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Neutrosophic point and its neighbourhood structure

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Abstract. In this article we define neutrosophic point, neutrosophic crisp point, neighbourhood of a neutrosophic point and investigate some properties of neutrosophic point as well as neighbourhood of a neutrosophic point. We also study the characterization of neutrosophic topological space in terms of neighbourhoods.

Keywords: Neutrosophic set ; Neutrosophic point; Neutrosophic crisp point ; Neighbourhood of a neutrosophic point.

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1. Introduction

In the year 1965, L.A.Zadeh [18] introduced a revolutionary concept, Fuzzy set theory. But after some decades a new branch of philosophy, known as Neutrosophic set theory, was developed and studied by Florentin Smarandache [10–12]. Smarandache [12] proved that neutrosophic set was a generalisation of intuitionistic fuzzy set which was developed by K.Atanassov [1] in 1986 as a generalisation of fuzzy set. In an intuitionistic fuzzy set an element belonging to the universe of discourse has the degree of membership and the degree of non-membership. But in case of neutrosophic set an element has another grade of membership known as degree of indeterminacy besides the degree of membership and the degree of non-membership. After Smarandache had introduced the concept of neutrosophy, it was studied by many researchers [13, 14, 17]. In the year 2002, Smarandache [11] introduced the notion of neutrosophic topology on the non-standard interval. F.G.Lupiáñez [6, 8, 9] studied and investigated many properties of neutrosophic topological space. In [6] F.G.Lupiáñez showed that an intuitionistic fuzzy topology may not be a neutrosophic topology. The author [7] also developed the concept of interval neutrosophic sets and topology. A.A.Salma and S.Alblowi [13, 14] studied neutrosophic topological space and generalised neutrosophic topological space. A.A.Salma et.al. [15, 16] also investigated neutrosophic filters and neutrosophic continuous functions. Later, neutrosophic topology was studied by many mathematicians [2–4]. In the year 2016, Serkan Karatas and Cemil Kuru [5] redefined the set operations and introduced a new neutrosophic topology and then investigated some important properties of general topology on the redefined neutrosophic topological space. Since in neutrosphic set theory, the indeterminacy-membership is given the same importance as the truth-membership and falsehood-membership and since all the three neutrosophic components are independent of one another, so this theory is more flexible and effective than all the previous set theories. For this reason this theory is attracting the researchers throughout the world and is very useful not only in the developement of science and technology but also in various other fields. For instance, Abdel-Basset et.al. [19–23] studied the applications of neutrosophic theory in various scientific fields. Pramanik and Roy [25] in 2014 studied on the conflict between India and Pakistan over Jammu-Kashmir through neutrosophic game Theory. Mondal and Pramanik [26] studied the problems of eunuchs in West Bengal(India) based on neutrosophic cognitive maps. Very recently some studies on COVID-19 [21, 24] had been done under neutrosophic environment. But the theory still has many concepts to be developed.

In this article we try to introduce neutrosophic point and its neighbourhood structure on the neutrosophic topological space defined by Serkan Karatas and Cemil Kuru [5]. We investigate some results on neutrosophic points, neighbourhood of a neutrosophic point and study the characterization of neutrosophic topological space in terms of the neighbourhoods of neutrosophic points.

2. **Preliminaries**

In this section we discuss some concepts related with neutrosophic sets.

2.1. **Definition: [5]**

Let X be the universe of discourse. A neutrosophic set (NS for short) A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0, 1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

The set of all neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

2.2. **Definition: [5]**

Let $A, B \in \mathcal{N}(X)$. Then

(i) (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

(ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

(iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle : x \in X\}$.

(iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle : x \in X\}$.

(v) (Complement): The complement of the neutrosophic set A, denoted by A^c, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle : x \in X\}$

(vi) (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

(vii) (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

2.3. **Definition: [14]**

Let $\{A_i : i \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Then

(i) $\cup_{i\in\triangle} A_i = \{\langle x, \vee_{i\in\triangle}\mathcal{T}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{I}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}$.
i.e., $\cup_{i\in\triangle} A_i = \{\langle x, \sup_{i\in\triangle} \mathcal{T}_{A_i}(x), \inf_{i\in\triangle} \mathcal{I}_{A_i}(x), \inf_{i\in\triangle} \mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

(ii) $\cap_{i\in\triangle} A_i = \{\langle x, \wedge_{i\in\triangle}\mathcal{T}_{A_i}(x), \vee_{i\in\triangle}\mathcal{I}_{A_i}(x), \vee_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}$.
i.e., $\cap_{i\in\triangle} A_i = \{\langle x, \inf_{i\in\triangle} \mathcal{T}_{A_i}(x), \sup_{i\in\triangle} \mathcal{I}_{A_i}(x), \sup_{i\in\triangle} \mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

2.4. **Neutrosophic topological space : [5]**

2.4.1. **Definition: [5]**

Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if

(i) $\tilde{\emptyset}$ and $\tilde{X}$ belong to τ.

