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Decision-making techniques based on similarity measures of possibility neutrosophic soft expert sets

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Abstract. A neutrosophic set (NS) is a novel computing technique that accesses uncertain information by using three memberships. The main goal of this study is to come up with a novel approach called the "possibility neutrosophic soft expert set" (PNSE-set), which is based on the idea that each element of the universe of discourse has a certain level of possibility. Based on this new approach, the set-theoretical operations on PNSE-set (i.e complement, subset, equality, union, intersection, DeMorgans laws, AND-product, and OR-product operations) are introduced, along with illustrative examples and relevant laws. A generalized algorithm is proposed and applied to decision-making problems. Meanwhile, a similarity measure of two PNSE-sets is offered, and it's tested in real-life applications involving medical diagnosis applications. Finally, this work is supported by a comparative analysis of three recent methods.

Keywords: neutrosophic set; similarity measure; decision making; possibility neutrosophic soft expert sets.

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1. Introduction

With the rapid development that our world is witnessing in all areas of our daily life, we face several practical problems that include uncertain, inconsistent, and incomplete information, and this requires a new and effective mathematical tools to deal with problems. Smarandache managed to overcome the weaknesses that appeared in both [3] and [4] by establishing an idea of a neutrosophic set (NS). An NS is considered a more comprehensive mathematical tool for human thinking, as it covers the aspects of right and wrong and the indeterminacy between them through its mathematical structure, which contains three functions, namely $T(u)$ true function, $I(u)$ indeterminacy function, and $F(u)$ falsity function, such that image of all of them belong $]^{-}0, {}^{+}1[$

However, NS and its extensions [5], [6] have their own intrinsic difficulties and weaknesses in precisely expressing their preferences. To overcome this drawback, Molodtsov [7] established a new parameterization tool named soft set (SS). After Molodtsov, a lot of researchers combined the SS with the NS and its extension; for instance,Maji [8] introduced a neutrosophic soft set (NSS), which can be considered a new track of thinking that opens the horizons for researchers in engineering, computer science, and others.Peng [9] proposed similarity measures on neutrosophic soft sets to measure level soft sets based on some algorithms. Broumi [10] tested the notions of relation between NS-sets in decision-making applications. Some techniques of MAGDM and MADM tested on neutrosophic environment by [11]- [13]. Naeem et al. [14]- [17] discussed fuzzy, soft, and m-polar neutrosophic environments with decision-making. Al-Sharqi et al. [18]- [22] merged all of NS and SS into a complex environment and applied it in some real-life applications. In addition, researchers have applied these mathematical tools in various fields [23]- [32]. Alkhazaleh pointed out all these theories have their own shortcomings. One of these shortcomings is the soft set's inability to absorb users' opinions (experts) simultaneously. To overcome these difficulties, Alkhazaleh et al. [33] created a new technique for modelling uncertainty called a "soft expert set" (SES) based on the merged concept of a "soft set" with an expert system. This approach has now been applied in many fields, such as intelligent systems, game theory, measurement theory, cybernetics, probability theory, and so on. Research on SES is progressing rapidly up to now. This concept has been studied and combined with fuzzy set theory and its extensions by researchers. Alkhazaleh et al. were the first to introduce the model fuzzy-ESSs [34] and neutrosophic-SESs [35]. Alhazaymeh and Hassan merged a soft expert set with a vague set and gave some new hybrid notions [36].Ihsan et al. [37] have developed m-polar fuzzy SESs with the same properties. Hassan et al. [38], [39] demonstrated the properties of the Q-NSE-set. Pramanik et al. [40]compiend SNS and SES and they proposed the idea SNSES. Subsequently, more general properties and applications of soft expert set theory have been investigated by Hassan and others, for instance, see [41]- [44]. From a scientific point of view, an element's probability degree will significantly influence modelling some applications under multiple attribute decision-making problems. Therefore, several researchers studied this idea in fuzzy set theory and its extinctions. For instance, Alkhazaleh et al. [45]first established the possibility setting on fuzzy soft sets and defined similarity measures for two possibility fuzzy soft sets. Alhazaymeh and Hassan then presented the concepts of possibility vague soft set [46] and possibility interval-valued vague soft set [47]. Al-Quran and Hassan [48] proposed the possibility neutrosophic vague soft set and employed it in medical diagnosis applications. Karaaslan [49] suggested the theory of possibility of NSSs as an extension of [50] and illustrated its application in decision-making. Selvachandran and Salleh [51] established the idea of possibility intuitionistic fuzzy-SESs by a develops the structures in [45]- [47]. But there are some limitations in [49]- [51]. In the first one can only be used by one user, while more than one user can use the second, but it lacks an important tool, which is the indefiniteness found in NS. To overcome these limitations, we will organize in this work a new hybrid concept called possibility neutrosophic soft expert sets (PNSE-sets) by assigning a possibility degree to each approximate member of an NSE-set. This model keeps the advantages of the SESs by allowing users to understand the experts' opinions without the requirement for further operations. Also, Similarity measures [52], [53] are layered extensively in the fuzzy environment . Therefore, based on this model, we define the measure of similarity between two PNSE-sets and show how this measure can be used in medical diagnosis.

This article is divided into eight parts, which are as follows: we review some important definitions and properties in Section 2. The general framework of the proposed concept, some properties, and numerical examples in Section 3.. Then, in section 4, we present basic operations on the PNSE-set together with some propositions and numerical examples. Some applications in decision-making are solved by PNSE-setting in Section 5. In Section 6, we define the similarity measure between two PNSE-sets and show the importance of this measure by one application in medical diagnosis. Finally, Section 7 contains a brief comparison between PNSE-set and some other methods to show the reader the importance of this work. In addition, conclusions of this work showed in Section 8.

2. Preliminaries

In this part, we give the most important definitions and properties of $[1, 7]$ a that will be used in later parts of this work.

Definition 2.1. Neutrosophic Set (N-set) $[1, 2]$ An N-set $\ddot{\mathfrak{N}}$ is characterized by $\ddot{\mathfrak{N}} = \left\{\left\langle v, \dot{\mathfrak{T}}_{\ddot{\mathfrak{N}}}(v), \dot{\mathfrak{I}}_{\ddot{\mathfrak{N}}}(v), \dot{\mathfrak{F}}_{\ddot{\mathfrak{N}}}(v), \forall v \in \mathfrak{V}\right\rangle\right\}$ such that $\dot{\mathfrak{T}}_{\ddot{\mathfrak{N}}}(v), \dot{\mathfrak{I}}_{\ddot{\mathfrak{N}}}(v), \dot{\mathfrak{F}}_{\ddot{\mathfrak{N}}}(v) : \mathfrak{V} \rightarrow [0, 1]$ are real-valued truth-membership, indeterminacy-membership, and non-membership, respectively

Definition 2.2. (Properties of N-set) [1,2] If $\ddot{\mathfrak{N}}$ and $\ddot{\mathfrak{M}}$ are two N-sets on $\mathfrak{V}$ then for $v \in \mathfrak{V}$, we have:

(i) $\ddot{\mathfrak{N}} \subseteq \ddot{\mathfrak{M}}$ if $\dot{\mathfrak{T}}_{\ddot{\mathfrak{N}}}(v) \leq \dot{\mathfrak{T}}_{\ddot{\mathfrak{M}}}(v), \dot{\mathfrak{I}}_{\ddot{\mathfrak{N}}}(v) \geq \dot{\mathfrak{I}}_{\ddot{\mathfrak{M}}}(v)$ and $\dot{\mathfrak{F}}_{\ddot{\mathfrak{N}}}(v) \geq \dot{\mathfrak{F}}_{\ddot{\mathfrak{M}}}(v)$ for all $v \in \mathfrak{V}$.

(ii) $\ddot{\mathfrak{N}}^c = \left\{\left\langle v, \dot{\mathfrak{T}}_{\ddot{\mathfrak{N}}^c}(v), \dot{\mathfrak{I}}_{\ddot{\mathfrak{N}}^c}(v), \dot{\mathfrak{F}}_{\ddot{\mathfrak{N}}^c}(v)\right\rangle\right\} = \left\{\left\langle v, \dot{\mathfrak{F}}_{\ddot{\mathfrak{N}}}(v), 1 - \dot{\mathfrak{I}}_{\ddot{\mathfrak{N}}}(v), \dot{\mathfrak{T}}_{\ddot{\mathfrak{N}}}(v)\right\rangle\right\}$.

(iii) If $\ddot{\mathfrak{N}} \cup (\cap) \ddot{\mathfrak{M}} = \ddot{\mathfrak{D}}$ and defined as follows
$\ddot{\mathfrak{D}} = \left\{\left\langle v, \dot{\mathfrak{T}}_{\ddot{\mathfrak{D}}}(v), \dot{}_{\ddot{\mathfrak{D}}}(v), \dot{\mathfrak{F}}_{\ddot{\mathfrak{D}}}(v)\right\rangle\right\}$

where
$\dot{\mathfrak{T}}_{\ddot{\mathfrak{D}}}(v) = max\,(min)\left[\dot{\mathfrak{T}}_{\ddot{\mathfrak{N}}}(v), \dot{\mathfrak{T}}_{\ddot{\mathfrak{M}}}(v)\right],$
$\dot{\mathfrak{I}}_{\ddot{\mathfrak{D}}}(v) = min\,(max)\left[\dot{\mathfrak{I}}_{\ddot{\mathfrak{N}}}(v), \ddot{\mathfrak{I}}_{\ddot{\mathfrak{M}}}(v)\right],$
$\dot{\mathfrak{F}}_{\ddot{\mathfrak{D}}}(v) = min\,(max)\left[\mathfrak{F}_{\ddot{\mathfrak{N}}}(v), \mathfrak{F}_{\ddot{\mathfrak{M}}}(v)\right].$

Definition 2.3. Soft Set (SS) [7] A pair $\left(\tilde{\mathfrak{F}}, \mathfrak{E}\right)$ is a SS on fixed set $\mathfrak{V}$, where $\tilde{\mathfrak{F}} : \mathfrak{E} \to \mathfrak{P}\left(\mathfrak{V}\right)$ such that $\mathfrak{A}$ is a subset of attributes set $\mathfrak{E}$.

3. Possibility Neutrosophic Soft Expert Sets(PNSE-set)

In the current section, we will establish the main definition of possibility neutrosophic soft expert sets (PNSE-sets) and the elementary properties of PNSE-sets are conceptualized with some numerical examples.

Definition 3.1. The pair$(\mathfrak{F}_\mu, \mathfrak{Z})$is called the possibility neutrosophic soft expert set (PNSE-set) over a nonempty soft universe $(\mathfrak{V}, \mathfrak{Z})$ if

$$\mathfrak{F}_\mu : A \to N^V \times I^V$$

defined by

$$\mathfrak{F}_\mu\left(z_i\right) = \left\{\mathfrak{F}\left(z_i\right)\left(v_n\right), \mu\left(z_i\right)\left(v_n\right)\right\}$$

with

$$\mathfrak{F}\left(z_i\right)\left(v_n\right) = \left\langle\rho\left(z_i\right)\left(v_n\right), \eta\left(z_i\right)\left(v_n\right), \psi\left(z_i\right)\left(v_n\right)\right\rangle \forall z_i \in \mathfrak{P} \subseteq Z, v_n \in V.$$

Where,

(1) For $\mathfrak{V} = \{v_1, v_2, v_3, ..., v_n\}$be a non-empty initial universe, $\mathfrak{P} = \{p_1, p_2, p_3, ..., p_j\}$be a parameters set, $\mathfrak{M} = \{m_1, m_2, m_3, ..., m_k\}$be a set of experts, $\mathfrak{O} = \{1 = agree, 0 = disagree\}$be a set of opinions, and $\mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{O}\}$.
(2) $\Im : \mathfrak{Z} \to N^{\mathfrak{V}}$and $\mu : \mathfrak{Z} \to I^{\mathfrak{V}}$,$N^{\mathfrak{V}}$ and $I^{\mathfrak{V}}$ indicates the collection of all neutrosophic and fuzzy subset of $\mathfrak{V}$ respectively.
(3) $\mathfrak{F}\left(z\right)\left(v_n\right)$ is the degree of neutrosophic membership of $v \in \mathfrak{V}$ in $\mathfrak{F}(z)$, i.e$(\rho\left(z\right)\left(v_n\right), \eta\left(z\right)\left(v_n\right), \psi\left(z\right)\left(v_n\right))$ denotes to three neutrosophic memberships receptively.
(4) $\mu\left(z\right)\left(v_n\right)$ is a degree of possibility membership of $v \in \mathfrak{V}$ in $\mathfrak{F}(z)$.

so $\mathfrak{F}_\mu\left(z_i\right)$ can be written as below:
$\left\{\left(\frac{v_1}{F(z)(v_1)}, \mu\left(z\right)\left(v_1\right)\right), \left(\frac{v_2}{F(z)(v_2)}, \mu\left(z\right)\left(v_2\right)\right), \left(\frac{v_3}{F(z)(v_3)}, \mu\left(z\right)\left(v_3\right)\right) ..., \left(\frac{v_n}{F(z)(v_n)}, \mu\left(z\right)\left(v_n\right)\right)\right\}$
for $i = 1, 2, 3, ..., n$

Remark 3.2.

