Neutrosophic Φ-open sets and neutrosophic Φ-continuous functions

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Abstract: We introduce the notion of neutrosophic Φ-open set and neutrosophic Φ-continuous mapping via neutrosophic topological spaces and investigate several properties of it. By defining neutrosophic Φ-open set, neutrosophic Φ-continuous mapping, and neutrosophic Φ-open mapping, we prove some remarks, theorems on neutrosophic topological spaces.

Keywords: Neutrosophic set; Neutrosophic topology; Neutrosophic supra topology; Neutrosophic α-open set; Neutrosophic Φ-open set.

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1. Introduction

Smarandache [53] defined the Neutrosophic Set (NS) in 1998 by extending fuzzy set [58], and intuitionistic fuzzy set [2] to deal with uncertain, inconsistent and indeterminate information. An NS Θ defined over the universe Ω, $\alpha = \alpha(\xi, \psi, \zeta) \in \Theta$ with ξ, ψ and ζ being the real standard or non-standard subsets of $]0^-, 1^+[$. ξ, ψ and ζ are the degrees of true membership function, indeterminate membership function and falsity membership function respectively in the set Θ. Wang, Smarandache, Zhang, and Sunderraman [56] defined Interval NS (INS) as an instance and a subclass of NS by considering the subunitary interval [0, 1]. An INS τ defined on universe Ω, $\alpha = \alpha(\xi, \psi, \zeta) \in \tau$ with ξ, ψ and ζ being the subinterval of [0, 1]. Wang, Smarandache, Zhang, and Sunderraman [57] defined Single Valued NS (SVNS) as an instance of NS. In SVNS, the degrees of truth-membership function, indeterminacy-membership function and falsity-membership function lie in the interval [0,1]. NS has drawn many researchers' much attention for theoretical as well as practical applications [3-18, 24, 26-34, 36-46, 54-55].

Salama and Alblowi [49] grounded the concept of Neutrosophic Topological Space (NTS). Salama and Alblowi [50] also studied the generalized NS and generalized NTS. Salama, Smarandache and Alblowi [51] presented a new concept on neutrosophic crisp topology. Iswaraya and Bageerathi [23] presented the neutrosophic semi-closed set and neutrosophic semi-open set. Arokiarani, Dhavaseelan, Jafari, and Parimala [1] present the neutrosophic semi-open functions and established some relations between them. Rao and Srinivasa [48] presented neutrosophic pre-open set and pre-closed set. Dhavaseelan, Parimala, Jafari, and Smarandache [20] presented the neutrosophic semi-supra open set and neutrosophic semi-supra continuous functions.

Dhavaseelan, Ganster, Jafari, and Parimala [21] presented the neutrosophic α-supra open set and neutrosophic α-supra continuous functions. Parimala, Karthika, Dhavaseelan, & Jafari [35] presented the neutrosophic supra pre- continuous functions, the neutrosophic supra pre-open maps, and the neutrosophic supra pre-closed maps in terms of neutrosophic supra pre-open sets and neutrosophic supra pre-closed sets. Dhavaseelan, and Jafari [21] studied Generalized Neutrosophic Closed Set (GNCS). Pushpalatha and Nandhini [47] defined the GNCS in NTSs. Ebenanjar, Immaculate, and Wilfred [22] studied neutrosophic *b*-open sets in NTSs. Maheswari, Sathyabama, and Chandrasekar [25] studied the neutrosophic generalized b- closed sets in NTSs. Das and Pramanik [17] presented the generalized neutrosophic b-open sets in NTSs.

Research gap: No study on neutrosophic Φ-open sets and neutrosophic Φ-continuous functions neutrosophic generalized *b*-open set has been reported in the recent literature.

Motivation: To fill the research gap, we introduce the neutrosophic Φ-open set.

In this paper, we develop the notion of neutrosophic Φ-open set and neutrosophic Φ-continuous mapping, neutrosophic Φ-open mapping, and neutrosophic Φ-closed mapping via NTSs.

The rest of the paper is designed as follows:

Section 2 recalls the definitions neutrosophic set, neutrosophic topological space, neutrosophic supra topological space, neutrosophic α-open sets, and neutrosophic α-closed sets. Section 3 introduces neutrosophic Φ-open set, neutrosophic Φ-continuous mapping, and neutrosophic Φ-open mapping and proofs of some remarks, and theorems on neutrosophic Φ-open sets and neutrosophic Φ-continuous mapping. Section 4 presents concluding remarks.

2. Preliminaries and some properties

In this section, we discuss some existing definitions and theorems which are already defined by many researchers.