(ii) The union of any number of neutrosophic sets in τ belongs to τ.

(iii) The intersection of any two neutrosophic sets in τ belongs to τ.

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS for short) over X. The members of τ are called neutrosophic open sets in X. If for a neutrosophic set A, $A^c \in \tau$ then A is said to be a neutrosophic closed set in X.

2.4.2. **Theorem: [5]**

Let (X, τ) be a neutrosophic topological space over X. Then

(i) $\tilde{\emptyset}$ and $\tilde{X}$ are neutrosophic closed sets over X.

(ii) The intersection of any number of neutrosophic closed sets is a neutrosophic closed set over X.

(iii) The union of any two neutrosophic closed sets is a neutrosophic closed set over X.

3. **Main Results**

In this section we introduce and study the following concepts. Throughout this discussion we have considered the neutrosophic topological space defined by Serkan Karatas and Cemil Kuru [5] in the year 2016.

3.1. **Definition:**

Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X. A NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic point (NP for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = \alpha, \mathcal{I}_P(y) = \beta, \mathcal{F}_P(y) = \gamma$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$.

A neutrosophic point $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ will be denoted by $P^x_{\alpha,\beta,\gamma}$ or $P < x, \alpha, \beta, \gamma >$ or simply by $x_{\alpha,\beta,\gamma}$. For the NP $x_{\alpha,\beta,\gamma}$, x will be called its support.

The complement of the NP $P^x_{\alpha,\beta,\gamma}$ will be denoted by $(P^x_{\alpha,\beta,\gamma})^c$ or by $x^c_{\alpha,\beta,\gamma}$.

A NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic crisp point (NCP for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = 1, \mathcal{I}_P(y) = 0, \mathcal{F}_P(y) = 0$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$.

3.2. **Definition:**

Let A be a neutrosophic set over X. Also let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha^/,\beta^/,\gamma^/}$ be two neutrosophic points in X. Then

(i) $x_{\alpha,\beta,\gamma}$ is said to be contained in A, denoted by $x_{\alpha,\beta,\gamma} \subseteq A$, iff $\alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x)$.

(ii) $x_{\alpha,\beta,\gamma}$ is said to belong to A, denoted by $x_{\alpha,\beta,\gamma} \in A$, iff $\alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x)$.

(iii) $x_{\alpha,\beta,\gamma}$ is said to be contained in $y_{\alpha^/,\beta^/,\gamma^/}$, denoted by $x_{\alpha,\beta,\gamma} \subseteq y_{\alpha^/,\beta^/,\gamma^/}$, iff $x = y$ and $\alpha \leq \alpha^/, \beta \geq \beta^/, \gamma \geq \gamma^/$.

(iv) $x_{\alpha,\beta,\gamma}$ is said to belong to $y_{\alpha^/,\beta^/,\gamma^/}$, denoted by $x_{\alpha,\beta,\gamma} \in y_{\alpha^/,\beta^/,\gamma^/}$, iff $x = y$ and $\alpha \leq \alpha^/, \beta \geq \beta^/, \gamma \geq \gamma^/$.

(v) A NCP $x_{1,0,0} \subseteq A$ iff $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$.

(vi) A NCP $x_{1,0,0} \in A$ iff $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$.

3.3. **Remark:**

In 3.2, the definitions of inclusion and belongingness are being the same between a NP and a NS as well as between two neutrosophic points.The clarification behind that is given below :

Suppose the inclusion and belongingness between a NP and a NS are defined as follows :

$$x_{\alpha,\beta,\gamma} \subseteq A \text{ iff } \alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x).$$
$$x_{\alpha,\beta,\gamma} \in A \text{ iff } \alpha < \mathcal{T}_A(x), \beta > \mathcal{I}_A(x), \gamma > \mathcal{F}_A(x).$$

Let $X = \{x, y\}$ and $A = \{\langle x, 0.5, 0.4, 0.3\rangle, \langle y, 0.4, 0.5, 0.6\rangle\}$. Also we consider the neutrosophic points $P\langle x, 0.5, 0.4, 0.3\rangle$ and $Q\langle x, 0.3, 0.6, 0.7\rangle$. According to the above definitions, $P \subseteq A$, $Q \subseteq A$, $Q \in A$, but $P \notin A$.

