Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers, Absorbance Law, and the Multiplication of Neutrosophic Quadruple Numbers

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Abstract. In this paper, we introduce for the first time the neutrosophic quadruple numbers (of the form \(a + bT + cI + dF\)) and the refined neutrosophic quadruple numbers. Then we define an absorbance law, based on a prevalence order, both of them in order to multiply the neutrosophic components \(T, I, F\) or their sub-components \(T_p, I_k, F_l\) and thus to construct the multiplication of neutrosophic quadruple numbers.

Keywords: neutrosophic quadruple numbers, refined neutrosophic quadruple numbers, absorbance law, multiplication of neutrosophic quadruple numbers, multiplication of refined neutrosophic quadruple numbers.

1 Neutrosophic Quadruple Numbers

Let’s consider an entity (i.e. a number, an idea, an object, etc.) which is represented by a known part \(a\) and an unknown part \((bT + cI + dF)\).

Numbers of the form:

\[ \text{NQ} = a + bT + cI + dF, \]  

where \(a, b, c, d\) are real (or complex) numbers (or intervals or in general subsets), and

- \(T = \text{truth / membership / probability,}\)
- \(I = \text{indeterminacy,}\)
- \(F = \text{false / membership / improbability,}\)

are called Neutrosophic Quadruple (Real respectively Complex) Numbers (or Intervals, or in general Subsets).

“\(a\)” is called the known part of \(\text{NQ}\), while “\(bT + cI + dF\)” is called the unknown part of \(\text{NQ}\).

2 Operations

Let’s consider that Neutrosophic Quadruple Numbers are numbers of the form:

\[ \text{RNQ} = a + \sum_{i=1}^{p} b_i T_i + \sum_{j=1}^{r} c_j I_j + \sum_{k=1}^{s} d_k F_k, \]  

where \(a, b_i, c_j, d_k\) are real (or complex) numbers, intervals, or, in general Subsets.

Then:

2.1 Addition

\[ \text{NQ}_1 + \text{NQ}_2 = (a_1 + a_2) + (b_1 + b_2)T + (c_1 + c_2)I + (d_1 + d_2)F. \]  

2.2 Subtraction

\[ \text{NQ}_1 - \text{NQ}_2 = (a_1 - a_2) + (b_1 - b_2)T + (c_1 - c_2)I + (d_1 - d_2)F. \]  

2.3 Scalar Multiplication

\[ a \cdot \text{NQ} = \text{NQ} \cdot a = a(a + abT + acI + adF). \]  

One has:

\[ 0 \cdot T = 0 \cdot I = 0 \cdot F = 0, \]  

\[ mT + nT = (m + n)T, \]  

\[ mI + nI = (m + n)I, \]  

\[ mF + nF = (m + n)F. \]  

3 Refined Neutrosophic Quadruple Numbers

Let us consider that Refined Neutrosophic Quadruple Numbers are numbers of the form:

\[ \text{RNQ} = a + \sum_{i=1}^{p} b_i T_i + \sum_{j=1}^{r} c_j I_j + \sum_{k=1}^{s} d_k F_k, \]  

where \(a, b_i, c_j, d_k\) are real (or complex) numbers, intervals, or, in general Subsets.

Then:

2.1 Addition

\[ \text{RNQ}_1 + \text{RNQ}_2 = (a_1 + a_2) + (b_1 + b_2)T + (c_1 + c_2)I + (d_1 + d_2)F. \]  

2.2 Subtraction

\[ \text{RNQ}_1 - \text{RNQ}_2 = (a_1 - a_2) + (b_1 - b_2)T + (c_1 - c_2)I + (d_1 - d_2)F. \]  

There are cases when the known part \((a)\) can be refined as well as \(a_1, a_2, \ldots\)

The operations are defined similarly.

Let
\[ RNQ^{(u)} = a^{(u)} + \sum_{i=1}^{p} b_i^{(u)}T_i + \sum_{j=1}^{r} c_j^{(u)}I_j + \sum_{k=1}^{s} d_k^{(u)}F_k, \]

for \( u = 1 \) or \( 2 \).

### 3.1 Addition

\[
RNQ^{(1)} + RNQ^{(2)} = \left[ a^{(1)} + a^{(2)} \right] + \sum_{i=1}^{p} \left[ b_i^{(1)} + b_i^{(2)} \right] T_i + \sum_{j=1}^{r} \left[ c_j^{(1)} + c_j^{(2)} \right] I_j + \sum_{k=1}^{s} \left[ d_k^{(1)} + d_k^{(2)} \right] F_k.
\]

### 3.2 Substraction

\[
RNQ^{(1)} - RNQ^{(2)} = \left[ a^{(1)} - a^{(2)} \right] + \sum_{i=1}^{p} \left[ b_i^{(1)} - b_i^{(2)} \right] T_i + \sum_{j=1}^{r} \left[ c_j^{(1)} - c_j^{(2)} \right] I_j + \sum_{k=1}^{s} \left[ d_k^{(1)} - d_k^{(2)} \right] F_k.
\]

### 3.3 Scalar Multiplication

For \( \alpha \in \mathbb{R} \) (or \( \alpha \in \mathbb{C} \)) one has:

\[
\alpha \cdot RNQ^{(1)} = \alpha \cdot a^{(1)} + \alpha \cdot \sum_{i=1}^{p} b_i^{(1)}T_i + \alpha \cdot \sum_{j=1}^{r} c_j^{(1)}I_j + \alpha \cdot \sum_{k=1}^{s} d_k^{(1)}F_k.
\]

