\{F_N(x \cdot (y \cdot z)), F_N(y)\}$. Thus $F_N(x \cdot (y \cdot z)) \leq \gamma$ and $F_N(y) \leq \gamma$, so $x \cdot (y \cdot z), y \in L(F_N; \gamma) \neq \emptyset$. By assumption, we have $L(F_N; \gamma)$ is a UP-ideal of $X$ and so $x \cdot z \in L(F_N; \gamma)$. Thus $F_N(x \cdot z) \leq \gamma = \max\{F_N(x \cdot (y \cdot z)), F_N(y)\}$.

Therefore, $X_N$ is a neutrosophic $N$-UP-ideal of $X$.

**Theorem 4.6** A neutrosophic $N$-structure $X_N$ over $X$ is a neutrosophic $N$-strongly UP-ideal of $X$ if and only if the sets $E(T_N; T_N(0)), E(I_N; I_N(0))$, and $E(F_N; F_N(0))$ are strongly UP-ideals of $X$.

**Proof.** Assume that $X_N$ is a neutrosophic $N$-strongly UP-ideal of $X$. By Theorem 3.17, we have $X_N$ is constant, that is, $T_N, I_N$, and $F_N$ are constant. Thus
Hence, \( E(T_N; T_N(0)) = X, E(I_N; I_N(0)) = X \), and \( E(F_N; F_N(0)) = X \) and so \( E(T_N; T_N(0)), E(I_N; I_N(0)), \) and \( E(F_N; F_N(0)) \) are strongly UP-ideals of \( X \).

Conversely, assume that \( E(T_N; T_N(0)), E(I_N; I_N(0)), \) and \( E(F_N; F_N(0)) \) are strongly UP-ideals of \( X \). Then \( E(T_N; T_N(0)) = X, E(I_N; I_N(0)) = X \), \( E(F_N; F_N(0)) = X \) and so

\[
\begin{aligned}
\forall x \in X & \quad T_N(x) = T_N(0), \\
& \quad I_N(x) = I_N(0), \\
& \quad F_N(x) = F_N(0).
\end{aligned}
\]

Thus \( T_N, I_N, \) and \( F_N \) are constant, that is \( X_N \) is constant. By Theorem 3.17, we have \( X_N \) is a neutrosophic \( N \)-strongly UP-ideal of \( X \).

5. Neutrosophic \( N \)-structures of special type

In this section, we introduce the notions of special neutrosophic \( N \)-UP-subalgebras, special neutrosophic \( N \)-near UP-filters, special neutrosophic \( N \)-UP-filters, special neutrosophic \( N \)-UP-ideals, and special neutrosophic \( N \)-strongly UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

**Definition 5.1** A neutrosophic \( N \)-structure \( X_N \) over \( X \) is called a special neutrosophic \( N \)-UP-subalgebra of \( X \) if it satisfies the following conditions:

\[
\begin{align}
(\forall x, y \in X) & \quad (T_N(x \cdot y) \geq \min\{T_N(x), T_N(y)\}), \\
(\forall x, y \in X) & \quad (I_N(x \cdot y) \leq \max\{I_N(x), I_N(y)\}), \\
(\forall x, y \in X) & \quad (F_N(x \cdot y) \geq \min\{F_N(x), F_N(y)\}).
\end{align}
\]

**Example 5.2** Let \( X = \{0,1,2,3,4\} \) be a set with a binary operation \( \cdot \) defined by the following Cayley table:

<table>
<thead>
<tr>
<th>( \cdot )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Then \( (X, \cdot, 0) \) is a UP-algebra. We define a neutrosophic \( N \)-structure \( X_N \) over \( X \) as follows:

\[
\begin{align}
T_N(0) & = -0.2, \quad I_N(0) = -0.9, \quad F_N(0) = -0.2, \\
T_N(1) & = -0.4, \quad I_N(1) = -0.8, \quad F_N(1) = -0.4, \\
T_N(2) & = -0.8, \quad I_N(2) = -0.7, \quad F_N(2) = -0.6, \\
T_N(3) & = -0.3, \quad I_N(3) = -0.5, \quad F_N(3) = -0.7, \\
T_N(4) & = -0.8, \quad I_N(4) = -0.3, \quad F_N(4) = -0.8.
\end{align}
\]

Hence, \( X_N \) is a special neutrosophic \( N \)-UP-subalgebra of \( X \).
Definition 5.3 A neutrosophic $\mathcal{N}$-structure $X_\mathcal{N}$ over $X$ is called a special neutrosophic $\mathcal{N}$-near UP-filter of $X$ if it satisfies the following conditions:

\begin{align*}
(\forall x \in X)(T_\mathcal{N}(0) & \geq T_\mathcal{N}(x)), \quad (5.4) \\
(\forall x \in X)(I_\mathcal{N}(0) & \leq I_\mathcal{N}(x)), \quad (5.5) \\
(\forall x \in X)(F_\mathcal{N}(0) & \geq F_\mathcal{N}(x)), \quad (5.6) \\
(\forall x, y \in X)(T_\mathcal{N}(x \cdot y) & \geq T_\mathcal{N}(y)), \quad (5.7) \\
(\forall x, y \in X)(I_\mathcal{N}(x \cdot y) & \leq I_\mathcal{N}(y)), \quad (5.8) \\
(\forall x, y \in X)(F_\mathcal{N}(x \cdot y) & \geq F_\mathcal{N}(y)). \quad (5.9)
\end{align*}

Example 5.4 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

\begin{align*}
\cdot & 0 1 2 3 4 \\
0 & 0 1 2 3 4 \\
1 & 0 0 0 3 0 \\
2 & 0 1 0 3 0 \\
3 & 0 1 2 0 0 \\
4 & 0 1 2 3 0
\end{align*}

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic $\mathcal{N}$-structure $X_\mathcal{N}$ over $X$ as follows:

\begin{align*}
T_\mathcal{N}(0) &= 0.2, \quad I_\mathcal{N}(0) = 0.8, \quad F_\mathcal{N}(0) = 0.3, \\
T_\mathcal{N}(1) &= 0.5, \quad I_\mathcal{N}(1) = 0.5, \quad F_\mathcal{N}(1) = 0.7, \\
T_\mathcal{N}(2) &= 0.4, \quad I_\mathcal{N}(2) = 0.4, \quad F_\mathcal{N}(2) = 0.4, \\
T_\mathcal{N}(3) &= 0.5, \quad I_\mathcal{N}(3) = 0.5, \quad F_\mathcal{N}(3) = 0.5, \\
T_\mathcal{N}(4) &= 0.8, \quad I_\mathcal{N}(4) = 0.2, \quad F_\mathcal{N}(4) = 0.8.
\end{align*}

Hence, $X_\mathcal{N}$ is a special neutrosophic $\mathcal{N}$-near UP-filter of $X$.

Definition 5.5 A neutrosophic $\mathcal{N}$-structure $X_\mathcal{N}$ over $X$ is called a special neutrosophic $\mathcal{N}$-UP-filter of $X$ if it satisfies the following conditions: (5.4), (5.5), (5.6), and

\begin{align*}
(\forall x, y \in X)(T_\mathcal{N}(y) & \geq \min(T_\mathcal{N}(x \cdot y), T_\mathcal{N}(x))), \quad (5.10) \\
(\forall x, y \in X)(I_\mathcal{N}(y) & \leq \max(I_\mathcal{N}(x \cdot y), I_\mathcal{N}(x))), \quad (5.11) \\
(\forall x, y \in X)(F_\mathcal{N}(y) & \geq \min(F_\mathcal{N}(x \cdot y), F_\mathcal{N}(x))). \quad (5.12)
\end{align*}

Example 5.6 Let $X = \{0,1,2,3,4\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

\begin{align*}
\cdot & 0 1 2 3 4 \\
0 & 0 1 2 3 4 \\
1 & 0 1 0 3 0 \\
2 & 0 1 0 3 0 \\
3 & 0 1 2 0 0 \\
4 & 0 1 2 3 0
\end{align*}

Then $(X, \cdot, 0)$ is a UP-algebra. We define a neutrosophic $\mathcal{N}$-structure $X_\mathcal{N}$ over $X$ as follows:

\begin{align*}
T_\mathcal{N}(0) &= 0.2, \quad I_\mathcal{N}(0) = 0.8, \quad F_\mathcal{N}(0) = 0.2, \\
T_\mathcal{N}(1) &= 0.8, \quad I_\mathcal{N}(1) = 0.5, \quad F_\mathcal{N}(1) = 0.8, \\
T_\mathcal{N}(2) &= 0.6, \quad I_\mathcal{N}(2) = 0.4, \quad F_\mathcal{N}(2) = 0.5, \\
T_\mathcal{N}(3) &= 0.7, \quad I_\mathcal{N}(3) = 0.6, \quad F_\mathcal{N}(3) = 0.7, \\
T_\mathcal{N}(4) &= 0.8, \quad I_\mathcal{N}(4) = 0.2, \quad F_\mathcal{N}(4) = 0.8.
\end{align*}
\[ T_N(4) = -0.5, \quad I_N(4) = -0.7, \quad F_N(4) = -0.4. \]

Hence, \( X_N \) is a special neutrosophic \( N \)-UP-filter of \( X \).