Having observed the neutrosophic points P and Q and the neutrosophic set A, it is really difficult to accept that $P \notin A$ whereas $Q \in A$. Similar is the case between two neutrosophic points.

3.4. **Proposition:**

Every NS $A \in \mathcal{N}(X)$ can be expressed as the union of all neutrosophic points contained in A.

Proof: Let $B = \bigcup\{x_{\alpha,\beta,\gamma} : x_{\alpha,\beta,\gamma} \in A\}$, where $x \in X$. If $\mathcal{T}_A(x) \neq 0, \mathcal{I}_A(x) \neq 1, \mathcal{F}_A(x) \neq 1$ then

$$\begin{aligned}\mathcal{T}_A(x) &= \sup\{\alpha : x_{\alpha,\beta,\gamma} \text{ is a NP and } \alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x)\}\\ &= \mathcal{T}_{\cup x_{\alpha,\beta,\gamma}}(x)\\ &= \mathcal{T}_B(x).\end{aligned}$$

$$\begin{aligned}\mathcal{I}_A(x) &= \inf\{\beta : x_{\alpha,\beta,\gamma} \text{ is a NP and } \alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x)\}\\ &= \mathcal{I}_{\cup x_{\alpha,\beta,\gamma}}(x)\\ &= \mathcal{I}_B(x).\end{aligned}$$

$$\begin{aligned}\mathcal{F}_A(x) &= \inf\{\gamma : x_{\alpha,\beta,\gamma} \text{ is a NP and } \alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x)\}\\ &= \mathcal{F}_{\cup x_{\alpha,\beta,\gamma}}(x)\\ &= \mathcal{F}_B(x).\end{aligned}$$

Therefore $A = B$, i.e., $A = \bigcup\{x_{\alpha,\beta,\gamma} : x_{\alpha,\beta,\gamma} \in A\}$. Hence proved.

3.5. **Proposition:**

Let $A, B \in \mathcal{N}(X)$. Then $A = B$ iff $P \in A \Longleftrightarrow P \in B$ for every NP $P \in \mathcal{N}(X)$.

Proofs: Let $A = B$ and let $x_{\alpha,\beta,\gamma}$ be a NP. Then

$$\begin{aligned} & x_{\alpha,\beta,\gamma} \in A \\ \Leftrightarrow & \alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x) \\ \Leftrightarrow & \alpha \leq \mathcal{T}_B(x), \beta \geq \mathcal{I}_B(x), \gamma \geq \mathcal{F}_B(x) [\because \text{A = B}] \\ \Leftrightarrow & x_{\alpha,\beta,\gamma} \in B \end{aligned}$$

Therefore the proposition is necessary.

Converse part : From proposition 3.4 we can write $A = \bigcup\{x_{\alpha,\beta,\gamma} : x_{\alpha,\beta,\gamma} \in A\}$ and $B = \bigcup\{y_{\alpha^/,\beta^/,\gamma^/} : y_{\alpha^/,\beta^/,\gamma^/} \in B\}$. Now

$$\begin{aligned} & x_{\alpha,\beta,\gamma} \in A \\ \Rightarrow & x_{\alpha,\beta,\gamma} \in B \text{ [by the hypothesis]} \\ \Rightarrow & \cup\, x_{\alpha,\beta,\gamma} \subseteq B \\ \Rightarrow & A \subseteq B \end{aligned}$$

Exactly in the same manner we can show that $B \subseteq A$. Therefore $A = B$, i.e.,the proposition is sufficient.

Hence proved.

3.6. **Proposition:**

If $x_{\alpha,\beta,\gamma} \in A$ and $A \subseteq B$, where $A, B \in \mathcal{N}(X)$ then $x_{\alpha,\beta,\gamma} \in B$.

Proof: Since $A \subseteq B$, so $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$.

$$\begin{aligned} & x_{\alpha,\beta,\gamma} \in A \\ \Rightarrow & \alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x) \\ \Rightarrow & \alpha \leq \mathcal{T}_B(x), \beta \geq \mathcal{I}_B(x), \gamma \geq \mathcal{F}_B(x) \\ \Rightarrow & x_{\alpha,\beta,\gamma} \in B \end{aligned}$$

Hence proved.

3.7. **Proposition:**

Let $\{A_i : i \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha^/,\beta^/,\gamma^/}$ be any two neutrosophic points over X. Then the following hold good.

(i) $x_{\alpha,\beta,\gamma} \in \bigcap\{A_i : i \in\triangle\} \Longleftrightarrow x_{\alpha,\beta,\gamma} \in A_i \,\forall\, i \in\triangle$.