### 4 Absorbance Law

Let \( S \) be a set, endowed with a total order \( x \ll y \), named “\( x \) prevailed by \( y \)” or “\( x \) less stronger than \( y \)” or “\( x \) less preferred than \( y \)”. We consider \( x \preceq y \) as “\( x \) prevailed by or equal to \( y \)” “\( x \) less strong than or equal to \( y \)”, or “\( x \) less preferred than or equal to \( y \)”. For any elements \( x,y \in S \), with \( x \ll y \), one has the absorbance law:

\[ x \cdot y = y \cdot x = \text{absorb} (x,y) = \max \{ x, y \} = y, \]

which means that the bigger element absorbs the smaller element (the big fish eats the small fish!). Clearly,

\[ x \cdot x = \text{absorb} (x,x) = \max \{ x, x \} = x, \]

and

\[
x_1 \cdot x_2 \cdot \ldots \cdot x_n = \text{absorb} (\ldots \text{absorb} (\text{absorb} (x_1, x_2), x_3), \ldots, x_n) = \max \{ \ldots \max \{ \max \{ x_1, x_2 \}, x_3 \}, \ldots, x_n \} = \max \{ x_1, x_2, \ldots, x_n \}.
\]

Analogously, we say that “\( x > y \)” and we read: “\( x \) prevails to \( y \)” or “\( x \) is stronger than or equal to \( y \)”.

Also, \( x 
precedes \) \( y \), and we read: “\( x \) prevails or is equal to \( y \)” “\( x \) is stronger than or equal to \( y \)”, or “\( x \) is preferred or equal to \( y \)”.

### 5 Multiplication of Neutrosophic Quadruple Numbers

It depends on the prevalence order defined on \( \{ T, I, F \} \). Suppose in an optimistic way the neutrosophic expert considers the prevalence order \( T > I > F \). Then:

\[
NQ_1 \cdot NQ_2 = (a_1 + b_1T + c_1I + d_1F) \cdot (a_2 + b_2T + c_2I + d_2F) = a_1a_2 + (a_1b_2 + a_2b_1 + b_1b_2 + b_1c_2 + c_1b_2 + b_1d_2 + d_1b_2)T + (a_1c_2 + a_2c_1 + c_1d_2 + c_2d_1)I + (a_1d_2 + a_2d_1 + d_1d_2)F,
\]

since \( TI = IT = T, TF = FT = T, IF = FI = I \), while \( T^2 = I, F^2 = F \).

Suppose in a pessimistic way the neutrosophic expert considers the prevalence order \( F > I > T \). Then:

\[
NQ_1 \cdot NQ_2 = (a_1 + b_1T + c_1I + d_1F) \cdot (a_2 + b_2T + c_2I + d_2F) = a_1a_2 + (a_1b_2 + a_2b_1 + b_1b_2 + b_1c_2 + c_1b_2 + b_1d_2 + d_1b_2)T + (a_1c_2 + a_2c_1 + c_1d_2 + c_2d_1)I + (a_1d_2 + a_2d_1 + d_1d_2)F,
\]

since \( F \cdot I = I \cdot F = F, F \cdot T = T \cdot F = F, I \cdot T = T \cdot I = I \) while similarly \( F^2 = F, I^2 = I, T^2 = T \).

### 5.1 Remark

Other prevalence orders on \( \{ T, I, F \} \) can be proposed, depending on the application/problem to solve, and on other conditions.
6 Multiplication of Refined Neutrosophic Quadruple Numbers

Besides a neutrosophic prevalence order defined on \( \{T, I, F\} \), we also need a sub-prevalence order on \( \{T_1, T_2, \ldots, T_p\} \), a sub-prevalence order on \( \{I_1, I_2, \ldots, I_r\} \), and another sub-prevalence order on \( \{F_1, F_2, \ldots, F_s\} \).

We assume that, for example, if \( T > I > F \), then \( T_j > I_k > F_l \) for any \( j \in \{1, 2, \ldots, p\} \), \( k \in \{1, 2, \ldots, r\} \), and \( l \in \{1, 2, \ldots, s\} \). Therefore, any prevalence order on \( \{T, I, F\} \) imposes a prevalence suborder on their corresponding refined components.

Without loss of generality, we may assume that \( T_1 > T_2 > \cdots > T_p \)
(if this was not the case, we re-number the subcomponents in a decreasing order).

Similarly, we assume without loss of generality that:

\[ I_1 > I_2 > \cdots > I_r \]
\[ F_1 > F_2 > \cdots > F_s \]

6.1 Exercise for the Reader

Let’s have the neutrosophic refined space
\( NS = \{T_1, T_2, T_3, I, F_1, F_2\} \),
with the prevalence order \( T_1 > T_2 > T_3 > I > F_1 > F_2 \).

Let’s consider the refined neutrosophic quadruples
\( NA = 2 - 3T_1 + 2T_2 + T_3 - I + 5F_1 - 3F_2 \), and
\( NB = 0 + T_1 - T_2 + 0 \cdot T_3 + 5I - 8F_1 + 5F_2 \).

By multiplication of sub-components, the bigger absorbs the smaller. For example:
\[ T_2 \cdot T_3 = T_2, \]
\[ T_1 \cdot F_1 = T_1, \]
\[ I \cdot F_2 = I, \]
\[ T_2 \cdot F_1 = T_2, \text{ etc.} \]
Multiply NA with NB.

References


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