Neutrosophic Soft Cubic Set in Topological Spaces

R. Anitha Cruz\textsuperscript{1}, F. Nirmala Irudayam \textsuperscript{2}

\textsuperscript{1}Department of Mathematics, Nirmala College for Women, Coimbatore-641018, India.
E-mail: anithacruz@gmail.com

\textsuperscript{2}Assistant Professor, Department of Mathematics, Nirmala College for Women, Coimbatore-641018, India.
E-mail: nirmalairudayam@ymail.com

Abstract: This research article lays the foundation to propose the new concept of neutrosophic soft cubic topology. Here we focus on the systematic study of neutrosophic soft cubic sets and deduce various properties which are induced by them. This enables us to introduce some equivalent characterizations and brings out the inter relations among them.

Keywords: Neutrosophic Soft Cubic Set, Neutrosophic Soft Cubic Topological Space.

1 Introduction

Molodtsov [11] proposed the concept of soft set theory in 1999 which is an entirely new approach for modeling various forms of vagueness and uncertainty of real life situations. Soft set theory has a rich potential to penetrate itself in several directions which is clearly figured out in Molodtsov’s pioneer work[11]. Zadeh [19] in 1965 came out with a novel concept of fuzzy set which deals with the degree of membership function between [0,1]. In [9] Maji et al. initiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union, intersection, complement of fuzzy soft set. Moreover in [10] Maji et al. extended the idea of soft sets to Neutrosophic setting. Neutrosophic Logic has been proposed by Florentine Smarandache[15] which is based on nonstandard analysis that was given by Abraham Robinson[14] in 1960s.

Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision undefined, incompleteness, inconsistency, redundancy, contradiction. The neutrosophic logic is formal frame to measure truth, indeterminacy and falsehood values. In Neutrosophic set, indeterminacy is quantified explicitly and the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors.


As a further development Anitha et al.[1] proposed the notion of neutrosophic soft cubic set and defined internal neutrosophic soft cubic set, external neutrosophic soft cubic set and studied some new type of internal...
neutrosophic cubic set (INSCS) and external neutrosophic cubic set (ENSCS) namely, $\frac{1}{3}$ INCS or $\frac{2}{3}$ ENSCS, $\frac{2}{3}$ INSCS or $\frac{1}{3}$ ENSCS. Anitha et al.[2-3] has discussed various operations on Neutrosophic soft cubic sets and investigated several related properties. In [4] the author has presented an application of Neutrosophic soft Cubic set in pattern recognition. Neutrosophic soft Cubic set theory was applied in BCI/BCK algebra[5]. In this paper we define neutrosophic soft cubic topological space and we discuss some of its properties.

2 Preliminaries

Definition 2.1. [19]
Let $E$ be a universe. Then a fuzzy set over $E$ is defined by $X = \{\mu_x(x) : x \in E\}$ where $\mu_x$ is called membership function of $X$ and defined by $\mu_x : E \rightarrow [0, 1]$. For each $x \in E$, the value $\mu_x(x)$ represents the degree of $x$ belonging to the fuzzy set $X$.

Definition 2.2. [17]
Let $X$ be a non-empty set. By a cubic set, we mean a structure $\Xi = \{\langle A(x), \mu(x) \rangle : x \in X\}$ in which $A$ is an interval valued fuzzy set (IVF) and $\mu(x)$ is a fuzzy set. It is denoted by $\langle A, \mu\rangle$.

Definition 2.3. [8]
Let $U$ be an initial universe set and $E$ be a set of parameters. Consider $A \subset E$. Let $P(U)$ denotes the set of all neutrosophic sets of $U$. The collection $(F, A)$ is termed to be the soft neutrosophic set over $U$, where $F$ is a mapping given by $F : A \rightarrow P(U)$.

Definition 2.4. [15]
Let $X$ be an universe. Then a neutrosophic set (NS) $\lambda$ is an object having the form $\lambda = \{\langle x : T(x), I(x), F(x) \rangle : x \in X\}$ where the functions $T, I, F : X \rightarrow [0, 1]^+\} define respectively the degree of Truth, the degree of indeterminacy, and the degree of falsehood of the element $x \in X$ to the set $\lambda$ with the condition $\sum 0 \leq T(x) + I(x) + F(x) \leq 3$.

Definition 2.5. [16]
Let $X$ be a non-empty set. An interval neutrosophic set (INS) $A$ in $X$ is characterized by the truth-membership function $A_T$, the indeterminacy-membership function $A_I$ and the falsity-membership function $A_F$. For each point $x \in X$, $A_T(x), A_I(x), A_F(x) \subseteq [0, 1]$.

Definition 2.6. [1]
Let $U$ be an the initial universal set. Let $NC(U)$ denote the set of all neutrosophic cubic sets and $E$ be a set of parameters. Let $M \subseteq E$ then $$(P, M) = \{P(e) = \{\langle x, A_e(x), \lambda_e(x) \rangle : x \in U\} : e \in M\}$$ where $A_e(x) = \{\langle x, A_e^T(x), A_e^I(x), A_e^F(x) \rangle : x \in U\}$ is an interval neutrosophic set and $\lambda_e(x) = \{\langle x, \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x) \rangle : x \in U\}$ is an neutrosophic set. Let $P(U)$ denote the set of all neutrosophic cubic sets of $U$. The collection $(P, M)$ is termed to be the neutrosophic soft cubic set over $U$, where $F$ is a mapping given by $F : A \rightarrow P(U)$. The neutrosophic soft cubic set is denoted by NSCset / NSCS. The collection of all neutrosophic soft cubic set over $U$ is denoted by NSCS(U).
3 Some Results On Neutrosophic Soft Cubic Set

**Definition 3.1.** Let \((P, E)\) be neutrosophic soft cubic set over \(U\).

(i) \((P, E)\) is called absolute or universal neutrosophic soft cubic set \(U\) if \(P(e) = \hat{1} = \{(\tilde{0}, \tilde{1}, \tilde{1}), (1, 1, 1)\}\) for all \(e \in E\). We denote it by \(U\).

(ii) \((P, E)\) is called null or empty neutrosophic soft cubic set \(\Phi\) if \(P(e) = \hat{0} = \{(\tilde{0}, \tilde{0}, \tilde{0}), (0, 0, 0)\}\) for all \(e \in E\). We denote it by \(\Phi\).

Obviously \(\Phi^c = U\) and \(U^c = \Phi\).

**Definition 3.2.** A neutrosophic soft cubic set \((P, M)\) is said to be a subset of a neutrosophic soft cubic set \((Q, N)\) if \(M \subseteq N\) and \(P(e) \subseteq Q(e) \forall e \in M, x \in U\) if and only if \(A_e(x) \subseteq B_e(x)\) and \(\lambda_e(x) \subseteq \mu_e(x) \forall e \in M, u \in U\). We denote it by \((P, M) \subseteq (Q, N)\). Where \(P(e) \subseteq Q(e) \forall e \in M, u \in U\) if and only if \(A_e(x) \subseteq B_e(x)\) and \(\lambda_e(x) \subseteq \mu_e(x)\).

\[
A_e(x) \subseteq B_e(x) \implies A_e^{-T}(x) \leq B_e^{-T}(x),
A_e^+(x) \leq B_e^+(x),
A_e^{-I}(x) \geq B_e^{-I}(x),
A_e^{+I}(x) \geq B_e^{+I}(x),
A_e^{-F}(x) \geq B_e^{-F}(x),
A_e^{+F}(x) \geq B_e^{+F}(x)
\]

and \(\lambda_e(x) \subseteq \mu_e(x) \implies \lambda_e^T(x) \leq \mu_e^T(x),\)
\[
\lambda_e^I(x) \geq \mu_e^I(x),
\lambda_e^F(x) \geq \mu_e^F(x),
\]

**Definition 3.3.** The complement of neutrosophic soft cubic set \((P, M)\) is denoted by \((P, M)^c\) and defined as \((P, M)^c = (P^e, \neg M)\) where \(P^e : \neg A \rightarrow P(U)\) is a mapping given by

\[
(P, M)^c = \{(x, 1 - A_e^T(x), 1 - A_e^I(x), 1 - A_e^F(x)), 1 - \lambda_e^T(x), 1 - \lambda_e^I(x), 1 - \lambda_e^F(x)) : xeU\} e\in M\}
\]

**Remark 3.4.** The complement of a neutrosophic soft cubic set \((P, M)\) can also be defined as \((P, M)^c = U \setminus P(e)\) for all \(e \in M\).

**Definition 3.5.** The union of two neutrosophic soft cubic sets \((P, M)\) and \((Q, N)\) is \(H(e) = \begin{cases} P(e) & \text{if } e \in A - B \\ Q(e) & \text{if } e \in B - A \\ P(e) \cup Q(e) & \text{if } e \in A \cap B \end{cases}\) over \((U, E)\) is neutrosophic soft cubic set where \(C = M \cup N, \forall e \in C\).
and is written as \((P, M) \cup (Q, N) = (H, C)\).

where \(H(e) = \langle x, \max(\tilde{A}_e(x), \tilde{B}_e(x)), \max(\lambda_e(x), \mu_e(x)) \rangle\).

**Definition 3.6.** The intersection of two neutrosophic soft cubic sets \((P, M)\) and \((Q, N)\) over \((U, E)\) is neutrosophic soft cubic set where \(C = M \cap N\) \forall e \in C\) \(H(e) = P(e) \cap Q(e)\) and is written as \((P, M) \cap (Q, N) = (H, C)\).

where \(H(e) = \langle x, \min(\tilde{A}_e(x), \tilde{B}_e(x)), \min(\lambda_e(x), \mu_e(x)) \rangle\).