**Definition 5.7** A neutrosophic \( N \)-structure \( X_N \) over \( X \) is called a special neutrosophic \( N \)-UP-ideal of \( X \) if it satisfies the following conditions: (5.4), (5.5), (5.6), and
\[
(\forall x, y, z \in X)(T_N(x \cdot z) \geq \min\{T_N(x), T_N(y)\}, T_N(y)), (5.13)
\]
\[
(\forall x, y, z \in X)(I_N(x \cdot z) \leq \max\{I_N(x), I_N(y)\}, I_N(y)), (5.14)
\]
\[
(\forall x, y, z \in X)(F_N(x \cdot z) \geq \min\{F_N(x), F_N(y)\}, F_N(y)). (5.15)
\]

**Example 5.8** Let \( X = \{0, 1, 2, 3, 4\} \) be a set with a binary operation \( \cdot \) defined by the following Cayley table:

\[
\begin{array}{ccccc}
0 & 1 & 2 & 3 & 4 \\
0 & 0 & 1 & 2 & 3 & 4 \\
1 & 0 & 0 & 2 & 0 & 4 \\
2 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 3 & 2 & 0 & 4 \\
4 & 0 & 3 & 2 & 0 & 0
\end{array}
\]

Then \( (X, \cdot, 0) \) is a UP-algebra. We define a neutrosophic \( N \)-structure \( X_N \) over \( X \) as follows:
\[
T_N(0) = -0.3, \quad I_N(0) = -0.8, \quad F_N(0) = -0.2,
T_N(1) = -0.6, \quad I_N(1) = -0.6, \quad F_N(1) = -0.3,
T_N(2) = -0.8, \quad I_N(2) = -0.4, \quad F_N(2) = -0.8,
T_N(3) = -0.6, \quad I_N(3) = -0.6, \quad F_N(3) = -0.3,
T_N(4) = -0.7, \quad I_N(4) = -0.5, \quad F_N(4) = -0.7.
\]

Hence, \( X_N \) is a special neutrosophic \( N \)-UP-ideal of \( X \).

**Definition 5.9** A neutrosophic \( N \)-structure \( X_N \) over \( X \) is called a special neutrosophic \( N \)-strongly UP-ideal of \( X \) if it satisfies the following conditions: (5.4), (5.5), (5.6), and
\[
(\forall x, y, z \in X)(T_N(x \cdot z) \geq \min\{T_N((z \cdot y) \cdot (z \cdot x)), T_N(y)\}), (5.16)
\]
\[
(\forall x, y, z \in X)(I_N(x \cdot z) \leq \max\{I_N((z \cdot y) \cdot (z \cdot x)), I_N(y)\}), (5.17)
\]
\[
(\forall x, y, z \in X)(F_N(x \cdot z) \geq \min\{F_N((z \cdot y) \cdot (z \cdot x)), F_N(y)\}). (5.18)
\]

**Example 5.10** Let \( X = \{0, 1, 2, 3, 4\} \) be a set with a binary operation \( \cdot \) defined by the following Cayley table:

\[
\begin{array}{ccccc}
0 & 1 & 2 & 3 & 4 \\
0 & 0 & 1 & 2 & 3 & 4 \\
1 & 0 & 0 & 2 & 3 & 0 \\
2 & 0 & 1 & 0 & 0 & 4 \\
3 & 0 & 1 & 2 & 0 & 4 \\
4 & 0 & 4 & 2 & 3 & 0
\end{array}
\]

Then \( (X, \cdot, 0) \) is a UP-algebra. We define a neutrosophic \( N \)-structure \( X_N \) over \( X \) as follows:
\[
\begin{cases}
T_N(x) = -0.5 \\
I_N(x) = -1 \\
F_N(x) = -0.3
\end{cases}
\]

Hence, \( X_N \) is a special neutrosophic \( N \)-strongly UP-ideal \( X \).
Theorem 5.11 Every special neutrosophic $\mathcal{N}$-UP-subalgebra of $X$ satisfies the conditions (5.4), (5.5), and (5.6).

Proof. Assume that $X_s$ is a special neutrosophic $\mathcal{N}$-UP-subalgebra of $X$. Then for all $x \in X$, by Proposition 2.5 (1), (5.1), (5.2), and (5.3), we have

$$T_s(0) = T_s(x \cdot x) \geq \min\{T_s(x), T_s(x)\} = T_s(x), \quad I_s(0) = I_s(x \cdot x) \leq \max\{I_s(x), I_s(x)\} = I_s(x),$$

$$F_s(0) = F_s(x \cdot x) \geq \min\{F_s(x), F_s(x)\} = F_s(x).$$

Hence, $X_s$ satisfies the conditions (5.4), (5.5), and (5.6).

By Lemma 3.4 (1) and (4), we have the following five theorems.

Theorem 5.12 A neutrosophic $\mathcal{N}$-structure $X_s$ over $X$ is a neutrosophic $\mathcal{N}$-UP-subalgebra of $X$ if and only if $X_s$ is a special neutrosophic $\mathcal{N}$-UP-subalgebra of $X$.

Theorem 5.13 A neutrosophic $\mathcal{N}$-structure $X_s$ over $X$ is a neutrosophic $\mathcal{N}$-near UP-filter of $X$ if and only if $X_s$ is a special neutrosophic $\mathcal{N}$-near UP-filter of $X$.

Theorem 5.14 A neutrosophic $\mathcal{N}$-structure $X_s$ over $X$ is a neutrosophic $\mathcal{N}$-UP-filter of $X$ if and only if $X_s$ is a special neutrosophic $\mathcal{N}$-UP-filter of $X$.

Theorem 5.15 A neutrosophic $\mathcal{N}$-structure $X_s$ over $X$ is a neutrosophic $\mathcal{N}$-UP-ideal of $X$ if and only if $X_s$ is a special neutrosophic $\mathcal{N}$-UP-ideal of $X$.

Theorem 5.16 A neutrosophic $\mathcal{N}$-structure $X_s$ over $X$ is a neutrosophic $\mathcal{N}$-strongly UP-ideal of $X$ if and only if $X_s$ is a special neutrosophic $\mathcal{N}$-strongly UP-ideal of $X$.

Theorem 5.17 A neutrosophic $\mathcal{N}$-structure $X_s$ over $X$ is constant if and only if it is a special neutrosophic $\mathcal{N}$-strongly UP-ideal of $X$.

Proof. It is straightforward by Remark 3.2 and Theorems 3.17 and 5.16.

By Remark 3.2 and Theorems 5.12, 5.13, 5.14, 5.15, and 5.16, we have that the notion of special neutrosophic $\mathcal{N}$-UP-subalgebras is a generalization of special neutrosophic $\mathcal{N}$-near UP-filters, special neutrosophic $\mathcal{N}$-near UP-filters is a generalization of special neutrosophic $\mathcal{N}$-UP-filters, special neutrosophic $\mathcal{N}$-UP-filters is a generalization of special neutrosophic $\mathcal{N}$-UP-ideals, and special neutrosophic $\mathcal{N}$-UP-ideals is a generalization of special neutrosophic $\mathcal{N}$-strongly UP-ideals. Moreover, by Theorem 5.17, we obtain that special neutrosophic $\mathcal{N}$-strongly UP-ideals and constant neutrosophic $\mathcal{N}$-structures coincide.

Theorem 5.18 If $X_s$ is a special neutrosophic $\mathcal{N}$-UP-subalgebra of $X$ satisfying the following condition:

$$\forall x, y \in X \left\{ x \cdot y \neq 0 \Rightarrow \begin{cases} T_s(x) \geq T_s(y) \\ I_s(x) \leq I_s(y) \\ F_s(x) \geq F_s(y) \end{cases} \right. \tag{5.19}$$

then $X_s$ is a special neutrosophic $\mathcal{N}$-near UP-filter of $X$.

Proof. Assume that $X_s$ is a special neutrosophic $\mathcal{N}$-UP-subalgebra of $X$ satisfying the condition (5.19). By Theorem 5.11, we have $X_s$ satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then, by (5.4), (5.5), and (5.6), we have

$$T_s(x \cdot y) = T_s(0) \geq T_s(y), \quad I_s(x \cdot y) = I_s(0) \leq I_s(y), \quad F_s(x \cdot y) = F_s(0) \geq F_s(y).$$

Case 2: $x \cdot y \neq 0$. Then, by (5.1), (5.2), (5.3), and (5.19), we have

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Theorem 5.19 If $X_N$ is a special neutrosophic $N$-near UP-filter of $X$ satisfying the condition (3.21), then $X_N$ is a special neutrosophic $N$-UP-filter of $X$.

Proof. Assume that $X_N$ is a special neutrosophic $N$-near UP-filter of $X$ satisfying the condition (3.21). Then $X_N$ satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y, z \in X$. By (5.7), (5.8), and (3.21), we have

$$
\begin{align*}
T_N(x \cdot y) & \geq \min(T_N(x), T_N(y)) = T_N(y), \quad I_N(x \cdot y) \leq \max(I_N(x), I_N(y)) = I_N(y), \\
F_N(x \cdot y) & \geq \min(F_N(x), F_N(y)) = F_N(y).
\end{align*}
$$

Hence, $X_N$ is a special neutrosophic $N$-near UP-filter of $X$.

Theorem 5.20 If $X_N$ is a special neutrosophic $N$-UP-filter of $X$ satisfying the condition (3.22), then $X_N$ is a special neutrosophic $N$-UP-ideal of $X$.

Proof. Assume that $X_N$ is a special neutrosophic $N$-UP-filter of $X$ satisfying the condition (3.22). Then $X_N$ satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y, z \in X$. By (5.10), (5.11), (5.12), and (3.22), we have

$$
\begin{align*}
T_N(x \cdot z) & \geq \min[T_N(y \cdot (x \cdot z)), T_N(y)] = \min[T_N(x \cdot (y \cdot z)), T_N(y)], \\
I_N(x \cdot z) & \leq \max[I_N(y \cdot (x \cdot z)), I_N(y)] = \max[I_N(x \cdot (y \cdot z)), I_N(y)], \\
F_N(x \cdot z) & \geq \min[F_N(y \cdot (x \cdot z)), F_N(y)] = \min[F_N(x \cdot (y \cdot z)), F_N(y)].
\end{align*}
$$

Hence, $X_N$ is a special neutrosophic $N$-UP-ideal of $X$.

Theorem 5.21 If $X_N$ is a neutrosophic $N$-structure over $X$ satisfying the following condition:

$$
\left( \forall x, y, z \in X \right) \left\{ \begin{array}{l}
z \leq x \cdot y \Rightarrow \left\{ \begin{array}{l}
T_N(z) \geq \min(T_N(x), T_N(y)) \\
I_N(z) \leq \max(I_N(x), I_N(y)) \\
F_N(z) \geq \min(F_N(x), F_N(y))
\end{array} \right. 
\end{array} \right.,
$$

then $X_N$ is a special neutrosophic $N$-UP-subalgebra of $X$.