(ii) If $x_{\alpha,\beta,\gamma} \in A_i$ for some $i \in\triangle$ then $x_{\alpha,\beta,\gamma} \in \bigcup\{A_i : i \in\triangle\}$.

(iii) If $x_{\alpha,\beta,\gamma} \in \bigcup\{A_i : i \in\triangle\}$ then there exists a NS $A(x_{\alpha,\beta,\gamma})$ such that $x_{\alpha,\beta,\gamma} \in A(x_{\alpha,\beta,\gamma}) \subseteq \bigcup\{A_i : i \in\triangle\}$.

(iv) If $x_{\alpha,\beta,\gamma} \in A$, where $A \in \mathcal{N}(X)$, then there exist $\alpha^{/}, \beta^{/}, \gamma^{/}$ such that $\alpha \leq \alpha^{/}, \beta \geq \beta^{/}, \gamma \geq \gamma^{/}$ and $x_{\alpha^{/},\beta^{/},\gamma^{/}} \in A$.

Proofs: (i)

$$
\begin{aligned}
& x_{\alpha,\beta,\gamma} \in \bigcap\{A_i : i \in\triangle\}. \\
\Leftrightarrow & x_{\alpha,\beta,\gamma} \in \{\langle x, \wedge_{i\in\triangle}\mathcal{T}_{A_i}(x), \vee_{i\in\triangle}\mathcal{I}_{A_i}(x), \vee_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}. \\
\Leftrightarrow & \alpha \leq \wedge_{i\in\triangle}\mathcal{T}_{A_i}(x), \beta \geq \vee_{i\in\triangle}\mathcal{I}_{A_i}(x), \gamma \geq \vee_{i\in\triangle}\mathcal{F}_{A_i}(x). \\
\Leftrightarrow & \alpha \leq \inf_{i\in\triangle} \mathcal{T}_{A_i}(x), \beta \geq \sup_{i\in\triangle} \mathcal{I}_{A_i}(x), \gamma \geq \sup_{i\in\triangle} \mathcal{F}_{A_i}(x). \\
\Leftrightarrow & \alpha \leq \mathcal{T}_{A_i}(x), \beta \geq \mathcal{I}_{A_i}(x), \gamma \geq \mathcal{F}_{A_i}(x) \,\forall\, i \in\triangle\ . \\
\Leftrightarrow & x_{\alpha,\beta,\gamma} \in A_i \,\forall\, i \in\triangle\ .
\end{aligned}
$$

Hence Proved.

(ii)

$$
\begin{aligned}
& x_{\alpha,\beta,\gamma} \in A_i \text{ for some } i \in\triangle\ . \\
\Rightarrow & x_{\alpha,\beta,\gamma} \in \{\langle x, \mathcal{T}_{A_i}(x), \mathcal{I}_{A_i}(x), \mathcal{F}_{A_i}(x)\rangle : x \in X\} \text{ for some } i \in\triangle\ . \\
\Rightarrow & \alpha \leq \mathcal{T}_{A_i}(x), \beta \geq \mathcal{I}_{A_i}(x), \gamma \geq \mathcal{F}_{A_i}(x) \text{ for some } i \in\triangle\ . \\
\Rightarrow & \alpha \leq \sup_{i\in\triangle} \mathcal{T}_{A_i}(x), \beta \geq \inf_{i\in\triangle} \mathcal{I}_{A_i}(x), \gamma \geq \inf_{i\in\triangle} \mathcal{F}_{A_i}(x). \\
\Rightarrow & x_{\alpha,\beta,\gamma} \in \{\langle x, \vee_{i\in\triangle}\mathcal{T}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{I}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}. \\
\Rightarrow & x_{\alpha,\beta,\gamma} \in \bigcup\{A_i : i \in\triangle\}.
\end{aligned}
$$

Hence Proved.

(iii)

$$
\begin{aligned}
& x_{\alpha,\beta,\gamma} \in \bigcup\{A_i : i \in\triangle\} \\
\Rightarrow & \alpha \leq \sup_{i\in\triangle} \mathcal{T}_{A_i}(x), \beta \geq \inf_{i\in\triangle} \mathcal{I}_{A_i}(x), \gamma \geq \inf_{i\in\triangle} \mathcal{F}_{A_i}(x). \\
\Rightarrow & \alpha \leq \mathcal{T}_{A_r}(x), \beta \geq \mathcal{I}_{A_s}(x), \gamma \geq \mathcal{F}_{A_t}(x) \text{ for some } r, s, t \in\triangle\ . \\
\Rightarrow & x_{\alpha,\beta,\gamma} \in A_r \cup A_s \cup A_t. \\
\Rightarrow & x_{\alpha,\beta,\gamma} \in A(x_{\alpha,\beta,\gamma}), \text{where } A(x_{\alpha,\beta,\gamma}) = A_r \cup A_s \cup A_t.
\end{aligned}
$$

Obviously $A(x_{\alpha,\beta,\gamma}) = A_r \cup A_s \cup A_t \subseteq \bigcup\{A_i : i \in\triangle\}$.