**Definition 3.7.** If \((P, M)\) and \((Q, N)\) be two neutrosophic soft cubic sets then \((P, M) AND (Q, N)\) is a NSCS denoted by \((P, M) \wedge (Q, N) \equiv (H, M \times N)\), where \((H, A \times B) = P(\alpha_i) \cap F(\beta_i)\) the truth membership, indeterminacy membership and the falsity membership of \((H, A \times B)\) are as follows:

\[\begin{align*}
H^T(a_i, b_i)(h) &= \min(\tilde{A}_e^T(a)(h), \tilde{B}_e^T(b)(h)), \min(\lambda_e^T(a)(h), \lambda_e^T(b)(h)) \\
H^I(a_i, b_i)(h) &= \min(\tilde{A}_e^I(a)(h), \tilde{B}_e^I(b)(h)), \min(\lambda_e^I(a)(h), \lambda_e^I(b)(h)) \\
H^F(a_i, b_i)(h) &= \min(\tilde{A}_e^F(a)(h), \tilde{B}_e^F(b)(h)), \min(\lambda_e^F(a)(h), \lambda_e^F(b)(h))
\end{align*}\]

forall \((a_i, b_i)\) \epsilon M \times N.

**Definition 3.8.** [2] If \((P, M)\) and \((Q, N)\) be two Neutrosophic soft cubic sets then \((P, M) OR (Q, N)\) is a NSCS denoted by \((P, M) \vee (Q, N) \equiv (H, A \times B)\), where the truth membership, indeterminacy membership and the falsity membership of \((H, A \times B)\) are as follows:

\[\begin{align*}
H^T(a_i, b_i)(h) &= \max(\tilde{A}_e^T(a)(h), \tilde{B}_e^T(b)(h)), \max(\lambda_e^T(a)(h), \lambda_e^T(b)(h)) \\
H^I(a_i, b_i)(h) &= \max(\tilde{A}_e^I(a)(h), \tilde{B}_e^I(b)(h)), \max(\lambda_e^I(a)(h), \lambda_e^I(b)(h)) \\
H^F(a_i, b_i)(h) &= \max(\tilde{A}_e^F(a)(h), \tilde{B}_e^F(b)(h)), \max(\lambda_e^F(a)(h), \lambda_e^F(b)(h))
\end{align*}\]

forall \((a_i, b_i)\) \epsilon M \times N.

**Proposition 3.9.** Let \(U\) be an initial universal set and \(E\) be a set of parameters. Let \((P, E)\) and \((Q, E)\) be NSCS over \(U\). Then the following are true.

(i) \((P, E) \subseteq (Q, E)\) iff \((P, E) \cap (Q, E) = (P, E)\)

(ii) \((P, E) \subseteq (Q, E)\) iff \((P, E) \cup (Q, E) = (Q, E)\)

**Proof:**

(i) Suppose that \((P, E) \subseteq (Q, E)\) then \(P(e) \subseteq Q(e)\) for all \(e \in E\).

Let \((P, E) \cap (Q, E) = (H, E)\). Since \(H(e) = P(e) \cap Q(e) = P(e)\) for all \(e \in E\) then \((H, E) = (P, E)\).

Suppose that \((P, E) \cap (Q, E) = (P, E)\) and let \((P, E) \cap (Q, E) = (H, E)\).

Since \(H(e) = P(e) \cap Q(e)\) for all \(e \in E\), we know that \(P(e) \subseteq Q(e)\) for all \(e \in E\).

Hence \((P, E) \subseteq (Q, E)\).

(ii) Suppose that \((P, E) \subseteq (Q, E)\) then \(P(e) \subseteq Q(e)\) for all \(e \in E\).

Let \((P, E) \cup (Q, E) = (H, E)\). Since \(H(e) = P(e) \cup Q(e) = Q(e)\) for all \(e \in E\) then \((H, E) = (Q, E)\).

Suppose that \((P, E) \cup (Q, E) = (Q, E)\) and let \((P, E) \cup (Q, E) = (H, E)\).

Since \(H(e) = P(e) \cup Q(e)\) for all \(e \in E\), we know that \(P(e) \subseteq Q(e)\) for all \(e \in E\).

Hence \((P, E) \subseteq (Q, E)\).

**Proposition 3.10.** Let \(U\) be an initial universal set and \(E\) be a set of parameters. Let \((P, E), (Q, E), (H, E)\) and \((K, E)\) be NSCS over \(U\). Then the following are true.

---

*R. Anitha Cruz and F. Nirmala Irudayam. Neutrosophic Soft Cubic Set In Topological Spaces*
(i) If \((P, E) \cap (Q, E) = \emptyset\), then \((P, E) \subseteq (Q, E)^c\)

(ii) If \((P, E) \subseteq (Q, E)\) and \((Q, E) \subseteq (H, E)\) then \((P, E) \subseteq (H, E)\)

(iii) If \((P, E) \subseteq (Q, E)\) and \((H, E) \subseteq (K, E)\) then \((P, E) \cap (H, E) \subseteq (Q, E) \cap (K, E)\)

(iv) \((P, E) \subseteq (Q, E)\) iff \((Q, E)^c \subseteq (P, E)^c\)

Proof:

(i) Suppose that \((P, E) \cap (Q, E) = \emptyset\), then \(P(e) \cap Q(e) = \emptyset\).
So \(P(e) \subseteq U \setminus Q(e) = Q^c(e)\) for all \(e \in E\).
Therefore we have \((P, E) \subseteq (Q, E)^c\).

(ii) \((P, E) \subseteq (Q, E)\) and \((Q, E) \subseteq (H, E)\)
\(\Rightarrow P(e) \subseteq Q(e) \subseteq H(e)\) for all \(e \in E\)
\(\Rightarrow P(e) \subseteq Q(e) \subseteq H(e)\) for all \(e \in E\)
\(\Rightarrow P(e) \subseteq H(e)\) for all \(e \in E\).

(iii) \((P, E) \subseteq (Q, E)\) and \((H, E) \subseteq (K, E)\)
\(\Rightarrow P(e) \subseteq Q(e) \subseteq H(e)\) for all \(e \in E\)
\(\Rightarrow P(e) \subseteq Q(e) \subseteq K(e)\) for all \(e \in E\)
\(\Rightarrow (P, E) \cap (H, E) \subseteq (Q, E) \cap (K, E)\) for all \(e \in E\).

(iv) \((P, E) \subseteq (Q, E)\) \iff \(P(e) \subseteq Q(e)\) for all \(e \in E\)
\(\iff (Q(e))^c \subseteq (P(e))^c\) for all \(e \in E\)
\(\iff Q^c(e) \subseteq P^c(e)\) for all \(e \in E\)
\(\iff (Q, E)^c \subseteq (P, E)^c\).

Definition 3.11. Let \(I\) be an arbitrary indexed set and \(\{(P_i, E)\}_{i \in I}\) be a subfamily of NSCS over \(U\) with parameter set \(E\).

(i) The union of these NSCS is the NSCS \((H, E)\) where \(H(e) = \bigcup_{i \in I} P_i(e)\) for each \(e \in E\). We write
\[\bigcup_{i \in I} (P_i, E) = (H, E).\]

(ii) The intersection of these NSCS is the NSCS \((K, E)\) where \(K(e) = \bigcap_{i \in I} P_i(e)\) for each \(e \in E\). We write
\[\bigcap_{i \in I} (P_i, E) = (K, E).\]

Proposition 3.12. Let \(I\) be an arbitrary indexed set and \(\{(P_i, E)\}_{i \in I}\) be a subfamily of NSCS over \(U\) with parameter set \(E\). Then

(i) \(\left( \bigcup_{i \in I} (P_i, E) \right)^c = \bigcap_{i \in I} (P_i, E)^c\)

(ii) \(\left( \bigcap_{i \in I} (P_i, E) \right)^c = \bigcup_{i \in I} (P_i, E)^c\)

Proof:
Proposition 3.13. Let \( U \) be an initial universal set and \( E \) be a set of parameters.

(i) \((\Phi, E)^c = (U, E)\).

(ii) \((U, E)^c = (\Phi, E)\).

Proof:

(i) Let \((\Phi, E) = (P, E)\), then for all \( e \in E \),

\[
\begin{align*}
P(e) &= \{ (x, \bar{A}_e(T)(x), \hat{A}_e(I)(x), \bar{A}_e(F)(x)), \\
&\quad \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x) : x \in U \} \\
&= \{ x, \hat{0}, \hat{0}, \hat{0}, (0, 0, 0) : x \in U \}
\end{align*}
\]

\((\Phi, E)^c = (P, E)^c\), then for all \( e \in E \)

\[
\begin{align*}
&= \{ (x, \bar{A}_e(T)(x), \hat{A}_e(I)(x), \bar{A}_e(F)(x)), \\
&\quad \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x) : x \in U \}^c \\
&= \{ x, 1 - \bar{A}_e(T)(x), 1 - \hat{A}_e(I)(x), \\
&\quad 1 - \bar{A}_e(F)(x), 1 - \lambda_e^T(x), \\
&\quad 1 - \lambda_e^I(x), 1 - \lambda_e^F(x) : x \in U \} \\
&= \{ x, [1 - A_e^{+T}, 1 - A_e^{-T}](x), \\
&\quad [1 - A_e^{+I}, 1 - A_e^{-I}](x), \\
&\quad [1 - A_e^{+F}, 1 - A_e^{-F}](x), \\
&\quad 1 - \lambda_e^T(x), 1 - \lambda_e^I(x), \\
&\quad 1 - \lambda_e^F(x) : x \in U \} \\
&= \{ (x, (1, \hat{1}, \hat{1}), (1, 1, 1)) : x \in U \}.
\end{align*}
\]

Thus \((\Phi, E)^c = (U, E)\).
(ii) Let \((U, E) = (P, E)\), then for all \(e \in E\),

\[
P(e) = \{(x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x)), \\
\lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x) : x \in U\}
\]

\[
(U, E)^c = (P, E)^c, \text{ then for all } e \in E
\]

\[
(x, 1 - \tilde{A}^T_e(x), 1 - \tilde{A}^I_e(x), 1 - \tilde{A}^F_e(x) : x \in U)
\]

Thus \((U, E)^c = (\Phi, E)\)

**Proposition 3.14.** Let \(U\) be an initial universal set and \(E\) be a set of parameters.

(i) \((P, E) \cup (\Phi, E) = (P, E)\).