Proof. Assume that $X_N$ is a neutrosophic $N$-structure over $X$ satisfying the condition (5.20). Let $x, y \in X$. By Proposition 2.5 (1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (5.20) that

$$
\begin{align*}
T_N(x \cdot y) & \geq \min(T_N(x), T_N(y)), \quad I_N(x \cdot y) \leq \max(I_N(x), I_N(y)), \quad F_N(x \cdot y) \geq \min(F_N(x), F_N(y)).
\end{align*}
$$

Hence, $X_N$ is a special neutrosophic $N$-UP-subalgebra of $X$.

Theorem 5.22 If $X_N$ is a neutrosophic $N$-structure over $X$ satisfying the following condition:

$$
\left( \forall x, y, z \in X \right) \left\{ \begin{array}{l}
z \leq x \cdot y \Rightarrow \left\{ \begin{array}{l}
T_N(z) \geq T_N(y) \\
I_N(z) \leq I_N(y) \\
F_N(z) \geq F_N(y)
\end{array} \right. 
\end{array} \right.,
$$

then $X_N$ is a special neutrosophic $N$-near UP-filter of $X$.
Proof. Assume that \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the condition (5.21). Let \( x \in X \). By (UP-2) and Proposition 2.5 (1), we have 0 \( (x \cdot x) = 0 \), that is, \( 0 \leq x \cdot x \). It follows from (5.21) that \( T_N(0) \geq T_N(x) \), \( I_N(0) \leq I_N(x) \), and \( F_N(0) \geq F_N(x) \). Next, let \( x, y \in X \). By Proposition 2.5 (1), we have \( (x \cdot y) \cdot (x \cdot y) = 0 \), that is, \( x \cdot y \leq x \cdot y \). It follows from (5.21) that \( T_N(x \cdot y) \geq T_N(y), I_N(x \cdot y) \leq I_N(y) \), and \( F_N(x \cdot y) \geq F_N(y) \). Hence, \( X_N \) is a special neutrosophic \( N \) -near UP-filter of \( X \).

**Theorem 5.23** If \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the following condition:

\[
(\forall x, y, z \in X) \quad \begin{cases} 
z \leq x \cdot y \Rightarrow \left[ 
T_N(y) \geq \min\{T_N(z), T_N(x)\} 
\right] \\
I_N(y) \leq \max\{I_N(z), I_N(x)\} \\
F_N(y) \geq \min\{F_N(z), F_N(x)\}
\end{cases},
\]

(5.22)

then \( X_N \) is a special neutrosophic \( N \) -UP-filter of \( X \).

Proof. Assume that \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the condition (5.22). Let \( x \in X \). By (UP-3), we have \( x \cdot (x \cdot 0) = 0 \), that is, \( x \leq x \cdot 0 \). It follows from (5.22) that \( T_N(0) \geq \min\{T_N(x), T_N(x)\} = T_N(x), I_N(0) \leq \max\{I_N(x), I_N(x)\} = I_N(x), F_N(0) \geq \min\{F_N(x), F_N(x)\} = F_N(x) \).

Next, let \( x, y \in X \). By Proposition 2.5 (1), we have \( (x \cdot y) \cdot (x \cdot y) = 0 \), that is, \( x \cdot y \leq x \cdot y \). It follows from (5.22) that \( T_N(y) \geq \min\{T_N(x \cdot y), T_N(x)\}, I_N(y) \leq \max\{I_N(x \cdot y), I_N(x)\}, F_N(y) \geq \min\{F_N(x \cdot y), F_N(x)\} \).

Hence, \( X_N \) is a special neutrosophic \( N \) -UP-filter of \( X \).

**Theorem 5.24** If \( X_N \) is a neutrosophic \( N \)-structure over \( X \) satisfying the following condition:

\[
(\forall a, x, y, z \in X) \quad \begin{cases} 
a \leq x \cdot (y \cdot z) \Rightarrow \left[ 
T_N(x \cdot z) \geq \min\{T_N(a), T_N(y)\} 
\right] \\
I_N(x \cdot z) \leq \max\{I_N(a), I_N(y)\} \\
F_N(x \cdot z) \geq \min\{F_N(a), F_N(y)\}
\end{cases},
\]

(5.23)

then \( X_N \) is a special neutrosophic \( N \) -UP-ideal of \( X \).

Proof. Assume that \( X_N \) is a neutrosophic \( N \) -structure over \( X \) satisfying the condition (5.23). Let \( x \in X \). By (UP-3), we have \( x \cdot (0 \cdot x) = 0 \), that is, \( x \leq 0 \). It follows from (5.23) and (UP-2) that \( T_N(0) = T_N(0 \cdot 0) \geq \min\{T_N(x), T_N(x)\} = T_N(x), I_N(0) = I_N(0 \cdot 0) \leq \max\{I_N(x), I_N(x)\} = I_N(x), F_N(0) = F_N(0 \cdot 0) \geq \min\{F_N(x), F_N(x)\} = F_N(x) \).

Next, let \( x, y, z \in X \). By Proposition 2.5 (1), we have \( (x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0 \), that is, \( x \cdot (y \cdot z) \leq x \cdot (y \cdot z) \). It follows from (5.23) that \( T_N(x \cdot z) \geq \min\{T_N(x \cdot (y \cdot z)), T_N(y)\}, I_N(x \cdot z) \leq \max\{I_N(x \cdot (y \cdot z)), I_N(y)\}, F_N(x \cdot z) \geq \min\{F_N(x \cdot (y \cdot z)), F_N(y)\} \).

Hence, \( X_N \) is a special neutrosophic \( N \) -UP-ideal of \( X \).

For any fixed numbers \( \alpha^-, \alpha^+, \beta^-, \beta^+, \gamma^-, \gamma^+ \in [-1, 0] \) such that \( \alpha^- < \alpha^+, \beta^- < \beta^+, \gamma^- < \gamma^+ \) and a nonempty subset \( G \) of \( X \), a neutrosophic \( N \)-structure \( g_{X_N}^{\alpha^-, \beta^-, \gamma^-} = (X, g_{T_N}^{\alpha^-, \beta^-, \gamma^-}, g_{I_N}^{\alpha^-, \beta^-, \gamma^-}, g_{F_N}^{\alpha^-, \beta^-, \gamma^-}) \) over \( X \) where \( g_{T_N}^{\alpha^-, \beta^-, \gamma^-}, g_{I_N}^{\alpha^-, \beta^-, \gamma^-}, g_{F_N}^{\alpha^-, \beta^-, \gamma^-} \) are \( N \)-functions on \( X \) which are given as follows:
\[\alpha_{T_N^{\alpha^+}}(x) = \begin{cases} \alpha^+ & \text{if } x \in G, \\ \alpha^- & \text{otherwise}, \end{cases}\]

\[\alpha_{I_N^{\beta^+}}(x) = \begin{cases} \beta^+ & \text{if } x \in G, \\ \beta^- & \text{otherwise}, \end{cases}\]

\[\alpha_{F_N^{\gamma^+}}(x) = \begin{cases} \gamma^+ & \text{if } x \in G, \\ \gamma^- & \text{otherwise}. \end{cases}\]

**Lemma 5.25** Let \( \alpha^-, \alpha^+, \beta^-, \beta^+, \gamma^-, \gamma^+ \in [-1,0] \). Then the following statements hold:

1. \[\alpha_{X_N^{G_{\alpha^-,\beta^-,\gamma^-}}} = \alpha_{X_N^{[-1-\alpha^-, -1-\beta^-,-1-\gamma^-]}}, \]

2. \[\alpha_{X_N^{G_{\alpha^+,\beta^+,\gamma^+}}} = \alpha_{X_N^{[-1-\alpha^+, -1-\beta^+, -1-\gamma^+]}}. \]

**Proof.**

1. Let \( \alpha_{X_N^{G_{\alpha^+,\beta^+,\gamma^+}}} \) be a neutrosophic \( \mathcal{N} \)-structure over \( X \). Then

\[\alpha_{X_N^{G_{\alpha^+}}}(x) = \begin{cases} \alpha^+ & \text{if } x \in G, \\ \alpha^- & \text{otherwise}, \end{cases}\]

\[\alpha_{T_N^{\alpha^+}}(x) = \begin{cases} -1-\alpha^+ & \text{if } x \in G, \\ -1-\alpha^- & \text{otherwise}, \end{cases}\]

\[\alpha_{I_N^{\beta^+}}(x) = \begin{cases} \beta^+ & \text{if } x \in G, \\ \beta^- & \text{otherwise}, \end{cases}\]

\[\alpha_{F_N^{\gamma^+}}(x) = \begin{cases} \gamma^+ & \text{if } x \in G, \\ \gamma^- & \text{otherwise}. \end{cases}\]

Hence, \( (X, \alpha_{\alpha_{T_N^{\alpha^+}}}, \alpha_{\alpha_{I_N^{\beta^+}}}, \alpha_{\alpha_{F_N^{\gamma^+}}}) = \alpha_{X_N^{[-1-\alpha^+, -1-\beta^+, -1-\gamma^+]}}. \)

2. Let \( \alpha_{X_N^{G_{\alpha^+,\beta^+,\gamma^+}}} \) be a neutrosophic \( \mathcal{N} \)-structure over \( X \). Then

\[\alpha_{X_N^{G_{\alpha^+,\beta^+,\gamma^+}}} = (X, \alpha_{\alpha_{T_N^{\alpha^+}}}, \alpha_{\alpha_{I_N^{\beta^+}}}, \alpha_{\alpha_{F_N^{\gamma^+}}}). \]

\[\alpha_{T_N^{\alpha^+}}(x) = \begin{cases} \alpha^+ & \text{if } x \in G, \\ \alpha^- & \text{otherwise}, \end{cases}\]

\[\alpha_{I_N^{\beta^+}}(x) = \begin{cases} \beta^+ & \text{if } x \in G, \\ \beta^- & \text{otherwise}, \end{cases}\]

\[\alpha_{F_N^{\gamma^+}}(x) = \begin{cases} \gamma^+ & \text{if } x \in G, \\ \gamma^- & \text{otherwise}. \end{cases}\]

Hence, \( (X, \alpha_{\alpha_{T_N^{\alpha^+}}}, \alpha_{\alpha_{I_N^{\beta^+}}}, \alpha_{\alpha_{F_N^{\gamma^+}}}) = \alpha_{X_N^{[-1-\alpha^+, -1-\beta^+, -1-\gamma^+]}}. \)
Lemma 5.26 If the constant 0 of X is in a nonempty subset G of X, then a neutrosophic \(N\)-structure \(G X_{\alpha, \beta, \gamma} \) over \(X\) satisfies the conditions (5.4), (5.5), and (5.6).

