Neutrosophic Semi-Baire Spaces

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Abstract: In this paper, we introduce the concept of Neutrosophic Semi Baire spaces in Neutrosophic Topological Spaces. Also we define Neutrosophic Semi-nowhere dense, Neutrosophic Semi-first category and Neutrosophic Semi-second category sets. Some of its characterizations of Neutrosophic Semi-Baire spaces are also studied. Several examples are given to illustrate the concepts

Keywords: Neutrosophic semi-open set, Neutrosophic semi-nowhere dense set, Neutrosophic semi-first category, Neutrosophic semi-second category and Neutrosophic semi-Baire spaces

1. Introduction and Preliminaries

The fuzzy idea has invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [29]. The important concept of fuzzy topological space was offered by C. L. Chang [9] and from that point forward different ideas in topology have been reached out to fuzzy topological space. The concept of "intuitionistic fuzzy set" was first presented by Atanassov [5]. He and his associates studied this useful concept [6 - 8]. Afterward, this idea was generalized to "intuitionistic L – fuzzy sets" by Atanassov and Stoeva [6]. The idea of somewhat fuzzy continuous functions and somewhat fuzzy open hereditarily irresolvable were introduced and investigated by by G. Thangaraj and G. Balasubramanian in [25]. The idea of intuitionistic fuzzy nowhere dense set in intuitionistic fuzzy topological space presented and studied by Dhavaseelan and et al. in [16]. The concepts of neutrosophy and Neutrosophic set were introduced by F. Smarandache [[22], [23]]. Afterwards, the works of Smarandache inspired A. A. Salama and S. A. Alblowi[21] to introduce and study the concepts of Neutrosophic crisp set and Neutrosophic crisp topological spaces. The Basic definitions and Proposition related to Neutrosophic topological spaces was introduced and discussed by Dhavaseelan et al. [17]. The concepts of Neutrosophic Baire spaces are introduced by R. Dhavaseelan, S. Jafari ,R. Narmada Devi, Md. Hanif Page [16]

Definition 1.1. [22, 23] Let T,I,F be real standard or non standard subsets of $]0^{-}, 1^{+}[$, with

$sup_T = t_{sup}$ T ; $inf_T = t_{inf}$

$Sup_I = i_{sup}$; $inf_I = i_{inf}$

$Sup_F = f_{sup}$; $inf_F = f_{inf}$

$n\text{-}sup = t_{sup} + i_{sup} + f_{sup}$

$n\text{-}inf = t_{inf}+i_{inf}+f_{inf}$.   T, I, F are Neutrosophic components.

Definition 1.2. [22, 23] Let X is a nonempty fixed set. A Neutrosophic set [briefly Ne.S] K is an object having the form $K = \{\langle x, \mu_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$ where $\mu_K(x), \sigma_K(x) and\, \gamma_K(x)$ which represents the degree of membership function (namely $\mu_K(x)$), the degree of indeterminacy (namely $\sigma_k(x)$) and the degree of non-membership (namely $\gamma_K(x)$) respectively of each element $x \in X$ to the set K.

Remark 1.2. [22, 23]

(1) A Ne.S $K = \{\langle x, \mu_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_K, \sigma_K, \gamma_K\rangle$ in $]0^-, 1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol K = $\langle \mu_K, \sigma_K, \gamma_K\rangle$ for the Ne.S $K = \{\langle x, \mu_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$

Definition 1.3. [22, 23] Let X be a nonempty set and the Ne.Sets K and L in the form $K = \{\langle x, \mu_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$, L= $\{\langle x, \mu_L(x), \sigma_L(x), \gamma_L(x)\rangle : x \in X\}$. Then

(a) $K \subseteq L$ iff $\mu_K(x) \leq \mu_L(x)$, $\sigma_K(x) \leq \sigma_L(x)$, $\gamma_K(x) \geq \gamma_L(x)$ for all $x \in X$;

(b) $K = L$ iff $K \subseteq L$ and $L \subseteq K$;

(c) $\overline{K} = \{\langle x, \gamma_L(x), \sigma_K(x), \mu_L(x)\rangle : x \in X\}$; [Complement of K]

(d) K ∩ L= $\left\{\langle x, \mu_K(x) \wedge \mu_L(x), \sigma_K(x) \wedge \sigma_L(x), \gamma_K(x) \vee \gamma_L(x)\rangle : x \in X\right\}$;

