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Neutrosophic RHO –Ideal with Complete Neutrosophic RHO–Ideal in RHO–Algebras

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Abstract: In real-life structures, indeterminacy is always present. Neuosophic sets theory is a well-known mathematical tool for dealing with indeterminacy. Smarandache proposed the neutrosophic set approach. Neutrosophic sets deal with vague data. In this study, we introduced and investigated several types of $\rho -$algebra ideals, which we called neutrosophic $\rho -$subalgebra, complete neutrosophic $\rho -$subalgebra, neutrosophic $\rho -$ideal, complete neutrosophic $\rho -$ideal, neutrosophic $\bar{\rho} -$ideal, and complete neutrosophic $\bar{\rho} -$ideal, respectively. We also proposed some hypotheses to explain some of the relationships between these ideal types.

Keywords: Neutrosophic $\rho -$subalgebra; neutrosophic $\rho -$ideal; neutrosophic $\bar{\rho} -$ideal.

1. Introduction

Many different problems in our lives, such as engineering and medical sciences, necessitate uncertainty. Non-classical sets, like fuzzy sets ([19,]-[24]), soft sets ([25]-[31]), and permutation sets ([32]-[37]) are used to solve some problems in decision making. Smarandache [2] investigates neutrosophic sets as a method for dealing with issues involving unreliable, indeterminate, and persistent data. Imai & Iseki [6] introduce the concepts of $BCK -$algebra and $BCI -$algebra. The $d -$algebra was then introduced by Negger & Kim [9] as a generalization of $BCK -$algebra. In $d -$algebra, Negger et al. [8] discussed the ideal theory. In 1965, Zadeh proposed the concept of a fuzzy set [12]. Following that, Atanassov introduced the intuitionistic fuzzy set [1], which is a natural generalization of fuzzy set. Jun et al. [7] later applied the intuitionistic fuzzy set concept to $d -$algebra. Hasan [4] developed the concept of an intuitionistic fuzzy $d -$ideal of $d -$algebra in 2017. After that, Hasan [5] in 2020 introduced the concept of intuitionistic fuzzy $d -$filter. Smarandache [3] proposed the concept of a neutrosophic set. Next, some basic properties of this notion are studied ([13]-[18]). Also, Smarandache and Rezaei studied the neutrosophic triplet of BI-

algebras[38]. In 2021, some notions of neutrosophic ideals in BCK-algebras are discussed [39].The ρ −algebra was first introduced by Khalil and Abud Alradha[10]. In this paper, we define neutrosophic ρ −subalgebra, complete neutrosophic ρ −subalgebra, neutrosophic ρ −ideal, full neutrosophic ρ −ideal, neutrosophic ρ −ideal, neutrosophic $\bar{\rho}$ −ideal, and complete neutrosophic $\bar{\rho}$ −ideal of ρ −algebra, and investigate the relationship between these types.

2. Preliminaries and Some Results.

Here, we will recall basic ideas and results that are necessary in this research.

Definition 2.1.[10] A ρ-algebra is a non-empty set $\mho$ with a constant 0 and a binary operation "$\oint$" satisfying the following axioms:

(1) $\alpha \oint \alpha = 0$,

(2) $0 \oint \alpha = 0$,

(3) $\alpha \oint \beta = 0 = \beta \oint \alpha$ imply that $\alpha = \beta$,

(4) For all $\alpha \neq \beta \in \mho - \{0\}$ imply that $\alpha \oint \beta = \beta \oint \alpha \neq 0$.

Definition 2.2. [10] A non-empty subset Υ of a ρ-algebra $(\mho, \oint, 0)$ is called ρ- subalgebra of $\mho$ if $\alpha \oint \beta \in \Upsilon$ for any $\alpha, \beta \in \Upsilon$.

Definition 2.3.[10] A non- empty subset Υ of a ρ-algebra $\mho$ is called an ρ- ideal of $\mho$ if satisfies:

(1) $\alpha, \beta \in \Upsilon \Rightarrow \alpha \oint \beta \in \Upsilon$,

(2) $\alpha \oint \beta \in \Upsilon$ & $\beta \in \Upsilon \Rightarrow \alpha \in \Upsilon$.

Remark 2.4[10]. If Υ is any a ρ- Ideal, then it is easy to show that Υ is ρ- subalgebra. However, the convers maybe not true.

Definition2.5.[10] A non- empty subset Υ of a ρ-algebra $\mho$ is called an $\bar{\rho}$- ideal of $\mho$ if satisfies:

(1) $0 \in \Upsilon$,

(2) $\alpha \in \Upsilon$ & $\beta \in \mho \Rightarrow \alpha \oint \beta \in \Upsilon$.

Proposition 2.6. [10] Let $\emptyset \neq \Upsilon \subseteq \mho$ where $\mho$ is ρ-algebra. Then Υ is a ρ- subalgebra of $\mho$ if it is $\bar{\rho}$-Ideal.

Definition 2.7. [2] A Neutrosophic set $\mathcal{N}$ (briefly, NS) over the universal $\mho$ is defined by $\mathcal{N} = \{< \alpha, \mathcal{N}_T(\alpha), \mathcal{N}_I(\alpha), \mathcal{N}_F(\alpha) >| \alpha \in \mho\}$, where $N_T(\alpha), N_I(\alpha), N_F(\alpha): \mho \longrightarrow [0,1]$ are maps, with $\mathcal{N}_T(\alpha), \mathcal{N}_I(\alpha)$ and $\mathcal{N}_F(\alpha)$ are real numbers and their values represent the degree of membership, indeterminate and non- membership of α to $\mathcal{N}$ respectively.

Definition 2.8 . [2] A complement neutrosophic set $\mathcal{N}^c$ over the universal $\mho$ is defined by

$\mathcal{N}^c = 1 - \mathcal{N} = 1 - \{< \alpha, \mathcal{N}_T(\alpha), \mathcal{N}_I(\alpha), \mathcal{N}_F(\alpha) >| \alpha \in \mho\} =$

$\{< \alpha, 1 - \mathcal{N}_T(\alpha), 1 - \mathcal{N}_I(\alpha), 1 - \mathcal{N}_F(\alpha) >| \alpha \in \mho\} = \{< \alpha, \mathcal{N}_{T^c}(\alpha), \mathcal{N}_{I^c}(\alpha), \mathcal{N}_{F^c}(\alpha) >| \alpha \in \mho\}$.

Definition 2.9. [2] Let $\mathcal{N}$ be (NS) over the universal $\mho$ and $t \in [0,1]$ then the set $\mathcal{N}_t = \{\alpha \in \mho | \mathcal{N}_T(\alpha) \geq t, \mathcal{N}_I(\alpha) \leq t, \mathcal{N}_F(\alpha) \geq t\}$ is called neutrosophic set t-cut, (briefly, NS-t-cut) .

Definition 2.10. [2] Let $(\mho, *, 0)$ be a ρ-algebra and $\mathcal{N} = \{< \alpha, \mathcal{N}_T(\alpha), \mathcal{N}_I(\alpha), \mathcal{N}_F(\alpha) >| \alpha \in \mho\}$ be a neutrosophic set (NS) of $\mho$. We say $\mathcal{N}$ is a neutrosophic ρ -constant of $\mho$ if all the maps $\mathcal{N}_T$, $\mathcal{N}_I$, $\mathcal{N}_F$: $\mho \rightarrow [0,1]$ are constant maps.

3. Neutrosophic ρ – Subalgebra and Complete Neutrosophic ρ – Subalgebra:

Definition 3.1. A (NS) $\mathcal{N}$ in $\mho$ is called a neutrosophic ρ −subalgebra (briefly, NS $-\rho - SA$) of $\mho$ if such that:

(i) $\mathcal{N}_T(\alpha \oint \beta) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$,

(ii) $\mathcal{N}_I(\alpha \oint \beta) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$,

(iii) $\mathcal{N}_F(\alpha \oint \beta) \geq min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$, for any $\alpha, \beta \in \mho$.

Example 3.2. Assume $\mho = \{0,1,2,3\}$ is a set and $\oint$ is defined by table (1). So, we get $(\mho, \oint, 0)$ is a ρ-algebra, We define a (NS) $\mathcal{N}$ in $\mho$ as follows:

$$\mathcal{N}_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.5 & 0.4 & 0.4 & 0.4 \end{pmatrix}, \mathcal{N}_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.2 & 0.3 & 0.3 & 0.3 \end{pmatrix}, \mathcal{N}_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.3 & 0.2 & 0.2 & 0.2 \end{pmatrix}$$

Hence, $\mathcal{N}$ is (NS $-\rho - SA$).