**Proof.** If \(0 \in G\), then \(G T_{\alpha}^{x}(0) = \alpha^{+}, G I_{\beta}^{z}(0) = \beta^{-},\) and \(G F_{\gamma}^{y}(0) = \gamma^{+}\). Thus
\[
\left\{ \begin{array}{l}
G T_{\alpha}^{x}(0) = \alpha^{+} \geq G T_{\alpha}^{x}(x) \\
G I_{\beta}^{z}(0) = \beta^{-} \leq G I_{\beta}^{z}(x) \\
G F_{\gamma}^{y}(0) = \gamma^{+} \geq G F_{\gamma}^{y}(x)
\end{array} \right. \quad (\forall x \in X)
\]
Hence, \(G X_{\alpha, \beta, \gamma} \) satisfies the conditions (5.4), (5.5), and (5.6).

Lemma 5.27 If a neutrosophic \(N\)-structure \(G X_{\alpha, \beta, \gamma} \) over \(X\) satisfies the condition (5.4) (resp., (5.5), (5.6)), then the constant 0 of \(X\) is in a nonempty subset \(G\) of \(X\).

**Proof.** Assume that a neutrosophic \(N\)-structure \(G X_{\alpha, \beta, \gamma} \) over \(X\) satisfies the condition (5.4).

Then \(G T_{\alpha}^{x}(0) \geq G T_{\alpha}^{x}(x)\) for all \(x \in X\). Since \(G\) is nonempty, there exists \(g \in G\). Thus \(G T_{\alpha}^{x}(g) = \alpha^{+}\), so \(G T_{\alpha}^{x}(0) \geq G T_{\alpha}^{x}(g) = \alpha^{+}\), that is, \(G T_{\alpha}^{x}(0) = \alpha^{+}\). Hence, \(0 \in G\).

Theorem 5.28 A neutrosophic \(N\)-structure \(G X_{\alpha, \beta, \gamma} \) over \(X\) is a special neutrosophic \(N\)-UP-subalgebra of \(X\) if and only if a nonempty subset \(G\) of \(X\) is a UP-subalgebra of \(X\).

**Proof.** Assume that \(G X_{\alpha, \beta, \gamma} \) is a special neutrosophic \(N\)-UP-subalgebra of \(X\). Let \(x, y \in G\).

Then \(G T_{\alpha}^{x}(0) = \alpha^{+}\). Thus
\[
G T_{\alpha}^{x}(x,y) = \text{min}(G T_{\alpha}^{x}(x), G T_{\alpha}^{x}(y)) = \alpha^{+} \geq G T_{\alpha}^{x}(x,y)
\]
and so \(G T_{\alpha}^{x}(x,y) = \alpha^{+}\). Thus \(x, y \in G\). Hence, \(G\) is a UP-subalgebra of \(X\).

Conversely, assume that \(G\) is a UP-subalgebra of \(X\). Let \(x, y \in X\).

**Case 1:** \(x, y \in G\). Then
\[
G T_{\alpha}^{x}(y) = \alpha^{+} \geq G T_{\alpha}^{x}(y), \quad G I_{\beta}^{z}(x) = \beta^{-} \geq G I_{\beta}^{z}(y), \quad G F_{\gamma}^{y}(x) = \gamma^{+} \geq G F_{\gamma}^{y}(y)
\]
Thus
\[
\text{min}(G T_{\alpha}^{x}(x), G T_{\alpha}^{x}(y)) = \alpha^{+}, \quad \text{max}(G I_{\beta}^{z}(x), G I_{\beta}^{z}(y)) = \beta^{-}, \quad \text{min}(G F_{\gamma}^{y}(x), G F_{\gamma}^{y}(y)) = \gamma^{+}
\]
Since $G$ is a UP-subalgebra of $X$, we have $x \cdot y \in G$ and so $G_{T_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}[(x \cdot y)] = \alpha^+ \tau$, $G_{I_{\beta^+}^{\beta^{-}}}[(x \cdot y)] = \beta^-$, and $G_{F_{\gamma^+}^{\gamma^{-}}}[(x \cdot y)] = \gamma^+$. Hence,

$$G_{T_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}(x \cdot y) = \alpha^+ \geq \alpha^+ = \min\{G_{T_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}[(x)], G_{T_a^\tau_{\alpha^{-}}}[(y)]\},$$

$$G_{I_{\beta^+}^{\beta^{-}}}(x \cdot y) = \beta^- \leq \beta^- = \max\{G_{I_{\beta^+}^{\beta^{-}}}[(x)], G_{I_{\beta^+}^{\beta^{-}}}[(y)]\},$$

$$G_{F_{\gamma^+}^{\gamma^{-}}}(x \cdot y) = \gamma^+ \geq \gamma^+ = \min\{G_{F_{\gamma^+}^{\gamma^{-}}}[(x)], G_{F_{\gamma^+}^{\gamma^{-}}}[(y)]\}. $$

**Case 2:** $x \not\in G$ or $y \not\in G$. Then

$$G_{T_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}(x) = \alpha^- \text{ or } G_{T_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}(y) = \alpha^- \text{ or } G_{I_{\beta^+}^{\beta^{-}}}[(x)] = \beta^+ \text{ or } G_{I_{\beta^+}^{\beta^{-}}}[(y)] = \beta^+, \min\{G_{F_{\gamma^+}^{\gamma^{-}}}[(x)], G_{F_{\gamma^+}^{\gamma^{-}}}[(y)]\} = \gamma^-.$$

Thus

$$\min\{G_{T_a^\tau_{\alpha^{-}}}[(x)], G_{T_a^\tau_{\alpha^{-}}}[(y)]\} = \alpha^-, \max\{G_{I_{\beta^+}^{\beta^{-}}}[(x)], G_{I_{\beta^+}^{\beta^{-}}}[(y)]\} = \beta^+, \min\{G_{F_{\gamma^+}^{\gamma^{-}}}[(x)], G_{F_{\gamma^+}^{\gamma^{-}}}[(y)]\} = \gamma^-.$$ 

Therefore,

$$G_{T_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}(x \cdot y) \geq \alpha^- = \min\{G_{T_a^\tau_{\alpha^{-}}}[(x)], G_{T_a^\tau_{\alpha^{-}}}[(y)]\},$$

$$G_{I_{\beta^+}^{\beta^{-}}}[(x \cdot y)] \leq \beta^- = \max\{G_{I_{\beta^+}^{\beta^{-}}}[(x)], G_{I_{\beta^+}^{\beta^{-}}}[(y)]\},$$

$$G_{F_{\gamma^+}^{\gamma^{-}}}(x \cdot y) \geq \gamma^- = \min\{G_{F_{\gamma^+}^{\gamma^{-}}}[(x)], G_{F_{\gamma^+}^{\gamma^{-}}}[(y)]\}. $$

Hence, $G_{X_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}$ is a special neutrosophic $\mathcal{N}$-UP-subalgebra of $X$.

**Theorem 5.29** A neutrosophic $\mathcal{N}$-structure $G_{X_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}$ over $X$ is a special neutrosophic $\mathcal{N}$-near UP-filter of $X$ if and only if a nonempty subset $G$ of $X$ is a near UP-filter of $X$.

**Proof.** Assume that $G_{X_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}$ is a special neutrosophic $\mathcal{N}$-near UP-filter of $X$. Since $G_{X_a^\tau_{\alpha^{-}\beta^{-}\gamma^{-}}}$ satisfies the condition (5.4), it follows from Lemma 5.27 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then $G_{T_a^\tau_{\alpha^{-}}}[(y)] = \alpha^+$. Thus, by (5.7), we have

$$G_{T_a^\tau_{\alpha^{-}}}[(x \cdot y)] \geq G_{T_a^\tau_{\alpha^{-}}}[(y)] = \alpha^+ \geq G_{T_a^\tau_{\alpha^{-}}}[(x \cdot y)]$$

and so $G_{T_a^\tau_{\alpha^{-}}}[(x \cdot y)] = \alpha^+$. Thus $x \cdot y \in G$. Hence, $G$ is a near UP-filter of $X$.
Conversely, assume that $G$ is a near UP-filter of $X$. Since $0 \in G$, it follows from Lemma 5.26 that $^G X_{\alpha^+,\beta^+,\gamma^+}$ satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y \in X$.

Case 1: $y \in G$. Then $^G T_{\alpha^+}(y) = \alpha^+, ^G I_{\beta^+}(y) = \beta^+$, and $^G F_{\gamma^+}(y) = \gamma^+$. Since $G$ is a near UP-filter of $X$, we have $x \cdot y \in G$ and so $^G T_{\alpha^+}(x \cdot y) = \alpha^+, ^G I_{\beta^+}(x \cdot y) = \beta^+$, and $^G F_{\gamma^+}(x \cdot y) = \gamma^+$. Thus

$$^G T_{\alpha^+}(x \cdot y) \geq \alpha^+ = ^G T_{\alpha^+}(y), \quad ^G I_{\beta^+}(x \cdot y) \leq \beta^+ = ^G I_{\beta^+}(y), \quad ^G F_{\gamma^+}(x \cdot y) \geq \gamma^+ = ^G F_{\gamma^+}(y).$$

Case 2: $y \notin G$. Then $^G T_{\alpha^+}(y) = \alpha^-, ^G I_{\beta^+}(y) = \beta^+$, and $^G F_{\gamma^+}(y) = \gamma^-$. Thus

$$^G T_{\alpha^+}(x \cdot y) \geq \alpha^- = ^G T_{\alpha^+}(y), \quad ^G I_{\beta^+}(x \cdot y) \leq \beta^- = ^G I_{\beta^+}(y), \quad ^G F_{\gamma^+}(x \cdot y) \geq \gamma^- = ^G F_{\gamma^+}(y).$$

Hence, $^G X_{\alpha^+,\beta^+,\gamma^+}$ is a special neutrosophic $N$-near UP-filter of $X$.