Definition 2.1. Assume that *W* be a universal set. Then *D*, an NS [53] over *W* is denoted as follows: $D= \{(m, T_D(m), I_D(m), F_D(m)): m \in W$ and $T_D(m), I_D(m), F_D(m) \in]^-0,1^+[\}$ where T_D, I_D and F_D are the functions from *D* to $]^-0,1^+[$ and for each $y \in W$, $^-0 \leq T_D(m)+I_D(m)+F_D(m) \leq 3^+$.

Definition 2.2. Assume that $D = \{(m, T_D(m), I_D(m), F_D(m)): m \in W\}$ and $K = \{(m, T_K(m), I_K(m), F_K(m)): m \in W\}$ are any two NS over *W*, then $D\cup K$ and $D\cap K$ [53] are defined by

i. $D\cup K = \{(m, T_D(m)\vee T_K(m), I_D(m)\wedge I_K(m), F_D(m)\wedge F_K(m)): m \in W\}$;

ii. $D\cap K = \{(m, T_D(m)\wedge T_K(m), I_D(m)\vee I_K(m), F_D(m)\vee F_K(m)): m \in W\}$.

Definition 2.3. Assume that $D = \{(m, T_D(m), I_D(m), F_D(m)): m \in W\}$ is an NS over *W*. Then the complement [53] of *D* is defined by $D^c = \{(m, 1-T_D(m), 1-I_D(m), 1-F_D(m)): m \in W\}$.

Definition 2.4. Assume that $D = \{(m, T_D(m), I_D(m), F_D(m)): m \in W\}$ and $K = \{(m, T_K(m), I_K(m), F_K(m)): m \in W\}$ are any two NSs over W. Then D is contained in K [53] if and only if $T_D(m) \leq T_K(m)$, $I_D(m) \geq I_K(m)$, $F_D(m) \geq F_K(m)$, for all $m \in W$.

Now we may consider two NSs 0_N and 1_N over W as follows:

1) $0_N = \{(m, 0, 1, 1): m \in W\}$;
2) $1_N = \{(m, 1, 0, 0): m \in W\}$.

Clearly, $0_N \subseteq 1_N$.

Definition 2.5. Assume that W is a universe of discourse and τ is the collection of some NSs over W. Then the collection τ is said to be a Neutrosophic Topology (NT) [49] on W if the following axioms hold:

1. $0_N, 1_N \in \tau$
2. $C_1, C_2 \in \tau \Rightarrow C_1 \cap C_2 \in \tau$
3. $\cup C_i \in \tau$, for every $\{C_i: i \in \Delta\} \subseteq \tau$.

The pair (W,τ) is said to be an NTS. If $H \in \tau$, then H is called a Neutrosophic Open Set (NOS) and the complement of H i.e. H^c is called a Neutrosophic Closed Set (NCS).

Example 2.1. Assume that $W = \{s_1, s_2, s_3\}$ is a set with three NSs over W as follows:
$M_1 = \{(s_1, 0.9, 0.5, 0.7), (s_2, 0.7, 0.6, 0.8), (s_3, 0.7, 0.4, 0.7): s_1, s_2, s_3 \in W\}$;
$M_2 = \{(s_1, 1.0, 0.3, 0.4), (s_2, 0.9, 0.5, 0.5), (s_3, 1.0, 0.1, 0.5): s_1, s_2, s_3 \in W\}$;
$M_3 = \{(s_1, 0.9, 0.3, 0.5), (s_2, 0.8, 0.5, 0.8), (s_3, 0.9, 0.3, 0.5): s_1, s_2, s_3 \in W\}$;
Then (W,τ) is an NTS, where $\tau = \{0_N, 1_N, M_1, M_2, M_3\}$ is an NT on W.

Remark 2.1. The collection of all NOSs and NCSs in (W, τ) may be denoted as NOS(W) and NCS(W) respectively. The neutrosophic interior and neutrosophic closure [49] of a neutrosophic subset H of W is denoted by $N_{int}(H)$ and $N_{cl}(H)$ respectively and defined as follows:
$N_{int}(H) = \cup\{D: D$ is an NOS in W and $D \subseteq H\}$,
$N_{cl}(H) = \cap\{L: L$ is an NCS in W and $H \subseteq L\}$.
Clearly $N_{int}(H) \subseteq H \subseteq N_{cl}(H)$.

Definition 2.6. Assume that (W,τ) is an NTS and H be an NS over W. Then H is

1) Neutrosophic Pre-Open (NPO) set [48] iff $H \subseteq N_{int}N_{cl}(H)$;
2) Neutrosophic Semi-Open (NSO) set [23] iff $H \subseteq N_{cl}N_{int}(H)$;
3) Neutrosophic α-Open (Nα-O) set [1] iff $H \subseteq N_{int}N_{cl}N_{int}(H)$.