Thus $x_{\alpha,\beta,\gamma} \in A(x_{\alpha,\beta,\gamma}) \subseteq \bigcup\{A_i : i \in\triangle\}$.

Hence proved.

(iv) Since $A \in \mathcal{N}(x)$, so from the proposition 3.4, $A = \bigcup\{x_{p,q,r} : x_{p,q,r} \in A\}$. Now

$$
\begin{aligned}
& x_{\alpha,\beta,\gamma} \in A. \\
\Rightarrow & \alpha \leq \mathcal{T}_A(x),\ \beta \geq \mathcal{I}_A(x),\ \gamma \geq \mathcal{F}_A(x) \\
\Rightarrow & \alpha \leq \sup_{x_{p,q,r} \in A} p,\ \beta \geq \inf_{x_{p,q,r} \in A} q,\ \gamma \geq \inf_{x_{p,q,r} \in A} r.
\end{aligned}
$$

Let $\sup\{p : x_{p,q,r} \in A\} = \alpha^/$, $\inf\{q : x_{p,q,r} \in A\} = \beta^/$, $\inf\{r : x_{p,q,r} \in A\} = \gamma^/$. Then $\alpha \leq \alpha^/, \quad \beta \geq \beta^/, \quad \gamma \geq \gamma^/$. Obviously $x_{\alpha^/,\beta^/,\gamma^/} \in A$

Thus there exist $\alpha^/, \beta^/, \gamma^/$ such that $\alpha \leq \alpha^/, \beta \geq \beta^/, \gamma \geq \gamma^/$ and $x_{\alpha^/,\beta^/,\gamma^/} \in A$. Hence proved.

3.8. **Remark:**

The converse of the proposition 3.7(ii) is not true.We shall establish it by the following counter example.

Let $X = \{x, y\}$ and $A = \{\langle x, 0.5, 0.7, 0.6\rangle, \langle y, 0.6, 0.7, 0.7\rangle\}, B = \{\langle x, 0.4, 0.6, 0.6\rangle, \langle y, 0.3, 0.8, 0.7\rangle\}$ and $C = \{\langle x, 0.3, 0.4, 0.5\rangle, \langle y, 0.6, 0.1, 0.7\rangle\}$ be three neutrosophic sets over X.Then $A \cup B \cup C = \{\langle x, 0.5, 0.4, 0.5\rangle, \langle y, 0.6, 0.1, 0.7\rangle\}$ Let us consider the neutrosophic point $P\langle x, 0.4, 0.6, 0.5\rangle$. It is clear that $P\langle x, 0.4, 0.6, 0.5\rangle \in A \cup B \cup C$ but $P\langle x, 0.4, 0.6, 0.5\rangle$ belongs to neither of the neutrosophic sets A, B and C.

3.9. **Definition:**

Let (X, τ) be a neutrosophic topological space. A NS $A \in \mathcal{N}(X)$ is called a neutrosophic neighbourhood or simply neighbourhood (nhbd for short) of a NP $x_{\alpha,\beta,\gamma}$ iff there exists a NS $B \in \tau$ such that $x_{\alpha,\beta,\gamma} \in B \subseteq A$.

A neighbourhood A of the NP $x_{\alpha,\beta,\gamma}$ is said to be a neutrosophic open neighbourhood of $x_{\alpha,\beta,\gamma}$ if A is a neutrosophic open set.

The family consisting of all the neighbourhoods of the NP $x_{\alpha,\beta,\gamma}$ is called the system of neighbourhoods(or neighbourhood system) of $x_{\alpha,\beta,\gamma}$. This family is denoted by $\mathbf{N}(x_{\alpha,\beta,\gamma})$.

3.10. **Proposition:**

A NS in a NTS is neutrosophic open iff it is a nhbd of each of its neutrosophic points.

Proof: Let (X, τ) be a NTS and let $A \in \mathcal{N}(X)$.

Suppose that A is a τ-open set. Then for every NP $x_{\alpha,\beta,\gamma} \in A$, we have $x_{\alpha,\beta,\gamma} \in A \subseteq A$ and so A is a nhbd of $x_{\alpha,\beta,\gamma}$.Thus A is nhbd of each of its neutrosophic points.