Theorem 5.30 A neutrosophic $N$-structure $^G X_{\alpha^+,\beta^+,\gamma^+}$ over $X$ is a special neutrosophic $N$-UP-filter of $X$ if and only if a nonempty subset $G$ of $X$ is a UP-filter of $X$.

Proof. Assume that $^G X_{\alpha^+,\beta^+,\gamma^+}$ is a special neutrosophic $N$-UP-filter of $X$. Since $^G X_{\alpha^+,\beta^+,\gamma^+}$ satisfies the condition (5.4), it follows from Lemma 5.27 that $0 \in G$. Next, let $x, y \in X$ be such that $x \cdot y \in G$ and $x \in G$. Then $^G T_{\alpha^+}(x \cdot y) = \alpha^+ = ^G T_{\alpha^+}(x)$. Thus, by (5.10), we have

$$^G T_{\alpha^+}(x \cdot y) \geq \min\{^G T_{\alpha^+}(x), ^G T_{\alpha^+}(x)\} = \alpha^+ = ^G T_{\alpha^+}(x)$$

and so $^G T_{\alpha^+}(y) = \alpha^+$. Thus $y \in G$. Hence, $G$ is a UP-filter of $X$.

Conversely, assume that $G$ is a UP-filter of $X$. Since $0 \in G$, it follows from Lemma 5.26 that $^G X_{\alpha^+,\beta^+,\gamma^+}$ satisfies the conditions (5.4), (5.5), and (5.6). Next, let $x, y \in X$.

Case 1: $x \cdot y \in G$ and $x \in G$. Then

$$^G T_{\alpha^+}(x \cdot y) = \alpha^+ = ^G T_{\alpha^+}(x), ^G I_{\beta^+}(x \cdot y) = \beta^+ = ^G I_{\beta^+}(x), ^G F_{\gamma^+}(x \cdot y) = \gamma^+ = ^G F_{\gamma^+}(x).$$

Since $G$ is a UP-filter of $X$, we have $y \in G$ and so $^G T_{\alpha^+}(y) = \alpha^+ = ^G T_{\alpha^+}(y)$, and $^G F_{\gamma^+}(y) = \gamma^+$. Thus

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\[ gT_{\alpha+}(x, y) = \alpha^+ \geq \alpha^+ = \min\{gT_{\alpha+}(x, y), gT_{\alpha-}(x)\}, \]
\[ gI_{\beta+}(x, y) = \beta^+ \leq \beta^+ = \max\{gI_{\beta+}(x, y), gI_{\beta-}(x)\}, \]
\[ gF_{\gamma+}(x, y) = \gamma^+ \geq \gamma^+ = \min\{gF_{\gamma+}(x, y), gF_{\gamma-}(x)\}. \]

**Case 2:** \( x \cdot y \not\in G \) or \( x \not\in G \). Then
\[ gT_{\alpha+}(x, y) = \alpha^- \text{ or } gT_{\alpha-}(x) = \alpha^- \text{ or } gI_{\beta+}(x, y) = \beta^- \text{ or } gI_{\beta-}(x) = \beta^+, \}
\[ gF_{\gamma+}(x, y) = \gamma^- \text{ or } gF_{\gamma-}(x) = \gamma^-. \]

Thus
\[ \min\{gT_{\alpha+}(x, y), gT_{\alpha-}(x)\} = \alpha^-, \]
\[ \max\{gI_{\beta+}(x, y), gI_{\beta-}(x)\} = \beta^+, \]
\[ \min\{gF_{\gamma+}(x, y), gF_{\gamma-}(x)\} = \gamma^- \]

Therefore,
\[ gT_{\alpha+}(x, y) \geq \alpha^+ = \min\{gT_{\alpha+}, gT_{\alpha-}\}(x, y), \]
\[ gI_{\beta+}(x, y) \leq \beta^+ = \max\{gI_{\beta+}, gI_{\beta-}\}(x, y), \]
\[ gF_{\gamma+}(x, y) \geq \gamma^- = \min\{gF_{\gamma+}, gF_{\gamma-}\}(x, y) \]

Hence, \( gX_{\alpha+, \beta+, \gamma+} \) is a special neutrosophic \( N \)-UP-filter of \( X \).

**Theorem 5.31** A neutrosophic \( N \)-structure \( gX_{\alpha+, \beta+, \gamma+} \) over \( X \) is a special neutrosophic \( N \)-UP-ideal of \( X \) if and only if a nonempty subset \( G \) of \( X \) is a UP-ideal of \( X \).

**Proof.** Assume that \( gX_{\alpha+, \beta+, \gamma+} \) is a special neutrosophic \( N \)-UP-ideal of \( X \). Since \( gX_{\alpha+, \beta+, \gamma+} \) satisfies the condition (5.4), it follows from Lemma 5.27, that \( 0 \in G \). Next, let \( x, y, z \in X \) be such that \( x \cdot (y \cdot z) \in G \) and \( y \in G \). Then \( gT_{\alpha+}(x, (y \cdot z)) = \alpha^+ = gT_{\alpha+}(y) \).

Thus, by (5.13), we have
\[ gT_{\alpha+}(x, (y \cdot z)) \geq \min\{gT_{\alpha+}(x, (y \cdot z)), gT_{\alpha+}(y)\} = \alpha^+ \geq gT_{\alpha+}(x, y) \]
and so \( gT_{\alpha+}(x, z) = \alpha^+ \). Thus \( x \cdot z \in G \). Hence, \( G \) is a UP-ideal of \( X \).

Conversely, assume that \( G \) is a UP-ideal of \( X \). Since \( 0 \in G \), it follows from Lemma 5.26 that \( gX_{\alpha+, \beta+, \gamma+} \) satisfies the conditions (5.4), (5.5), and (5.6). Next, let \( x, y, z \in X \).

**Case 1:** \( x \cdot (y \cdot z) \in G \) and \( y \in G \). Then...
\[ G T_{\alpha}^{\gamma}(x \cdot (y \cdot z)) = \alpha^+ \quad G T_{\beta}^{\gamma}(x \cdot (y \cdot z)) = \beta^+ \quad G I_{\mu}^{\gamma}(x \cdot (y \cdot z)) = \beta^- \quad G I_{\nu}^{\gamma}(x \cdot (y \cdot z)) = \beta^- \quad G F_{\lambda}^{\gamma}(x \cdot (y \cdot z)) = \gamma^+ \]

Thus
\[ \min\{G T_{\alpha}^{\gamma}(x \cdot (y \cdot z)), G T_{\alpha}^{\gamma}(y)\} = \alpha^+ \quad \max\{G I_{\mu}^{\gamma}(x \cdot (y \cdot z)), G I_{\mu}^{\gamma}(y)\} = \beta^+ \quad \min\{G I_{\mu}^{\gamma}(x \cdot (y \cdot z)), G I_{\mu}^{\gamma}(y)\} = \beta^- \quad \min\{G F_{\lambda}^{\gamma}(x \cdot (y \cdot z)), G F_{\lambda}^{\gamma}(y)\} = \gamma^+ \]

Since \( G \) is a \( G \)-ideal of \( X \), we have \( x \cdot z \in G \) and so \( G T_{\alpha}^{\gamma}(x \cdot z) = \alpha^+ \quad G I_{\mu}^{\gamma}(x \cdot z) = \beta^- \), and

\[ G F_{\lambda}^{\gamma}(x \cdot z) = \gamma^+ \]

Thus
\[ G T_{\alpha}^{\gamma}(x \cdot z) = \alpha^+ \quad G T_{\beta}^{\gamma}(x \cdot z) = \beta^+ \quad G I_{\mu}^{\gamma}(x \cdot z) = \beta^- \quad G I_{\nu}^{\gamma}(x \cdot z) = \beta^- \quad G F_{\lambda}^{\gamma}(x \cdot z) = \gamma^+ \]

Thus
\[ \min\{G T_{\alpha}^{\gamma}(x \cdot (y \cdot z)), G T_{\alpha}^{\gamma}(y)\} = \alpha^+ \quad \max\{G I_{\mu}^{\gamma}(x \cdot (y \cdot z)), G I_{\mu}^{\gamma}(y)\} = \beta^+ \quad \min\{G I_{\mu}^{\gamma}(x \cdot (y \cdot z)), G I_{\mu}^{\gamma}(y)\} = \beta^- \quad \min\{G F_{\lambda}^{\gamma}(x \cdot (y \cdot z)), G F_{\lambda}^{\gamma}(y)\} = \gamma^- \]

Therefore,
\[ G T_{\alpha}^{\gamma}(x \cdot z) = \alpha^- \quad G T_{\beta}^{\gamma}(x \cdot z) \leq \beta^+ \quad G I_{\mu}^{\gamma}(x \cdot z) = \beta^- \quad G I_{\nu}^{\gamma}(x \cdot z) = \beta^- \quad G F_{\lambda}^{\gamma}(x \cdot z) \geq \gamma^- \]

Hence, \( G X_{\alpha, \beta, \gamma, \cdot} \) is a special neutrosophic \( G \)-UP-ideal of \( X \).