Definition 2.7. Assume that W is a universal set and Ω be the collection of some NSs over W. Then Ω is said to be a Neutrosophic Supra Topology (NST) [19] on W if the following axioms hold:

1) $0_N, 1_N \in \Omega$
2) $\cup C_i \in \Omega$, for every $\{C_i: i \in \Delta\} \subseteq \Omega$.

The pair (W, Ω) is said to be a Neutrosophic Supra Topological Space (NSTS). If $H\in$ Ω, then H is called a Neutrosophic-Supra Open (N-SO) set and its complement H^c is called a Neutrosophic-Supra Closed (N-SC) set in (W, Ω). The neutrosophic-supra interior and neutrosophic-supra closure of an NS H is denoted by $N_{int}^{\Omega}(H)$ and $N_{cl}^{\Omega}(H)$ respectively and are defined as follows:

$N_{int}^{\Omega}(H)=\cup\{D: D$ is an N-SO set in W and $D\subseteq H\}$,

$N_{cl}^{\Omega}(H)=\cap\{L: L$ is an N-SC set in W and $H\subseteq L\}$.

Definition 2.8. Assume that (W, Ω) be an NSTS and H is an NS over W. Then H is

1) Neutrosophic-Pre Supra Open (N-PSO) set [35] iff $H \subseteq N_{int}^{\Omega}(N_{cl}^{\Omega}(H))$;
2) Neutrosophic-Semi Supra Open (N-SSO) set [20] if and only if $H \subseteq N_{cl}^{\Omega}(N_{int}^{\Omega}(H))$;
3) Neutrosophic-α-Supra Open (N-αSO) set [19] if and only if $H \subseteq N_{int}^{\Omega}(N_{cl}^{\Omega}(N_{int}^{\Omega}(H)))$.

The complement of N-PSO set, N-SSO set and N-αSO set are called Neutrosophic Pre Supra-Closed (N-PSC) set, Neutrosophic Semi Supra-Closed (N-SSC) set and Neutrosophic α-Supra-Closed (N-αSC) set respectively.

Theorem 2.1. Assume that (W, Ω) be an NSTS.

Then

i. Every N-SO set is an N-αSO set.
ii. Every N-αSO set is an N-PSO set (N-SSO set).

For proof, see Parimala, Karthika, Dhavaseelan, and Jafari (2018).

Theorem 2.2. Assume that (W, Ω) be an NSTS.

Then

i. Union of two N-αSO sets is an N-αSO set.
ii. Intersection of two N-αSO sets may not be an N-αSO set in general.

For proof, see [19].

Definition 2.9. Let (W, Ω) and (M,Π) be any two NTSs. Then a function ξ:(W, Ω)→(Y, M) is called a neutrosophic continuous function [52] if the inverse image of each NOS G in M is an NOS in W.

Definition 2.10. Let (W, Ω) and (M,Π) be any two NSTSs. Then a function ξ:(W, Ω)→(Y, M) is called a neutrosophic supra continuous [19} if and only if the inverse image of each N-SO set G in M is an N-SO set in W.

Definition 2.11. A function ξ:(W, Ω)→(M,Π), where (W, Ω) and (M,Π) are two NSTSs is said to be a neutrosophic α-supra [19] continuous iff $\xi^{-1}(G)$ is an N-αSO set in W whenever G is an N-SO set in M.

Theorem 2.3. Assume that ξ be a function from an NSTS (W, Ω) to another NSTS (M,Π). Then the following statements [19] are equivalent:

i. ξ is a neutrosophic α-supra continuous mapping.

ii. $\xi^{-1}(G)$ is an N-αSC set in W whenever G is an N-SC set in M.

3. Neutrosophic Φ-open set and neutrosophic Φ-continuous mapping

Definition 3.1. Assume that (W, τ) is an NTS and H s an NS over W. Then H is called a Neutrosophic Φ-Open (N-Φ-O) set iff there exist an Nα-O set K such that $K \subseteq H \subseteq N_{cl}(K)$, where $N_{cl}(K)$ denotes the neutrosophic closure of K with respect to the NT τ on W.

Theorem 3.1. In an NTS (W, τ),

1) Every NOS is a neutrosophic Φ-open set;
2) Every Nα-O set is a neutrosophic Φ-open set.

Proof.