1. If we have $\mathfrak{P} \subseteq \mathfrak{Z}$ it is also possible to write a PNSE-set as $(\mathfrak{F}_\mu, \mathfrak{P})$ and to essay way the PNSE-set can be write as $\mathfrak{F}_\mu$.
2. Here in this work, we suppose that the set of opinions consists of only two values (i,e agree and disagree), but it is possible to include other options that match the nature of the problem.

Example 3.3. Let $\mathfrak{V} = \{v_1, v_2, v_3\}$be the universal set of elements, let $\mathfrak{P} = \{p_1, p_2\}$be a parameters set, whee p_1 =cheap, p_2 =beautiful and let $\mathfrak{M} = \{m_1, m_2\}$ be a set containing two experts. Assume that $\mathfrak{F}_\mu : A \to N^V \times I^V$ is a function represented as follows:

$$\mathfrak{F}_\mu (p_1, m_1, 1) = \left\{ \left(\frac{v_1}{\langle 0.5,0.3,0.1 \rangle}, 0.2 \right), \left(\frac{v_2}{\langle 0.4,0.2,0.7 \rangle}, 0.3 \right), \left(\frac{v_3}{\langle 0.6,0.1,0.6 \rangle}, 0.5 \right) \right\}$$
$$\mathfrak{F}_\mu (p_2, m_1, 1) = \left\{ \left(\frac{v_1}{\langle 0.6,0.3,0 \rangle}, 0.5 \right), \left(\frac{v_2}{\langle 0.1,0.3,0.9 \rangle}, 0.7 \right), \left(\frac{v_3}{\langle 0.9,0.4,0.2 \rangle}, 0.6 \right) \right\}$$
$$\mathfrak{F}_\mu (p_1, m_2, 1) = \left\{ \left(\frac{v_1}{\langle 0.3,0.4,0.6 \rangle}, 0.1 \right), \left(\frac{v_2}{\langle 0.4,0.5,0.3 \rangle}, 0.5 \right), \left(\frac{v_3}{\langle 0.2,0.4,0.4 \rangle}, 0.8 \right) \right\}$$
$$\mathfrak{F}_\mu (p_2, m_2, 1) = \left\{ \left(\frac{v_1}{\langle 0.1,0.1,0.4 \rangle}, 0.3 \right), \left(\frac{v_2}{\langle 0.6,0.2,0.4 \rangle}, 0.8 \right), \left(\frac{v_3}{\langle 0.3,0.2,0.5 \rangle}, 0.6 \right) \right\}$$
$$\mathfrak{F}_\mu (p_1, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.2,0.8,0.3 \rangle}, 0.5 \right), \left(\frac{v_2}{\langle 0.3,0.4,0.2 \rangle}, 0.5 \right), \left(\frac{v_3}{\langle 0.3,0.2,0.6 \rangle}, 0.8 \right) \right\}$$
$$\mathfrak{F}_\mu (p_2, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.4,0.9,2 \rangle}, 0.9 \right), \left(\frac{v_2}{\langle 0.4,0.3,0.2 \rangle}, 0.7 \right), \left(\frac{v_3}{\langle 0.3,0.4,0.7 \rangle}, 0.2 \right) \right\}$$
$$\mathfrak{F}_\mu (p_1, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.4,0.3,0,3 \rangle}, 0.4 \right), \left(\frac{v_2}{\langle 0.2,0.6,0.6 \rangle}, 0.7 \right), \left(\frac{v_3}{\langle 0.7,0.3,0.5 \rangle}, 0.5 \right) \right\}$$
$$\mathfrak{F}_\mu (p_2, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.2,0.4,0.8 \rangle}, 0.3 \right), \left(\frac{v_2}{\langle 0.5,0.5,0.2 \rangle}, 0.9 \right), \left(\frac{v_3}{\langle 0.1,0.1,0.7 \rangle}, 0.1 \right) \right\}$$

Now, we can present PNSE-set $(\mathfrak{F}_\mu, \mathfrak{Z})$ as be formed of the following aggregate of approximations:

$$(\mathfrak{F}_\mu, \mathfrak{Z}) = \Big\{ (p_1, m_1, 1) = \left\{ \left(\frac{v_1}{\langle 0.5,0.3,0.1 \rangle}, 0.2 \right), \left(\frac{v_2}{\langle 0.4,0.2,0.7 \rangle}, 0.3 \right), \left(\frac{v_2}{\langle 0.6,0.1,0.6 \rangle}, 0.5 \right) \right\},$$

$$(p_2, m_1, 1) = \left\{ \left(\frac{v_1}{\langle 0.6,0.3,0 \rangle}, 0.5 \right), \left(\frac{v_2}{\langle 0.1,0.3,0.9 \rangle}, 0.7 \right), \left(\frac{v_3}{\langle 0.9,0.4,0.2 \rangle}, 0.6 \right) \right\},$$

$$(p_1, m_2, 1) = \left\{ \left(\frac{v_1}{\langle 0.3,0.4,0.6 \rangle}, 0.1 \right), \left(\frac{v_2}{\langle 0.4,0.5,0.3 \rangle}, 0.5 \right), \left(\frac{v_3}{\langle 0.2,0.4,0.4 \rangle}, 0.8 \right) \right\},$$

$$(p_2, m_2, 1) = \left\{ \left(\frac{v_1}{\langle 0.1,0.1,0.4 \rangle}, 0.3 \right), \left(\frac{v_2}{\langle 0.6,0.2,0.4 \rangle}, 0.8 \right), \left(\frac{v_3}{\langle 0.3,0.2,0.5 \rangle}, 0.6 \right) \right\},$$

$$(p_1, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.2,0.8,0.3 \rangle}, 0.5 \right), \left(\frac{v_2}{\langle 0.3,0.4,0.2 \rangle}, 0.5 \right), \left(\frac{v_3}{\langle 0.3,0.2,0.6 \rangle}, 0.8 \right) \right\},$$

$(p_2, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.4,0.9,0.2 \rangle}, 0.9 \right), \left(\frac{v_2}{\langle 0.4,0.3,0.2 \rangle}, 0.7 \right), \left(\frac{v_3}{\langle 0.3,0.4,0.7 \rangle}, 0.2 \right) \right\},$

$(p_1, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.4,0.3,0.3 \rangle}, 0.4 \right), \left(\frac{v_2}{\langle 0.2,0.6,0.6 \rangle}, 0.7 \right), \left(\frac{v_3}{\langle 0.7,0.3,0.5 \rangle}, 0.5 \right) \right\},$

$(p_2, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.2,0.4,0.8 \rangle}, 0.3 \right), \left(\frac{v_2}{\langle 0.5,0.5,0.2 \rangle}, 0.9 \right), \left(\frac{v_3}{\langle 0.1,0.1,0.7 \rangle}, 0.1 \right) \right\} \}$

Then we say that $(\mathfrak{F}_\mu, \mathfrak{Z})$ is a is said to be possibility neutrosophic soft expert set (PNSE-set) over soft universe $(\mathfrak{V}, \mathfrak{Z})$

Definition 3.4. For two PNSE-sets $(\mathfrak{F}_\mu, A)$ and $(\mathfrak{G}_\varphi, B)$ over $(\mathfrak{V}, \mathfrak{Z})$.Then $(\mathfrak{F}_\mu, A)$ is said to be be a PNSE-subset of $(\mathfrak{G}_\varphi, B)$ if $A \subseteq B$, and $\forall z \in A \subseteq \mathfrak{Z}$ the next conditions are fulfilled:
1. $\mu(z)$ is fuzzy subset of $\varphi(z)$.
2. $\mathfrak{F}_\mu(z)$ is neutrosophic subset of $\mathfrak{G}_\varphi(z)$.
And we denoted this relation as $(\mathfrak{F}_\mu, A) \subseteq (\mathfrak{G}_\varphi, B)$.In this issue, $(\mathfrak{G}_\varphi, B)$ is named a PNSE-superset of $(\mathfrak{F}_\mu, A)$.

Definition 3.5. If $(\mathfrak{F}_\mu, A)$ and $(\mathfrak{G}_\varphi, B)$ be two PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$.Then $(\mathfrak{F}_\mu, A)$ is equal to $(\mathfrak{G}_\varphi, B)$ if $\forall z \in A \subseteq \mathfrak{Z}$ the next conditions are fulfilled:
1. $\mu(z)$ is equal of $\varphi(z)$.
2. $\mathfrak{F}_\mu(z)$ is equal of $\mathfrak{G}_\varphi(z)$.
And we denoted this relation as $(\mathfrak{F}_\mu, A) = (\mathfrak{G}_\varphi, B)$.In this words, $(\mathfrak{G}_\varphi, B)$ is equal of $(\mathfrak{F}_\mu, A)$ if $(\mathfrak{G}_\varphi, B)$ is PNSE-subset of $(\mathfrak{F}_\mu, A)$ and $(\mathfrak{F}_\mu, A)$ is PNSE-subset of $(\mathfrak{G}_\varphi, B)$.

Definition 3.6. A PNSE-set $(\mathfrak{F}_\mu, A)$ is named null-PNSE-set, indicated by $\left(\ddot{\Phi}_\mu, A\right)$ and given as follows

$$\ddot{\Phi}_\mu(z_i) = \{\mathfrak{F}(z_i)(v_n), \mu(z_i)(v_n)\}, \forall z_i \in A \subseteq \mathfrak{Z}$$

where $\mathfrak{F}(z_i)(v_n) = \langle 0, 1, 1 \rangle$ such that $\forall z_i \in A \subseteq \mathfrak{Z}, v \in \mathfrak{V}$ we have $\rho(z_i)(v_n) = 0$,$\eta(z_i)(v_n) = 1$,$\psi(z_i)(v_n) = 1$ and $\mu(z_i)(v_n) = 0$.

Definition 3.7. A PNSE-set $(\mathfrak{F}_\mu, A)$ is named to be absolute-PNSE-set, indicate by $(\mathfrak{F}_\mu, A)_{Abso}$ and given as follows

$$\mathfrak{F}_\mu(z_i) = \{\mathfrak{F}(z_i)(v_n), \mu(z_i)(v_n)\}, \forall z_i \in A \subseteq \mathfrak{Z}$$

where $\mathfrak{F}(z_i)(v_n) = \langle 1, 0, 0 \rangle$ such that $\forall z_i \in A \subseteq \mathfrak{Z}, v \in \mathfrak{V}$ we have $\rho(z_i)(v_n) = 1$,$\eta(z_i)(v_n) = 0$,$\psi(z_i)(v_n) = 0$ and $\mu(z_i)(v_n) = 1$.