Next suppose that A is a nhbd of each of its neutrosophic points. If $A = \tilde{\emptyset}$ then A is open as $\tilde{\emptyset} \in \tau$. But if $A \neq \tilde{\emptyset}$ then for each $x_{\alpha,\beta,\gamma} \in A$ there exists a τ-open set $B(x_{\alpha,\beta,\gamma})$ such that

$x_{\alpha,\beta,\gamma} \in B(x_{\alpha,\beta,\gamma}) \subseteq A$. Obviously $A = \cup B(x_{\alpha,\beta,\gamma})$ and so A is τ-open set, being a union of τ-open sets.

Hence proved.

3.11. **Proposition:**

Two neutrosophic topologies on the same set are identical iff they admit the same neighbourhoods.

Proof: Necessary part is very obvious. Conversely suppose that τ_1 and τ_2 are two neutrosophic topologies on X having the same neighbourhood system of the neutrosophic points over X. Let $A \in \mathcal{N}(X)$. Now

$$\begin{aligned} & A \text{ is a } \tau_1 - \text{open set} \\ \Leftrightarrow & A \text{ is a } \tau_1 - \text{neighbourhood of each of its neutrosophic points} \\ \Leftrightarrow & A \text{ is a } \tau_2 - \text{neighbourhood of each of its neutrosophic points} \\ \Leftrightarrow & A \text{ is a } \tau_2 - \text{open set} \end{aligned}$$

Therefore $\tau_1 = \tau_2$.
Hence proved.

3.12. **Remark:**

It is very clear that for every NP $x_{\alpha,\beta,\gamma}$, $\ x_{\alpha,\beta,\gamma} \in \mathcal{N}(X) \Longleftrightarrow x_{\alpha,\beta,\gamma} \in \tilde{X}$.

3.13. **Properties of neutrosophic neighbourhoods:**

Let (X, τ) be a neutrosophic topological space and let $x \in X$.If $\mathbf{N}(x_{\alpha,\beta,\gamma})$ be the collection of all nhbds of the neutrosophic point $x_{\alpha,\beta,\gamma}$ then

N1) $\mathbf{N}(x_{\alpha,\beta,\gamma}) \neq \emptyset$ for every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$.
N2) $N \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Rightarrow x_{\alpha,\beta,\gamma} \in N$.
N3) $N \in \mathbf{N}(x_{\alpha,\beta,\gamma}), N \subseteq M \Rightarrow M \in \mathbf{N}(x_{\alpha,\beta,\gamma})$.
N4) $M, N \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Rightarrow M \cap N \in \mathbf{N}(x_{\alpha,\beta,\gamma})$.
N5) $N \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Rightarrow$ there exists a $M \in \mathbf{N}(x_{\alpha,\beta,\gamma})$ such that $M \subseteq N$ and $M \in \mathbf{N}(y_{\alpha',\beta',\gamma'})$ for all $y_{\alpha',\beta',\gamma'} \in M$.

Proofs:

N1) Since $\tilde{X}$ is an open set, so it is a nhbd of every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$. Thus there exists at least one nhbd for every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$. Therefore $\mathbf{N}(x_{\alpha,\beta,\gamma}) \neq \emptyset$ for every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$.

N2) $N \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Rightarrow N$ is a nhbd of $x_{\alpha,\beta,\gamma} \Rightarrow x_{\alpha,\beta,\gamma} \in N$.

N3)

$$\begin{aligned}
&N \in \mathbf{N}(x_{\alpha,\beta,\gamma})\\
&\Rightarrow N \text{ is a neighbourhood of } x_{\alpha,\beta,\gamma}\\
&\Rightarrow \exists \text{ an open set } G \text{ such that } x_{\alpha,\beta,\gamma} \in G \subseteq N.\\
&\Rightarrow \exists \text{ an open set } G \text{ such that } x_{\alpha,\beta,\gamma} \in G \subseteq M.[\because N \subseteq M]\\
&\Rightarrow M \text{ is a neighbourhood of } x_{\alpha,\beta,\gamma}\\
&\Rightarrow M \in \mathbf{N}(x_{\alpha,\beta,\gamma})
\end{aligned}$$

N4) $M, N \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Rightarrow M, N$ are neighbourhoods of $x_{\alpha,\beta,\gamma} \Rightarrow \exists\ G_1, G_2 \in \tau$ such that $x_{\alpha,\beta,\gamma} \in G_1 \subseteq N$ and $x_{\alpha,\beta,\gamma} \in G_2 \subseteq M$. But $G_1, G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau$. Therefore $x_{\alpha,\beta,\gamma} \in G_1 \cap G_2 \subseteq M \cap N$ and so $M \cap N$ is a nhbd of $x_{\alpha,\beta,\gamma}$, i.e., $M \cap N \in \mathbf{N}(x_{\alpha,\beta,\gamma})$.