1) Assume that Q is an NOS in an NTS (W, τ). Since every NOS is an Nα-O set, so Q is an Nα-O set in (W, τ). Clearly $Q \subseteq Q \subseteq N_{cl}(Q)$. Therefore, Q is a neutrosophic Φ-open set. Hence every NOS in (W, τ) is a neutrosophic Φ-open set.
2) Assume that R is an Nα-O set in an NTS (W, τ). For any neutrosophic set R, $R \subseteq R \subseteq N_{cl}(R)$. Therefore, there exists an Nα-O set R in (W, τ) such that $R \subseteq R \subseteq N_{cl}(R)$.
Hence R is a neutrosophic Φ-open set.
Thus, every Nα-O set in (W, τ) is a neutrosophic Φ-open set.

Theorem 3.2. Assume that (W,τ) is an NTS and θ is a neutrosophic supra topology such that $\tau \subseteq \theta$. Then

1) Every neutrosophic Φ-open set in (W,τ) is a neutrosophic Φ-supra open set in (W, θ);
2) Every NOS in (W,τ) is a neutrosophic Φ-supra open set in (W, θ).

Proof.

1) Assume that (W,τ) is an NTS and θ is an NST such that $\tau \subseteq \theta$.
Assume that Q is an arbitrary neutrosophic Φ-open set in (W,τ).
Then there exists an Nα-O set K such that $K \subseteq Q \subseteq N_{cl}(K)$, where $N_{cl}(K)$ denotes the neutrosophic closure of K with respect to the topology τ.
Since $\tau \subseteq \theta$ and θ is an NST on W, so $N_{cl}(K) \subseteq N_{cl}^{\theta}(K)$, where $N_{cl}^{\theta}(K)$ denotes the neutrosophic supra-closure of K with respect to the NST θ.
Therefore $K \subseteq Q \subseteq N_{cl}^{\theta}(K)$.

Hence Q is a neutrosophic Φ-supra open set in (W, θ).

2) Assume that (W,τ) is an NTS and θ be an NST on W such that $\subseteq \theta$.
Assume that Q be an arbitrary NOS in (W,τ). From Theorem 3.1, it is clear that every NOS in an NTS (W,τ) is a neutrosophic Φ-open set. So, Q is a neutrosophic Φ-open set in (W,τ). From the first part of the theotem 3.2, it is clear that Q is a neutrosophic Φ-open set in (W, θ). Hence every NOS in an NTS (W, τ) is a neutrosophic Φ-supra open set in the NSTS (W, θ).

Lemma 3.1. In an NTS (W,τ), the union of two neutrosophic Φ-open sets is a neutrosophic Φ-open set.

Proof.
Assume that K and L are any two neutrosophic Φ-open sets in an NTS (W,τ). Then there exist two Nα-O sets Q_1 and Q_2 in (W,τ) such that $Q_1 \subseteq K \subseteq N_{cl}(Q_1)$, $Q_2 \subseteq L \subseteq N_{cl}(Q_2)$.
Now, $Q_1 \cup Q_2 \subseteq K \cup L \subseteq N_{cl}(Q_1) \cup N_{cl}(Q_2) = N_{cl}(Q_1 \cup Q_2)$ and $Q_1 \cup Q_2$ is an Nα-O set in (W,τ). Therefore $K \cup L$ is a neutrosophic Φ-open set in (W,τ). Hence the union of two neutrosophic Φ-open sets in an NTS (W,τ) is a neutrosophic Φ-open set.

Theorem 3.3. Assume that (W,τ) is an NTS. Then

1) Union of an NOS and a neutrosophic Φ-open set is a neutrosophic Φ-open set.
2) Union of an Nα-O set and a neutrosophic Φ-open set is a neutrosophic Φ-open set.

Proof. Let Q be an NOS and R be a neutrosophic Φ-open set in an NTS (W,τ). From Theorem 3.1, Q is a neutrosophic Φ-open set. Again, from Lemma 3.1, it is clear that $Q \cup R$ is a neutrosophic Φ-open set in (W,τ).

1) Assume that H is an Nα-O set and G is a neutrosophic Φ-open set in an NTS (W,τ). From Theorem 3.1, it is clear that H is a neutrosophic Φ-open set. Again, from Remark 3.1, it is clear that $H \cup G$ is a neutrosophic Φ-open set in (W,τ).

Definition 3.2. Assume that (W,τ) and (M,δ) are two NTSs. Then a function $\xi: (W, \tau) \rightarrow (M,\delta)$ is called a neutrosophic Φ-continuous function iff the inverse image of every NOS G in M is a neutrosophic Φ-open set in W.

Definition 3.3. Assume that (W,τ), and (M,δ) are two NTSs and θ is an NST on W such that $\tau \subseteq \theta$. Then a function $\xi: (W, \tau) \rightarrow (M,\delta)$ is called a neutrosophic Φ-supra continuous function iff the inverse image of every NOS G in M is a neutrosophic Φ-supra open set in W with respect to the NST θ on W.