(ii) \((P, E) \cup (U, E) = (U, E)\).

**Proof:**

(i) \((P, E) = \{(x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x)), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x) : x \in U\} \forall e \in E

\(\Phi, E) = \{(x, (\bar{0}, \bar{0}, 0)(0, 0, 0)) : x \in U\} \forall e \in E

(P, E) \cup (\Phi, E)

\[
= \{(x, \max(\tilde{A}^T_e(x), \bar{0}), \max(\tilde{A}^I_e(x), \bar{0}), \max(\tilde{A}^F_e(x), \bar{0}), \max(\lambda_e^T(x), 0), \max(\lambda_e^I(x), 0), \max(\lambda_e^F(x), 0) : x \in U\} \forall e \in E
\]

\[
= \{(x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x)), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)) : x \in U\} \forall e \in E
\]

Thus \((P, E) \cup (\Phi, E) = (P, E)\)

(ii) \((P, E) = \{(x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x)), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x) : x \in U\} \forall e \in E

\(\Phi, E) = \{(x, (\bar{1}, \bar{1}, \bar{1})(1, 1, 1)) : x \in U\} \forall e \in E

(P, E) \cup (U, E)

\[
= \{(x, \max(\tilde{A}^T_e(x), \bar{1}), \max(\tilde{A}^I_e(x), \bar{1}), \max(\tilde{A}^F_e(x), \bar{1}), \max(\lambda_e^T(x), 1), \max(\lambda_e^I(x), 1), \max(\lambda_e^F(x), 1)) : x \in U\} \forall e \in E
\]

\[
= \{(x, (\bar{1}, \bar{1}, \bar{1})(1, 1, 1)) : x \in U\} \forall e \in E
\]

Thus \((P, E) \cup (U, E) = (U, E)\)
Proposition 3.15. Let $U$ be an initial universal set and $E$ be a set of parameters.

(i) $(P, E) \cap (\Phi, E) = (\Phi, E)$.

(ii) $(P, E) \cap (U, E) = (P, E)$.

Proof:

(i) $(P, E) = \{ (x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x)), \lambda^T_e(x), \lambda^I_e(x), \lambda^F_e(x) : x \in U \} \forall e \in E$

$(P, E) \cap (\Phi, E) = \{ (x, (0, 0, 0)(0, 0, 0)) : x \in U \} \forall e \in E$

$(P, E) \cap (U, E) = \{ (x, \min(\tilde{A}^T_e(x), 0), \min(\tilde{A}^I_e(x), 0), \min(\tilde{A}^F_e(x), 0), \min(\lambda^T_e(x), 0), \min(\lambda^I_e(x), 0), \min(\lambda^F_e(x), 1) : x \in U \} \forall e \in E$

Thus $(P, E) \cap (\Phi, E) = (\Phi, E)$

Thus $(P, E) \cap (U, E) = (P, E)$

(ii) $(P, E) = \{ (x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x)), \lambda^T_e(x), \lambda^I_e(x), \lambda^F_e(x) : x \in U \} \forall e \in E$

$(U, E) = \{ (x, (1, 1, 1)(1, 1, 1)) : x \in U \} \forall e \in E$

$(P, E) \cap (U, E) = \{ (x, \min(\tilde{A}^T_e(x), 1), \min(\tilde{A}^I_e(x), 1), \min(\tilde{A}^F_e(x), 1), \min(\lambda^T_e(x), 1), \min(\lambda^I_e(x), 1), \min(\lambda^F_e(x), 1) : x \in U \} \forall e \in E$

Thus $(P, E) \cap (U, E) = (P, E)$

Proposition 3.16. Let $U$ be an initial universal set, $E$ be a set of parameters and $M, N \subseteq E$.

(i) $(P, M) \cup (\Phi, N) = (P, M)$ iff $N \subseteq M$.

(ii) $(P, M) \cup (U, N) = (P, M)$ iff $M \subseteq N$.

Proof:

(i) For $(P, M)$ we have

$P(e) = \{ (x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x)), \lambda^T_e(x), \lambda^I_e(x), \lambda^F_e(x) : x \in U \} \forall e \in M$

Also let $(\Phi, N) = (Q, N)$, then

$Q(e) = \{ (x, (0, 0, 0)(0, 0, 0)) : x \in U \} \forall e \in N$

Let $(P, M) \cup (\Phi, N) = (P, M) \cup (Q, N) = (H, C)$

where $C = M \cup N$ and for all $e \in C$

$H(e) = \begin{cases} P(e) & \text{if } e \in M - N \\ Q(e) & \text{if } e \in N - M \\ P(e) \cap Q(e) & \text{if } e \in M \cap N \end{cases}$

$H(e) = \{ (x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x), \lambda^T_e(x), \lambda^I_e(x), \lambda^F_e(x), x \in U \} \text{ if } e \in M - N$

$\{ (x, B^T_e(x), B^I_e(x), B^F_e(x), \mu^T_e(x), \mu^I_e(x), \mu^F_e(x), x \in U \} \text{ if } e \in N - M$

$\{ (x, \max(\tilde{A}^T_e(x), B_e(x))) \text{ if } e \in M \cap N}$
Let \( N \subseteq M \), then

\[
H(e) = \begin{cases} 
\{ \langle x, \tilde{A}_e^T(x), \tilde{A}_e^L(x), \tilde{A}_e^F(x) \rangle, \\
\lambda_e^T(x), \lambda_e^L(x), \lambda_e^F(x) : x \in U \} & \text{if } e \in M - N \\
\{ \langle x, (0, 0, 0)(0, 0, 0)(0, 0, 0) \rangle : x \in U \} & \text{if } e \in N - M \\
\{ \langle x, \tilde{A}_e^T(x), \tilde{A}_e^L(x), \tilde{A}_e^F(x) \rangle, \\
\lambda_e^T(x), \lambda_e^L(x), \lambda_e^F(x) : x \in U \} & \text{if } e \in M \cap N 
\end{cases}
\]

\[= P(e), \forall e \in M. \]

Conversely, Let \( (P, M) \cup (\Phi, N) = (P, M) \)

Then \( M = M \cup N \Rightarrow N \subseteq M \)

(ii) For \((P, M)\) we have

\[
P(e) = \{ \langle x, \tilde{A}_e^T(x), \tilde{A}_e^L(x), \tilde{A}_e^F(x) \rangle, \lambda_e^T(x), \lambda_e^L(x), \lambda_e^F(x) : x \in U \} \forall e \in M
\]

Also let \((U, N) = (Q, N),\) then

\[
Q(e) = \{ \langle x, (1, 1, 1)(1, 1, 1) \rangle : x \in U \} \forall e \in N
\]

Let \((P, M) \cup (U, N) = (P, M) \cup (Q, N) = (H, C)\)

where \( C = M \cup N \) and for all \( e \in C \)

\[
H(e) = \begin{cases} 
P(e) & \text{if } e \in M - N \\
Q(e) & \text{if } e \in N - M \\
P(e) \cup Q(e) & \text{if } e \in M \cap N 
\end{cases}
\]
Let $U$ be an initial universal set, $E$ be a set of parameters and $M, N \subseteq E$.

(i) $(P, M) \cap (\Phi, N) = (\Phi, M \cap N)$.

(ii) $(P, M) \cap (U, N) = (P, M \cap N)$.

Proof:

(i) For $(P, M)$ we have $P(e) = \{ (x, A^T_e(x), A^I_e(x), A^F_e(x), \lambda^T_e(x), \lambda^I_e(x), \lambda^F_e(x)) : x \in U \}$ if $e \in M - N$.

Let $M \subseteq N$, then $H(e) = \{ (x, (\bar{\bar{1}}, \bar{\bar{1}}, \bar{\bar{1}})(1, 1, 1)(x)) : x \in U \}$ if $e \in M \cap N$.

Conversely, Let $(P, M) \cup (U, N) = (U, N)$

Then $N = M \cup N \Rightarrow M \subseteq N$.

**Proposition 3.17.** Let $U$ be an initial universal set, $E$ be a set of parameters and $M, N \subseteq E$.

(i) $(P, M) \cap (\Phi, N) = (\Phi, M \cap N)$.

(ii) $(P, M) \cap (U, N) = (P, M \cap N)$.

Proof:

(i) For $(P, M)$ we have $P(e) = \{ (x, A^T_e(x), A^I_e(x), A^F_e(x), \lambda^T_e(x), \lambda^I_e(x), \lambda^F_e(x)) : x \in U \}$ if $e \in M - N$.

Also let $(\Phi, N) = (Q, N)$, then $Q(e) = \{ x, (0, 0, 0), (0, 0, 0) : x \in U \}$ $\forall e \in N$.

Let $(P, M) \cap (\Phi, N) = (P, M) \cap (Q, N) = (H, C)$

where $C = M \cap N$ and for all $e \in C$.
Proposition 3.18. Let \( U \) be an initial universal set, \( E \) be a set of parameters and \( \mathcal{R} \). Anitha Cruz and F. Nirmala Irudayam. Neutrosophic Soft Cubic Set In Topological Spaces.