(e) K ∪ L= $\left\{\langle x, \mu_K(x) \vee \mu_L(x), \sigma_K(x) \vee \sigma_L(x), \gamma_K(x) \wedge \gamma_L(x)\rangle : x \in X\right\}$;

(f) [ ]K = $\{\langle x, \mu_K(x), \sigma_K(x), 1 - \mu_K(x)\rangle : x \in X\}$;

(g) $\langle\,\rangle K = \{\langle x, 1 - \gamma_K(x), \sigma_K(x), \gamma_K(x)\rangle : x \in X\}$

Definition 1.4. [22, 23] Let $\{K_i : i \in J\}$ be an arbitrary family of Ne.Sets in X. Then

(a) $\cap K_i = \{\langle x, \wedge \mu_{Ki}(x), \wedge \sigma_{Ki}(x), \vee \gamma_{Ki}(x)\rangle : x \in X\}$,

(b) $\cup K_i = \{\langle x, \vee \mu_{Ki}(x), \vee \sigma_{Ki}(x), \wedge \gamma_{Ki}(x)\rangle : x \in X\}$,

Since our main purpose is to construct the tools for developing Ne.T.Spaces, we introduce the Ne.Sets 0_N and 1_N in X as follows:

Definition 1.5. [22, 23]

$$0_N = \{\langle x, 0,0,1\rangle : x \in X\}\ and\ \ 1_N = \{\langle x, 1,1,0\rangle : x \in X\}$$

Definition 1.6. [21]

A Neutrosophic topology (Ne.T) on a nonempty set X is a family N_T of Ne.Sets in X satisfying the following axioms:

(i) $0_N, 1_N \in N_T$,

(ii) $G_1 \cap G_2 \in N_T$ for any $G_1, G_2 \in N_T$.

(iii)$\cup G_i$ for arbitrary family $\{G_i | i \in \wedge\}$.

In this case the ordered pair (X, N_T) or simply X is called a Neutrosophic Topological Space (briefly Ne.T.S) and each Ne.S in N_T is called a Neutrosophic open set (briefly Ne.O.S). The complement K of a Ne.O.S K in X is called a Neutrosophic closed set (briefly Ne.C.S) in X.

Definition 1.7. [9]

Let K be a Ne.S in a Ne.T.S X. Then

Ne.int(K) = $\cup\{G \mid G\ is\ Neutrosophic\ open\ set\ in\ X\ and\ G \subseteq K\}$

is called the Neutrosophic interior of K;

Ne.cl(K) = $\cap\{G \mid G \text{ is Neutrosophic closed set in } X \text{ and } G \supseteq K\}$

is called the Neutrosophic closure of K.

Definition 1.8: [13] A Ne.S K in a Ne.T.S X is said to a Neutrosophic Semi Open set (Ne.S.O.S) if $K \subseteq Ne.cl(Ne.\text{int}(K))$ and Neutrosophic Semi Closed set (Ne.S.C.S) if $Ne.\text{int}(Ne.cl(K)) \subseteq K$.

Definition 1.9:[13] Let K be a Ne.S in a Ne.T.S X. Then

Ne.S.int(K) = $\cup\{G \mid G \text{ is Neutrosophic semi open set in } X \text{ and } G \subseteq K\}$

is called the Neutrosophic semi interior of K;

Ne.S.cl(K) = $\cap\{G \mid G \text{ is Neutrosophic semi closed set in } X \text{ and } G \supseteq K\}$

is called the Neutrosophic semi closure of K;

Result: 1.9 Let K be a Ne.S in a Ne.T.S X. Then

Ne.S.cl(K) = $K \cup Ne.\text{int}(Ne.cl(K))$

Ne.S.int(K) = $K \cap Ne.cl(Ne.\text{int}(K))$

2. Neutrosophic Semi-nowhere dense sets

Definition 2.1 A Ne.S K in Ne.T.S (X, N_T) is called Neutrosophic semi nowhere dense (briefly Ne.S.N.D) if there exists no non-zero Ne.S.O.S L in (X; N_T) such that $L \subset Ne.S.cl(K)$. That is $Ne.S.\text{int}(Ne.S.cl(K)) = 0_N$

Example 2.1 Let *X = {k, l}*. Define the Ne.S K, L and M on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.6}\right), \left(\frac{k}{0.5}, \frac{l}{0.2}\right), \left(\frac{k}{0.4}, \frac{l}{0.5}\right)\right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.2}, \frac{l}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.1}\right)\right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L, K \cup L, K \cap L\}$ is Ne.T on X. Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, \overline{K \cup L}$ are Ne.S.N.D set

Proposition 2.1. If K is a Ne.S.N.D set in (X; N_T), then $\overline{K}$ is a Ne.S.D set in (X, T)

Proposition 2.2. Let K be a set. If K is a Ne.S.C.S in (X, N_T) with Ne.S.int(K) = 0_N, then K is a Ne.S.N.D set in (X; N_T).