Definition 3.8. Let $(\mathfrak{F}_\mu, A)$ be a PNSE-set over $(\mathfrak{V}, \mathfrak{Z})$. Then an agree-PNSE-set over non-empty universal $\mathfrak{V}$ denoted $(\mathfrak{F}_\mu, A)_1$ is a PNSE-subset of $(\mathfrak{V}, \mathfrak{Z})$ and its given as follows:

$$\mathfrak{F}_\mu(z_i)_1 = \{\mathfrak{F}(z_i)(v_n), \mu(z_i)(v_n)\}, \forall z_i \in A \subseteq \mathfrak{Z} = \mathfrak{P} \times \mathfrak{M} \times 1.$$

Definition 3.9. Let $(\mathfrak{F}_\mu, A)$ be a PNSE-set over $(\mathfrak{V}, \mathfrak{Z})$. Then a disagree-PNSE-set over non-empty universal $\mathfrak{V}$ denoted $(\mathfrak{F}_\mu, A)_0$ is a PNSE-subset of $(\mathfrak{V}, \mathfrak{Z})$ and its given as follows:

$$\mathfrak{F}_\mu (z_i)_0 = \{\mathfrak{F} (z_i) (v_n) , \mu (z_i) (v_n)\}, \forall z_i \in A \subseteq \mathfrak{Z} = \mathfrak{P} \times \mathfrak{M} \times 0.$$

4. Fundamental set theoretic operations of PNSE-set

In the next part, we offer some fundamental mathematical operations on PNSE-set, namely complement on one set of PNSE-set, union, and intersection on two or more sets of PNSE-set, followed by AND, OR operations on two or more sets of PNSE-set. Finally, we offer some properties related to these operations with suitable examples.

Definition 4.1. Let $\left(\mathfrak{F}_\mu, \dot{A}\right)$ be a PNSE-set over fixed set (soft universe) $(\mathfrak{V}, \mathfrak{Z})$. Then the complement of a PNSE-set $\left(\mathfrak{F}_\mu, \dot{A}\right)$ indicated by $\left(\mathfrak{F}_\mu, \dot{A}\right)^c$ is given as follows:

$$\left(\mathfrak{F}_\mu, \dot{A}\right)^c = \mathfrak{F}^c_\mu (z_i) = \{\ddot{c}(\mathfrak{F} (z) (v_n)), \dot{c}(\mu (z) (v_n))\}$$

where $\ddot{c}$ indicates a neutrosophic complement and $\dot{c}$ indicates a fuzzy complement.

Example 4.2. Take the part given in Example 3.3 where,
$\mathfrak{F}_\mu (p_1, m_1, 1)$
$= \left\{ \left(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\right), \left(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\right), \left(\frac{v_3}{\langle 0.6,0.1,0.6\rangle}, 0.8\right) \right\}$
Now, by employing the neutrosophic complement and fuzzy complement, we get the complement of the part that is given by $\mathfrak{F}^c_\mu (p_1, m_1, 1)$
$= \left\{ \left(\frac{v_1}{\langle 0.1,0.7,0.5\rangle}, 0.8\right), \left(\frac{v_2}{\langle 0.7,0.8,0.4\rangle}, 0.7\right), \left(\frac{v_3}{\langle 0.6,0.9,0.6\rangle}, 0.2\right) \right\}$

Proposition 4.3. *Let $\left(\mathfrak{F}_\mu, \dot{A}\right)$ be a PNSE-set over fixed set $(\mathfrak{V}, \mathfrak{Z})$. Then the following property applies:*

$$\left(\left(\mathfrak{F}_\mu, \dot{A}\right)^c\right)^c = \left(\mathfrak{F}_\mu, \dot{A}\right)$$

Proof. Assume that $\left(\mathfrak{F}_\mu, \dot{A}\right)$ be a PNSE-set over fixed set $(\mathfrak{V}, \mathfrak{Z})$ and defined as $\left(\mathfrak{F}_\mu, \dot{A}\right) = \mathfrak{F}_\mu(z_i) = (\mathfrak{F} (z_i) , \mu (z_i))$.
Now, let $\left(\mathfrak{F}_\mu, \dot{A}\right)^c = \left(\mathfrak{G}_\varphi, \dot{B}\right)$.
Then based on definition 4.1 $\left(\mathfrak{G}_\mu, \dot{B}\right) = \mathfrak{G}_\varphi(z_i) = (\mathfrak{G} (z_i) , \varphi (z_i))$.Such that $\mathfrak{G} (z_i) = \ddot{c}(\mathfrak{F} (z_i))$ and $\varphi(z_i) = \dot{c}(\mu(z_i))$. Thus it leads us to
$\left(\mathfrak{G}_\mu, \dot{B}\right)^c = \mathfrak{G}^c_\varphi(z_i) = (\ddot{c}(\mathfrak{G} (z_i)), \dot{c}(\varphi (z_i))) = (\ddot{c}(\ddot{c}(\mathfrak{F} (z_i))), \dot{c}(\dot{c}(\mu (z_i)))) = (\mathfrak{F} (z_i) , \mu (z_i)) = \left(\mathfrak{F}_\mu, \dot{A}\right)$.
Thus $\left(\left(\mathfrak{F}_\mu, \dot{A}\right)^c\right)^c = (\mathfrak{G}_\varphi, B)^c = \left(\mathfrak{F}_\mu, \dot{A}\right)$. Hence we get $\left(\left(\mathfrak{F}_\mu, \dot{A}\right)^c\right)^c = \left(\mathfrak{F}_\mu, \dot{A}\right)$. □

Definition 4.4. If $\left(\mathfrak{F}_{\mu}, \dot{A}\right)$ and $\left(\mathfrak{G}_{\varphi}, \dot{B}\right)$ two PNSE-sets on fixed set (soft universe) $(\mathfrak{V}, \mathfrak{Z})$. Then the union operation of these sets is also PNSE-set $\left(\mathfrak{H}_{\Psi}, \dot{C}\right)$ and denoted by $\left(\mathfrak{H}_{\Psi}, \dot{C}\right) = \left(\mathfrak{F}_{\mu}, \dot{A}\right) = \ddot{\cup}\,(\mathfrak{G}_{\varphi}, B)$. Where $\dot{C} = \dot{A} \cup \dot{B}$ and
$\Psi(z_i) = max(\mu(z_i), \varphi(z_i)), \qquad \forall z_i \in \dot{C} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}$.
$\mathfrak{H}(z_i) = \mathfrak{F}(z_i) \ddot{\cup} \mathfrak{G}(z_i), \qquad \forall z_i \in \dot{C} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}$.

where

$$\mathfrak{H}(z_i) = \begin{cases} \mathfrak{F}(z_i) & ,\text{if} z_i \in \dot{A} - \dot{B} \\ \mathfrak{G}(z_i) & ,\text{if} z_i \in \dot{B} - \dot{A} \\ max(\mathfrak{F}(z_i), \mathfrak{G}(z_i)) & ,\text{if} z_i \in \dot{A} \cap \dot{B} \end{cases}$$

Proposition 4.5. *Let* $\left(\mathfrak{F}_{\mu}, \dot{A}\right)$, $\left(\mathfrak{G}_{\varphi}, \dot{B}\right)$ *and* $\left(\mathfrak{H}_{\Psi}, \dot{C}\right)$ *be any three optional PNSE-sets over* $(\mathfrak{V}, \mathfrak{Z})$. *Then the following results are achieved:*

(i). $\left(\mathfrak{F}_{\mu}, \dot{A}\right) \ddot{\cup} \left(\mathfrak{G}_{\varphi}, \dot{B}\right) = \left(\mathfrak{G}_{\varphi}, \dot{B}\right) \ddot{\cup} \left(\mathfrak{F}_{\mu}, \dot{A}\right)$. *(Aommutative Condition)*
(ii) $\left(\mathfrak{F}_{\mu}, \dot{A}\right) \ddot{\cup} \left(\left(\mathfrak{G}_{\varphi}, \dot{B}\right) \ddot{\cup} \left(\mathfrak{H}_{\Psi}, \dot{C}\right)\right) = \left(\left(\mathfrak{F}_{\mu}, \dot{A}\right) \ddot{\cup} \left(\mathfrak{G}_{\varphi}, \dot{B}\right)\right) \ddot{\cup} \left(\mathfrak{H}_{\Psi}, \dot{C}\right)$. *(Associative Condition)*

Proof. Assume that $\left(\mathfrak{F}_{\mu}, \dot{A}\right) \ddot{\cup} \left(\mathfrak{G}_{\varphi}, \dot{B}\right) = \left(\mathfrak{H}_{\Psi}, \dot{C}\right)$.Then based on Definition 4.4, $\forall z_i \in \dot{C} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}$. we have

$$\left(\mathfrak{H}_{\Psi}, \dot{C}\right) = \mathfrak{H}_{\Psi}(z_i) = (\mathfrak{H}(z_i), \Psi(z_i))$$

where $\mathfrak{H}(z_i) = \mathfrak{F}(z_i) \ddot{\cup} \mathfrak{G}(z_i)$ and $\Psi(z_i) = max(\mu(z_i), \varphi(z_i))$. So,$\mathfrak{H}(z_i) = \mathfrak{F}(z_i) \ddot{\cup} \mathfrak{G}(z_i) = \mathfrak{G}(z_i) \ddot{\cup} \mathfrak{F}(z_i)$ and $\Psi(z_i) = max(\mu(z_i), \varphi(z_i)) = max(\varphi(z_i), \mu(z_i))$. we have the union of these sets is commutative by Definition 4.4.
Therefore,$\left(\mathfrak{H}_{\Psi}, \dot{C}\right) = \left(\mathfrak{G}_{\varphi}, \dot{B}\right) \ddot{\cup} \left(\mathfrak{F}_{\varphi}, \dot{A}\right)$.
Then we get the union of two PNSE-sets is commutative, such that $\left(\mathfrak{F}_{\mu}, \dot{A}\right) \ddot{\cup} \left(\mathfrak{G}_{\varphi}, \dot{B}\right) = \left(\mathfrak{G}_{\varphi}, \dot{B}\right) \ddot{\cup} \left(\mathfrak{F}_{\mu}, \dot{A}\right)$.
(ii) The proof of this part is equivalent to (i) and is therefore overlooked. □

Definition 4.6. If $\left(\mathfrak{F}_{\mu}, \dot{A}\right)$ and $\left(\mathfrak{G}_{\varphi}, \dot{B}\right)$ two PNSE-sets on fixed set (soft universe) $(\mathfrak{V}, \mathfrak{Z})$. Then the intersection operation of these sets is also PNSE-set $\left(\mathfrak{H}_{\Psi}, \dot{C}\right)$ and denoted by $\left(\mathfrak{H}_{\Psi}, \dot{C}\right) = \left(\mathfrak{F}_{\mu}, \dot{A}\right) = \ddot{\cap} \left(\mathfrak{G}_{\varphi}, \dot{B}\right)$. Where $\dot{C} = \dot{A} \cup \dot{B}$ and
$\Psi(z_i) = min(\mu(z_i), \varphi(z_i)), \qquad \forall z_i \in \dot{C} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}$.
$\mathfrak{H}(z_i) = \mathfrak{F}(z_i) \ddot{\cap} \mathfrak{G}(z_i), \qquad \forall z_i \in \dot{C} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}$.

where

$$\mathfrak{H}(z_i) = \begin{cases} \mathfrak{F}(z_i) & ,\text{if} z_i \in \dot{A} - dotB \\ \mathfrak{G}(z_i) & ,\text{if} z_i \in \dot{B} - \dot{A} \\ min(\mathfrak{F}(z_i), \mathfrak{G}(z_i)) & ,\text{if} z_i \in \dot{A} \cap \dot{B} \end{cases}$$