Thus \((P, M) \cap (\Phi, N) = (\Phi, M \cap N)\).

(ii) For \((P, M)\) we have
\[
P(e) = \{ (x, \tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x), \lambda_e(x), \mu_e(x)) : x \in U \} \quad e \in M
\]
Also let \((U, N) = (Q, N)\), then
\[
Q(e) = \{ x, (\tilde{\Lambda}_e(x), 1, 1, 1) : x \in U \} \quad \forall e \in N
\]
Let \((P, M) \cap (U, N) = (P, M) \cap (Q, N) = (H, C)\)
where \(C = M \cap N\) and for all \(e \in C\)
\[
H(e) = \{ (x, \min(\tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x)), \lambda_e(x), \mu_e(x)) : x \in U \}
\]
\[
= \{ (x, \min(\tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x)), \lambda_e(x), \mu_e(x)) : x \in U \}
\]
\[
= \{ (x, \tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x), \lambda_e(x), \mu_e(x)) : x \in U \}
\]
\[
= \{ (x, \max(\tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x)), \lambda_e(x), \mu_e(x)) : x \in U \}
\]
Thus \((P, M) \cap (U, N) = (P, M \cap N)\).

Proposition 3.18. Let \( U \) be an initial universal set, \( E \) be a set of parameters and \( M, N \subseteq E \).

(i) \( (P, M) \cup (Q, N))^e \subseteq (P, M)^e \cup (Q, N)^e \).

(ii) \( (P, M)^e \cap (Q, N)^e \subseteq ((P, M) \cap (Q, N))^e \).

Proof:

(i) Let \((P, M) \cup (Q, N) = (H, C)\) where \(C = M \cup N\) and \(\forall e \in C\)
\[
H(e) = \begin{cases} 
\{ (x, \tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x), \lambda_e(x), \lambda_e(x), \lambda_e(x)) : x \in U \}, & \text{if } e \in M - N \\
\{ (x, \tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x), \lambda_e(x), \mu_e(x)) : x \in U \}, & \text{if } e \in N - M \\
\{ (x, \max(\tilde{\Lambda}_e(x), \tilde{\Lambda}_e(x)), \lambda_e(x), \mu_e(x)) : x \in U \}, & \text{if } e \in M \cap N 
\end{cases}
\]
Thus \((P, M) \cup (Q, N))^e = (H, C)^e\), where \(C = M \cup N\) and \(\forall e \in C\)
\[
(H(e))^e = \begin{cases} 
\{ (P(e))^e \}, & \text{if } e \in M - N \\
\{ (Q(e))^e \}, & \text{if } e \in N - M \\
\{ (P(e) \cup Q(e))^e \}, & \text{if } e \in N \cap M 
\end{cases}
\]
\[
= \begin{cases} 
\{ (x, 1 - \tilde{\Lambda}_e(x), 1 - \tilde{\Lambda}_e(x), 1 - \tilde{\Lambda}_e(x)), 1 - \lambda_e(x), e(x)) : x \in U \}, & \text{if } e \in M - N \\
\{ (x, 1 - \tilde{\Lambda}_e(x), 1 - \tilde{\Lambda}_e(x), 1 - \tilde{\Lambda}_e(x), 1 - \lambda_e(x), e(x)) : x \in U \}, & \text{if } e \in N - M \\
\{ (x, 1 - \mu_e(x), 1 - \mu_e(x), 1 - \mu_e(x)) : x \in U \}, & \text{if } e \in M \cap N 
\end{cases}
\]
Again \((P, M)^c \cup (Q, N)^c = (I, J)\) say \(J = M \cup N\) and \(\forall e \in J\)

\[
I(e) = \begin{cases} 
\{(P(e))^c\} & \text{if } e \in M - N \\
\{(Q(e))^c\} & \text{if } e \in N - M \\
\{(P(e) \cup Q(e))^c\} & \text{if } e \in M \cap N
\end{cases}
\]

\[
= \begin{cases} 
\{(x, 1 - P^T_e(x), 1 - P^I_e(x), 1 - P^F_e(x)) : x \in U\}, & \text{if } e \in M - N \\
\{(x, 1 - Q^T_e(x), 1 - Q^I_e(x), 1 - Q^F_e(x)) : x \in U\}, & \text{if } e \in N - M \\
\{(x, 1 - \max(A^T_e(x), B^T_e(x)), 1 - \max(A^I_e(x), B^I_e(x)), 1 - \max(A^F_e(x), B^F_e(x)), \\
1 - \max(\lambda^T_e(x), \mu^T_e(x)), 1 - \max(\lambda^I_e(x), \mu^I_e(x)), 1 - \max(\lambda^F_e(x), \mu^F_e(x)) : x \in U\} & \text{if } e \in M \cap N
\end{cases}
\]

\(C \subseteq J \forall e \in J. (H(e))^c \subseteq I(e)\). Thus \(((P, M) \cup (Q, N))^c \subseteq (P, M)^c \cup (Q, N)^c\)

(ii) Let \((P, M) \cap (Q, N) = (H, C)\) where \(C = M \cap N\) and \(\forall e \in C\) and 

\[
H(e) = P(e) \cap Q(e) = \{(x, \min(P^T_e(x), Q^T_e(x)), \min(P^I_e(x), Q^I_e(x)), \min(P^F_e(x), Q^F_e(x)))\}
\]

where

\[
\begin{align*}
\min(P^T_e(x), Q^T_e(x)) &= \min(A^T_e(x), B^T_e(x)), \\
\min(P^I_e(x), Q^I_e(x)) &= \min(A^I_e(x), B^I_e(x)), \\
\min(P^F_e(x), Q^F_e(x)) &= \min(\lambda^F_e(x), \mu^F_e(x))
\end{align*}
\]

Thus \(((P, M) \cap (Q, N))^c = (H, C)^c\), where \(C = M \cap N\) and \(\forall e \in C\)

\[
(H(e))^c = \{(x, 1 - \min(P^T_e(x), Q^T_e(x)), 1 - \min(P^I_e(x), Q^I_e(x)), 1 - \min(P^F_e(x), Q^F_e(x)))\}
\]

Again \((P, M)^c \cap (Q, N)^c = (I, J)\) say \(J = M \cup N\) and \(\forall e \in J\)

\[
I(e) = (P(e))^c \cap (Q(e))^c
\]

\[
= \{(x, 1 - \min(P^T_e(x), Q^T_e(x)), 1 - \min(P^I_e(x), Q^I_e(x)), 1 - \min(P^F_e(x), Q^F_e(x)))\}
\]

We see that \(C = J\) and \(\forall e \in J, I(e) \subseteq (H(e))^c\).

Thus \((P, M)^c \cap (Q, N)^c \subseteq ((P, M) \cup (Q, N))^c\).

**Proposition 3.19.** (De Morgan’s Law) For neutrosophic soft cubic sets \((P, E)\) and \((Q, E)\) over the same universe \(U\) with parameter set \(E\), we have the following.

(i) \(((P, E) \cup (Q, E))^c = (P, E)^c \cap (Q, E)^c\).

(ii) \(((P, E) \cap (Q, E))^c = (P, E)^c \cup (Q, E)^c\).

**Proof:**

(i) Let \((P, E) \cup (Q, E) = (H, E)\) where \(\forall e \in E\)

\[
H(e) = P(e) \cup Q(e)
\]

\[
= \{(x, \max(P^T_e(x), Q^T_e(x)), \max(P^I_e(x), Q^I_e(x)), \max(P^F_e(x), Q^F_e(x)))\}
\]

Thus \(((P, E) \cup (Q, E))^c = (H, E)^c\) where \(\forall e \in E\)

\[
H(e)^c = (P(e) \cup Q(e))^c
\]

\[
= \{(x, \min(P^T_e(x), Q^T_e(x)), \min(P^I_e(x), Q^I_e(x)), \min(P^F_e(x), Q^F_e(x)))\}
\]

Again \((P, E)^c \cap (Q, E)^c = (I, E)\) say \(\forall e \in E\)
Proposition 3.20. Let \((P, E) \cap (Q, E) = (H, E)\) where \(\forall \in E\)

\[H(e) = P(e) \cap Q(e)\]

Thus \(((P, E) \cap (Q, E))^c = (P, E)^c \cap (Q, E)^c\).

(ii) Let \((P, E) \cap (Q, E) = (H, E)\) where \(\forall \in E\)

\[H(e) = P(e) \cap Q(e)\]

Thus \(((P, E) \cap (Q, E))^c = (H, E)^c\) where \(\forall \in E\)

\[H(e)^c = (P(e) \cap Q(e))^c\]

\[I(e)^c = (P(e)^c \cup (Q(e)^c)\]

Again \((P, E)^c \cup (Q, E)^c = (I, E)\) say \(\forall \in E\)

\[I(e) = P(e)^c \cup (Q(e)^c)\]

\[I(e)^c = (P(e) \cap Q(e))^c\]

\[[((P, M) \wedge (Q, N))^c = (P, M)^c \vee (Q, N)^c\].

(iii) \(((P, M) \wedge (Q, N))^c = (P, M)^c \wedge (Q, N)^c\).

Proof:

(i) \(((P, M) \wedge (Q, N))^c = (P, M)^c \vee (Q, N)^c\).

(ii) \(((P, M) \wedge (Q, N))^c = (P, M)^c \wedge (Q, N)^c\).