$\oint$	**0**	**1**	**2**	**3**
0	0	0	0	0
1	1	0	1	2
2	2	1	0	2
3	3	2	2	0

Table (1) , $\mathcal{N}$ is (NS $-\rho - SA$)

Lemma 3.3. Let $\mathcal{N}$ be (NS $-\rho - SA$) of $\mho$ then:

(i) $\mathcal{N}_T(0) \geq \mathcal{N}_T(\alpha)$, (ii) $\mathcal{N}_I(0) \leq \mathcal{N}_I(\alpha)$, (iii) $\mathcal{N}_F(0) \geq \mathcal{N}_F(\alpha)$, for any $\alpha \in \mho$.

Proof: Let $\mathcal{N}$ be (NS $-\rho - SA$) then

(i) $\mathcal{N}_T(0) = \mathcal{N}_T(\alpha \oint \alpha) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\alpha)\} = \mathcal{N}_T(\alpha)$.

(ii) $\mathcal{N}_I(0) = \mathcal{N}_I(\alpha \oint \alpha) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\alpha)\} = \mathcal{N}_I(\alpha)$.

(iii) $\mathcal{N}_F(0) = \mathcal{N}_F(\alpha \oint \alpha) \geq min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\alpha)\} = \mathcal{N}_F(\alpha)$.

Lemma 3.4. Let $\mathcal{N}$ be (NS $-\rho - SA$) of $\mho$ then:

(i) $\mathcal{N}_{T^c}(\alpha) \geq \mathcal{N}_{T^c}(0)$, (ii) $\mathcal{N}_{I^c}(\alpha) \leq \mathcal{N}_{I^c}(0)$, (iii) $\mathcal{N}_{F^c}(\alpha) \geq \mathcal{N}_{F^c}(0)$, for any $\alpha \in \mho$.

Proof: Let $\mathcal{N}$ be (NS $-\rho - SA$) , then from lemma (3.3) we obtain: $\mathcal{N}_T(0) \geq \mathcal{N}_T(\alpha)$, $\mathcal{N}_I(0) \leq \mathcal{N}_I(\alpha)$, $\mathcal{N}_F(0) \geq \mathcal{N}_F(\alpha)$, for any $\alpha \in \mho$. Since, $\mathcal{N}^c = 1 - \mathcal{N}$, thus

$\mathcal{N}_{T^c}(\alpha) = 1 - \mathcal{N}_T(\alpha) \geq 1 - \mathcal{N}_T(0) = \mathcal{N}_{T^c}(0)$,

$\mathcal{N}_{I^c}(\alpha) = 1 - \mathcal{N}_I(\alpha) \leq 1 - \mathcal{N}_I(0) = \mathcal{N}_{I^c}(0)$,

$\mathcal{N}_{F^c}(\alpha) = 1 - \mathcal{N}_F(\alpha) \geq 1 - \mathcal{N}_F(0) = \mathcal{N}_{F^c}(0)$, This completes proof.

Proposition 3.5: Let $\mathcal{N}$ be (NS) of $\rho -$ algebra $(\mho, \oint, 0)$, then $\mathcal{N}$ is (NS $-\rho - SA$) if it is $\mathcal{N} = \{\prec \alpha, \mathcal{N}_T(\alpha) = \mathcal{N}_T(0), \mathcal{N}_I(\alpha) = \mathcal{N}_I(0), \mathcal{N}_F(\alpha) = \mathcal{N}_F(0) \succ \mid \alpha \in \mho\}$.

Proof: Let $\mathcal{N}$ be (NS) and $\mathcal{N}_T(\alpha) = \mathcal{N}_T(0), \mathcal{N}_I(\alpha) = \mathcal{N}_I(0), \mathcal{N}_F(\alpha) = \mathcal{N}_F(0)$, for any $\alpha \in \mathcal{N}$. Now, Let $\alpha, \beta \in \mho$, then $\mathcal{N}_T(\alpha \oint \beta) = \mathcal{N}_T(0) = min\{\mathcal{N}_T(0), \mathcal{N}_T(0)\} = min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$, thus $\mathcal{N}_T(\alpha \oint \beta) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$, $\mathcal{N}_I(\alpha \oint \beta) = \mathcal{N}_I(0) = max\{\mathcal{N}_I(0), \mathcal{N}_I(0)\} = max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$,

thus $\mathcal{N}_I(\alpha \oint \beta) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$,

$\mathcal{N}_F(\alpha \oint \beta) = \mathcal{N}_F(0) = min\{\mathcal{N}_F(0), \mathcal{N}_F(0)\} = min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$, thus $\mathcal{N}_F(\alpha \oint \beta) \geq min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$.
Hence $\mathcal{N}$ is (NS $-\rho - SA$).

Proposition 3.6. Let $\mathcal{N}$ be (NS) of $\rho -$ algebra $(\mho, \oint, 0)$, then $\mathcal{N}^c$ is(NS $-\rho - SA$).
If $\mathcal{N} = \{\prec \alpha, \mathcal{N}_{T^c}(\alpha) = \mathcal{N}_{T^c}(0), \mathcal{N}_{I^c}(\alpha) = \mathcal{N}_{I^c}(0), \mathcal{N}_{F^c}(0) = \mathcal{N}_{F^c}(0) \succ \mid \alpha \in \mho\}$

Proof: Let $\mathcal{N}^c = \{\prec \alpha, \mathcal{N}_{T^c}(\alpha) = \mathcal{N}_{T^c}(0), \mathcal{N}_{I^c}(\alpha) = \mathcal{N}_{I^c}(0), \mathcal{N}_{F^c}(0) = \mathcal{N}_{F^c}(0) \succ \mid \alpha \in \mho\}$ and let $\alpha, \beta \in \mho$, then $\mathcal{N}_{T^c}(\alpha \oint \beta) = \mathcal{N}_{T^c}(0) = min\{\mathcal{N}_{T^c}(0), \mathcal{N}_{T^c}(0)\} = min\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$, thus $\mathcal{N}_{T^c}(\alpha \oint \beta) \geq min\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$, $\mathcal{N}_{I^c}(\alpha \oint \beta) = \mathcal{N}_{I^c}(0) = max\{\mathcal{N}_{I^c}(0), \mathcal{N}_{I^c}(0)\} = max\{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$, thus $\mathcal{N}_{I^c}(\alpha \oint \beta) \leq max\{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$, $\mathcal{N}_{F^c}(\alpha \oint \beta) = \mathcal{N}_{F^c}(0) = min\{\mathcal{N}_{F^c}(0), \mathcal{N}_{F^c}(0)\} = min\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\beta)\}$, thus $\mathcal{N}_{F^c}(\alpha \oint \beta) \geq min\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\beta)\}$. Hence $\mathcal{N}^c$ is (NS $-\rho - SA$).

Definition 3.7. Let $\mathcal{N}$ be (NS) of $\rho -$ algebra $(\mho, \oint, 0)$, then $K(\mathcal{N}) = \{\alpha \in \mho \mid \mathcal{N}_T(\alpha) = \mathrm{N_T}(0), \mathrm{N_I}(\alpha) = \mathrm{N_I}(0)$ and $\mathrm{N_F}(\alpha) = \mathrm{N_F}(0)\}$ is a subset of $\mho$ and it is called neutrosophic ρ −kernel of $\mathcal{N}$ over $\mho$.

Example 3.8. Let $\mho = \{a, b, c, d\}$ and define $\oint$ on the set $\mho$ as table (2). Then $(\mho, \oint, a)$ is a ρ-algebra, we define a (NS) $\mathcal{N}$ in $\mho$ as follows:

$\mathcal{N}_\mathrm{T} = \begin{pmatrix} a & b & c & d \\ 0.1 & 0.2 & 0.5 & 0.1 \end{pmatrix}$, $\mathcal{N}_\mathrm{I} = \begin{pmatrix} a & b & c & d \\ 0.1 & 0.3 & 0.4 & 0.1 \end{pmatrix}$,

$\mathcal{N}_\mathrm{I} = \begin{pmatrix} a & b & c & d \\ 0.1 & 0.3 & 0.5 & 0.1 \end{pmatrix}$, $K(\mathcal{N}) = \{a, d\}$.

$\oint$	a	b	c	d
$\boldsymbol{a}$	a	a	a	a
$\boldsymbol{b}$	b	a	b	d
$\boldsymbol{c}$	c	b	a	d
$\boldsymbol{d}$	d	d	d	a

Table (2) , $K(\mathcal{N}) = \{a, d\}$

Proposition 3.9. If $\mathcal{N}$ is (NS $-\rho - SA$) of $(\mho, \oint, 0)$, then $K(\mathcal{N}^c)$ is a $(\rho - SA)$.