**Theorem 5.32** A neutrosophic \( N \)-structure \( G X_{\alpha, \beta, \gamma, \cdot} \) over \( X \) is a special neutrosophic \( N \)-strongly UP-ideal of \( X \) if and only if a nonempty subset \( G \) of \( X \) is a strongly UP-ideal of \( X \).
**Proof.** Assume that \( g X N [\alpha, \beta, \gamma] \) is a special neutrosophic \( N \)-strongly UP-ideal of \( X \). By Theorem 5.17, we have \( g T \alpha N [\alpha] \) is constant, that is, \( g T \alpha N [\alpha] \) is constant. Since \( G \) is nonempty, we have \( g T \alpha N [\alpha] \) for all \( x \in X \). Thus \( G \) is X. Hence, \( G \) is a strongly UP-ideal of \( X \).

Conversely, assume that \( G \) is a strongly UP-ideal of \( X \). Then \( G = X \), so

\[
\begin{align*}
gT \alpha N [\alpha] (x) &= \alpha^+ \\
(\forall x \in X) \quad gI \beta N [\beta] (x) &= \beta^- \\
gF \gamma N [\gamma] (x) &= \gamma^-
\end{align*}
\]

Thus \( g T \alpha N [\alpha] \), \( g I \beta N [\beta] \), and \( g F \gamma N [\gamma] \) are constant, that is, \( g X N [\alpha, \beta, \gamma] \) is constant. By Theorem 5.17, we have \( g X N [\alpha, \beta, \gamma] \) is a special neutrosophic \( N \)-strongly UP-ideal of \( X \).

6. Level subset of a neutrosophic \( N \)-structure of special type

In the last section of this paper, we discuss the relationships among special neutrosophic \( N \)-UP-subalgebras (resp., special neutrosophic \( N \)-near UP-filters, special neutrosophic \( N \)-UP-filters, special neutrosophic \( N \)-UP-ideals, special neutrosophic \( N \)-strongly UP-ideals) of UP-algebras and their level subsets.

**Theorem 6.1** A neutrosophic \( N \)-structure \( X_N \) over \( X \) is a special neutrosophic \( N \)-UP-subalgebra of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [-1,0] \), the sets \( UT(\alpha; \alpha), LI(\beta; \beta) \), and \( UF(\gamma; \gamma) \) are UP-subalgebras of \( X \) if \( UT(\alpha; \alpha), LI(\beta; \beta), \) and \( UF(\gamma; \gamma) \) are nonempty.

**Proof.** Assume that \( X_N \) is a special neutrosophic \( N \)-UP-subalgebra of \( X \). Let \( \alpha, \beta, \gamma \in [-1,0] \) be such that \( UT(\alpha; \alpha), LI(\beta; \beta), \) and \( UF(\gamma; \gamma) \) are nonempty.

Let \( x, y \in UT(\alpha; \alpha) \). Then \( T_\alpha (x) \geq \alpha \) and \( T_\alpha (y) \geq \alpha \), so \( \alpha \) is a lower bound of \( \{ T_\alpha (x), T_\alpha (y) \} \).

By (5.1), we have \( T_\alpha (x \cdot y) \geq \min(\{ T_\alpha (x), T_\alpha (y) \}) \). Thus \( x \cdot y \in UT(\alpha; \alpha) \).

Let \( x, y \in LI(\beta; \beta) \). Then \( I_\beta (x) \leq \beta \) and \( I_\beta (y) \leq \beta \), so \( \beta \) is an upper bound of \( \{ I_\beta (x), I_\beta (y) \} \).

By (5.2), we have \( I_\beta (x \cdot y) \leq \max(\{ I_\beta (x), I_\beta (y) \}) \). Thus \( x \cdot y \in LI(\beta; \beta) \).

Let \( x, y \in UF(\gamma; \gamma) \). Then \( F_\gamma (x) \geq \gamma \) and \( F_\gamma (y) \geq \gamma \), so \( \gamma \) is a lower bound of \( \{ F_\gamma (x), F_\gamma (y) \} \).

By (5.3), we have \( F_\gamma (x \cdot y) \geq \min(\{ F_\gamma (x), F_\gamma (y) \}) \). Thus \( x \cdot y \in UF(\gamma; \gamma) \).

Hence, \( UT(\alpha; \alpha), LI(\beta; \beta), \) and \( UF(\gamma; \gamma) \) are UP-subalgebras of \( X \).

Conversely, assume that for all \( \alpha, \beta, \gamma \in [-1,0] \), the set \( UT(\alpha; \alpha), LI(\beta; \beta), \) and \( UF(\gamma; \gamma) \) are UP-subalgebras if \( UT(\alpha; \alpha), LI(\beta; \beta), \) and \( UF(\gamma; \gamma) \) are nonempty.

Let \( x, y \in X \). Then \( T_\alpha (x), T_\alpha (y) \in [-1,0] \) Choose \( \alpha = \min(\{ T_\alpha (x), T_\alpha (y) \}) \). Thus \( T_\alpha (x) \geq \alpha \) and \( T_\alpha (y) \geq \alpha \), so \( x, y \in UT(\alpha; \alpha) \). By assumption, we have \( UT(\alpha; \alpha) \) is a UP-subalgebra of \( X \) and so \( x, y \in UT(\alpha; \alpha) \). Thus \( T_\alpha (x \cdot y) \geq \alpha = \min(\{ T_\alpha (x), T_\alpha (y) \}) \).
Let \( x, y \in X \). Then \( I_\alpha(x), I_\alpha(y) \in [-1,0] \). Choose \( \beta = \max\{I_\alpha(x), I_\alpha(y)\} \). Thus \( I_\alpha(x) \leq \beta \) and \( I_\alpha(y) \leq \beta \), so \( x, y \in L(I_\alpha; \beta) \neq \emptyset \). By assumption, we have \( L(I_\alpha; \beta) \) is a UP-subalgebra of \( X \) and so \( x, y \in L(I_\alpha; \beta) \). Thus \( I_\alpha(x \cdot y) \leq \beta = \max\{I_\alpha(x), I_\alpha(y)\} \).

Let \( x, y \in X \). Then \( F_\alpha(x), F_\alpha(y) \in [-1,0] \). Choose \( \gamma = \min\{F_\alpha(x), F_\alpha(y)\} \). Thus \( F_\alpha(x) \geq \gamma \) and \( F_\alpha(y) \geq \gamma \), so \( x, y \in U(F_\alpha; \gamma) \neq \emptyset \). By assumption, we have \( U(F_\alpha; \gamma) \) is a UP-subalgebra of \( X \) and so \( x, y \in U(F_\alpha; \gamma) \). Thus \( F_\alpha(x \cdot y) \leq \gamma = \min\{F_\alpha(x), F_\alpha(y)\} \).

Therefore, \( X_\alpha \) is a special neutrosophic \( N \)-UP-subalgebra of \( X \).

**Theorem 6.2** A neutrosophic \( N \)-structure \( X_\alpha \) over \( X \) is a special neutrosophic \( N \)-near UP-filter of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [-1,0] \), the sets \( U(T_\alpha; \alpha), L(I_\alpha; \beta), \) and \( U(F_\alpha; \gamma) \) are near UP-filters of \( X \) if \( U(T_\alpha; \alpha), L(I_\alpha; \beta), \) and \( U(F_\alpha; \gamma) \) are nonempty.

**Proof.** Assume that \( X_\alpha \) is a special neutrosophic \( N \)-near UP-filter of \( X \). Let \( \alpha, \beta, \gamma \in [-1,0] \) be such that \( U(T_\alpha; \alpha), L(I_\alpha; \beta), \) and \( U(F_\alpha; \gamma) \) are nonempty.

Let \( x \in U(T_\alpha; \alpha) \). Then \( T_\alpha(x) \geq \alpha \). By (5.4), we have \( T_\alpha(0) \geq T_\alpha(x) \geq \alpha \). Thus \( 0 \in U(T_\alpha; \alpha) \).

Next, let \( y \in U(T_\alpha; \alpha) \). Then \( T_\alpha(y) \geq \alpha \). By (5.7), we have \( T_\alpha(x \cdot y) \geq T_\alpha(y) \geq \alpha \). Thus \( x \cdot y \in U(T_\alpha; \alpha) \).

Let \( x \in L(I_\alpha; \beta) \). Then \( I_\alpha(x) \leq \beta \). By (5.5), we have \( I_\alpha(0) \leq I_\alpha(x) \leq \beta \). Thus \( 0 \in L(I_\alpha; \beta) \).

Next, let \( y \in L(I_\alpha; \beta) \). Then \( I_\alpha(y) \leq \beta \). By (5.8), we have \( I_\alpha(x \cdot y) \leq I_\alpha(y) \leq \beta \). Thus \( x \cdot y \in L(I_\alpha; \beta) \).

Let \( x \in U(F_\alpha; \gamma) \). Then \( F_\alpha(x) \geq \gamma \). By (5.6), we have \( F_\alpha(0) \geq F_\alpha(x) \geq \gamma \). Thus \( 0 \in U(F_\alpha; \gamma) \).

Next, let \( y \in U(F_\alpha; \gamma) \). Then \( F_\alpha(y) \geq \gamma \). By (5.9), we have \( F_\alpha(x \cdot y) \geq F_\alpha(y) \geq \gamma \). Thus \( x \cdot y \in U(F_\alpha; \gamma) \).

Hence, \( U(T_\alpha; \alpha), L(I_\alpha; \beta), \) and \( U(F_\alpha; \gamma) \) are near UP-filters of \( X \).

Conversely, assume that for all \( \alpha, \beta, \gamma \in [-1,0] \), the set \( U(T_\alpha; \alpha), L(I_\alpha; \beta), \) and \( U(F_\alpha; \gamma) \) are near UP-filters if \( U(T_\alpha; \alpha), L(I_\alpha; \beta), \) and \( U(F_\alpha; \gamma) \) are nonempty.

Let \( x \in X \). Then \( T_\alpha(0) \in [-1,0] \). Choose \( \alpha = T_\alpha(x) \). Thus \( T_\alpha(x) \geq \alpha \), so \( x \in L(T_\alpha; \alpha) \neq \emptyset \). By assumption, we have \( U(T_\alpha; \alpha) \) is a near UP-filter of \( X \) and so \( 0 \in U(T_\alpha; \alpha) \).