Let \((P, M) \wedge (Q, N) = (H, M \times N)\) where

\[H(m, n) = \{(x, min(P^e(x), Q^e(x)), min(P^f(x), Q^f(x)))\}

\(\forall m \in M\) and \(\forall n \in N\)

\[((P, M) \wedge (Q, N))^c = (H, M \times N)^c \forall (a, b) \in M \times N\]

\[(H(m, n))^c = \{(x, 1 - min(P^e(x), Q^e(x)), 1 - min(P^f(x), Q^f(x)))\} \forall m \in M\) and \(\forall n \in N\)

Let \((P, M)^c \vee (Q, N)^c = (R, M \times N)\) where

\[R(m, n) = \{(x, max(1 - P^e(x), 1 - Q^e(x)), max(1 - P^f(x), 1 - Q^f(x)))\} \forall m \in M\) and \(\forall n \in N\)

\[max(1 - P^e(x), 1 - Q^e(x)), max(1 - P^f(x), 1 - Q^f(x)))\} \forall m \in M\) and \(\forall n \in N\)

\[H(m, n) = \{(x, max(P^e(x), Q^e(x)), max(P^f(x), Q^f(x)))\}

\(\forall m \in M\) and \(\forall n \in N\)

\[((P, M) \wedge (Q, N))^c = (H, M \times N)^c \forall (a, b) \in M \times N\]

\[(H(m, n))^c = \{(x, max(P^e(x), Q^e(x)), max(P^f(x), Q^f(x)))\} \forall m \in M\) and \(\forall n \in N\)

Thus \(((P, M) \wedge (Q, N))^c = (P, M)^c \wedge (Q, N)^c\).
\[(x, 1 - \max(P^T_e(x), Q^T_e(x)), 1 - \max(P^I_e(x), Q^I_e(x)), 1 - \max(P^F_e(x), Q^F_e(x))) \} \forall m \in M \text{ and } \forall n \in N \]

Let \((P, M)^c \land (Q, N)^c = (R, M \times N)\) where
\[R(m, n) = \{(x, \min(1 - P^T_e(x), 1 - Q^T_e(x)), \min(1 - P^I_e(x), 1 - Q^I_e(x)), \min(1 - P^F_e(x), 1 - Q^F_e(x))) \} \forall m \in M \text{ and } \forall n \in N \]

Thus \(((P, M) \lor (Q, N))^c = (P, M)^c \land (Q, N)^c\)

4 Neutrosophic soft cubic topological spaces

In this section, we give the definition of neutrosophic soft cubic topological spaces with some examples and results.

Let \(U\) be an universe set, \(E\) be the set of parameters, \(\varphi(U)\) be the set of all subsets of \(U\), \(\text{NSCS}(U)\) be the set of all neutrosophic soft cubic sets in \(U\) and \(\text{NSCS}(U; E)\) be the family of all neutrosophic soft cubic sets over \(U\) via parameters in \(E\).

**Definition 4.1.** Let \((X, E)\) be an element of \(\text{NSCS}(U; E)\), \(\varphi(X; E)\) be the collection of all neutrosophic soft cubic subsets of \((X, E)\). A sub family \(\tau\) of \(\varphi(X; E)\) is called neutrosophic soft cubic topology (in short NSCTS-topology) on \((X, E)\) if the following conditions hold.

1. \((\Phi, E), (X, E) \in \tau\).
2. \((P, E), (Q, E) \in \tau\) implies \((P, E) \cap (Q, E) \in \tau\).
3. \(\{(P_\alpha, E); \alpha \in \Gamma\} \in \tau\) implies \(\bigcup \{(P_\alpha, E); \alpha \in \Gamma\} \in \tau\)

The triplet \((X, \tau, E)\) is called a neutrosophic soft cubic topological space (in short NSCTS) over \((X, E)\). Every member of \(\tau\) is called a neutrosophic soft cubic open set in \((X, E)\) (in short NSCTOS)(X).

\(\Phi: A \rightarrow \text{NSCS}(U)\) is defined as
\(\Phi(e) = \{x, ([0,0], [0,0], [0,0], (0,0),(0,0),(0,0)), x \in X\} \forall e \in A \subseteq E\).

A neutrosophic soft cubic subset \((P, E)\) of \((X, E)\) is called a neutrosophic soft cubic closed set in \((X, E)\) (in short NSCCS(X)) if \((P, E) \in \tau^c\) where \(\tau^c = \{(P, E)^c; (P, E) \in \tau\}\)

**Example 4.2.** Let \(X = \{x_1, x_2, x_3\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1, e_2, e_3\}\). The tabular representations which are shown in Table 1-6.

<table>
<thead>
<tr>
<th>(X)</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(c_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_1)</td>
<td>(0.5, 0.8), (0.3, 0.5), (0.2, 0.7)</td>
<td>(0.8, 0.4, 0.5),</td>
<td>(0.8, 0.4, 0.5),</td>
</tr>
<tr>
<td>(x_2)</td>
<td>(0.5, 0.1), (0.4, 0.6), (0.4, 0.6)</td>
<td>(0.8, 0.4, 0.5),</td>
<td>(0.8, 0.4, 0.5),</td>
</tr>
<tr>
<td>(x_3)</td>
<td>(0.4, 0.7), (0.3, 0.4), (0.1, 0.2)</td>
<td>(0.9, 0.5, 0.6),</td>
<td>(0.6, 0.9), (0.1, 0.2), (0.10, 0.2)</td>
</tr>
</tbody>
</table>

Table 1: The tabular representation of \((X, E)\).

Here the sub-family \(\tau_1 = \{(\Phi, E), (X, E), (P, E), (Q, E), (H, E), (L, E)\}\) of \(\varphi(X, E)\) is a neutrosophic soft cubic topology of \((X, E)\), as it satisfies the necessary three axioms of topology and \((X, \tau, E)\) is a NSCTS. But the sub-family \(\tau_2 = \{(\Phi, E), (X, E), (P, E), (Q, E)\}\) of \(\varphi(X, E)\) is not a neutrosophic soft cubic topology on \((X, E)\), as the union \((P, E) \cup (Q, E)\) does not belong to \(\tau_2\).

---

*R. Anitha Cruz and F. Nirmala Irudayam. Neutrosophic Soft Cubic Set In Topological Spaces*
Let \((H, E) = (P, E) \cap (Q, E)\) The tabular representation of \((H, E)\) is given by

<table>
<thead>
<tr>
<th>(x)</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(c_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>[0.1, 0.1, 0.1, 0.1, 0.1]</td>
<td>[0.1, 0.1, 0.1, 0.1, 0.1]</td>
<td>[0.1, 0.1, 0.1, 0.1, 0.1]</td>
</tr>
<tr>
<td>0.2</td>
<td>[0.2, 0.2, 0.2, 0.2, 0.2]</td>
<td>[0.2, 0.2, 0.2, 0.2, 0.2]</td>
<td>[0.2, 0.2, 0.2, 0.2, 0.2]</td>
</tr>
<tr>
<td>0.3</td>
<td>[0.3, 0.3, 0.3, 0.3, 0.3]</td>
<td>[0.3, 0.3, 0.3, 0.3, 0.3]</td>
<td>[0.3, 0.3, 0.3, 0.3, 0.3]</td>
</tr>
<tr>
<td>0.4</td>
<td>[0.4, 0.4, 0.4, 0.4, 0.4]</td>
<td>[0.4, 0.4, 0.4, 0.4, 0.4]</td>
<td>[0.4, 0.4, 0.4, 0.4, 0.4]</td>
</tr>
<tr>
<td>0.5</td>
<td>[0.5, 0.5, 0.5, 0.5, 0.5]</td>
<td>[0.5, 0.5, 0.5, 0.5, 0.5]</td>
<td>[0.5, 0.5, 0.5, 0.5, 0.5]</td>
</tr>
</tbody>
</table>

Table 5: The tabular representation of \((H, E)\)

Let \((L, E) = (P, E) \cup (Q, E)\) The tabular representation of \((L, E)\) is given by

<table>
<thead>
<tr>
<th>(x)</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(c_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>[0.1, 0.1, 0.1, 0.1, 0.1]</td>
<td>[0.1, 0.1, 0.1, 0.1, 0.1]</td>
<td>[0.1, 0.1, 0.1, 0.1, 0.1]</td>
</tr>
<tr>
<td>0.2</td>
<td>[0.2, 0.2, 0.2, 0.2, 0.2]</td>
<td>[0.2, 0.2, 0.2, 0.2, 0.2]</td>
<td>[0.2, 0.2, 0.2, 0.2, 0.2]</td>
</tr>
<tr>
<td>0.3</td>
<td>[0.3, 0.3, 0.3, 0.3, 0.3]</td>
<td>[0.3, 0.3, 0.3, 0.3, 0.3]</td>
<td>[0.3, 0.3, 0.3, 0.3, 0.3]</td>
</tr>
<tr>
<td>0.4</td>
<td>[0.4, 0.4, 0.4, 0.4, 0.4]</td>
<td>[0.4, 0.4, 0.4, 0.4, 0.4]</td>
<td>[0.4, 0.4, 0.4, 0.4, 0.4]</td>
</tr>
<tr>
<td>0.5</td>
<td>[0.5, 0.5, 0.5, 0.5, 0.5]</td>
<td>[0.5, 0.5, 0.5, 0.5, 0.5]</td>
<td>[0.5, 0.5, 0.5, 0.5, 0.5]</td>
</tr>
</tbody>
</table>

Table 6: The tabular representation of \((L, E)\)
Definition 4.3. As every NSC-topology on \((X, E)\) must contain the sets \((\Phi, E), (X, E)\) \(\in \tau\) so the family \(\tau = \{(\Phi, E), (X, E)\}\) forms a NSC-topology on \((X, E)\). The topology is called indiscrete NSC-topology and the triplet \((X, \tau, E)\) is called an indiscrete neutrosophic soft cubic topological space (or simply indiscrete NSC-topological space).