Definition 2.2. Let K be a Neutrosophic semi first category set (Ne.S.F.C.) in (X, N_T). Then $\overline{K}$ is called a Neutrosophic residual set in (X; N_T).

Proposition 2.3. The complement of a Ne.S.N.D. set in a Ne.T.S (X, N_T) need not be Ne.S.N.D. set.

Proof: For, in example 2.1, $\overline{K}$ is a Ne.S.N.D. set in (X, N_T) whereas K is not a Ne.S.N.D. set in (X, N_T).

Proposition 2.4. If K & L are Ne.S.N.D. sets in a Ne.T.S (X, N_T), then $K \cup L$ need not be Ne.S.N.D. set in (X, N_T).

Proof: For, in example 2.1, $\overline{K} \& \overline{L}$ is Ne.S.N.D. sets in (X, N_T). But $\overline{K} \cup \overline{L}$ implies that Ne.S.int(Ne.S.cl($\overline{K} \cup \overline{L}$) ≠ 0_N. Therefore union of Ne.S.N.D. sets need not be Ne.S.N.D. set in (X, N_T).

Proposition 2.5: If the Ne.Sets K and L are Ne.S.N.D. sets in a Ne.T.S (X, N_T) then $K \cap L$ is a Ne.S.N.D. set in (X, N_T).

Proof: Let the fuzzy sets K and L be Ne.S.N.D. sets in (X, N_T). Now Ne.S.int (Ne.S.cl ($K \cap L$)) ⊆ Ne.S.int (Ne.S.cl (K)) ∩ Ne.S.int (Ne.S.cl (L)) = 0_N ∩ 0_N (since Ne.S.int (Ne.S.cl (K)) = 0_N and Ne.S.int(Ne.S.cl(B)) = 0_N). That is, Ne.S.int(Ne.S.cl ($K \cap L$) = 0_N. Hence ($K \cap L$) is a Ne.S.N.D. set in (X, N_T).

Proposition 2.6: If K is a Ne.S.N.D. set in a Ne.T.S (X, N_T) then Ne. S.int (K) = 0_N.

Proof: Let K be a Ne.S.N.D. set in (X, N_T). Then, we have Ne.S.int (Ne.S.cl (K)) = 0_N. Now K ⊆ Ne.S.cl (K) we have Ne.S.int (K) ⊆ Ne.S.int (Ne.S.cl (K))= 0_N. Hence Ne.S.int (K) = 0_N

Proposition 2.7:

If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T) then Ne.int (Ne.S.cl (K)) = 0.

Proof: Let K be a Ne.S.N.D. sets in (X, N_T). Then, we have Ne.int(Ne.cl (K)) = 0_N and Ne.int (K) = 0_N. Now Ne.S.cl (K) = K, since K is fuzzy semi-closed set in (X, N_T) implies that Ne.int (Ne.S.cl(K)) =Ne.int (K) = 0_N. Hence Ne.int (Ne.S.cl (K)) = 0_N.

Proposition 2.8: If K is a Ne.S.N.D. set and L is any Ne.Set in a Ne.T.S. (X, N_T), then ($K \cap L$) is a Ne.S.N.D. set in (X, N_T).

Proof: Let K be a Ne.S.N.D. set in (X, N_T). Then, Ne.S.int (Ne.S.cl (K)) = 0. Now Ne.S.int (Ne.S.cl ($K \cap L$)) ⊆ Ne.S.int (Ne.S.cl (K)) ∩ Ne.S.int (Ne.S.cl (L)) ⊆ 0_N ∩ Ne.S.int (Ne.S.cl (L)) = 0_N. That is, Ne.S.int (Ne.S.cl ($K \cap L$) = 0_N. Hence ($K \cap L$) is a Ne.S.N.D. set in (X, N_T).