Proof: Let $\alpha, \beta \in K(\mathcal{N}^c)$. Then; $\mathcal{N}_{T^c}(\alpha) = \mathcal{N}_{T^c}(\beta) = \mathcal{N}_{T^c}(0), \mathcal{N}_{I^c}(\alpha) = \mathcal{N}_{I^c}(\beta) = \mathcal{N}_{I^c}(0)$ and $\mathcal{N}_{F^c}(\alpha) = \mathcal{N}_{F^c}(\beta) = \mathcal{N}_{F^c}(0)$.

Also $\mathcal{N}_{T^c}(\alpha \oint \beta) = 1 - \mathcal{N}_T(\alpha \oint \beta) \leq 1 - min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$

[since $\mathcal{N}_T(\alpha \oint \beta) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$].

$= max\{1 - \mathcal{N}_T(\alpha), 1 - \mathcal{N}_T(\beta)\}$

$= max\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$

$= max\{\mathcal{N}_{T^c}(0), \mathcal{N}_{T^c}(0)\} = \mathcal{N}_{T^c}(0)$,

$\mathcal{N}_{I^c}(\alpha \oint \beta) = 1 - \mathcal{N}_I(\alpha \oint \beta) \geq 1 - max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$[since $\mathcal{N}_I(\alpha \oint \beta) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$].

$= min\{1 - \mathcal{N}_I(\alpha), 1 - \mathcal{N}_I(\beta)\}$

$= min\ \{\mathcal{N}_{I^c}(\alpha),\ \mathcal{N}_{I^c}(\beta)\}$

$= min\{\mathcal{N}_{I^c}(0),\ \mathcal{N}_{I^c}(0)\} = \mathcal{N}_{I^c}(0),$

$\mathcal{N}_{F^c}(\alpha \oint \beta\ ) = 1 - \mathcal{N}_F(\alpha \oint \beta\ ) \leq 1 - min\{\mathcal{N}_F(\alpha),\mathcal{N}_F(\beta)\}$

[since $\mathcal{N}_F\left(\alpha \oint \beta\ \right) \geq min\{\mathcal{N}_F(\alpha),\mathcal{N}_F(\beta)\}$].

$= max\{1 - \mathcal{N}_F(\alpha),1 - \mathcal{N}_F(\beta$

$= max\{\mathcal{N}_{F^c}(\alpha),\mathcal{N}_{F^c}(\beta)\}$

$= max\{\mathcal{N}_{F^c}(0),\mathcal{N}_{F^c}(0)\} = \mathcal{N}_{F^c}(0),$

and from lemma (3.4), W obtain:

$\mathcal{N}_{T^c}(\alpha \oint \beta\ ) \geq \mathcal{N}_{T^c}(0)$, $\mathcal{N}_{I^c}\ (\alpha \oint \beta\ ) \leq \mathcal{N}_{I^c}(0)$, $\mathcal{N}_{F^c}\ (\alpha \oint \beta\ ) \geq \mathcal{N}_{F^c}\ (0\ )$

thus $\mathcal{N}_{T^c}(\alpha \oint \beta\ ) = \mathcal{N}_{T^c}(0)$, $\mathcal{N}_{I^c}(\alpha \oint \beta\ ) = \mathcal{N}_{I^c}(0)$, $\mathcal{N}_{F^c}(\alpha \oint \beta\ ) = \mathcal{N}_{F^c}(0)$,

this implies $\alpha \oint \beta \in K(\mathcal{N}^c)$, hence $K(\mathcal{N}^c)$ is ρ −subalgebra.

Proposition 3.10. Let $\mathcal{N}$ be (NS $-\rho - SA$) then $\mathcal{N}_t$ is ρ −subalgebra.

Proof: Assume that $\mathcal{N}$ is (NS $-\rho - SA$) and $\alpha, \beta \in \mathcal{N}_t$, then $(\mathcal{N}_T(\alpha) \geq \mathrm{t}, \mathcal{N}_I(\alpha) \leq \mathrm{t}, \mathcal{N}_F(\alpha) \geq \mathrm{t})$ and $(\mathcal{N}_T(\beta) \geq \mathrm{t}, \mathcal{N}_I(\beta) \leq \mathrm{t}, \mathcal{N}_F(\beta) \geq \mathrm{t})$. Also, $\mathcal{N}_T(\alpha \oint \beta\ ) \geq min\{\mathcal{N}_T(\alpha),\ \mathcal{N}_T(\beta)\} \geq t$, $\mathcal{N}_I(\alpha \oint \beta\ ) \leq max\{\mathcal{N}_I(\alpha),\ \mathcal{N}_I(\beta)\} \leq t$, $\mathcal{N}_F(\alpha \oint \beta\ ) \geq min\{\mathcal{N}_F(\alpha),\mathcal{N}_F(\beta)\} \geq t$, this implies $\alpha \oint \beta \in \mathcal{N}_t$, hence $\mathcal{N}_t$ subalgebra.

Proposition 3.11. Let $(\mho,\oint,0)$ be a ρ-algebra and $\mathcal{N}$ be a (NS) of $\mho$. Then $\mathcal{N}$ is (NS $-\rho - SA$)if it is neutrosophic ρ −constant.

Proof: Assume that $\mathcal{N}$ is constant. Then for all $\alpha \in \mho$, $\mathcal{N}_T(\alpha) = \mathcal{N}_T(0)$, $\mathcal{N}_I(\alpha) = \mathcal{N}_I(0)$ and $\mathcal{N}_F(\alpha) = \mathcal{N}_F(0)$, and so $\mathcal{N}_T(0) \geq \mathcal{N}_T(\alpha)$, $\mathcal{N}_I(0) \leq \mathcal{N}_I(\alpha)$and $\mathcal{N}_F(0) \geq \mathcal{N}_F(\alpha)$. Next, for all α, $\beta \in \mho$, $\mathcal{N}_T(\alpha \oint \beta\ ) = \mathcal{N}_T(0) = min\{\mathcal{N}_T(0),\ \mathcal{N}_T(0)\} \geq min\{\mathcal{N}_T(\alpha),\ \mathcal{N}_T(\beta)\}$, $\mathcal{N}_I(\alpha \oint \beta\ ) = \mathcal{N}_I(0\ ) = max\{\mathcal{N}_I(0),\ \mathcal{N}_I(0)\} \leq max\{\mathcal{N}_I(\alpha),\ \mathcal{N}_I(\beta)\}$, $\mathcal{N}_F(\alpha \oint \beta\ ) = \mathcal{N}_F(0\ ) = min\{\mathcal{N}_F(0),\mathcal{N}_F(0)\} \geq min\{\mathcal{N}_F(\alpha),\mathcal{N}_F(\beta)\}$, hence $\mathcal{N}$ is (NS $-\rho - SA$).

Proposition 3.12. Let $\mathcal{N}$ be (NS $-\rho - SA$). Then $0 \in \mathcal{N}_t$, if $\mathcal{N}_t \neq \emptyset$.

Proof: Assume that $\mathcal{N}$ is (NS $-\rho - SA$) and $\mathcal{N}_t \neq \emptyset$ then there is at least $\alpha \in \mathcal{N}_t$, also from Lemma (3.3) and Definition (2.9) we obtain, $\mathcal{N}_T(0\ ) \geq \mathcal{N}_T(\alpha) \geq \mathrm{t}$, $\mathcal{N}_I\ (0\ ) \leq \mathcal{N}_I(\alpha) \leq \mathrm{t}$, $\mathcal{N}_F\ (0) \geq \mathcal{N}_F(\alpha) \geq \mathrm{t}$, this means $0 \in \mathcal{N}_t$.

Corollary 3.13. If $\mathcal{N}$ neutrosophic ρ -constant then $\mathcal{N}_t$ is ρ −subalgebra.

Proof: From proposition (3.11) and proposition (3.10).

Definition 3.14. Let $\mathcal{N}$ be (NS) in $\mho$ then it is called a complete neutrosophic ρ −subalgebra (briefly, CNS $-\rho - SA$) of $\mho$ if it satisfies the following conditions:

(i) $\mathcal{N}_T(\alpha \oint \beta\ ) \leq max\{\mathcal{N}_T(\alpha),\ \mathcal{N}_T(\beta)\}$,

(ii) $\mathcal{N}_I(\alpha \oint \beta\ ) \geq min\{\mathcal{N}_I(\alpha),\ \mathcal{N}_I(\beta)\}$,

(iii) $\mathcal{N}_F(\alpha \oint \beta\ ) \leq max\{\mathcal{N}_F(\alpha),\mathcal{N}_F(\beta)\}$, for any $\alpha, \beta \in \mho$.

