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Single Valued Neutrosophic R-dynamic Vertex Coloring of Graphs

Aparna V[1], Mohanapriya N[2,*] and Said Broumi[3]

[1,2]PG and Research Department of Mathematics, Kongunadu Arts and Science College, Coimbatore-641 029, Tamil Nadu, India.; aparnav18794@gmail.com

[3]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman Casablanca, Morocco; broumisaid78@gmail.com

[*]Correspondence: Mohanapriya N, phdmohana@gmail.com

Abstract. In 1998, Smarandache introduced the new theory - Neutrosophic sets. In order to achieve the best results in a current situation, policy makers must contend with uncertainty and unpredictability. The neutrosophic definition aids in the investigation of ambiguous or indeterminate values. Here, we have the amalgamated the theory of Single Valued Neutrosophic Vertex Coloring and r-dynamic coloring to introduce a new thought Single Valued Neutrosophic R-dynamic Vertex Coloring (SVNRVC) and have shown example. Further we have determined the Single Valued Neutrosophic R-dynamic chromatic number $\chi_R^v(G)$ for some graphs.

Keywords: Single Valued Neutrosophic Graph; Single Valued Neutrosophic Vertex Coloring; Single Valued Neutrosophic R-dynamic Vertex Coloring.

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1. Introduction

Graph Theory dates back to the year 1736 when the famous Mathematician Leonard Euler solved the Problem of Seven Bridges of Konigsberg. Graphs are mathematical structures made up of a set of vertices connected by edges. Many complex real-world problems can be successfully analysed using graphs as mathematical models. It can be used in a variety of fields such as chemical and physical sciences, networks, maps, sudoku, operations research, and so on. Graph coloring is a sub-discipline with graph theory. The famous Four Color Problem, posed by graduate student Francis Guthrie in 1852, inspired the problem of graph coloring. Is it possible to color the countries on the map with four or fewer colors so that any two countries sharing a border are colored differently? It was later on demonstrated by Appel and Haken in

1976. Graph coloring is the process of assigning colors to the elements of graph while keeping some constraints in mind.

Zadeh [26] proposed the theory of fuzzy sets way back in 1965, and ten years hence A. Rosenfeld [21] developed further work on fuzzy graph theory. Munoz et al. [24] first proposed the fuzzy chromatic number in 2004, and C. Eslahchi et al. [13] expanded it further in 2006. The idea of fuzzy total coloring was first suggested by S. Lavanya and R. Sattanathan [15] in 2009. In 2012 Arindam Dey and Anita Pal discussed fuzzy vetex coloring using α-cut of fuzzy graphs in [6]. In a research paper published in 2014, the strong chromatic number of such graphs was addressed by Anjaly Kishore, M.S.Sunitha [4].

Intuitionistic fuzzy sets are used to deal with data on membership and non-membership values. In 1986, Kassimir T. Atanassov [5] proposed the theory of intuitionistic fuzzy sets, and in 1999, he proposed the notion of intuitionistic fuzzy graphs. In 2015, Ismail and Rifayathali [14] examined intuitionistic fuzzy graph coloring using (α, β) cuts, while Rifayathali et al. [17] in 2017 and 2018 published articles on intuitionistic fuzzy and strong intuitionistic fuzzy coloring.

The membership and non-membership principles are inadequate to determine the outcome of all real-time scenarios. Where the vagueness or indeterminacy qualities of a decision need to be weighed, intuitionistic fuzzy logic is inadequate to provide a solution. As a consequence of this condition, F. Smarandache devised a solution: "Neutrosophic logic." Neutrosophic logic is important in a number of real-world problems, including law, business, medicine, finance, information technology and so on. Thus in 1998 Smarandache [22] introduced the thought of Neutrosophic sets which is a generalized version of intuitionistic fuzzy set which includes three types of values: truth, indeterminacy and false membership values. In 2010, Wang et al. [25] investigated single valued neutrosophic sets. Dhavaseelan et al. [12] put forward and discussed the Strong Neutrosophic graphs in 2015, and Akram and Shahzadi [1–3] introduced and discussed the Single Valued Neutrosophic definition in 2016. Broumi et al. [7–11] built on their previous work in the areas of single-valued neutrosophic graphs. In their paper published in 2018, Dhavaseelan et al. [12] addressed single valued co-neutrosophic graphs. In 2018, Sinha et al. [23] widened the scope of the single-valued work for signed digraphs.