Definition 2.3 A Ne.S. K in Ne.T.S. (X; N_T) is called Neutrosophic semi dense(Ne.S.D.) if there exists no Ne.S.C.set L in (X; N_T) such that $K \subset L \subset 1_N$ .That is $Ne.S.cl(K) = 1_N$

Proposition2.9 If K is a Ne.S.D. and Ne.S.O. set in a Ne.T.S. (X, N_T) and if L ⊆ 1 - K then L is a Ne.S.N.D. set in (X, N_T).

Proof: Let K be a Ne.S.D. set in (X, N_T). Then we have Ne.S.cl (K) = 1_N and Ne.S.int (K) = K. Now L ⊆1-K implies that Ne.S.cl (L) ⊆ Ne.S.cl (1 - K). Then Ne.S.cl (L) ⊆ 1- Ne.S.int (K) = 1 - K. Hence Ne.S.cl (L) ⊆ (1 - K), which implies that Ne.S.int (Ne.S.cl (L)) ⊆ Ne.S.int(1- K) = 1-Ne.S.cl (K) = 1 – 1 = 0_N. That is, Ne.S.int(Ne.S.cl (L))= 0_N. Hence L is a Ne.S.N.D. set in (X, N_T).

Proposition 2.10: If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T), then 1 - K is a Ne.S.D. set in (X, N_T).

Proof: Let K b e a Ne.S.N.D. set in (X, N_T). Then, Ne.S.int (Ne.S.cl(K) = 0_N. Now K ⊆ Ne.S.cl (K) implies that Ne.S.int(K) ⊆ Ne.S.int (Ne.S.cl(K) = 0_N. Then Ne.S.int (K) = 0_N and Ne.S.cl(1 - K) = 1 – Ne.S.int(K) = 1 – 0_N = 1_N and hence 1 - K is a fuzzy semi-dense set in (X, N_T).

Proposition 2.11: If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T), then Ne.S.cl (K) is also a Ne.S.N.D. set in (X, N_T).

Proof: Let K be a Ne.S.N.D. set in (X, N_T). Then, Ne.S.int (Ne.S.cl (K) = 0_N. Now Ne.S.cl (Ne.S.cl (K)) = Ne.S.cl (K). Hence Ne.S.int(Ne.S.cl (Ne.S.cl (K))) = Ne.S.int (Ne.S.cl (K)) = 0_N. Therefore Ne.S.cl (K) is also a Ne.S.N.D. set in (X, N_T).

Proposition 2.12: If K is a Ne.S.N.D. set in a Ne.T.S. (X, N_T), then 1 – Ne.S.cl (K) is a Ne.S.D. set in (X, N_T).

Proof: Let K be a Ne.S.N.D. set in (X, N_T). Then, by proposition 2.11, Ne.S.cl (K) is a Ne.S.N.D. set in (X, T). Also by proposition 2.10, 1 – Ne.S.cl (K) is a Ne.S.D. set in (X, N_T).

Proposition 2.13: Let K be a Ne.S.D. set in a Ne.T.S. (X, N_T). If L is any Ne. set in (X, N_T), then L is a Ne.S.N.D. set in (X, N_T) if and only if K∩ L is a Ne.S.N.D. set in (X, N_T).

Proof: Let L be a Ne.S.N.D. set in (X, N_T). Then, Ne.S.int (Ne.S.cl (L) = 0_N. Now Ne.S.int (Ne.S.cl (K∩ L)) $\subseteq$ Ne.S.int (Ne.S.cl (K) ∩ Ne.S.int (Ne.S.cl (L)) $\subseteq$ Ne.S.int (Ne.S.cl (K)) ∩ 0_N = 0_N. That is, Ne.S.int(Ne.S.cl (K∩ L)) = 0_N. Hence (K∩ L) is a Ne.S.N.D. set in (X, N_T). Conversely, let (K∩ L) be a Ne.S.N.D. set in (X, N_T). Then Ne.S.int Ne.S.cl (K∩ L) = 0_N. Then, Ne.S.int (Ne.S.cl (K)) ∩ Ne.S.int(Ne.S.cl(L)) = 0_N. Since K is a Ne.S.D. set in (X, N_T), Ne.S.cl (K) = 1_N. Then, Ne.S.int (1_N) ∩ Ne.S.int (Ne.S.cl (L))= 0_N. That is, (1_N) ∩ Ne.S.int (Ne.S.cl (L)) = 0_N. Hence Ne.S.int (Ne.S.cl (L)) = 0_N, which means that L is a Ne.S.N.D. set in (X, N_T).