Example 3.15 .Assume $\mho$ = {0,1,2,3} is a set and $\oint$ is defined by table (3). So, we get ($\mho$, $\oint$,0) is a ρ-algebra, we define a (NS) $\mathcal{N}$ in $\mho$ as follows:

$\mathcal{N}_T = \begin{pmatrix} 0 & 1 & 2 \\ 0.1 & 0.3 & 0.3 \end{pmatrix}$, $\mathcal{N}_I = \begin{pmatrix} 0 & 1 & 2 \\ 0.6 & 0.3 & 0.3 \end{pmatrix}$, $\mathcal{N}_F = \begin{pmatrix} 0 & 1 & 2 \\ 0.2 & 0.4 & 0.4 \end{pmatrix}$

Hence, $\mathcal{N}$ is (NS $-\rho - SA$).

$\oint$	0	1	2
0	0	0	0
1	1	0	1
2	2	1	0

Table (3) , $\mathcal{N}$ is (NS $-\rho - SA$)

Lemma 3.16. Let $\mathcal{N}$ be (CNS $-\rho - SA$) of $\mho$ then:
(i) $\mathcal{N}_T(0) \leq \mathcal{N}_T(\alpha)$, (ii) $\mathcal{N}_I(0) \geq \mathcal{N}_I(\alpha)$, (iii) $\mathcal{N}_F(0) \leq \mathcal{N}_F(\alpha)$, for any $\alpha \in \mho$.

Proof: Let $\mathcal{N}$ be (CNS $-\rho - SA$) then,
(i) $\mathcal{N}_T(0) = \mathcal{N}_T(\alpha \oint \alpha) \leq max\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\alpha)\} = \mathcal{N}_T(\alpha)$.
(ii) $\mathcal{N}_I(0) = \mathcal{N}_I(\alpha \oint \alpha) \geq min\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\alpha)\} = \mathcal{N}_I(\alpha)$.
(iii) $\mathcal{N}_F(0) = \mathcal{N}_F(\alpha \oint \alpha) \leq max\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\alpha)\} = \mathcal{N}_F(\alpha)$.This completes proof.

Proposition 3.17. If $\mathcal{N}$ is a (CNS $-\rho - SA$), then $K(\mathcal{N})$ is $\rho -$ subalgebra.

Proof: Let $\alpha, \beta \in K(\mathcal{N})$, then $\mathcal{N}_T(\alpha) = \mathcal{N}_T(\beta) = \mathcal{N}_T(0)$, $\mathcal{N}_I(\alpha) = \mathrm{N_I}(\beta) = \mathrm{N_I}(0)$ and $\mathcal{N}_F(\alpha) = \mathcal{N}_F(\beta) = \mathcal{N}_F(0)$. Also, $\mathcal{N}_T(\alpha \oint \beta) \leq max\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\} = max\{\mathcal{N}_T(0), \mathcal{N}_T(0)\} = \mathcal{N}_T(0)$, $\mathcal{N}_I(\alpha \oint \beta) \geq min\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\} = min\{\mathcal{N}_I(0), \mathcal{N}_I(0)\} = \mathcal{N}_I(0)$, $\mathcal{N}_F(\alpha \oint \beta) \leq max\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\} = max\{\mathcal{N}_F(0), \mathcal{N}_F(0)\} = \mathcal{N}_F(0)$, and from lemma (3.16) $\mathcal{N}_T(0) \leq \mathcal{N}_T(\alpha \oint \beta)$, $\mathcal{N}_I(0) \geq \mathcal{N}_I(\alpha \oint \beta)$, $\mathcal{N}_F(0) \leq \mathcal{N}_F(\alpha \oint \beta)$, thus $\mathcal{N}_T(\alpha \oint \beta) = \mathcal{N}_T(0)$, $\mathcal{N}_I(\alpha \oint \beta) = \mathcal{N}_I(0)$, $\mathcal{N}_F(\alpha \oint \beta) = \mathcal{N}_F(0)$, and $\alpha \oint \beta \in K(\mathcal{N})$ hence $K(\mathcal{N})$ is $\rho -$ subalgebra.

Proposition 3.18. Let $\mathcal{N}$ be (NS) then $\mathcal{N}$ is (NS $-\rho - SA$) if and only if $\mathcal{N}^c$ is (CNS $-\rho - SA$).

Proof: Let $\mathcal{N}$ be (NS $-\rho - SA$) then $\mathcal{N}_T(\alpha \oint \beta) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$, $\mathcal{N}_I(\alpha \oint \beta) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$, $\mathcal{N}_F(\alpha \oint \beta) \geq min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$, for any $\alpha, \beta \in \mho$.

Now, $\mathcal{N}_{T^c}(\alpha \oint \beta) = 1 - \mathcal{N}_T(\alpha \oint \beta) \leq 1 - min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$
$= max\{1 - \mathcal{N}_T(\alpha), 1 - \mathcal{N}_T(\beta)\}$
$= max\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$,

$\mathcal{N}_{I^c}(\alpha \oint \beta) = 1 - \mathcal{N}_I(\alpha \oint \beta) \geq 1 - max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\} = min\{1 - \mathcal{N}_I(\alpha), 1 - \mathcal{N}_I(\beta)\}$
$= min\{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$,

$\mathcal{N}_{F^c}(\alpha \oint \beta) = 1 - \mathcal{N}_F(\alpha \oint \beta) \leq 1 - min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$
$= max\{1 - \mathcal{N}_F(\alpha), 1 - \mathcal{N}_F(\beta)\}$
$= max\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\beta)\}$,

Hence $\mathcal{N}^c$ is (CNS $-\rho - SA$).

Conversely: Let $\mathcal{N}^c$ be (CNS $-\rho-SA$) then $\mathcal{N}_{T^c}(\alpha \oint \beta) \le max\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$, $\mathcal{N}_{I^c}(\alpha \oint \beta) \ge min\{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$, $\mathcal{N}_{F^c}(\alpha \oint \beta) \le max\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\beta)\}$, for any $\alpha, \beta \in \mho$.

Now, $\mathcal{N}_T(\alpha \oint \beta) = 1 - \mathcal{N}_{T^c}(\alpha \oint \beta) \ge 1 - max\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$

$= min\{1 - \mathcal{N}_{T^c}(\alpha), 1 - \mathcal{N}_{T^c}(\beta)\}$

$= min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$,

$\mathcal{N}_I(\alpha \oint \beta) = 1 - \mathcal{N}_{I^c}(\alpha \oint \beta) \le 1 - min\{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$

$= max\{1 - \mathcal{N}_{I^c}(\alpha), 1 - \mathcal{N}_{I^c}(\beta)\}$

$= max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$

$\mathcal{N}_F(\alpha \oint \beta) = 1 - \mathcal{N}_{F^c}(\alpha \oint \beta) \ge 1 - max\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\beta)\}$

$= min\{1 - \mathcal{N}_{F^c}(\alpha), 1 - \mathcal{N}_{F^c}(\beta)\}$

$= min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$, Hence $\mathcal{N}$ is (NS $-\rho-SA$).

Corollary 3.19.

1- Let $\mathcal{N}^c$ be is (CNS $-\rho-SA$) then $\mathcal{N}_t$ is $\rho-$ subalgebra.

2- Let $\mathcal{N}$ be a neutrosophic ρ -constant then $\mathcal{N}_t$ is $\rho-$ subalgebra.

Proof (1): From Proposition (3.18) and Proposition (3.10).

Proof (2): From Proposition (3.11) and Proposition (3.10).

4. Neutrosophic ρ -Ideal and Complete Neutrosophic ρ -Ideal:

Definition 4.1. Assume $(\mho, \oint, 0)$ is a ρ-algebra and $\mathcal{N}$ is (NS) of $\mho$. We say $\mathcal{N}$ is a neutrosophic ρ - ideal of $\mho$ (briefly, NS $-\rho-I$))if such that:

(i) $\mathcal{N}_T(\alpha \oint \beta) \ge min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$,

(ii) $\mathcal{N}_I(\alpha \oint \beta) \le max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$,

(iii) $\mathcal{N}_F(\alpha \oint \beta) \ge min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$,

(iv) $\mathcal{N}_T(\alpha) \ge min\{\mathcal{N}_T(\alpha \oint \beta), \mathcal{N}_T(\beta)\}$,

(v) $\mathcal{N}_I(\alpha) \le max\{\mathcal{N}_I(\alpha \oint \beta), \mathcal{N}_I(\beta)\}$,

(vi) $\mathcal{N}_F(\alpha) \ge min\{\mathcal{N}_F(\alpha \oint \beta), \mathcal{N}_F(\beta)\}$, for any $\alpha, \beta \in \mho$.