In the research articles [18, 19] published in 2019 Rohini et al. introduced the thought of single valued neutrosophic vertex, edge and total coloring of SVNG with examples. Further in [20] Rohini et al. have extended their work on single valued neutrosophic vertex coloring and put forward the new idea of single valued neutrosophic irregular vertex coloring.

The idea of r-dynamic coloring was put forward by Bruce Montgomery in [16]. The r-dynamic coloring of a graph is a proper coloring of the graph such that for each vertex u, the neighbors of the vertex u receives $min\{r, d(v)\}$ different colors. Here we have integrated the thought of single valued neutrosophic vertex coloring and r-dynamic coloring to introduce the new idea of Single Valued Neutrosophic R-dynamic Vertex Coloring and have shown example. Further we have determined the Single Valued Neutrosophic R-dynamic chromatic number $\chi_R^v(G)$ for some graphs.

2. Preliminaries

Definition 2.1. [22] Assume S be a collection of points(objects). A **neutrosophic set** X in S is represented by truth membership function $t_X(s)$, an indeterminacy function $i_X(s)$ and a falsity membership (non-membership) function $f_X(s)$. $t_X(s)$, $i_X(s)$ and $f_X(s)$ are real standard or non-standard subsets of $]0^-, 1^+[$ which means $t_X(s) : S \rightarrow]0^-, 1^+[$, $i_X(s) : S \rightarrow]0^-, 1^+[$ and $f_X(s) : S \rightarrow]0^-, 1^+[$. Also $0^- \leq t_X(s) + i_X(s) + f_X(s) \leq 3^+$.

Definition 2.2. [2] A **Single Valued Neutrosophic Graph (SNVG)** $G = (P, Q)$ is a pair where $P : N \rightarrow [0, 1]$ is a single valued neutrosophic set on N and $Q : N \times N \rightarrow [0, 1]$ is a single valued neutrosophic relation on N with the following properties:
$t_Q(uv) \leq min\{t_P(u), t_P(v)\}$
$i_Q(uv) \leq min\{i_P(u), i_P(v)\}$
$f_Q(uv) \leq max\{f_P(u), f_P(v)\}$
for all $u, v \in N$. The sets P and Q are said to be single valued neutrosophic vertex set and edge set of G respectively. The single valued neutrosophic relation Q is symmetric if it satisfies $t_Q(uv) = t_Q(vu)$, $i_Q(uv) = i_Q(vu)$ and $f_Q(uv) = f_Q(vu)$ for all $u, v \in N$.

Definition 2.3. [3] An SVNG $G = (P, Q)$ is called a **complete neutrosophic graph (CSVNG)** if it complies criteria below:
$t_Q(uv) = min\{t_P(u), t_P(v)\}$
$i_Q(uv) = min\{i_P(u), i_P(v)\}$
$f_Q(uv) = max\{f_P(u), f_P(v)\}$
for all $u, v \in P$.

Definition 2.4. [3] The **complement** of a SVNG $G = (P, Q)$ is a SNVG $G' = (P', Q')$ where
$i) P' = P$
$ii) t'_P(u) = t_P(u), i'_P(u) = i_P(u) \ and \ f'_P(u) = f_P(u)$
$$iii) t'_Q(uv) = \begin{matrix} min\{t_P(u), t_P(v)\} & if \quad t_Q(uv) = 0 \\ min\{t_P(u), t_P(v)\} - t_Q(uv) & if \quad t_Q(uv) > 0 \end{matrix}$$
$$iv) i'_Q(uv) = \begin{cases} min\{i_P(u), i_P(v)\} & if \quad i_Q(uv) = 0 \\ min\{i_P(u), i_P(v)\} - i_Q(uv) & if \quad i_Q(uv) > 0 \end{cases}$$
$$v) f'_Q(uv) = \begin{cases} max\{f_P(u), f_P(v)\} & if \quad f_Q(uv) = 0 \\ max\{f_P(u), f_P(v)\} - f_Q(uv) & if \quad f_Q(uv) > 0 \end{cases}$$
for all $u, v \in P$.

Definition 2.5. [3] An SVNG $G = (P, Q)$ is said to be a **strong neutrosophic graph (SSVNG)** if it complies criteria:
$t_Q(uv) = min\{t_P(u), t_P(v)\}$
$i_Q(uv) = min\{i_P(u), i_P(v)\}$
$f_Q(uv) = max\{f_P(u), f_P(v)\}$
for all $(u, v) \in Q$.