Theorem 3.3. Every neutrosophic continuous function from an NTS (W,τ) to another NTS (M,δ) is a neutrosophic Φ-continuous function.

Proof. Assume that ξ:(W, τ)→(M,δ) is a neutrosophic continuous function and K be an arbitrary NOS in M. Then by hypothesis, $\xi^{-1}(K)$ is an NOS in W. Since each NOS is a neutrosophic Φ-open set, so $\xi^{-1}(K)$ is a neutrosophic Φ-open set in W. Therefore, for each NOS K in M, $\xi^{-1}(K)$ is a neutrosophic Φ-open set in W. Hence ξ is a neutrosophic Φ-continuous function. Therefore, every neutrosophic continuous function is a neutrosophic Φ-continuous function.

Theorem 3.4. Assume that (W,τ) and (M,δ) are two NTSs and $\tau \subseteq \theta$, where θ is an NST on W. Then every neutrosophic Φ-continuous function from (W,τ) to (M,δ) is a neutrosophic Φ-supra continuous function from (W,θ) to (M,δ).

Proof. Assume that ξ:(W, τ)→(M,δ) is a neutrosophic Φ-continuous mapping. Let θ be an NST such that $\tau \subseteq \theta$. Let T be an NOS in M. Then by hypothesis $\xi^{-1}(T)$ is a neutrosophic Φ-open set in W. Since each neutrosophic Φ-open set (W,τ) is a neutrosophic Φ-supra open set in (W,θ), so $\xi^{-1}(T)$ is a neutrosophic Φ-supra open set in (W,θ). Therefore ξ is a neutrosophic Φ-supra continuous mapping from (W, θ) to (M, δ).

Definition 3.4. Let (W,τ) and (M,δ) be two NTSs. A function ξ:(W, τ)→ (M, δ) is called a neutrosophic Φ-open function if $\xi(Q)$ is a neutrosophic Φ-open set in M for each NOS Q in W.

Definition 3.5. Let (W,τ) and (M,δ) be two NTSs. A function ξ:(W,τ)→(M,δ) is called a neutrosophic Φ-closed function if $\xi(Q)$ is a neutrosophic Φ-closed set in M for each NCS Q in W.

Theorem 3.5. Assume that (W,τ) and (M,δ) are any two NTSs. Then ξ:(W,τ)→(M,δ) is a neutrosophic Φ-open function iff $\xi(N_{int}(K))\subseteq N_{int}(\xi(K))$, for each neutrosophic subset K of W.

Proof. Let ξ:(W,τ)→(M,δ) be a neutrosophic Φ-open function and K be a neutrosophic subset of W. Clearly $N_{int}(K)$ is an NOS in W and $N_{int}(K) \subseteq K$. Since ξ is a neutrosophic Φ-open function, so $\xi(N_{int}(K))$ is a neutrosophic Φ-open set in M and $\xi(N_{int}(K))\subseteq \xi(K)$. Since each NOS is a neutrosophic Φ-open set and $N_{int}(\xi(K))$ is the largest NOS contained in $\xi(K)$, so $N_{int}(\xi(K))$ is the largest neutrosophic Φ-open set contained in $\xi(K)$. Therefore $\xi(N_{int}(K)) \subseteq N_{int}(\xi(K)) \subseteq \xi(K)$ i.e. $\xi(N_{int}(K)) \subseteq N_{int}(\xi(K))$. Hence for each neutrosophic subset K of W, $\xi(N_{int}(K))\subseteq N_{int}(\xi(K))$.

Conversely, let L be an NOS in (W, τ). Therefore, $N_{int}(L) = L$. Now by hypothesis $\xi(N_{int}(L)) \subseteq N_{int}(\xi(L))$. This implies $\xi(L) \subseteq N_{int}(\xi(L))$. We know that $N_{int}(\xi(L)) \subseteq \xi(L)$. Therefore $\xi(L) = N_{int}(\xi(L))$. This means that $\xi(L)$ is an NOS in (M,δ). Since each NOS is a neutrosophic Φ-open set, so $\xi(L)$ is a neutrosophic Φ-open set in (M,δ). Hence for each NOS L in (W,τ), $\xi(L)$ is a neutrosophic Φ-open set in (M,δ). Therefore ξ is a neutrosophic Φ-open function.

Theorem 3.6. Assume that ξ is a bijective function from an NTS (W,τ) to another NTS (M,δ). Then the following mathematical statements are equivalent:

1) ξ is a neutrosophic Φ-continuous function;
2) ξ is a neutrosophic Φ-closed function;
3) ξ is a neutrosophic Φ-open function.