3. Neutrosophic Semi Baire Spaces

Definition 3.1. Let (X, N_T) be a Ne.T.S. A Ne. Set K in (X, N_T) is called Neutrosophic semi first category(Ne.S.F.C.) if A = $\bigcup_{i=1}^{\infty} A_i$ where A_i's are Ne.S.N.D. sets in (X, N_T). Any other Ne. set in (X, N_T) is said to be of Neutrosophic semi second category(Ne.S.S.C.).

Example 3.1: Let $X = \{k, l\}$. Define the Ne. set K, L ,M and N on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right)\right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.6}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right)\right\rangle$$

$$M = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.5}\right)\right\rangle$$

$$N = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.7}\right)\right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L\}$ is Ne.T. on X. Thus (X, N_T) is a Ne.T.S.. Now the sets $\overline{K}, \overline{L}, M, N$ are Ne.S.N.D. set and $\left[\overline{K} \cup \overline{L} \cup M \cup N\right] = \overline{L}$ is Ne.S.F.C. set in (X, N_T)

Definition 3.2: Let K be a Ne.S.F.C. set in a Ne..S. (X, N_T). Then 1 - K is called a Neutrosophic semi-residual (Ne.S.R.) set in (X, N_T).

Proposition 3.1: If K is a Ne.S.F.C. set in a Ne.T.S. (X, N_T), then 1–K = $\bigcap_{i=1}^{\infty} K_i$, where Ne.S.cl(L_i) = 1_N.

Proof: Let K be a Ne.S.F.C. set in (X, N_T). Then we have K = $\bigcup_{i=1}^{\infty} K_i$), where K_i 's are Ne.S.N.D. in (X, N_T). Now 1– K = $\bigcap_{i=1}^{\infty} K_i$. Let L_i = 1 – K_i. Then 1-K = $\bigcap_{i=1}^{\infty} L_i$. Since K_i's are Ne.S.N.D. sets in (X, N_T), by proposition 2.10, we have 1-K 's are Ne.S.D. sets in (X, N_T). Hence Ne.S.cl (L_i) = Ne.S.cl (1- K_i) =1_N. Therefore we have 1-K = $\bigcap_{i=1}^{\infty} L_i$ where Ne.S.cl (L_i) = 1_N.

Definition 3.3: A Ne.T.S. (X, N_T) is called a Ne.S.F.C. space if the Ne. set 1_N is a Ne.S.F.C. set in (X, N_T). That is, 1_N = $\bigcup_{i=1}^{\infty} K_i$ where K_i's are Ne.S.N.D. sets in (X, N_T). Otherwise (X, N_T) will be called a Ne.S.S.C. space.

Proposition 3.2: If K is a Ne.S.C. set in a Ne.T.S. (X, N_T) and if Ne.S.int (K) = 0_N, then K is a NeS.N.D. set in (X, N_T).

Proof: Let K be a Ne.S.C. set in (X, N_T). Then we have Ne.S.cl (K) = K. Now Ne.S.int (Ne.S.cl (K) = Ne.S.int (K) and Ne.S.int(K) = 0_N, implies that Ne.S.int(Ne.S.cl(K))= 0_N. Hence K is a Ne.S.N.D. set in (X, N_T).

Definition 3.4: Let (X, N_T) be a Ne.T.S.. Then (X, N_T) is called a Neutrosophic semi-Baire space(Ne.S.B.) if Ne.S.int [$\bigcup_{i=1}^{\infty} K_i$] = 0_N, where K_i's are Ne.S.N.D. sets in (X, N_T).

Example 3.2: Let *X = {k, l}*. Define the Ne. set k, L ,M and N on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.6}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right) \right\rangle$$

$$M = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.5}\right) \right\rangle$$

$$N = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.7}\right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L\}$ is Ne.T. on X. Thus (X, N_T) is a Ne.T.S.. Now the sets $\overline{K}, \overline{L}, M, N$ are Ne.S.N.D. set and $\left[\overline{K} \cup \overline{L} \cup M \cup N\right]$=Ne.S.int ($\overline{L}$) = 0_N is Ne.S.B. space.