Definition 4.4. Let \(\xi\) denote the family of all NSC-subsets of \((X, E)\). Then we observe that \(\xi\) satisfies all the axioms of topology on \((X, E)\). This topology is called discrete neutrosophic soft cubic topology and the triplet \((X, \xi, E)\) is called discrete neutrosophic soft cubic topological space (or simply discrete NSCTS).

Definition 4.5. Let \((X, \tau, E)\) be an NSC-topological space over \((X, E)\). A neutrosophic soft cubic subset \((P, E)\) of \((X, E)\) is called neutrosophic soft cubic set (in short NSC-closed set) if its complement \((P, E)^c\) is a member of \(\tau\).

Example 4.6. Let us consider Example 4.2 then the NSC-closed set in \((X, \tau_1, E)\) are shown in Table7-12.

<table>
<thead>
<tr>
<th>X</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(c_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_1)</td>
<td>[0.5, 0.8, 0.3, 0.5, 0.2, 0.7]</td>
<td>[0.8, 0.4, 0.5]</td>
<td>[0.4, 0.7, 0.2, 0.3, 0.1, 0.6, 0.7, 0.5]</td>
</tr>
<tr>
<td>(x_2)</td>
<td>[0.4, 0.7, 0.3, 0.4, 0.1, 0.2]</td>
<td>[0.6, 0.9, 0.1, 0.2, 0.1, 0.2]</td>
<td>[0.6, 0.8, 0.1, 0.2, 0.1, 0.2, 0.8, 0.5]</td>
</tr>
</tbody>
</table>

Table 7: The tabular representation of \((X, E)^c\)

<table>
<thead>
<tr>
<th>X</th>
<th>(c_1)</th>
<th>(c_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_1)</td>
<td>[1, 1, 1, 1, 1, 1, 1, 1]</td>
<td>[1, 1, 1, 1, 1, 1, 1, 1]</td>
</tr>
<tr>
<td>(x_2)</td>
<td>[1, 1, 1, 1, 1, 1, 1, 1]</td>
<td>[1, 1, 1, 1, 1, 1, 1, 1]</td>
</tr>
<tr>
<td>(x_3)</td>
<td>[1, 1, 1, 1, 1, 1, 1, 1]</td>
<td>[1, 1, 1, 1, 1, 1, 1, 1]</td>
</tr>
</tbody>
</table>

Table 8: The tabular representation of \((\Phi, E)^c\)

<table>
<thead>
<tr>
<th>X</th>
<th>(c_1)</th>
<th>(c_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_1)</td>
<td>[0.4, 0.9, 0.2, 0.6, 0.0, 0.7]</td>
<td>[0.2, 0.6, 0.0, 0.7]</td>
</tr>
<tr>
<td>(x_2)</td>
<td>[0.7, 0.9, 0.4, 1.0, 0.4, 0.0, 0.5, 0.5]</td>
<td>[0.6, 0.3, 0.0, 0.2, 0.8]</td>
</tr>
<tr>
<td>(x_3)</td>
<td>[0.7, 0.9, 0.4, 0.0, 0.4, 0.3, 0.4, 0.7]</td>
<td>[0.4, 0.5, 0.6, 0.4, 0.3, 0.4, 0.6]</td>
</tr>
</tbody>
</table>

Table 9: The tabular representation of \((P, E)^c\)

Proposition 4.7. Let \((X, \tau_1, E)\) and \((X, \tau_2, E)\) be two neutrosophic soft cubic topological spaces. Denote \(\tau_1 \cap \tau_2 = \{(P, E) : (P, E) \in \tau_1 and (P, E) \in \tau_2\}\). Then \(\tau_1 \cap \tau_2\) is a neutrosophic soft cubic topology.

Proof:

(i) Since \((X, \tau_1, E)\) is a neutrosophic soft cubic topological space then \((\Phi, E) \in \tau_1\), \((X, \tau_2, E)\) is a neutrosophic soft cubic topological space then \((\Phi, E) \in \tau_2\).

Therefore \((\Phi, E) \in \tau_1 \cap \tau_2\)

Since \((X, \tau_1, E)\) is a neutrosophic soft cubic topological space then \((X, E) \in \tau_1\), \((X, \tau_2, E)\) is a neutrosophic soft cubic topological space then \((X, E) \in \tau_2\).

Therefore \((X, E) \in \tau_1 \cap \tau_2\)

(ii) Let \((P, E), (Q, E) \in \tau_1 \cap \tau_2\). Then \((P, E), (Q, E) \in \tau_1\) and \((P, E), (Q, E) \in \tau_2\), \(\tau_1\) and \(\tau_2\) are two neutrosophic soft cubic topologies on \(X\). Then \((P, E) \cap (Q, E) \in \tau_1\) and \((P, E) \cap (Q, E) \in \tau_2\). Hence \((P, E) \cap (Q, E) \in \tau_1 \cap \tau_2\).
Let \( \{(P_\alpha, E) ; \alpha \in \Gamma \} \in \tau_1 \cap \tau_2 \). Then \( \{(P_\alpha, E) ; \alpha \in \Gamma \} \in \tau_1 \) and \( \{(P_\alpha, E) ; \alpha \in \Gamma \} \in \tau_2 \). Since \( \tau_1 \) and \( \tau_2 \) are two neutrosophic soft cubic topologies on \( X \). Then \( \bigcup \{(P_\alpha, E) ; \alpha \in \Gamma \} \in \tau_1 \) and \( \bigcup \{(P_\alpha, E) ; \alpha \in \Gamma \} \in \tau_2 \).

Thus \( \bigcup \{(P_\alpha, E) ; \alpha \in \Gamma \} \in \tau_1 \cap \tau_2 \).

**Theorem 4.8.** Let \( \{\tau_i : i \in I\} \) be any collection of NSC-topology on \((X, E)\). Then their intersection \( \bigcap_{i \in I} \tau_i \) is also a NSC-topology on \((X, E)\).

**Proof:**

(i) Since \((\Phi, E), (X, E) \in \tau_i \) for each \( i \in I \). Hence \((\Phi, E), (X, E) \in \bigcap_{i \in I} \tau_i \).

(ii) Let \( \{(P_\alpha, E) ; \alpha \in \Gamma \} \) be an arbitrary family of neutrosophic soft cubic sets where \( \{(P_\alpha, E) ; \alpha \in \Gamma \} \) for each \( \alpha \in \Gamma \). Then for each \( i \in I \) \( \{(P_\alpha, E) ; \alpha \in \Gamma \} \) is a NSC-topology, therefore for each \( i \in I \) \( \{(P_\alpha, E) ; \alpha \in \Gamma \} \) is a NSC-topology. Hence \( \bigcup_{i \in I} \{(P_\alpha, E) ; \alpha \in \Gamma \} \) is a NSC-topology.

(iii) Let \( (P, E), (Q, E)X \in \bigcap_{i \in I} \tau_i \), then \( (P, E), (Q, E)X \in \tau_i \) for each \( i \in I \). Since for each \( i \in I \) \( \tau_i \) is an NSC-topology, therefore \( (P, E) \cap (Q, E)X \in \bigcap_{i \in I} \tau_i \).

Thus \( \bigcap_{i \in I} \tau_i \) satisfies all the axioms of topology. Hence \( \bigcap_{i \in I} \tau_i \) forms a NSC-topology. But union of NSC-topologies need not be a NSC-topology. Let us show this with the following example

**Remark 4.9.** If \( \tau_1 \) and \( \tau_2 \) be two neutrosophic soft cubic topologies on \((X, E)\).

(i) \( \tau_1 \cap \tau_2 = \{(P, E) \cup (Q, E) : (P, E) \in \tau_1 \) and \( (Q, E) \in \tau_2 \} \).

(ii) \( \tau_1 \cap \tau_2 = \{(P, E) \cap (Q, E) : (P, E) \in \tau_1 \) and \( (Q, E) \in \tau_2 \} \).

**Example 4.10.** Let \((P, E)\) and \((Q, E)\) be neutrosophic soft cubic set as in Example 28.

Define \( \tau_1 = \{(\Phi, E), (X, E), (P, E)\} \) and \( \tau_2 = \{(\Phi, E), (X, E), (Q, E)\} \).

Then \( \tau_1 \cap \tau_2 = \{(\Phi, E), (X, E), (P, E) \cup (Q, E)\} \) is neutrosophic soft cubic topology on \( X \).

But \( \tau_1 \cup \tau_2 = \{(\Phi, E), (X, E), (P, E) \cup (Q, E)\} \).

**Definition 4.11.** Let \((X, \tau, E)\) be a neutrosophic soft cubic topological space on \((X, E)\) and \((Y, E) \in \varphi(X, E)\).

Then the collection \( \tau_Y = \{(Y, E) \cap (Q, E) : (Q, E) \in \tau\} \) is called a neutrosophic soft cubic subspace topology on \((Y, E)\).

Hence \((Y, \tau_Y, E)\) is called a neutrosophic soft cubic topological subspace of \((X, \tau, E)\).

**Theorem 4.12.** Let \((X, \tau, E)\) be a neutrosophic soft cubic topological space and \(e \in E\), \( \tau(e) = \{P(e) : (P, E) \in \tau\} \) is a neutrosophic soft cubic topology on \((X, E)\).