Proof.

(1)⇒(2) Assume that $\xi:(W,\tau)\rightarrow(M,\delta)$ is a neutrosophic Φ-continuous function. Let Q be any arbitrary NCS in (W,τ). Then Q^c is an NOS in (W,τ). Since each NOS is a neutrosophic Φ-open set, so Q^c is a neutrosophic Φ-open set in (W,τ). Since ξ is a bijective function, so $\xi(Q^c)=(\xi(Q))^c$ is an NOS in (M,δ). Hence $\xi(Q)$ is an NCS in (M,δ). Therefore, for each NCS Q in (W,τ), $\xi(Q)$ is a neutrosophic Φ-closed set in (M,δ). Hence ξ is a neutrosophic Φ-closed function.

(2)⇒(3) Assume that $\xi:(W,\tau)\rightarrow(M,\delta)$ be a neutrosophic Φ-closed function. Let L be any arbitrary NOS in (W,τ). Then L^c is an NCS in (W,τ). Since ξ is a neutrosophic Φ-closed function, so $\xi(L^c)=(\xi(L))^c$ is a neutrosophic Φ-closed set in (M,δ). Then $\xi(L)$ is a neutrosophic Φ-open set in (M,δ). Therefore, for each NOS L in (W,τ), $\xi(L)$ is a neutrosophic Φ-open set in (M,δ). Hence ξ is a neutrosophic Φ-open function.

(3)⇒(1) Assume that $\xi:(W,\tau)\rightarrow(M,\delta)$ is a neutrosophic Φ-open function. Let P be any arbitrary NOS in (M,δ). Then P is a neutrosophic Φ-open set in (M,δ). Since ξ is a bijective function, so $\xi^{-1}(P)$ is an NOS in (W,τ). Again, since each NOS is a neutrosophic Φ-open set, so $\xi^{-1}(P)$ is a neutrosophic Φ-open set in (W,τ). Therefore, for each NOS P in (M,δ), $\xi^{-1}(P)$ is a neutrosophic Φ-open set in (W,τ). Hence ξ is a neutrosophic Φ-continuous function.

4. Conclusion

In this study we have introduced neutrosophic Φ-open set, neutrosophic Φ-continuous mapping via NTSs and investigated their several properties. By defining neutrosophic Φ-open set, neutrosophic Φ-continuous mapping, we have proved some remarks, and theorems on NTSs. In the future, we hope that based on Φ-open set, neutrosophic Φ-continuous mapping via NTSs, many new investigations can be carried out.

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References

1. Arokiarani, I., Dhavaseelan, R., Jafari, S., & Parimala, M. (2017). On some new notations and functions in neutrosophic topological spaces. Neutrosophic Sets and Systems, 16,16-19.
2. Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87-96.
3. Biswas, P, Pramanik, S. & Giri, B.C. (2014). Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2, 102-110.
4. Biswas, P, Pramanik, S. & Giri, B.C. (2014). A new methodology for neutrosophic multi-attribute decision-making with unknown weight information. Neutrosophic Sets and Systems, 3, 42-50.
5. Biswas, P., Pramanik, S., & Giri, B. C. (2014). Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Neutrosophic Sets and Systems, 8, 46-56.
6. Biswas, P, Pramanik, S. & Giri, B. C. (2016). Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. Neutrosophic Sets and Systems, 12, 20-40.
7. Biswas, P, Pramanik, S. & Giri, B. C. (2016). Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets and Systems, 12, 127-138.
8. Biswas, P, Pramanik, S. & Giri, B. C. (2016). TOPSIS method for multi-attribute group decision making under single-valued neutrosophic environment. Neural Computing and Applications, 27(3), 727-737.
9. Biswas, P., Pramanik, S., & Giri, B. C. (2018). Distance measure based MADM strategy with interval trapezoidal neutrosophic numbers. Neutrosophic Sets and Systems, 19, 40-46.
10. Biswas, P., Pramanik, S., & Giri, B. C. (2018). TOPSIS strategy for multi-attribute decision making with trapezoidal neutrosophic numbers. Neutrosophic Sets and Systems,19, 29-39.
11. Biswas, P., Pramanik, S., & Giri, B. C. (2018). Multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. In F. Smarandache, & S. Pramanik (Eds., vol.2), New trends in neutrosophic theory and applications (pp. 103-124). Brussels: Pons Editions.
12. Biswas, P., Pramanik, S., & Giri, B. C. (2019). NH-MADM strategy in neutrosophic hesitant fuzzy set environment based on extended GRA. Informatica, 30 (2), 1–30.
13. Biswas, P., Pramanik, S., & Giri, B. C. (2019). Non-linear programming approach for single-valued neutrosophic TOPSIS method. New Mathematics and Natural Computation, 15 (2), 307-326.