Example 3.3: Let *X = {k, l}*. Define the Ne.Sets K, L and M on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.6}\right), \left(\frac{k}{0.5}, \frac{l}{0.2}\right), \left(\frac{k}{0.4}, \frac{l}{0.5}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.2}, \frac{l}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.1}\right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L, K \cup L, K \cap L\}$ is Ne.T on X. Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, \overline{K \cup L}$ are Ne.S.N.D set and Ne.S.int $(\overline{K} \cup \overline{L} \cup \overline{(K \cup L)}) = Ne.S.\text{int}(\overline{K \cap L}) \neq 0_N$. Hence the Ne.T.S. (X, N_T) is not Ne.S.B. space.

Proposition 3.3:` If Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$), = 0_N, where Ne.S.int (K_i) = 0_N and K_i 's are Ne.S.C. sets in a Ne.T.S. (X, N_T), then (X, N_T) is a Ne.S.B. space.

Proof: Let K_i 's be Ne.S.C. sets in (X, N_T). Since Ne.S.int (K_i) = 0_N, by proposition 3.2, the K_i 's are Ne.S.N.D. sets in (X, N_T). Therefore we have Ne.S.int ($\bigcup_{i=1}^{\infty}(K_i)$) = 0_N, where K_i 's are fuzzy semi-nowhere dense sets in (X, N_T). Hence (X, N_T) is a Ne.S.B. space.

Proposition 3.4:

If Ne.S.cl($\bigcap_{i=1}^{\infty}(K_i$)) = 1_N, where K_i's are Ne.S.D. and Ne.S.O. sets in a Ne.T.S. (X, N_T), then (X, N_T) is a Ne.S.B. space.

Proof:

Now Ne.S.cl ($\bigcap_{i=1}^{\infty}(K_i)$) = 1_N implies that 1-Ne.S.cl ($\bigcap_{i=1}^{\infty}(K_i)$) = 0_N. Then we have

Ne.S.int (1-$\bigcap_{i=1}^{\infty} K_i$) = 0_N,which implies that Ne.S.int ($\bigcup_{i=1}^{\infty} 1 - K_i$)= 0_N. Since K_i's are Ne.S.D. sets in (X, N_T), Ne.S.cl (K_i) = 1_N and Ne.S.int(1- K_i) = 1-Ne.S.cl (K_i) = 1-1_N = 0_N. Hence we have Ne.S.int ($\bigcup_{i=1}^{\infty}(1-K_i)$) = 0_N, where Ne.S.int (1- K_i) = 0 and (1- K_i)'s are Ne.S.C. sets in (X, N_T). Then, by proposition 3.3, (X, N_T) is a Ne.S.B. space.

Proposition 3.5: Let (X, N_T) be a Ne.T.S. The$\bigcup_{i=1}^{\infty} K_i$ n the following are equivalent:

(1). (X, N_T) is a Ne.S.B. space.

(2). Ne.S.int (K) = 0_N for everyone.S.F.C. set K in (X, N_T).

(3). Ne.S.cl (L) = 1_N for every Ne.S.R. set in (X, N_T).

Proof: (1) → (2). Let K be a Ne.S.F.C. set in (X, N_T). Then K =$\bigcup_{i=1}^{\infty} K_i$, where K_i's are Ne.S.N.D. sets in (X, N_T). Now Ne.S.int (K) = Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = 0_N (since (X, N_T) is a Ne.S.B. space). Therefore

Ne.S.int (K) = 0_N.

(2) → (3). Let L be a Ne.S.R. set in (X, N_T). Then 1-L is a Ne.S.F.C set in (X, N_T). By hypothesis, Ne.S.int (1-L) = 0_N which implies that 1- Ne.S.cl (L) = 0_N.

Hence we have Ne.S.cl (L) = 1_N.

(3)→ (1). Let K be a Ne.S.F.C.set in (X, N_T). Then K = $\bigcup_{i=1}^{\infty} K_i$ where K_i's are Ne.S.N.D.sets in (X, N_T). 1-K is a Ne.S.R. set in (X, N_T). Since K is a Ne.S.F.C. set in (X, N_T), By hypothesis, we have Ne.S.cl (1-

K) = 1_N. Then 1-Ne.S.int (K) = 1_N, which implies that Ne.S.int (K) = 0_N. Hence Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = 0_N where K_i's are Ne.S.N.D. sets in (X, N_T). Hence (X, N_T) is a Ne.S.B. space.

Proposition 3.6: If a fuzzy topological space (X, N_T) is a Ne.S.B. space, then (X, N_T) is a Ne.S.S.C.space.