Proposition 4.7. *Let $\left(\mathfrak{F}_\mu, \dot{A}\right)$, $\left(\mathfrak{G}_\varphi, \dot{B}\right)$ and $\left(\mathfrak{H}_\Psi, \dot{A}\right)$ be any three optional PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$. Then the following results are achieved:*

(i). $\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cap} (\mathfrak{G}_\varphi, B) = \left(\mathfrak{G}_\varphi, \dot{B}\right) \ddot{\cap} \left(\mathfrak{F}_\mu, \dot{B}\right)$. *(Aommutative Condition)*
(ii) $\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cap} \left(\left(\mathfrak{G}_\varphi, \dot{B}\right) \ddot{\cap} \left(\mathfrak{H}_\Psi, \dot{C}\right)\right) = \left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cap} \left(\mathfrak{G}_\varphi, \dot{B}\right)\right) \ddot{\cap} \left(\mathfrak{H}_\Psi, \dot{C}\right)$. *(Associative Condition)*

Proof. The proof of these two parts (i, ii) is equivalent to (i, ii) in proposition 4. 5 and are and are overlooked. □

Proposition 4.8. *Let $\left(\mathfrak{F}_\mu, \dot{A}\right)$, $\left(\mathfrak{G}_\varphi, \dot{B}\right)$ and $\left(\mathfrak{H}_\Psi, \dot{C}\right)$ be any three optional PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$. Then the following results are satisfying:*
(i). $\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cup} \left(\left(\mathfrak{G}_\varphi, \dot{B}\right) \ddot{\cap} \left(\mathfrak{H}_\Psi, \dot{C}\right)\right) = \left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cup} \left(\mathfrak{G}_\varphi, \dot{B}\right)\right) \ddot{\cap} \left((\mathfrak{F}_\mu, A) \ddot{\cup} \left(\mathfrak{H}_\Psi, \dot{C}\right)\right)$
(ii). $\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cap} \left(\left(\mathfrak{G}_\varphi, \dot{B}\right) \ddot{\cup} \left(\mathfrak{H}_\Psi, \dot{C}\right)\right) = \left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cap} \left(\mathfrak{G}_\varphi, \dot{B}\right)\right) \ddot{\cup} \left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cap} \left(\mathfrak{H}_\Psi, \dot{C}\right)\right)$

Proof. The proof of these propositions clear dependency Definitions 4.4 and 4.6 and is therefore overlooked. □

Proposition 4.9. *Let $\left(\mathfrak{F}_\mu, \dot{A}\right)$ and $\left(\mathfrak{G}_\varphi, \dot{A}\right)$ be any two optional PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$. Then De Morgans laws satisfying:*
(i). $\left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cup} \left(\mathfrak{G}_\varphi, \dot{A}\right)\right)^c = \left(\left(\mathfrak{F}_\mu, \dot{A}\right)^c \ddot{\cap} \left(\mathfrak{G}_\varphi, \dot{A}\right)^c\right)$.
(ii). $\left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cap} \left(\mathfrak{G}_\varphi, \dot{A}\right)\right)^c = \left(\left(\mathfrak{F}_\mu, \dot{A}\right)^c \ddot{\cup} \left(\mathfrak{G}_\varphi, \dot{A}\right)^c\right)$.

Proof. (i) Assume that $\left(\mathfrak{F}_\mu, \dot{A}\right)$ and $\left(\mathfrak{G}_\varphi, \dot{A}\right)$ be any two optional PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$ defined as following:

$$\left(\mathfrak{F}_\Psi, \dot{A}\right) = \mathfrak{F}_\mu(z_i) = (\mathfrak{F}(z_i), \mu(z_i)), \qquad \forall z_i \in \dot{C} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}.$$
$$\left(\mathfrak{G}_\varphi, \dot{B}\right) = \mathfrak{G}_\varphi(z_i) = (\mathfrak{G}(z_i), \varphi(z_i)), \qquad \forall z_i \in \dot{C} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}.$$

Now, since the commutative and associative properties are fulfilled with PNSE-set, it follows that

$\left(\mathfrak{F}_\mu, \dot{A}\right)^c \ddot{\cup} \left(\mathfrak{G}_\varphi, \dot{B}\right)^c$
$= (\mathfrak{F}(z_i), \mu(z_i))^c \ddot{\cup} (\mathfrak{G}(z_i), \varphi(z_i))^c$
$= (\ddot{c}(\mathfrak{F}(z_i)), \dot{c}(\mu(z_i))) \ddot{\cup} (\ddot{c}(\mathfrak{G}(z_i)), \dot{c}(\varphi(z_i)))$
$= (\ddot{c}(\mathfrak{F}(z_i)), \ddot{\cup} \ddot{c}(\mathfrak{G}(z_i))) \max(\dot{c}(\mu(z_i)), \dot{c}(\varphi(z_i)))$

$= (\ddot{c}\left(\mathfrak{F}\left(z_i\right) \ddot{\cap} \mathfrak{G}\left(z_i\right)\right)), \dot{c}\left(\min\left(\mu\left(z_i\right), \varphi\left(z_i\right)\right)\right)$
$= \left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\cap} \left(\mathfrak{G}_\varphi, \dot{B}\right)\right)^c.$

(ii) (ii)The proof of the(ii) is comparable to the proof of the (i) and therefore overlooked. □

Definition 4.10. Let $\left(\mathfrak{F}_\mu, \dot{A}\right)$and $\left(\mathfrak{G}_\varphi, \dot{B}\right)$ be any two optional PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$.Then $\left(\mathfrak{F}_\mu, \dot{A}\right) AND \left(\mathfrak{G}_\varphi, \dot{B}\right)$ indicated by $\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\wedge} \left(\mathfrak{G}_\varphi, \dot{B}\right)$ is a PNSE-set and defined as:

$$\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\wedge} \left(\mathfrak{G}_\varphi, \dot{B}\right) = \left(\mathfrak{H}_\Psi, \dot{A} \times \dot{B}\right)$$

where $\left(\mathfrak{H}_\Psi, \dot{A} \times \dot{B}\right) = (\mathfrak{H}(z_i, z_j), \Psi(z_i, z_j))$, such that $\mathfrak{H}(z_i, z_j) = \mathfrak{F}(z_i) \ddot{\cap} \mathfrak{G}(z_j)$ and $\Psi(z_i, z_j) = min(\mu(z_i), \varphi(z_j))$, $\forall (z_i, z_j) \in A \times \dot{B} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}$ $and \ddot{\cap}$ depicts the basic intersection operation.

Definition 4.11. Let $\left(\mathfrak{F}_\mu, \dot{A}\right)$and $\left(\mathfrak{G}_\varphi, \dot{B}\right)$ be any two optional PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$.Then $\left(\mathfrak{F}_\mu, \dot{A}\right) OR \left(\mathfrak{G}_\varphi, \dot{B}\right)$ indicated by $\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\vee} \left(\mathfrak{G}_\varphi, \dot{B}\right)$ is a PNSE-set and defined as:

$$\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\vee} \left(\mathfrak{G}_\varphi, \dot{B}\right) = \left(\mathfrak{H}_\Psi, \dot{A} \times \dot{B}\right)$$

where $\left(\mathfrak{H}_\Psi, \dot{A} \times \dot{B}\right) = (\mathfrak{H}(z_i, z_j), \Psi(z_i, z_j))$, such that $\mathfrak{H}(z_i, z_j) = \mathfrak{F}(z_i) \ddot{\cup} \mathfrak{G}(z_j)$ and $\Psi(z_i, z_j) = max(\mu(z_i), \varphi(z_j))$, $\forall (z_i, z_j) \in \dot{A} \times \dot{B} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}$ $and \ddot{\cup}$ depicts the basic union.

Proposition 4.12. *Let* $\left(\mathfrak{F}_\mu, \dot{A}\right)$ *and* $\left(\mathfrak{G}_\varphi, \dot{B}\right)$ *be any two optional PNSE-sets over* $(\mathfrak{V}, \mathfrak{Z})$*.Then De Morgans laws satisfying:*
(i). $\left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\vee} \left(\mathfrak{G}_\varphi, \dot{B}\right)\right)^c = \left(\left(\mathfrak{F}_\mu, \dot{A}\right)^c \ddot{\wedge} \left(\mathfrak{G}_\varphi, \dot{B}\right)^c\right).$
(ii). $\left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\wedge} \left(\mathfrak{G}_\varphi, \dot{B}\right)\right)^c = \left(\left(\mathfrak{F}_\mu, \dot{A}\right)^c \ddot{\vee} \left(\mathfrak{G}_\varphi, B\right)^c\right).$

Proof. (i) Assume that $\left(\mathfrak{F}_\mu, \dot{A}\right)$ and $\left(\mathfrak{G}_\varphi, \dot{B}\right)$ be any two optional PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$ defined as following:

$$\left(\mathfrak{F}_\Psi, \dot{A}\right) = \mathfrak{F}_\mu(z_i) = (\mathfrak{F}(z_i), \mu(z_i)), \qquad \forall z_i \in \dot{C} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}.$$
$$\left(\mathfrak{G}_\varphi, \dot{B}\right) = \mathfrak{G}_\varphi(z_i) = (\mathfrak{G}(z_i), \varphi(z_i)), \qquad \forall z_i \in \dot{C} \subseteq \mathfrak{Z} = \{\mathfrak{P} \times \mathfrak{M} \times \mathfrak{Q}\}.$$

Now, since the commutative and associative properties are fulfilled with PNSE-set, it follows that
$\left(\mathfrak{F}_\mu, \dot{A}\right)^c \ddot{\vee} \left(\mathfrak{G}_\varphi, \dot{B}\right)^c$
$= \left(\mathfrak{F}\left(z_i\right), \mu\left(z_i\right)\right)^c \ddot{\vee} \left(\mathfrak{G}\left(z_i\right), \varphi\left(z_i\right)\right)^c$
$= \left(\ddot{c}\left(\mathfrak{F}\left(z_i\right)\right), \dot{c}\left(\mu\left(z_i\right)\right)\right) \ddot{\vee} \left(\ddot{c}\left(\mathfrak{G}\left(z_i\right)\right), \dot{c}\left(\varphi\left(z_i\right)\right)\right)$
$= \left(\ddot{c}\left(\mathfrak{F}\left(z_i\right)\right), \ddot{\vee} \ddot{c}\left(\mathfrak{G}\left(z_i\right)\right)\right) \max\left(\dot{c}\left(\mu\left(z_i\right)\right), \dot{c}\left(\varphi\left(z_i\right)\right)\right)$
$= \left(\ddot{c}\left(\mathfrak{F}\left(z_i\right) \ddot{\wedge} \mathfrak{G}\left(z_i\right)\right)\right), \dot{c}\left(\min\left(\mu\left(z_i\right), \varphi\left(z_i\right)\right)\right)$
$= \left(\left(\mathfrak{F}_\mu, \dot{A}\right) \ddot{\wedge} \left(\mathfrak{G}_\varphi, \dot{B}\right)\right)^c.$

(ii)The proof of the second part is similar to the proof of the first part therefore omitted. □

Proposition 4.13. *Let $(\mathfrak{F}_\mu, A)$, $(\mathfrak{G}_\varphi, B)$ and $(\mathfrak{H}_\Psi, C)$ be any three optional PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$. Then the following results are achieved:*

(i). $(\mathfrak{F}_\mu, A)\ddot{\vee}\ ((\mathfrak{G}_\varphi, B)\ddot{\vee}(\mathfrak{H}_\Psi, C)) = ((\mathfrak{F}_\mu, A)\ddot{\vee}(\mathfrak{G}_\varphi, B))\ddot{\vee}(\mathfrak{H}_\Psi, C)$.
(ii). $(\mathfrak{F}_\mu, A)\ddot{\wedge}\ ((\mathfrak{G}_\varphi, B)\ddot{\wedge}(\mathfrak{H}_\Psi, C)) = ((\mathfrak{F}_\mu, A)\ddot{\wedge}(\mathfrak{G}_\varphi, B))\ddot{\wedge}(\mathfrak{H}_\Psi, C)$.
(iii). $(\mathfrak{F}_\mu, A)\ddot{\vee}\ ((\mathfrak{G}_\varphi, B)\ddot{\wedge}(\mathfrak{H}_\Psi, C)) = ((\mathfrak{F}_\mu, A)\ddot{\vee}(\mathfrak{G}_\varphi, B))\ddot{\wedge}((\mathfrak{F}_\mu, A)\ddot{\vee}(\mathfrak{H}_\Psi, C))$.
(iV). $(\mathfrak{F}_\mu, A)\ddot{\wedge}\ ((\mathfrak{G}_\varphi, B)\ddot{\vee}(\mathfrak{H}_\Psi, C)) = ((\mathfrak{F}_\mu, A)\ddot{\wedge}(\mathfrak{G}_\varphi, B))\ddot{\vee}((\mathfrak{F}_\mu, A)\ddot{\wedge}(\mathfrak{H}_\Psi, C))$.