N5) Since $N \in \mathbf{N}(x_{\alpha,\beta,\gamma})$, so there exists a τ-open set M such that $x_{\alpha,\beta,\gamma} \in M \subseteq N$. Since M is an open set and since $x_{\alpha,\beta,\gamma} \in M \subseteq M$, so $M \in \mathbf{N}(x_{\alpha,\beta,\gamma})$. Thus $M \in \mathbf{N}(x_{\alpha,\beta,\gamma})$ and $M \subseteq N$.

Again since M is an open set, so M is a nhbd of each of its neutrosophic points. Therefore $M \in \mathbf{N}(y_{\alpha^/,\beta^/,\gamma^/})$ for all $y_{\alpha^/,\beta^/,\gamma^/} \in M$.

Hence proved.

3.14. **Characterization of NTS in terms of neutrosophic neighbourhoods:**

Let X be the universe of discourse and $x \in X$. Let $\mathbf{N}(x_{\alpha,\beta,\gamma})$ be a family of neutrosophic sets over X satisfying the following five conditions :

N1) $\mathbf{N}(x_{\alpha,\beta,\gamma}) \neq \emptyset$ for every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$.

N2) $N \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Rightarrow x_{\alpha,\beta,\gamma} \in N$.

N3) $N \in \mathbf{N}(x_{\alpha,\beta,\gamma}), N \subseteq M \Rightarrow M \in \mathbf{N}(x_{\alpha,\beta,\gamma})$.

N4) $M, N \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Rightarrow M \cap N \in \mathbf{N}(x_{\alpha,\beta,\gamma})$.

N5) $N \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Rightarrow$ there exists a $M \in \mathbf{N}(x_{\alpha,\beta,\gamma})$ such that $M \subseteq N$ and $M \in \mathbf{N}(y_{\alpha^/,\beta^/,\gamma^/})$ for all $y_{\alpha^/,\beta^/,\gamma^/} \in M$.

Then there exists a unique neutrosophic topology τ on X in such a way that if $\mathbf{N}^*(x_{\alpha,\beta,\gamma})$ is the collection of all nhbds of the NP $x_{\alpha,\beta,\gamma}$, defined by the topology τ, then $\mathbf{N}(x_{\alpha,\beta,\gamma}) = \mathbf{N}^*(x_{\alpha,\beta,\gamma})$.

Proof: We define τ as follows :

A NS $G \in \tau$ iff $G \in \mathbf{N}(x_{\alpha,\beta,\gamma})$ for every NP $x_{\alpha,\beta,\gamma} \in G$.

We claim that τ is a neutrosophic topology on X.

T1) $\tilde{\emptyset} \in \tau$ as $\tilde{\emptyset}$ contains no NP. By (N1) $\mathbf{N}(x_{\alpha,\beta,\gamma}) \neq \emptyset$ for every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$. Therefore there exists a $G(x_{\alpha,\beta,\gamma}) \in \mathbf{N}(x_{\alpha,\beta,\gamma})$ for every NP $x_{\alpha,\beta,\gamma} \in \tilde{X}$. Since $G(x_{\alpha,\beta,\gamma}) \subseteq \tilde{X}$, so by (N3), $\tilde{X} \in \mathbf{N}(x_{\alpha,\beta,\gamma})$ for every $x_{\alpha,\beta,\gamma} \in \tilde{X}$. Therefore $\tilde{X} \in \tau$. Thus $\tilde{\emptyset}, \tilde{X} \in \tau$.

T2) Suppose $G_1, G_2 \in \tau$. Then

$$G_1 \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \ \forall \ x_{\alpha,\beta,\gamma} \in G_1 \text{ and } G_2 \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \ \forall \ x_{\alpha,\beta,\gamma} \in G_2$$

$$\Rightarrow G_1 \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \text{ and } G_2 \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \ \forall \ x_{\alpha,\beta,\gamma} \in G_1 \cap G_2$$

$$\Rightarrow G_1 \cap G_2 \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \ \forall \ x_{\alpha,\beta,\gamma} \in G_1 \cap G_2 \text{ [by (N4)]}$$

$$\Rightarrow G_1 \cap G_2 \in \tau \text{ [by the definition of } \tau]$$

T3) Suppose $\{G_i : i \in \triangle\} \subseteq \tau$. We show that $\cup\{G_i : i \in \triangle\} \in \tau$. Now

$$G_i \in \tau \ \forall \ i \in \triangle$$

$$\Rightarrow G_i \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \ \forall \ x_{\alpha,\beta,\gamma} \in G_i \text{ and } \forall \ i \in \triangle$$

$$\Rightarrow \cup \{G_i : i \in \triangle\} \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \ \forall \ x_{\alpha,\beta,\gamma} \in \cup\{G_i : i \in \triangle\} \text{[by (N2)and (N3)]}$$

$$\Rightarrow \cup \{G_i : i \in \triangle\} \in \tau \text{ [by the definition of } \tau]$$

Therefore τ is a neutrosophic topology on X.