Definition 2.6. [18] The collection $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_k\}$ of SVN fuzzy sets is termed as **k-Single Valued Neutrosophic Vertex Coloring(SVNVC)** of a SVNG $G = (P, Q)$ if the following criteria hold:
1. $\vee\gamma_j(u) = P, \forall u \in P$
2. $\gamma_j \wedge \gamma_h = 0$
3. For each incident vertices of the edge uv of G, $min\{\gamma_j(t_P(u)), \gamma_j(t_P(v))\} = 0$, $min\{\gamma_j(i_P(u)), \gamma_j(i_P(v))\} = 0$ and $max\{\gamma_j(f_P(u)), \gamma_j(f_P(v))\} = 1, (1 \leq j \leq k)$.
This is indicated as $\chi^v(G)$ and is termed as the SVN chromatic number of the SVNG G.

Example: Consider the following SVNG $G_1 = (P, Q)$ with SVN vertex set $P = \{v_1, v_2, v_3, v_4\}$ and SVN edge $Q = \{v_j v_k | jk = 12, 13, 14, 23, 25, 34\}$ with

$$(t_P(v_j), i_P(v_j), f_P(v_j)) = \begin{cases} (0.2, 0.3, 0.7) & j = 1 \\ (0.7, 0.2, 0.8) & j = 2 \\ (0.6, 0.5, 0.9) & j = 3 \\ (0.5, 0.4, 0.6) & j = 4 \end{cases}$$

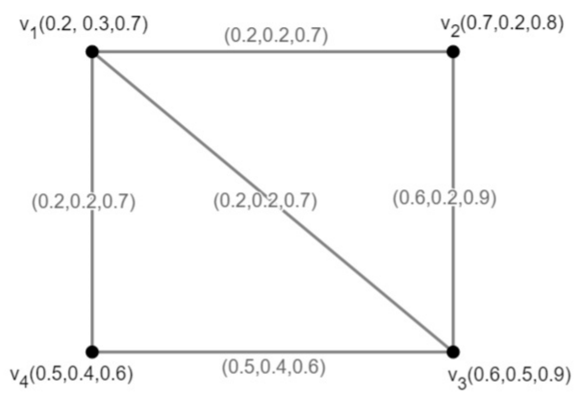

Figure 1. G_1

$$(t_Q(v_j v_k), i_Q(v_j v_k), f_Q(v_j v_k)) = \begin{cases} (0.2, 0.2, 0.7) & jk = 12, 13, 14 \\ (0.6, 0.2, 0.9) & jk = 23 \\ (0.5, 0.4, 0.6) & jk = 34 \end{cases}$$

Figure 1 depicts the SVNG G_1.
Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be collection of SVN fuzzy sets determined on P as below:

$$\gamma_1(v_j) = \begin{cases} (0.2, 0.3, 0.7) & for \quad j = 1 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_2(v_j) = \begin{cases} (0.7, 0.2, 0.8) & for \quad j = 2 \\ (0.5, 0.4, 0.6) & for \quad j = 4 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_3(v_j) = \begin{cases} (0.6, 0.5, 0.9) & for \quad j = 3 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ assures the criteria of SVNVC of the graph G. Any collection with points less than three points will not fulfill our definition. Hence $\chi^v(G_1) = 3$.

Definition 2.7. [20] The collection $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_k\}$ of SVN fuzzy sets is called a **k-Single Valued Neutrosophic Irregular Vertex Coloring (SVNIVC)** of a SVNG $G = (P, Q)$ if the following criteria hold:

1. $\vee\, \gamma_j$ (u) $= P, \forall u \in P$

2.$\gamma_j \wedge \gamma_h = 0$

3. For each incident vertices of edge uv of G, $min\{\gamma_j(t_P(u)), \gamma_j(t_P(v))\} = 0$, $min\{\gamma_j(i_P(u)),$ $\gamma_j(i_P(v))\} = 0$ and $max\{\gamma_j(f_P(u)), \gamma_j(f_P(v))\} = 1, (1 \leq j \leq k)$.

4. All the vertices have different color codes.

This is depicted as $\chi^v{}_{ir}\ (G)$ and is termed as the SVNI chromatic number of the SVNG G.