Proof. The proof of these propositions are clear by Definitions 4.10 and 4.11 and therefore omitted. □

Remark 4.14. Due $A \times B \neq B \times A$, therefore AND operation and OR operation don't satisfy commutative law.

Example 4.15. Let $(\mathfrak{F}_\mu, A)$ and $(\mathfrak{G}_\varphi, B)$ be any two optional PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$ and let $A = \{(p_1, m_1, 1), (p_2, m_2, 1)\}$,$B = \{(p_2, m_2, 1), (p_1, m_1, 0)\}$. Then the PNSE-set defined as bellow:

$$(\mathfrak{F}_\mu, A) = \Big\{(p_1, m_1, 1) = \Big\{\Big(\tfrac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\Big), \Big(\tfrac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big), \Big(\tfrac{v_3}{\langle 0.6,0.1,0.6\rangle}, 0.5\Big)\Big\},$$
$$(p_2, m_2, 1) = \Big\{\Big(\tfrac{v_1}{\langle 0.6,0.3,0\rangle}, 0.5\Big), \Big(\tfrac{v_2}{\langle 0.5,0.3,0.8\rangle}, 0.4\Big), \Big(\tfrac{v_3}{\langle 0.1,0.5,0.2\rangle}, 0.9\Big)\Big\}\Big\}$$

and

$$(\mathfrak{G}_\mu, B) = \Big\{(p_2, m_2, 1) = \Big\{\Big(\tfrac{v_1}{\langle 0.3,0.4,0\rangle}, 0.7\Big), \Big(\tfrac{v_2}{\langle 0.3,0.7,0.2\rangle}, 0.4\Big), \Big(\tfrac{v_3}{\langle 0.1,0.4,0.8\rangle}, 0.6\Big)\Big\},$$
$$(p_1, m_1, 0) = \Big\{\Big(\tfrac{v_1}{\langle 0.3,0.7,0.5\rangle}, 0.8\Big), \Big(\tfrac{v_2}{\langle 0.6,0.3,0.2\rangle}, 0.7\Big), \Big(\tfrac{v_3}{\langle 0.3,0.4,0.8\rangle}, 1\Big)\Big\}\Big\}$$

Then,

$$(\mathfrak{F}_\mu, A)\ddot{\cup}(\mathfrak{G}_\mu, B) = \Big\{(p_1, m_1, 1) = \Big\{\Big(\tfrac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\Big), \Big(\tfrac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big), \Big(\tfrac{v_3}{\langle 0.6,0.1,0.6\rangle}, 0.5\Big)\Big\},$$
$$(p_2, m_2, 1) = \Big\{\Big(\tfrac{v_1}{\langle 0.6,0.3,0\rangle}, 0.7\Big), \Big(\tfrac{v_2}{\langle 0.5,0.3,0.2\rangle}, 0.4\Big), \Big(\tfrac{v_3}{\langle 0.1,0.4,0.2\rangle}, 0.9\Big)\Big\},$$
$$(p_1, m_1, 0) = \Big\{\Big(\tfrac{v_1}{\langle 0.3,0.7,0.5\rangle}, 0.8\Big), \Big(\tfrac{v_2}{\langle 0.6,0.3,0.2\rangle}, 0.7\Big), \Big(\tfrac{v_3}{\langle 0.3,0.4,0.8\rangle}, 1\Big)\Big\}\Big\}.$$

$$(\mathfrak{F}_\mu, A)\ddot{\cap}(\mathfrak{G}_\mu, B) = \Big\{(p_1, m_1, 1) = \Big\{\Big(\tfrac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\Big), \Big(\tfrac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big), \Big(\tfrac{v_3}{\langle 0.6,0.1,0.6\rangle}, 0.5\Big)\Big\},$$
$$(p_2, m_2, 1) = \Big\{\Big(\tfrac{v_1}{\langle 0.3,0.4,0\rangle}, 0.5\Big), \Big(\tfrac{v_2}{\langle 0.3,0.7,0.8\rangle}, 0.4\Big), \Big(\tfrac{v_3}{\langle 0.1,0.5,0.8\rangle}, 0.6\Big)\Big\},$$
$$(p_1, m_1, 0) = \Big\{\Big(\tfrac{v_1}{\langle 0.3,0.7,0.5\rangle}, 0.8\Big), \Big(\tfrac{v_2}{\langle 0.6,0.3,0.2\rangle}, 0.7\Big), \Big(\tfrac{v_3}{\langle 0.3,0.4,0.8\rangle}, 1\Big)\Big\}\Big\}.$$

$(\mathfrak{F}_\mu, A)\ddot{\vee}(\mathfrak{G}_\mu, B) = (\mathfrak{H}_\Psi, C = A \times B) =$

$\Big\{(p_1, m_1, 1), (p_1, m_2, 1) = \Big\{\Big(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.7\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.4\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.6\Big)\Big\},$
$(p_1, m_1, 1), (p_1, m_1, 0) = \Big\{\Big(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.8\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.7\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.1\Big)\Big\},$
$(p_2, m_2, 1), (p_2, m_2, 1) = \Big\{\Big(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.7\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.4\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.9\Big)\Big\},$
$(p_2, m_2, 1), (p_1, m_1, 0) = \Big\{\Big(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.8\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.7\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 1\Big)\Big\}\Big\}$

and
$(\mathfrak{F}_\mu, A)\,\ddot{\curlywedge}\,(\mathfrak{G}_\mu, B) = (\mathfrak{H}_\Psi, C = A \times B) =$
$\Big\{(p_1, m_1, 1), (p_1, m_2, 1) = \Big\{\Big(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big)\Big\},$
$(p_1, m_1, 1), (p_1, m_1, 0) = \Big\{\Big(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big)\Big\},$
$(p_2, m_2, 1), (p_2, m_2, 1) = \Big\{\Big(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big)\Big\},$
$(p_2, m_2, 1), (p_1, m_1, 0) = \Big\{\Big(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big)\Big\}\Big\}$

5. **Decision-Making Application on PNSE-sets**

In this part, we introduce a new generalized algorithm to show the efficiency of the proposed model to help the decision maker (user) make the right decision from available alternatives based on hypothetical data, as in the following example.

Example 5.1. Suppose Mr. Xu wants to choose a primary school for his daughter out of three schools available in the universe, $\mathfrak{V} = \{v_1, v_2, v_3\}$. Mr. Xu asked for the opinion of three of his friends (experts) and could represent his friends (experts) by the set $\mathfrak{M} = \{m_1, m_2, m_3\}$ and the opines set $\mathfrak{Q} = \{1 = agree, 0 = disagree\}$ describes the opinions set of Mr. Xu friends. Mr. Xu friends consider a set of attributes $\mathfrak{P} = \{p_1, p_2, p_3\}$ where the attributes represent the characteristics that depend on selecting the suitable school namely, $p_1 = teachingquality$, $p_2 = cost$, and $p_3 = environment$, respectively. According to the evaluation of experts, the PNSE-set $(\mathfrak{F}_\mu, \mathfrak{Z} = \mathfrak{P})$ is obtained.

$(\mathfrak{F}_\mu, \mathfrak{P}) =$
$\Big\{(p_1, m_1, 1) = \Big\{\Big(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\Big), \Big(\frac{v_2}{\langle 0.4,0.2,0.7\rangle}, 0.3\Big), \Big(\frac{v_2}{\langle 0.6,0.1,0.6\rangle}, 0.5\Big)\Big\},$

$(p_2, m_1, 1) = \Big\{\Big(\frac{v_1}{\langle 0.6,0.3,0\rangle}, 0.5\Big), \Big(\frac{v_2}{\langle 0.1,0.3,0.9\rangle}, 0.3\Big), \Big(\frac{v_3}{\langle 0.9,0.4,0.2\rangle}, 0.6\Big)\Big\},$

$(p_3, m_1, 1) = \Big\{\Big(\frac{v_1}{\langle 0.4,0.1,0.2\rangle}, 0.3\Big), \Big(\frac{v_2}{\langle 0,0.1,0.7\rangle}, 0.5\Big), \Big(\frac{v_3}{\langle 0.7,0.2,0.4\rangle}, 0.5\Big)\Big\},$

$(p_1, m_2, 1) = \Big\{\Big(\frac{v_1}{\langle 0.3,0.4,0.6\rangle}, 0.1\Big), \Big(\frac{v_2}{\langle 0.4,0.5,0.3\rangle}, 0.5\Big), \Big(\frac{v_3}{\langle 0.2,0.4,0.4\rangle}, 0.8\Big)\Big\},$

$$(p_2, m_2, 1) = \left\{ \left(\frac{v_1}{\langle 0.3,0.3,0.6 \rangle}, 0.6 \right), \left(\frac{v_2}{\langle 0.8,0.4,0.6 \rangle}, 0.9 \right), \left(\frac{v_3}{\langle 0.5,0.5,0.7 \rangle}, 0.8 \right) \right\},$$

$$(p_3, m_2, 1) = \left\{ \left(\frac{v_1}{\langle 0.6,0.3,0 \rangle}, 0.5 \right), \left(\frac{v_2}{\langle 0.1,0.3,0.9 \rangle}, 0.7 \right), \left(\frac{v_3}{\langle 0.9,0.4,0.2 \rangle}, 0.6 \right) \right\},$$

$$(p_1, m_3, 1) = \left\{ \left(\frac{v_1}{\langle 0.3,0.4,0.6 \rangle}, 0.1 \right), \left(\frac{v_2}{\langle 0.4,0.5,0.3 \rangle}, 0.5 \right), \left(\frac{v_3}{\langle 0.2,0.4,0.4 \rangle}, 0.8 \right) \right\},$$

$$(p_2, m_3, 1) = \left\{ \left(\frac{v_1}{\langle 0.1,0.1,0.4 \rangle}, 0.3 \right), \left(\frac{v_2}{\langle 0.6,0.2,0.4 \rangle}, 0.8 \right), \left(\frac{v_3}{\langle 0.3,0.2,0.5 \rangle}, 0.6 \right) \right\},$$

$$(p_3, m_3, 1) = \left\{ \left(\frac{v_1}{\langle 0.5,0.4,0.7 \rangle}, 0.2 \right), \left(\frac{v_2}{\langle 0.3,0.5,0.6 \rangle}, 0.5 \right), \left(\frac{v_3}{\langle 0,0.3,0.6 \rangle}, 0.7 \right) \right\},$$

$$(p_1, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.2,0.8,0.3 \rangle}, 0.5 \right), \left(\frac{v_2}{\langle 0.3,0.4,0.2 \rangle}, 0.5 \right), \left(\frac{v_3}{\langle 0.3,0.2,0.6 \rangle}, 0.8 \right) \right\},$$

$$(p_2, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.4,0.9,0.2 \rangle}, 0.9 \right), \left(\frac{v_2}{\langle 0.4,0.3,0.2 \rangle}, 0.7 \right), \left(\frac{v_3}{\langle 0.3,0.4,0.7 \rangle}, 0.2 \right) \right\},$$