Example 4.2. Let $\mho = \{\alpha, \gamma, \beta, \delta\}$ be a set with the following table (4), it is clear that $(\mho, \oint, \alpha)$ is a ρ-algebra, We define a (NS) $\mathcal{N}$ in $\mho$ as follows:

$\mathcal{N}_T = \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ 0.8 & 0.7 & 0.7 & 0.7 \end{pmatrix}$, $\mathcal{N}_I = \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ 0.4 & 0.6 & 0.6 & 0.6 \end{pmatrix}$, $\mathcal{N}_F = \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ 0.6 & 0.5 & 0.5 & 0.5 \end{pmatrix}$

Hence, $\mathcal{N}$ is (NS $-\rho-I$).

$\oint$	α	β	γ	δ
α	α	α	α	α
β	β	α	β	γ
γ	γ	β	α	γ
δ	δ	γ	γ	α

Table (4): $\mathcal{N}$ is (NS $-\rho - I$).

Lemma 4.3: Every (NS $-\rho - I$)is (NS $-\rho - SA$).

Proof: Let $\mathcal{N}$ be (NS $-\rho - I$). Then from Definition (4.1)-[(i),(ii),(iii)], we get $\mathcal{N}$ is (NS $-\rho - SA$).

Corollary 4.4.

1- Let $\mathcal{N}$ be (NS $-\rho - I$) then $\mathcal{N}_t$ is ρ −subalgebra.

2-Let $\mathcal{N}$ be (NS $-\rho - I$) then $\mathcal{N}^c$is (CNS $-\rho - SA$).

Proof 1: From Lemma (4.3**)** and Proposition (3.10)**.**

Proof 2: From Lemma (4.3**)** and Proposition (3.18)**.**

Lemma 4.5. Let $\mathcal{N}$ be (NS $-\rho - I$) of Ʊ. Then;

(i) $\mathcal{N}_T(0) \geq \mathcal{N}_T(\alpha)$, (ii) $\mathcal{N}_I(0) \leq \mathcal{N}_I(\alpha)$, (iii) $\mathcal{N}_F(0) \geq \mathcal{N}_F(\alpha)$,for any $\alpha \in$ Ʊ.

Proposition 4.6. If $\mathcal{N}$ is (NS $-\rho - I$), then $K(\mathcal{N})$ is $\rho -$ ideal.

Proof: Let $\mathcal{N}$ be (NS $-\rho - I$) and let $\alpha, \beta \in K(\mathcal{N})$, then $\mathcal{N}_T(\alpha) = \mathcal{N}_T(\beta) = \mathcal{N}_T(0), \mathcal{N}_I(\alpha) = \mathcal{N}_I(\beta) = \mathcal{N}_I(0)$ and $\mathcal{N}_F(\alpha) = \mathcal{N}_F(\beta) = \mathcal{N}_F(0)$. Also, $\mathcal{N}_T(\alpha \oint \beta) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\} = min\{\mathcal{N}_T(0), \mathcal{N}_T(0)\} = \mathcal{N}_T(0)$,

$\mathcal{N}_I(\alpha \oint \beta) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\} = max\{\mathcal{N}_I(0), \mathcal{N}_I(0)\} = \mathcal{N}_I(0)$,

$\mathcal{N}_F(\alpha \oint \beta) \geq min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\} = min\{\mathcal{N}_F(0), \mathcal{N}_F(0)\} = \mathcal{N}_F(0)$, and from lemma (4.5) we obtain $\mathcal{N}_T(0) \geq \mathcal{N}_T(\alpha \oint \beta)$, $\mathcal{N}_I(0) \leq \mathcal{N}_I(\alpha \oint \beta)$, $\mathcal{N}_F(0) \geq \mathcal{N}_F(\alpha \oint \beta)$. Hence, $\alpha \oint \beta \in K(\mathcal{N})$. Now, assume that, $\alpha \oint \beta \in K(\mathcal{N})$ & $\beta \in K(\mathcal{N})$ then $\mathcal{N}_T(\alpha \oint \beta) = \mathcal{N}_T(0), \mathcal{N}_I(\alpha \oint \beta) = \mathcal{N}_I(0)$, $\mathcal{N}_F(\alpha \oint \beta) = \mathcal{N}_F(0)$, and $\mathcal{N}_T(\beta) = \mathcal{N}_T(0), \mathcal{N}_I(\beta) = \mathcal{N}_I(0)$, $\mathcal{N}_F(\beta) = \mathcal{N}_F(0)$, thus $\mathcal{N}_T(\alpha) \geq min\{\mathcal{N}_T(\alpha \oint \beta), \mathcal{N}_T(\beta)\} = \mathcal{N}_T(0)$, $\mathcal{N}_I(\alpha) \leq max\{\mathcal{N}_I(\alpha \oint \beta), \mathcal{N}_I(\beta)\} = \mathcal{N}_I(0)$, $\mathcal{N}_F(\alpha) \geq min\{\mathcal{N}_F(\alpha \oint \beta), \mathcal{N}_F(\beta)\} = \mathcal{N}_F(0)$, and from lemma (4.5) We obtain $\mathcal{N}_T(0) = \mathcal{N}_T(\alpha)$, $\mathcal{N}_I(0) = \mathcal{N}_I(\alpha)$, $\mathcal{N}_F(0) = \mathcal{N}_F(\alpha)$, thus $\alpha \in K(\mathcal{N})$, hence $K(\mathcal{N})$ is $\rho -$ ideal.

Proposition 4.7. If $\mathcal{N}$ is (NS $-\rho - I$), then $K(\mathcal{N}^c)$ is $\rho -$ ideal.

Proof: Let $\mathcal{N}$ be (NS $-\rho - I$) and let $\alpha, \beta \in K(\mathcal{N}^c)$, then

$\mathcal{N}_{T^c}(\alpha) = \mathcal{N}_{T^c}(\beta) = \mathcal{N}_{T^c}(0), \mathcal{N}_{I^c}(\alpha) = \mathcal{N}_{I^c}(\beta) = \mathcal{N}_{I^c}(0)$ and $\mathcal{N}_{F^c}(\alpha) = \mathcal{N}_{F^c}(\beta) = \mathcal{N}_{F^c}(0)$.

Also, $\mathcal{N}_{T^c}(\alpha \oint \beta) = 1 - \mathcal{N}_T(\alpha \oint \beta) \leq 1 - min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$

$$= max\{1 - \mathcal{N}_T(\alpha), 1 - \mathcal{N}_T(\beta)\}$$
$$= max\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$$
$$= max\{\mathcal{N}_{T^c}(0), \mathcal{N}_{T^c}(0)\} = \mathcal{N}_{T^c}(0),$$

$\mathcal{N}_{I^c}(\alpha \oint \beta) = 1 - \mathcal{N}_I(\alpha \oint \beta) \geq 1 - max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$

$$= min\{1 - \mathcal{N}_I(\alpha), 1 - \mathcal{N}_I(\beta)\}$$
$$= min\ \{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$$
$$= min\{\mathcal{N}_{I^c}(0), \mathcal{N}_{I^c}(0)\} = \mathcal{N}_{I^c}(0),$$

$\mathcal{N}_{F^c}(\alpha \oint \beta) = 1 - \mathcal{N}_F(\alpha \oint \beta) \leq 1 - min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$

$$= max\{1 - \mathcal{N}_F(\alpha), 1 - \mathcal{N}_F(\beta)\}$$
$$= max\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\alpha)\} = \mathcal{N}_{F^c}(0)\}$$
$$= max\{\mathcal{N}_{F^c}(0), \mathcal{N}_{F^c}(0)\} = \mathcal{N}_{F^c}(0)\},$$

and from lemma (3.4),we obtain

$\mathcal{N}_{T^c}(\alpha \oint \beta) \geq \mathcal{N}_{T^c}(0)$, $\mathcal{N}_{I^c}(\alpha \oint \beta) \leq \mathcal{N}_{I^c}(0)$, $\mathcal{N}_{F^c}(\alpha \oint \beta) \geq \mathcal{N}_{F^c}(0)$

thus $\mathcal{N}_{T^c}(\alpha \oint \beta) = \mathcal{N}_{T^c}(0), \mathcal{N}_{I^c}(\alpha \oint \beta) = \mathcal{N}_{I^c}(0), \mathcal{N}_{F^c}(\alpha \oint \beta) = \mathcal{N}_{F^c}(0)$,
this implies $\alpha \oint \beta \in K(\mathcal{N}^c)$. Now, let $\alpha \oint \beta, \beta \in K(\mathcal{N}^c)$, then $\mathcal{N}_{T^c}(\alpha \oint \beta) = \mathcal{N}_{T^c}(0)$, $\mathcal{N}_{I^c}(\alpha \oint \beta) = \mathcal{N}_{I^c}(0)$, $\mathcal{N}_{F^c}(\alpha \oint \beta) = \mathcal{N}_{F^c}(0)$.
And $\mathcal{N}_{T^c}(\beta) = \mathcal{N}_{T^c}(0), \mathcal{N}_{I^c}(\beta) = \mathcal{N}_{I^c}(0), \mathcal{N}_{F^c}(\beta) = \mathcal{N}_{F^c}(0)$. Since $\mathcal{N}$ is (NS $-\rho - I$) then,