Proof: Let (X, N_T) be a Ne.S.B. space. Then Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = 0_N where K_i's are Ne.S.N.D. sets in (X, N_T). Then $\bigcup_{i=1}^{\infty} K_i \neq 1_N$. (Suppose, $\bigcup_{i=1}^{\infty} K_i = 1_N$ implies that Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = Ne.S.int($1_N$) which implies that $0_N = 1_N$, a contradiction). Hence (X, N_T) is a Ne.S.S.C. space.

Remarks 3.6: The converse of the above proposition need not be true. A Ne.S.S.C. space need not be Ne.S.B. space.

Example 3.4: Let $X = \{k, l\}$. Define the Ne.Sets K and L on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.6}\right), \left(\frac{k}{0.5}, \frac{l}{0.2}\right), \left(\frac{k}{0.4}, \frac{l}{0.5}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.2}, \frac{l}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.3}\right), \left(\frac{k}{0.7}, \frac{l}{0.1}\right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L, K \cup L, K \cap L\}$ is Ne.T on X. Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, \overline{K \cup L}$ are Ne.S.N.D set and $(\overline{K} \cup \overline{L} \cup \overline{(K \cup L)}) = \overline{(K \cap L)} \neq 1_N$ & $Ne.S.\text{int}\overline{(K \cap L)} \neq 0_N$.

Hence the Ne.S.S.C. space need not be Ne.S.B.space.

Proposition 3.7: If a Ne.T.S. (X, N_T) is a Ne.S.B. space, then no non-zero Ne.S.O. set in (X, N_T) is a fuzzy semi-first category set in (X, N_T).

Proof: Suppose that K is a non-zero Ne.S.O. set in (X, N_T) such that K = $\bigcup_{i=1}^{\infty} K_i$, where K_i 's are Ne.S.N.D. sets in (X, N_T). Then we have Ne.S.int (K) = Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$). Since K is a non-zero Ne.S.O. set in (X, N_T) Ne.S.int(K) = K. Then Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = K ≠ 0. But this is a contradiction to (X, N_T) being a Ne.S.B. space, in which Ne.S.int ($\bigcup_{i=1}^{\infty} K_i$) = 0, where K_i 's are Ne.S.N.D. sets in (X, N_T). Hence we must have A ≠ ($\bigcup_{i=1}^{\infty} K_i$).

Therefore no non-zero Ne.S.O. set in (X, N_T) is a Ne.S.F.C. set in (X, N_T).

Proposition 3.8: A Ne.S.B. space is a Ne.B. space. For consider the following example:

Example 3.5: Let $X = \{k, l\}$. Define the Ne. set K, L ,M and N on X as follows:

$$K = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3}\right) \right\rangle$$

$$L = \left\langle x, \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.5}\right), \left(\frac{k}{0.6}, \frac{l}{0.6}, \frac{m}{0.6}\right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4}\right) \right\rangle$$

$$M = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4} \right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.4} \right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.5} \right) \right\rangle$$

$$N = \left\langle x, \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3} \right), \left(\frac{k}{0.3}, \frac{l}{0.3}, \frac{m}{0.3} \right), \left(\frac{k}{0.7}, \frac{l}{0.7}, \frac{m}{0.7} \right) \right\rangle$$

Then the families $N_T = \{0_N, 1_N, K, L\}$ is Ne.T. on X. Thus (X, N_T) is a Ne.T.S. Now the sets $\overline{K}, \overline{L}, M, N$ are Ne.S.N.D. set and Ne.S.int$\left[\overline{K} \cup \overline{L} \cup M \cup N\right]$= Ne.S.int($\overline{L}$)= 0_N Hence the Ne.T.S. (X, N_T) is Ne.S.B. space.

Here the sets $\overline{K}, \overline{L}, M, N$ are Ne.N.D. set and Ne.int$\left[\overline{K} \cup \overline{L} \cup M \cup N\right]$= Ne.int($\overline{L}$)= 0_N .Hence Ne.S.B. space is a Ne.B. space

Conclusions

Many different forms of closed sets have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, : In this paper, we introduced the concept of Neutrosophic Semi Baire spaces in Neutrosophic Topological Spaces. Also we define Neutrosophic Semi-nowhere dense, Neutrosophic Semi-first category and Neutrosophic Semi-second category sets. Some of its characterizations of Neutrosophic Semi-Baire spaces are also studied. This shall be extended in the future Research with some applications

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Conflicts of Interest

The authors declare no conflict of interest.

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