$$(p_3, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.6,0.4,0.1 \rangle}, 0.6 \right), \left(\frac{v_2}{\langle 0.5,0.4,0.3 \rangle}, 0.8 \right), \left(\frac{v_3}{\langle 0.4,0.6,0.5 \rangle}, 0.6 \right) \right\},$$

$$(p_1, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.4,0.3,0.3 \rangle}, 0.7 \right), \left(\frac{v_2}{\langle 0.2,0.6,0.6 \rangle}, 0.3 \right), \left(\frac{v_3}{\langle 0.7,0.3,0.5 \rangle}, 0.9 \right) \right\},$$

$$(p_2, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.7,0.5,0.4 \rangle}, 0.7 \right), \left(\frac{v_2}{\langle 0.4,0.3,0.7 \rangle}, 0.8 \right), \left(\frac{v_3}{\langle 0.1,0.3,0.6 \rangle}, 0.4 \right) \right\},$$

$$(p_3, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.5,0.3,0.6 \rangle}, 0.5 \right), \left(\frac{v_2}{\langle 0.1,0.5,0.8 \rangle}, 0.4 \right), \left(\frac{v_3}{\langle 0.5,0.3,0.7 \rangle}, 0.8 \right) \right\},$$

$$(p_1, m_3, 0) = \left\{ \left(\frac{v_1}{\langle 0.2,0.5,0.6 \rangle}, 0.5 \right), \left(\frac{v_2}{\langle 0.6,0.9,0.5 \rangle}, 0.2 \right), \left(\frac{v_3}{\langle 0.4,0,0.7 \rangle}, 0.8 \right) \right\},$$

$$(p_2, m_3, 0) = \left\{ \left(\frac{v_1}{\langle 0.5,0.4,0.2 \rangle}, 0.6 \right), \left(\frac{v_2}{\langle 0.3,0.7,0.3 \rangle}, 0.4 \right), \left(\frac{v_3}{\langle 0.8,0.3,0.6 \rangle}, 0.3 \right) \right\},$$

$$(p_3, m_3, 0) = \left\{ \left(\frac{v_1}{\langle 0.2,0.4,0.8 \rangle}, 0.3 \right), \left(\frac{v_2}{\langle 0.5,0.5,0.2 \rangle}, 0.9 \right), \left(\frac{v_3}{\langle 0.1,0.1,0.7 \rangle}, 0.1 \right) \right\} \Bigg\}$$

Next, by using the proposed algorithm given below together with the PNSE-set model $(\mathfrak{F}_\mu, \mathfrak{P})$, we will solve the problem noted at the beginning of this part to help Mr. Xu choose the appropriate school. The generalised algorithm is shown below.

Algorithm 1

Step 1: Build a PNSE-set model $(\mathfrak{F}_\mu, \mathfrak{P})$ depending on opinion of Experts.

Step 2: Find the values of $\rho(z)(v_n) - \eta(z)(v_n) + \psi(z)(v_n) \ \forall v_n \in \mathfrak{V}.$, where

$\rho(z)(v_n), \eta(z)(v_n)$ and $\psi(z)(v_n)$ are the three neutrosophic membership functions(truth, indeterminacy and falsehood) $\forall v \in \mathfrak{V}$, respectively and $\mu(z)(v_n)$ indicated to possibility grade of $v \in \mathfrak{V}$.

Step 3: For both agree-PNSES and disagree-PNSES values, take the greatest numerical degree.

Step 4: Calculate values of the score $\mathfrak{R}_i = \mathfrak{M}_i - \mathfrak{N}_i$, where $\mathfrak{M}_i, \mathfrak{N}_i$ are degree for agree-PNSES and disagree-PNSES $\forall v_i \in \mathfrak{V}$

Step 5: Choose the value of the highest score in $\mathfrak{Z}_i = \max_{v_i \in \mathfrak{V}} \{\mathfrak{R}_i\}$.Then the decision is to choose an alternative v_i as the optimal or most suitable solution to the problem.

Now, from Table 1, we get the values $\rho(z)(v_n) - \eta(z)(v_n) + \psi(z)(v_n)$ $\forall v_n \in \mathfrak{V}$. It is to be noted that the first column and second column in Table 1 symbolize the values of $\rho(z)(v_n) - \eta(z)(v_n) + \psi(z)(v_n)$ and the degree of PNSE-set for all $v_n \in \mathfrak{V}$ respectively.
Tables 2 and 3 present the highest numerical degree for the elements in the agree-PNSE-set and disagree-PNSE-set, respectively.
The values of $\mathfrak{M}_i$ and $\mathfrak{N}_i$ are gaven in Table 4 and represent numerical grades for both the agree-PNSE-set and disagree-PNSE-set, respectively.
Then $\mathfrak{D}_i = \max_{v_i \in \mathfrak{V}} \{\mathfrak{R}_i\} = \{\mathfrak{R}_3\}$. Therefore, based on the opinions of experts, the appropriate school is v_3.

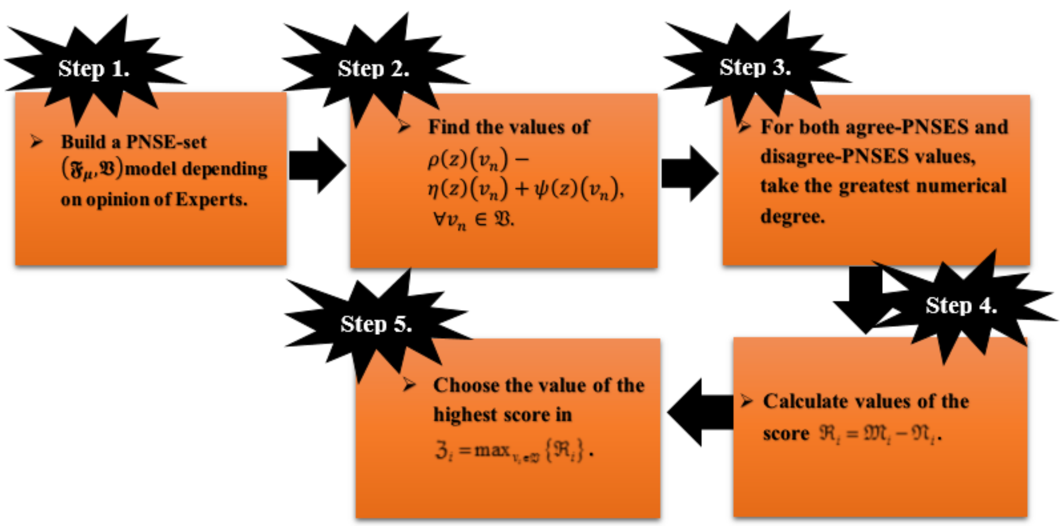

Figure 1: Representation of algorithm 1.

Remark 5.2. If we have more than one alternative with the highest $\mathbb{R}_i$ grade, then any of those alternatives can be selected as the best solution to the problem.

TABLE 1. Values of $\rho(z)(v_n) - \eta(z)(v_n) + \psi(z)(v_n)\ \forall v_n \in \mathfrak{V}$.

V_n	v_1	v_2	v_3
$(p_1, m_1, 1)$	0.3,0.2	0.9,0.3	-0.1,0.5
$(p_2, m_1, 1)$	0.3,0.5	0.7,0.3	0.7,0.6
$(p_3, m_1, 1)$	0.5,0.3	0.6,0.5	0.9,0.5
$(p_1, m_2, 1)$	0.5,0.1	0.2,0.5	0.2,0.8
$(p_2, m_2, 1)$	0.6,0.6	-0.2,0.9	0.7,0.8
$(p_3, m_2, 1)$	0.3,0.5	0.7,0.7	0.7,0.6
$(p_1, m_3, 1)$	0.5,0.1	0.2,0.5	0.2,0.8
$(p_2, m_3, 1)$	0.4,0.3	0,0.8	0.6,0.6
$(p_3, m_3, 1)$	0.8,0.2	0.4,0.5	0.3,0.7
$(p_1, m_1, 0)$	-0.3,0.5	0.1,0.5	0.7,0.8
$(p_2, m_1, 0)$	-0.3,0.9	0.1,0.7	0.6,0.2
$(p_3, m_1, 0)$	0.1,0.6	0.4,0.8	0.3,0.6
$(p_1, m_2, 0)$	0.4,0.7	0.2,0.3	-0.1,0.9
$(p_2, m_2, 0)$	0.6,0.7	0.8,0.8	0.4,0.4
$(p_3, m_2, 0)$	0.8,0.5	0.4,0.4	0.9,0.8
$(p_1, m_3, 0)$	0.3,0.5	0.2,0.2	0.1,0.4
$(p_2, m_3, 0)$	0.3,0.6	0.7,0.4	0.2,0.3
$(p_3, m_3, 0)$	0.6,0.3	0.2,0.9	0.3,0.1

TABLE 2. Numerical grade for agree-PNSES.

	V_n	**Highest numerical grade**	**Degree of possibility**
(p_1, m_1)	v_2	0.9	0.3
(p_2, m_1)	v_3	0.7	0.6
(p_3, m_1)	v_3	0.9	0.5
(p_1, m_2)	v_1	0.5	0.1
(p_2, m_2)	v_3	0.7	0.8
(p_3, m_2)	v_2	0.7	0.7
(p_1, m_3)	v_1	0.5	0.1
(p_2, m_3)	v_3	0.6	0.6
(p_3, m_3)	v_1	0.8	0.2
Score(v_1)=0.26		Score(v_2)=0.76	Score(v_3)=1.79

6. Similarity Measure on PNSE-Sets

Similarity measures are considered essential tools in fuzzy set theory and its extensions, where numerous researchers have extensively studied it and employed it in many areas of our daily life, such as medical diagnosis, decision making, pattern recognition, and so forth. In

TABLE 3. Numerical grade for disagree-PNSES.

	V_n	**Highest numerical grade**	**Degree of possibility**
(p_1, m_1)	v_3	0.7	0.8
(p_2, m_1)	v_3	0.6	0.2
(p_3, m_1)	v_2	0.4	0.8
(p_1, m_2)	v_1	0.4	0.7
(p_2, m_2)	v_2	0.8	0.8
(p_3, m_2)	v_3	0.9	0.2
(p_1, m_3)	v_3	0.1	0.4
(p_2, m_3)	v_2	0.7	0.4
(p_3, m_3)	v_1	0.6	0.3
Score(v_1)=0.46		Score(v_2)=1.24	Score(v_3)=1.44

TABLE 4. The score of $\mathfrak{R}_i = \mathfrak{M}_i - \mathfrak{N}_i$

$\mathfrak{M}_i$	$\mathfrak{N}_i$	$\mathfrak{R}_i$
Score(v_1)=0.26	Score(v_1)=0.46	-0.2
Score(v_2)= 0.76	Score(v_2)=1.24	-0.48
Score(v_3)=1.79	Score(v_3)=1.44	0.35

this part, we illustrate the similarity measure between two PNSE-sets and use a medical diagnosis example to demonstrate the importance of the proposed similarity measures in solving real-world problems.