14. Biswas, P., Pramanik, S., & Giri, B. C. (2019). Neutrosophic TOPSIS with group decision making. In C. Kahraman & I. Otay (Eds.), Fuzzy multicriteria decision making using neutrosophic sets, studies in fuzziness and soft computing 369. doi. https://doi.org/10.1007/978-3-030-00045-5_21.
15. Broumi, S., Bakali, A., Talea, M., Smarandache, F., Uluçay, V., Sahin, S, & Pramanik, S. (2018). Neutrosophic sets: An overview. In F. Smarandache, & S. Pramanik (Eds., vol.2), New trends in neutrosophic theory and applications (pp. 403-434). Brussels: Pons Editions.
16. Das, S., Roy, B. K., Kar, M. B., Kar, S., & Pamučar, D. (2020). Neutrosophic fuzzy set and its application in decision making. Journal of Ambient Intelligence and Humanized Computing. doi:10.1007/s12652-020-01808-3
17. Das, S., & Pramanik, S. (2020). Generalized neutrosophic b-open sets in neutrosophic topological space. Neutrosophic Sets and Systems, 35, 522-530.
18. Das, S., & Pramanik, S. (2020). Neutrosophic simply soft open set in neutrosophic soft topological space. Neutrosophic Sets and Systems. In Press.
19. Dhavaseelan, R., Ganster, M., Jafari, S., & Parimala, M. (2018). On neutrosophic α-supra open sets and neutrosophic α-supra continuous functions. In F. Smarandache, & S. Pramanik (Eds., vol.2), New trends in neutrosophic theory and applications (pp. 289-297). Brussels: Pons Editions.
20. Dhavaseelan, R., Parimala, M., Jafari, S., & Smarandache, F. (2017). On neutrosophic semi-supra open set and neutrosophic semi-supra continuous functions. Neutrosophic Sets and Systems, 16, 39-43.
21. Dhavaseelan, R., & Jafari, S. (2018). Generalized neutrosophic closed sets. In F. Smarandache, & S. Pramanik (Eds., vol.2), New trends in neutrosophic theory and applications (pp. 261-273). Brussels: Pons Editions.
22. Ebenanjar, E., Immaculate, J., & Wilfred, C. B. (2018). On neutrosophic b -open sets in neutrosophic topological space. Journal of Physics Conference Series, 1139(1), 012062. doi: 10.1088/1742-6596/1139/1/012062
23. Iswarya, P., & Bageerathi, K. (2016). On neutrosophic semi-open sets in neutrosophic topological spaces. International Journal of Mathematical Trends and Technology, 37(3), 214-223.
24. Khalid, H. E. (2020). Neutrosophic geometric programming (NGP) with (max-product) operator, an innovative model. Neutrosophic Sets and Systems, 32, 269-281.
25. Maheswari, C., Sathyabama, M., & Chandrasekar, S. (2018). Neutrosophic generalized b-closed sets in neutrosophic topological spaces. Journal of Physics Conference Series, 1139(1), 012065. doi: 10.1088/1742-6596/1139/1/012065

26. Mallick, R., & Pramanik, S. (2020). Pentapartitioned neutrosophic set and its properties. Neutrosophic Sets and Systems, 36, 184-192.
27. Mondal, K., &Pramanik, S. (2015). Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7, 62-68.
28. Mondal, K., & Pramanik, S. (2015). Neutrosophic tangent similarity measure and its application to multiple attribute decision making. Neutrosophic Sets and Systems, 9, 80-87.
29. Mondal, K., & Pramanik, S. (2015). Neutrosophic decision making model for clay-brick selection in construction field based on grey relational analysis. Neutrosophic Sets and Systems, 9, 64-71.
30. Mondal, K., Pramanik, S., & Giri, B. C. (2018). Single valued neutrosophic hyperbolic sine similarity measure based MADM strategy. Neutrosophic Sets and Systems, 20, 3-11.
31. Mondal, K., Pramanik, S., & Giri, B. C. (2018). Hybrid binary logarithm similarity measure for MAGDM problems under SVNS assessments. Neutrosophic Sets and Systems, 20, 12-25.
32. Mondal, K., Pramanik, S. & Smarandache, F. (2016). Rough neutrosophic TOPSIS for multi-attribute group decision making. Neutrosophic Sets and Systems, 13, 105-117.
33. Mondal, K., Pramanik, S. & Smarandache, F. (2016). Multi-attribute decision making based on rough neutrosophic variational coefficient similarity measure. Neutrosophic Sets and Systems,13, 3-17.
34. Nguyen, G.N., Son, L.H., Ashour, A.S., & Dey, N. (2019). A survey of the state-of-the-arts on neutrosophic sets in biomedical diagnoses. International Journal of Machine Learning and Cybernetics, 10, 1–13.
35. Parimala, M., Karthika, M., Dhavaseelan, R., & Jafari, S. (2018). On neutrosophic supra p re-continuous functions in neutrosophic topological spaces. In F. Smarandache, & S. Pramanik (Eds., vol.2), New trends in neutrosophic theory and applications (pp. 356-368.
36. Peng, X., & Dai, J. (2020). A bibliometric analysis of neutrosophic set: Two decades review from 1998 to 2017. Artificial Intelligence Review, 53(1), 199-255.
37. Pramanik, S. (2013). A critical review of Vivekanada's educational thoughts for women education based on neutrosophic logic. MS Academic, 3(1), 191-198.
38. Pramanik, S. (2020). Rough neutrosophic set: an overview. In F. Smarandache, & Broumi, S. (Eds.), Neutrosophic theories in communication, management and information technology (pp.275-311). New York. Nova Science Publishers.
39. Pramanik, S., Biswas, P., & Giri, B. C. (2017). Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Computing and Applications, 28 (5), 1163-1176.