Thus \( T_\alpha(x \cdot y) \geq \alpha = T_\alpha(y) \).

Let \( x \in X \). Then \( I_\alpha(0) \in [-1,0] \). Choose \( \beta = I_\alpha(x) \). Thus \( I_\alpha(x) \leq \beta \), so \( x \in L(I_\alpha; \beta) \neq \emptyset \). By assumption, we have \( L(I_\alpha; \beta) \) is a near UP-filter of \( X \) and so \( 0 \in L(I_\alpha; \beta) \).

Thus \( I_\alpha(x \cdot y) \leq \beta = I_\alpha(y) \).

Therefore, \( X_\alpha \) is a special neutrosophic \( N \)-near UP-filter of \( X \).
Theorem 6.3 A neutrosophic \( N \)-structure \( X_N \) over \( X \) is a special neutrosophic \( N \)-UP-filter of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [-1,0] \), the sets \( U(T_N;\alpha), L(I_N;\beta) \), and \( U(F_N;\gamma) \) are UP-filters of \( X \) if \( U(T_N;\alpha), L(I_N;\beta), \) and \( U(F_N;\gamma) \) are nonempty.

**Proof.** Assume that \( X_N \) is a special neutrosophic \( N \)-UP-filter of \( X \). Let \( \alpha, \beta, \gamma \in [-1,0] \) be such that \( U(T_N;\alpha), L(I_N;\beta), \) and \( U(F_N;\gamma) \) are nonempty.

Let \( x \in U(T_N;\alpha) \). Then \( T_N(x) \geq \alpha \). By (5.4), we have \( T_N(0) \geq T_N(x) \geq \alpha \). Thus \( 0 \in U(T_N;\alpha) \). Next, let \( x \cdot y \in U(T_N;\alpha) \) and \( x \in U(T_N;\alpha) \). Then \( T_N(x \cdot y) \geq \alpha \) and \( T_N(x) \leq \alpha \), so \( \alpha \) is a lower bound of \( \{T_N(x \cdot y), T_N(x)\} \). By (5.10), we have \( T_N(y) \geq \min\{T_N(x \cdot y), T_N(x)\} \). Thus \( y \in U(T_N;\alpha) \).

Let \( x \in L(I_N;\beta) \). Then \( I_N(x) \leq \beta \). By (5.5), we have \( I_N(0) \leq I_N(x) \leq \beta \). Thus \( 0 \in L(I_N;\beta) \). Next, let \( x \cdot y \in L(I_N;\beta) \) and \( x \in L(I_N;\beta) \). Then \( I_N(x \cdot y) \leq \beta \) and \( I_N(x) \leq \beta \), so \( \beta \) is an upper bound of \( \{I_N(x \cdot y), I_N(x)\} \). By (5.11), we have \( I_N(y) \leq \max\{I_N(x \cdot y), I_N(x)\} \). Thus \( y \in L(I_N;\beta) \).

Let \( x \in U(F_N;\gamma) \). Then \( F_N(x) \geq \gamma \). By (5.6), we have \( F_N(0) \geq F_N(x) \geq \gamma \). Thus \( 0 \in U(F_N;\gamma) \). Next, let \( x \cdot y \in U(F_N;\gamma) \) and \( x \in U(F_N;\gamma) \). Then \( F_N(x \cdot y) \geq \gamma \) and \( F_N(x) \geq \gamma \), so \( \gamma \) is a lower bound of \( \{F_N(x \cdot y), F_N(x)\} \). By (5.12), we have \( F_N(y) \geq \min\{F_N(x \cdot y), F_N(x)\} \). Thus \( y \in U(F_N;\gamma) \).

Hence, \( U(T_N;\alpha), L(I_N;\beta), \) and \( U(F_N;\gamma) \) are UP-filters of \( X \).

Conversely, assume that for all \( \alpha, \beta, \gamma \in [-1,0] \), the set \( U(T_N;\alpha), L(I_N;\beta), \) and \( U(F_N;\gamma) \) are UP-filters if \( U(T_N;\alpha), L(I_N;\beta), \) and \( U(F_N;\gamma) \) are nonempty.

Let \( x \in X \). Then \( T_N(x) \in [-1,0] \). Choose \( \alpha = T_N(x) \). Thus \( T_N(x) \geq \alpha \), so \( x \in U(T_N;\alpha) \). By assumption, we have \( U(T_N;\alpha) \) is a UP-filter of \( X \) and so \( 0 \in U(T_N;\alpha) \). Thus \( T_N(x) \geq \alpha \). Next, let \( x \cdot y \in X \). Then \( T_N(x \cdot y), T_N(x) \in [-1,0] \). Choose \( \alpha = \min\{T_N(x \cdot y), T_N(x)\} \). Thus \( T_N(x \cdot y) \geq \alpha \) and \( T_N(x) \geq \alpha \), so \( x \cdot y, x \in U(T_N;\alpha) \). By assumption, we have \( U(T_N;\alpha) \) is a UP-filter of \( X \) and so \( y \in U(T_N;\alpha) \). Thus \( T_N(x) \geq \alpha \). Next, let \( x \cdot y \in X \). Then \( I_N(x \cdot y), I_N(x) \in [-1,0] \). Choose \( \beta = \max\{I_N(x \cdot y), I_N(x)\} \) . Thus \( I_N(x \cdot y) \leq \beta \) and \( I_N(x) \leq \beta \), so \( x \cdot y, x \in L(I_N;\beta) \). By assumption, we have \( L(I_N;\beta) \) is a UP-filter of \( X \) and so \( y \in L(I_N;\beta) \). Thus \( I_N(x) \leq \beta \). Next, let \( x \cdot y \in X \). Then \( F_N(x \cdot y), F_N(x) \in [-1,0] \). Choose \( \gamma = \min\{F_N(x \cdot y), F_N(x)\} \) . Thus \( F_N(x \cdot y) \geq \gamma \) and \( F_N(x) \geq \gamma \), so \( x \cdot y, x \in U(F_N;\gamma) \). By assumption, we have \( U(F_N;\gamma) \) is a UP-filter of \( X \) and so \( y \in U(F_N;\gamma) \). Thus \( F_N(x) \geq \gamma \). Therefore, \( X_N \) is a special neutrosophic \( N \)-UP-filter of \( X \).

Theorem 6.4 A neutrosophic \( N \)-structure \( X_N \) over \( X \) is a special neutrosophic \( N \)-UP-ideals of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [-1,0] \), the sets \( U(T_N;\alpha), L(I_N;\beta), \) and \( U(F_N;\gamma) \) are UP-ideals of \( X \) if \( U(T_N;\alpha), L(I_N;\beta), \) and \( U(F_N;\gamma) \) are noneempty.

**Proof.** Assume that \( X_N \) is a special neutrosophic \( N \)-UP-ideal of \( X \). Let \( \alpha, \beta, \gamma \in [-1,0] \) be such that \( U(T_N;\alpha), L(I_N;\beta), \) and \( U(F_N;\gamma) \) are noneempty.

Let \( x \in U(T_N;\alpha) \). Then \( T_N(x) \geq \alpha \). By (5.4), we have \( T_N(0) \geq T_N(x) \). Thus \( 0 \in U(T_N;\alpha) \). Next, let \( x \cdot (y \cdot z) \in U(T_N;\alpha) \) and \( y \in U(T_N;\alpha) \). Then \( T_N(x \cdot (y \cdot z)) \geq \alpha \) and \( T_N(y) \geq \alpha \), so \( \alpha \) is a
lower bound of \( \{T_N(x \cdot (y \cdot z)), T_N(y)\} \). By (5.13), we have \( T_N(x \cdot z) \geq \min[T_N(x \cdot (y \cdot z)), T_N(y)] \geq \alpha \). Thus \( x \cdot z \in U(T_N; \alpha) \).

Let \( x \in L(I_N; \beta) \). Then \( I_N(x) \leq \beta \). By (5.5), we have \( I_N(0) \leq I_N(x) \leq \beta \). Thus \( 0 \in L(I_N; \beta) \). Next, let \( x \cdot (y \cdot z) \in L(I_N; \beta) \) and \( y \in L(I_N; \beta) \). Then \( I_N(x \cdot (y \cdot z)) \leq \beta \) and \( I_N(y) \leq \beta \), so \( \beta \) is an upper bound of \( \{I_N(x \cdot (y \cdot z)), I_N(y)\} \). By (5.14), we have \( I_N(x \cdot z) \leq \max[I_N(x \cdot (y \cdot z)), I_N(y)] \leq \beta \). Thus \( x \cdot z \in L(I_N; \beta) \).

Let \( x \in U(F_N; \gamma) \). Then \( F_N(x) \geq \gamma \). By (5.6), we have \( F_N(0) \leq F_N(x) \geq \gamma \). Thus \( 0 \in U(F_N; \gamma) \). Next, let \( x \cdot (y \cdot z) \in U(F_N; \gamma) \) and \( y \in U(F_N; \gamma) \). Then \( F_N(x \cdot (y \cdot z)) \geq \gamma \) and \( F_N(y) \geq \gamma \), so \( \gamma \) is a lower bound of \( \{F_N(x \cdot (y \cdot z)), F_N(y)\} \). By (5.15), we have \( F_N(x \cdot z) \geq \min[F_N(x \cdot (y \cdot z)), F_N(y)] \geq \gamma \). Thus \( x \cdot z \in U(F_N; \gamma) \).

Hence, \( U(T_N; \alpha), L(I_N; \beta) \), and \( U(F_N; \gamma) \) are UP-ideals of \( X \).

Conversely, assume that for all \( \alpha, \beta, \gamma \in [-1,0] \), the set \( U(T_N; \alpha), L(I_N; \beta) \), and \( U(F_N; \gamma) \) are UP-ideals if \( U(T_N; \alpha), L(I_N; \beta) \), and \( U(F_N; \gamma) \) are nonempty.