$$\mathcal{N}_{T^c}(\alpha) = 1 - \mathcal{N}_T(\alpha) \leq 1 - min\{\mathcal{N}_T(\alpha \oint \beta), \mathcal{N}_T(\beta)\}$$
$$= max\{1 - \mathcal{N}_T(\alpha \oint \beta), 1 - \mathcal{N}_T(\beta)\}$$
$$= max\{\mathcal{N}_{T^c}(\alpha \oint \beta), \mathcal{N}_{T^c}(\beta)\}$$
$$= max\{\mathcal{N}_{T^c}(0), \mathcal{N}_{T^c}(0)\} = \mathcal{N}_{T^c}(0),$$
$$\mathcal{N}_{I^c}(\alpha) = 1 - \mathcal{N}_I(\alpha) \geq 1 - max\{\mathcal{N}_I(\alpha \oint \beta), \mathcal{N}_I(\beta)\}$$
$$= min\{1 - \mathcal{N}_I(\alpha \oint \beta), 1 - \mathcal{N}_I(\beta)\}$$
$$= min\ \{\mathcal{N}_{I^c}(\alpha \oint \beta), \mathcal{N}_{I^c}(\beta)\}$$
$$= min\{\mathcal{N}_{I^c}(0), \mathcal{N}_{I^c}(0)\} = \mathcal{N}_{I^c}(0),$$
$$\mathcal{N}_{F^c}(\alpha) = 1 - \mathcal{N}_F(\alpha) \leq 1 - min\{\mathcal{N}_F(\alpha \oint \beta), \mathcal{N}_F(\beta)\}$$
$$= max\{1 - \mathcal{N}_F(\alpha \oint \beta), 1 - \mathcal{N}_F(\beta)\}$$
$$= max\{\mathcal{N}_{F^c}(\alpha \oint \beta), \mathcal{N}_{F^c}(\alpha)\} = \mathcal{N}_{F^c}(0)\}$$
$$= max\{\mathcal{N}_{F^c}(0), \mathcal{N}_{F^c}(0)\} = \mathcal{N}_{F^c}(0),$$

and from lemma (3.4),we obtain
$\mathcal{N}_{T^c}(\alpha) \geq \mathcal{N}_{T^c}(0), \mathcal{N}_{I^c}(\alpha) \leq \mathcal{N}_{I^c}(0), \mathcal{N}_{F^c}(\alpha) \geq \mathcal{N}_{F^c}(0)$
thus $\mathcal{N}_{T^c}(\alpha) = \mathcal{N}_{T^c}(0), \mathcal{N}_{I^c}(\alpha) = \mathcal{N}_{I^c}(0), \mathcal{N}_{F^c}(\alpha) = \mathcal{N}_{F^c}(0)$,
this implies $\alpha \in K(\mathcal{N}^c)$, hence $K(\mathcal{N}^c)$is $\rho -$ ideal.

Proposition 4.8. If $\mathcal{N}$ is (NS $-\rho - I$), then $\mathcal{N}_t$ is $\rho -$ideal.

Proof: Assume that $\mathcal{N}$ is (NS $-\rho - I$) and $\alpha, \beta \in \mathcal{N}_t$, then $\mathcal{N}_T(\alpha \oint \beta) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\} \geq t, \mathcal{N}_I(\alpha \oint \beta) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\} \leq t$, $\mathcal{N}_F(\alpha \oint \beta) \geq min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\} \geq t$, this implies $\alpha \oint \beta \in \mathcal{N}_t$. Now, assume that $\alpha \oint \beta \in \mathcal{N}_t$ & $\beta \in \mathcal{N}_t$, and [since $\mathcal{N}$ is (NS $-\rho - I$)]. We obtain $\mathcal{N}_T(\alpha) \geq min\{\mathcal{N}_T(\alpha \oint \beta), \mathcal{N}_T(\beta)\} \geq t$, $\mathcal{N}_I(\alpha) \leq max\{\mathcal{N}_I(\alpha \oint \beta), \mathcal{N}_I(\beta) \leq t$, $\mathcal{N}_F(\alpha) \geq min\{\mathcal{N}_F(\alpha \oint \beta), \mathcal{N}_F(\beta)\} \geq t$, thus $\alpha \in \mathcal{N}_t$, hence $\mathcal{N}_t$ is $\rho -$ideal.

Definition4.9. Assume $(\mho, \oint, 0)$ is a ρ-algebra and $\mathcal{N}$ is (NS) of $\mho$. We say $\mathcal{N}$ is a complete neutrosophic ρ -ideal of $\mho$ (briefly, CNS$-\rho - I$) if such that:
(i) $\mathcal{N}_T(\alpha \oint \beta) \leq max\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$,
(ii) $\mathcal{N}_I(\alpha \oint \beta) \geq min\ \{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$,
(iii) $\mathcal{N}_F(\alpha \oint \beta) \leq max\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$,
(iv) $\mathcal{N}_T(\alpha) \leq max\{\mathcal{N}_T(\alpha \oint \beta), \mathcal{N}_T(\beta)\}$,
(v) $\mathcal{N}_I(\alpha) \geq min\{\mathcal{N}_I(\alpha \oint \beta), \mathcal{N}_I(\beta)\}$,
(vi) $\mathcal{N}_F(\alpha) \leq max\{\mathcal{N}_F(\alpha \oint \beta), \mathcal{N}_F(\beta)\}$, for any $\alpha, \beta \in \mho$.

Example 4.10. Let $\mho = \{0, 1, 2, 3, 4\}$ be a set with the following table (5), it is clear that $(\mho, \oint, 0)$ is a ρ-algebra, We define a (NS) $\mathcal{N}$ in $\mho$ as follows:

$\mathcal{N}_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.4 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}$, $\mathcal{N}_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.7 & 0.3 & 0.3 & 0.3 & 0.3 \end{pmatrix}$, and

$\mathcal{N}_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.2 & 0.4 & 0.4 & 0.4 & 0.4 \end{pmatrix}$. Hence, $\mathcal{N}$ is (CNS $-\rho - I$).

$\oint$	**0**	**1**	**2**	**3**	4
0	0	0	0	0	0
1	1	0	1	2	4
2	2	1	0	2	2
3	3	2	2	0	3
4	4	4	2	3	0

Table (5): $\mathcal{N}$ is (CNS $-\rho - I$).

Proposition 4.11: Let $\mathcal{N}$ be NS then $\mathcal{N}$ is (NS $-\rho - I$) if and only if $\mathcal{N}^c$ is (CNS $-\rho - I$).

Proof: Let $\mathcal{N}$ be (NS $-\rho - I$) ,From proof proposition (3.18) we obtain $\mathcal{N}_{T^c}(\alpha \oint \beta) \leq max\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$, $\mathcal{N}_{I^c}(\alpha \oint \beta) \geq min\{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$, $\mathcal{N}_{F^c}(\alpha \oint \beta) \leq max\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\beta)\}$.

Now, $\mathcal{N}_{T^c}(\alpha) = 1 - \mathcal{N}_T(\alpha) \leq 1 - min\{\mathcal{N}_T(\alpha \oint \beta), \mathcal{N}_T(\beta)\}$

$$= max\{1 - \mathcal{N}_T(\alpha \oint \beta), 1 - \mathcal{N}_T(\beta)\}$$
$$= max\{\mathcal{N}_{T^c}(\alpha \oint \beta), \mathcal{N}_{T^c}(\beta)\},$$

$\mathcal{N}_{I^c}(\alpha) = 1 - \mathcal{N}_I(\alpha) \geq 1 - max\{\mathcal{N}_I(\alpha \oint \beta), \mathcal{N}_I(\beta)\}$

$$= min\{1 - \mathcal{N}_I(\alpha), 1 - \mathcal{N}_I(\beta)\}$$
$$= min\{\mathcal{N}_{I^c}(\alpha \oint \beta), \mathcal{N}_{I^c}(\beta)\},$$

$\mathcal{N}_{F^c}(\alpha) = 1 - \mathcal{N}_F(\alpha) \leq 1 - min\{\mathcal{N}_F(\alpha \oint \beta), \mathcal{N}_F(\beta)\}$

$$= max\{1 - \mathcal{N}_F(\alpha \oint \beta), 1 - \mathcal{N}_F(\beta)\}$$
$$= max\{\mathcal{N}_{F^c}(\alpha \oint \beta), \mathcal{N}_{F^c}(\beta)\}.$$

Hence $\mathcal{N}^c$ is (CNS $-\rho - I$).