40. Pramanik, S., & Chackrabarti, S.N. (2013). A study on problems of construction workers in West Bengal based on neutrosophic cognitive maps. International Journal of Innovative Research in Science, Engineering and Technology, 2(11), 6387-6394.
41. Pramanik, S., Dey, P.P., & Smarandache, F. (2018). Correlation coefficient measures of interval bipolar neutrosophic sets for solving multi-attribute decision making problems. Neutrosophic Sets and Systems, 19,70-79.
42. Pramanik, S., & Mallick, R. (2018). VIKOR based MAGDM strategy with trapezoidal neutrosophic numbers. Neutrosophic Sets and Systems, 22, 118-130.
43. Pramanik, S., and Mallick, R. (2019). TODIM strategy for multi-attribute group decision making in trapezoidal neutrosophic number environment. Complex & Intelligent Systems, 5 (4), 379–389.
44. Pramanik, S., & Mallick, R. (2020). MULTIMOORA strategy for solving multi-attribute group decision making (MAGDM) in trapezoidal neutrosophic number environment. CAAI Transactions on Intelligence Technology, 5(3), 150-156.
45. Pramanik, S., & Mondal, K. (2015). Interval neutrosophic multi-attribute decision-making based on grey relational analysis. Neutrosophic Sets and Systems, 9, 13-22.
46. Pramanik, S., & Roy, T.K. (2014). Neutrosophic game theoretic approach to Indo-Pak conflict over Jammu-Kashmir. Neutrosophic Sets and Systems, 2, 82-101.
47. Pushpalatha, A., & Nandhini, T. (2019). Generalized closed sets via neutrosophic topological spaces. Malaya Journal of Matematik, 7(1), 50-54.
48. Rao, V. V., & Srinivasa, R. (2017). Neutrosophic pre-open sets and pre-closed sets in neutrosophic topology. International Journal of Chem Tech Research, 10(10), 449-458.
49. Salama, A. A., & Alblowi, S. A. (2012). Neutrosophic set and neutrosophic topological space. ISOR Journal of Mathematics, 3(4), 31-35.
50. Salama, A. A., & Alblowi, S. A. (2012). Generalized neutrosophic set and generalized neutrosophic topological space. Computer Science and Engineering, 2 (7), 129-132.
51. Salama, A. A., Smarandache, F., & Alblowi, S. A. (2014). New neutrosophic crisp topological concepts. Neutrosophic Sets and System, 4, 50-54.
52. Salama, A. A., Smarandache, F., & Kromov, V. (2014). Neutrosophic closed set and neutrosophic continuous functions. Neutrosophic Sets and Systems, 4, 4-8.
53. Smarandache, F. (1998). A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. Rehoboth: American Research Press.
54. Smarandache, F. & Pramanik, S. (Eds.). (2016). New trends in neutrosophic theory and applications. Brussels: Pons Editions.
55. Smarandache, F. & Pramanik, S. (Eds.). (2018). New trends in neutrosophic theory and applications, Vol.2. Brussels: Pons Editions.

56. Wang, H., Smarandache, F., Zhang, Y.Q., & Sunderraman, R. (2005). Interval neutrosophic sets and logic: theory and applications in computing. Arizona, Hexis.
57. Wang, H., Smarandache F., Zhang, Y.Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. Multispace and Multistructure, 4, 410-413.
58. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338-353.

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