**Proof:** Let \( e \in E \)

<table>
<thead>
<tr>
<th>( e )</th>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>( e_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>([0.3, 0.6], [0.3, 0.5], [0.1, 0.6], [0.5, 0.6, 0.4])</td>
<td>([0.7, 0.8], [0.5, 0.6], [0.1, 0.4], [0.5, 0.6, 0.4])</td>
<td>([0.3, 0.7], [0.2, 0.5], [0.8, 0.9], [0.6, 0.5, 0.6])</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>([0.1, 0.7], [0.8, 0.9], [0.6, 0.4], [0.4, 0.3, 0.7])</td>
<td>([0.4, 0.5], [0.3, 0.4], [0.6, 0.7], [0.7, 0.6, 0.3])</td>
<td>([0.4, 0.8], [0.5, 0.6], [0.2, 0.5], [0.6, 0.5, 0.3])</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>([0.8, 0.9], [0.2, 0.6], [0.6, 0.9], [0.5, 0.6, 0.5])</td>
<td>([0.4, 0.6], [0.5, 0.7], [0.5, 0.8], [0.6, 0.4, 0.7])</td>
<td>([0.7, 0.9], [0.5, 0.7], [0.2, 0.4], [0.7, 0.5, 0.6])</td>
</tr>
</tbody>
</table>

Table 10: The tabular representation of \((Q, E)^c\)

R. Anitha Cruz and F. Nirmala Irudayam. Neutrosophic Soft Cubic Set In Topological Spaces
(i) Let $(\Phi, E), (X, E) \in \tau$, $(\Phi, E), (X, E) \in \tau(e)$.

(ii) Let $V, W \in \tau$, then there exist $(P, E), (Q, E) \in \tau$ such that $V = P(e)$ and $W = Q(e)$.
Since $\tau$ is a neutrosophic soft cubic topology on $X, (P, E) \cap (Q, E) \in \tau$.
Put $(H, E) = (P, E) \cap (Q, E)$. Then $(H, E) \in \tau$.
We have $V \cap W = P(e) \cap Q(e) = H(e)$ and $\tau(e) = \{P(e) : (P, E) \in \tau\}$.
Then $V \cap W \in \tau$.

(iii) Let $\{(V_{\alpha}, E) : \alpha \in \Gamma\} \in \tau(e)$.
Then for every $\alpha \in \Gamma$, there exist $(P_{\alpha}, E) \in \tau$ such that $V_{\alpha} = P_{\alpha}(e)$.
Since $\tau$ is a neutrosophic soft cubic topology on $X, \bigcup \{\{(P_{\alpha}, E) : \alpha \in \Gamma\} \in \tau\}$.
Put $(P, E) = \bigcup \{(P_{\alpha}, E) : \alpha \in \Gamma\}$ then $(P, E) \in \tau$.
Note that $\bigcup_{\alpha \in \Gamma} V_{\alpha} = \bigcup_{\alpha \in \Gamma} \{P_{\alpha}(e) : \alpha \in \Gamma\} = P(e)$ and $\tau(e) = \{P(e) : (P, E) \in \tau\}$.
Then $\bigcup_{\alpha \in \Gamma} V_{\alpha} \in \tau(e)$.
Therefore $\tau(e) = \{P(e) : (P, E) \in \tau\}$ is a neutrosophic soft cubic topology on $X$.

**Definition 4.13.** Let $(X, \tau, E)$ be a neutrosophic soft cubic topological space over $(X, E)$ and $\mathcal{B} \subseteq \tau$. $\mathcal{B}$ is a basis on $\tau$ if for each $(Q, E) \in \tau$, there exist $\mathcal{B}' \subseteq \mathcal{B}$ such that $(Q, E) = \bigcup \mathcal{B}'$

**Example 4.14.** Let $\tau$ be a neutrosophic soft cubic topology as in Example 28. Then
$\mathcal{B} = \{(P, E), (Q, E), (L, E), (\Phi, E), (X, E)\}$ is a basis for $\tau$

**Theorem 4.15.** Let $\mathcal{B}$ be a basis for neutrosophic soft cubic topology $\tau$. Denote $\mathcal{B}_e = \{P(e) : (P, E) \in \mathcal{B}\}$ and $\tau(e) = \{P(e) : (P, E) \in \tau\}$ for any $e \in E$. Then $\mathcal{B}_e$ is a basis for neutrosophic soft cubic topology $\tau(e)$.

**Proof:** Let $e \in E$. For any $V \in \tau(e), V = Q(e)$ for $(Q, E) \in \tau$. Here $\mathcal{B}$ is a basis for $\tau$. Then there exists $\mathcal{B}' \subseteq \mathcal{B}$ such that $(Q, E) = \bigcup \mathcal{B}'$. So $V = \bigcup \mathcal{B}'_e$ where $\mathcal{B}'_e = \{P(e) : (P, E) \in \mathcal{B}'\} \subseteq \mathcal{B}_e$. Thus $\mathcal{B}_e$ is a basis for neutrosophic soft cubic topology $\tau(e)$ for any $e \in E$

**Definition 4.16.** Let $(X, \tau, E)$ be a neutrosophic soft cubic topological space and let $(P, E)$ be a neutrosophic soft cubic set over $(X, E)$. Then the interior and closure of $(P, E)$ denoted respectively by $\text{int}(P, E)$ and $\text{cl}(P, E)$ are defined as follows. $\text{int}(P, E) = \bigcup\{(Q, E) : (Q, E) \subseteq (P, E)\}$

ie., $\text{int}(P, E) = \bigcup\{(Q, E) : (Q, E) \subseteq (P, E)\}$ and $(Q, E)$ is NSCOS

$\text{cl}(P, E) = \bigcap\{(Q, E) : (P, E) \subseteq (Q, E)\}$

ie., $\text{cl}(P, E) = \bigcap\{(Q, E) : (P, E) \subseteq (Q, E)\}$ and $(Q, E)$ is NSCCS

**Example 4.17.** We consider the Example 4.2 and take NSCS $(G, E)$ as shown in Table 13.
\[\text{int}(G, E) = (P, E)\] and \[\text{cl}(G, E) = (P, E)^c\]

**Theorem 4.18.** Let $(X, \tau, E)$ be a neutrosophic soft cubic topological space. Then the following properties hold.

(i) $(\Phi, E), (X, E) \in \tau$

(ii) The intersection of any number of neutrosophic soft cubic closed sets is a neutrosophic soft cubic closed set over $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$e_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>[$0.3, 0.9$, $0.5, 0.6$, $0.1, 0.7$]</td>
<td>[$0.5, 0.6$, $0.1, 0.7$]</td>
<td>[$0.5, 0.6$, $0.1, 0.7$]</td>
</tr>
<tr>
<td>$x_2$</td>
<td>[$0.5, 0.6$, $0.1, 0.7$]</td>
<td>[$0.5, 0.6$, $0.1, 0.7$]</td>
<td>[$0.5, 0.6$, $0.1, 0.7$]</td>
</tr>
<tr>
<td>$x_3$</td>
<td>[$0.5, 0.6$, $0.1, 0.7$]</td>
<td>[$0.5, 0.6$, $0.1, 0.7$]</td>
<td>[$0.5, 0.6$, $0.1, 0.7$]</td>
</tr>
</tbody>
</table>

Table 11: The tabular representation of $(L, E)^c$
(iii) The union of any two neutrosophic soft cubic closed sets is a neutrosophic soft cubic closed set over X.

**Proof:**

(i) Since $(\Phi, E), (X, E) \in \tau_0$, therefore $(\Phi, E)^c, (X, E)^c$ are NSC-closed set.

(ii) Let $\{(P_\alpha, E); \alpha \in \Gamma\}$ be an arbitrary family of NSC-closed sets in $(X, \tau, E)$ and let $(P, E) = \bigcap_{\alpha \in \Gamma} (P_\alpha, E)$.

Now $(P, E)^c = \bigcap_{\alpha \in \Gamma} (P_\alpha, E)^c = \bigcup_{\alpha \in \Gamma} (P_\alpha, E)^c$ and $(P_\alpha, E)^c \in \tau$ for each $\alpha \in \Gamma$, so $\bigcup_{\alpha \in \Gamma} (P_\alpha, E)^c \in \tau$.

Hence $(P_\alpha, E)^c \in \tau$. Thus $(P_\alpha, E)^c$ is NSC-closed set.

(iii) Let $\{(P_i, E) : i = 1, 2, 3, \ldots n\}$ be the family of NSCCS

**Theorem 4.19.** Let $(X, \tau, E)$ be a neutrosophic soft cubic topology on X and let $(P, E)$ be neutrosophic soft cubic set over $(X, E)$. Then the following properties hold.

(i) $int(P, E) \subseteq (P, E)$.

(ii) $(Q, E) \subseteq (P, E) \Rightarrow int(Q, E) \subseteq int(P, E)$.

(iii) $(P, E)$ is a neutrosophic soft cubic open set $\iff int(P, E) = (P, E)$.

(iv) $int(int(P, E)) = int(P, E)$.

(v) $int((\Phi, E)) = (\Phi, E)$, $int((X, E)) = (X, E)$.

**Proof:** (i) and (v) follows from definition [4.16].

(ii) $int(Q, E) = \bigcup \{(K, E) : (K, E) \subseteq (Q, E) \text{ and } (K, E) \text{ is NCSOS in } X\}$

$int(P, E) = \bigcup \{(S, E) : (S, E) \subseteq (P, E) \text{ and } (S, E) \text{ is NCSOS in } X\}$

Now $int(Q, E) \subseteq (Q, E) \subseteq (P, E) \Rightarrow int(Q, E) \subseteq (P, E)$.