Definition 6.1. Let $\mathfrak{F}_\mu$ and $\mathfrak{G}_\varphi$ be two PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$. Similarity measure between $\mathfrak{F}_\mu$ and $\mathfrak{G}_\varphi$ indicated by $\hat{S}\left(\mathfrak{F}_\mu, \mathfrak{G}_\varphi\right)$ is defined as follows:

$$\hat{S}\left(\mathfrak{F}_\mu, \mathfrak{G}_\varphi\right) = \ddot{M}\left(\mathfrak{F}\left(z\right), \mathfrak{G}\left(z\right)\right) \times \ddot{M}\left(\mu\left(z\right), \varphi\left(z\right)\right),$$

such that

$$\ddot{M}\left(\mathfrak{F}\left(z\right), \mathfrak{G}\left(z\right)\right) = max \ddot{M}_i\left(\mathfrak{F}\left(z\right), \mathfrak{G}\left(z\right)\right),$$
$$\ddot{M}\left(\mu\left(z\right), \varphi\left(z\right)\right) = max \ddot{M}_i\left(\mu\left(z\right), \varphi\left(z\right)\right),$$

where

$$\ddot{M}_i\left(\mathfrak{F}\left(z\right), \mathfrak{G}\left(z\right)\right) = 1 - \frac{1}{\sqrt{n}}\sqrt{\sum_{i=1}^{n}\left(\dot{\phi}_{\mathfrak{F}(z_i)}\left(v_j\right) - \dot{\phi}_{\mathfrak{G}(z_i)}\left(v_j\right)\right)^2},$$

such that and,
$\dot{\phi}_{\mathfrak{F}_\mu(z)}\left(v_j\right) = \frac{\rho_{\mathfrak{F}_\mu(z_i)}(v_j)+\eta_{\mathfrak{F}_\mu(z_i)}(v_j)+\psi_{\mathfrak{F}_\mu(z_i)}(v_j)}{3}$, $\dot{\phi}_{\mathfrak{G}_\mu(z)}\left(v_j\right) = \frac{\rho_{\mathfrak{G}_\mu(z_i)}(v_j)+\eta_{\mathfrak{G}_\mu(z_i)}(v_j)+\psi_{\mathfrak{G}_\mu(z_i)}(v_j)}{3}$.

$$\ddot{M}\left(\mu\left(z_i\right), \varphi\left(z_i\right)\right)=1-\frac{\sum\limits_{j=1}^{n}\left|\mu_j(z_i)-\varphi_j(z_i)\right|}{\sum\limits_{j=1}^{n}\left|\mu_j(z_i)+\varphi_j(z_i)\right|}$$

Definition 6.2. Let $\mathfrak{F}_\mu$ and $\mathfrak{G}_\varphi$ be two PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$.We say that $\mathfrak{F}_\mu$ and $\mathfrak{G}_\varphi$ are signicantly similar if $\hat{S}\left(\mathfrak{F}_\mu, \mathfrak{G}_\varphi\right) \geq \frac{1}{2}$.

Proposition 6.3. *Let $\mathfrak{F}_\mu$, $\mathfrak{G}_\varphi$ and $\mathfrak{H}_\lambda$ be three PNSE-sets over $(\mathfrak{V}, \mathfrak{Z})$.Then the following results are achieved:*

(i). $\hat{S}\left(\mathfrak{F}_\mu, \mathfrak{G}_\varphi\right)=\hat{S}\left(\mathfrak{G}_\mu, \mathfrak{F}_\varphi\right)$.
(ii). $0 \leq \hat{S}\left(\mathfrak{F}_\mu, \mathfrak{G}_\varphi\right) \leq 1$.
(iii).If $\mathfrak{F}_\mu=\mathfrak{G}_\varphi$ *then* $\hat{S}\left(\mathfrak{F}_\mu, \mathfrak{G}_\varphi\right)=1$.
(iv).$\mathfrak{F}_\mu \subseteq \mathfrak{G}_\varphi \subseteq \mathfrak{H}_\lambda$ *then* $\hat{S}\left(\mathfrak{F}_\mu, \mathfrak{G}_\varphi\right) \leq \hat{S}\left(\mathfrak{G}_\varphi, \mathfrak{H}_\lambda\right)$.
(v).If $\mathfrak{F}_\mu \cap \mathfrak{G}_\varphi=\Phi \Leftrightarrow \hat{S}\left(\mathfrak{F}_\mu, \mathfrak{G}_\varphi\right)=0$.

Proof. The proof of these propositions are clear by Definitions 6.1 and therefore omitted. □

6.1. *Application in Medical Diagnosis based on Similarity Measure of PNSE-set*

In this subsection, we crete an algorithm works to measure similarity ratio of two PNSE-sets. This proposed algorithm employ to estimate whether a sick person has dengue fever based on the accompanying symptoms. To run this algorithm, we created two models of PNSE-sets depends on the assistance of physicians (experts) such that the first PNSE-set represent illness stat and the second PNSE-set represent the ill person state. Based on similarity degree, if it is ≥ 0.5, then the ill person may have dengue fever.

Algorithm 2

Step 1: Create a PNSE-set $\mathfrak{F}_\mu$ for the disease (dengue fever), based on assistance of physicians (experts).

Step 2: Build PNSE-set $\mathfrak{G}_\varphi$ for the patient person describes the severity of the symptoms experienced by the sick person by helping a medical expert person.

Step 3: Calculate similarity measure between a PNSE-set $\mathfrak{F}_\mu$ for illness and a PNSE-set $\mathfrak{G}_\varphi$ for the patient person, and if the similarity ratio is ≥ 0.5, then the person might have dengue fever. Meanwhile, if the similarity ratio is $\prec 0.5$, the person might not have dengue fever.

Now, to test this proposed algorithm, we present an applied example to ascertain whether a person has dengue fever or not.

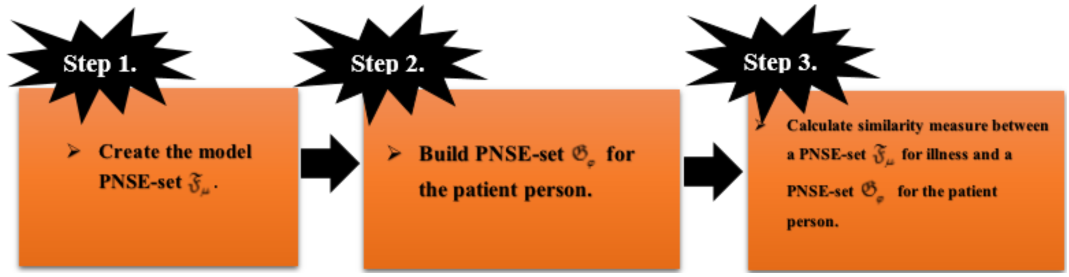

Figure 2: Representation of algorithm 2.

Example 6.4. Consider our universal set include only two alternatives, Yes and No, that is, $\mathfrak{V} = \{v_1 = Yes, v_2 = No\}$ and attributes set that includes a set of symptoms $\mathfrak{P} = \{p_1, p_2, p_3\}$ where p_1 =body temperature, p_2 =cough with chest congestion, and p_3 =headache.

Now, we apply our proposed algorithm.

Step 1: Create the model PNSE-set $\mathfrak{F}_\mu$ for dengue fever by the assistance of two physicians (experts), can be expressed with $\mathfrak{M} = \{m_1, m_2\}$ while the set $\mathfrak{Q} = \{1 = agree, 0 = disagree\}$ describes the set of opinions of two physicians (experts). :

$\mathfrak{F}_\mu =$

$$\left\{(p_1, m_1, 1) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\}, (p_2, m_1, 1) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\},\right.$$

$$(p_3, m_1, 1) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\}, (p_1, m_2, 1) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\},$$

$$(p_2, m_2, 1) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\}, (p_3, m_2, 1) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\},$$

$$(p_1, m_1, 0) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\}, (p_2, m_1, 0) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\},$$

$$(p_3, m_1, 0) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\}, (p_1, m_2, 0) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\},$$

$$\left.(p_2, m_2, 0) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\}, (p_3, m_2, 0) = \left\{\left(\frac{v_1}{\langle 1,0,0\rangle}, 1\right), \left(\frac{v_2}{\langle 0,1,1\rangle}, 1\right)\right\}\right\}$$

Step 2: Create a model of PNSE-set $\mathfrak{G}_\varphi$ for sick person X as following:

$\mathfrak{G}_\varphi =$

$$\left\{(p_1, m_1, 1) = \left\{\left(\frac{v_1}{\langle 0.5,0.3,0.1\rangle}, 0.2\right), \left(\frac{v_2}{\langle 0.6,0.1,0.6\rangle}, 0.5\right)\right\},\right.$$

$$(p_2, m_1, 1) = \left\{\left(\frac{v_1}{\langle 0.6,0.3,0\rangle}, 0.5\right), \left(\frac{v_3}{\langle 0.9,0.4,0.2\rangle}, 0.6\right)\right\},$$

$$(p_3, m_1, 1) = \left\{\left(\frac{v_1}{\langle 0.3,0.4,0.6\rangle}, 0.1\right), \left(\frac{v_3}{\langle 0.2,0.4,0.4\rangle}, 0.8\right)\right\},$$

$(p_1, m_2, 1) = \left\{ \left(\frac{v_1}{\langle 0.1,0.1,0.4\rangle}, 0.3 \right), \left(\frac{v_3}{\langle 0.3,0.2,0.5\rangle}, 0.6 \right) \right\},$

$(p_2, m_2, 1) = \left\{ \left(\frac{v_1}{\langle 0.2,0.8,0.3\rangle}, 0.5 \right), \left(\frac{v_3}{\langle 0.3,0.2,0.6\rangle}, 0.8 \right) \right\},$

$(p_3, m_2, 1) = \left\{ \left(\frac{v_1}{\langle 0.4,0.9,0.2\rangle}, 0.9 \right), \left(\frac{v_3}{\langle 0.3,0.4,0.7\rangle}, 0.2 \right) \right\},$

$(p_1, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.4,0.3,0.3\rangle}, 0.4 \right), \left(\frac{v_3}{\langle 0.7,0.3,0.5\rangle}, 0.5 \right) \right\},$

$(p_2, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.8,0.1,0.3\rangle}, 0.1 \right), \left(\frac{v_3}{\langle 0.2,0.1,0.5\rangle}, 0.7 \right) \right\},$

$(p_3, m_1, 0) = \left\{ \left(\frac{v_1}{\langle 0.3,0.4,0.3\rangle}, 0.4 \right), \left(\frac{v_3}{\langle 0.9,0.3,0.5\rangle}, 0.9 \right) \right\},$

$(p_1, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.4,0.3,0.3\rangle}, 0.6 \right), \left(\frac{v_3}{\langle 0.7,0.3,0.5\rangle}, 0.6 \right) \right\},$

$(p_2, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.5,0.5,0.7\rangle}, 0.4 \right), \left(\frac{v_3}{\langle 0.8,0.4,0.5\rangle}, 0.5 \right) \right\},$

$(p_3, m_2, 0) = \left\{ \left(\frac{v_1}{\langle 0.2,0.4,0.8\rangle}, 0.3 \right), \left(\frac{v_3}{\langle 0.1,0.1,0.7\rangle}, 0.1 \right) \right\}$

Step 3: Calculate similarity between $\mathfrak{F}_\varphi$ and $\mathfrak{G}_\varphi$ according to Definition 6.1 given above. Then,

$$\ddot{M}\left(\mu\left(z_1 = (p_1, m_1, 1)\right), \varphi\left(z_1 = (p_1, m_1, 1)\right)\right) = 1 - \frac{\sum_{j=1}^{2} |\mu_1(z_1) - \varphi_1(z_1)|}{\sum_{j=1}^{2} |\mu_1(z_1) + \varphi_1(z_1)|}$$
$$= 1 - \frac{|1-0.2| + |1-0.5|}{|1+0.2| + |1+0.5|} = 0.52$$