Conversely: Let $\mathcal{N}^c$ be (CNS $-\rho - I$) then from proof proposition, (3.18) we obtain,

$\mathcal{N}_T(\alpha \oint \beta) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$, $\mathcal{N}_I(\alpha \oint \beta) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$, $\mathcal{N}_F(\alpha \oint \beta) \geq min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$.

Now, $\mathcal{N}_T(\alpha) = 1 - \mathcal{N}_{T^c}(\alpha) \geq 1 - max\{\mathcal{N}_{T^c}(\alpha \oint \beta), \mathcal{N}_{F^c}(\beta)\}$

$$= min\{1 - \mathcal{N}_{T^c}(\alpha \oint \beta), 1 - \mathcal{N}_{T^c}(\beta)\}$$
$$= min\{\mathcal{N}_T(\alpha \oint \beta), \mathcal{N}_T(\beta)\},$$

$\mathcal{N}_I(\alpha) = 1 - \mathcal{N}_{T^c}(\alpha) \leq 1 - min\{\mathcal{N}_{I^c}(\alpha \oint \beta), \mathcal{N}_{I^c}(\beta)\}$

$$= max\{1 - \mathcal{N}_{I^c}(\alpha \oint \beta), 1 - \mathcal{N}_{I^c}(\beta)\}$$
$$= max\{\mathcal{N}_I(\alpha \oint \beta), \mathcal{N}_I(\beta)\}$$

$\mathcal{N}_F(\alpha) = 1 - \mathcal{N}_{F^c}(\alpha) \geq 1 - max\{\mathcal{N}_{F^c}(\alpha \oint \beta), \mathcal{N}_{F^c}(\beta)\}$

$$= min\{1 - \mathcal{N}_{F^c}(\alpha \oint \beta), 1 - \mathcal{N}_{F^c}(\beta)\}$$

$= min\{\mathcal{N}_F(\alpha \oint \beta), \mathcal{N}_F(\beta)\}$.

Hence $\mathcal{N}$ is $(\text{NS} - \rho - I)$

Corollary 4.12: If $\mathcal{N}^c$ is $(\text{CNS} - \rho - I)$. Then;

1- $\mathcal{N}_t$ is ρ −subalgebra,

2- $\mathcal{N}^c$ is $(\text{CNS} - \rho - SA)$,

3- $\mathcal{N}_t$ is ρ −ideal.

Proof 1: From proposition (4.11) and corollary (4.4)-1.

Proof 2: From proposition (4.11) and corollary (4.4)-2.

Proof 3: From Proposition (4.11) and Proposition (4.8).

5. Neutrosophic $\bar{\rho}$ -Ideal and Complete Neutrosophic $\bar{\rho}$ -Ideal

Definition 5.1. Assume $(\mho, \oint, 0)$ is a ρ-algebra and $\mathcal{N}$ is (NS) of $\mho$. We say $\mathcal{N}$ is a neutrosophic $\bar{\rho}$ -ideal of $\mho$ (briefly, $\text{NS} - \bar{\rho} - I$) if such that:

(i) $\mathcal{N}_T(0) \geq \mathcal{N}_T(\alpha)$,

(ii) $\mathcal{N}_I(0) \leq \mathcal{N}_I(\alpha)$,

(iii) $\mathcal{N}_F(0) \geq \mathcal{N}_F(\alpha)$,

(iv) $\mathcal{N}_T(\alpha \oint \beta) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$,

(v) $\mathcal{N}_I(\alpha \oint \beta) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$,

(vi) $\mathcal{N}_F(\alpha \oint \beta) \geq min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$, for any $\alpha, \beta \in \mho$.

Example 5.2. Let $\mho = \{x, y, z, w\}$ be a set with the following table(6), it is clear that $(\mho, \oint, x)$ is a ρ-algebra. We define a (NS) $\mathcal{N}$ in $\mho$ as follows:

$\mathcal{N}_T = \begin{pmatrix} x & y & z & w \\ 0.6 & 0.4 & 0.4 & 0.4 \end{pmatrix}$, $\mathcal{N}_I = \begin{pmatrix} x & y & z & w \\ 0.1 & 0.2 & 0.2 & 0.2 \end{pmatrix}$, $\mathcal{N}_F = \begin{pmatrix} x & y & z & w \\ 0.2 & 0.1 & 0.1 & 0.1 \end{pmatrix}$.

Hence, $\mathcal{N}$ is $(\text{NS} - \bar{\rho} - I)$.

$\oint$	x	y	z	w
x	x	x	x	x
y	y	x	z	w
z	z	z	x	z
w	w	w	z	x

Table (6): $\mathcal{N}$ is $(\text{NS} - \bar{\rho} - I)$.

Lemma 5.3. If $\mathcal{N}$ is $(\text{NS} - \bar{\rho} - I)$, then $\mathcal{N}$ is $(\text{NS} - \rho - SA)$.

Corollary 5.4. Let $\mathcal{N}$ be $(\text{NS} - \bar{\rho} - I)$. Then;

1- $\mathcal{N}_t$ is ρ −subalgebra,

2- $\mathcal{N}^c$is $(\text{CNS} - \rho - S)$.

Proof (1): From lemma (5.3) and proposition (3.10).

Proof (2): From lemma (5.3) and proposition (3.18).

Proposition 5.5. If $\mathcal{N}$ is (NS $-\bar{\rho}-I$), then $\mathcal{N}_t$ is $\bar{\rho}$ −ideal.

Proof: Assume that $\mathcal{N}$ is (NS $-\bar{\rho}-I$) and $\alpha, \beta \in \mathcal{N}_t$, then $\mathcal{N}_T(\alpha \oint \beta) \geq min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\} \geq t, \mathcal{N}_I(\alpha \oint \beta) \leq max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\} \leq t$, $\mathcal{N}_F(\alpha \oint \beta) \geq min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\} \geq t$, this implies $\alpha \oint \beta \in \mathcal{N}_t$. Since [$\mathcal{N}$ is (NS $-\bar{\rho}-I$)], and $\mathcal{N}$ ={$\prec \alpha, \mathcal{N}_T(\alpha) \geq$ t , $\mathcal{N}_I(\alpha) \leq$ t , $\mathcal{N}_F(\alpha) \geq$ t $\succ$| $\alpha \in$ Ʊ}. We obtain $\mathcal{N}_T(0) \geq \mathcal{N}_T(\alpha) \geq$ t, $\mathcal{N}_I(0) \leq \mathcal{N}_I(\alpha) \leq t$, $\mathcal{N}_F(0) \geq \mathcal{N}_F(\alpha) \geq$ t, thus $0 \in \mathcal{N}_t$, hence $\mathcal{N}_t$ is $\bar{\rho}$ −ideal.

Definition5.6. Assume (Ʊ,$\oint$, 0) is a ρ-algebra and $\mathcal{N}$ is (NS) of Ʊ. We say $\mathcal{N}$ is a complete neutrosophic $\bar{\rho}$ -ideal of Ʊ (briefly, CNS $-\bar{\rho}-I$). If such that:

(i)$\mathcal{N}_T(0) \leq \mathcal{N}_T(\alpha)$,

(ii) $\mathcal{N}_I(0) \geq \mathcal{N}_I(\alpha)$,

(iii) $\mathcal{N}_F(0) \leq \mathcal{N}_F(\alpha)$,

(iv) $\mathcal{N}_T(\alpha \oint \beta) \leq max\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$,

(v) $\mathcal{N}_I(\alpha \oint \beta) \geq min\{\mathcal{N}_I(\alpha),\mathcal{N}_I(\beta)\}$,

(vi) $\mathcal{N}_F(\alpha \oint \beta) \leq max\{\mathcal{N}_F(\alpha),\mathcal{N}_F(\beta)\}$,for any α, $\beta \in$ Ʊ.

Example 5.7. Let Ʊ = { 0 ,1 ,2,3 } be a set with the following table(7), it is clear that (Ʊ, $\oint$, 0) is a ρ-algebra. We define a (NS) $\mathcal{N}$ in Ʊ as follows:

$\mathcal{N}_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.4 & 0.5 & 0.5 & 0.5 \end{pmatrix}, \mathcal{N}_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.3 & 0.2 & 0.2 & 0.2 \end{pmatrix}, \mathcal{N}_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.1 & 0.2 & 0.2 & 0.2 \end{pmatrix}$.