Since $int(P, E)$ is the largest NCSOS contained in $(P, E)$, Therefore $int(Q, E) \subseteq int(P, E)$

(iii) Let $(P, E)$ be a neutrosophic soft cubic open set. Then it is the largest neutrosophic soft cubic open set contained in $(P, E)$ and hence $(P, E) = int(P, E)$. Conversely let $(P, E) = int(P, E)$, since $int(P, E)$ is the union of neutrosophic soft cubic open sets which is neutrosophic soft cubic open set. Hence $(P, E)$ is neutrosophic soft cubic open set.

(iv) $int(P, E) = \bigcup \{(S, E) : (S, E) \subseteq int(P, E) \text{ and } (S, E) \text{ is NCSOS in } X\}$

Since $int(P, E)$ is the largest neutrosophic soft cubic open set contained in $int(P, E)$.

Therefore $int(int(P, E)) = int(P, E)$.

**Theorem 4.20.** Let $(X, \tau, E)$ be a neutrosophic soft cubic topological space and $(P, E)$ be a neutrosophic soft cubic set over $(X, E)$. Then the following properties hold.

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 12: The tabular representation of $(H, E)^c$
Proof:

(i) From the definition [4.16] $(P, E) \subseteq \text{cl}(P, E)$.

(ii) $\text{cl}(Q, E) = \bigcap\{(K, E) : (Q, E) \subseteq (K, E) \text{ and } (K, E) \text{ is NCSCS in } X\}$

\[ \text{cl}(P, E) = \bigcap\{(S, E) : (P, E) \subseteq (S, E) \text{ and } (S, E) \text{ is NCSCS in } X\} \]

Since $(Q, E) \subseteq \text{cl}(Q, E)$ and $(P, E) \subseteq \text{cl}(P, E) \Rightarrow (Q, E) \subseteq (P, E) \subseteq \text{cl}(P, E)$

Since $\text{cl}(P, E)$ is the smallest neutrosophic soft cubic closed set containing $(P, E)$.

Therefore $\text{cl}(\text{cl}(P, E)) = (P, E)$.

(vi) $\text{cl}((\Phi, E)) = (\Phi, E)$, $\text{cl}((X, E)) = (X, E)$ are follows from definition[4.16].

Theorem 4.21. Let $(X, \tau, E)$ be neutrosophic soft cubic topological space and let $(P, E)$ and $(Q, E)$ are neutrosophic soft cubic sets over $(X, E)$. Then the following properties hold.

(i) $\text{int}(Q, E) \cap \text{int}(P, E) = \text{int}((Q, E) \cap (P, E))$

(ii) $\text{int}(Q, E) \cup \text{int}(P, E) \subseteq \text{int}((Q, E) \cup (P, E))$

(iii) $\text{cl}(Q, E) \cup \text{cl}(P, E) = \text{cl}((Q, E) \cup (P, E))$

(iv) $\text{cl}(Q, E) \cap (P, E) \subseteq \text{cl}(Q, E) \cap \text{cl}(P, E)$

(v) $[\text{int}(Q, E)]^c = \text{cl}[(Q, E)^c]$

(vi) $[\text{cl}(Q, E)]^c = \text{int}[(Q, E)]^c$

<table>
<thead>
<tr>
<th>$r_1$</th>
<th>$r_2$</th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\varepsilon_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>$[0.2, 0.4]$, $[0.3, 0.6]$, $[0.2, 0.8]$</td>
<td>$[0.2, 0.4]$, $[0.4, 0.6]$, $[0.2, 0.4]$</td>
<td>$[0.2, 0.4]$, $[0.6, 0.4]$, $[0.6, 0.4]$</td>
<td>$[0.0, 0.6]$, $[0.7, 0.8]$, $[0.3, 0.4]$</td>
</tr>
<tr>
<td>$r_2$</td>
<td>$[0.5, 0.8]$, $[0.5, 0.6]$, $[0.5, 0.8]$</td>
<td>$[0.4, 0.6]$, $[1.0, 0.5]$, $[0.7, 0.3]$</td>
<td>$[0.2, 0.4]$, $[0.5, 0.5]$, $[0.5, 0.5]$</td>
<td>$[0.0, 0.5]$, $[0.2, 0.4]$, $[0.5, 0.5]$</td>
</tr>
<tr>
<td>$r_3$</td>
<td>$[0.1, 0.6]$, $[0.4, 0.3]$, $[0.2, 0.7]$</td>
<td>$[0.2, 0.6]$, $[0.5, 0.7]$, $[0.1, 0.5]$</td>
<td>$[0.1, 0.4]$, $[0.2, 0.5]$, $[0.1, 0.5]$</td>
<td>$[0.7, 0.4]$, $[0.4, 0.4]$, $[0.7, 0.4]$</td>
</tr>
</tbody>
</table>

Table 13: The tabular representation of $(Q, E)$
References

This paper lays the foundation for the further study on different separation axioms via this sets. Further various types of relation between neutrosophic soft cubic topological sets can be analysed under various mappings.

5 Conclusions

This paper lays the foundation for the further study on different separation axioms via this sets. Further various types of relation between neutrosophic soft cubic topological sets can be analysed under various mappings.

Proof:

(i) Since \((Q, E) \cap (P, E) \subseteq (Q, E)\) and \((Q, E) \cap (P, E) \subseteq (Q, E)\),
by Theorem 4.19(ii), \((Q, E) \subseteq (P, E) \Rightarrow \text{int}(Q, E) \subseteq \text{int}(P, E)\),
then \(\text{int}((Q, E) \cap (P, E)) \subseteq \text{int}(Q, E)\) and \(\text{int}((Q, E) \cap (P, E)) \subseteq \text{int}(P, E)\)
\(\Rightarrow \text{int}((Q, E) \cap (P, E)) \subseteq \text{int}(Q, E) \cap \text{int}(P, E)\).

Now \(\text{int}(Q, E)\) and \(\text{int}(P, E)\) are NCSOSs
\(\Rightarrow \text{int}(Q, E) \cap \text{int}(P, E)\) is NCSOS.
then \(\text{int}(Q, E) \subseteq (Q, E)\) and \(\text{int}(P, E) \subseteq (P, E) \Rightarrow \text{int}(Q, E) \cap \text{int}(P, E) \subseteq \text{int}((Q, E) \cap (P, E))\).
Therefore \(\text{int}(Q, E) \cap \text{int}(P, E) = \text{int}((Q, E) \cap (P, E))\).

(ii) \((Q, E) \subseteq (P, E) \cup (Q, E)\) and \((P, E) \subseteq (P, E) \cup (Q, E)\)
by Theorem 4.19(ii), \((Q, E) \subseteq (P, E) \Rightarrow \text{int}(Q, E) \subseteq \text{int}(P, E)\),
then \(\text{int}(Q, E) \subseteq \text{int}((P, E) \cup (Q, E))\) and \(\text{int}(P, E) \subseteq \text{int}((P, E) \cup (Q, E))\).
Hence \(\text{int}(Q, E) \cup \text{int}(P, E) \subseteq \text{int}((Q, E) \cup (P, E))\)

(iii) Since \((Q, E) \subseteq \text{cl}(Q, E)\) and \((P, E) \subseteq \text{cl}(P, E)\).
We have \((Q, E) \cup (P, E) = \text{cl}(Q, E) \cup \text{cl}(P, E)\)
\(\Rightarrow \text{cl}((Q, E) \cup (P, E)) = \text{cl}(Q, E) \cup \text{cl}(P, E)\) ....(1)
\(\text{cl}(Q, E) \cup \text{cl}(P, E) = \text{cl}((Q, E) \cup (P, E))\) And since \((Q, E) \subseteq \text{cl}(Q, E)\) and \((P, E) \subseteq \text{cl}(P, E)\)
so \(\text{cl}(Q, E) \subseteq \text{cl}((Q, E) \cup (P, E))\) and \(\text{cl}(P, E) \subseteq \text{cl}((Q, E) \cup (P, E))\) ......(2)
Therefore \(\text{cl}(Q, E) \cup \text{cl}(P, E) \subseteq \text{cl}((Q, E) \cup (P, E))\)
From (1) and (2) \(\text{cl}(Q, E) \cup \text{cl}(P, E) = \text{cl}((Q, E) \cup (P, E))\).

(iv) Since \((Q, E) \cap (P, E) \subseteq (Q, E)\) and \((Q, E) \cap (P, E) \subseteq (P, E)\)
and so \(\text{cl}((Q, E) \cap (P, E)) \subseteq \text{cl}(Q, E)\) and \(\text{cl}((Q, E) \cap (P, E)) \subseteq \text{cl}(P, E)\)
Hence \(\text{cl}(Q, E) \cap \text{cl}(P, E) \subseteq \text{cl}(Q, E) \cap \text{cl}(P, E)\)

(v) \([\text{int}(Q, E)]^c = [\bigcup \{(K, E) : (K, E) \subseteq (Q, E)\} \text{ and } (K, E)\ \text{is NCSOS in X}\}]^c\)
\(= \cap \{(K, E)^c : (Q, E)^c \subseteq (K, E)^c\} \text{ and } (K, E)^c\ \text{is NCSCS in X}\}\)
\(= \text{cl}((Q, E)^c)\).

(vi) \([\text{cl}(Q, E)]^c = [\bigcap \{(K, E) : (K, E) \subseteq (Q, E)\} \text{ and } (K, E)\ \text{is NCSCS in X}\}]^c\)
\(= \cup \{(K, E)^c : (Q, E)^c \subseteq (K, E)^c\} \text{ and } (K, E)^c\ \text{is NCSOS in X}\}\]^c
\(= \text{int}((Q, E)^c)\).

5 Conclusions

This paper lays the foundation for the further study on different separation axioms via this sets. Further various types of relation between neutrosophic soft cubic topological sets can be analysed under various mappings.

References


Received: October 17, 2018. Accepted: November 12, 2018