Let \( x \in X \). Then \( T_N(x) \in [-1,0] \). Choose \( \alpha = T_N(x) \). Thus \( T_N(x) \geq \alpha \), so \( x \in U(T_N; \alpha) \) \( \neq \emptyset \). By assumption, we have \( U(T_N; \alpha) \) is a UP-ideal of \( X \) and so \( 0 \in U(T_N; \alpha) \). Thus \( T_N(0) \geq \alpha = T_N(x) \). Next, let \( x, y, z \in X \). Then \( T_N(x \cdot (y \cdot z)) \), \( T_N(y) \in [-1,0] \). Choose \( \alpha = \min[T_N(x \cdot (y \cdot z)), T_N(y)] \). Thus \( T_N(x \cdot (y \cdot z)) \geq \alpha \) and \( T_N(y) \geq \alpha \), so \( x \cdot (y \cdot z), y \in U(T_N; \alpha) \) \( \neq \emptyset \). By assumption, we have \( U(T_N; \alpha) \) is a UP-ideal of \( X \) and so \( x \cdot z \in L(I_N; \beta) \). Thus \( T_N(x \cdot z) \geq \alpha = \min[T_N(x \cdot (y \cdot z)), T_N(y)] \).

Let \( x \in X \). Then \( I_N(x) \in [-1,0] \). Choose \( \beta = I_N(x) \). Thus \( I_N(x) \leq \beta \), so \( x \in L(I_N; \beta) \) \( \neq \emptyset \). By assumption, we have \( L(I_N; \beta) \) is a UP-ideal of \( X \) and so \( 0 \in L(I_N; \beta) \). Thus \( I_N(0) \leq \beta = I_N(x) \). Next, let \( x, y, z \in X \). Then \( I_N(x \cdot (y \cdot z)), I_N(y) \in [-1,0] \). Choose \( \beta = \max[I_N(x \cdot (y \cdot z)), I_N(y)] \). Thus \( I_N(x \cdot (y \cdot z)) \leq \beta \) and \( I_N(y) \leq \beta \), so \( x \cdot (y \cdot z), y \in L(I_N; \beta) \) \( \neq \emptyset \). By assumption, we have \( L(I_N; \beta) \) is a UP-ideal of \( X \) and so \( x \cdot z \in L(I_N; \beta) \). Thus \( I_N(x \cdot z) \leq \beta = \max[I_N(x \cdot (y \cdot z)), I_N(y)] \).

Let \( x \in X \). Then \( F_N(x) \in [-1,0] \). Choose \( \gamma = F_N(x) \). Thus \( F_N(x) \geq \gamma \), so \( x \in U(F_N; \gamma) \) \( \neq \emptyset \). By assumption, we have \( U(F_N; \gamma) \) is a UP-ideal of \( X \) and so \( 0 \in U(F_N; \gamma) \). Thus \( F_N(0) \geq \gamma = F_N(x) \). Next, let \( x, y, z \in X \). Then \( F_N(x \cdot (y \cdot z)), F_N(y) \in [-1,0] \). Choose \( \gamma = \min[F_N(x \cdot (y \cdot z)), F_N(y)] \). Thus \( F_N(x \cdot (y \cdot z)) \geq \gamma \) and \( F_N(y) \geq \gamma \), so \( x \cdot (y \cdot z), y \in U(F_N; \gamma) \) \( \neq \emptyset \). By assumption, we have \( U(F_N; \gamma) \) is a UP-ideal of \( X \) and so \( x \cdot z \in U(F_N; \gamma) \). Thus \( F_N(x \cdot z) \geq \gamma = \min[F_N(x \cdot (y \cdot z)), F_N(y)] \).

Therefore, \( X_N \) is a special neutrosophic \( N \)-UP-ideal of \( X \).

**Definition 6.5** Let \( X_N \) be a neutrosophic \( N \)-structure over \( X \). For \( \alpha, \beta, \gamma \in [-1,0] \), the sets

\[
ULU_N(\alpha, \beta, \gamma) = \{ x \in X \mid T_N(x) \geq \alpha, I_N \leq \beta, F_N \geq \gamma \}
\]

\[
\text{LUL}_N(\alpha, \beta, \gamma) = \{ x \in X \mid T_N \leq \alpha, I_N \geq \beta, F_N \leq \gamma \}
\]

\[
E_N(\alpha, \beta, \gamma) = \{ x \in X \mid T_N = \alpha, I_N = \beta, F_N = \gamma \}
\]

are called a \( ULU \) - \( (\alpha, \beta, \gamma) \) -level subset, an \( LUL \) - \( (\alpha, \beta, \gamma) \) -level subset, and an \( E \) - \( (\alpha, \beta, \gamma) \) -level subset of \( X_N \), respectively. Then we see that

\[
ULU_N(\alpha, \beta, \gamma) = U(T_N; \alpha) \cap L(I_N; \beta) \cap U(F_N; \gamma)
\]

\[
\text{LUL}_N(\alpha, \beta, \gamma) = L(T_N; \alpha) \cap U(I_N; \beta) \cap L(F_N; \gamma)
\]
Corollary 6.6 A neutrosophic $\mathcal{N}$-structure $X_N$ over $X$ is a neutrosophic $\mathcal{N}$-UP-subalgebra of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, $LUL_{x_N}(\alpha, \beta, \gamma)$ is a UP-subalgebra of $X$ where $LUL_{x_N}(\alpha, \beta, \gamma)$ is nonempty.

**Proof.** It is straightforward by Theorem 4.2.

Corollary 6.7 A neutrosophic $\mathcal{N}$-structure $X_N$ over $X$ is a neutrosophic $\mathcal{N}$-near UP-filter of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, $LUL_{x_N}(\alpha, \beta, \gamma)$ is a near UP-filter of $X$ where $LUL_{x_N}(\alpha, \beta, \gamma)$ is nonempty.

**Proof.** It is straightforward by Theorem 4.3.

Corollary 6.8 A neutrosophic $\mathcal{N}$-structure $X_N$ over $X$ is a neutrosophic $\mathcal{N}$-UP-filter of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, $LUL_{x_N}(\alpha, \beta, \gamma)$ is a UP-filter of $X$ where $LUL_{x_N}(\alpha, \beta, \gamma)$ is nonempty.

**Proof.** It is straightforward by Theorem 4.4.

Corollary 6.9 A neutrosophic $\mathcal{N}$-structure $X_N$ over $X$ is a neutrosophic $\mathcal{N}$-UP-ideal of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, $LUL_{x_N}(\alpha, \beta, \gamma)$ is a UP-ideal of $X$ where $LUL_{x_N}(\alpha, \beta, \gamma)$ is nonempty.

**Proof.** It is straightforward by Theorem 4.5.

Corollary 6.10 A neutrosophic $\mathcal{N}$-structure $X_N$ over $X$ is a neutrosophic $\mathcal{N}$-strongly UP-ideal of $X$ if and only if $E(T_N, T_N(0)) = X$, $E(I_N, I_N(0)) = X$, and $E(F_N, F_N(0)) = X$.

**Proof.** It is straightforward by Theorem 4.6.

Corollary 6.11 A neutrosophic $\mathcal{N}$-structure $X_N$ over $X$ is a special neutrosophic $\mathcal{N}$-UP-subalgebra of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, $ULU_{x_N}(\alpha, \beta, \gamma)$ is a UP-subalgebra of $X$ where $ULU_{x_N}(\alpha, \beta, \gamma)$ is nonempty.

**Proof.** It is straightforward by Theorem 6.1.

Corollary 6.12 A neutrosophic $\mathcal{N}$-structure $X_N$ over $X$ is a special neutrosophic $\mathcal{N}$-near UP-filter of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, $ULU_{x_N}(\alpha, \beta, \gamma)$ is a near UP-filter of $X$ where $ULU_{x_N}(\alpha, \beta, \gamma)$ is nonempty.
Proof. It is straightforward by Theorem 6.2.

Corollary 6.13 A neutrosophic $\mathcal{N}$-structure $X_{\mathcal{N}}$ over $X$ is a special neutrosophic $\mathcal{N}$-UP-filter of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, $ULU_X(\alpha, \beta, \gamma)$ is a UP-filter of $X$ where $ULU_X(\alpha, \beta, \gamma)$ is nonempty. 
Proof. It is straightforward by Theorem 6.3.

Corollary 6.14 A neutrosophic $\mathcal{N}$-structure $X_{\mathcal{N}}$ over $X$ is a special neutrosophic $\mathcal{N}$-UP-ideal of $X$ if and only if for all $\alpha, \beta, \gamma \in [-1,0]$, $ULU_X(\alpha, \beta, \gamma)$ is a UP-ideal of $X$ where $ULU_X(\alpha, \beta, \gamma)$ is nonempty. 
Proof. It is straightforward by Theorem 6.4.

7. Conclusions

In this paper, we have introduced the notions of (special) neutrosophic $\mathcal{N}$-UP-subalgebras, (special) neutrosophic $\mathcal{N}$-near UP-filters, (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-ideals, and (special) neutrosophic $\mathcal{N}$-strongly UP-ideals of UP-algebras and investigated some of their important properties. Then we have that the notion of (special) neutrosophic $\mathcal{N}$-UP-subalgebras is a generalization of (special) neutrosophic $\mathcal{N}$-near UP-filters, (special) neutrosophic $\mathcal{N}$-near UP-filters is a generalization of (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-filters is a generalization of (special) neutrosophic $\mathcal{N}$-UP-ideals, and (special) neutrosophic $\mathcal{N}$-UP-ideals is a generalization of (special) neutrosophic $\mathcal{N}$-strongly UP-ideals. Moreover, we obtain that (special) neutrosophic $\mathcal{N}$-strongly UP-ideals and constant neutrosophic $\mathcal{N}$-structures coincide.

In our future study, we will apply these notion/results to other type of neutrosophic $\mathcal{N}$-structures in UP-algebras. Also, we will study the soft set theory/cubic set theory of such neutrosophic $\mathcal{N}$-structures.

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