We now show that $\mathbf{N}(x_{\alpha,\beta,\gamma}) = \mathbf{N}^*(x_{\alpha,\beta,\gamma})$, i.e., $N \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Leftrightarrow N$ is a nhbd of $x_{\alpha,\beta,\gamma}$.

Let $N \in \mathbf{N}(x_{\alpha,\beta,\gamma})$. Then by (N5) there exists $M \in \mathbf{N}(x_{\alpha,\beta,\gamma})$ such that $M \subseteq N$ and $M \in \mathbf{N}(y_{\alpha^/,\beta^/,\gamma^/})$ for all $y_{\alpha^/,\beta^/,\gamma^/} \in M$. Now $M \in \mathbf{N}(x_{\alpha,\beta,\gamma}) \Rightarrow x_{\alpha,\beta,\gamma} \in M$ [by (N2)]. Also $M \in \mathbf{N}(y_{\alpha^/,\beta^/,\gamma^/})$ for all $y_{\alpha^/,\beta^/,\gamma^/} \in M \Rightarrow M \in \tau$. Thus M is a τ-open set such that $x_{\alpha,\beta,\gamma} \in M \subseteq N$. Therefore N is a neighbourhood of $x_{\alpha,\beta,\gamma}$, i.e., $N \in \mathbf{N}^*(x_{\alpha,\beta,\gamma})$, i.e., $\mathbf{N}(x_{\alpha,\beta,\gamma}) \subseteq \mathbf{N}^*(x_{\alpha,\beta,\gamma})$. Conversely let $N \in \mathbf{N}^*(x_{\alpha,\beta,\gamma})$ so that N is a nhbd of $x_{\alpha,\beta,\gamma}$. Then there exists a τ-open set G such that $x_{\alpha,\beta,\gamma} \in G \subseteq N$. Now $G \in \tau \Rightarrow G \in \mathbf{N}(x_{\alpha,\beta,\gamma})$ for all $x_{\alpha,\beta,\gamma} \in G$. But $G \in \mathbf{N}(x_{\alpha,\beta,\gamma})$ and $G \subseteq N$ together imply by (N3) that $N \in \mathbf{N}(x_{\alpha,\beta,\gamma})$. Therefore $\mathbf{N}^*(x_{\alpha,\beta,\gamma}) \subseteq \mathbf{N}(x_{\alpha,\beta,\gamma})$.Therefore $\mathbf{N}(x_{\alpha,\beta,\gamma}) = \mathbf{N}^*(x_{\alpha,\beta,\gamma})$.

Next we show the uniqueness of the topology.

Le τ and $\tau^/$ be two topologies on X having the same system of neighbourhoods. Let $G \in \mathcal{N}(X)$. Then

$$G \in \tau.$$

$$\Leftrightarrow G \text{ is a } \tau - \text{open set.}$$

$$\Leftrightarrow G \text{ is a } \tau - \text{neighbourhood of } x_{\alpha,\beta,\gamma} \text{ for all NP } x_{\alpha,\beta,\gamma} \in G.$$

$$\Leftrightarrow G \text{ is a } \tau^/ - \text{neighbourhood of } x_{\alpha,\beta,\gamma} \text{ for all NP } x_{\alpha,\beta,\gamma} \in G.$$

$$\Leftrightarrow G \text{ is a } \tau^/ - \text{open set.}$$

$$\Leftrightarrow G \in \tau^/$$

Therefore $\tau = \tau^/$. Thus the topology is unique.

Hence proved.

4. **Conclusion**

Like fuzzy and intuitionistic fuzzy set theories, neutrosophic set theory also deals with imprecise situation. But neutrosophic theory also handles the situation of neutrality which keeps this theory ahead of those theories. In this article we tried to introduce the concept of neutrosophic point and neighbourhood of a neutrosophic point. We discussed some properties of neutrosophic points and their neighbourhoods. We also studied about the characterization of neutrosophic topological space in terms of the neighbourhoods of the neutrosophic points.

5. **Conflict of Interest**

We certify that there is no actual or potential conflict of interest in relation to this article.

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