Similarly we get, $\ddot{M}(\mu(z_2), \varphi(z_2)) = 0.71$, $\ddot{M}(\mu(z_3), \varphi(z_3)) = 0.62$, $\ddot{M}(\mu(z_4), \varphi(z_4)) = 0.62$, $\ddot{M}(\mu(z_5), \varphi(z_5)) = 0.62$, $\ddot{M}(\mu(z_6), \varphi(z_6)) = 0.79$, $\ddot{M}(\mu(z_7), \varphi(z_7)) = 0.62$, $\ddot{M}(\mu(z_8), \varphi(z_8)) = 0.62$, $\ddot{M}(\mu(z_9), \varphi(z_9)) = 0.58$, $\ddot{M}(\mu(z_{10}), \varphi(z_{10})) = 0.85$, $\ddot{M}(\mu(z_{11}), \varphi(z_{11})) = 0.75$, $\ddot{M}(\mu(z_{12}), \varphi(z_{12})) = 0.34$,then
$\ddot{M}(\mu(z), \varphi(z)) = max\ddot{M}_i(\mu(z), \varphi(z))$,

$$\ddot{M}_1(\mathfrak{F}(z_1), \mathfrak{G}(z_1)) = 1 - \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1}^{n} \left(\dot{\phi}_{\mathfrak{F}(z_1)}(v_j) - \dot{\phi}_{\mathfrak{G}(z_1)}(v_j) \right)^2},$$
$$= 1 - \frac{1}{\sqrt{2}} \sqrt{(1-0.3)^2 + (1-0.43)^2} = 0.36$$

Similarly, we get the rest of the values in Table 5

TABLE 5. Valudes of $\ddot{M}_1(\mathfrak{F}(z_i), \mathfrak{G}(z_i))$ and $\ddot{M}(\mu(z_i), \varphi(z_i))$

$\ddot{M}_1(\mathfrak{F}(z_i), \mathfrak{G}(z_i))$	**Degree**	$\ddot{M}(\mu(z_i), \varphi(z_i))$	**Degree**
$\ddot{M}_1(\mathfrak{F}(z_1), \mathfrak{G}(z_1))$	0.36	$\ddot{M}(\mu(z_2), \varphi(z_2))$	0.52
$\ddot{M}_1(\mathfrak{F}(z_2), \mathfrak{G}(z_2))$	0.39	$\ddot{M}(\mu(z_2), \varphi(z_2))$	0.71
$\ddot{M}_1(\mathfrak{F}(z_3), \mathfrak{G}(z_3))$	0.38	$\ddot{M}(\mu(z_3), \varphi(z_3))$	0.62
$\ddot{M}_1(\mathfrak{F}(z_4), \mathfrak{G}(z_4))$	0.26	$\ddot{M}(\mu(z_4), \varphi(z_4))$	0.62
$\ddot{M}_1(\mathfrak{F}(z_5), \mathfrak{G}(z_5))$	0.40	$\ddot{M}(\mu(z_5), \varphi(z_5))$	0.62
$\ddot{M}_1(\mathfrak{F}(z_6), \mathfrak{G}(z_6))$	0.48	$\ddot{M}(\mu(z_6), \varphi(z_6))$	0.79
$\ddot{M}_1(\mathfrak{F}(z_7), \mathfrak{G}(z_7))$	0.41	$\ddot{M}(\mu(z_7), \varphi(z_7))$	0.62
$\ddot{M}_1(\mathfrak{F}(z_8), \mathfrak{G}(z_8))$	0.41	$\ddot{M}(\mu(z_8), \varphi(z_8))$	0.62
$\ddot{M}_1(\mathfrak{F}(z_9), \mathfrak{G}(z_9))$	0.48	$\ddot{M}(\mu(z_9), \varphi(z_9))$	0.58
$\ddot{M}_1(\mathfrak{F}(z_{10}), \mathfrak{G}(z_{10}))$	0.56	$\ddot{M}(\mu(z_{10}), \varphi(z_{10}))$	0.85
$\ddot{M}_1(\mathfrak{F}(z_{11}), \mathfrak{G}(z_{11}))$	0.49	$\ddot{M}(\mu(z_{11}), \varphi(z_{11}))$	0.75
$\ddot{M}_1(\mathfrak{F}(z_{12}), \mathfrak{G}(z_{12}))$	0.64	$\ddot{M}(\mu(z_{12}), \varphi(z_{12}))$	0.34
$\ddot{M}(\mathfrak{F}(z), \mathfrak{G}(z)) = 0.64$		$\ddot{M}(\mu(z), \varphi(z)) = 0.85.$	

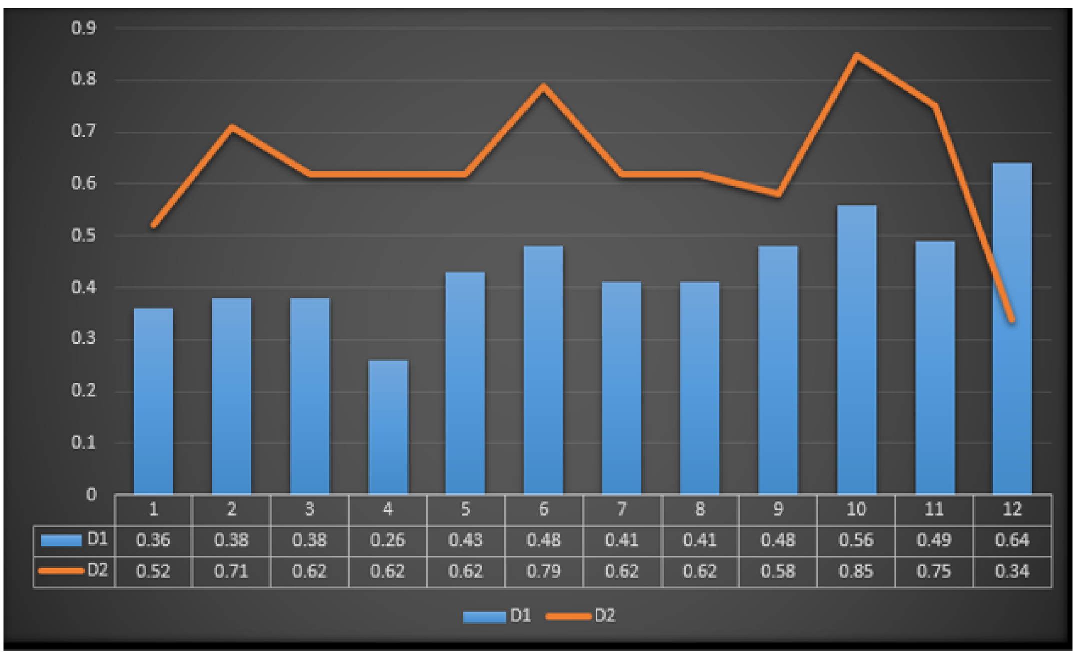

Figure 3: Statistical chart.

Then, the similarity measure between a PNSE-set $\mathfrak{F}_\mu$ for illness and a PNSE-set $\mathfrak{G}_\varphi$ for the patient person:

$\hat{S}(\mathfrak{F}_\mu, \mathfrak{G}_\varphi) = 0.64 \times 0.85 = 0.54$ (The patient has dengue fever).

From Table 5, we give the following statistical chart (Figure 3), which shows the differing opinions of experts (physicians) about the condition of patients based on the strength of symptoms. Where we will point to $\ddot{M}_1\left(\mathfrak{F}\left(z_i\right), \mathfrak{G}\left(z_i\right)\right)$ by symbol D_1 and $\ddot{M}\left(\mu\left(z_i\right), \varphi\left(z_i\right)\right)$ by symbol D_2.

7. Comparison with Some Methods in Literature

In the literature section, we mentioned that there are many contributions discussed based on fuzzy-like, intuitionistic fuzzy-like and neutrosophic-like. As a result, in this section, we will compare our proposed PNSE-set to other existing models that aim to find the relationship between the degree of probability and the fuzzy environment. First of all, the PNSE-set is an extension of PIFSE-set and PFSESet. With three neutrosopic membership functions, the PNSE-set can deal with alternatives and attributes in an alternatives set V and a set of attributes E in greater detail, whereas the PIFSE-set appears to have some weaknesses in dealing with alternatives and attributes that exist in an alternatives set V and an attributes set E. It can only get a handle on the uncertainty issues considering both the membership and non-membership values, whereas PNSE-set can get a handle on these issues as well as the issues containing indeterminacy and inconsistent data. These tools makes it more flexible and practical than the PIFSE-set. On the other hand, it is worthwhile to note that the PNSE-set was created to overcome one of the main shortcomings of the PNS-set so that it is more advantageous to deal with expert set opinions about alternatives and attributes that exist in an alternatives set V and a attributes set E.

To further clarify the usefulness and difference of our concept with other methods, we present Figure 4, which contains some basic criteria to back up this comparison.

Where the symbols (TM,FM,IM,PT,DOP,and ES) indicate to true membership, false membership, indeterminate membership, Parameterization tools, Degree of Possibility, and Expert set respectively. Finally, based on all that has been mentioned above, it can be said that our proposed concept is a generalization of all the concepts mentioned above.

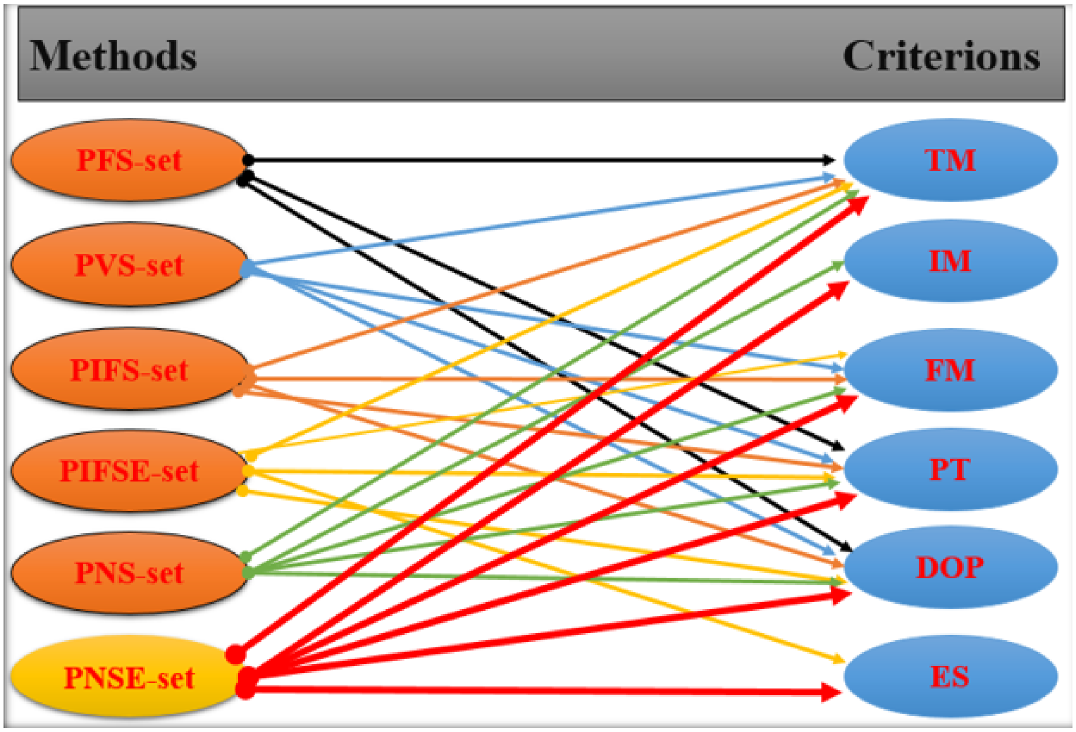

Figure 4: Comparison with current models under suitable criteria.

8. Conclusion

In this work, in the first part, the possibility neutrosophic soft expert set (PNSE-set) is developed in order to fix some weaknesses in [49]- [51]. Some properties and some fundamental set-theory were set up on PNSE-set. Also, using this method, we proposed an algorithm to solve the assumed problem in the decision-making problem. In the second part, we succeeded in applying similarity measures to this method by computing the similarity ratio between PNSE-sets. Then, these measures are applied to medical diagnosis to discover if the patient has dengue fever or not. In addition, a comparison between the existing methods and the PNSE-set was given. Finally, for further work on these topics, We recommend developing these tools by integrating them with some other mathematical structures, such as the hypersoft set [54]- [56], algebraic structures, topological structures, and other ideas [57]- [63].

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