Definition 2.8. [8] **Path** P_n in a single valued neutrosophic graph $G = (P, Q)$ is an ar rangement of distinct vertices $v_1, v_2, \cdots, v_n$ which complies the criteria $t_Q(v_{j-1}, v_j) > 0$, $i_Q(v_{j-1}, v_j) > 0$ and $f_Q(v_{j-1}, v_j) > 0$ for $2 \leq j \leq n$.

Definition 2.9. [8] A **cycle** C_n in a single valued neutrosophic graph $G = (P, Q)$ is a sequence of distinct vertices $v_1, v_2, \cdots, v_n, v_1$ which satisfies the condition $t_Q(v_{i-1}, v_i) > 0$, $i_Q(v_{i-1}, v_i) > 0$ and $f_Q(v_{i-1}, v_i) > 0$ for $2 \leq i \leq n$ together with $t_Q(v_n, v_1) > 0$, $i_Q(v_n, v_1) > 0$ and $f_Q(v_n, v_1) > 0$.

3. Single Valued Neutrosophic R-dynamic Vertex Coloring

Definition 3.1. A family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_k\}$ of SVN fuzzy sets is termed as **k-Single Valued Neutrosophic R-dynamic Vertex Coloring (SVNRVC)** of a SVNG $G = (P, Q)$ if the following criteria hold:

1. $\vee\, \gamma_j$ (u) $= P, \forall u \in P$

2.$\gamma_j \wedge \gamma_h = 0$

3. For each incident vertices of edge uv of G, $min\{\gamma_j(t_P(u)), \gamma_j(t_P(v))\} = 0$, $min\{\gamma_j(i_P(u)),$ $\gamma_j(i_P(v))\} = 0$ and $max\{\gamma_j(f_P(u)), \gamma_j(f_P(v))\} = 1, (1 \leq j \leq k)$.

4. Every vertex u with m number of incident edges, the corresponding incident vertices of the vertex u receives atleast $min\{R, m\}$ different members(colors) from the set Γ.

Here, $1 \leq R \leq M$ where M represents the maximum number of incident edges of the vertices of SVNG G.

The least value of k is the SVNRVC of SVNG G is denoted as $\chi_R^v(G)$, is called the Single Valued Neutrosophic R-dynamic chromatic number.

Example: Examine SVNG $G_2 = (P, Q)$ with SVN vertex set and edge set $P = \{v_1, v_2, \cdots, v_5\}$ and $Q = \{v_jv_k | jk = 12, 13, 14, 23, 25, 34, 35, 45\}$ respectively:

$$(t_P(v_j), i_P(v_j), f_P(v_j)) = \begin{cases} (0.4, 0.2, 0.7) & j = 1 \\ (0.6, 0.3, 0.4) & j = 2, 3 \\ (0.3, 0.1, 0.6) & j = 4 \\ (0.7, 0.4, 0.3) & j = 5 \end{cases}$$

$$(t_Q(v_jv_k), i_Q(v_jv_k), f_Q(v_jv_k)) = \begin{cases} (0.4, 0.2, 0.6) & jk = 12, 13 \\ (0.3, 0.1, 0.6) & jk = 14, 34, 45 \\ (0.6, 0.3, 0.4) & jk = 23, 25, 35 \end{cases}$$

Figure 2 depicts the SVNG G_2.

Here $M = 4$ so $1 \leq R \leq 4$

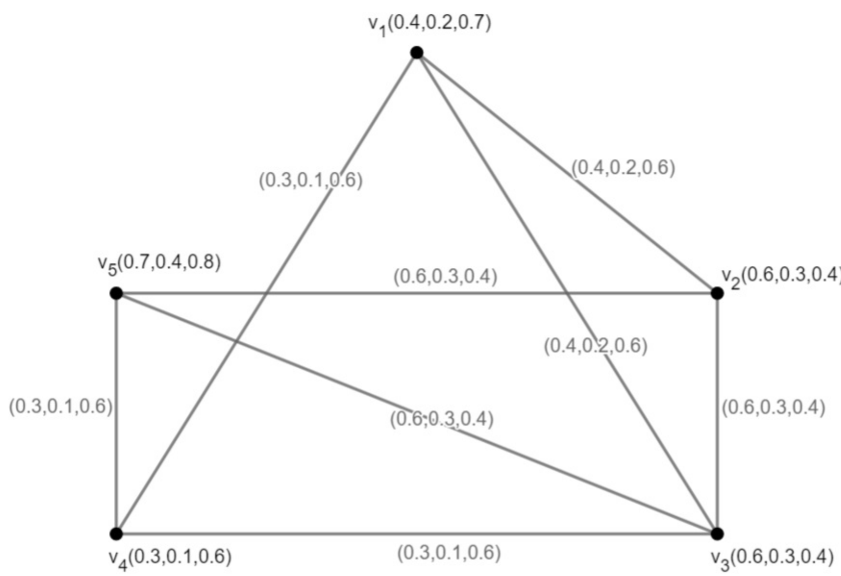

FIGURE 2. G_2

For $1 \leq R \leq 2$ let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ denote collection of SVN fuzzy sets determined on P as below:

$$\gamma_1(v_j) = \begin{cases} (0.4, 0.2, 0.7) & for \quad j = 1 \\ (0.7, 0.4, 0.8) & for \quad j = 5 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_2(v_j) = \begin{cases} (0.6,0.3,0.4) & for \quad j=2 \\ (0.3,0.1,0.6) & for \quad j=4 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(v_j) = \begin{cases} (0.6,0.3,0.4) & for \quad j=3 \\ (0,0,1) & for \quad otherwise \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ assures criteria of SVNRVC of the graph G. Any collection with points lesser than three points will not fulfill our definition. Hence $\chi_R^v(G_2) = 3$ for $1 \leq R \leq 2$.

For $3 \leq R \leq 4$ let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ be collection of SVN fuzzy sets determined on P.

$$\gamma_1(v_j) = \begin{cases} (0.4,0.2,0.7) & for \quad j=1 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(v_j) = \begin{cases} (0.6,0.3,0.4) & for \quad j=2 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(v_j) = \begin{cases} (0.6,0.3,0.4) & for \quad j=3 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(v_j) = \begin{cases} (0.3,0.1,0.6) & for \quad j=4 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_5(v_j) = \begin{cases} (0.7,0.4,0.3) & for \quad j=5 \\ (0,0,1) & for \quad otherwise \end{cases}$$

Hence the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ assures criteria of SVNRVC of the graph G. Any collection with points lesser than below five points will not fulfill our definition. Hence $\chi_R^v(G_2) = 5$ for $3 \leq R \leq 4$. Hence $\chi_R^v(G) = \begin{cases} 3 & for \quad 1 \leq R \leq 2 \\ 4 & for \quad 3 \leq R \leq 4 \end{cases}$

Remark 3.2. For any SVNG G we have $\chi^v(G) \leq \chi_R^v(G)$.

Theorem 3.3. *Let $n \geq 3$, P_n be a path graph then* $\chi_R^v(P_n) = \begin{cases} 2 & for \quad R=1 \\ 3 & for \quad R=2 \end{cases}$

Proof:

For the path graph P_n, $1 \leq R \leq 2$.

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be collection of fuzzy sets determined on vertices $V(P_n) = \{u_1, u_2, \cdots, u_n\}$ for $R = 1$ as follows:

$$\gamma_1(u_k) = \begin{cases} (t_P(u_k), i_P(u_k), f_P(u_k)) & for \quad k \text{ is odd} \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(u_k) = \begin{cases} (t_P(u_k), i_P(u_k), f_P(u_k)) & for \quad k \text{ is even} \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2\}$ assures the conditions of SVNRVC of P_n. Any families with less than two points did not meet our criteria of the definition.

Thus $\chi_1^v(P_n) = 2$.

When $R = 2$, let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be collection of fuzzy sets determined on vertices $V(P_n)$:

$$\gamma_1(u_k) = \begin{cases} (t_P(u_k), i_P(u_k), f_P(u_k)) & for \quad k \equiv 1(mod\ 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2(u_k) = \begin{cases} (t_P(u_k), i_P(u_k), f_P(u_k)) & for \quad k \equiv 2(mod\ 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_3(u_k) = \begin{cases} (t_P(u_k), i_P(u_k), f_P(u_k)) & for \quad k \equiv 0(mod\ 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ assures the conditions of SVNRVC of P_n. Any families with less than three points did not meet our criteria of the definition.

Thus $\chi_2^v(P_n) = 3$.