Hence, $\mathcal{N}$ is (CNS $-\bar{\rho}-I$).

$\oint$	**0**	**1**	**2**	**3**
0	0	0	0	0
1	1	0	2	3
2	2	2	0	2
3	3	3	2	0

Table (7): $\mathcal{N}$ is (CNS $-\bar{\rho}-I$).

Lemma 5.8. If $\mathcal{N}$ is (CNS $-\bar{\rho}-I$), then $\mathcal{N}$ is (CNS $-\rho-SA$).

Proposition 5.9: Let $\mathcal{N}$ be (CNS $-\bar{\rho}-I$), then $K(\mathcal{N})$ is $\rho -$ subalgebra.

Proof: Assume $\alpha, \beta \in K(N)$, then $N_T(\alpha) = N_T(\beta) = N_T(0), N_I(\alpha) = N_I(\beta) = N_I(0)$ and $N_F(\alpha) = N_F(\beta) = N_F(0)$. Also, $N_T(\alpha \oint \beta) \leq max\{N_T(\alpha), N_T(\beta)\} = max\{N_T(0), \mathcal{N}_T(0)\} = \mathcal{N}_T(0)$, $\mathcal{N}_I(\alpha \oint \beta) \geq min\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\} = min\{\mathcal{N}_I(0), \mathcal{N}_I(0)\} = \mathcal{N}_I(0)$, $\mathcal{N}_F(\alpha \oint \beta) \leq max\{\mathcal{N}_F(\alpha),\mathcal{N}_F(\beta)\} = max\{\mathcal{N}_F(0),\mathcal{N}_F(0)\} = \mathcal{N}_F(0)$, and $\mathcal{N}_T(0) \leq \mathcal{N}_T(\alpha \oint \beta)$, $\mathcal{N}_I(0) \geq \mathcal{N}_I(\alpha \oint \beta)$, $\mathcal{N}_F(0) \leq \mathcal{N}_F(\alpha \oint \beta)$, thus $\mathcal{N}_T(\alpha \oint \beta) = \mathcal{N}_T(0)$, $\mathcal{N}_I(\alpha \oint \beta) = \mathcal{N}_I(0)$, $\mathcal{N}_F(\alpha \oint \beta) = \mathcal{N}_F(0)$, and $\alpha \oint \beta \in K(\mathcal{N})$ hence $K(\mathcal{N})$ is ρ −subalgebra.

Proposition 5.10. Let $\mathcal{N}$ be (NS) then $\mathcal{N}$ is (NS $-\bar{\rho}-I$) if and only if $\mathcal{N}^c$ is (CNS $-\bar{\rho}-I$).

Proof: Let $\mathcal{N}$ be (NS $-\bar{\rho}-I$), we obtain $\mathcal{N}_T(0) \geq \mathcal{N}_T(\alpha)$, $\mathcal{N}_I(0) \leq \mathcal{N}_I(\alpha)$, $\mathcal{N}_F(0) \geq \mathcal{N}_F(\alpha)$, thus $\mathcal{N}_{T^c}(\alpha) = 1 - \mathcal{N}_T(\alpha) \geq 1 - \mathcal{N}_T(0) = \mathcal{N}_{T^c}(0)$, $\mathcal{N}_{I^c}(\alpha) = 1 - \mathcal{N}_I(\alpha) \leq 1 - \mathcal{N}_I(0) = \mathcal{N}_{I^c}(0)$, $\mathcal{N}_{F^c}(\alpha) = 1 - \mathcal{N}_F(\alpha) \geq 1 - \mathcal{N}_F(0) = \mathcal{N}_{F^c}(0)$.

Now, $\mathcal{N}_{T^c}(\alpha \oint \beta) = 1 - \mathcal{N}_T(\alpha \oint \beta) \leq 1 - min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$

$= max\{1 - \mathcal{N}_T(\alpha), 1 - \mathcal{N}_T(\beta)\}$

$= max\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$,

$\mathcal{N}_{I^c}(\alpha \oint \beta) = 1 - \mathcal{N}_I(\alpha \oint \beta) \geq 1 - max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$

$= min\{1 - \mathcal{N}_I(\alpha), 1 - \mathcal{N}_I(\beta)\} = min\{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$,

$\mathcal{N}_{F^c}(\alpha \oint \beta) = 1 - \mathcal{N}_F(\alpha \oint \beta) \leq 1 - min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$

$= max\{1 - \mathcal{N}_F(\alpha), 1 - \mathcal{N}_F(\beta)\} = max\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\beta)\}$,

Hence $\mathcal{N}^c$ is $(\text{CNS} - \bar{\rho} - I)$.

Conversely: Let $\mathcal{N}^c$ be $(\text{CNS} - \bar{\rho} - I)$, then $\mathcal{N}_{T^c}(0) \leq \mathcal{N}_{T^c}(\alpha)$, $\mathcal{N}_{I^c}(0) \geq \mathcal{N}_{I^c}(\alpha)$, $\mathcal{N}_{F^c}(0) \leq \mathcal{N}_{F^c}(\alpha)$, $\mathcal{N}_T(0)$ $= 1 - \mathcal{N}_{T^c}(0) \geq 1 - \mathcal{N}_{T^c}(\alpha) = \mathcal{N}_T(\alpha)$,

$\mathcal{N}_I(0) = 1 - \mathcal{N}_{I^c}(0) \leq 1 - \mathcal{N}_{I^c}(\alpha) = \mathcal{N}_I(\alpha)$,

$\mathcal{N}_F(0) = 1 - \mathcal{N}_{F^c}(0) \geq 1 - \mathcal{N}_{F^c}(\alpha) = \mathcal{N}_F(\alpha)$,

and from the following

$\mathcal{N}_{T^c}(\alpha \oint \beta) \leq max\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$, $\mathcal{N}_{I^c}(\alpha \oint \beta) \geq min\{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$,

$\mathcal{N}_{F^c}(\alpha \oint \beta) \leq max\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\beta)\}$, for any $\alpha, \beta \in \mho$,

We obtain,

$\mathcal{N}_T(\alpha \oint \beta) = 1 - \mathcal{N}_{T^c}(\alpha \oint \beta) \geq 1 - max\{\mathcal{N}_{T^c}(\alpha), \mathcal{N}_{T^c}(\beta)\}$

$= min\{1 - \mathcal{N}_{T^c}(\alpha), 1 - \mathcal{N}_{T^c}(\beta)\} = min\{\mathcal{N}_T(\alpha), \mathcal{N}_T(\beta)\}$,

$\mathcal{N}_I(\alpha \oint \beta) = 1 - \mathcal{N}_{I^c}(\alpha \oint \beta) \leq 1 - min\{\mathcal{N}_{I^c}(\alpha), \mathcal{N}_{I^c}(\beta)\}$

$= max\{1 - \mathcal{N}_{I^c}(\alpha), 1 - \mathcal{N}_{I^c}(\beta)\} = max\{\mathcal{N}_I(\alpha), \mathcal{N}_I(\beta)\}$

$\mathcal{N}_F(\alpha \oint \beta) = 1 - \mathcal{N}_{F^c}(\alpha \oint \beta) \geq 1 - max\{\mathcal{N}_{F^c}(\alpha), \mathcal{N}_{F^c}(\beta)\}$

$= min\{1 - \mathcal{N}_{F^c}(\alpha), 1 - \mathcal{N}_{F^c}(\beta)\} = min\{\mathcal{N}_F(\alpha), \mathcal{N}_F(\beta)\}$.

Hence $\mathcal{N}$ is $(\text{NS} - \bar{\rho} - I)$.

Corollary 5.11. If $\mathcal{N}^c$ is $(\text{CNS} - \bar{\rho} - I)$. Then;

1- $\mathcal{N}_t$ is ρ −subalgebra,

2- $\mathcal{N}^c$ is $(\text{CNS} - \rho - SA)$,

3- $\mathcal{N}_t$ is $\bar{\rho}$ −ideal.

Proof 1: From proposition (5.10) and corollary (5.4)-1.

Proof 2: From Proposition (5.10) and Corollary (5.4)-2.

Proof 3: From Proposition (5.10) and Proposition (5.5).

6. Conclusion

We presented and examined several kinds of ρ −algebra ideals in this research, which we called neutrosophic ρ −subalgebra, complete neutrosophic ρ −subalgebra, neutrosophic ρ −ideal, complete neutrosophic ρ −ideal, neutrosophic ρ −ideal, neutrosophic ρ −ideal, neutrosophic $\bar{\rho}$ −ideal, and complete neutrosophic $\bar{\rho}$ −ideal, respectively. We also suggested some theories try to explain some of these ideal type relationships. In future work, we will use soft set theory to study our notions and results in neutrosophic soft sets.

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