Hence $\chi_R^v(P_n) = \begin{cases} 2 & for \quad R = 1 \\ 3 & for \quad R = 2 \end{cases}$

Theorem 3.4. *Let $k \geq 3$, C_k be a cycle then* $\chi_R^v(C_k) = \begin{cases} 2 & for \quad R = 1 \quad and\ k\ is\ even \\ 3 & for \quad R = 1 \quad and\ k\ is\ odd \\ 5 & for \quad R = 2 \quad and\ k = 5 \\ 3 & for \quad R = 2 \quad and\ k = 3m \\ 4 & for \quad R = 2 \quad and\ otherwise \end{cases}$

Proof:

For a cycle C_k, $1 \leq R \leq 2$.

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be collection of fuzzy sets determined on vertices $V(C_k) = \{c_1, c_2, \cdots, c_k\}$ for $R = 1$ and k is even as follows:

$$\gamma_1(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j\ is\ odd \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j\ is\ even \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2\}$ assures the conditions of SVNRVC of C_k. Any families with less than two points did not meet our criteria of the definition.

Thus $\chi_1^v(C_k) = 2$ when k is even.

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be collection of fuzzy sets determined on vertices $V(C_k)$ for $R = 1$ and k is odd:

$$\gamma_1(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 1(mod\ 2)\ but\ j \neq k \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 0(mod\ 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_3(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j = k \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ assures the conditions of SVNRVC of C_k. Any families with less than three points did not meet our criteria of the definition.

Thus $\chi_1^v(C_k) = 3$ when k is odd.

For $R = 2$ and $n = 5$, let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ be a family of fuzzy sets determined on vertices $V(C_5)$:

$$\gamma_1(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j = 1 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j = 2 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j = 3 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j = 4 \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_5(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j = 5 \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ assures the conditions of SVNRVC of C_5. Any families with less than five points did not meet our criteria of the definition.

Thus $\chi_2^v(C_5) = 3$.

For $R = 2$ and $k = 3m, m = 1, 2, \cdots$ let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be a family of fuzzy sets determined on vertices:

$$\gamma_1(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 1(mod\ 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 2(mod\ 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 0(mod\ 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ assures the conditions of SVNRVC of C_n. Any families with less than three points did not meet our criteria of the definition.

Thus $\chi_1^v(C_k) = 3$ when $k = 3m$.

For $R = 2$ and otherwise let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be a family of fuzzy sets determined on vertices as follows:

When $k = 3m + 1, m = 1, 2, \cdots$.

$$\gamma_1(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 1(mod\ 3)\ but\ j \neq k \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 2(mod\ 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 0(mod\ 3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j = k \\ (0,0,1) & for \quad otherwise \end{cases}$$

When $k = 3m + 2, m = 1, 2, \cdots$

$$\gamma_1(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 1(mod\ 3)\ and\ j = k-2 \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 2(mod\ 3)\ and\ j = k-3 \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_3(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j \equiv 0(mod\ 3)\ and\ j = k \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_4(c_j) = \begin{cases} (t_P(c_j), i_P(c_j), f_P(c_j)) & for \quad j = k-1, k-4 \\ (0,0,1) & for \quad otherwise \end{cases}$$

Thus the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ assures the conditions of SVNRVC of C_n. Any families with less than four points did not meet our criteria of the definition.
Thus $\chi_2^v(C_k) = 4$ when otherwise.

Hence $$\chi_R^v(C_k) = \begin{cases} 2 & for \quad R = 1 \quad and\ k\ is\ even \\ 3 & for \quad R = 1 \quad and\ k\ is\ odd \\ 5 & for \quad R = 2 \quad and\ k = 5 \\ 3 & for \quad R = 2 \quad and\ k = 3m \\ 4 & for \quad R = 2 \quad and\ otherwise \end{cases}$$

Theorem 3.5. *For the CSVNG with n vertices,* $\chi_R^v(K_n) = n$.

Proof:
For the CSVNG $M = n-1$ and hence $1 \leq R \leq n-1$. One can notice that all vertices are incident to one another. Let $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ be a family of fuzzy sets determined on vertices such that each set contains exactly one vertex with value $t_P(w), i_P(w), f_P(w) > 0$ and all the other vertices have the value $(0,0,1)$. By this the criteria of SVNRVC will be assured and hence $\chi_R^v(K_n) = n$.

4. Conclusion

We have the amalgamated the theory of Single Valued Neutrosophic Vertex Coloring and r-dynamic coloring to build a new thought Single Valued Neutrosophic R-dynamic Vertex Coloring (SVNRVC). We have defined the new coloring and provided examples. Further we have looked onto Single Valued Neutrosophic R-dynamic Chromatic Number of certain graphs.
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